Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
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How to analyze the music objectively?

Summary

Music is a form of art that profoundly influences humans and societies. Given the four data sets, we are motivated to develop a series of objective measurements to analyze the music from four perspectives.

First of all, the measurement of music influence between artists is given by the combination of three indicators: 1) the indicator related to artists active in one certain decade being followed by artists active in other decades, 2) the indicator related to artists of one certain genre being followed by artists of other genres, and 3) the out-degree of the influencers. A weighted directed graph $G(V,E)$ is constructed to represent the influence network, where an edge is directed from the influencer to the follower. Therefore, the calculated influence between the two artists is the weight of a directed edge.

Secondly, the music similarity is quantified. Since the provided data sets *full_music_data* contains high-dimensional data, thus the dimension reduction is needed. We are motivated to use the modified randomly distributed embedding (RDE) framework to take good advantage of interaction information of high-dimensional data. Moreover, the Bayes error rate is used to identify whether the distributions of within-genre and between-genre are significantly different.

Thirdly, we analyzed two kinds of roles of musical characteristics by examining the p-values from Kruskal-Wallis test. We removed each characteristic one at a time, and compared the p-values before and after the certain characteristics is removed. From the experiment results, the most influential characteristics in distinguishing genres are *Energy*, *Valence*, and *Speechiness*. The most *contagious* characteristics among artists are *Acousticness*, *Danceability*, and *Valence*.

Finally, we detected the most revolutionary artists by detecting the important nodes in the influence network using the modified closeness centrality (the synthesized centrality of in-closeness and out-closeness centrality). The calculated top 12 evolutionary artists are listed, including Bob Dylan, Chuck Berry, The Beatles, etc. In addition, the dynamic process of evolution is analyzed. By conducting discrete Fourier transform (DFT) and detecting the time series which is reconstructed from low-frequency components, we can observe the low-frequency components in a musical characteristic over time. An indicator σ is proposed to measure the proportion of low-frequency components among all the frequency components. *Energy* and *Popularity* have large σ values according to the experiment, meaning that there are rich low-frequency components in the time series of these two characteristics.

Keywords: Complex Network; Bayes Error Rate; Closeness Centrality; Kruskal-Wallis Test; Discrete Fourier Transform

Contents

1 Introduction

1.1 Background

Music has had a profound effect on humans and societies throughout history. The evolution of music occurs constantly, influenced by advanced technology, personal experiences, social events, etc. The internal influence between artists also accounts for the music changes. Analysis of music is usually deemed relatively perceptual and subjective. However, music that is more abstract in itself can be quantified by various characteristics, which makes it possible for us to analyze the music systematically and objectively.

One of our goals in this paper is to develop appropriate measures to quantify the influence and similarity of music. Based on the quantification, more thorough analyses could be conducted, including identifying the significant artists and characteristics. We believe that a better understanding of the music evolved could be also be obtained with a series of carefully designed models and experiments.

1.2 Restatement of the Problem

The analysis of music influence involves detail ranging from methods for quantification to identification of decisive factors. Thus, in order to analyze the influence of music in a clear and systematic way, we restate the problem as follows.

- Develop a measurement of music influence between artists, and construct a directed influence network that connects influencers and followers accordingly;
- Quantify the music similarities and compare the music similarities between and within genres;
- Testify whether there are significant differences between the distributions of within-genre and between-genre similarities;
- Discover the decisive music characteristics that distinguish genres;
- Determine the most contagious music characteristics among artists;
- Identify the revolutionary artists, and analyze the evolution of music.

1.3 An Overview of Our work

To address the problem, this paper proposed five models to solve the respective sub-problems. The rest of the paper is structured as follows. Section 2 states a thorough problem analysis and our motivations. In Section 3, we give the assumptions and notations of the model. Section 4 provides sufficient details of our model. In Section 5, we test the proposed model. Section 6 analyzed the strengths and weaknesses of our model. At lat, a conclusion is given in Section 7. A one-page document to the ICM Society is given in the appendices???.

2 Problem Analysis

Based on the detailed information provided by the four data sets, we first systematically analyze the problem. After exploring and reviewing the relevant literature research, we then construct a detailed model.

To begin with, the a directed network is required to be constructed from the *influence_data* data set. In *influence_data*, a detailed relationship between influencers and followers is given. For an influence network, an edge is directed from the influencer to the follower. Though a directed graph is easy to construct, a measurement of influence should be developed to describe the strength of the influence. Therefore, three factors are taking into account to measure the influence:1) the indicator related to artists active in one certain decade being followed by artists active in other decades, 2) the indicator related to artists of one certain genre being followed by artists of other genres, and 3) the out-degree of the influencers.

On top of that, the music similarity also needs to be quantified. An abundant amount of data are provided in *full_music_data* data set, and the quantification of high-dimensional data is therefore essential. Classical methods, such as the principal components analysis (PCA), are commonly used for dimension reduction. However, we are motivated to use the modified the randomly distributed framework (RDE) instead, which can take good advantage of interaction information provided by high-dimensional data. We also testify difference between the distribution of within-genre and between-genre similarities to justify whether the influencers indeed influence the followers.

Thirdly, we need to analyze roles of characteristics under different circumstances. The *influential characteristics* are defined to be the ones that are effective in distinguishing the genres, and the *contagious* characteristics are defined to represent the common inherent music features among artists. We are inspired to test the influence of a characteristics by comparing the p-values (obtained by conducting non-parametric test) before and after it is removed from other characteristics.

Afterwards, the artists who signify the revolutionaries is detected. Empirically, we assume that the revolutionary artists have great impact on others while receiving small influence from others. Therefore, we are motivated to calculate closeness centrality of the directed influence network, and use the difference between out-closeness and in-closeness centrality to measure how revolutionary an artist is.

Finally, the analyze of dynamic process of evolution is needed. With the date given in *data_year*, we aim at analyzing the evolution trend of each musical characteristics from the perspective of frequency domain. Therefore, we are motivated to conduct the discrete Fourier transform in order to observe the different frequency components. Since it is not objective to observe the series by the curve shapes, we developed an indicator σ that measures the proportion of low-frequency components.

3 Model Assumptions and Notations

3.1 Assumptions and Justifications

- The musical characteristics chosen in the data set are representative;
- The data collected are reliable;

- The revolutionary artist are the ones who have large impact on others while receiving small influence from others;

3.2 Notations

Symbol	Definition
G	A directed graph describing the influence among artists
v_i	The ith node in the graph G
e_{ij}	The edge in graph G directed from node v_i to node v_j
w_{ij}	The weight of edge e_{ij}
n_0	The number of nodes in graph G
n_j	The in-degree of node v_j
I_{ij}	The influence of influencer v_i on follower v_j
w_{ij}	The weight on edge e_{ij} in graph G
S_{ij}	The musical similarity between two nodes
c_i	The ith musical characteristics, $i = 1, 2, ...13$
F_i	The set of labels of artists that followed artist v_i
$CC_i(i)$	The in-closeness centrality of node v_i
$CC_o(i)$	The out-closeness centrality of node v_i
$C(i)$	The synthesized closeness centrality of node v_i

4 Model Construction

4.1 Measurement of Influence between Artists

According to the influence relationship revealed in the *influence_data* data set, a directed graph $G(V, E)$ can be constructed, where V is a set of nodes and E is a set of links directed from one node to another. Let n_0 be the number of nodes (the number of artists mentioned in *influence_data*). The n_0 nodes are labeled from v_1 to v_{n_0}, and e_{ij} denotes the edge directed from v_i to v_j. For each follower v_j, let n_j represent its in-degree (i. e. the number of artists that influence v_j). A subgraph showing the relation among one follower and its influencers is illustrated in Fig. 1.

