Nonlinear Model Predictive Control for Longitudinal and Lateral Dynamic of Autonomous Car

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Abstract—**Autonomous cars, when two essential variables influence tracks to determine their stability, namely position control and speed control. Speed is a control variable in longitudinal motion because it controls car movement along the X axis. Steering is a control variable in the lateral control system because it controls car movement along the Y axis. Some of the studies before, most of them only paid attention to one variable between speed and steering, and other variables are considered constant. Therefore, in this study, multivariable control was carried out using Nonlinear Model Predictive Control (NMPC) to determine the speed and steering of the car. Nonlinear Model Predictive Control (NMPC) predicts control inputs in the next few steps. The use of the predicted value results is to minimize the execution time required by the car control system. Because the system's complexity in autonomous cars is high, thus it is better to process data faster. The NMPC process produces a car's steering angle and speed value that must be achieved when tracking on a predetermined track. The results showed that the proposed control strategy could precisely set the vehicle to follow the target's trajectory.**

***Keyword*—autonomous car, lateral control, longitudinal control, nonlinear model predictive control**

I. INTRODUCTION

In these recent years, there has been a lot of research and development of autonomous vehicle technology ranging from air, sea and ground vehicles. Industry and academia have widely carried out research on autonomous cars in the past two decades to improve road traffic safety. Technology embedded in automotive vehicles is a significant factor that must be considered [1]. Some of the studies carried out on road terrain include a control that presents a two-layer controller for accurate lateral path tracking. Automatic vehicle control. The top screen controller results in the value of the steering angle of the front wheels. Implemented with Linear Time-Varying MPC (LTV-MPC), whose prediction and control range are both optimized offline with particle swarm optimization [2]. Other studies also discuss two-layer control, but the upper layer control produces the desired torque value in longitudinal movement. The lower layer control controls the actuator to produce the torque as desired. In this study, a combination system designed for lateral and longitudinal control works together to increase vehicle stability when tracking. Combining these controls uses the Predictive Control (MPC) Model [3]. Further research discusses longitudinal and lateral control combined with increasing stability and minimizing the error value between the path passed by the car and the reference path longitudinal control works together to increase vehicle stability when tracking. Combining these controls uses the Predictive Control (MPC) Model [4]. Further research discusses longitudinal and lateral control combined with increasing stability and minimizing the error value between the path passed by the car and the reference path [5].

In this study, the control design focus on the autonomous car by calculating the prediction of the control input value of the steering wheel angle and the speed value that must be achieved based on the car's condition when tracking. The contribution of this study is to build multivariable control in autonomous car in order to minimize lateral and longitudinal errors between the car and the reference path so that the car maneuvers well. A predictive control strategy based on the plant model gets the desired input control system. Because the plant model has high non-linearity properties, it will be more efficient to use non-linear control so as not to do linearization by assuming the parameters contained in the car. This study uses the Nonlinear Model Predictive Control (NMPC) control strategy as an autonomous car control method.

The next part of this paper will compile as follows. Part 2 describes autonomous cars' lateral, longitudinal, and dynamic yaw models, and Part 3 describes the design of a nonlinear controller model predictive controller (NMPC). Part 4 describes the results of multivariable steering and speed control with a predetermined road scenario. Part 5 concludes the results of the study.

II. VEHICLE MODELLING

Modelling on an autonomous car describes the modelling of the movement of its car or often called the kinematics model, and modelling in terms of the entire force acting on an autonomous car or often called a dynamics model [6].

