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Model Predictive Control Design for Overtaking Maneuvers for Multi-Vehicle Scenarios

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Abstract— **The paper proposes the design of an MPC-based method for overtaking maneuvers of autonomous vehicles. The strategy incorporates a graph-based optimization algorithm, in which the probability of collisions with the surrounding vehicles is incorporated. The purpose of the graph search is to determine the road and velocity profile of the autonomous vehicle, which is guaranteed by the proposed MPC control. The novel algorithm is able to consider the multi-vehicle scenarios during the overtaking maneuver. The effectiveness of the method is presented in CarMaker simulation scenarios.**

I. Introduction and motivation

Overtaking is one of the most risky maneuvers for drivers due to the high velocity of the participants and the many unexpected events and uncertainties involved in the maneuver. Since overtaking maneuvers are a salient cause of accidents, a crucial goal of autonomous vehicle control is to design a strategy with which emergency situations are avoided.

The Model Predictive Control (MPC) strategy is a frequently applied method for the solution of this vehicle control problem, in which the multi-vehicle scenarios can be incorporated. A survey about the actual autonomous overtaking problems which summarizes the results in the MPC-based control approach is found in [1]. Although the MPC control has some weaknesses in the solution of the overtaking problem (e.g. the high computation requirements in the case of nonlinearities, the sensitivity of its performance on the model and the high number of tuning parameters), it provides a systematic design procedure and the constraints can easily be incorporated in the design. Moreover, an advantage of the selection of the MPC control is the incorporation of the future prediction in the design, which is crucial in the overtaking in multi-vehicle scenarios. A further advantage of the MPC control through the design of the acceleration and the steering change ratio was proposed by [2], which resulted in the vehicle motion close to the human behaviour. In [3] it was proved that the non-convex optimal control problem of vehicle positioning can be transformed into a convex quadratic program, which can be solved efficiently. A stochastic model predictive control in which the velocities of the surrounding vehicles are considered was designed by [4]. The work of [5] focused on the modeling of the decision process through a mixed logical dynamical system which is solved through model predictive control using mixed integer program formulation. A combination of continuation and generalized minimum residual method for MPC design was presented in [6]. The aim of this work is to optimize the system respect to the minimum safe distance and minimum time of the overtaking action. Moreover, a hierarchical control structure in which MPC and gain-scheduled control were combined was proposed in [7]. The MPC technique for path planning together with collision avoidance with the application of overtaking was proposed by [8]. Similarly, an MPC-based algorithm in which velocity and steering were optimized simultaneously without a priori knowledge about the obstacles was reported in [9]. The obstacles were detected through LiDAR, by which a safe and quick navigation was guaranteed without the need for a preloaded map of the environment.

In this paper an overtaking strategy based on the MPC framework is proposed. The novelty of the paper related to the existing results is the graph-based route and velocity search algorithm, whose results are incorporated in the MPC design. As a benefit of the proposed method, the multi-vehicle scenarios can be handled through the algorithm. Moreover, in the control design the comfort factor of the passengers in the autonomous vehicle is considered as limitations on the longitudinal jerk. The effectiveness of the method is presented in a CarMaker simulation example, in which two surrounding vehicles are considered.

The structure of the paper is the following. The selection method of the route and velocity of the autonomous vehicle based on a graph search algorithm is presented in Section II. Based on its results, Section III proposes the control design to perform the velocity profile and the selected route of the vehicle. Section IV demonstrates the effectiveness of the method through a multi-vehicle scenario. Finally, the results of the paper are concluded in Section V.

II. Graph-based route selection algorithm

The aim of this section is to show the graph-based route and velocity selection algorithm briefly. The algorithm is

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based on a map of the forthcoming road sections, in which the motions of the surrounding vehicles are predicted. In this paper the motion prediction is built on a probability-based method, whose details can be found in [10]. In this method three probability values are distinguished. P_{dec} is the probability that the vehicle decides to change lane and $1 - P_{dec}$ is the probability that the vehicle moves straight, $P_{lat}(y_{min}, y_{max}, t)$ refers to the probability that the lateral position of the vehicle at time t lies between y_{min} and y_{max} related to its selected route and $P_{long}(s_{min}, s_{max}, t)$ refers to the probability that the longitudinal position of the vehicle is between s_{min} and s_{max} at time t.