In a addition to construct a directed network by simply connecting the influencer to the follower, we aim at measuring the influence by three indicators from the perspective of a pair of the follower and influencer on edge e_{ij}: 1) frequency $\widetilde{k}_{ij}$ that indicates the influence according to the period where influencer v_i and follower v_j are active, 2) frequency $\widetilde{m}_{ij}$ that describes the influence due to

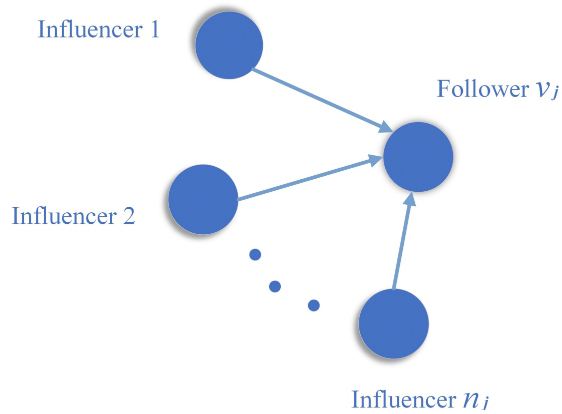


Fig. 1. The subgraph of one follower v_j and its n_j influencers

the main genres of influencer v_i and follower v_j, and 3) the out degree O_i of the influencer v_i. To better explain the three indicator, more explicit definitions of $\widetilde{k}_{ij}$ and $\widetilde{m}_{ij}$ are given as follows.

Let D_i be the decade when the artist v_i begin his or her music career. For influencer v_i and follower v_j on edge e_{ij}, k_{ij} is defined to be the total number of times that artists in period D_i are followed by artists in period D_j.

Definition 4.1. *For edge e_{ij} that is directed from influencer v_i to follower v_j, the indicator $\widetilde{k}_{ij}$ is defined as:*

$$\widetilde{k}_{ij} = \frac{k_{ij}}{\sum_{l \in \boldsymbol{S_j}} k_{lj}} \tag{1}$$

where $\boldsymbol{S_j}$ is the set of labels of influencers of follower v_j.

Similarly, let R_i be the main genre of artist v_i. Then m_{ij} is defined as the total number of times that artists of genre R_i are followed by artists in genre R_j.

Definition 4.2. *For edge e_{ij} that is directed from influencer v_i to follower v_j, the indicator $\widetilde{m}_{ij}$ is defined as:*

$$\widetilde{m}_{ij} = \frac{m_{ij}}{\sum_{l \in \boldsymbol{S_j}} m_{lj}} \tag{2}$$

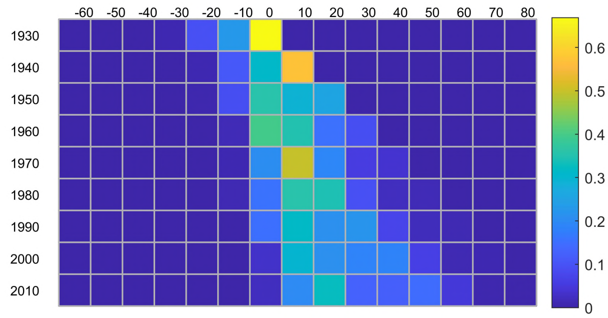
where $\boldsymbol{S_j}$ is the set of labels of influencers of follower v_j.

Applying the data from *influence_data*), the $\widetilde{k}_{ij}$ and $\widetilde{m}_{ij}$ for influencers and followers of each type (including the decade when they began their music career and their main genres) are shown in Fig. 2.

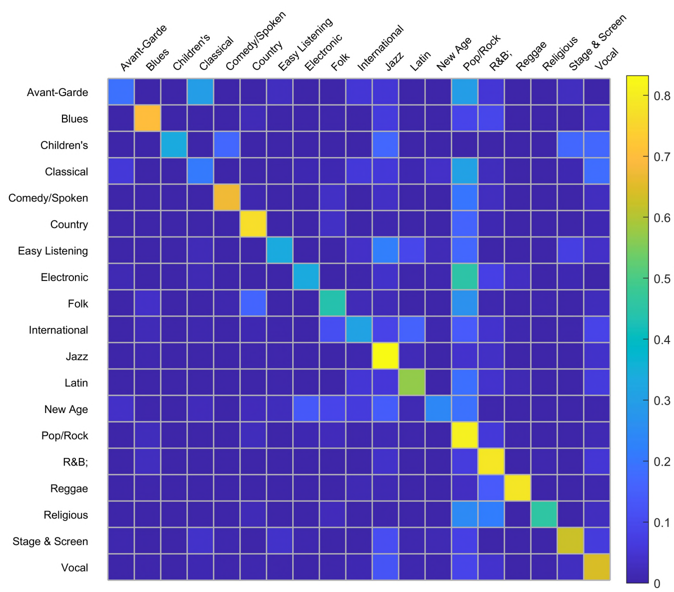
Therefore, the influence I_{ij} of influencer v_i on follower v_j should be able to aggregate the data and information provided by the three indicators, i. e. $\widetilde{k}_{ij}$, $\widetilde{m}_{ij}$ and O_i.

Definition 4.3. *The influence I_{ij} of influencer v_i on follower v_j on edge e_{ij} is defined as:*

$$I_{ij} = \widetilde{k}_{ij} \cdot \widetilde{m}_{ij} \cdot O_i \tag{3}$$



(a) The indicator k_{ij}



(b) The indicator m_{ij}

Fig. 2. The calculation results of m_{ij} and k_{ij}. (a) The indicator k_{ij} for any pair of influencers and followers, where the vertical axis denotes the decade when the followers began their music career, and the horizontal axis denotes the time interval of the decade of followers and influencers. (b) The indicator m_{ij} for any pair of influencers and followers, where the vertical and horizontal axes represent the main genre of the followers and influencers, respectively.

Here, in Example 1, we give a brief example to better illustrate the calculation of influence $\widetilde{m}_{ij}$ (the indicator $\widetilde{k}_{ij}$ can be calculated by using the exact same method). Since the number of influencer is huge and the out-degree O_i for the influencers in each edge can be easily found out from the data set, we will avoid displaying the cumbersome data here.

Example 1 Consider an influencer v_i and a follower v_j whose main genres are *Avant_garde* and *Classical*, respectively. Through the statistics in *influence_data* data set, the total number of times for artists of genre *Classical* to be influenced by artists of genre *Avant_garde* is 11, i. e. $m_{ij} = 11$. Additionally, there are in total 37 artists of various genres being followed by the artists of genre *Avant_garde*. Therefore, it can be calculated that the indicator $\widetilde{m}_{ij}$ between these two artists is $\widetilde{m}_{ij} = 11/37 \approx 0.2973$. It should be noted that the $\widetilde{m}_{ij}$ remains the same for any pair of influencer and follower whose genres are respectively *Avant_garde* and *Classical*.

