



## A Road-Matching Method for Precise Vehicle Localization Using Belief Theory and Kalman Filtering

MAAN E. EL NAJJAR AND PHILIPPE BONNIFAIT

*Heudiasyc UMR 6599 CNRS/Université de Technologie de Compiègne, BP 20529,  
60205 Compiègne Cedex, France*

maan.el-najjar@hds.utc.fr

**Abstract.** This paper describes a novel road-matching method designed to support the real-time navigational function of cars for advanced systems applications in the area of driving assistance. This method provides an accurate estimation of position for a vehicle relative to a digital road map using Belief Theory and Kalman filtering. Firstly, an Extended Kalman Filter combines the DGPS and ABS sensor measurements to produce an approximation of the vehicle's pose, which is then used to select the most likely segment from the database. The selection strategy merges several criteria based on distance, direction and velocity measurements using Belief Theory. A new observation is then built using the selected segment, and the approximate pose adjusted in a second Kalman filter estimation stage. The particular attention given to the modeling of the system showed that incrementing the state by the bias (also called absolute error) of the map significantly increases the performance of the method. Real experimental results show that this approach, if correctly initialized, is able to work over a substantial period without GPS.

**Keywords:** localization, sensor fusion, belief theory, geographical information system, global positioning system

### 1. Introduction

Many modern in-vehicle navigation and safety applications require real-time positioning of the vehicle with respect to a given set of digital map data. Real-time positioning allows the driving assistance module to accurately depict the position of the vehicle on the map, facilitates operations such as route calculation, supports Advanced Driver Assistance System applications (ADAS) such as Adaptive Cruise Control (ACC), adaptive lighting control, collision warning and lane departure warning. For driving assistance applications, the positioning module is of crucial importance to reach the ADAS attributes stored in the database, like the radius of curvature, the width of the road or the speed limits.

The quality of the localization process depends mainly on the quality of the road-matching which is a complicated problem when seeking to obtain reliable, precise and robust vehicle positioning on the road network (Bernstein and Kornhauser, 1998; Zhao, 1997).

Positioning systems often rely on GPS, because of its affordability and convenience. However, GPS suffers from satellite masks occurring in urban environments, under bridges, tunnels or in forests. GPS appears then as an intermittently-available positioning system that needs to be backed up by a dead-reckoning system (Abbott and Powell, 1999). In this paper, we use the car's rear wheel ABS sensors for this purpose. Given that modern cars are often equipped with ABS braking systems, it seems to us judicious to re-use these sensors to measure elementary rotations of the wheels and to estimate the displacement of the car rather than to add sensors like gyrometers or magnetic compasses. Thus, a dead-reckoned estimated pose is obtained by integrating the elementary rotations of the wheels using a differential odometric model. The multisensor fusion of GPS and odometry is performed by an Extended Kalman Filter (denoted EKF in the following).

The selection of candidate roads is the first stage of the road-matching problem (Taylor and Blewitt, 2000). Generally, this involves applying a first filter which

selects all the segments close to the estimated position of the vehicle. The goal is then to select the most likely segment(s) from this subset. Nowadays, since the geometry of roadmaps is more and more detailed, the number of segments representing roads is increasing. The road selection module is an important stage in the vehicle localization process because the robustness of the localization depends mainly on this stage. The road selection stage is also important because it reduces the number of roads to be processed, which is essential for a real time implementation. In order to be focused on this point, an accurate map *Géoroute V2* provided by the French National Institute of Geography (IGN) was used in this work. Our strategy is based on the merging of several criteria using distance, direction and velocity measurements within the framework of Belief Theory. A connectivity test with the latest matched road is subsequently applied. Finally, the use of the one way restrictions, available in the database that has been used, allows less likely solutions to be eliminated by supposing that the driver respects the Highway Code.

A more accurate location of the vehicle can be obtained by combining the selected segment with the pose estimated jointly by GPS and odometry. The key idea is to model the fact that the true position of the vehicle is located around the centerline of the most likely road. This region depends mainly on the width of the road, which is an ADAS attribute also stored in the database. We propose using the most likely road in order to build a new Kalman observation with its estimated associated error.

The outline is as follows. An overview of related work is firstly given in Section 2. Then, the architecture of the road-matching method is described. The state space formulation and the observation equations are detailed. We propose constructing a map observation akin to the GPS observation to be used in the Kalman filter. In Section 4, we discuss the problem of road selection and we present the formulation of the problem in the framework of Belief Theory. Our approach is illustrated with an example, and experimental results corresponding to real situations are presented. Finally, real data results are analyzed in Section 5.

## 2. Related Work

The road-matching problem is a localization problem which can be tackled in two different ways: global localization and/or pose tracking. The latter is the recur-

sive estimation of the pose of the car starting from an approximate solution. Several aspects of this problem are linked to dynamic robot localization.

Studies of navigation systems for driving assistance are often heuristic and not published in the literature because of patents. The early map-matching algorithms of the 1970s were deterministic. In this context, errors are not explicitly modeled and methods consist of correlating absolute or dead-reckoned positions and road geometry (Bernstein and Kornhauser, 1998). Topology is used to eliminate outliers (Taylor and Blewitt, 2000; Greenfeld). In Zhao (1997), several approaches based on fuzzy logic are described and the problem of sensor and map errors is addressed. Difficult problems occur when the vehicle is on a road not digitalized in the database or when the situation is ambiguous like in junctions or when several roads are close and parallel.

The localization problem of a land motorised vehicle on a digital roadmap can be seen as a robot localization problem. In the last ten years, a large number of approaches have been proposed in robotics and rely on the following key concepts (Borenstein et al., 1996). Localization sensors are generally imperfect and provide only uncertain information. Additionally, sensor readings generally contain noise. Moreover, readings can be ambiguous, that is, the environment may contain situations which cannot be distinguished. A localization method that seeks to be reliable must use a methodology able to handle uncertain and ambiguous information.

Localization techniques can be distinguished according to the type of problem they address. *Global localization* (also called the *wake-up* robot problem) has to estimate the pose of the robot without any prior information. Such methods can handle the *kidnapped robot* problem, in which a robot is carried to an arbitrary location during its operation. *Pose-tracking* techniques (also called dynamic localization techniques) aim at compensating, by using absolute sensors, accumulated dead-reckoned or odometric errors that occur during vehicle navigation. They require the initial location of the vehicle to be approximately known and they cannot recover if they lose the track of the vehicle pose. An outdoor vehicle equipped with a GPS receiver is faced with a pose-tracking problem since the probability of not having a GPS fix for a long distance is very low, even though the GPS system can be intermittent in urban environments for example. Therefore, the work reported in this paper is motivated by the need to build a robust pose tracking system based on a road-matching

module. The objective is to localize the vehicle on a digital roadmap with a quantification of the accuracy of the positioning given the quality of sensor measurements. To manipulate uncertainty, we make use of Belief Theory (El Najjar and Bonnifait, 2002).

The pose-tracking problem is well described by a state space description (Dissanayake et al., 2001). The state vector contains the pose to be estimated along with other parameters like derivatives, bias or beacon coordinates in the case of a simultaneous localization and map building (Fox et al., 1999; Thrun et al., 2000). The evolution model integrates the inertial and odometric sensors while the absolute sensors are used in the observation model to correct the drift of the estimates. The combination process is often done in the context of Bayes recursive estimation. In the linear case and if the perturbations are white and Gaussian, Bayes filtering reduces to Kalman filtering (Arulampalam et al., 2002). If the equations are non linear and the noises non Gaussian, the probability density function can be estimated by particle filters. This methodology can then handle multi-hypothesis situations (Jensfelt and Kristensen, 2001) if the resampling of the particles is well adapted to the problem and able to maintain the convergence of the estimation process. Gustafsson et al. recently completed a localization approach that has been successfully verified in a real environment with a digital roadmap using a rao-blackwelised particular filter (Gustafsson et al., 2002). This method is well adapted to the global localization problem because it can output several particle clouds while the situation is ambiguous. But this method can give rise to many calculations which are not adapted for a real time implementation of the pose-tracking problem.

### 3. Principle of the Road-Matching Method

The road-matching problem probably does not have an ideal solution. All developed methods have their advantages and their disadvantages and are optimized for the applications they were designed for Tanaka et al. (1990) and Zhao (1997). The performances of many navigation systems seem to be sufficient. However, safety applications need a reliable road-matching process.

In addition, the techniques used to address this problem are in permanent evolution. Some problems solved today can disappear and other can appear. For example, improvements in satellite positioning systems have tended to reduce absolute positioning errors. On the other hand, making an accurate road network increases

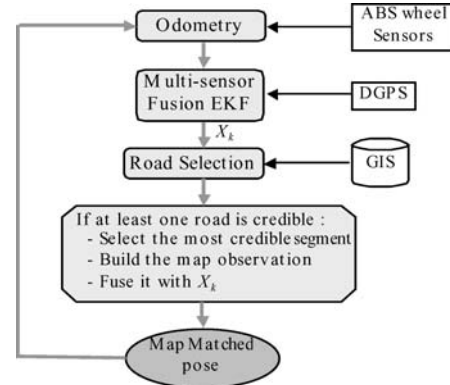


Figure 1. Synoptic of the road-matching method.

the number of points describing arcs, thus making more complicated the segment selection problem.