Since the driver has the possibility to decide about the overtaking maneuver, the probability of the combined longitudinal-lateral motion has two components. Both of them are expressed as geometric probabilities. First, the leading vehicle can be driven straightforward, whose probability is $1 - P_{dec}$. In this way the vehicle covers the road in the longitudinal straight direction with probability $P_{long}(s_i(t_i), s_j(t_j))$. In the lateral direction the probability is determined by the width of the vehicle, whose probability is 1. Thus, the combined probability is $(1 - P_{dec})P_{long}(s_i(t_i), s_j(t_j))$. Second, the driver can decide about the start of the overtaking maneuver, which is represented by P_{dec}. In this case the probability of the lateral motion is expressed through $P_{lat}(y_{min}, y_{max})$, which results in the combined probability $P_{dec}P_{long}(s_i(t_i), s_j(t_j))P_{lat}(y_{min}, y_{max})$. Finally, it is necessary to sum up the probability values of the two components, such as

$$\begin{aligned} &P(s_i(t_i), s_j(t_j), y_{min}, y_{max}) = \\ &\quad = (1 - P_{dec})P_{long}(s_i(t_i), s_j(t_j)) + \\ &\quad + P_{dec}P_{long}(s_i(t_i), s_j(t_j))P_{lat}(y_{min}, y_{max}). \end{aligned} \tag{1}$$

The probability can be computed to all time segments between t_i, t_j. It ensures that the vehicle is positioned between s_i, s_j, its longitudinal position has a normal distribution, the lateral position has a Gamma distribution, considering the probability of the beginning of the overtaking maneuver. It results in a map for the examined human-driven vehicle, which provides information about the probability of the forthcoming positioning. In multiple vehicle scenarios it is necessary to determine the areas of $P(s_i(t_i), s_j(t_j), y_{min}, y_{max})$ for all human-driven vehicles. Since the probability of a collision P_c increases with the number of human-driven vehicles, each probability from the prediction must be summed up, such as

$$P_c(t_i, t_j) = \sum_{l=1}^{N} \frac{P(s_i(t_i), s_j(t_j), y_{min}, y_{max})}{N}, \tag{2}$$

where N is the number of human-driven vehicles in the region of interest. The division with N guarantees that the value of $P_c(t_i, t_j)$ is between 0 and 1.

In the following a directed graph $G = (V, \bar{E})$ is built on the predicted road section, whose vertices V represent the possible route points and velocity profile of the vehicle. The edges $\bar{E}$ connect the vertices together, which represent the route and the acceleration of the vehicle. Through the directions of the edges the constraints of the route and the vehicle motion are considered.

The graph is combined with the probabilities of a collision at different velocities of the autonomous vehicle. Since the purpose of the route selection is to guarantee the minimum probability of a collision for the autonomous vehicle, the edges of the graph are weighted. The weight of the edge between vertices $V_i, V_j \in E(V_i, V_j)$, $j > i$ is formed as follows

$$S(i, j) = P_c(t_i, t_j) + S_c(i, j) + S_v(i, j), \tag{3}$$

where $P_c(t_i, t_j)$ is computed from (2). S_c is a weight which represents the difference from the center of the lane, while S_v is a weight which represents the difference from the velocity.

The idea of the motion priorization in the centerline is based on the potential field method in the lateral control design, see e.g. [11]. It means that the vehicle should be driven close to the centerline, by which the safety of the vehicle is guaranteed. The weight $S_c(i, j)$ is based on the difference in the lateral position from the center of the lane, such as

$$S_c(i, j) = S_c \cdot ((y(V_j) - y_c(V_j))^2 - (y(V_i) - y_c(V_i))^2) \tag{4}$$

where $S_c(i, j)$ is a design parameter for the scaling of the weight, which guarantees that $S_c(i, j)$ is between 0 and 1. $y(V_j)$ is the lateral position of V_j and $y_c(V_j)$ is the lateral position of the centerline, which is related to the lane of V_j. For example, if V_j is farther from the centerline as V_i then $S_c(i, j) > 0$, thus $S(i, j)$ is increased.

