Finite complement topology

X is nonempty set

$\mathcal{T} = \{\phi, G \subseteq X : G^c \text{ is finite}\}$

is a topology

$\phi^c = X$

① $\phi \in \mathcal{T}$ [Given]

$X \in \mathcal{T}$ [$X \subseteq X$, $X^c = \phi$, which is finite]

$\{G_i : i \in I\} \subset \mathfrak{J} \Rightarrow \bigcup \{G_i : i \in I\} \in \mathfrak{J}$

$\Downarrow$

$G_i \in \mathfrak{J}, \ \forall i \in I$

$\Downarrow$

$G_i \subseteq X$ & G_i^c is finite

$\Downarrow$

$\bigcup\{G_i : i \in I\} \subseteq X$

$\bigcap\{G_i^c : i \in I\}$ is finite

(i) $\bigcup\{G_i : i \in I\} \subseteq X$

(ii) $\left[\bigcup\{G_i : i \in I\}\right]^c$ is finite

$\Downarrow$

$\bigcap\{G_i^c : i \in I\}$ is finite

$\bigcup\{G_i : i \in I\} \in \mathfrak{J}$

(iii) $G_1, G_2 \in \mathcal{T} \Rightarrow G_1 \cap G_2 \in \mathcal{T}$

$\Downarrow$

$G_1, G_2 \subseteq X \Rightarrow G_1 \cap G_2 \subseteq X$

$G_1, G_2 \in \mathcal{T} \Rightarrow G_1^c \,\&\, G_2^c$ are finite

$\Rightarrow G_1^c \cup G_2^c$ is finite

$\Rightarrow (G_1 \cap G_2)^c$ is finite

$\Rightarrow G_1 \cap G_2 \in \mathcal{T}$

$\mathcal{T} = \{\phi, G \subseteq X ; G^c \text{ is finite}\}$

is a topology

finite complement topology

Cofinite topology $\simeq$

X taken as a finite set

$X = \{a, b, c\}$

$\mathcal{T} = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\} \{b,c\}$
$\{a,c\}, X\}$

$G \subseteq X$ → finite
G^c is finite

G^c is finite

discrete topology

X is finite ⇒ cofinite topology is discrete

$X = \mathbb{N}$

$\mathcal{T} = \{\phi, \mathbb{N}, \{n, n+1, n+2, \dots\} ; n \in \mathbb{N}\}$

$\Downarrow$ topology

$\phi \neq \{1, 2, 3, \dots, n-1\}$

finite set

a finite topology

but not discrete

$X = \mathbb{N}$

$\mathcal{T} = \{\phi, \mathbb{N}, \{1, 2, \dots, n\}\}$

cofinite ??

topology

$\{1,2,3,4\} \in \mathcal{T}$

$\{5,6,7,\dots\}$

Not cofinite

$X = \mathbb{N}$

$\mathcal{T} = D \rightsquigarrow$ discrete

$E \in D$

$\{2, 4, 6, 8, 10, \ldots\} \in D$

Not closed

$\{1, 3, 5, 7, \ldots\}$ is not finite.

X = Finite

Co-finite $\equiv$ Discrete topo

$X \equiv$ infinite

infinite

Cofinite $\times$ Disc

$(X, \mathcal{T}) \rightarrow$ cofinite topology

$\rightarrow$ $X \rightsquigarrow$ is any nonempty set

$\mathcal{T}$ is a topology on X s.t

$\{x\} \in \mathcal{T}$, $\forall x \in X$

nature ??

$X = \{a, b, c\}$

$\{a\}, \{b\}, \{c\} \in \mathcal{T}$

$\phi, X \in \mathcal{T}$

$\{a, b\} = \{a\} \cup \{b\}$

$\{b, c\} = \{b\} \cup \{c\}$, $\{a\} \cup \{c\} = \{a, c\}$

$\mathcal{T} = P(X)$

$\downarrow$ Discrete

Ex X

$\mathcal{T} = \{\phi, G \subseteq X : G^c \text{ is Countable}\}$

topology [Yes]

Co-Countable topology

$X \to \{1, 2, \dots, n\}$

$X \to \mathbb{N}$

ϕ

$\Downarrow$

Finite OR bijective correspondence with $\mathbb{N}$

X = Countable set

Co-countable topology is discrete topology

$G \subseteq X \Rightarrow G^c$ is countable

countable

$X = \mathbb{R}$

$\tau = \{\emptyset, \mathbb{R}\} \rightarrow$ indiscrete

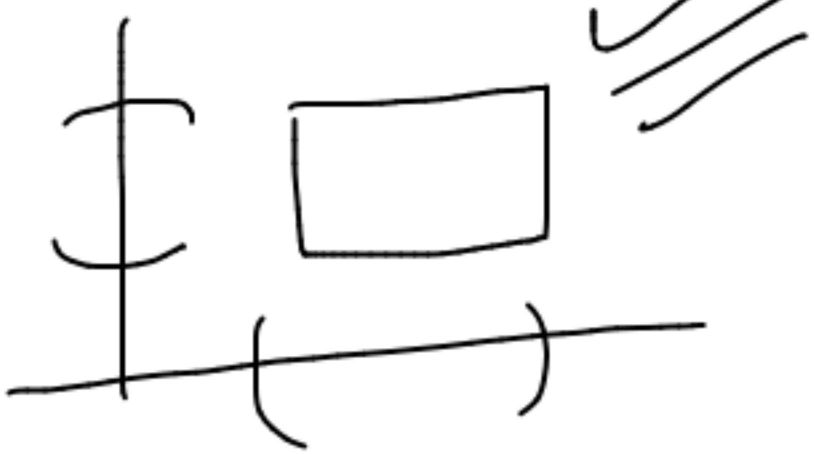
$\tau = P(\mathbb{R}) \rightarrow$ discrete

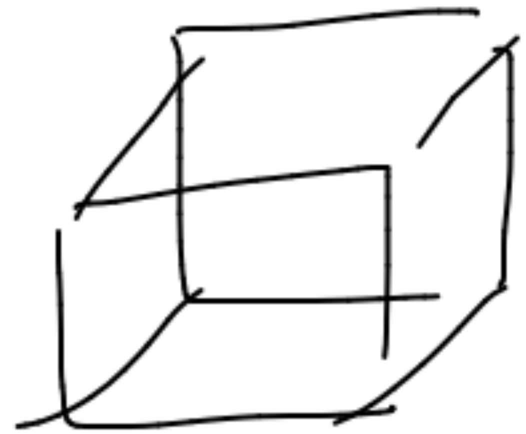
(a, b)

$[a, b)$

$(a, b]$

$[a, b]$





$X = \mathbb{R}$

$\tau = \{ G \subseteq \mathbb{R} : \forall x \in G \ \exists (a,b) \text{ s.t } x \in (a,b) \subseteq G \}$

① $\phi \in \tau$ $\qquad \phi \subseteq \mathbb{R}$

$\mathbb{R} \in \tau$

$x \in \mathbb{R}$

$\epsilon > 0$

$x \in (x-\epsilon, x+\epsilon) \subset \mathbb{R}$ $\quad O$

$\mathbb{R} \in \tau$

$X = \{a, b, c\}$

$T = \{\phi, X, \{a\}\}$

$T = \{\phi, G \subseteq X : G^c \text{ is finite}\}$