By employing the calculation method illustrated in Example 1, let the weight w_{ij} of each directed edge in graph G be the quantified influence I_{ij} (i. e. $w_{ij} = I_{ij}$). Therefore, a weight matrix $\boldsymbol{W}$ is obtained, and graph G can be extended to $G(V, E, \boldsymbol{W})$. Fig. 3 demonstrated the overall structure of graph G.

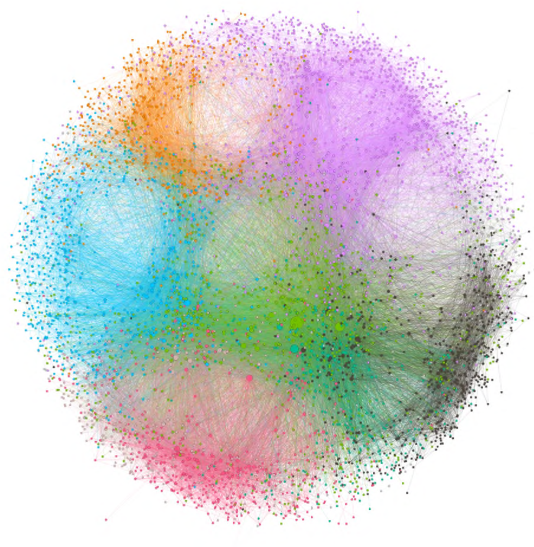


Fig. 3. The structure of graph G. The size of nodes represents the out-degree. The largest two nodes are Bod Dylan and The Beatles, whose subnetwork is presented in Fig. 4

In Fig. 3, the color of nodes represent the genre they belong to, and the thickness of an edge denote the its weight (the thicker the edge is, the larger its weight is). Fig. 3 gives a rough distribution of genres and influence among artist, but is hard to discover details from. To give a better insight into the influence among artists, a subnetwork shown in Fig. 4 is provided.

4.2 The Music Similarity

With an abundant amount of data regarding the music-related characteristics, it is essential to develop a measurement to quantify the similarity with these high-dimensional data. Moreover, it is also of great importance to analysis whether the genres are distinguishable according to the similarity measurement that we developed. In this section, we proposed a modified randomly distributed embedding (MRDE) framework to measure the music similarity, which takes good advantage of interaction information among high-dimensional data. Afterwards, we analyze the similarity distribution between and within genres to identify whether the difference between within-genre similarity and between-genre similarity is significant. Finally, we test the effectiveness of the proposed measurement of influence in Section 4.1 by analyzing the similarity distribution among artist who do or do nor have mutual influence.

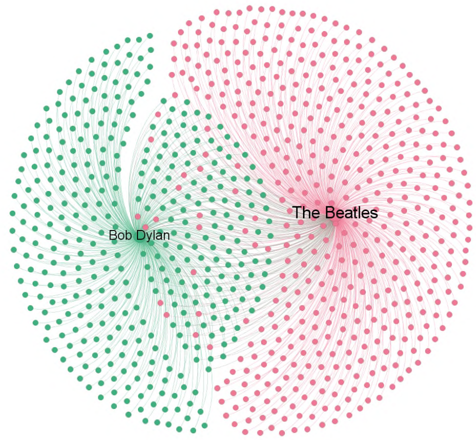


Fig. 4. A subnetwork of Bod Dylan, The Beatles and their followers

4.2.1 Quantification of Similarities Based on the MRDE Framework

In this section, we demonstrate our model for quantifying the musical similarity. The proposed MRDE framework is explained carefully, and the quantification result is then given and visualized.

To begin with, a brief introduction of the original RDE (randomly distributed embedding) framework is given. The RDE framework is first proposed by Ma et al. in 2018 published on PNAS [1]. When dealing with high-dimensional data, dimension reduction techniques are usually applied, but the application is likely to overlook the interactions among high-dimensional variables. To make good use of the interaction information, the RDE framework builds a distribution of a sufficient number of embeddings of low-dimension. While each low-dimensional embedding preserves a part a information of the whole system, these low-dimensional embeddings form a probability distribution which can be used to obtain the final one-dimensional variable (or the value). Readers could refer to more details in [1]. Hence, we proposed the MRDE framework to adapt the original RDE framework for the problem discussed in this paper, which provides another perspective for the comprehensive evaluation.

Among the fourteen musical characteristics summarized in *data_by_artist*, we eliminate the factor *count* for the experiment in this section, since *count* is not music-related and do not provide any information about an artist's music style. Then we are left with 13 musical characteristics from danceability to popularity. In the MRDE framework, we choose the embedding dimension e to be $e = 3$ empirically according to the experiment in [1]. In other words, for each 3-dimension embedding, we randomly choose 3 characteristics out of the 13 musical characteristics. There are in total q kinds of combination of the chosen 3 characteristics, which can be calculated by Eq. (4).

$$q = \binom{E}{e} \tag{4}$$

where $E = 13$ is the total number of musical characteristics involved in the analysis, $e = 3$ is the embedding dimension.

Each of the 13 musical characteristic is label from c_1 to c_{13}. For each 3-dimensional embedding, we construct an index tuple $\boldsymbol{l} = (l_1, l_2, l_2), (l_k = 1, 2, ...13, k = 1, 2, 3)$, where l_1, l_2 and l_3 are the indexes of characteristics in this tuple. Let $\boldsymbol{C_j}$ represent the tuple of values of 3 chosen characteristics for node v_j, then the three characteristics of artist v_j is represented as $\boldsymbol{C_j} = (c_{l_1}, c_{l_2}, c_{l_3})$.

When measuring the similarity between artist v_i and artist v_j, we need to first identify their *rough similarity* calculated by each embedding. For the kth 3-dimensional embedding (k=1, 2, ..., q), we define its *rough similarity* sr_k as

$$sr_k = 1 - \|\boldsymbol{C_i} - \boldsymbol{C_j}\|_2 \tag{5}$$

Let Sr be the random variable where $sr_k (k = 1, 2, ..., q)$ are the specific value. Therefore, the q *rough similarity* labeled from sr_1 to sr_q can form a probability distribution. The actual similarity S_{ij} between v_i and v_j is defined as the expectation value of the distribution, formulated in Eq. (6).

$$S_{ij} = E(Sr) \tag{6}$$

where $S_{ij} \in [0, 1]$, and $E(Sr)$ is the expectation of random variable Sr. When S_{ij} is large, it indicates that the these two artist are more similar, and vice versa.

By using Eq. (5) and Eq. (6), the similarity S_{ij} between v_i and v_j can be calculated. The proposed MRDE framework can be better illustrated by the following steps:

Step 1: Randomly choose q tuples. Each tuple contains 3 index numbers denoted as $\boldsymbol{l} = (l_1, l_2, l_2)$, and the musical characteristics in this tuple are denoted as $(c_{l_1}, c_{l_2}, c_{l_3})$;

Step 2: For the q tuples (namely the 3-dimensional embeddings), calculated the *rough similarity* $sr_k, k = 1, 2, ..., q$ by using Eq. (5);

Step 3: Aggregate the *rough similarities* calculated in each embedding to form the probability distribution of the random variable Sr;

Step 4: Given the distribution of Sr, obtain the similarity $S_{ij} = E(Sr)$ by calculating the expectation value of random variable Sr.