The road-matching method described in this section relies on Kalman filtering like in Krakiwsky et al. (1988). The proposed approach can be described by Fig. 1. Firstly, the algorithm combines the ABS measurements with a GPS position, if it is available. Then, using this estimate, the credible roads are selected. If at least one segment is credible, a map observation is built and merged with the other data in a second Kalman filter estimation stage. We suppose that the reader is familiar with this formalism, so only the state-space representation will be detailed, i.e. the state vector, the motion model, the observation model and the covariance of the errors.

#### 3.1. Localization and Heading Estimation by Combining Odometry and GPS

Let us consider a car-like vehicle with front-wheel drive. The mobile frame is chosen with its origin  $M$  attached to the center of the rear axle. The  $x$ -axis is aligned with the longitudinal axis of the car (see Fig. 2).

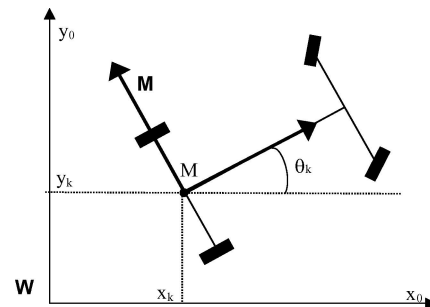


Figure 2. The mobile frame attached to the car.

The vehicle's position is represented by the  $(x_k, y_k)$  Cartesian coordinates of  $M$  in a world frame. The heading angle is denoted  $\theta_k$ . If the road is perfectly planar and horizontal, and if the motion is locally circular, the motion model can be expressed as Ming Wang and Bonnifait et al., (2001):

$$\begin{cases} x_{k+1} = x_k + \delta_s \cdot \cos(\theta_k + \delta_\theta/2) \\ y_{k+1} = y_k + \delta_s \cdot \sin(\theta_k + \delta_\theta/2) \\ \theta_{k+1} = \theta_k + \delta_\theta \end{cases} \quad (1)$$

where  $\delta_s$  is the length of the circular arc followed by  $M$  and  $\delta_\theta$  the elementary rotation of the mobile frame. These values are computed using the ABS measurements of the rear wheels.

### 3.2. Observation Equations: GPS and Map

When a GPS position is available, a correction of the odometric estimation is performed using an Extended Kalman Filter. If the GPS satellites signal is blocked by buildings or tunnels, for example, the motion model provides an odometric estimate of pose.

This approximation of pose is used to select the most likely segments from the database. These segments are then used to build a second observation (this approach will be presented in Section 3). If several segments are candidates, the observation function is non-linear (see Fig. 3).

$$Y = f(X_k) + \beta_k \quad (2)$$

where  $X_k$  is the state and  $\beta_k$  represents the observation error.

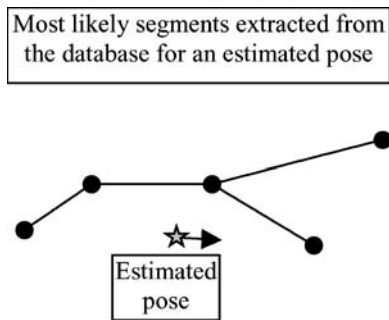


Figure 3. Non-linearity of the map observation.

Two main strategies can deal with this non-linearity:

- The management of multi-hypotheses
- The selection of the most likely segment from the segment set.

In this paper, we consider the second solution because of the simplicity of processing. The major drawback of this strategy is that the estimated location can be attributed to the wrong road, particularly when GPS measurements are not available. The management of multi-hypotheses is theoretically the ideal solution. Nevertheless, implementation is complicated because of combinatorial problems. In our method the most likely segment is used to construct a map observation, denoted  $(x_h, y_h)$ , and its associated error. Therefore, the complete observation equation becomes linear:

$$Y = \begin{bmatrix} x_{\text{gps}} \\ y_{\text{gps}} \\ x_h \\ y_h \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \beta_k \quad (3)$$

Where  $(x_{\text{gps}}, y_{\text{gps}})$  is the GPS position measurement and  $(x_h, y_h)$  is the map observation.

The GPS measurement error can be estimated in real time using the NMEA sentence "GST" provided by the Trimble AgGPS132 receiver which has been used in the experiments. Therefore, the GPS noise is not stationary.

If we assume that the GPS position and the map observation errors are not correlated, the covariance matrix of the complete measurement  $Y$  can be separated into two parts:

- $Q_{\text{gps}}$ : covariance matrix of the GPS error
- $Q_h$ : covariance matrix of the map observation error.

$$Q_k = \begin{pmatrix} \boxed{Q_{\text{gps}}} & & & \\ \begin{matrix} \sigma_{x,\text{gps}}^2 & Q_{xy,\text{gps}} \\ Q_{xy,\text{gps}} & \sigma_{y,\text{gps}}^2 \end{matrix} & & 0 & 0 \\ 0 & 0 & \begin{matrix} \sigma_{x,h}^2 & Q_{xy,h} \\ Q_{xy,h} & \sigma_{y,h}^2 \end{matrix} & \\ 0 & 0 & & \boxed{Q_h} \end{pmatrix}_k \quad (4)$$

Since  $Q_k$  is diagonal, the GPS and map observations can be used in two separated Kalman filter estimation

stages. This is an important issue for the real time implementation of the filter.

### 3.3. Map Observation

One way of combining the most likely segment with the other sensors is to treat it as an observation that is a function of the state vector. Much effort has been spent on modeling the map observation error in a realistic way. It has turned out that a Gaussian mixture which encloses the road works well.

To build the map observation, we consider two cases.

**3.3.1. Principle: Case of a Straight Road.** The simplest case occurs when the most likely segment corresponds to a straight road. In Fig. 4, the road selection stage provides three segments (bold characters). Their masses of Belief are 0.9, 0.7 and 0.55 for segments 1, 2 and 3, respectively (we will see in Section 3 how this masses can be assigned). As all these segments are credible (decision reached at the road selection stage), it is an ambiguous situation and the map observation is non-linear. One way of circumventing this difficulty is to select the segment which has the highest Belief value (segment 1 here). The map observation  $(x_h, y_h)$  is defined as the orthogonal projection of the estimated position  $(x_k, y_k)$  onto segment 1 (Bétaille and Bonnifait, 2000). Please note that the map observation can be constructed in a different way. In Gustafsson et al. (2002), the observation is the distance to the nearest road. In the recursive Bayesian estimation context (i.e. a particle filter), this (non-linear) measurement should be equal to zero.

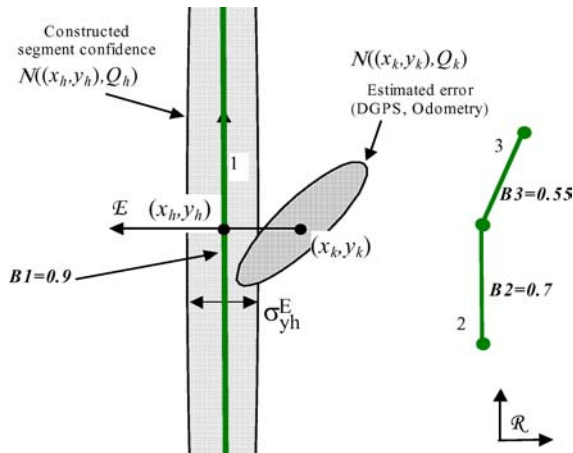


Figure 4. Case of a straight road.

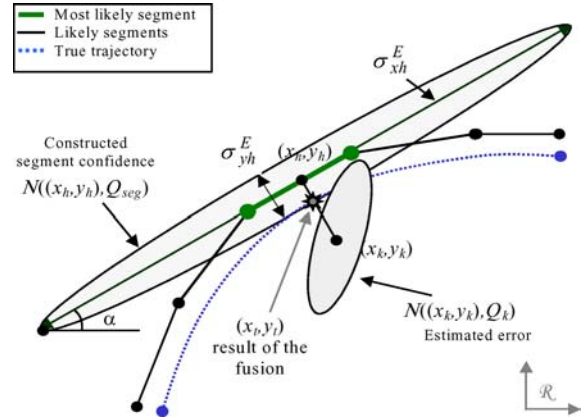


Figure 5. Case of a curved road.

The problem is now to estimate the error of this observation.

The box surrounding the segment and representing the road determines the maximum error of the map observation. The resulting probability density function is then theoretically spatially truncated. In the Kalman filtering context, this box is approximated by a Gaussian ellipse as shown in Figs. 4 and 5.

Let consider a local frame attached to the segment. Its  $x$  axis is collinear to the segment. In this frame, the Gaussian ellipse is oriented along the road segment and the co-ordinates of its center are  $(x_h, y_h)$ .

