Assignment #04:

Q1 (a) $f(x) = x^2 - 4x + 5 \quad 0 \le x \le 3$, find the Riemann sum with $n = 6$, taking the simple points to be

(i) left endpoints (ii) Right endpoints (iii) midpoints

(b) Use the midpoint Rule with $n = 4$ to approximate the integral.

$$\int_0^{\pi} x \sin^2 x \, dx$$

Q2: (a) Express the integral as a limit of Riemann sum. Don't evaluate the limit.

$$\int_2^5 \left(x^2 + \frac{1}{x}\right) dx$$

(b) Express the limit as a definit integral on the given Interval.

(i) $$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{\sin x_i}{1 + x_i} \Delta x \; ; \; [0, \pi]$$

(ii) $$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{x_i^*}{(x_i^*)^2 + 4} \Delta x \; ; \; [1, 3].$$

Q3 (a) Find $\int_0^5 f(x)\,dx$ if $f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x + e^x & \text{for } x \ge 3 \end{cases}$

Q4 Evaluate the integral ~~using fundamental theorem of calculus~~ (2).

(i) $\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}}\,dx$ (ii) $\int_0^{\pi} f(x)\,dx$ where $f(x) = \begin{cases} \sin x & \text{if } 0 \le x \le \frac{\pi}{2} \\ \cos x & \text{if } 0 \le x \le 2 \end{cases}$

(iii) $F(x) = \int_{2\pi}^{x} \sqrt{1+\sec t}\,dt$ (iv) $\int_1^3 \frac{y^3 - 2y^2 - y}{y^2}\,dy$

(v) $\int_0^{\pi/3} \frac{\sin\theta + \sin\theta \tan^2\theta}{\sec^2\theta}\,d\theta$ (vi) $\int_0^2 |2x-1|\,dx$.

Q5 (a) Use part 1 of Fundamental theorem of calculus to find the derivative of

(i) $g(x) = \int_x^0 \sqrt{t+t^2}\,dt$. (ii) $h(x) = \int_0^{e^x} \ln t\,dt$.

(iii) $h(x) = \int_2^{\sqrt{x}} \frac{z^2}{z^4+1}\,dz$. (iv) $h(x) = \int_{2x}^{3x} \frac{y^2-1}{y^2+1}\,dy$

(b) What is wrong with the equation?

(i) $\int_{-1}^{2} \frac{4}{x^3}\,dx = -\frac{2}{x^2}\Big]_{-1}^{2} = 3/2$.

(ii) $\int_0^{\pi} \sec^2 x\,dx = \tan x\Big]_0^{\pi} = 0$.

Q6: Evaluate the Integral

(i) $\int \sin^5(2t)\cos^2(2t)\,dt$ (ii) $\int \frac{\sin^2(1/t)}{t^2}\,dt$

(iii) $\int \tan^3 x \sec^6 x\,dx$ (iv) $\int_0^{\pi/4} \sqrt{1-\cos 4\theta}\,d\theta$

(v)

Q7: (a) Evaluate the Integral by Trigonometric substitutions.

(i) $\int \frac{\sqrt{x^2-4}}{x}\,dx$ (ii) $\int \frac{x^3}{\sqrt{x^2+4}}\,dx$

(b) Evaluate.

(i) $\int x^2\sqrt{3+2x-x^2}\,dx$ (ii) $\int \frac{x^2+1}{(x^2-2x+2)^2}\,dx$

(iii) $\int_0^a \frac{dx}{(a^2+x^2)^{3/2}}$ $a>0$ (iv) $\int \frac{dx}{\sqrt{x^2+a^2}}$

Q8. Evaluate the Integral using partial fractions.

(i) $\int \frac{5x+1}{(2x+1)(x-1)^2}\,dx$ (ii) $\int \frac{dx}{(x+a)(x+b)}$

(iii) $\frac{x^5+9x^2+x+2}{(x^2+9)(x+2)^2}\,dx$ (vi) $\int \frac{x^3-2x^2+2x+5}{x^4+4x^2+3}\,dx$

(4)

Q.9 Use the substitution to express the integrand as a rational function and then evaluate the Integral.

(i) $\int \frac{dx}{(1+\sqrt{x})^2}$ (ii) $\int \frac{1}{\sqrt{x}-\sqrt[3]{x}}\,dx$

(iii) $\int \frac{\sec^2 t}{\tan^2 t + 3\tan t + 2}\,dt$ (iv) $\int \frac{\sin x}{\cos^2 x - 3\cos x}\,dx$

Q10. Evaluate the Integral.

(i) $\int \frac{4^x + 10^x}{9^x}\,dx$ (ii) $\int_0^1 x\sqrt{2-\sqrt{1-x^2}}\,dx$

(iii) $\int \frac{x \ln x}{\sqrt{x^2-1}}\,dx$

END

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