SECTION 8.2 *n*-ary Relations and Their Applications

This section is a brief introduction to relational models for data bases. The exercises are straightforward and similar to the examples. Projections are formed by omitting certain columns, and then eliminating duplicate rows. Joins are analogous to compositions of relations.

1. We simply need to find solutions of the inequality, which we can do by common sense. The set is $\{(1,2,3),(1,2,4),(1,3,4),(2,3,4)\}$.

3. The 5-tuples are just the lines of the table. Thus the relation is
$\{(\text{Nadir}, 122, 34, \text{Detroit}, 08{:}\,10), (\text{Acme}, 221, 22, \text{Denver}, 08{:}\,17), (\text{Acme}, 122, 33, \text{Anchorage}, 08{:}\,22),$
$(\text{Acme}, 323, 34, \text{Honolulu}, 08{:}\,30), (\text{Nadir}, 199, 13, \text{Detroit}, 08{:}\,47), (\text{Acme}, 222, 22, \text{Denver}, 09{:}\,10),$
$(\text{Nadir}, 322, 34, \text{Detroit}, 09{:}\,44)\}$.

5. We need to find a field that, when used along with the *Airline* field uniquely specifies a row of the table. Certainly *Flight_number* is one such field, since there is only one line of the table for each pair (*Airline*, *Flight_number*); no airline has the same flight number for two different flights. *Gate* and *Destination* do not qualify, however, since Nadir has two flights leaving from Gate 34 going to Detroit. Finally, *Departure_time* is a key by itself (no two flights leave at the same time), so it and *Airline* form a composite key as well.

7. **a)** A school would be giving itself a lot of headaches if it didn't make the student ID number different for each student, so student ID is likely to be a primary key.
b) Name could very easily not be a primary key. Two people named Jennifer Johnson might easily both be students.
c) Phone number is unlikely to be a primary key. Two roommates, or two siblings living at home, will likely have the same phone number, and they might both be students at that school.

9. **a)** Everyone has a different Social Security number, so that field will be a primary key.
b) It is unlikely that (name, street address) will be a composite key. Somewhere in the United States there could easily be two people named Jennifer Johnson both living at 123 Washington Street. In order for this to work, there must never be two people with the same name who happen to have the same street address.
c) For this to work, we must never have two people with the same name living together. Given the size of the country, one would doubt that this would work. For example, a husband and wife each named Morgan White would make this not a composite key, as would a mother and daughter living at home with the same name.

11. The selection operator picks out all the tuples that match the criteria. The 5-tuples in Table 8 that have Detroit as their destination are $(\text{Nadir}, 122, 34, \text{Detroit}, 08{:}\,10)$, $(\text{Nadir}, 199, 13, \text{Detroit}, 08{:}\,47)$, and $(\text{Nadir}, 322, 34, \text{Detroit}, 09{:}\,44)$.

13. The selection operator picks out all the tuples that match the criteria. The 5-tuples in Table 8 that have Nadir as their airline are (Nadir, 122, 34, Detroit, 08: 10), (Nadir, 199, 13, Detroit, 08: 47), and (Nadir, 322, 34, Detroit, 09: 44). The 5-tuples in Table 8 that have Denver as their destination are (Acme, 221, 22, Denver, 08: 17) and (Acme, 222, 22, Denver, 09: 10). We need the union of these two lists: (Nadir, 122, 34, Detroit, 08: 10), (Nadir, 199, 13, Detroit, 08: 47), (Nadir, 322, 34, Detroit, 09: 44), (Acme, 221, 22, Denver, 08: 17), and (Acme, 222, 22, Denver, 09: 10).

15. The subscripts on the projection mapping notation indicate which columns are to be retained. Thus if we want to delete columns 1, 2, and 4 from a 6-tuple, we need to use the projection $P_{3,5,6}$.

17. The table uses columns 1 and 4 of Table 8. We start by deleting columns 2, 3, and 5 from Table 8. At this point, rows 5, 6 and 7 are duplicates of earlier rows, so they are omitted (rather than being listed twice). Therefore the answer is as follows.

Airline	*Destination*
Nadir	Detroit
Acme	Denver
Acme	Anchorage
Acme	Honolulu

19. We need to find rows of Table 9 the last two entries of which are identical to the first two entries of rows of Table 10. We combine each such pair of rows into one row of our new table. For instance, the last two entries in the first row of Table 9 are 1092 and 1. The first two entries in the second row of Table 10 are also 1092 and 1. Therefore we combine them into the row 23, 1092, 1, 2, 2 of our new table, whose columns represent *Supplier*, *Part_number*, *Project*, *Quantity*, and *Color_code*. The new table consists of all pairs found in this way.

Supplier	*Part_number*	*Project*	*Quantity*	*Color_code*
23	1092	1	2	2
23	1101	3	1	1
23	9048	4	12	2
31	4975	3	6	2
31	3477	2	25	2
32	6984	4	10	1
32	9191	2	80	4
33	1001	1	14	8

21. Both sides of this equation pick out the subset of R consisting of those n-tuples satisfying both conditions C_1 and C_2. This follows immediately from the definition of the selection operator.

23. Both sides of this equation pick out the set of n-tuples that satisfy three conditions: they are in R, they are in S, and they satisfy condition C. This follows immediately from the definitions of intersection and the selection operator.

25. Both sides of this equation pick out the m-tuples consisting of i_1^{th}, i_2^{th}, ..., i_m^{th} components of n-tuples in either R or S (or, of course, both). This follows immediately from the definitions of union and the projection operator.

27. Note that we lose information when we delete columns. Therefore we might be taking something away when we form the second set of m-tuples that might not have been taken away if all the original information is there (forming the first set of m-tuples). A simple example would be to let $R = \{(a,b)\}$ and $S = \{(a,c)\}$, $n = 2$, $m = 1$, and $i_1 = 1$. Then $R - S = R$, so $P_1(R - S) = P_1(R) = \{(a)\}$. On the other hand, $P_1(R) = P_1(S) = \{(a)\}$, so $P_1(R) - P_1(S) = \emptyset$.

29. This is similar to Example 13.

a) Since two databases are listed in the "FROM" field, the first operation is to form the join of these two databases, specifically the join J_2 of these two databases. We then apply the selection operator with the condition "Quantity $\leq$ 10." This join will have eight 5-tuples in it. Finally we want just the Supplier and Project, so we are forming the projection $P_{1,3}$.

b) Four of the 5-tuples in the joined database have a quantity of no more than 10. The output, then, is the set of the four 2-tuples corresponding to these fields: $(23,1), (23,3), (31,3), (32,4)$.

31. A primary key is a domain whose value determines the values of all the other domains. For this relation, this does not happen. The third domain (the modulus) is not a primary key, because, for example, $1 \equiv 11 \pmod{10}$ and $2 \equiv 12 \pmod{10}$, so the triples $(1,11,10)$ and $(2,12,10)$ are both in the relation. Knowing that the third component of a triple is 10 does not tell us what the other two components are. Similarly, the triples $(1,11,10)$ and $(1,21,10)$ are both in the relation, so the first domain is not a key; and the triples $(1,11,10)$ and $(11,11,10)$ are both in the relation, so the second domain is not a key.