The error is then given by (in the ellipse co-ordinate system):

$$Q_h^E = \begin{bmatrix} (\sigma_{xh}^E)^2 & 0 \\ 0 & (\sigma_{yh}^E)^2 \end{bmatrix} \quad (5)$$

where  $\sigma_{xh}^E$  and  $\sigma_{yh}^E$  are the longitudinal and the transversal standard deviations. If the segment is infinite, then  $\sigma_{xh}^E = \infty$ . This means that the map measurement can only correct the position in the  $y$  direction.

The transversal standard deviation is given by:

$$\sigma_{yh}^E = (w_y/2k) + e_c \quad (6)$$

where

- $w_y$  is the width of the road segment ( $w_y$  is stored in the database as an ADAS attribute)
- $k$  is the constant associated with the chosen probability error ellipse given by:

$$k = \sqrt{-2 \ln(1 - P)} \quad (7)$$

- $e_c$  is the map error.

**3.3.2. Case of a Curved Road.** In general, roads are not straight and, because recent digital roadmaps are more detailed, segments have a smaller length (see Fig. 5).

The map observation is the nearest point from  $(x_k, y_k)$  making part of the most likely segment. Depending on the case, it is the orthogonal projection of  $(x_k, y_k)$  onto the segment or one of its extremities. It should be noticed that the most likely segment represents a linearization of the curved road.

In this case,  $\sigma_{xh}^E$  is not infinite, but it needs to be big enough in comparison with  $\sigma_{yh}^E$  to indicate that the adjustment is greater in the  $y$  direction (in the local frame attached to the segment).

In the reference frame, the covariance matrix  $Q_h$  of the map observation is obtained from  $Q_h^E$  by a simple rotation. If  $\alpha$  is the orientation of the segment with respect to the  $x$  axis of the reference frame, then:

$$Q_h = \begin{bmatrix} \sigma_{xh}^2 & \sigma_{xyh}^2 \\ \sigma_{xyh}^2 & \sigma_{yh}^2 \end{bmatrix} \quad (8)$$

where

$$\begin{aligned} \sigma_{xh}^2 &= (\sigma_{xh}^E)^2 \cos^2(\alpha) + (\sigma_{yh}^E)^2 \sin^2(\alpha) \\ \sigma_{yh}^2 &= (\sigma_{xh}^E)^2 \sin^2(\alpha) + (\sigma_{yh}^E)^2 \cos^2(\alpha) \\ \sigma_{xyh}^2 &= ((\sigma_{xh}^E)^2 - (\sigma_{yh}^E)^2) \cos(\alpha) \sin(\alpha) \end{aligned} \quad (9)$$

### 3.4. Augmenting the State Vector

The roads are symbolized by arcs whereas the car is moving with respect to a surface centered on these arcs. Moreover, the geometrical transformation between the GPS reference frame and the French Lambert projection frame can have an offset of several meters ( $<5$  m). Finally, the segment co-ordinates contain errors because of plotting inaccuracies and because the co-ordinates are stored as integers in the database (values rounded to the nearest meter).

For all these reasons, combining GPS with odometry has a variable offset with respect to the map data. A solution to this problem is to add two offsets (denoted  $\delta_x$  and  $\delta_y$ ) in the state vector and to observe them.

As the goal of the positioning module is to localize the car on the road network (because the ADAS attributes are attached to this network), it is the GPS measurement that presents an offset rather than the map

observation.

$$Y = \begin{bmatrix} x_{\text{gps}} \\ y_{\text{gps}} \\ x_h \\ y_h \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ \theta \\ \delta_x \\ \delta_y \end{pmatrix} + \beta_k \quad (10)$$

$\delta_x$  and  $\delta_y$  are observable because they can be expressed as a combination of the measurements. The evolution of  $\delta_x$  and  $\delta_y$  is modeled by a constant. The evolution is made possible thanks to a non-zero state noise  $\alpha_k$ . Equation (1) becomes:

$$\begin{cases} x_{k+1} = x_k + \delta_s \cdot \cos(\theta_k + \delta_\theta/2) \\ y_{k+1} = y_k + \delta_s \cdot \sin(\theta_k + \delta_\theta/2) \\ \theta_{k+1} = \theta_k + \delta_\theta \\ \delta_{x,k+1} = \delta_{x,k} \\ \delta_{y,k+1} = \delta_{y,k} \end{cases} \quad (11)$$

Let  $X = [x, y, \theta, \delta_x, \delta_y]^T$  and  $U = [\delta_s, \delta_\theta]^T$ . By rewriting Eqs. (10) and (11), we obtain the state-space representation where the model error  $\alpha$  and observation error  $\beta$  appear:

$$\begin{cases} X_{k+1} = f(X_k, U_k) + \alpha_k \\ Y_k = C \cdot X_k + \beta_k \end{cases} \quad (12)$$

The observation model is linear whereas the evolution model is non linear. An Extended Kalman Filter with a measured input provides a means of combining all this data.

## 4. Road Selection Using Multi-Criteria Fusion

The road selection process can be described as in Fig. 6. The multi-sensor fusion gives an estimation of the pose  $X = (x, y, \theta)^t$ . In order to take into account of the estimation error, a Gaussian ellipse is built using the co-variance matrix  $P$  of the state vector  $X$  (El Najjar and Bonnifait, 2002). The speed  $v$  is the mean speed of the rear wheels.

The question is now to select the most likely segment(s) using a Geographical Information System (GIS). In order to speed up the treatments (a map contains thousands of roads, each one having several segments), a first filter selects the  $n$  road segments

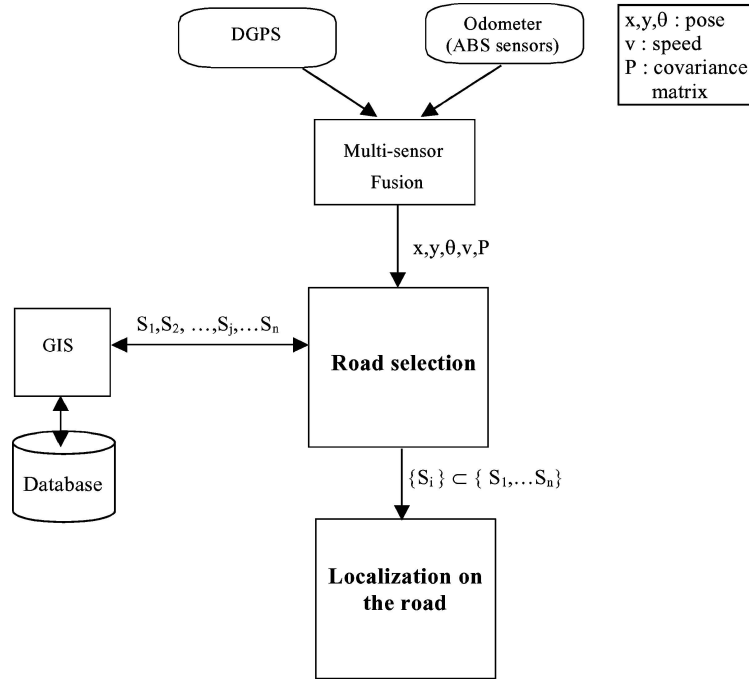


Figure 6. Architecture of road-matching.

$\{S_1, \dots, S_n\}$  that are located within a radius of 100 meters, for example. The center of the circle is the estimation of the current position  $(x, y)$  of the car.

The problem is to select the 'good' segments from the subset  $\{S_1, \dots, S_n\}$ : this is the road selection problem, also called *Road Reduction Filter* (El Najjar and Bonnifait, 2002; Taylor and Blewitt, 2000).

This stage is difficult because,

- The position is estimated with errors which can be increased by multi-path effects. In addition, the transformation between the GPS coordinates (WGS84 system of reference) and the French Lambert coordinates of the roadmap introduces errors ( $< 5$  m),
- The coordinates of the segments contain errors due to inaccurate terrain measurements by cartographers and because of numerical approximation,
- The road network of the database does not always correspond to reality, i.e. it can contain old roads which no longer exist, and newly-built roads might not yet be included in the database,
- The map does not contain all road network details. For example, a roundabout can be represented as a simple point,
- The vehicle is moving on a 3D surface whereas the map represents a plane sight,

- The vehicle does not run exactly on the segments representing the roads.

Our road selection method combines several criteria using Belief Theory. This approach is very flexible and allows partial knowledge to be taken into account. This section first presents the concepts of Belief Theory. The criteria for selection will then be described, and finally the combination of data will be illustrated by a simple example and some real experiments.

#### 4.1. Belief Theory

Belief Theory allows uncertainties to be incorporated into calculations and provides a way of combining uncertain data. This theory was introduced by Dempster (1976) and mathematically formalized by Shafer in 1976. It is a generalization of Bayes Theory in the treatment of uncertainty. Generally, this theory is used in a multi-sensor context to merge heterogeneous information in order to obtain the best decision.

