SECTION 1.3 Predicates and Quantifiers

The reader may find quantifiers hard to understand at first. Predicate logic (the study of propositions with quantifiers) is one level of abstraction higher than propositional logic (the study of propositions without quantifiers). Careful attention to this material will aid you in thinking more clearly, not only in mathematics but in other areas as well, from computer science to politics. Keep in mind exactly what the quantifiers mean: $\forall x$ *means "for all* x*" or "for every* x*," and* $\exists x$ *means "there exists an* x *such that" or "for some* x*." It is good practice to read every such sentence aloud, paying attention to English grammar as well as meaning. It is very important to understand how the negations of quantified statements are formed, and why this method is correct; it is just common sense, really.*

The word "any" in mathematical statements can be ambiguous, so it is best to avoid using it. In negative contexts it almost always means "some" (existential quantifier), as in the statement "You will be suspended from school if you are found guilty of violating any of the plagiarism rules" (you don't have to violate all the rules to get into trouble—breaking one is sufficient). In positive contexts, however, it can mean either "some" (existential quantifier) or "every" (universal quantifier), depending on context. For example, in the sentence "The fraternity will be put on probation if any of its members is found intoxicated," the use is existential (one drunk brother is enough to cause the sanction); but in the sentence "Any member of the sorority will be happy to lead you on a tour of the house," the use is universal (every member is able to be the guide). Another interesting example is an exercise in a mathematics textbook that asks you to show that "the sum of any two odd numbers is even." The author clearly intends the universal interpretation here—you need to show that the sum of two odd numbers is always even. If you interpreted the question existentially, you might say, "Look, $3+5=8$*, so I've shown it is true—you said I could do it for any numbers, and those are the ones I chose."*

1. **a)** T, since $0 \le 4$ **b)** T, since $4 \le 4$ **c)** F, since $6 \not\le 4$

3. **a)** This is true.
b) This is false, since Lansing, not Detroit, is the capital.
c) This is false (but $Q(\text{Boston}, \text{Massachusetts})$ is true).
d) This is false, since Albany, not New York, is the capital.

5. **a)** There is a student who spends more than five hours every weekday in class.
b) Every student spends more than five hours every weekday in class.
c) There is a student who does not spend more than five hours every weekday in class.
d) No student spends more than five hours every weekday in class. (Or, equivalently, every student spends less than or equal to five hours every weekday in class.)

7. **a)** This statement is that for every x, if x is a comedian, then x is funny. In English, this is most simply stated, "Every comedian is funny."
b) This statement is that for every x in the domain (universe of discourse), x is a comedian *and* x is funny. In English, this is most simply stated, "Every person is a funny comedian." Note that this is not the sort of thing one wants to say. It really makes no sense and doesn't say anything about the existence of boring comedians; it's surely false, because there exist lots of x for which $C(x)$ is false. This illustrates the fact that you rarely want to use conjunctions with universal quantifiers.
c) This statement is that there exists an x in the domain such that if x is a comedian then x is funny. In English, this might be rendered, "There exists a person such that if s/he is a comedian, then s/he is funny." Note that this is not the sort of thing one wants to say. It really makes no sense and doesn't say anything about the existence of funny comedians; it's surely true, because there exist lots of x for which $C(x)$ is false (recall

the definition of the truth value of $p \to q$). This illustrates the fact that you rarely want to use conditional statements with existential quantifiers.

d) This statement is that there exists an x in the domain such that x is a comedian and x is funny. In English, this might be rendered, "There exists a funny comedian" or "Some comedians are funny" or "Some funny people are comedians."

9. a) We assume that this sentence is asserting that the same person has both talents. Therefore we can write $\exists x(P(x) \wedge Q(x))$.

b) Since "but" really means the same thing as "and" logically, this is $\exists x(P(x) \wedge \neg Q(x))$

c) This time we are making a universal statement: $\forall x(P(x) \vee Q(x))$

d) This sentence is asserting the nonexistence of anyone with either talent, so we could write it as $\neg\exists x(P(x) \vee Q(x))$. Alternatively, we can think of this as asserting that everyone fails to have either of these talents, and we obtain the logically equivalent answer $\forall x \neg(P(x) \vee Q(x))$. Failing to have either talent is equivalent to having neither talent (by De Morgan's law), so we can also write this as $\forall x((\neg P(x)) \wedge (\neg Q(x))$. Note that it would *not* be correct to write $\forall x((\neg P(x)) \vee (\neg Q(x))$ nor to write $\forall x \neg(P(x) \wedge Q(x))$.

