Lecture - 19 12. Feb. 21

Matrix Representation

1. Adjacency Matrix
2. Incidence Matrix
3. Cut-set Matrix
4. Circuit (Cycle Matrix)
5. Path matrix

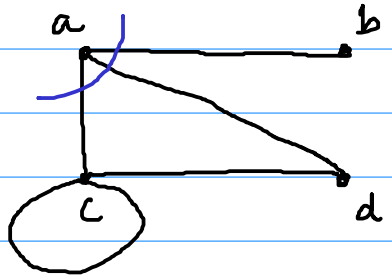
(1) Adjacency Matrix: A.M. of G with n vertices and no parallel edges is n x n binary (0-1) matrix

$M(G) = [m_{ij}]$

s.t. $m_{ij} = 1$ if $\exists e(v_i, v_j)$

$= 0$ otherwise

	v_1	v_2	...	v_n
v_1	.	.		.
v_2	.	.		.
⋮				
v_n	.	.		.



	a	b	c	d
a	0	1	1	1
b	1	0	0	0
c	1	0	1	1
d	1	0	1	0

(2) Incidence Matrix :

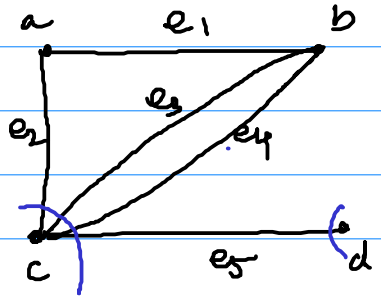
If G, $|V| = n$, $|E| = m$, without self-loop

then incidence Matrix of G, is $n \times m$ matrix (binary)

$$A(G) = [a_{ij}]$$

s.t. a_{ij} = e_i —— on v_i

$$a_{ij} = \begin{cases} 1 & \text{if } j^{th} \text{ edge is incident on } i^{th} \text{ vertex} \\ 0 & \text{otherwise} \end{cases}$$



identical columns due to Parallel edges

$$A(G) = \begin{array}{c|ccccc} & e_1 & e_2 & e_3 & e_4 & e_5 \\ a & 1 & 1 & 0 & 0 & 0 \\ b & 1 & 0 & 1 & 1 & 0 \\ c & 0 & 1 & 1 & 1 & 1 \\ d & 0 & 0 & 0 & 0 & 1 \end{array}$$

—— x ——

Notes: (Adjacency):

1) If principal diagonal is all zero → there is no self loop

2) it can't represent || edges

3) for simple graph degree of vertex is no. of 1's in corresponding row.

4) for disconnected graph G (with q, h components) adjacency Matrix

$$M(G) = \begin{bmatrix} M(g) & 0 \\ 0 & M(h) \end{bmatrix}$$

II Note (Incidency Matrix):

1) it can't represent self loops

2) deg. of Vi is no. of 1's in that row

3) the row with all 0's stand for isolated vertex

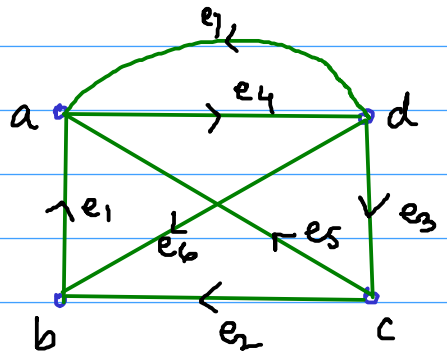
4) Incidence Matrix for disconnected graph G with q, h components

$$A(G) = \begin{bmatrix} A(g) & 0 \\ 0 & A(h) \end{bmatrix}$$

2(a) Incidence Matrix (Di-graph)
If G a di-graph

$$A(G) = [a_{ij}]$$

$$a_{ij} = \begin{cases} 1 & \text{if } e_j \text{ is outgoing edge from } v_i \\ 0 & \text{if } e_j \text{ is not incident on } v_i \\ -1 & \text{if } e_j \text{ is incident into } v_i \end{cases}$$



	e_1	e_2	e_3	e_4	e_5	e_6	e_7
a	-1	0	0	1	-1	0	-1
b	1	-1	0	0	0	-1	0
c	0	1	-1	0	1	0	0
d	0	0	1	-1	0	1	1