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Assigment # 1 :-

Question # 1 :-

Output from		Purchased by
Goods	serv-ices	
0.8	0.3	services
0.2	0.7	goods

Let : p_g as annual output of goods sector
p_s as annual output of servic sector

Equation of row one will be :

$$p_s = 0.8p_g = 0.3p_s$$

Equation of row two will be :

$$p_g = 0.2p_g + 0.7p_s$$

To solve system, we will move all the unknowns to left;

$$-0.8p_g + 0.7p_s = 0 \quad - \text{①}$$

$$0.8p_g + (-0.7) = 0 \quad - \text{②}$$

Augmented matrix will be,

$$\left[\begin{array}{cc:c} -0.8 & 0.7 & 0 \\ 0.8 & -0.7 & 0 \end{array}\right]$$

adding R_2 & R_1

$$\left[\begin{array}{cc:c} -0.8 & 0.7 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Adding R_1 by -0.8

$$\left[\begin{array}{cc:c} 1 & -0.875 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

* General solution is $p_g = 0.875 p_s$ and where p_s is free.
* Equation will remain unaffected with prices until the ratio $p_g = 0.875 p_s$ satisfies.

Consider $p_s = 1000$ than $p_g =$

$0.875 \times 1000 \Rightarrow p_g = 875$

Question # 2:

consider $p_s = 300$ million dollar

than $p_c = 0.94 p_s$, $p_e = 0.85 p_s$

$p_c = 282$, $p_e = 255$

No, there will be on change in equilibrim until and unless the ratio are not satisfied.

Question # 3:-

Output from			Purchased by
chemi--cals	Feul	Machi--nery	
0.2	0.8	0.4	chemicals
0.3	0.1	0.4	Feul
0.5	0.1	0.2	machinery

Let,

pc as annual output for chemical sector

pf as annual output for feul sector

pm as annual output for machinery sector

Equation for row1 will be,

$pc = 0.2pc + 0.8pf + 0.4pm$

Equation for row2 will be,

$pf = 0.3pc + 0.1pf + 0.4pm$

Equation for row3 will be,

$pm = 0.5pc + 0.1pf + 0.2pm$

To solve system of equation, move all unknowns to left.

$$0.8p_c + -0.2p_f - 0.4p_m = 0$$
$$-0.3p_c + 0.9p_f - 0.4p_m = 0$$
$$-0.5p_c \cancel{+} - 0.1p_f + 0.8p_m = 0$$

Augmented matrix will be,

$$\left[\begin{array}{ccc:c} 0.8 & -0.8 & -0.4 & 0 \\ -0.3 & 0.9 & -0.4 & 0 \\ -0.5 & 0.1 & 0.8 & 0 \end{array}\right]$$

Multiply each row by 10

$$\left[\begin{array}{ccc:c} 8 & -8 & -4 & 0 \\ -3 & 9 & -4 & 0 \\ -5 & -1 & 8 & 0 \end{array}\right]$$

$R_2 + (+3R_1)$: $R_3 + (-5R_1)$

$$= \left[\begin{array}{ccc:c} 1 & -1 & -0.5 & 0 \\ 0 & -5 & -5.5 & 0 \\ 0 & -6 & 5.5 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc:c} 1 & -1 & 0.5 & 0 \\ 0 & 1 & -0.917 & 0 \\ 0 & -6 & 5.5 & 0 \end{array}\right]$$

$R_2 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1.417 & 0 \\ 0 & 1 & -0.917 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The general solution is,

$pc = 1.417pm$

$pf = 0.9417pm$

pm = free variable

If $pm = 100$

$pc = 1.147(100) = 114.7$

$pf = 0.9417(100) = 94.17$

$pm = 100$

Question # 04

Output				Purchased by
A	M	E	T	
0.65	.30	.30	.20	A
0.10	.10	.15	.10	E
0.25	.35	.15	.30	M
0	.25	.40	.40	T

Denoting total annual outcome of sectors by pa, pe, pm, pt

Equations for row 1:

$pa = 0.65pa + 0.3pe + 0.30pm + 0.20pt$

Eq for row: 2, 3, 4 are,

$pe = 0.10pm + 0.10pe + 0.15pm + 0.10pt$

$pm = 0.25pe + 0.35pe + 0.15pm + 0.03pt$

$pt = 0.25pe + 0.40pm + 0.40pt$

Augmented Matrix form,

$$\left[\begin{array}{cccc:c} 0.35 & -0.3 & -0.3 & -0.2 & 0 \\ 0 & 0.81 & -2.4 & -1.6 & 0 \\ 0 & 0 & 1.0 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Dividing r_1 by 0.35

$$\left[\begin{array}{cccc:c} 1 & -0.86 & -0.86 & -0.57 & 0 \\ 0 & 0.81 & -2.4 & -1.6 & 0 \\ 0 & 0 & 1.0 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 : 0.86 R_2 \rightarrow R_1$

$$\left[\begin{array}{cccc:c} 1 & 0 & 2.55 & 0.97 & 0 \\ 0 & 1 & -2.96 & 1.98 & 0 \\ 0 & 0 & 1.0 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2.55 & 0.97 & 0 \\ 0 & 1 & -2.96 & 1.98 & 0 \\ 0 & 0 & 1.0 & -1.11 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 + 2.96R_3$ & $R_1 - 2.55R_3$.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -0.97 & 0 \\ 0 & 1 & 0 & -1.48 & 0 \\ 0 & 0 & 1 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

General solution

$p_a = 2.03p_t$: $p_e = 0.53p_t$: $p_m = 1.17p_t$

If $p_t = 100$ then,

$p_a = 203$: $p_e = 0.53$, $p_m = 117$

Question # 5 :-

$B_2S_3 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \end{bmatrix}$, $H_2O \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $H_3BO_3 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 1 \end{bmatrix}$,

$H_2S \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$ $\begin{array}{l} \rightarrow \text{boron (B)} \\ \rightarrow \text{sulphus (S)} \\ \rightarrow \text{hydrogen (H)} \\ \rightarrow \text{oxygen (O)} \end{array}$

The co-effecient to equation,

$x_1 B_2S_3 + x_2 H_2O \rightarrow x_3 H_3BO_3 + x_4 H_2$

$$x_1 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 6 \\ 2 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 \\ 6 \\ 3 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

Moving right term to left side,

$$\left[\begin{array}{cccc:c} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & 3 & 0 & 0 \end{array}\right]$$

$R_1 \div 2$: $2R_2 - 3R_1$

$$\left[\begin{array}{cccc:c} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & 3 & 0 & 0 \end{array}\right]$$

Exchanging R_2 and R_3

$$\left[\begin{array}{cccc:c} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array}\right]$$

$\frac{1}{2}R_3 + R_1 \quad : \quad R_2 - R_4$

$$\left[\begin{array}{cccc:c} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 3 & -2 & 0 \end{array}\right]$$

$3R_3 - R_4$

$$\left[\begin{array}{cccc:c} 1 & 0 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Thus, the general solution is,

$R_1 + 2R_3$

$$\left[\begin{array}{cccc:c} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array}\right]$$

$R_4 - R_2 \quad : \quad R_5 - 2R_2$

$$\left[\begin{array}{cccc:c} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & -3 & -1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

General equation is,

$x_1 = 1/2\, x_4$; $x_3 = 1/6 \times 4$,

$x_2 = 1/2 \times 4$, x_4 = free variable

if $x_4 = 6$

than balance equation will be,

$2Na_3PO_4 + 3Bt(NO_3)_2 \rightarrow Ba_3(PO_4)_2 + 6NaNO_3$

general solution,

$pa = 2.03pt$: $pe = 0.53pt$ $pm = 1.17pt$

and pt = free variable

If $pt = 100$ than,

$pa = 203$: $pe = 053$ $pm = 117$

Question # 5:

When solutions of ...