Generally, the weight $S_v(i, j)$ in (3) represents the difference of the velocity in V_j from the velocity in V_i. In the route selection strategy the constant velocity cruising has priority, if it is possible without the significant increased risk of a collision. Thus, the velocity change is penalized as follows

$$S_v(i, j) = S_v \cdot |v(V_j) - v(V_i)|, \tag{5}$$

where $S_v(i, j)$ is a design parameter for the scaling of the weight, which guarantees that $S_v(i, j)$ is between 0 and 1. Moreover, $v(V_j), v(V_i)$ are the selected velocities in the vertices V_i, V_j.

The weights $S(i, j)$ on the edges with the directed graph $G = (V, \bar{E})$ results in a map for the autonomous vehicle, which incorporates the probability of a collision with the surrounding vehicles. Thus, it is necessary to find the route which guarantees the minimum probability of a collision on the graph. It has been aided by the Dijkstra algorithm, see [12], whose role is to find the shortest path from the initial vertex to the target vertex.

The result of the graph search algorithm is the trajectory of the vehicle, regarding the route and the velocity on the predicted horizon. The trajectory is given through the vertices V_i, from which the lateral reference position and the reference velocity are generated.

Lateral reference position $r(k)$ is depicted by the closest point V_c from the autonomous vehicle on the selected edge of the graph, which is between V_i and V_{i+1}, where V_i is the last and V_{i+1} is the next vertex of the route, see Figure 1. Thus, $r(k)$ is computed through a linear interpolation, such as

$$r(k) = (y(V_{i+1}) - y(V_i))\frac{|V_i\bar{V}_c|}{|V_iV_{i+1}|} + y(V_i), \tag{6}$$

where $y()$ notes the lateral position of the vertex and $|()|$ is the length of the vector.

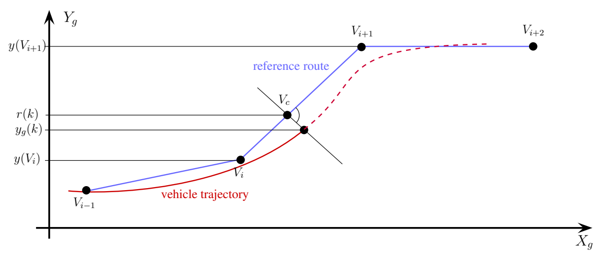


Fig. 1. Computation of $r(k)$

The reference velocity $v_{ref}(k)$ is yielded by the velocity profile. It is computed from the reference velocity profile, which is described by the vertices V_i. The reference velocity profile requires the acceleration in all V_i, such as $a(V_i)$. The velocity profile computation is based on the assumption that the acceleration of the vehicle $a(V_i)$ is between the vertices V_i and V_{i+1}. Therefore,

$$v_{ref}(i+1) = v_{ref}(i) + a(V_i)\Delta T, \tag{7}$$

where ΔT is the traveling time between V_i and V_{i+1}. Moreover, the distance $|V_iV_{i+1}|$ is computed as

$$|V_iV_{i+1}| = \frac{v_{ref}(i) + v_{ref}(i+1)}{2}\Delta T. \tag{8}$$

The reformulation of (7) and (8) leads to the the relation

$$v_{ref}(i+1) = \sqrt{v_{ref}(i)^2 + 2a(V_i)|V_iV_{i+1}|}. \tag{9}$$

Using (9) the reference velocity profile along the horizon can be computed. The $v_{ref}(k)$ current reference velocity in k is computed through an interpolation from the reference velocity profile.