A. Dynamic Model of Vehicle

A standard method of obtaining mathematical models of vehicle dynamics is using Newtonian equations of motion. Movements taken into account in vehicle dynamics are notified as degrees of freedom (DOF). The more axes the robot needs, the more degrees of freedom it has that allows it to access a more incredible amount of space. The dynamics equation of the autonomous car is described by the longitudinal, lateral, and yaw acceleration equations as follows:

$$\begin{aligned}
\ddot{x} &= -\dot{y}\cdot\dot{\theta} + \tfrac{1}{m}[\left(F_{x_{fl}} + F_{x_{fr}}\right)\cdot\cos(\delta) - \left(F_{y_{fl}} + F_{y_{fr}}\right) \\
&\quad \cdot\sin(\delta) + F_{x_{rl}} + F_{x_{rr}} - F_{\text{drag}}\,] \\
\ddot{y} &= -\dot{x}\cdot\dot{\theta} + \tfrac{1}{m}[\left(F_{x_{fl}} + F_{x_{fr}}\right)\cdot\sin(\delta) \pm \left(F_{y_{fl}} + F_{y_{fr}}\right) \\
&\quad \cdot\cos(\delta) + F_{y_{rl}} + F_{y_{rr}}] \\
\ddot{\theta} &= \tfrac{1}{I_z}[\left(F_{x_{fl}} + F_{x_{fr}}\right)\cdot\sin(\delta)\cdot l_f - \left(F_{y_{fl}} + F_{y_{fr}}\right) \\
&\quad .\cos(\delta)\cdot l_f + \left(F_{y_{rl}} + F_{y_{rr}}\right)\cdot l_r] \qquad (1)
\end{aligned}$$

where $\ddot{x}$, $\ddot{y}$, $\ddot{\theta}$ are longitudinal, lateral, and yaw acceleration. F_x and F_y are longitudinal and lateral forces. δ is the angle of the front wheels, then f_{drag} is an aerodynamic force opposite to the longitudinal force. l_f and l_r are the car's length from the COG point to the front and rear of the car.

B. Tire Model

The wheel model is an essential factor that must be calculated when controlling the car's movement. This model calculates the force generated between the road and the wheels using the principles of Newton's laws [7]. The force generated on the tire is divided into longitudinal and lateral forces. Based on Fig. 1, it can be seen that the force on the wheel is generated from the input control value given to the system and calculated through the wheel slip equation.

The longitudinal force that moves along the x-axis is influenced by tire slip. Tire slip is the ratio between the longitudinal speed of the car v_x with the rotational speed of the wheel and the rolling radius R, which is the ratio between the length of the static radius and the nominal radius, illustrated in (2):

$$TS = \frac{\omega R - V_x}{\omega R} \tag{2}$$

So, the equation for the longitudinal force on the wheel is defined by (3):

$$F_x = C_l . TS \tag{3}$$

where C_l is the value of the tire's cornering stiffness.

Slip angle is the difference between the front wheel angle and the path followed by the car, often called the velocity vector influencing the lateral force of the wheel. The following equation defines the slip angle:

$$\begin{aligned} SA_f &= \delta - \psi_{v_f} \\ SA_r &= -\psi_{v_r} \end{aligned} \tag{4}$$

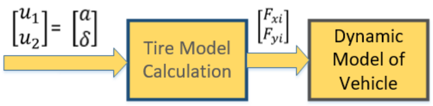


Fig. 1. Relation between input control and tire force

The value of cornering stiffness also affects the lateral force of the tire. Cornering stiffness is a function of the characteristics of the wheel itself for the road, so (5) obtained lateral force:

$$F_{y_f} = C_{\alpha f} \cdot SA_i \tag{5}$$

In this system, the output state will show velocity values for the x-axis (longitudinal), y-axis (lateral), and the z-axis (yaw), disturbance so that from the dynamics model described in (1), (6) will define the state:

$$dx = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \\ \dot{x_5} \\ \dot{x_6} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ \ddot{\theta} \\ \dot{x} \\ e_l \\ e_y \\ d \end{bmatrix} \tag{6}$$

In this study, the acceleration a and the front wheel angle δ are input control that will be given to the system to produce system output, output system also shows the magnitude of the lateral error value and the relative yaw angle, so the nonlinear state equation is defined as follows:

$$x = f \begin{pmatrix} \dot{y} \\ \ddot{\theta} \\ \dot{x} \\ e_l \\ e_y \\ d \end{pmatrix} + g \begin{pmatrix} a \\ \delta \end{pmatrix} \tag{7}$$

III. NONLINEAR MODEL PREDICTIVE CONTROL

The predictive control method proposed in this study is Predictive Control using the NMPC (Nonlinear Model Predictive Control) algorithm to determine position and velocity of the car. The purpose of implementing the NMPC control system on an autonomous car is that the car can perform trajectory tracking. Trajectory tracking is the car can follow the given trajectory with precision.

