# Уравнения Лагранжа II рода.

№1.

1) $L = \frac{(1+q^2)\dot{q}^2}{2} - \frac{q^2}{2}$

$$\left.\begin{aligned}
&\frac{\partial L}{\partial \dot{q}} = (1+q^2)\dot{q} \\
&\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}}\right) = 2q\dot{q}^2 + (1+q^2)\ddot{q} \\
&\frac{\partial L}{\partial q} = \dot{q}^2 q - q
\end{aligned}\right] \Rightarrow \boxed{\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = q(\dot{q}^2+1) + \ddot{q}(1+q^2)}$$

2) $L = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + (x\dot{y} - y\dot{x})$

$$\left.\begin{aligned}
&\frac{\partial L}{\partial \dot{x}} = \dot{x} - y \\
&\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) = \ddot{x} - \dot{y} \\
&\frac{\partial L}{\partial x} = \dot{y}
\end{aligned}\right] \Rightarrow \boxed{\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = \ddot{x} - 2\dot{y}}$$

$$\left.\begin{aligned}
&\frac{\partial L}{\partial \dot{y}} = \dot{y} + x \\
&\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{y}}\right) = \ddot{y} + \dot{x} \\
&\frac{\partial L}{\partial y} = -\dot{x}
\end{aligned}\right] \Rightarrow \boxed{\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = \ddot{y} + 2\dot{x}}$$

3) $L = \frac{1}{2}\dot{u}\dot{v} + \frac{1}{\sqrt{uv}}$

$$\left.\begin{aligned} \frac{\partial L}{\partial \dot{u}} &= \frac{1}{2}\dot{v} \\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{u}}\right) &= \frac{\ddot{v}}{2} \\ \frac{\partial L}{\partial u} &= \frac{1}{u\sqrt{uv}} \end{aligned}\right] \Rightarrow \boxed{\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{u}}\right) - \frac{\partial L}{\partial u} = \frac{\ddot{v}}{2} - \frac{1}{u\sqrt{uv}}}$$

$$\left.\begin{aligned} \frac{\partial L}{\partial \dot{v}} &= \frac{\dot{u}}{2} \\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{v}}\right) &= \frac{\ddot{u}}{2} \\ \frac{\partial L}{\partial v} &= \frac{1}{v\sqrt{uv}} \end{aligned}\right] \Rightarrow \boxed{\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{v}}\right) - \frac{\partial L}{\partial v} = \frac{\ddot{u}}{2} - \frac{1}{v\sqrt{uv}}}$$

№2.

1) Вывод ф. L

см. д.з. "Функ. Л." №4

$$\boxed{L=\frac{\dot{x}^2}{2}(m_1+m_2)+\frac{m_2}{2}\left(\dot{a}^2+(a\dot{\varphi})^2+2\dot{x}(\dot{a}\cos\varphi-a\sin\varphi\dot{\varphi})\right)-m_2 g a\sin\varphi}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=\frac{\partial}{\partial t}\left((m_1+m_2)\dot{x}+m_2(\dot{a}\cos\varphi-a\sin\varphi\dot{\varphi})\right)=$$

$$=(m_1+m_2)\ddot{x}+m_2\left(\ddot{a}\cos\varphi-\dot{a}\sin\varphi\dot{\varphi}-(\dot{a}\sin\varphi\dot{\varphi}+\right.$$

$$\left.+a\cos\varphi\dot{\varphi}^2+a\sin\varphi\ddot{\varphi})\right)=(m_1+m_2)\ddot{x}+m_2\left((\ddot{a}-a\dot{\varphi}^2)\cos\varphi-\right.$$

$$\left.-(2\dot{a}\dot{\varphi}+a\ddot{\varphi})\sin\varphi\right)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\varphi}}\right)-\frac{\partial L}{\partial \varphi}=\frac{\partial}{\partial t}\left(m_2 a^2\dot{\varphi}-m_2\dot{x}a\sin\varphi\right)+m_2\dot{x}(a\sin\varphi+$$