A undirected network $G_1(V_1, E_1)$ is constructed, where V_1 is the set of nodes and E_1 the set of undirected edges. Since nodes being analyzed are the same as in G, so the label of nodes remain the same (such v_1, v_2 etc.). It should be noted that even though Eq. (6) and Fig. **??** measures the musical similarity between two artists, the proposed MRDE scheme can also be applied to quantify the musical similarity between genres and songs, as long as the musical characteristics are available.

Discussion

In the original *data_by_artist* data set, the order of magnitude of each musical characteristics varies from -6 to 5 . However, since the Euclidean norm is used to measure the *rough similarity*, the influence of order of magnitude must be eliminated as much as possible before calculating the

Euclidean norm. Thus, the min-max normalization is utilized to normalize each characteristics. All of the normalized value of musical characteristics range from 0 to 1. Moreover, when we apply the 3-dimensional embedding, the maximum value of *rough similarity* is $\sqrt{3}$. Thus, the calculated similarities S_{ij} are reduced to $1/\sqrt{3}$ of the original scale.

4.2.2 Distribution of Similarities Between and Within Genres

From Section 4.2.1, we have obtained the similarity between every two artists. From the *influence_data* data set, we can easily label each artist with his or her main genre. Therefore, the similarity distribution among artists can easily be transformed into similarity distribution between and within genres. From our intuitive understanding, we would assume that the similarity within genres are smaller than between genres. We might assume that the similarity between genres might also varies. For instance, the genre of *Electronic* and *Country* may differ from each other from our daily experience. In this section, we will use the similarity distribution, Gaussian fitting and Bayes error rate for a thorough analysis.

Among the 19 genres (the genre *Unknown* is eliminated due to the possible interference of wrong or missing information), there are in total $\binom{19}{2}$ kinds of combinations. Fig. 5 is an example of the similarity distribution within the genre *Folk* and between *Folk* and *Reggae*.

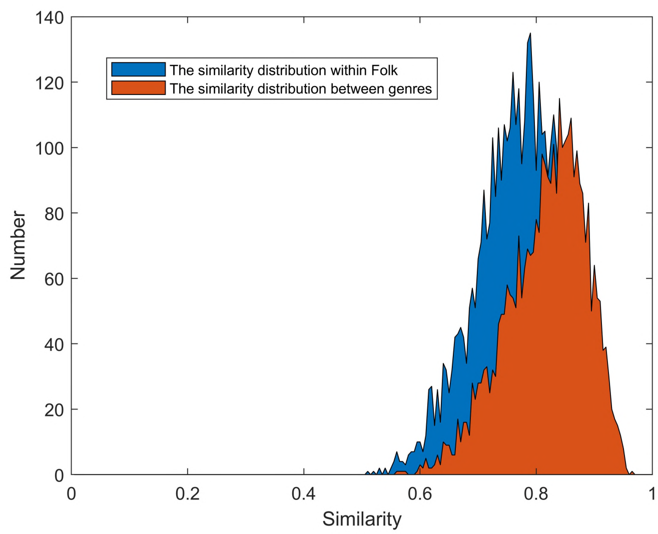


Fig. 5. The similarity within and between genres. The blue part denotes the similarity distribution within the genre of *Folk*, while the red part denotes the distribution between *Folk* and *Reggae*. The vertical axis represents the number of times that a certain value of similarity appears in the total calculated similarities, and the horizontal axis represents the similarity ranged from 0 to 1.

From Fig. 5, we can have an intuitive understanding that the similarity distribution within and between genres is different in the given example. But now a question arises: **how can this difference be measured?**

We are motivated to measure the above-mentioned *difference* by using Bayes error rate. Bayesian decision theory can address the patter classification problem from the perspective of statistics [2].

The theory quantifies the trade-off between the decision made and the costs that accompany the decision. Bayes error, on the other hand, is the lowest possible error rate for a pattern classification problem [3]. Let class ω_1 denote the *within genres*, class ω_2 denote the *between genres*, and s denote the similarity value. $P(\omega_i|s), i = 1, 2$ is the posterior probability. For the distribution illustrated in Fig. 6, the probability of error can be calculated by Eq. (7).

$$P(error|s) = \begin{cases} P(\omega_1|s) & when P(\omega_2|s) > P(\omega_1|s) \\ P(\omega_2|s) & when P(\omega_1|s) > P(\omega_2|s) \end{cases} \tag{7}$$

The Bayes error rate is therefore formulated in Eq. (8).

$$P(error) = \int_{-\infty}^{s_0} P(\omega_2|s)p(s)\mathrm{d}s + \int_{s_0}^{\infty} P(\omega_1|s)p(s)\mathrm{d}s \tag{8}$$

where s_0 is a certain point on the horizontal axis shown in Fig. 6 that divides the space into two parts. $P(error)$ can also be denoted as the size of shadow area.

Due to the large scale of data, we demonstrate some typical similarity distribution here. First of all, Fig. 7 shows the similarity within and between the genres of *Country* and *Eletronic*.

From Fig. 7, it can be observed that the overall similarity within *Country* is much smaller than the one within *Electronic* and between these two genres. The results indicate that value of musical characteristics of artists of genre *Country* are highly similar, whereas the ones of artists of genre *Electronic* are relatively scattered. Moreover, by performing Gaussian fitting, the estimated distribution for *within the genre Country*, *within the genre Electronic* and *between the two genres* are $N(0.85, 0.06)$, $N(0.74, 0.0.8)$ and $N(0.75, 0.09)$, respectively. With the Gaussian fitting results, the Bayes error of distributions in Fig. 7(a) and Fig. 7(b) are 0.486 and 0.916, respectively, indicating again that the similarity distributions in Fig. 7(a) are more different and distinguishable.

In order to have an overall understanding of the similarity within and between genres, Fig. 8 shows the similarity distribution by combining all the values of similarity within each genre and between every two genres.

The Gaussian fitting result of the within-genre and between-genre distribution are $N(0.806, 0.078)$ and $N(0.758, 0.095)$, respectively. The Bayes error rate calculated is 0.770. Combining the visualization result in Fig. 8, the Gaussian fitting result and the calculated Bayes error rate, we can

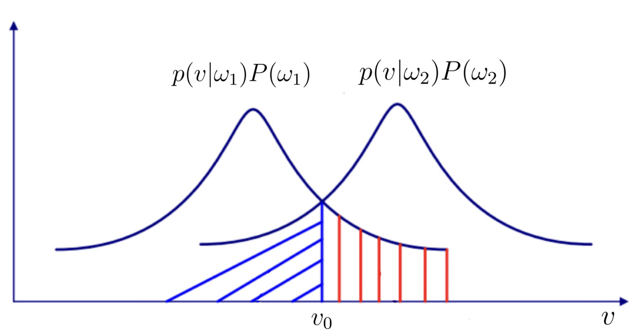
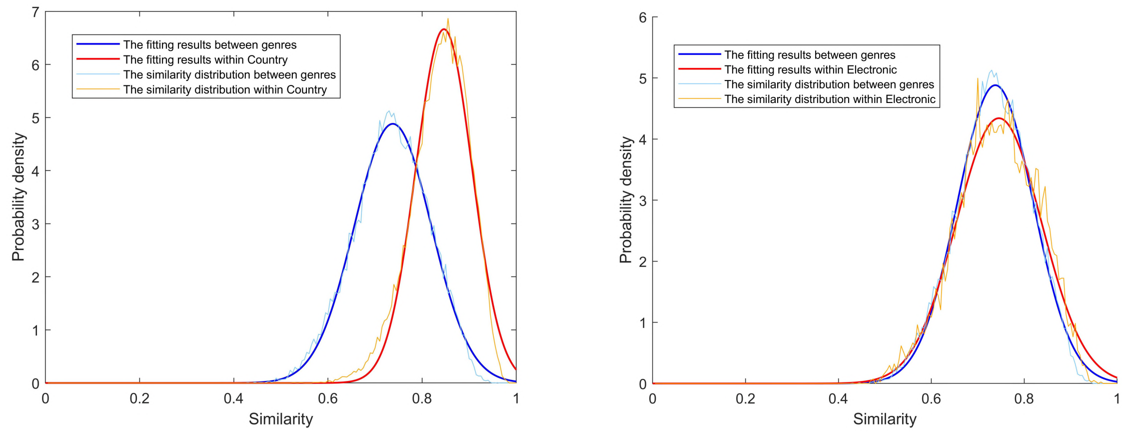