In order for the car to be able to follow the track well, the system uses the NMPC control algorithm. By taking back the state equation in (6), (8) defines nonlinear model prediction:

$$x(k+1) = f(x(k), u(k)) \tag{8}$$

where the value of the previous output state and the current output value depend on time changes that affect the predicted output state value as defined in equation above [8].

The acceleration of the wheel a_w and the steering angle δ are input control variables (u_1, u_2) which will be optimized through the objective function to produce an output so that output will be close to the reference value / the difference between the predicted output and the minimum reference. The following equation defined cost function (J) where the cost function in this problem is to minimize the error value between prediction state and current state:

$$\min_{u} J_N(x_u(k), u(k)) = \sum_{k=0}^{N-1} \ell(x_u(k), u(k)) \tag{9}$$

Subject to:

$$\begin{gathered} 0\,km/h \le \mu(t_k, X) \le 60\,km/h, t_k \in T_d \\ -0.2m \le \mu(t_k, X) \le 0.2m, t_k \in T_d \\ 0\,rad/s \le \mu(t_k, U) \le 200\text{ rad/s}, t_k \in T_d \\ -0.5\,rad \le \mu(t_k, U) \le 0.5\text{ rad}, t_k \in T_d \\ x(t_0) = x(0)\text{ given} \end{gathered} \tag{10}$$

This study provides constraints on calculating optimization controls on the output and input sides of the system [9]. At the output of the first system's constraint, there is speed. The maximum speed reached by the car is 60 km/h and at lateral deviation, which is the difference between the road points that the car must pass with the position of the car by 0.2 meters. In input control, the maximum limit on the wheel rotation speed of 200 rad/s and the range for the front angle limitation of the wheel is -0.5 radians to 0.5 radians.

The NMPC control method looks for the optimal value by minimizing the value of the cost function where the variable value limit is optimized fundamental so that the optimization calculation process does not expand. The control input value calculated in the optimization process is limited as in (8). The results of the calculation output will provide a new state output value at the closed-loop control system. The error value between the reference value and the feedback output state is the error value. The optimization will minimize the error value, so there will be an optimal value for the system. Fig. 2 depicts explanation of the closed loop control system. The system will continue to act until the car reaches the final tracking position.

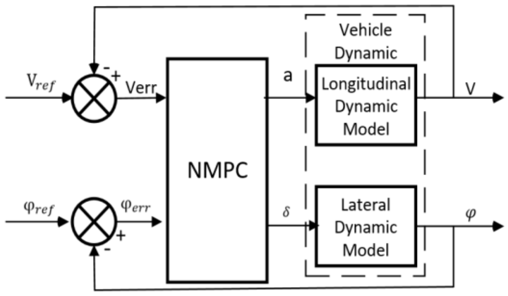


Fig. 2. Control architecture

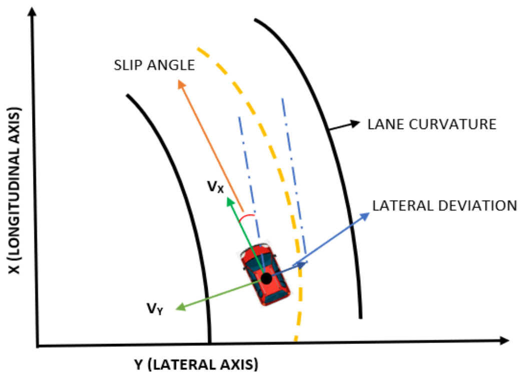


Fig. 3. Slip Angle and lateral deviation position

IV. Simulation Result

The test simulation uses the dynamics model described in (7) and the NMPC control design to calculate the value of the objective function in (9) against the constraints given in (10). Simulation of the NMPC multivariable control system on the autonomous car is carried out by signaling input in the form of a waypoint. The value of the waypoint will be used as a reference value that the car controller must achieve by calculating the input control value that must be given. Waypoint data is generated using the driving scenario designer on MATLAB. The road scenario is given shape of the number 8 to test the results of controlling the speed and position of the car.