The basic entity is a set of all possible answers (also called hypotheses) to a specific question. This set is called the *frame of discernment* and is denoted  $\Theta$ . All the hypotheses must be exclusive and exhaustive and

each subset of the frame of discernment can be a possible answer to the question. The degree of belief of each hypothesis is represented by a real number in  $[0, 1]$  called the mass function  $m(\cdot)$ . It satisfies the following rules:

$$\begin{aligned} m(\phi) &= 0 \\ \sum_{A \subseteq \Theta} m(A) &= 1 \end{aligned} \quad (13)$$

A mass function is defined for all the different evidences. Each evidence  $A$ , for which  $m(A) \neq 0$ , is called a focal element.

The two criteria chosen in this article can be formulated as follows:

- The vehicle location is close to a segment of the neighborhood. This criterion depends on the error ellipse,
- The segments on which the vehicle can be located are those which have an angle approximating to the heading of the vehicle. This criterion depends on the estimated  $3\sigma$  bound of the heading and on the speed of the car.

Belief Theory requires the assignment of elementary probabilistic masses defined on  $[0, 1]$ . The mass assignment is computed on the definition referential  $2^\Theta$ .

$$2^\Theta = \{\emptyset, H_1, H_2, \dots, H_n, H_1 \cup H_2, \dots, H_i \cup H_j \cup H_k \cup H_l \cup \dots, H_n\}.$$

This distribution is a function of the knowledge about the source. The total mass obtained is called the “basic mass assignment”. The sum of these masses is equal to one. Each expert—also called source of information—defines a mass assignment according to its opinion about the situation.

In order to build mass assignments, we shall examine the inaccuracy of the various information sources (GPS, odometer and digital map) and physical observations like, for example, a car traveling at 40 m/s cannot be orthogonal to the direction of the segment. With this approach, information sources (i.e. criteria) are worked out from sensors.

The problem of mass assignment of each criterion can be tackled in a global or local way. The global strategy involves examining simultaneously all the segments selected around an estimated position when as-

signing masses. The local strategy treats each segment separately with respect to the criterion under consideration. Both strategies have been studied. We have concluded that the local strategy is the more effective, especially for a real time application.

The frame of discernment that we use is  $\Theta = \{Yes, No\}$ , corresponding to the answer to the following question: *is this segment the good one?* The definition referential is then  $2^\Theta = \{Yes, No, Perhaps\}$ .

#### 4.2. Proximity Criterion

The proximity criterion is based on the measurement of Euclidean distance between the estimated position and each segment extracted from the road database.

The estimated error of the position is quantified by an ellipse of 99% equi-probability produced by the EKF (drawn in dark gray in Fig. 7). The estimated position  $E$  is at the center of the ellipse.

To allot a mass to a candidate segment  $[AB]$ , we proceed as follows. Let denote  $d$  the distance between the segment and point  $E$ .

The point  $S'$  occurs at the intersection between the segment  $[ES]$  and the ellipse. The distance  $d_{ES'}$  depends on the angle  $\beta$  of the segment  $[ES']$  in the ellipse co-ordinates system. In the zone  $d < d_{ES'}$ , with a fuzzy modeling obtained by a probability-possibility transformation (Dubois and Prade, 1993; Zadeh, 1986), the degree of membership is quantified. The upper curve in Fig. 8 assigns a mass to the *Yes* assumption.

By complementing the mass of *Yes*, the mass of the *Perhaps* assumption is allotted. Then, the mass of *Perhaps* remains constant (equal to one) for  $d_{ES'} < d < d_{ES'} + e$ , in order to consider the projection error and the errors on the co-ordinates of the segments of the database. Finally, the mass of the *No* assumption is a step function starting from the distance  $d = d_{ES'} + e$ .

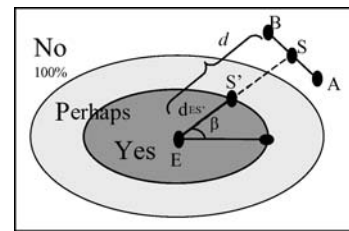


Figure 7. Case of a non-credible segment.



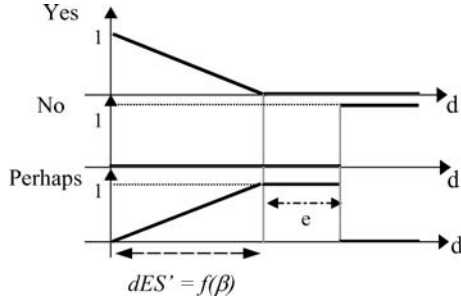


Figure 8. Mass assignment of the proximity criterion.

In conclusion, the mass assignment of the proximity criterion depends on two variables:

- The distance  $d$  between the center of the ellipse and the segment,
- The angle  $\beta$  between the distance support ( $ES$ ) and the major axis of the ellipse.

The problem becomes more complicated when the width of the road is taken into account. Our method involves modeling the road by a box centered on the segment, the length of which is equal to that of the segment. The exact influence of the width of the road  $l$  is difficult to take into account in the computations of the criterion because  $l$  modifies the values of  $\beta$  and  $d$ . To simplify, we have chosen the following strategy:

- (1) If the orthogonal projection of  $E$  exists inside segment  $[AB]$ ,  $d = d_{ortho} - l$  (Fig. 9(a))
- (2) If the orthogonal projection of  $E$  does not exist inside segment  $[AB]$ ,  $d = \min(d1, d2, d3)$  (Fig. 9(b)).

#### 4.3. Angular Criterion

In this section, a mass assignment function is proposed to express the fact that the most credible segments are those which have an angle approximating to the heading of the vehicle.

Figure 10 presents the fuzzy modeling of the absolute value of the difference between the heading of the vehicle and the direction of the candidate segment:

$$\Delta Heading = \min(|\alpha - \theta|, |\alpha - \theta + \pi|) \quad \text{with} \quad (14) \\ \theta \in [0, \pi].$$

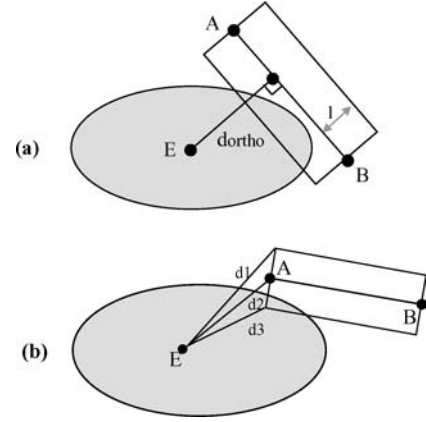


Figure 9. Computation of the distance  $d$  by considering the width of the road.

This curve depends on:

- The speed of the vehicle
- The standard deviation of the estimation error of the heading angle.

Let  $m$  be the maximum belief which can be assigned to the *Yes* hypothesis (see Fig. 10). Therefore,  $m$  varies according to  $\sigma_\theta$ :

$$m(\sigma_\theta) = 1 - \frac{6}{\pi} \sigma_\theta \quad (15)$$

The scalar value  $B$  fixes the angular limit tolerated at a given velocity  $v$ :

$$B(v) = 90 - kv, \quad \text{with } k = (90 - 10)/V_{\max}.$$

This strategy was developed to model the fact that an uncertain heading will not assign a significant mass to the *Yes* hypothesis.

The *Perhaps* mass assignment is done by computing the complement of the mass of *Yes*. The mass of *No* starts from the limit angle tolerated for a given speed i.e.  $B(v)$  and reaches one when the angle is equal to 90 degrees (Fig. 11).

#### 4.4. Criteria Fusion

To obtain more reliable information from two different single sources  $S_1$  and  $S_2$ , a combination of their mass assignments can be performed using Demspter-Shafer's rule. Let  $A$ ,  $A_i$  and  $B_i$  be assumptions of the

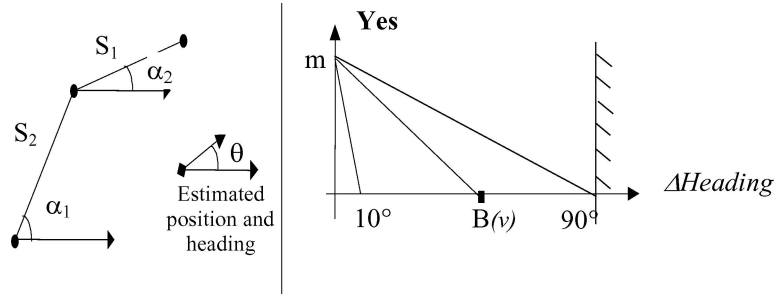
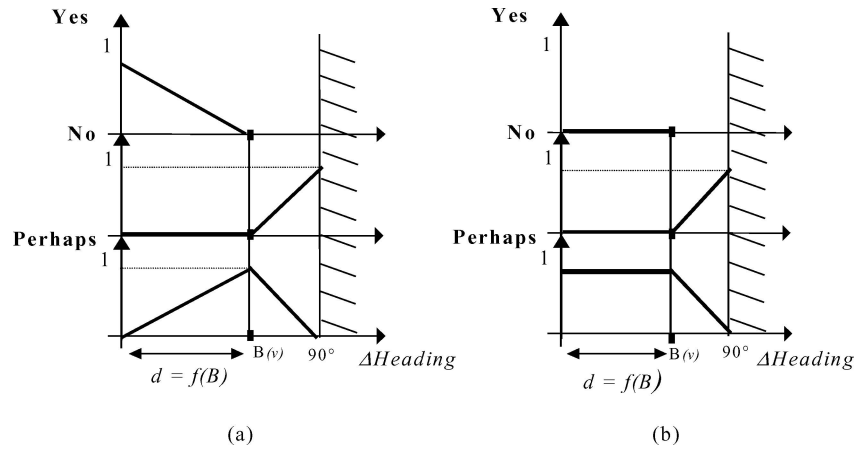


Figure 10. Mass assignment of Yes hypothesis for the angular criterion.