$Na_3PO_4 + Ba(NO_3)_2 \rightarrow Ba_3(PO_4)_2 + NaNO_3$

$Na_3PO_4: \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix}, \quad Ba(NO_3)_2 \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix} \begin{matrix} \text{sodium} \\ \text{phosphorus} \\ \text{oxygen} \\ \text{barium} \\ \text{nitrogen} \end{matrix}$

$Ba_3(PO_4)_2: \begin{bmatrix} 0 \\ 2 \\ 8 \\ 3 \\ 0 \end{bmatrix}, \quad NaNO_3: \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix}$

The co-effecients in equation

$x_1 \cdot NaNO_3 + x_2 \cdot Ba(NO_3)_2 \rightarrow x_3 \cdot Ba_3(PO_4)_2 +$
$x_4 \cdot NaNO_3$

$$x_1 \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 2 \\ 8 \\ 3 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

Move right terms to left side,

$$\begin{bmatrix} 3 & 0 & 0 & -1 & 0 \\ 1 & 0 & -2 & 0 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix}$$

Replacing R_1 and R_2

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix} \sim$$

$$\begin{array}{c} \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 6 & 0 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ -3R_1+R_2 \\ -4R_1+R_3 \\ \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 6 & 0 & -3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 18 & -3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 0 & 6 & -1 & 0 \end{bmatrix}$$

The general solution is

$$x_1 = \left(\frac{1}{3}\right) x_4$$

$x_2 \cdot \left(\frac{1}{2}\right) x_4$, $x_3 \left(\frac{1}{6}\right) x_4$ with x_4 as free

Taking $x_4 = 6$ then $x_1 = 2$, $x_2 = 3$

$x_3 = 1$ so balanced equation is,

$$2Na_2PO_3 + 3Ba(NO_3)_2 \rightarrow Ba_3(PO_4)_2 + 6NaNO_3$$

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Linear Algebra

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Question 8

- The following reaction...

$KMnO_4 + MnSO_4 + H_2O \to MnO_2 + K_2SO_4 + H_2SO_4$

Sol:

$KMnO_4 + MnSO_4 + H_2O \to MnO_2 + K_2SO_4 + H_2SO_4$

$$KMnO_4 \begin{bmatrix} 1 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix}, \quad MnSO_4 \begin{bmatrix} 0 \\ 1 \\ 4 \\ 1 \\ 0 \end{bmatrix} \begin{array}{l} \text{potassium} \\ \text{Manganese} \\ \text{Oxygen} \\ \text{Sulphur} \\ \text{Hydrogen} \end{array}$$

$$H_2O \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \quad MnO_2 \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} \quad K_2SO_4 \begin{bmatrix} 2 \\ 0 \\ 4 \\ 1 \\ 0 \end{bmatrix} \quad H_2SO_4 \begin{bmatrix} 0 \\ 0 \\ 4 \\ 1 \\ 2 \end{bmatrix}$$

Coefficients in chemical equation

$$x_1 KMnO_4 + x_2 MnSO_4 + x_3 H_2O \to x_4 MnO_2 + x_5 K_2SO_4 + x_6 H_2SO_4$$

Vector equation

$$x_1\begin{bmatrix}1\\1\\4\\0\\0\end{bmatrix}+x_2\begin{bmatrix}0\\1\\4\\1\\0\end{bmatrix}+x_3\begin{bmatrix}0\\0\\1\\2\\0\end{bmatrix}=x_4\begin{bmatrix}0\\1\\2\\0\\0\end{bmatrix}+x_5\begin{bmatrix}2\\0\\4\\1\\0\end{bmatrix}+x_6\begin{bmatrix}0\\0\\4\\1\\2\end{bmatrix}$$

Move the terms to left side:

Augmented Matrix

$$\begin{bmatrix}1&0&0&0&-2&0&0\\1&1&0&-1&0&0&0\\4&4&1&-2&-4&-4&0\\0&1&0&0&-1&-1&0\\0&0&2&0&0&-2&0\end{bmatrix}$$

$$\begin{array}{c}\begin{bmatrix}1&0&0&0&-2&0&0\\0&1&0&-1&2&0&0\\0&4&1&-2&4&-4&0\\0&1&0&0&-1&-1&0\\0&0&2&0&0&-2&0\end{bmatrix}\end{array}\begin{array}{l}\\R_2-R_1\\R_3-4R_1\\\\\\\end{array}$$

$$\begin{bmatrix}1&0&0&0&-2&0&0\\0&1&0&-1&2&0&0\\0&0&1&2&-4&-4&0\\0&0&0&1&-3&-1&0\\0&0&2&0&0&-2&0\end{bmatrix}\begin{array}{l}R_3-4R_1\\R_4-R_2\\\\\\\\\end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & -4 & 8 & 6 & 0 \end{bmatrix} \begin{array}{l} \\ \\ R_2+R_4 \\ R_5-2R_3 \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -5/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{bmatrix} \begin{array}{l} \\ \\ R_4+3R_5 \\ \\ \\ \end{array}$$

General solution is

$x_1 = x_6$, $x_2 = (1.5)x_6$, $x_3 = x_6$, $x_4 = (2.5)x_6$

$x_5 = 0.5x_6$ and x_6 is free

Taking $x_6 = 2$. Then

$x_1 = x_3 = 2$, $x_2 = 3$, $x_4 = 5$ and $x_5 = 1$

Balanced equation

$2KMnO_4 + 3MnSO_4 + 2H_2O \rightarrow 5MnO_2 + K_2SO_4 + 2H_2SO_4$

Q9 $PbN_6 + CrMn_2O_8 \rightarrow Pb_3O_4 + Cr_2O_3 + MnO_2$

Setting up vectors that list the atoms per molecule. The vector equation

$$x_1 \begin{bmatrix} 1 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 8 \end{bmatrix} = x_3 \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 3 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} + x_6 \begin{array}{c|l} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} & \begin{array}{l} \text{lead} \\ \text{nitrogen} \\ \text{chromium} \\ \text{manganese} \\ \text{oxygen} \end{array} \end{array}$$

Augmented matrix

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 18 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{bmatrix}$$ swap R_2 R_3

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 4 & -1 & 0 & 0 \\ 0 & 0 & -4 & 13 & -2 & -1 & 0 \end{array}\right] \begin{array}{l} \\ \\ R_1+3R_3 \\ \\ R_3/18 \end{array}$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & -1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5/4 & -11/9 & 0 \end{array}\right] \begin{array}{l} \\ R_2+2R_4 \\ \\ R_5-13R_4 \\ \\ \end{array}$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & 0 & 0 & -22/45 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & 0 & -11/45 & 0 \\ 0 & 0 & 0 & 0 & 1 & -44/45 & 0 \end{array}\right] \begin{array}{l} \\ R_4+\frac{R_5}{4} \\ \\ \\ \\ \end{array}$$

Solution is

$x_1 = \left(\frac{1}{6}\right)x_6$, $x_2 = \left(\frac{22}{45}\right)x_6$, $x_3 = \left(\frac{1}{18}\right)x_6$, $x_4 = \left(\frac{11}{45}\right)x_6$

$x_5 = \left(\frac{44}{45}\right)x_6$ and x_6 is free. $x_6 = 90$

then $x_1 = 15$, $x_2 = 44$, $x_3 = 5$, $x_4 = 22$, $x_5 = 88$

Balanced equation is :

$$15PbN_6 + 44CrMn_2O_8 \rightarrow 5Pb_3O_4 + 22Cr_2O_3 + 88MnO_2 + 90NO$$

Q10 $MnS + As_2Cr_{10}O_{35} + H_2SO_4 \rightarrow HMnO_4 + AsH_3 + CrS_3O_{12} + H_2O$

Sol:

$$x_1\begin{bmatrix}1\\1\\0\\0\\0\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\2\\10\\35\\0\end{bmatrix} + x_3\begin{bmatrix}0\\1\\0\\0\\4\\2\end{bmatrix} = x_4\begin{bmatrix}1\\0\\0\\0\\4\\1\end{bmatrix} + x_5\begin{bmatrix}0\\0\\1\\0\\0\\3\end{bmatrix} + x_6\begin{bmatrix}0\\3\\0\\1\\12\\0\end{bmatrix}$$

$$+x_7\begin{array}{c|l}\begin{bmatrix}0\\0\\0\\0\\1\\2\end{bmatrix} & \begin{matrix}\text{manganese}\\\text{sulphur}\\\text{arsenic}\\\text{chromium}\\\text{oxygen}\\\text{hydrogen}\end{matrix}\end{array}$$

Augmented matrix is.