Fig. 6. An example of probability distribution to illustrate the Bayes error rate



(a) The similarity within *Country* and between genres (b) The similarity within *Electronic* and between genres

Fig. 7. The similarity distribution within and between two typical genres. (a) The light blue and blue curves denote the uniformed similarity distribution and the corresponding Gaussian fitting result between *Country* and *Electronic*. The light orange and orange curves denote the uniformed similarity distribution and the corresponding Gaussian fitting result within *Country*. (b) The light blue and blue curves denote the same thing as in subgraph a. The light orange and orange curves denote the uniformed similarity distribution and the corresponding Gaussian fitting result within *Electronic*.

reach a conclusion: 1) the similarity between genres is generally larger than within genres, and 2) the smaller the between-genre similarity is, the less similar are the corresponding two genres.

4.2.3 Test the Measurement of Influence by the Similarity Network

In section 4.1, the measurement of influence among artists is proposed. In Section 4.2.1, we quantify the similarities of musical characteristics. To discover the interaction relationship between the above-mentioned two measurements, we conduct another experiment by *combining* the directed network G of influence and the undirected network G_1 of similarities. The experiment is aim at detecting whether the measurement of influence in Section 4.1 can actually reflect the real-world influence among artists.

First of all, we will explained how the *combination* works in this paper. Basically, we will reallocate the similarity distribution in G_1 in to two distribution, based on whether two artists have unilateral or mutual influence on each other in graph G. Let S_1 and S_2 be two random variables, where S_1 denotes the similarity distribution of edges in G_1 that are connected in graph G (the directed graph describing the influence), and S_2 denotes the similarity distribution of edges in G_1 that are not connected in graph G.

Step 1: Start traversing all edges in the undirected similarity network G_1;

Step 2: Consider an undirected edge that connect node v_i and v_j. If there exists a directed edge in the influence network G directed from v_i to v_j or from v_j to v_i, then the similarity value S_{ij} on this undirected edge is a possible value of the random variable S_1;

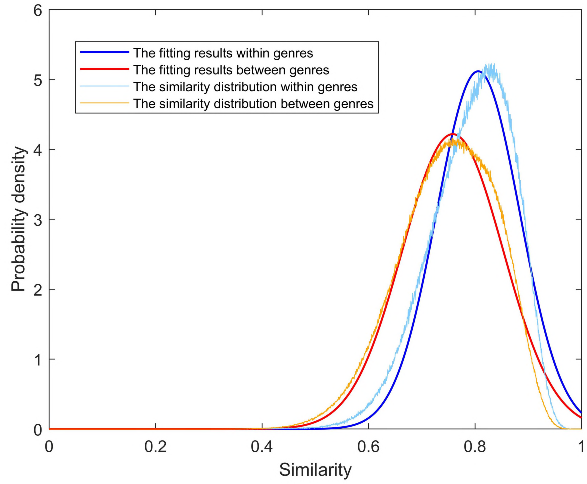


Fig. 8. The overall similarity distribution. The light orange and orange curves denote the distribution that concludes the between-genre similarity of any combination of two genres. The light blue and blue curves denote the distribution that concludes all the within-genre similarity of every genre.

Step 3: Consider the same edge in Step 2. If no directed edge exists between v_i and v_j in graph G, then the similarity value S_{ij} is a possible value of the random variable S_2;

Step 4: Repeat Step 2 and Step 3 until all the undirected edges in graph G_1 have been traversed;

Step 5: Compute the distributions of random variables S_1 and S_2.

The visualization of distributions of the two random variables are displayed in Fig. 9.

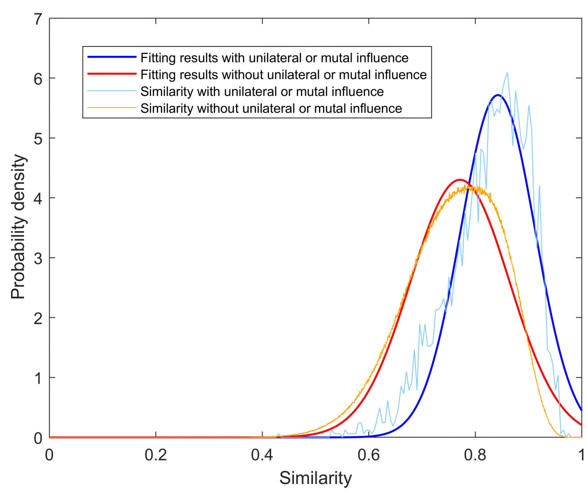


Fig. 9. The distributions of S_1 and S_2. The light orange and orange curves denote the distribution when there is no influence between to artists. The light blue and blue curves denote the distribution when two artists are unilaterally or mutually influence by each other.

The Gaussian fitting result for the distribution of S_1 and S_2 are $N(0.830, 0.067)$ and $N(0.781, 0.076)$, respectively. The Bayes error rate is 0.6452. The overall similarity between the artists who unilaterally or mutually influenced one another is significantly larger than similarity of artists that did not influence each other. The result demonstrates that the similarity of artists who have a influence relationship is in fact larger than the artists who do not. That is to say, the identified influencers indeed influence the respective followers from a statistical perspective.

4.3 The Roles of Characteristics

When measuring the music similarity, we have combined the information of 13 musical characteristics to form the quantification. However, what remains unknown is the roles that different musical characteristics plays in influencing the music. In other words, we wish to identify the one or several characteristics that are significant in distinguishing the genres. We also analyze which characteristics indicate the more general inherent features among artists. In this section, we first define the *central node* in each genre. Then we eliminate the characteristics one at a time or one by one, and conduct the non-parametric test to identify the roles of characteristics.

4.3.1 Characteristics that Distinguish Genres

Due to the lack of synthesized data for each genre, we first synthesize the information about musical characteristics by defining the central nodes. Then the Kruskal-Wallis test is utilized to identify the most influential characteristics that distinguish genres.

The Central Nodes of Each Genre:

We assume that each node in one genre is accounted for a certain percentage when deciding the central node of this genre. The percentage of node v_j is denoted as P_j, formulated in Eq. (9).

$$P_j = \frac{\sum_{k \in F_j} I_{jk} S_{jk}}{O_j} \tag{9}$$

where F_j is the set of labels of artists that follow artist v_j. I_{jk} and S_{jk} denote the influence and similarity between v_j and v_k, respectively.