III. Predictive optimal control design of trajectory tracking

The role of this section is to present the control design of the autonomous vehicle to guarantee the tracking of the selected route. During the control design of the longitudinal and lateral dynamics the MPC method is used. Since the route of the vehicle and the requested velocity profile of the vehicle are generated through the graph-based optimization strategy, the forthcoming reference signals of the control systems are known. Moreover, in the actuation of the vehicle some constraints must be guaranteed, e.g. the overshoot in the lateral position of the vehicle must be limited. Therefore, the MPC method provides an appropriate solution for the design of the control interventions. First, in the section the method of the derivation of the optimization problem in the MPC method is briefly introduced. Second, the longitudinal control design is presented, using the fundamentals of the MPC design. Third, the design of the lateral control is shown, using the MPC approach.

The control design of the overtaking maneuver is formed in a finite horizon length L_n ahead of the vehicle. The system is given in the form

$$x(k+1) = A_k x(k) + Bu(k), \tag{10}$$

where $x(k)$ is the state of the system, $u(k)$ is the control input and A_k is a time-dependent matrix which represents the dynamics of the system. The role of the control system is to guarantee the performances $z(k)$, which can be expressed through the following representation

$$z(k) = Cx(k) + D_1 r(k) + D_2 u(k), \tag{11}$$

where $r(k)$ represents the vector of the reference signals.

The performances can be formed in the horizon ahead of the vehicle in the following way

$$\begin{aligned}
Z(U) &= \begin{bmatrix} z(k) \\ z(k+1) \\ z(k+2) \\ \vdots \\ z(k+n) \end{bmatrix} = \begin{bmatrix} C \\ CA_k \\ CA_kA_{k+1} \\ \vdots \\ C\prod_{i=k}^{k+n} A_i \end{bmatrix} x(k) + D_1 \begin{bmatrix} r(k) \\ r(k+1) \\ r(k+2) \\ \vdots \\ r(k+n) \end{bmatrix} + \\
&+ \begin{bmatrix} D_2 & 0 & \cdots & 0 & 0 \\ CB & D_2 & \cdots & 0 & 0 \\ CA_kB & CB & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ C\prod_{i=k}^{k+n-1} A_iB & C\prod_{i=k}^{k+n-2} A_iB & \cdots & CB & D_2 \end{bmatrix} \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+n) \end{bmatrix} \\
&= \mathcal{A} + \mathcal{R} + \mathcal{B}U
\end{aligned} \tag{12}$$

where $\mathcal{A}$ contains the current states of the system with the varying system matrices, $\mathcal{R}$ is built by the reference values, $\mathcal{B}$ is built by the state matrices and U contains the control inputs. In the tracking problem it is necessary to minimize the following function

$$J(U) = \frac{1}{2}Z^T(U)QZ(U) + U^TRU, \tag{13}$$

where Q and R are weighting matrices. Substituting (12) into the function (13), the function is transformed as

$$\begin{aligned}
J(U) = &\frac{1}{2}U^T(\mathcal{B}^TQ\mathcal{B} + R)U + (\mathcal{A}^TQ\mathcal{B} + \mathcal{R}^TQ\mathcal{B})U + \\
&+ U^T(\mathcal{B}^TQ\mathcal{A} + \mathcal{B}^TQ\mathcal{R}) + \varepsilon = \\
&= \frac{1}{2}U^T\phi U + \beta^TU + U^T\gamma + \varepsilon
\end{aligned} \tag{14}$$

where ε contains all the constant components. Since ε is independent of the effect of U on $J(U)$, it can be canceled from the further optimization problem.

Through the minimization of the cost function $J(U)$ the tracking of the reference signals in $\mathcal{R}$ can be guaranteed. However, in practice several constraints must be guaranteed by the control system, as listed below.

A./ The control intervention has limits u_{min}, u_{max}, which bound the real actuation of the system. These constraints are given in the form

$$u_{min} \leq u(i) \leq u_{max}, \qquad \forall i = k \dots k+n. \tag{15}$$

B./ The states have constraints, which are not allowed to be reached or departed, e.g. the edges of the road. The form of the constraint is

$$A_{const} x(i) \leq b_{const}, \qquad \forall i = k \dots k+n, \tag{16}$$

where A_{const}, b_{const} are the matrices of the constraints.