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & -3 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 35 & 4 & -4 & 0 & -12 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 35 & 4 & -4 & 0 & -12 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 & 0 \end{bmatrix} \quad R_2 - R_1$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & -1 & 0 & 0 \\ 0 & 0 & 0 & -8 & 35/2 & 0 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 & 0 \end{bmatrix} \quad R_5 - 4R_3$$

$$\left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & -35/16 & 0 & 1/8 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -35/16 & 0 & 1/8 & 0 \\ 0 & 0 & 0 & 0 & 5 & -1 & 0 & 0 \\ 0 & 0 & 0 & -3 & -3 & 6 & -2 & 0 \end{array}\right] \begin{array}{l} R_1 + R_4 \\ -R_4/8 \end{array}$$

$$\left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & -7/16 & 1/8 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1/10 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -41/16 & -1/8 & 0 \\ 0 & 0 & 0 & 1 & -35/16 & 0 & 1/8 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -153/16 & 6 & -13/8 & 0 \end{array}\right] \begin{array}{l} R_1 + \frac{33R_5}{16} \\ R_2 + R_5/2 \\ R_3 - \frac{35R_5}{16} \end{array}$$

$$\left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & 0 & -16/327 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -13/327 & 0 \\ 0 & 0 & 1 & 0 & 0 & -41/16 & -1/8 & 0 \\ 0 & 0 & 0 & 1 & 0 & -7/16 & 1/8 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -130/327 & 0 \end{array}\right] \begin{array}{l} R_1 + \frac{7R_6}{16} \\ R_2 + \frac{R_6}{10} \\ \frac{80R_6}{327} \end{array}$$

$$\left[\begin{array}{cccccccc}1 & 0 & 0 & 0 & 0 & 0 & -16/327 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & -13/327 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & -347/327 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & -16/327 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & -26/327 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & -130/327 & 0\end{array}\right] \begin{array}{l} R_3+\frac{41R_6}{16} \\ R_4+\frac{7R_6}{16} \\ R_5+\frac{R_6}{5}\end{array}$$

General solution is

$x_1=\left(\frac{16}{327}\right)x_7,\ x_2=\left(\frac{13}{327}\right)x_7,\ x_3=\left(\frac{374}{327}\right)x_7,$

$x_4=\left(\frac{16}{327}\right)x_7,\ x_5=\left(\frac{26}{327}\right)x_7,\ x_6=\left(\frac{130}{327}\right)x_7$

and x_7 is free. if $x_7=327$ then

$x_1=16,\ x_2=13,\ x_3=374,\ x_4=16,\ x_5=26,$

$x_6=130$

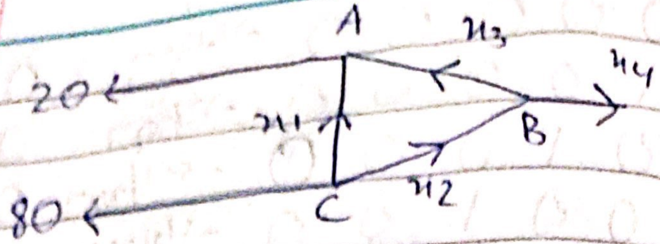
Balanced equation

$16MnS+13As_2Cr_{10}O_{35}+374H_2SO_4 \rightarrow$

$16HMnO_4+26AsH_3+130CrS_3O_{12}+$

$327H_2O$

Q11:



Sol:

eq for each node

Node	Flow In		Flow Out
A	$x_1 + x_3$	=	20
B	x_2	=	$x_3 + x_4$
C	80	=	$x_1 + x_2$

total flow $= 80 = x_4 + 20$

rearrange eqs

$$x_1 + x_3 = 20$$
$$x_2 - x_3 - x_4 = 0$$
$$x_1 + x_2 = 80$$
$$x_4 = 60$$

Reduce eq

$$\left[\begin{array}{cccc:c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 80 \\ 0 & 0 & 0 & 1 & 60 \end{array}\right]$$

$$\left[\begin{array}{cccc:c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & 0 & 60 \\ 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{cases} x_1 = 20 - x_3 \\ x_2 = 60 + x_3 \\ x_3 \text{ is free} \\ x_4 = 60 \end{cases}$$

Sol:

Node A : $x_1 = x_3 + x_4 + 40$

Node B : $200 = x_1 + x_2$

Node C : $x_2 + x_3 = x_5 + 100$

Node D : $x_4 + x_5 = 60$

Total flow: $200 = 200$

$$x_1 - x_3 - x_4 = 40$$
$$x_1 + x_3 \qquad = 200$$
$$x_2 + x_3 - x_5 = 100$$
$$x_4 + x_5 = 60$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & -1 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{array}\right] \sim \left[\begin{array}{ccccc|c} 1 & 0 & -1 & 0 & 1 & 10 \\ 0 & 1 & 1 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 & (\end{array}\right]$$

$$\begin{cases} x_1 = 100 + x_3 - x_5 \\ x_2 = 100 - x_3 + x_5 \\ x_3 \text{ is free} \\ x_4 = 60 - x_5 \\ x_5 \text{ is free} \end{cases}$$

when $x_4 = 0$, $x_5 = 60$

then
$$\begin{cases} x_1 = 40 + x_3 \\ x_2 = 160 - x_3 \\ x_3 \text{ is free} \\ x_4 = 0 \\ x_5 = 60 \end{cases}$$

Q13

Sol:

A : $x_2 + 30 = x_1 + 80$

B : $x_3 + x_5 = x_2 + x_4$

C : $x_6 + 100 = x_3 + 40$

D : $x_4 + 40 = x_6 + 90$

E : $x_1 + 60 = x_3 + 20$

Rearrange the eqs:

$x_1 - x_2 = -50$

$x_2 - x_3 + x_4 - x_5 = 0$

$x_3 - x_6 = 60$

$x_4 - x_6 = 50$

$x_1 - x_3 = -40$

Reduce augmented matrix:

$$\left[\begin{array}{cccccc:c} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 1 & 0 & -1 & 0 & 0 & 0 & -40 \end{array}\right]$$

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

general solution is
$$\begin{cases} x_1 = x_3 - 40 \\ x_2 = x_3 + 10 \\ x_3 = \text{is free} \\ x_4 = x_6 + 50 \\ x_5 = x_6 + 60 \\ x_6 = \text{is free} \end{cases}$$

Q.14) Sol:

A : $x_1 = x_2 + 100$

B : $x_2 + 50 = x_3$

C : $x_3 = x_4 + 120$

D : $x_4 + 150 = x_5$

E : $x_5 = x_6 + 80$

F : $x_6 + 100 = x_1$

Rearrange the equation.

$x_1 - x_2 = 100$

$x_2 - x_3 = -50$

$x_3 - x_4 = 120$

$x_4 - x_5 = -150$

$x_5 - x_6 = 80$

$-x_1 + x_6 = -100$

Reduce matrix is;

$$\left[\begin{array}{cccccc:c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ -1 & 0 & 0 & 0 & 0 & 1 & 100 \end{array}\right]$$

$$\begin{bmatrix} & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 100 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 50 \\ 0 & 0 & 0 & 1 & 0 & -1 & -70 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

General Solution is $\begin{cases} x_1 = 100 + x_6 \\ x_2 = x_6 \\ x_3 = 50 + x_6 \\ x_4 = -70 + x_6 \\ x_5 = 80 + x_6 \\ x_6 \text{ is free} \end{cases}$

Since x_4 cannot be negative, the minimum value of x_6 is 70.