A. Path tracking autonomous car control using NMPC with no disturbance

As explained before, the purpose of controlling the autonomous car system is to use prediction control to ensure that the car runs according to the predetermined waypoint. Simulations were carried out using MATLAB software to determine the magnitude of the lateral deviation and slip angle generated by the controller's influence on the system, as shown in Fig. 3. The lateral deviation is the difference between the car's center of gravity and the road's midpoint. The farther the distance between the center point of the car and the road line, the greater the lateral error generated. Slip angle is the angle difference between the direction the car is running and the velocity vector / Vx direction. The size of the slip angle is influenced by force exerted on the wheels and the influence of road conditions

The first test was carried out by assuming no measurement noise was contained in the system feedback output. The results obtained with values of control horizon is 10 and prediction horizon are 15 and 10 given in the following Fig. 4 and Fig. 5.

B. Path tracking autonomous car control using NMPC with disturbance (prediction horizon)

Interference with the system can be found in the measurement results, namely the presence of measurement noise from sensor readings. In this simulation, the noise provided is white noise 0.1. Henceforth, the output response will be compared with a system that is assumed to have no noise.

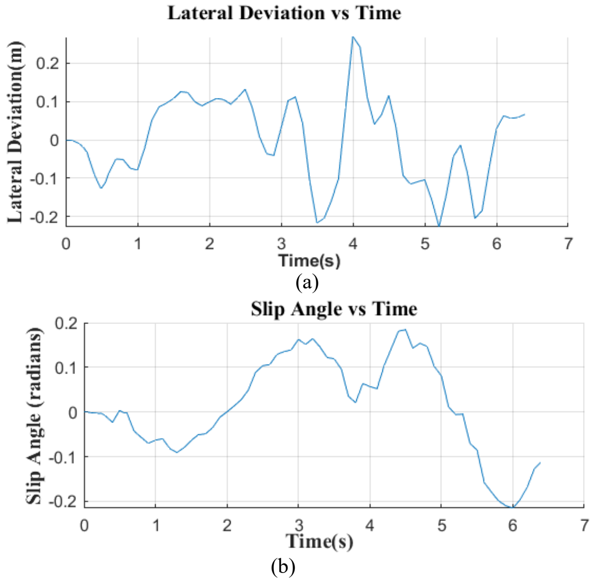


Fig. 4. System response when the prediction horizon value is 15

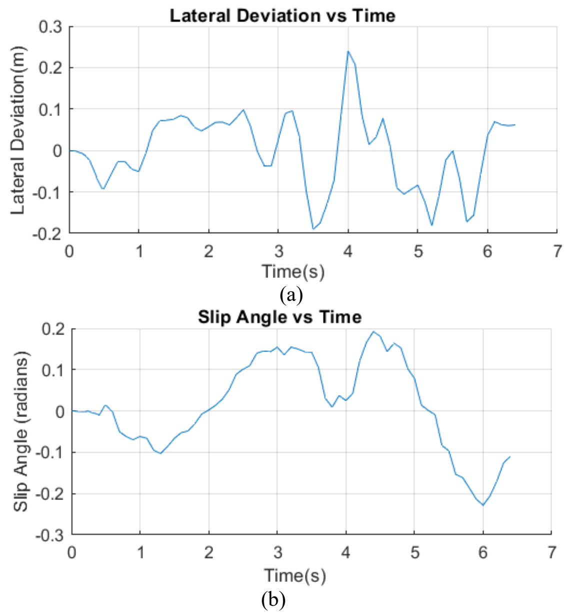
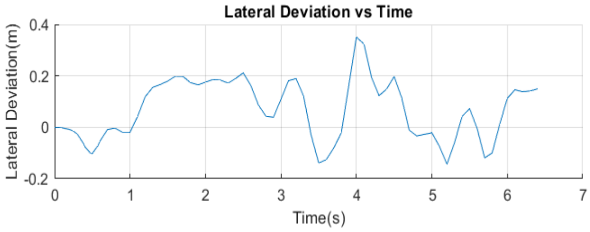
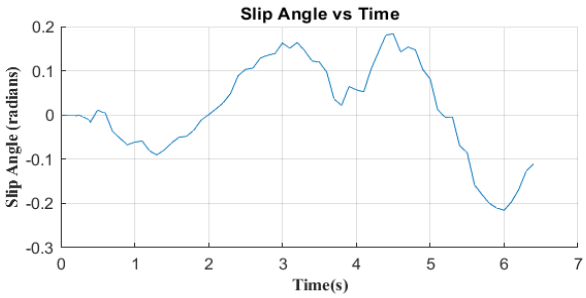


Fig. 5. System response when the prediction horizon value is 10

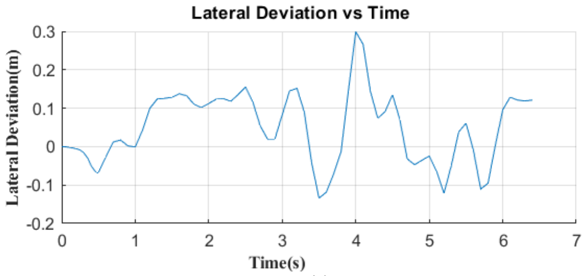


(a)

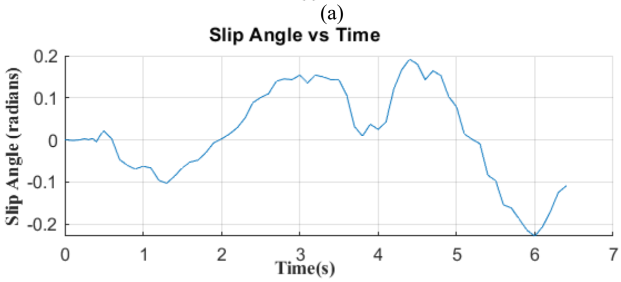


(b)

Fig. 6. System response when the prediction horizon value is 15



(a)



(b)