C./ The stability of the controlled system is guaranteed by an inequality constraint. It represents that the states of the system $x(k+n)$ must be incorporated in a terminal set $\mathcal{X}_{term}$, in which the system is stable:

$$x(k+n) \in \mathcal{X}_{term}. \tag{17}$$

Finally, the cost function (14) and the constraints lead to a constrained minimization problem, such as

$$\min_{u(k) \dots u(k+n)} \left(\frac{1}{2} U^T \phi U + \beta^T U + U^T \gamma \right) \tag{18}$$

such that (15), (16), (17) constraints are also guaranteed. The solution of (18) leads to a series of control inputs $u(\cdot)$ on the horizon L_n. The current actuation requires $u(k)$, which is yielded by the online solution of the MPC problem during the vehicle cruising.

Longitudinal control design

The purpose of the longitudinal control is to guarantee the velocity profile of the vehicle, which is generated by the graph-based optimization algorithm, see Section II. Moreover, another criterion against the control system is to guarantee a comfortable travel for the passengers, which leads to the limitation of the longitudinal acceleration and the longitudinal jerk. For example, in public transportation scenarios the maximum of a_{long} in non-emergency situation is up to 1.5 m/s^2, while in the case of longitudinal jerk $|\dot{a}_{long}| < 3$ m/s^3 is acceptable, see [13]. Thus, the longitudinal control task is formed as a tracking problem with constraints on the states and control input of the vehicle.

The longitudinal dynamics of the vehicle in discrete time is formed as $a_{long}(k+1) = a_{long}(k) + T_s \dot{a}_{long}(k)$, where T_s is the sampling time. Furthermore, the longitudinal velocity of the vehicle is formed as $v_{long}(k+1) = v_{long}(k) + T_s a_{long}(k)$. The relationships result in the state-space representation of the system

$$x_{long}(k+1) = A_{long} x_{long}(k) + B_{long} u_{long}(k), \tag{19a}$$

$$z_{long} = C_{long} x_{long} + D_{long,1} v_{ref}(k) + D_{long,2} u_{long}(k), \tag{19b}$$

where $x_{long} = \begin{bmatrix} a_{long}(k) & v_{long}(k) \end{bmatrix}^T$ and the performance is $z_{long} = v_{ref} - v_{long}(k)$, in which $v_{ref}(k)$ is the current reference velocity. The value of $v_{ref}(k)$ results from the reference velocity profile, see (9). The control input of the system is the longitudinal jerk $u_{long}(k) = \dot{a}_{long}(k)$.

The constraints of the system are on the jerk and the acceleration signals. Moreover, it is necessary to limit the error of the velocity tracking to avoid dangerous overshooting. Thus, the constraints are formed as

$$v_{ref}(k) - e_{long} \leq v_{long}(k); \;\; v_{ref}(k) + e_{long} \geq v_{long}(k); \tag{20a}$$

$$-a_{long,lim} \leq a_{long}(k); \;\; a_{long,lim} \geq a_{long}(k); \tag{20b}$$

$$-\dot{a}_{long,lim} \leq \dot{a}_{long}(k); \;\; \dot{a}_{long,lim} \geq \dot{a}_{long}(k), \tag{20c}$$

where $a_{long,lim}$ and $\dot{a}_{long,lim}$ are the limits of the signals, which are determined by traveling comfort criteria. e_{long} is defined as the limit of the velocity tracking error.

In the following (19) and (20) are used to design the velocity tracking of the system using the MPC method. This technique is advantageous, since the control problem contains constraints on the control input and the state $a_{long}(k)$. Note that the control-oriented model (19) contains simplified longitudinal dynamics, whose input is the jerk signal. However, in practice the generation of the jerk requires the control of the driveline, see e.g. [14], [15].

