Last date of submission

Assignment 05

Q1. Explaine why each of the following integrals

is Improper

(i) $\int_1^2 \frac{x}{x-1} \, dx$

(ii) $\int_0^\infty \frac{1}{1+x^3} \, dx$

(iii) $\int_{-\infty}^\infty x^2 e^{-x^2} \, dx$

(iv) $\int_0^{\pi/4} \cot x \, dx$

Q2. Determine whether each integral is convergent or divergent. Evaluate those that are convergent

(i) $\int_3^\infty \frac{1}{(x-2)^{3/2}} \, dx$

(ii) $\int_{-\infty}^0 \frac{z}{z^4+4} \, dz$

(iii) $\int_1^\infty \frac{1}{x(\ln x)^2} \, dx$

(iv) $\int_1^\infty \frac{dx}{\sqrt{x} + x\sqrt{x}}$

(v) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

(vi) $\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{\sin\theta}} \, d\theta$

(vii) $\int_0^1 \frac{e^{1/x}}{x^3} \, dx$

(viii) $\int_0^1 r \ln r \, dr$

Q3 :— Sketch the region enclosed by the given curves

and find it's area.

(i) $y = x^2$, $y = 4x - x^2$

(ii) $y = \sec^2 x$, $y = 8 \cos x$, $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$

(iii) $y = x^4$, $y = 2 - |x|$

Q#4 (i) Sketch the region enclosed by the given curves. and find its area.

$y = \dfrac{x}{\sqrt{1+x^2}}$, $y = \dfrac{x}{\sqrt{9-x^2}}$, $x \geq 0$

(ii) $y = \dfrac{\ln x}{x}$, $y = \dfrac{(\ln x)^2}{x}$.

Q#5 Find the limit, use L'Hospital

Rules where appropriate.

(i) $\displaystyle \lim_{x \to 4} \frac{x^2 - 2x - 8}{x - 4}$ (ii) $\displaystyle \lim_{x \to (\pi/2)^+} \frac{\cos x}{1 - \sin x}$

(iii) $\displaystyle \lim_{\theta \to \pi} \frac{1 + \cos x}{1 - \cos x}$ (iv) $\displaystyle \lim_{x \to 0} \frac{\cos mx - \cos nx}{x^2}$

(v) $\displaystyle \lim_{x \to 1} \frac{x^a - 1}{x^b - 1}$, $b \neq 0$, (vi) $\displaystyle \lim_{x \to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right)$

(vii) $\displaystyle \lim_{x \to 0} (\csc x - \cot x)$.