Lecture-12

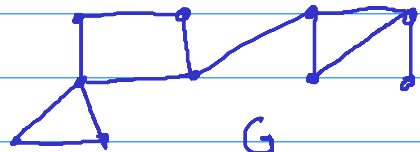
connectivity / seperable

↙ Vertex ↘ edges

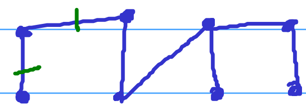
Cut - vertex
- if exist → seperable
- if not → non-seperable

Block / End block

<u>Th</u>. Every connected graph containing cut-vertices has at least 2 end blocks.



G



G_1

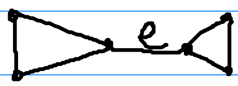
<u>Edge</u>:

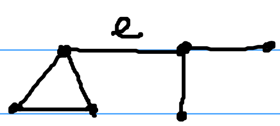
Cut-edge → Bridge

e (uv) in G is bridg if G-e is disconnected

$$\kappa(G-e) > \kappa(G)$$

$$\rightarrow \quad \kappa(G-e) = \kappa(G) + 1$$





Cut-vertex: every vertex of a connected graph G can not be a cut-vertex.

→ All the edges of a connected graph can be a bridge (cut edge).

Theorem: $e \in E(G)$ is bridge iff e lies on no cycle.

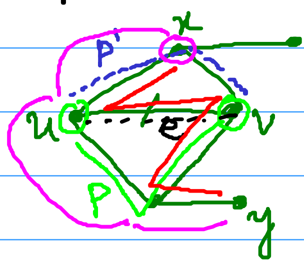
(e is not a bridge ⟺ e lies on cycle) ✓

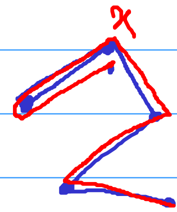
Part-I: let G is given connected graph

$e = uv \in E(G)$ is not a bridge.

⟹ $G-e$ is connected

⟹ ∃ uv-path 'P' in $G-e$





→ adding 'e' to P gives a cycle.

→ $e \in$ cycle 'C' in G

Part-II :- $e = uv \in$ cycle 'C' in G

⟹ ∃ uv path not containg 'e'

(to show e is not a bridge or $G-e$ is connected)

let $x, y \in V(G)$, G is connected

⟹ ∃ x-y-path 'Q'

Cases:

① If $e \notin Q$ (xy path)

② If $e \in Q$

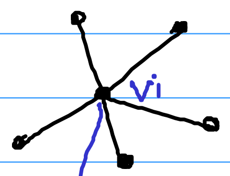
$G - e$ is connected

take case-② $e \in Q$ then by replacing 'e' in Q by uv path 'P' gives x-y walk

$\Rightarrow$ a path x-y

$\rightarrow$ $G - e$ is connected

Tree: An Acyclic Connected graph (every edge is a bridge)

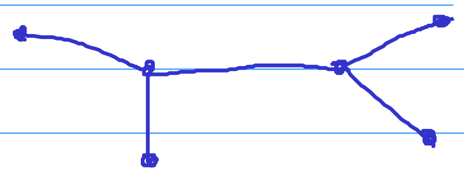


$K_{1,n-1}$ (or $K_{1,t}$)

$\deg(v_i) = t$

Centred vertex

Doublestar



tree with exact 2 vertices which are not leaves

Properties:

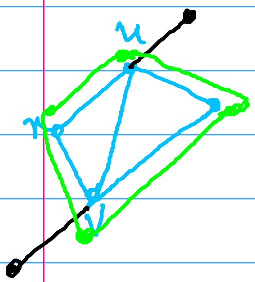
1) G is a tree iff every two vertices of G are connected by unique path.

Part - I : Let G is tree
(Neccessary)

$\rightarrow$ G is connected

$\rightarrow$ $\exists$ uv path ($\forall$ $uv \in V(G)$)

If there are more than one path (let 2)



$\rightarrow$ G contains a cycle.

$\rightarrow$ A contradiction

$\Rightarrow$ uv path is unique

Part - II (Sufficient) :- let G is given graph such that every pair of vertices are connected by unique path.

$\rightarrow$ G is connected

I) G has a cycle (C)

II) G don't contain cycle

$\Rightarrow$ every two vertices in G has 2 diff paths

Contradiction

$\rightarrow$ G has no cycle

G is a tree.