11. a) T, since $0 = 0^2$ b) T, since $1 = 1^2$ c) F, since $2 \neq 2^2$
d) F, since $-1 \neq (-1)^2$ e) T (let $x = 1$) f) F (let $x = 2$)

13. a) Since adding 1 to a number makes it larger, this is true.

b) Since $2 \cdot 0 = 3 \cdot 0$, this is true.

c) This statement is true, since $0 = -0$.

d) As was explained in Example 13, this is true for the integers.

15. Recall that the integers include the positive and negative integers and 0.

a) This is the well-known true fact that the square of a real number cannot be negative.

b) There are two *real* numbers that satisfy $n^2 = 2$, namely $\pm\sqrt{2}$, but there do not exist any *integers* with this property, so the statement is false.

c) If n is a positive integer, then $n^2 \geq n$ is certainly true; it's also true for $n = 0$; and it's trivially true if n is negative. Therefore the universally quantified statement is true.

d) Squares can never be negative; therefore this statement is false.

17. Existential quantifiers are like disjunctions, and universal quantifiers are like conjunctions. See Examples 11 and 16.

a) We want to assert that $P(x)$ is true for some x in the universe, so either $P(0)$ is true or $P(1)$ is true or $P(2)$ is true or $P(3)$ is true or $P(4)$ is true. Thus the answer is $P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4)$. The other parts of this exercise are similar. Note that by De Morgan's laws, the expression in part (c) is logically equivalent to the expression in part (f), and the expression in part (d) is logically equivalent to the expression in part (e).

b) $P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4)$

c) $\neg P(0) \vee \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$

d) $\neg P(0) \wedge \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$

e) This is just the uegation of part (a): $\neg(P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4))$

f) This is just the negation of part (b): $\neg(P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4))$

19. Existential quantifiers are like disjunctions, and universal quantifiers are like conjunctions. See Examples 11 and 16.

a) We want to assert that $P(x)$ is true for some x in the universe, so either $P(1)$ is true or $P(2)$ is true or $P(3)$ is true or $P(4)$ is true or $P(5)$ is true. Thus the answer is $P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5)$.

b) $P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5)$

c) This is just the negation of part (a): $\neg(P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5))$

d) This is just the negation of part (b): $\neg(P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5))$

e) The formal translation is as follows: $(((1 \neq 3) \to P(1)) \wedge ((2 \neq 3) \to P(2)) \wedge ((3 \neq 3) \to P(3)) \wedge ((4 \neq 3) \to P(4)) \wedge ((5 \neq 3) \to P(5))) \vee (\neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4) \vee \neg P(5))$. However, since the hypothesis $x \neq 3$ is false when x is 3 and true when x is anything other than 3, we have more simply $(P(1) \wedge P(2) \wedge P(4) \wedge P(5)) \vee (\neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4) \vee \neg P(5))$. Thinking about it a little more, we note that this statement is always true, since if the first part is not true, then the second part must be true.

21. a) One would hope that if we take the domain to be the students in your class, then the statement is true. If we take the domain to be all students in the world, then the statement is clearly false, because some of them are studying only other subjects.

b) If we take the domain to be United States Senators, then the statement is true. If we take the domain to be college football players, then the statement is false, because some of them are younger than 21.

c) If the domain consists of just Princes William and Harry of Great Britain (sons of the late Princess Diana), then the statement is true. It is also true if the domain consists of just one person (everyone has the same mother as him- or herself). If the domain consists of all the grandchildren of Queen Elizabeth II of Great Britain (of whom William and Harry are just two), then the statement is false.

d) If the domain consists of Bill Clinton and George W. Bush, then this statement is true because they do not have the same grandmother. If the domain consists of all residents of the United States, then the statement is false, because there are many instances of siblings and first cousins, who have at least one grandmother in common.