The MPC optimization problem of the longitudinal control design with the performance and the constraints (20) is formed through (18):

$$\min_{\dot{a}_{long}(k) \dots \dot{a}_{long}(k+n)} \Bigg(\frac{1}{2} U_{long}^T \phi_{long} U_{long} + \beta_{long}^T U_{long} + \\ + U_{long}^T \gamma_{long} \Bigg). \tag{21}$$

where $U_{long} = \begin{bmatrix} \dot{a}_{long}(k) \dots \dot{a}_{long}(k+n) \end{bmatrix}^T$ is the series of control input and $\phi_{long}, \beta_{long}, \gamma_{long}$ result from the reformulation of (19).

Lateral control design

The role of the lateral control is to guarantee the maneuvering of the vehicle. It is based on the bicycle model of the vehicle, which is formed as

$$m(\ddot{y} + v_x \dot{\psi}) = F_{y,f} \cos\delta + F_x \sin\delta + F_{y,r}, \tag{22a}$$

$$J\ddot{\psi} = l_f (F_{y,f} \cos\delta + F_x \sin\delta) - l_r F_{y,r}, \tag{22b}$$

where l_f, l_r are the distance of the front and rear axles from the chassis center of gravity, $F_{y,f}, F_{y,r}$ are the lateral force on the front and the rear axles and F_x is the longitudinal force on the front steered wheels. Moreover, δ is the steering angle on the front wheels, ψ is the yaw angle, v_x is the longitudinal velocity and y is the lateral motion of the vehicle.

Since the dynamical equations of (22) describe the motion of the vehicle in its own coordinate system, it must be transformed into the global coordinate system, which is related to the road. Thus, the relation between the velocity of the vehicle in the global and the local coordinates is

$$\dot{x}_g = v_x \cos(\psi) - v_y \sin(\psi), \tag{23a}$$

$$\dot{y}_g = v_x \sin(\psi) + v_y \cos(\psi), \tag{23b}$$

where v_x, v_y are the longitudinal and lateral velocities in the local system, while $\dot{x}_g, \dot{y}_g$ in the global, respectively.

Through the formulation of the vehicle model the following further considerations have been taken.

A./ The lateral tyre force characteristics are linearized around zero lateral slip α_f, α_r values, hence $F_{y,f} = C_f\alpha_f, F_{y,r} = C_r\alpha_r$, where $C_f = \delta - \frac{v_y + L_f\dot{\psi}}{v_x}$ and $C_r = \frac{-v_y + L_r\dot{\psi}}{v_x}$ are the cornering stiffness parameters.

B./ The steering angle has small value, which leads to the approximations $\cos\delta \approx 1$ and $\sin\delta \approx \delta$.

C./ It is assumed that the orientation of the vehicle is close to the orientation of the road, which means that small tracking error is achieved: $\cos\psi \approx 1$, $\sin\psi \approx \psi$.

The control-oriented formulation of the lateral vehicle model is based on the dynamical relationships (22) with the previous simplifications, while the performance equation is:

$$\dot{x}_{lat,c} = A_{lat,c}x_{lat,c} + B_{lat,c}\delta, \tag{24a}$$

$$z_{lat,c} = C_{lat,c}x_{lat,c} + D_{lat,c,2}\delta, \tag{24b}$$

where the state vector is $x_{lat,c}^T = \begin{bmatrix} v_y & \psi & \dot{\psi} \end{bmatrix}$ and $z_{lat,c} = y_g$. The system (24) is transformed into a discrete form using the sampling time T_s. Moreover, in the control design the variation of the steering angle can have limitations due to the physical capacities of the steering actuator. Therefore, the following relationship is added to the discrete form of the state-space equation

$$\delta(k+1) = \delta(k) + T_s\Delta\delta, \tag{25}$$

where $\Delta\delta$ is the variation of the steering command. Thus, the final form of the discrete state-space representation is

$$x_{lat}(k+1) = A_{lat}x_{lat}(k) + B_{lat}u_{lat}(k), \tag{26a}$$

$$z_{lat}(k) = C_{lat}x_{lat}(k) + D_{lat,2}u_{lat}(k), \tag{26b}$$

where $x_{lat}(k)^T = \begin{bmatrix} v_y(k) & \psi(k) & \dot{\psi}(k) \end{bmatrix}$, $u_{lat}(k) = \Delta\delta(k)$ and $z_{lat}(k)^T = \begin{bmatrix} y_g(k) & \Delta\delta(k) \end{bmatrix}$.

