# MASTER'S THEOREM

If $f(n) \in \theta(n^d)$ or $f(n) = c * n^d$, where $d \geq 0$ in recurrence $T(n) = aT(n/b) + f(n)$ then,

$$T(n) \in \begin{cases} \theta(n^d) & , \text{if } (a < b^d) \\ \theta(n^d \log n) & , \text{if } (a = b^d) \\ \theta(n \log_b a) & , \text{if } (a > b^d) \end{cases}$$

1) $T(n) = 8T(n/2) + 1000n^2$

$a = 8$, $b = 2$, $f(n) = 1000n^2 = c * n^d$

$\therefore d = 2$

$b^d = 2^2 = \underline{4}$.

Hence $a > b^d$

$T(n) \in O(n^{\log_b a})$

$\therefore T(n) \in \underline{O(n^3)}$.

$\log_b a = \log_2 8$

$= \underline{3}$.

2.) $T(n) = 2T(n/2) + n^2$

$a = 2$, $b = 2$, $d = 2$

$b^d = 2^2 = 4$

Hence $a < b^d$

$T(n) \in \theta(n^d)$

$\Rightarrow T(n) \in \underline{\theta(n^2)}$.

3.) $T(n) = 2T(n/2) + 10n$

$a = 2$, $b = 2$, $d = 1$

$b^d = 2^1 = \underline{2}$.

Hence

$a = b^d$

$T(n) \in \theta(n^d \log n)$

$\Rightarrow T(n) \in \underline{\theta(n \log n)}$.