Let R_i be the set of labels of nodes that belong to the ith genre, where $i = 1, 2, ...19$. Let Ch_i represent the 13 musical characteristics of the central node of the ith genre, then Ch_i is defined in Eq. (10).

$$\boldsymbol{Ch_i} = \frac{\sum_{j \in R_i} P_j \cdot \boldsymbol{C_j}}{\sum_{j \in R_i} P_j} \tag{10}$$

where $\boldsymbol{C_j}$ is the vector of 13 musical characteristics of node v_j.

We can then obtain the 13 musical characteristics of central nodes of each genre by using Eq. (10).

The Most Influential Characteristics:

Let $\boldsymbol{M_1}$ be the 19×13 matrix, where each row stores the vector of 13 musical characteristics of the central node of each genre. $\boldsymbol{M_1}$ is formulated in Eq. (11).

$$\boldsymbol{M_1} = \begin{bmatrix} \boldsymbol{Ch_1} \\ \boldsymbol{Ch_2} \\ \vdots \\ \boldsymbol{Ch_{19}} \end{bmatrix} \tag{11}$$

where $\boldsymbol{Ch_i}$ represents the 13 musical characteristics of the central node of the ith genre.

The uniformed values of matrix $\boldsymbol{M_1}$ is visualized in Fig. 10. For each column in Fig. 10, the corresponding characteristics has a great influence in distinguishing genres if the colors of each cube demonstrates a greater variability. We could roughly observe that the values of characteristics of *Energy, Valence, Speechiness* varies to a large extent, while values of *Acousticness, Instrumentalness* have relatively concentrated distribution. We will test the rough conclusion by conducting Kruskal-Wallis test on these 5 characteristics in the following part.

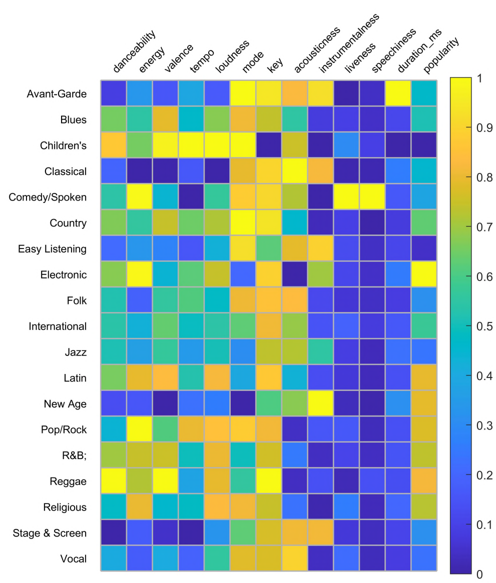


Fig. 10. The values of musical characteristics of the central nodes in each genre.

The Kruskal-Wallis test is a non-parametric test that can assess the the significant differences by two or more groups of variables [4]. The null hypothesis H_0 and the alternative hypothesis H_1 are defined. The null hypothesis represents no effect or no difference, while the alternative hypothesis denotes the presence of an effect or a difference [5]. A $p-$value is the probability that assumes H_0 is true. Namely, the smaller the p-value, the strong the evidence evidence against the null hypothesis.

By *the most influential characteristics*, we mean that theses characteristics play important roles in distinguishing the genres. In other words, the similarity between genres will significantly increase once theses characteristics are removed from matrix $\boldsymbol{M_1}$. Therefore, we removed each one of the 13 columns in M_1, and calculate the respective p-value before and after the elimination. The Kruskal-Wallis test is applied in calculating the p-value. The typical p-values calculated are presented in Table 2.

Table 2: The typical p-values when removing one of the characteristics

The removed characteristics	Energy	Valence	Speechiness	Acousticness	Instrumentalness
p-value before elimination	0.1284	0.1284	0.1284	0.1284	0.1284
p-value after elimination	0.6500	0.5319	0.3057	0.0086	0.0035

* The p-values in the second row denote the non-parametric results before elimination. The p-values in the third row is the result after the characteristics of this column has been eliminated.

From Table 2, the result shows that after eliminating *Energy, Valence* and *Speechiness*, the p-value increases significantly, meaning that the similarity distributions of each genre become less different after the elimination. Therefore, *Energy, Valence, Speechiness* are influential characteristics in distinguishing genres. The experiment result analysis is similar for other characteristics. On the other hand, the p-values become extremely small after eliminating *Acousticness* and *Instrumentalness*, meaning that these two characteristics are distributed similarly among genres. We can conclude that *Energy, Valence* and *Speechiness* are the relatively influential characteristics, while *Energy* is even more influential than other influential characteristics.

4.3.2 The *Contagious* Characteristics among Artists

In the last section, we discussed how influential the characteristics are when distinguishing the genres. In this section, we aim at discovering how *Contagious* the characteristics are when deciding the musical style of various artists.

The Selected Artists:

Since there are in total 5854 artists in the *data_by_artists* data set, it is neither practical nor necessary to perform the Kruskal-Wallis test among all the available artists. Therefore, we selected the top 25 artists who have the highest P_j value (who accounted for more percentage when deciding the central nodes) introduced in Section 4.3.1. Let $\boldsymbol{M_1}$ be the 25×13 matrix, where each row stores the vector of 13 musical characteristics of the central node of each genre. $\boldsymbol{M_1}$ is formulated in Eq. (11).

$$\boldsymbol{M_2} = \begin{bmatrix} \boldsymbol{Ch_1} \\ \boldsymbol{Ch_2} \\ \vdots \\ \boldsymbol{Ch_{25}} \end{bmatrix} \tag{12}$$

where $\boldsymbol{Ch_i}$ represents the 13 musical characteristics of the central node of the ith artists.

The Most *Contagious* Characteristics:

Different from the level of characteristics influence in Section 4.3.1, we define the characteristics to be *Contagious* when they represent **the common inherent features** of artists' musical styles. Following the idea of non-parametric test in Section 4.3.1, the Kruskal-Wallis test can also be used here to measure the level *Contagious*. The adapted method is illustrated as follows:

Step 1: For the 13 musical characteristics, calculate the corresponding 13 p-values. Each p-value is obtained by conducting Kruskal-Wallis test when the certain one out of 13 characteristics is temporally removed;

Step 2: Pick out the smallest p-value in step one and eliminate the respective characteristics permanently from $\boldsymbol{M_2}$. Name the characteristics as **the most contagious characteristics**;

Step 3: For the rest 12 characteristics, repeat the process in Step 1 and Step2. The characteristics that was picked out and eliminated is named as the **the second most contagious characteristics**;

Step 4: Repeat the process described from Step 1 to Step 3, until all the 13 characteristics have been sorted with an order.

Due to the limited length of the article, we will only demonstrate the 4 most contagious characteristics calculated from the above steps. Moreover, among the 25 artists we chose, their characteristics *Mode* all possess the value 1. Since $Mode$ is apparently the same for all the selected artists, it can be deemed as the most *contagious* characteristics. Table 3 listed the p-values of the other 3 most *contagious* characteristics before and after they are eliminated.