The goal of the control design is to minimize the tracking error of the lateral control, while the control input $u_{lat}(k)$ is also reduced and some constraints are guaranteed. Thus, in the MPC control design problem the following performance must be minimized

$$z_{lat}(k) = C_{lat}x_{lat}(k) + D_{lat,1}r(k) + D_{lat,2}u_{lat}(k), \tag{27}$$

where $r(k)$ is the reference lateral position of the vehicle from the route. $r(k)$ results from the graph-based route selection algorithm.

In the MPC control of the lateral dynamics there are two constraints. First, the lateral position of the vehicle is constrained. It is necessary to limit its lateral error from the reference route to avoid the dangerous motion of the vehicle on the road. Moreover, the lateral position of the vehicle is constrained by the width of the road:

$$\max(y_{road,min}, r(k) - \frac{W}{2} - e_{lim}) \leq y_g(k), \tag{28a}$$

$$\min(y_{road,max}, r(k) + \frac{W}{2} + e_{lim}) \geq y_g(k), \tag{28b}$$

where e_{lim} is the maximum lateral tracking error, $y_{road,min,max}$ represents the limits of the road and W is the width of the vehicle for all $k, k+1, k+2 \dots$ on the prediction horizon. Second, the variation of the steering angle $\Delta\delta(k)$ and the steering actuation $\delta(k)$ are constrained, such as

$$-\Delta\delta_{lim} \leq \Delta\delta(k); \quad \Delta\delta_{lim} \geq \Delta\delta(k); \tag{29a}$$

$$-\delta_{lim} \leq \delta(k); \quad \delta_{lim} \geq \delta(k), \tag{29b}$$

where $\Delta\delta_{lim}$ and δ_{lim} are the limits of the actuation, which are determined by the physical properties of the actuator and comfort criteria. Furthermore, $e_{term,lat}$ is the tracking error, which is allowed in time $k+n$. It represents the bound of the terminal set.

$$y_g(k+n) \leq r(k+n) + e_{term,lat}, \tag{30a}$$

$$y_g(k+n) \geq r(k+n) - e_{term,lat}. \tag{30b}$$

The MPC optimization problem of the lateral control design with the performance (27) and the constraints (28), (29) and (30) is formed through (18):

$$\min_{\Delta\delta(k)\dots\Delta\delta(k+n)} \left(\frac{1}{2}U_{lat}^T\phi_{lat}U_{lat} + \beta_{lat}^T U_{lat} + U_{lat}^T\gamma_{lat} \right). \tag{31}$$

In the formulation $U_{lat} = \begin{bmatrix} \Delta\delta(k) \dots \Delta\delta(k+n) \end{bmatrix}^T$ is the series of control input and $\phi_{lat}, \beta_{lat}, \gamma_{lat}$ are yielded through the reformulation of (26a) and (27). The result of the MPC control is $\Delta\delta(k)$, which must be actuated on the system to guarantee the tracking of the selected route.

IV. Simulation results

The effectiveness of the proposed prediction, route selection algorithm and control strategy is illustrated through a multi-vehicle simulation example using CarMaker vehicle control simulation software. The examined scenario is presented in Figure 2, which contains three vehicles. In $t = 0$ the autonomous vehicle stops $120m$ behind *vehicle I* and $150m$ from *vehicle II* in the right-hand side lane. Moreover, the faulty *vehicle III* in the left-hand side lane stops $300m$ ahead of the autonomous vehicle. Since the reference velocity of the autonomous vehicle is set at $20m/s$, it accelerates and approaches the slower vehicles *vehicle I* and *vehicle II*. However, *vehicle I* starts to overtake *vehicle II*, which leads to difficulties in the overtaking maneuver of the autonomous vehicle.

