

**Example 8 :** Divide  $3x^2 - x^3 - 3x + 5$  by  $x - 1 - x^2$ .

**Solution :** Note that the given polynomials are not in standard form. To carry out division, we first write both the dividend and divisor in decreasing orders of their degrees. So, dividend =  $-x^3 + 3x^2 - 3x + 5$  and divisor =  $-x^2 + x - 1$ .

Division process is shown on the right side.

We stop here since degree (3) =  $0 < 2$  = degree ( $-x^2 + x - 1$ ). So, quotient =  $x - 2$ , remainder = 3.

Now,

$$\begin{aligned} \text{Divisor} \times \text{Quotient} + \text{Remainder} \\ &= (-x^2 + x - 1)(x - 2) + 3 \\ &= -x^3 + 3x^2 - 3x + 2 + 3 \\ &= -x^3 + 3x^2 - 3x + 5 \\ &= \text{Dividend} \end{aligned}$$

$$\begin{array}{r} x-2 \\ -x^2+x-1 \overline{) -x^3+3x^2-3x+5} \\ \underline{-x^3+x^2-x} \phantom{+5} \\ 2x^2-2x+5 \\ \underline{2x^2-2x+2} \\ 3 \end{array}$$

$(-x^2 + x - 1).$

In this way, the division algorithm is verified.

**Example 9 :** Find all the zeroes of  $2x^4 - 3x^3 - 3x^2 + 6x - 2$ , if you know two of the zeroes are  $\sqrt{2}$  and  $\sqrt{-2}$ .

**Solution :** Since two of the zeroes are  $\sqrt{2}$  and  $-\sqrt{2}$ ,  $(x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$  is a factor of the given polynomial.

First term of the quotient is  $\frac{2x^4}{x^2} = 2x^2$

Second term of the quotient is  $\frac{-3x^3}{x^2} = -3x$

Third term of the quotient is  $\frac{x^2}{x^2} = 1$

$$\begin{array}{r} 2x^2-3x+1 \\ x^2-2 \overline{) 2x^4-3x^3-3x^2+6x-2} \\ \underline{-2x^4} \phantom{+6x-2} \\ 4x^3-3x^2+6x-2 \\ \underline{-4x^3+8x^2-4x+4} \\ x^2-2x-2 \\ \underline{-x^2+2x-2} \\ 0 \end{array}$$