Fig. 7. System response when the prediction horizon value is 10

As we can see in Fig. 6 and Fig. 7, Based on the comparative response between the uninterrupted system and the system with interference, it can be seen that the measurement noise affects the controller's performance. Systems that have noise produce a more significant lateral error when compared to a system with no noise. The noise value of 0.1 affects the system's response, exceeding the lateral error-tolerant limit with a prediction horizon value of 10. However, it can be overcome if the prediction horizon value is reduced.

The magnitude of the prediction horizon value also affects the controller's accuracy in predicting the next output state. The more significant the horizon prediction value, the controller's accuracy level decreases, and it can be seen from the magnitude of the lateral error generated.

V. Conclusion

This study explains the car's lateral and longitudinal control system or often called car position and speed control, because these two variables are essential factors in maintaining the car's movement to stay at the waypoint or track that has been determined. Based on this explanation, the vehicle's dynamics are modeled into three degrees of freedom with the dynamic equation of longitudinal, lateral, and yaw accelerations.

Because the car dynamics model is Nonlinear, the Nonlinear Model Predictive Control (NMPC) control method is used. This method predicts the state model for several steps in the future by optimizing the system input control values, namely acceleration and steering angle. NMPC aims to increase execution time because autonomous cars require a fast system response in their control.

Based on the simulations that have been carried out, it can be proven that this control method is effective in achieving the expected control objectivity by determining the appropriate prediction horizon value to predict the plant model in the next few steps.

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