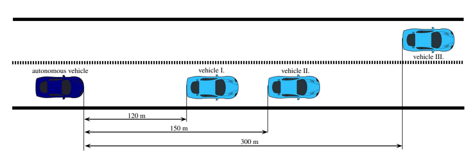


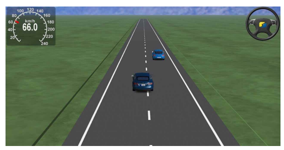
Fig. 2. Simulation example scenario

Figure 3 illustrates the different stages of the overtaking maneuvers of the autonomous vehicle. Moreover, some signals of the vehicle during the maneuvers are found in Figure 4. Figure 4(a) illustrates that the autonomous vehicle reaches the velocity target $20m/s$, but its motion must be modified due to the overtaking maneuver of *Vehicle I*, see Figure 3(a).

(a) Start of the first overtaking ($t = 31s$)

(b) End of the first overtaking ($t = 37s$)

(c) Bypassing *Vehicle III* ($t = 44s$)



(d) Start of the second overtaking ($t = 52s$)

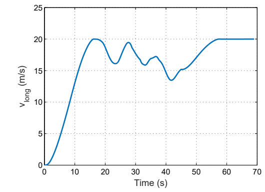
Fig. 3. Stages of the overtaking maneuvers

During the overtaking maneuver the velocity can also be increased slightly, until *Vehicle III* is approached. In Figure 3(b) the first overtaking maneuver is finished. Moreover, *Vehicle III* must be bypassed in the right-hand side lane, as illustrated in Figure 3(c). Since the target velocity of the autonomous vehicle is higher than the velocity of *Vehicle I*, a second overtaking maneuver must be performed, see Figure 3(d). It leads to the motion in the left-hand side lane and finally a motion back to the right-hand side lane.

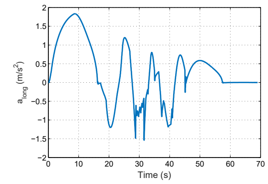
Vehicle dynamic and control signals of the autonomous vehicle are found in Figure 4. It can be seen that the longitudinal MPC control is able to track the target velocity $20m/s$ (Figure 4(a)), while the constraints are guaranteed, e.g. the limitation $\pm 2m/s^2$ on a_{long}. Moreover, the result of the lateral MPC control computation is the steering angle, which is shown in Figure 4(c). The resulting control input guarantees the tracking of the reference path with small error, which is computed by the graph-search algorithm, see Figure 4(e). Furthermore, a_{lat} has small value to reach the traveling comfort and it has smooth characteristics, see Figure 4(d).

V. Conclusions

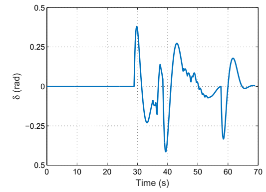
The paper has proposed a predictive control strategy for the design of overtaking maneuver in multi-vehicle scenarios. In the method a graph-based optimal route and velocity selection algorithm using a probability-based approach is applied. The route and velocity profile have been performed through MPC-based control strategies, in which some constraints are considered, e.g., the limits of the acceleration and jerk. Through the algorithm the motions of the surrounding vehicles can be incorporated in the safety design of the autonomous vehicle motion. The effectiveness of the method has been illustrated through an overtaking simulation example with three surrounding vehicles.



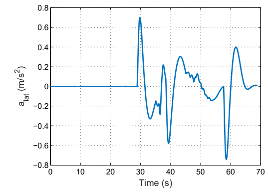
(a) v_{long}



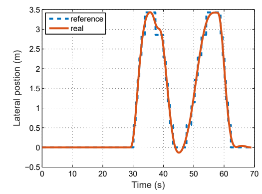
(b) a_{long}



(c) δ



(d) a_{lat}



(e) Lateral position of the vehicle

Fig. 4. Signals of the vehicle maneuvers

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