Figure 11. Examples of mass assignment at a given velocity (a):  $\sigma_\theta = 0$ . (b):  $\sigma_\theta = \frac{\pi}{2}$ .

definition referential  $2^\Theta$ . The merging of the knowledge of  $S_1$  and  $S_2$  is given by:

For all  $A$  in  $2^\Theta = \{Yes, No, Perhaps\}$

$$m_\Theta(A) = \sum_{A_i \cap B_j = A} m_\Theta^{S_1}(A_i) \cdot m_\Theta^{S_2}(B_j) \quad (16)$$

If conjunctions exist which are not focal elements, a re-normalization step is necessary to satisfy the rule that  $m(\phi) = 0$ . The coefficient of re-normalization is called  $k_\theta$  and is defined as:

$$k_\theta = \sum_{A_i \cap B_j = \phi} m_\Theta^{S_1}(A_i) \cdot m_\Theta^{S_2}(B_j) \quad (17)$$

It represents the incoherence between the different sources. If we set  $K_\theta = \frac{1}{1-k_\theta}$ , the normalized expression of the combination is given by:

$$m_\Theta(A) = K_\theta \cdot \sum_{A_i \cap B_j = A} m_\Theta^{S_1}(A_i) \cdot m_\Theta^{S_2}(B_j) \quad (18)$$

This combination rule is independent of the order in which evidences are combined, when more than two evidences are involved.

After the combination step, several decision rules can be used to obtain the final result. It is then possible to adjust a desired behavior. If an optimistic decision is desired, the maximum of *plausibility* has to be used. For a pessimistic decision, one can apply the maximum of *belief*.

Associated with each basic assignment, belief (*Bel*) and plausibility (*Pl*) are defined by:

$$\begin{aligned} Bel(A) &= \sum_{B \subseteq A} m(B) \\ Pl(A) &= \sum_{B \cap A \neq \phi} m(B) \end{aligned} \quad (19)$$

Belief and plausibility are interrelated by the relationship:

$$Pl(A) = 1 - Bel(\bar{A}) \quad (20)$$

where  $\bar{A}$  denotes the complement of  $A$ .

Many other decision rules exist in Belief Theory, especially for non-exhaustive frames of discernment. More information about them can be found in Fabiani (1996).

In decision-making, the strategy adopted here is to keep the most credible segments according to the law of *ideal* decision. The likelihood of a singleton assumption is characterized by two quantities (belief and plausibility) which are calculated using the set of masses. These quantities respectively correspond to the minimal probability and the maximum probability of that assumption's being true. Consequently, a law of decision without ambiguity is when an assumption has a belief higher than the plausibility of any other assumption.

The conflict computed in the Dempster-Shafer fusion rule is large when the two criteria are in total confusion. Therefore, we eliminate the segments which present a significant conflict. Experimentally, we have taken a threshold equal to 0.5.

#### 4.5. Illustrative Example: Approaching a Junction

Let us use a specific case study to illustrate the method. In Fig. 12, the vehicle is traveling on the road represented by the Segments 1 and 3, at a speed of 80 km/h. Estimation errors and digital map errors result in an erroneous estimated position which is closer to Segment 2 than to the others. In the following, the mass attribution, the combination and the decision stages are described for each segment.

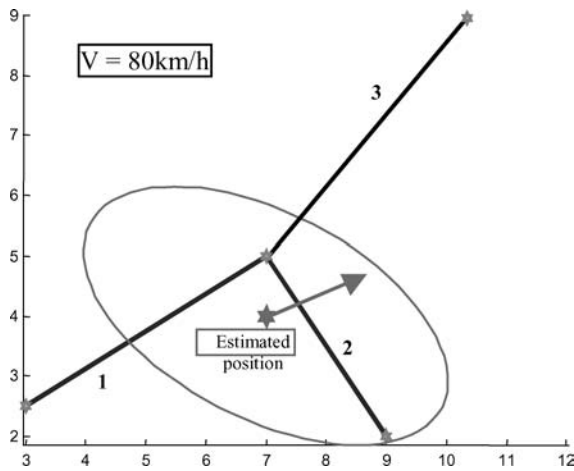


Figure 12. Estimated position and heading of the vehicle and 3 candidate segments.

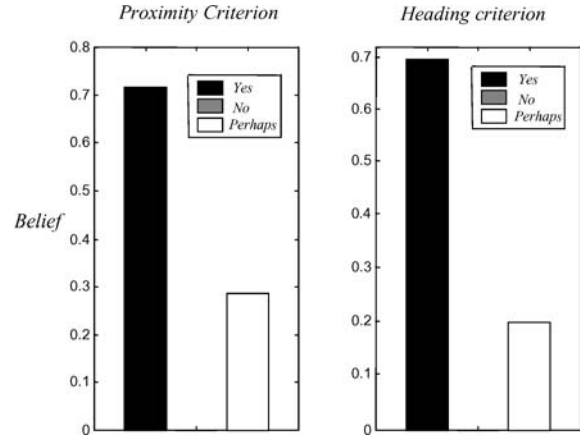


Figure 13. Mass assignment for segment 1.

Figures 13–15 show the mass assignments generated by the belief functions. It can be seen that in segments 1 and 3, the proximity criterion and the heading criterion are in agreement because both of them assign a large belief to the *Yes* hypothesis, a small belief to the *Perhaps* hypothesis and nothing to the *No* hypothesis. Conversely, segment 2 presents a total conflict between the two criteria.

Figure 16 shows the results of the fusion of the criteria with Dempster-Shafer rule without normalisation. It will be noticed that segment 2 contains a significant conflict, while the fusion of the criteria concerning segments 1 and 3 indicates a strong belief in the *Yes* hypothesis.

To decide if a segment is a good candidate, we first consider the conflict generated by the fusion stage. As it is important for segment 2, this segment is eliminated.

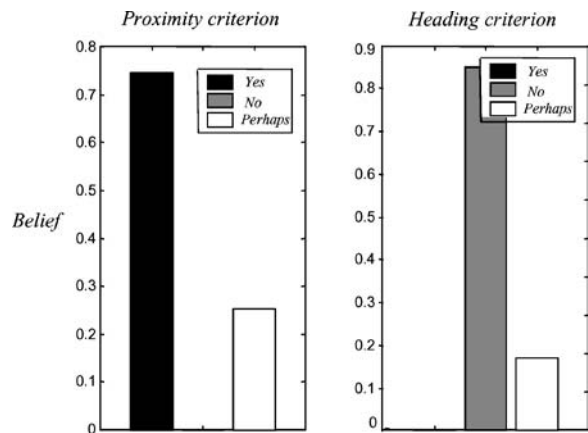


Figure 14. Mass assignment for segment 2.

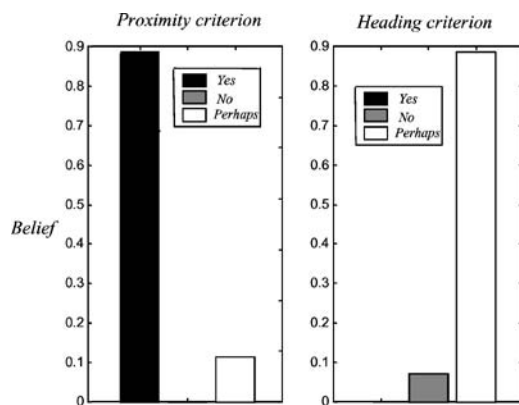


Figure 15. Mass assignment for segment 3.

Next, the ideal decision law is applied after normalisation of the masses. This law simply means here that if the Belief in the *Yes* hypothesis is larger than the sum of the *No* and *Perhaps* hypotheses, the segment in question is credible. Finally, Fig. 17 shows that Segments 1 et 3 are selected. This result concurs with the real situation.

#### 4.6. Making Use of Road Topology

It is important to distinguish, on the one hand, road-matching methods that use known facts about a driver's intended route, and, on the other hand, methods that do not use such information. Knowing the driver's in-

tended route can make the road-matching more easier since the search of possible segments is more restricted. For example, matching the location of a vehicle along its pre-calculated route is a relatively easy task since the vehicle is expected to follow a fixed set of segments in a predetermined sequence. However, confining the search space to only "expected to be traveled" segments is not always a good idea. Drivers can intentionally or unintentionally deviate from this itinerary. Various circumstances such as bad traffic conditions or inaccessibility of a given street segment can lead them to travel on an alternative route. Therefore, in our work, we avoid using route information in the selection of probable segments.