23. In order to do the translation the second way, we let $C(x)$ be the propositional function "x is in your class." Note that for the second way, we always want to use conditional statements with universal quantifiers and conjunctions with existential quantifiers.

a) Let $H(x)$ be "x can speak Hindi." Then we have $\exists x\, H(x)$ the first way, or $\exists x(C(x) \wedge H(x))$ the second way.

b) Let $F(x)$ be "x is friendly." Then we have $\forall x\, F(x)$ the first way, or $\forall x(C(x) \to F(x))$ the second way.

c) Let $B(x)$ be "x was born in California." Then we have $\exists x\, \neg B(x)$ the first way, or $\exists x(C(x) \wedge \neg B(x))$ the second way.

d) Let $M(x)$ be "x has been in a movie." Then we have $\exists x\, M(x)$ the first way, or $\exists x(C(x) \wedge M(x))$ the second way.

e) This is saying that everyone has failed to take the course. So the answer here is $\forall x\, \neg L(x)$ the first way, or $\forall x(C(x) \to \neg L(x))$ the second way, where $L(x)$ is "x has taken a course in logic programming."

25. Let $P(x)$ be "x is perfect"; let $F(x)$ be "x is your friend"; and let the domain (universe of discourse) be all people.

a) This means that everyone has the property of being not perfect: $\forall x\, \neg P(x)$. Alternatively, we can write this as $\neg \exists x\, P(x)$, which says that there does not exist a perfect person.

b) This is just the negation of "Everyone is perfect": $\neg \forall x\, P(x)$.

c) If someone is your friend, then that person is perfect: $\forall x(F(x) \to P(x))$. Note the use of conditional statement with universal quantifiers.

d) We do not have to rule out your having more than one perfect friend. Thus we have simply $\exists x(F(x) \wedge P(x))$. Note the use of conjunction with existential quantifiers.

e) The expression is $\forall x(F(x) \land P(x))$. Note that here we did use a conjunction with the universal quantifier, but the sentence is not natural (who could claim this?). We could also have split this up into two quantified statements and written $(\forall x\, F(x)) \land (\forall x\, P(x))$.
f) This is a disjunction. The expression is $(\neg\forall x\, F(x)) \lor (\exists x\, \neg P(x))$.

27. In all of these, we will let $Y(x)$ be the propositional function that x is in your school or class, as appropriate.
a) If we let $V(x)$ be "x has lived in Vietnam," then we have $\exists x\, V(x)$ if the universe is just your schoolmates, or $\exists x(Y(x) \land V(x))$ if the universe is all people. If we let $D(x,y)$ mean that person x has lived in country y, then we can rewrite this last one as $\exists x(Y(x) \land D(x, \text{Vietnam}))$.
b) If we let $H(x)$ be "x can speak Hindi," then we have $\exists x\, \neg H(x)$ if the universe is just your schoolmates, or $\exists x(Y(x) \land \neg H(x))$ if the universe is all people. If we let $S(x,y)$ mean that person x can speak language y, then we can rewrite this last one as $\exists x(Y(x) \land \neg S(x, \text{Hindi}))$.
c) If we let $J(x)$, $P(x)$, and $C(x)$ be the propositional functions asserting x's knowledge of Java, Prolog, and C++, respectively, then we have $\exists x(J(x) \land P(x) \land C(x))$ if the universe is just your schoolmates, or $\exists x(Y(x) \land J(x) \land P(x) \land C(x))$ if the universe is all people. If we let $K(x,y)$ mean that person x knows programming language y, then we can rewrite this last one as $\exists x(Y(x) \land K(x, \text{Java}) \land K(x, \text{Prolog}) \land K(x, \text{C++}))$.
d) If we let $T(x)$ be "x enjoys Thai food," then we have $\forall x\, T(x)$ if the universe is just your classmates, or $\forall x(Y(x) \to T(x))$ if the universe is all people. If we let $E(x,y)$ mean that person x enjoys food of type y, then we can rewrite this last one as $\forall x(Y(x) \to E(x, \text{Thai}))$.
e) If we let $H(x)$ be "x plays hockey," then we have $\exists x\, \neg H(x)$ if the universe is just your classmates, or $\exists x(Y(x) \land \neg H(x))$ if the universe is all people. If we let $P(x,y)$ mean that person x plays game y, then we can rewrite this last one as $\exists x(Y(x) \land \neg P(x, \text{hockey}))$.