Table 3: The p-values when removing the 4 most contagious characteristics

The removed characteristics	Acousticness	Danceability	Valence
p-value before elimination	0.7786	0.2631	0.0326
p-value after elimination	0.2631	0.0326	0.0205

* The p-values in the second row denote the non-parametric results before elimination of the characteristics of the corresponding column, while the third row denotes results after elimination.

From Table 3, we can draw a conclusion that *Mode*, *Acousticness*, *Danceability* and *Valence* are the most contagious characteristics among artists, while the *Mode* and *Acousticness* have the highest and the second highest level of *contagiousness*. The *Danceability* and *Valence* are the third and fourth most contagious characteristics, respectively. Since the smaller p-values represent larger significant difference, our experiment results are valid because: the elimination of our discovered *contagious* characteristics will lead to significant reduction of the p-value, indicating that characteristics have more concentrated distribution after the *contagious* characteristics are removed. The results is in correspondence with our definition of *contagiousness*.

4.4 Revolutionary Artists Based on Closeness Centrality

By *revolutionary artists*, we generally mean that their contribution to the field should be significant. Moreover, they ought to have great impact on others while receiving small influence from others. Inspired by the structure of the influence network G constructed in Section 4.1, we apply the evaluation of important nodes in discovering the revolutionary artists. To be more specific, we combine the in-closeness and out-closeness centrality to evaluate the significance of an artists in revolutionaries. Our goal is to discover the artists with small in-closeness centrality and large out-closeness centrality.

For the influence network $G(V, E)$, the closeness centrality $CC(i)$ of node v_i is the average farness to all the other nodes in the network, formulated in Eq. (13).

$$CC(i) = \frac{n-1}{\sum_{j \neq i} d(i,j)} \tag{13}$$

where n is the number of nodes in graph G.

Since closeness is a directional measure in the directed graph, the in-closeness centrality $CC_i(i)$ and the out-closeness centrality $CC_o(i)$ can be calculated accordingly. To evaluate the influence of the evolutionary artists who have large out-closeness centrality and small in-closeness centrality, the synthesized closeness centrality of artist v_i is defined in Eq. (14).

$$C(i) = CC_o(i) - CC_i(i) \tag{14}$$

The synthesized closeness centrality of the top 12 artists are shown in Fig. 11.

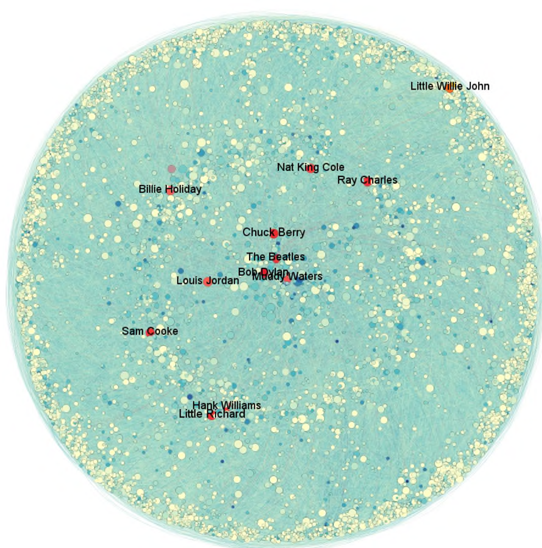


Fig. 11. The synthesized closeness centrality of the top 12 artists

The basic information of top 12 evolutionary artists are listed in Table 4.

Reader can easily find our results reliable according the name of these 12 artist. For instance, Dylan was awarded the Nobel Prize in Literature in 2016, Chuck Berry is one of the pioneers of Rock and Roll music, and The Beatles is regarded as the most influential band of all time.

4.5 Dynamic Process of Evolution

From *data_by_year* data set, we have known the evolution trend of the musical characteristics over time. Though the general trend is easy to observe by visualizing the data, deeper information

Table 4: The basic information of top 12 evolutionary artists

Artist ID	Name	Centrality	Artist ID	Name	Centrality
120521	Chuck Berry	0.2921	287604	Louis Jordan	0.2777
824022	Little Richard	0.2904	269972	Little Willie John	0.2731
66915	Bob Dylan	0.2838	549797	Hank Williams	0.2730
79016	Billie Holiday	0.2806	317093	Nat King Cole	0.2729
608701	Muddy Waters	0.2801	754032	The Beatles	0.2708
46861	Ray Charles	0.2783	238115	Sam Cooke	0.2702

* From left to right, top to bottom, the 12 artists are sorted according to their synthesized centrality.

could still be discovered. We are then motivated to analyze the frequency components of time series of each musical characteristics.

Discrete Fourier transform (DFT) could transform a sequence of samples with a finite length into a equally-spaced samples discrete-time Fourier transform with the same length [7]. Let $x[n]$ denote a time series. The DFT $X[k]$ of the series $x[n]$ is formulated in Eq. (15).

$$X[k] = \sum_{n=0}^{N-1} x[n]\mathrm{e}^{-j\frac{2\pi}{N}kn}, \quad k = 0, 1, 2, ..., N-1 \tag{15}$$

where N is the length of series $x[n]$.

An indicator σ is developed in Eq. (16), which measures the proportion of low-frequency components in the time series $x[n]$.

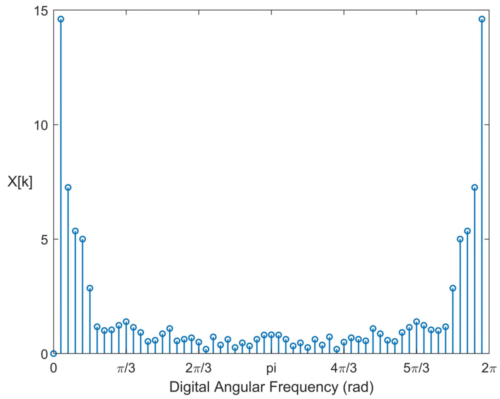
$$\sigma = \frac{2\sum_{m=1}^{4} |X[m]|^2}{\sum_{m=0}^{N-1} |X[m]|^2} \tag{16}$$

where N is the length of $x[n]$, $\frac{\sum_{m=0}^{N-1}|X[m]|^2}{N}$ is the energy of series $x[n]$. It should be noted that FFT (fast Fourier transform) is used instead of DFT when computing DFT by computers.

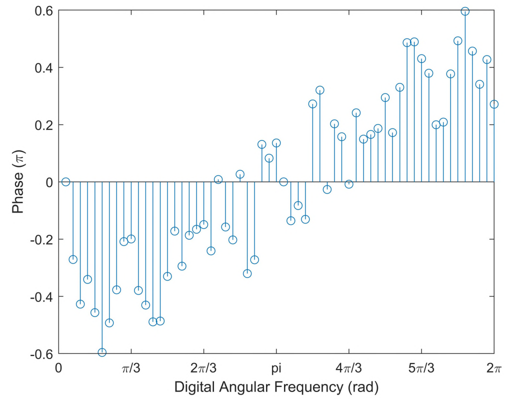
Eq. (16) is reasonable because $|X[0]| = 0$ (the DC component) according to data preprocessing, where $\hat{x}[n] = \frac{x[n]-\bar{x}[n]}{x[n]_{max}-\bar{x}[n]}$. $\bar{x}[n]$ and $x[n]_{max}$ denote the average and maximum value of series $x[n]$, respectively.