It is also customary to distinguish road-matching methods that use only geometric information (Bernstein and Kornhauser, 1998) from those that make use of topological information (Greenfeld). When using only geometric information, one can only make use of the "shape" of the segments and not of the way in which they are connected. Topological information makes use of the geometry of the arcs as well as the connectivity and the contiguity of the segments. This makes the topological solution much more reliable. Indeed, considering the topological characteristics of the network and the progression of the car along this network prevents the algorithm from jumping between one road and another. More generally, the integration of additional criteria in the road selection stage can improve the robustness of a road-matching algorithm.

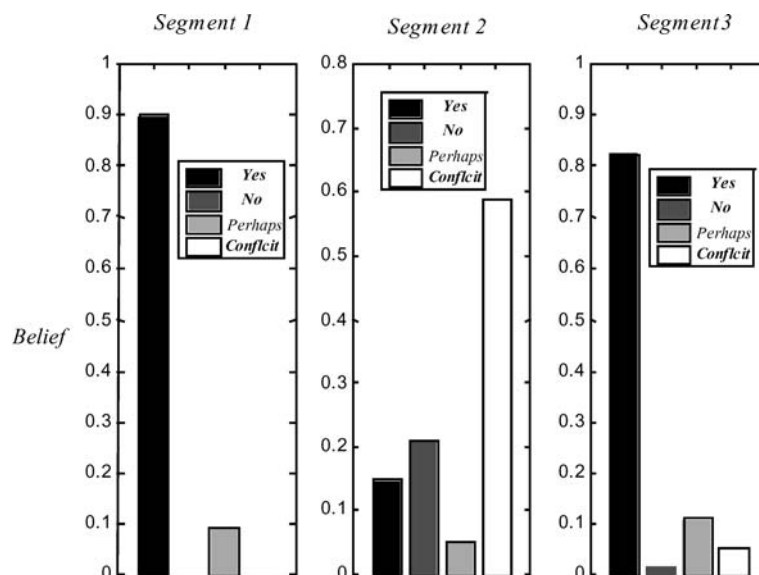


Figure 16. Fusion results without normalisation.

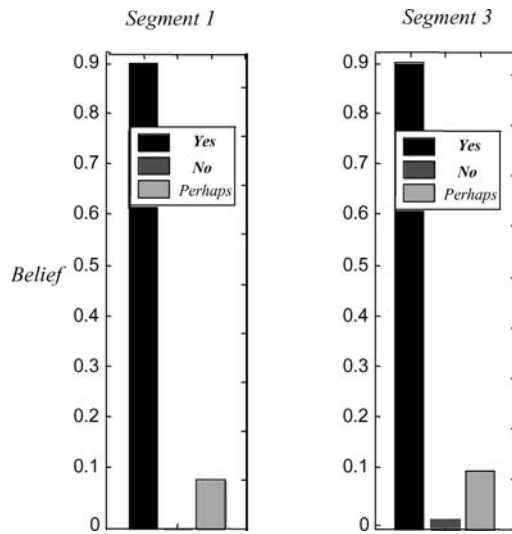


Figure 17. Fusion results with normalisation.

Thus, two binary criteria have been added to the two credibilist criteria presented in Sections 4.2 and 4.3:

- Test of connectivity to the segment on which the vehicle was matched at the previous stage, if this segment existed,
- Test of comparison between traffic direction, stored in the database and the estimated heading of the vehicle. This criterion is very effective for removing ambiguity in case of parallel roads.

#### 4.7. Experimental Results of the Road Selection Method

The road-selection method presented above works in real time conditions with a frequency of 1 Hz (under WIN NT/2000 Pentium III 700 MHz). The DGPS receiver used is a Trimble AgGPS132 with an *Omnistar* differential correction. It should be noticed that a 1 Hz sampling frequency is enough to compute an odometric estimation using the ABS sensors. In order to synchronize this sampling process with the GPS, we have used the PPS signal of the AgGPS132 receiver.

Figures 18 and 19 presents an aerial view of an experimental test performed in Compiègne, in France. The map database is managed and interfaced by the GIS software “Geoconcept”.

To illustrate the road-selection method, let consider how it treats ambiguous situations. In the test shown in

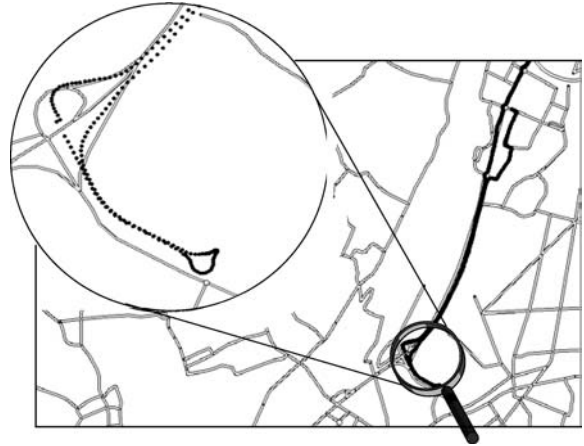


Figure 18. Experimental situation on the “IGN G  oroute” database. The estimated positions are dotted.

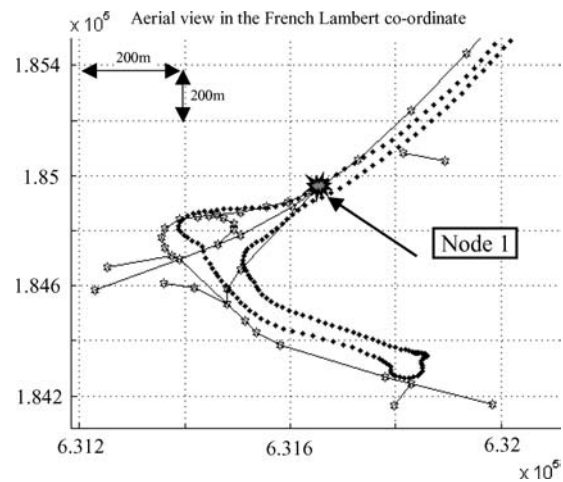


Figure 19. Candidate segments extracted from the “IGN G  oroute” database (the estimated positions are dotted).

Fig. 20, the vehicle exits a motorway. This situation is very ambiguous because the angles of three segments (the motorway, the exit ramp and the entrance ramp) are close to the heading of the car. Moreover, they have a common point very close to the estimated position.

At the beginning, three segments are selected (shown by bold lines in Fig. 20). Two of them correspond to the motorway and one to the exit ramp. The entrance ramp (located on the opposite side of the road) is not selected because of its angular criterion. Afterwards, the situation is still ambiguous (Fig. 21) until the difference between the car’s heading and the angles of the motorway segments becomes significant. Then, the

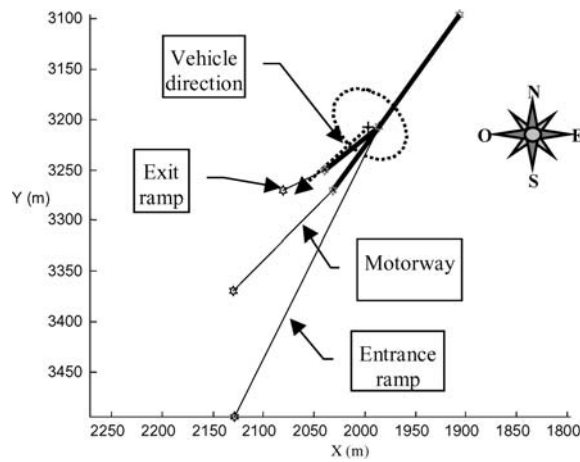


Figure 20. The car exiting the motorway (local frame).

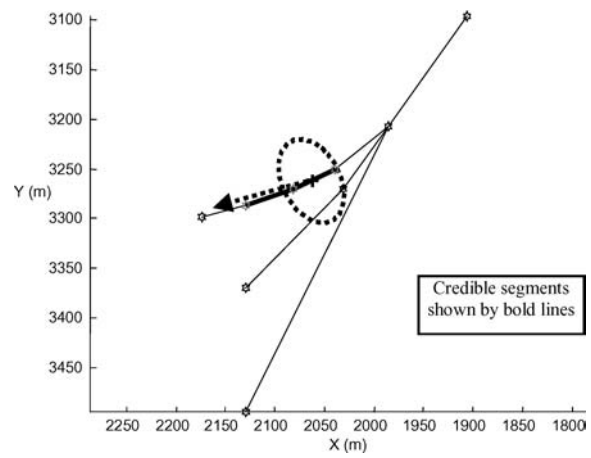


Figure 22. Vehicle on the exit ramp (b).