29. Our domain (universe of discourse) here is all propositions. Let $T(x)$ mean that x is a tautology and $C(x)$ mean that x is a contradiction. Since a contingency is just a proposition that is neither a tautology nor a contradiction, we do not need a separate predicate for being a contingency.
a) This one is just the assertion that tautologies exist: $\exists x\, T(x)$.
b) Although the word "all" or "every" does not appear here, this sentence is really expressing a universal meaning, that the negation of a contradiction is always a tautology. So we want to say that if x is a contradiction, then $\neg x$ is a tautology. Thus we have $\forall x(C(x) \to T(\neg x))$. Note the rare use of a logical symbol (negation) applied to a variable (x); this is purely a coincidence in this exercise because the universe happens itself to be propositions.
c) The words "can be" are expressing an existential idea—that there exist two contingencies whose disjunction is a tautology. Thus we have $\exists x \exists y(\neg T(x) \land \neg C(x) \land \neg T(y) \land \neg C(y) \land T(x \lor y))$. The same final comment as in part (b) applies here. Also note the explanation about contingencies in the preamble.
d) As in part (b), this is the universal assertion that whenever x and y are tautologies, then so is $x \land y$; thus we have $\forall x \forall y((T(x) \land T(y)) \to T(x \land y))$.

31. In each case we just have to list all the possibilities, joining them with $\lor$ if the quantifier is $\exists$, and joining them with $\land$ if the quantifier is $\forall$.
a) $Q(0,0,0) \land Q(0,1,0)$ b) $Q(0,1,1) \lor Q(1,1,1) \lor Q(2,1,1)$
c) $\neg Q(0,0,0) \lor \neg Q(0,0,1)$ d) $\neg Q(0,0,1) \lor \neg Q(1,0,1) \lor \neg Q(2,0,1)$

33. In each case we need to specify some predicates and identify the domain (universe of discourse).
a) Let $T(x)$ be the predicate that x can learn new tricks, and let the domain be old dogs. Our original statement is $\exists x\, T(x)$. Its negation is $\neg\exists x\, T(x)$, which we must to rewrite in the required manner as $\forall x\, \neg T(x)$. In English this reads "Every old dog is unable to learn new tricks" or "All old dogs can't learn new tricks."

(Note that this does *not* say that not all old dogs can learn new tricks—it is saying something stronger than that.) More colloquially, we can say "No old dogs can learn new tricks."

b) Let $C(x)$ be the predicate that x knows calculus, and let the domain be rabbits. Our original statement is $\neg\exists x\, C(x)$. Its negation is, of course, simply $\exists x\, C(x)$. In English this reads "There is a rabbit that knows calculus."

c) Let $F(x)$ be the predicate that x can fly, and let the domain be birds. Our original statement is $\forall x\, F(x)$. Its negation is $\neg\forall x\, F(x)$ (i.e., not all birds can fly), which we must to rewrite in the required manner as $\exists x\, \neg F(x)$. In English this reads "There is a bird who cannot fly."

d) Let $T(x)$ be the predicate that x can talk, and let the domain be dogs. Our original statement is $\neg\exists x\, T(x)$. Its negation is, of course, simply $\exists x\, T(x)$. In English this reads "There is a dog that talks."

e) Let $F(x)$ and $R(x)$ be the predicates that x knows French and knows Russian, respectively, and let the domain be people in this class. Our original statement is $\neg\exists x(F(x) \wedge R(x))$. Its negation is, of course, simply $\exists x(F(x) \wedge R(x))$. In English this reads "There is someone in this class who knows French and Russian."

35. **a)** As we saw in Example 13, this is true, so there is no counterexample.

b) Since 0 is neither greater than nor less than 0, this is a counterexample.

c) This proposition says that 1 is the only integer—that every integer equals 1. If is obviously false, and any other integer, such as -111749, provides a counterexample.

37. In each case we need to make up predicates. The answers are certainly not unique and depend on the choice of predicate, among other things.