$$+a\dot{\varphi}\cos\varphi)=m_2 2a\dot{a}\dot{\varphi}+m_2 a\ddot{\varphi}-m_2\ddot{x}a\sin\varphi-$$

$$-m_2\dot{x}\dot{a}\sin\varphi-m_2\dot{x}a\cancel{\cos\varphi\dot{\varphi}}+m_2\dot{x}(a\sin\varphi+$$

$$+\cancel{a\dot{\varphi}\cos\varphi})=m_2\left(a(2\dot{a}\dot{\varphi}+\ddot{\varphi})-\sin\varphi(\ddot{x}a-\dot{x}\ddot{a}+\dot{x}a)\right)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{a}}\right)-\frac{\partial L}{\partial a}=\frac{\partial}{\partial t}\left(m_2\dot{a}+2\dot{x}\cos\varphi\right)-m_2\dot{\varphi}^2a+2\dot{x}\sin\varphi\dot{\varphi}+m_2 g\sin\varphi$$

$$=m_2\ddot{a}+2\ddot{x}\cos\varphi-2\dot{x}\cancel{\sin\varphi\dot{\varphi}}-m_2\dot{\varphi}^2a+2\dot{x}\cancel{\sin\varphi\dot{\varphi}}+m_2 g\sin\varphi=$$

$$=m_2(\ddot{a}-\dot{\varphi}^2a+g\sin\varphi)+2\ddot{x}\cos\varphi$$

3) Дано:

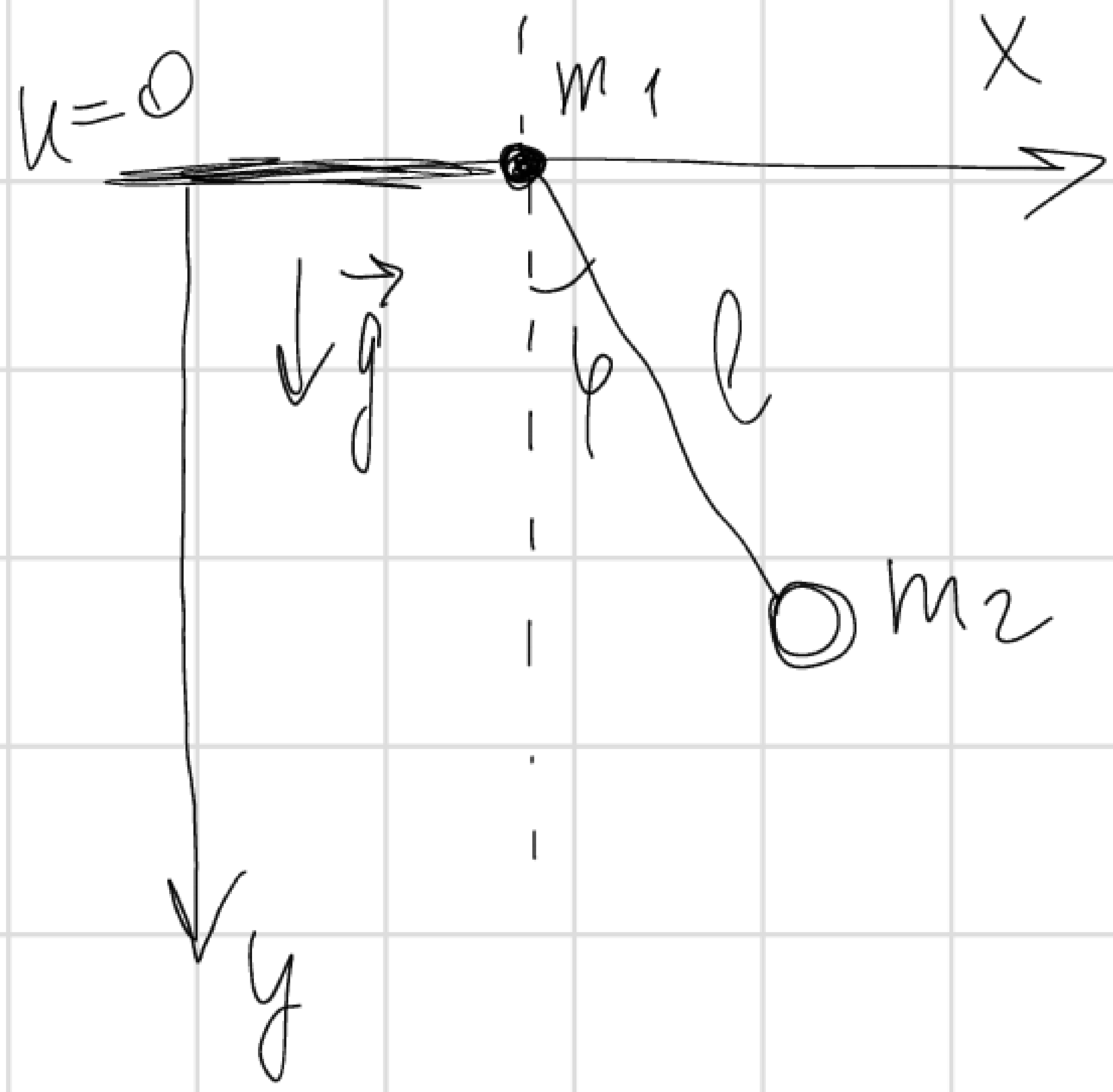
$m_1, m_2, g$

$L$, ур. Л II - ?

Решение:

Ур-е связи:

$$\begin{cases} y_1 = 0 \\ x_2^2 + y_2^2 = l \end{cases}$$




кол-во обобщ. коорд.

$S = 2\cdot 2 - 2 = 2$

$f(x, \varphi) = 0$

пусть $l$ - const, а то задолбалась уже.

3-и дв-е:

$$\begin{cases} x_1 = x(t) \\ y_1 = 0 \\ x_2 = x_1 + l\sin\varphi(t) \\ y_2 = l\cos\varphi(t) \end{cases}$$

$$T = \sum_i T_i = \frac{m_1}{2}\left(\dot{x}_1^2 + \dot{y}_1^2\right) + \frac{m_2}{2}\left(\dot{x}_2^2 + \dot{y}_2^2\right) =$$

$$= \frac{m_1}{2}\dot{x}^2 + \frac{m_2}{2}\left((\dot{x} + l\cos\varphi\,\dot{\varphi})^2 + (l\sin\varphi\,\dot{\varphi})^2\right) = \frac{\dot{x}^2}{2}(m_1 + m_2) + m_2\left(\dot{x}l\dot{\varphi}\cos\varphi + (l\dot{\varphi})^2\right)$$

$$U = \sum_i U_i = m_1 g y_1 + m_2 g y_2 = m_2 g l\cos\varphi$$

$$\boxed{L = \frac{\dot{x}^2}{2}(m_1+m_2) + m_2\left(\dot{x}l\dot{\varphi}\cos\varphi + (l\dot{\varphi})^2\right) + m_2 g l\cos\varphi}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) + \frac{\partial L}{\partial x} = \frac{\partial}{\partial t}\left(\dot{x}(m_1+m_2) + m_2 l\dot{\varphi}\cos\varphi\right) + 0 = \ddot{x}(m_1+m_2) + m_2 l\left(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi\right)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) + \frac{\partial L}{\partial \varphi} = \frac{\partial}{\partial t}\left(m_2 l\dot{x}\cos\varphi + 2l\dot{\varphi}\right) - m_2 l\dot{x}\dot{\varphi}^2\sin\varphi - m_2 g l\dot{\varphi}\sin\varphi =$$

$$= m_2 l\left(\ddot{x}\cos\varphi - \dot{x}\dot{\varphi}\sin\varphi\right) + 2l\ddot{\varphi} - m_2 l\sin\varphi\,\dot{\varphi}\left(\dot{x}\dot{\varphi} + g\right)$$