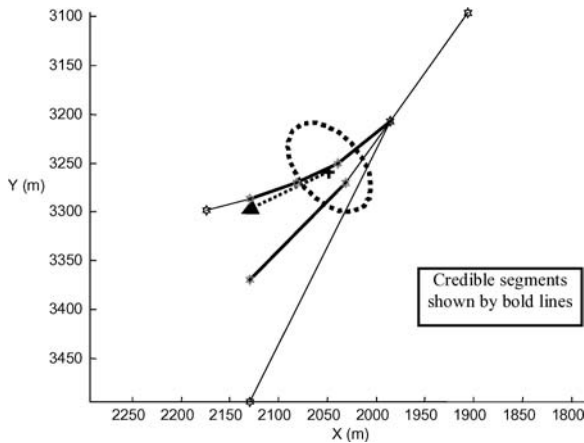


Figure 21. Vehicle on exit ramp (a).

system is able to assert that the car is on the exit ramp (Fig. 22).

Let us analyze the behavior of the method on another potentially-ambiguous situation. In Fig. 23, two critical situations occur. The first one corresponds to an intersection of three roads: two of them present the same direction and the third one has a 45-degree angle. In the second situation, three roads have the same direction and are very close to each other ( $< 10$  m). The speed of the vehicle is 70 km/h.

Figure 24 shows how the system treats the first critical situation: several credible roads are good candidates. First, it can be seen that only the segments which represent the parallel road are selected. Moreover, as these segments belong to two different roads, the situation is ambiguous. If the application which uses the

road-matching method can tolerate errors, the most credible segment can be output. In this particular case, the most credible segment corresponds to the right road, but it is a matter of chance.

Figure 25 shows the result for the second critical situation. In this situation, the vehicle is traveling on a wide road, represented by two arcs. A secondary road is parallel and very close to the main road.

The road selection method extracts 4 segments. Once more, the situation is ambiguous, because the segments belong to three different roads.

A possible strategy for handling the ambiguity of such a situation could be simply not to correct the pose using the map. In other words, the state observer, in this case, merges only the GPS and odometric measurements.

## 5. Experimental Results of the Road-Matching Method

In this section, we analyze the behavior of the complete method combining the data of the ABS, the GPS receiver and the map. Figure 26 presents an aerial view of a 4-km long experimental test performed in Compiègne. In the following, the map observation covariance matrix was computed with  $P = 0.9$ .

In Fig. 26, the gray path corresponds to the approximate absolute positions provided by the DGPS transformed in the French Lambert coordinate system of the map. The black path is the result of the fusion of the sensors with the roadmap. In this experiment, a DGPS signal mask was simulated (i.e. the DGPS

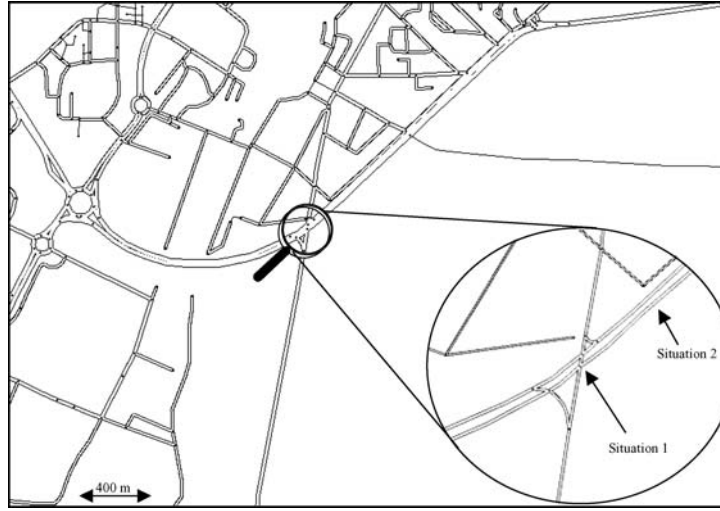


Figure 23. Top view of the test trajectory.

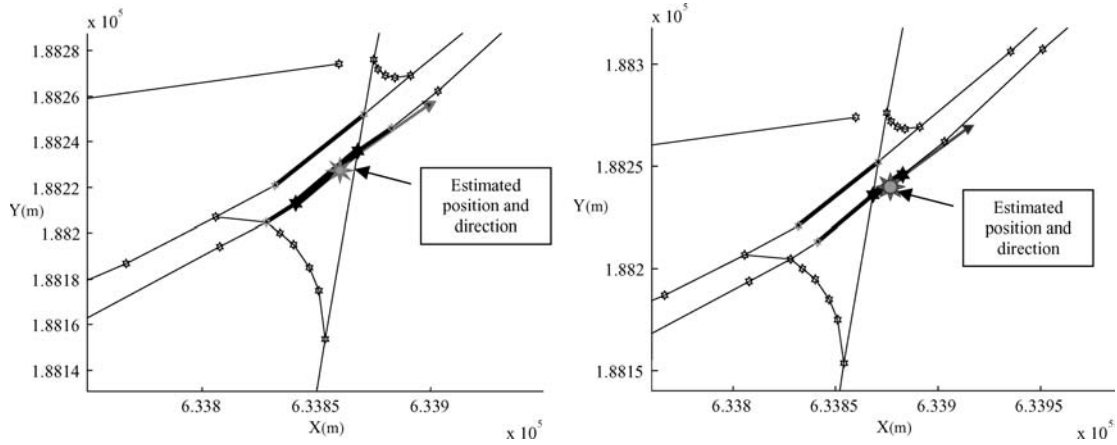


Figure 24. Credible segments are in bold and the most credible is in large bold (French Lambert coordinates).

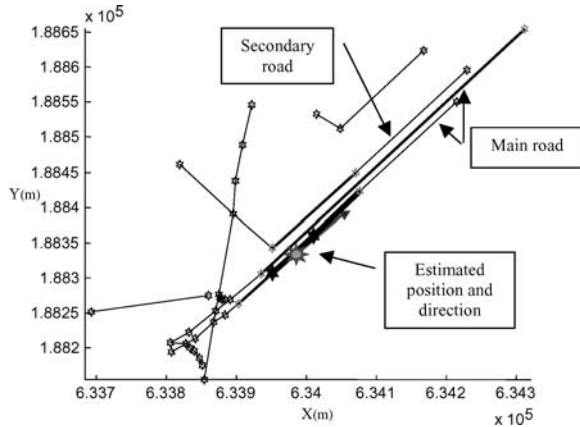


Figure 25. Credible segments are in bold and the most credible is in large bold.

measurements were not used). This signal mask starts at the exit of the first roundabout (in the bottom of Fig. 26). It can be seen that in spite of the long DGPS mask (about two kilometers), the vehicle location is matched correctly. In fact, the final estimated positions stay close to the DGPS points.

The fine performance of the road selection method is illustrated in Fig. 27. Between the two roundabouts, the journey is a  $2 \times 2$ -lane road: each roadway is represented by a one-way arc. In spite of the closeness of the DGPS positions to the wrong arc, the estimated positions are associated to the right arc. Moreover, the roundabouts succeed in correcting the estimation, even though they are represented by many segments, which leads to ambiguous situations.

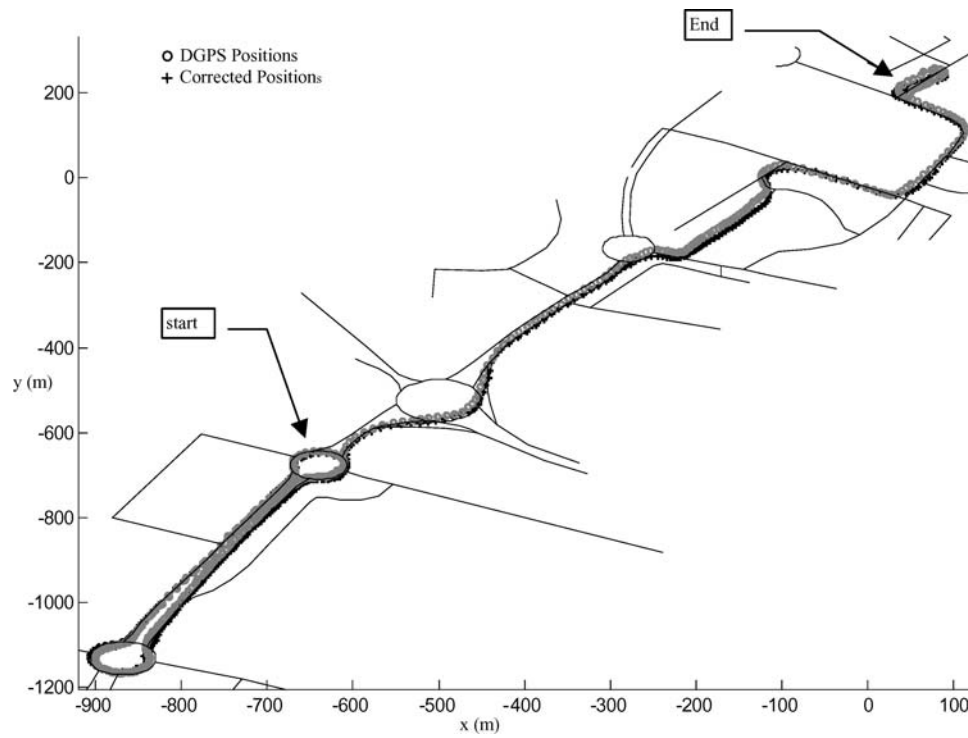


Figure 26. Experimental path and candidates roads.