a) $\forall x((F(x, 25000) \vee S(x, 25)) \rightarrow E(x))$, where $E(x)$ is "Person x qualifies as an elite flyer in a given year," $F(x, y)$ is "Person x flies more than y miles in a given year," and $S(x, y)$ is "Person x takes more than y flights in a given year"

b) $\forall x(((M(x) \wedge T(x, 3)) \vee (\neg M(x) \wedge T(x, 3.5))) \rightarrow Q(x))$, where $Q(x)$ is "Person x qualifies for the marathon," $M(x)$ is "Person x is a man," and $T(x, y)$ is "Person x has run the marathon in less than y hours"

c) $M \rightarrow ((H(60) \vee (H(45) \wedge T)) \wedge \forall y\, G(\mathrm{B}, y))$, where M is the proposition "The student received a masters degree," $H(x)$ is "The student took at least x course hours," T is the proposition "The student wrote a thesis," and $G(x, y)$ is "The person got grade x or higher in his course y"

d) $\exists x\, ((T(x, 21) \wedge G(x, 4.0))$, where $T(x, y)$ is "Person x took more than y credit hours" and $G(x, p)$ is "Person x earned grade point average p" (we assume that we are talking about one given semester)

39. In each case we pretty much just write what we see.

a) If there is a printer that is both out of service and busy, then some job has been lost.

b) If every printer is busy, then there is a job in the queue.

c) If there is a job that is both queued and lost, then some printer is out of service.

d) If every printer is busy and every job is queued, then some job is lost.

41. In each case we need to make up predicates. The answers are certainly not unique and depend on the choice of predicate, among other things.

a) $(\exists x\, F(x, 10)) \rightarrow \exists x\, S(x)$, where $F(x, y)$ is "Disk x has more than y kilobytes of free space," and $S(x)$ is "Mail message x can be saved"

b) $(\exists x\, A(x)) \rightarrow \forall x(Q(x) \rightarrow T(x))$, where $A(x)$ is "Alert x is active," $Q(x)$ is "Message x is queued," and $T(x)$ is "Message x is transmitted"

c) $\forall x((x \neq \text{main console}) \rightarrow T(x))$, where $T(x)$ is "The diagnostic monitor tracks the status of system x"

d) $\forall x(\neg L(x) \rightarrow B(x))$, where $L(x)$ is "The host of the conference call put participant x on a special list" and $B(x)$ is "Participant x was billed"

43. A conditional statement is true if the hypothesis is false. Thus it is very easy for the second of these propositions to be true—just have $P(x)$ be something that is not always true, such as "The integer x is a multiple of 2." On the other hand, it is certainly not always true that if a number is a multiple of 2, then it is also a multiple of 4, so if we let $Q(x)$ be "The integer x is a multiple of 4," then $\forall x(P(x) \to Q(x))$ will be false. Thus these two propositions can have different truth values. Of course, for some choices of P and Q, they will have the same truth values, such as when P and Q are true all the time.

45. Both are true precisely when at least one of $P(x)$ and $Q(x)$ is true for at least one value of x in the domain (universe of discourse).

47. We can establish these equivalences by arguing that one side is true if and only if the other side is true. For both parts, we will look at the two cases: either A is true or A is false.

a) Suppose that A is true. Then the left-hand side is logically equivalent to $\forall x P(x)$, since the conjunctiou of any proposition with a true proposition has the same truth value as that proposition. By similar reasoning the right-hand side is equivalent to $\forall x P(x)$. Therefore the two propositions are logically equivalent in this case; each one is true precisely when $P(x)$ is true for every x. On the other hand, suppose that A is false. Then the left-hand side is certainly false. Furthermore, for every x, $P(x) \wedge A$ is false, so the right-hand side is false as well. Thus in all cases, the two propositions have the same truth value.

b) This problem is similar to part (a). If A is true, then both sides are logically equivalent to $\exists x P(x)$. If A is false, then both sides are false.

49. We can establish these equivalences by arguing that one side is true if and only if the other side is true. For both parts, we will look at the two cases: either A is true or A is false.

a) Suppose that A is true. Then for each x, $P(x) \to A$ is true, becanse a conditional statement with a true conclusion is always true; therefore the left-hand side is always true in this case. By similar reasoning the right-hand side is always true in this case (here we used the fact that the domain is nonempty). Therefore the two propositions are logically equivalent when A is true. On the other hand, suppose that A is false. There are two subcases. If $P(x)$ is false for every x, then $P(x) \to A$ is vacuously true (a conditional statement with a false hypothesis is true), so the left-hand side is vacuously true. The same reasoning shows that the right-hand side is also true, because in this subcase $\exists x P(x)$ is false. For the second subcase, suppose that $P(x)$ is true for some x. Then for that x, $P(x) \to A$ is false (a conditioual statement with a true hypothesis and false conclusion is false), so the left-hand side is false. The right-hand side is also false, because in this subcase $\exists x P(x)$ is true but A is false. Thus in all cases, the two propositions have the same trnth value.