For computers, we apply IFFT (inverse fast Fourier transform) to the four low-frequency components illustrated in Eq. (16). The time series $x[n]$ is the data of *Acousticness* from 1955 to 2014 of the genre *Pop & Rock*. $N = 60$ is the length of $x[n]$. The results of FFT are shown in Fig. 12.

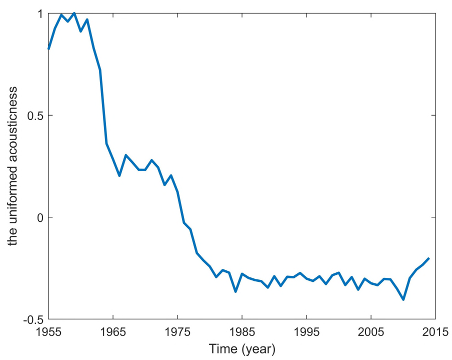
From Fig. 12, we can observe that the time series reconstructed from the low-frequency components can well represent the overall trend of the original time series. The result also indicates the the low-frequency components takes up more percentage in the time series of *Acousticness* of the genre *Pop & Rock*



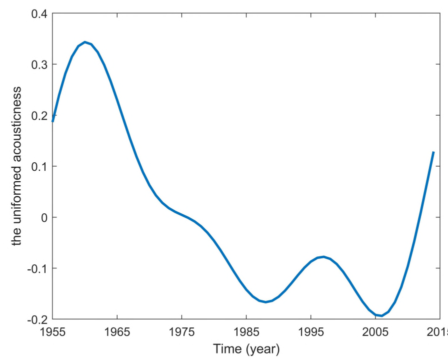
(a) Amplitude of the FFT result



(b) Phase of the FFT result



(c) The original time series



(d) The IFFT of low-frequency components of the original seires

Fig. 12. The process of reserving the low-frequency components of $x[n]$

We calculate σ for other typical characteristics of the genre *Pop & Rock*. The result is listed in Table 5.

Table 5: The σ values in for the time series of three typical characteristics

Characteristics	*Energy*	*Popularity*	*Liveness*
σ	0.7895	0.8481	0.4936

The results indicate that *Energy* and *Popularity* have larger proportion of low-frequency components, and have a smoother revolution over time. Meanwhile, *Liveness* has more high-frequency components. The dynamic process of evolution for other characteristics can be computed and analyzed in the exact same way.

5 Test the Model

In Section 4.3.1 we introduced the definition of *Central node* in each genre. Therefore, we will test the effectiveness of our *Central node* model in this section.

The within-genre similarity distribution of *Country* is given in Section 4.2.2, while the Gaussian fitting result is $N(0.85, 0.06)$. Different from the similarity distribution defined previously (where similarity denotes the similarity between every two nodes in the certain genre), we re-define the distribution of similarity values between the central node and other nodes within *Country*.

From the definition of central nodes, we can have an intuitive understanding that the overall similarity will become smaller after the re-definition, because the central node is likely to locate at the "center" of the genre. The new similarity distribution within *Country* is shown in Fig. 13.

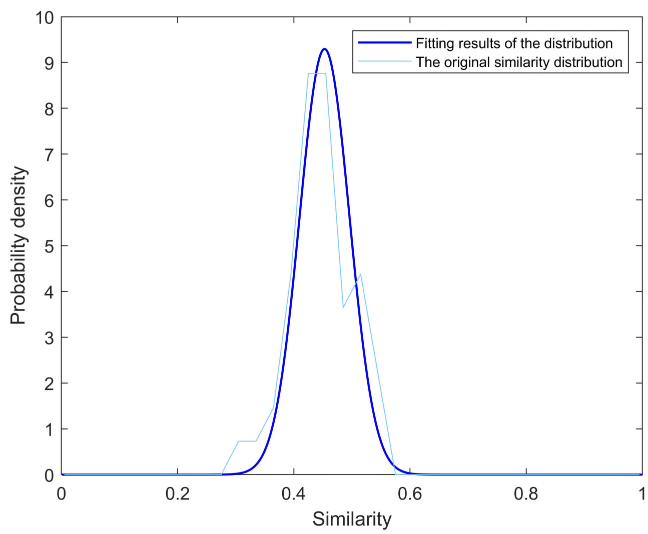


Fig. 13. The new similarity distribution.

The Gaussian fitting result for the new distribution is $N(0.45, 0.04)$. Compared with the original distribution (which is $N(0.85, 0.06)$), the expectation value of similarity is significantly reduced, meaning that our proposed model of central nodes is valid.

6 Strengths and Weaknesses

6.1 Strengths

1. For the measurement of influence, we have considered comprehensive aspects with the provided information in the influence relationship;
2. The MRDE framework we proposed for similarity measurement can well retain information of high-dimensional data;
3. Closeness centrality is used to detect the revolutionary artists. The idea of this method is tested to be effective by comparing the results with real-life information of corresponding

artists.

6.2 Weaknesses

1. The aggregation we used in the MRDE framework is not accurate enough for defining the actual similarity value from the distribution;

2. When we introduced the method for time series analysis, more state-of-the-art mechanism can be utilized.

7 Conclusion

In this paper, we proposed a detailed model in order to analyze the music objectively. The measurement of influence and quantification of similarity is proposed. Kruskal-Wallis test is performed to identify the influential characteristics in distinguishing genres, and the *contagious* nodes are detected using the similar method. Moreover, careful analyses of dynamic process evolution are done by transforming the characteristics' time series by discrete Fourier transform.

In the future, more state-of-the-art techniques could be utilized to analyze the evolution process. For example, the long short term memory (LSTM) framework to predict the time series, providing insight into the possible future trend of music evolution. Meanwhile, the external influence on the evolution of music could be examined or even quantified by calculating the correlation value (the Pearson correlation for instance) between the given the time series of external factors and a certain characteristics.

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A Valuable Work to Analyze the Influence of Music

KEYWORDS:

Complex Network

Bayes Error Rate

Closeness Centrality

Kruskal-Wallis Test

Discrete Fourier Transform

Understanding the influence of music is a skill that humans have always wanted to possess. Whether to explore the influence relationship between musicians or study the music similarity, the models and methods of complex network analysis are always valuable. By connecting influencers and followers and assigning weights to the corresponding edges, a directed influence network among artists can be established. Our work analyzes this network in detail, and obtains the quantified influence values.

At the same time, the similarity distribution within and between genres of is also analyzed. The results show that the similarity within genres is generally larger than that between-genre similarity. We obtained the representative vector of each genre that synthesized the music characteristics. The synthesized data has been tested to be indeed representative. Based on the synthesized result, we detected the main differentiating factors between different genres.

In fact, based on these data, we have also analyzed results other than similarity and influence. For example, we can analyze the revolutionary musicians by finding the important nodes by closeness centrality in the network. We have also established a model for analyzing dynamic influence indicators, and correspondingly, we have analyzed the dynamic process of the evolution for different genres of music in the past 100 years.

However, due to the limited data, our analysis of certain types of music may not be sufficient. With more data, we can have more effective information in the network analysis. Since our model mainly use the matrix-based linear calculation, the increased number of data will not significantly increase the computational complexity. In addition, when there is a larger order of magnitude of data, our model still retains computational advantages. For example, the proposed MRDE framework can take random tuple selection to obtain the same effective similarity distribution.