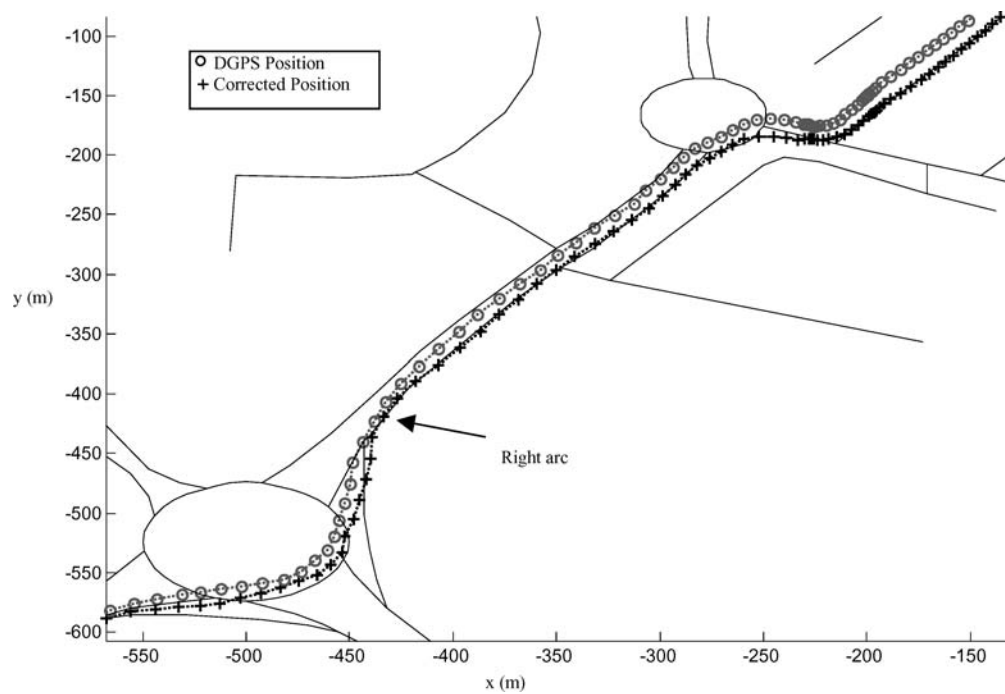


Figure 27. DGPS positions and  $\{ABS, DGPS, map\}$  fused positions.



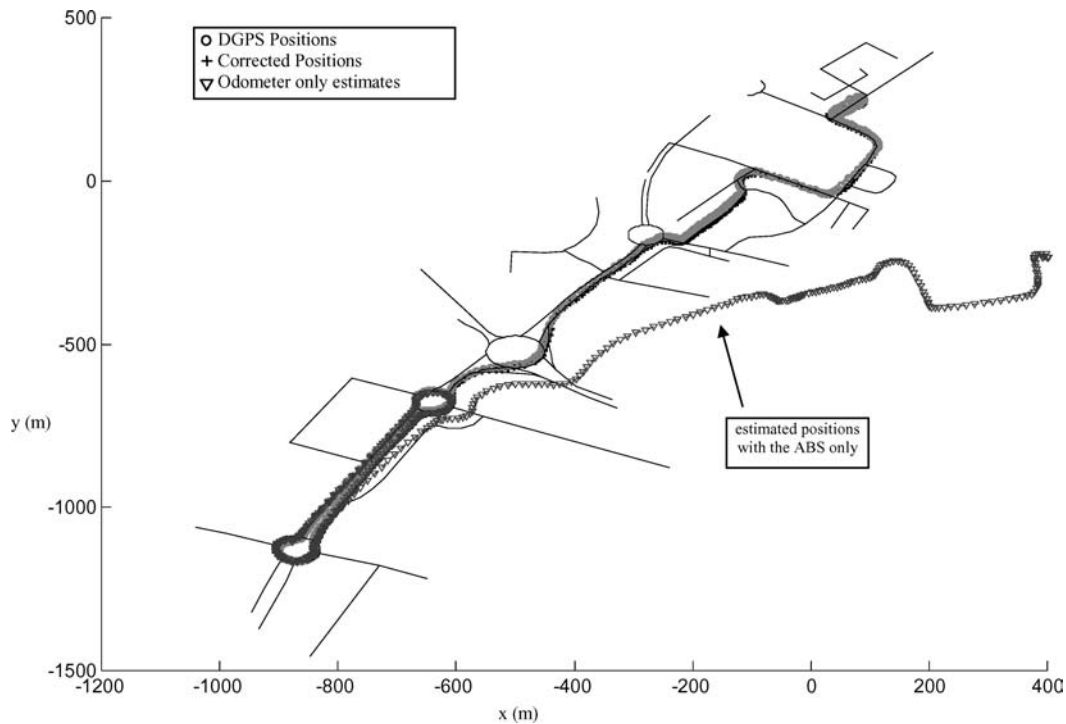


Figure 28. DGPS positions, {ABS, DGPS, map} fused positions and odometric estimation positions.

In order to prove the interest of the map observation, Fig. 28 shows the dead-reckoning results using the ABS sensors only, when the GPS signal is not available. One can see that the map observation corrects efficiently the odometric drift (the GPS mask still starts at the exit of the roundabout in the bottom of the figure).

## 6. Conclusion

This article has presented a road-matching method based on a multi-sensor fusion approach. The main contributions of this work are the formalization of a map observation in the Kalman filtering context, the use of a road selection method based on multi-criteria fusion using Belief Theory, and an experimental validation with real data. The selection of roads from a database is a key issue in the road-matching problem. A theoretical formalization of this problem in the framework of Belief Theory was proposed under the angle of data fusion of several criteria. Then, we presented the development of assignment functions, and an experimental validation was carried out with real data. Two criteria were developed. They use an estimation of the pose of

the car and segments extracted from the database. It should be noticed that these criteria take into account explicitly the estimation and geographical errors.

It is also interesting to note that, in Belief Theory, a lack of knowledge of a criterion can be quantified (in this particular case, it is the *Perhaps* hypothesis) and managed in the fusion process. Moreover, as different decision-rules can be applied, different behaviors can be obtained. If one wants a reliable behavior, the ideal decision-rule is to be used. This is the choice which was made in this work. An advantage of this strategy is that it is possible to detect an ambiguous situation, where several roads are not distinguishable. On the contrary, this method can also detect the fact that the vehicle is not on a road stored in the database. This kind of situation can be frequently met if the roadmap is not exhaustive.

Another interesting characteristic of this approach is that it is flexible and modular in the sense that it can easily integrate other criteria: the result of the combination of two criteria can be combined with the masses assigned by a third one, and so on. Therefore, it is possible, in the same framework, to build and combine other criteria testing, for example the compatibility between

the current speed and that recorded in the database. This feature is interesting because adding other criteria is a way to increase the robustness of the road selection.

Finally, a method to use the map as an observation of the state space representation has been introduced. This observation is used in the Kalman filter in the same way that the GPS data. It turned out in the experiments that the GPS measurements are not necessary all the time, since the merging of odometry and roadmap data can provide a good estimation of the position over a substantial period. Nevertheless, it was noticed that this estimation can sometimes diverge. This is due to the fact that the strategy presented in this paper keeps only the most likely segment. When approaching an intersection, several roads can be good candidates. If a wrong road is more credible than the right one, the method will diverge, because the GPS is not available to correct this wrong choice. A solution to this problem is to manage several hypotheses until the situation becomes unambiguous. We think that is the main perspective of this research.

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**Maan El Badaoui El Najjar** was born in Tripoli-Lebanon in 1975. He received his engineer diploma and his M.S. degree in control system and automation from the Institut National Polytechnique de Grenoble, France, in 1999 and 2000 respectively and his Ph.D. degree in control system from the Université de Technologie de Compiègne, France, in 2003. He is research and teaching associate at the Heudiasyc laboratory of the Université de Technologie de Compiègne. His current research interests include robot localization, map-aided navigation techniques, sensor fusion and Bayesian estimation techniques.



**Philippe Bonnifait** graduated from the Ecole Supérieure d'Electronique de l'Ouest, France, in 1992 and received the Ph.D. degree in automatic control and computer science from the Ecole Centrale de Nantes, France, in 1997. He joined the Institut de Recherche en Communications et Cybernétique de Nantes (IRCCyN UMR 6597), France, in 1993. Since September 1998, he is with Heudiasyc UMR 6599, France, and he is Assistant Professor at the Université de Technologie de Compiègne. His current research interests are in intelligent outdoor vehicles, with particular emphasis on applications to dynamic localisation.