b) This problem is similar to part (a). If A is true, then hoth sides are trivially true, because the conditional statements have true conclusions. If A is false, then there are two subcases. If $P(x)$ is false for some x, then $P(x) \to A$ is vacuously true for that x (a conditional statement with a false hypothesis is true), so the left-hand side is true. The same reasoning shows that the right-hand side is true, because in this subcase $\forall x P(x)$ is false. For the second subcase, suppose that $P(x)$ is trne for every x. Then for every x, $P(x) \to A$ is false (a conditional statement with a true hypothesis and false conclusion is false), so the left-hand side is false (there is no x making the couditional statement true). The right-hand side is also false, because it is a conditional statement with a true hypothesis and a false conclusion. Thus in all cases, the two propositions have the same truth value.

51. We can show that these are not logically equivalent by giving an example in which one is true and the other is false. Let $P(x)$ be the statement "x is odd" applied to positive integers. Similarly let $Q(x)$ be "x is even." Then since there exist odd numbers and there exist even nmmbers, the statement $\exists x P(x) \wedge \exists x Q(x)$ is true. On the other hand, no number is both odd and even, so $\exists x(P(x) \wedge Q(x))$ is false.

53. a) This is certainly true: if there is a unique x satisfying $P(x)$, then there certainly *is an* x satisfying $P(x)$.

b) Unless the domain (universe of discourse) has fewer than two items in it, the truth of the hypothesis implies that there is more than one x such that $P(x)$ holds. Therefore this proposition need not be true. (For example, let $P(x)$ be the proposition $x^2 \geq 0$ in the context of the real numbers. The hypothesis is true, but there is not a unique x for which $x^2 \geq 0$.)

c) This is true: if there is an x (unique or not) such that $P(x)$ is false, then we can conclude that it is not the case that $P(x)$ holds for all x.

55. A Prolog query returns a yes/no answer if there are no variables in the query, and it returns all values that make the query true if there are.

a) One of the facts was that Chan was the instructor of Math 273, so the response is `yes`.

b) None of the facts was that Patel was the instructor of CS 301, so the response is `no`.

c) Prolog returns the names of the people enrolled in CS 301, namely `juana` and `kiko`.

d) Prolog returns the names of the courses Kiko is enrolled in, namely `math273` and `cs301`.

e) Prolog returns the names of the students enrolled in courses which Grossman is the instructor for (which is just CS 301), namely `juana` and `kiko`.

57. Following the idea and syntax of Example 28, we have the following rule: `sibling(X,Y) :- mother(M,X), mother(M,Y), father(F,X), father(F,Y)`. Note that we used the comma to mean "and"; `X` and `Y` must have the same mother and the same father in order to be (full) siblings.

59. a) This is the statement that every person who is a professor is not ignorant. In other words, for every person, if that person is a professor, then that person is not ignorant. In symbols: $\forall x(P(x) \rightarrow \neg Q(x))$. This is not the only possible answer. We could equivalently think of the statement as asserting that there does not exist an ignorant professor: $\neg\exists x(P(x) \wedge Q(x))$.

b) Every person who is ignorant is vain: $\forall x(Q(x) \rightarrow R(x))$.

c) This is similar to part (a): $\forall x(P(x) \rightarrow \neg R(x))$.

d) The conclusion (part (c)) does not follow. There may well be vain professors, since the premises do not rule out the possibility that there are vain people besides the ignorant ones.

61. a) This is asserting that every person who is a baby is necessarily not logical: $\forall x(P(x) \rightarrow \neg Q(x))$.

b) If a person can manage a crocodile, then that person is not despised: $\forall x(R(x) \rightarrow \neg S(x))$.

c) Every person who is not logical is necessarily despised: $\forall x(\neg Q(x) \rightarrow S(x))$.

d) Every person who is a baby cannot manage a crocodile: $\forall x(P(x) \rightarrow \neg R(x))$.

e) The conclusion follows. Suppose that x is a baby. Then by the first premise, x is illogical, and hence, by the third premise, x is despised. But the second premise says that if x could manage a crocodile, then x would not be despised. Therefore x cannot manage a crocodile. Thus we have proved that babies cannot manage crocodiles.