Linear Algebra

Assignment 2

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- **Matrix Determinant:-**

The determinant is a special number that can be calculated from a square matrix using special method. It is also a function of entries of a square matrix.

Properties of a matrix:

The properties of a matrix determinant are as follows.

1. **Reflection Property**

The determinat remains unchanged if it rows are replaced with columns and vice-versa

$$\det A^T = \det A.$$

2. **Zero Property:**

Incase all elements of a row or column are zero then determinat will be zero

Example:

Let $A = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 4 & 5 & 6 \end{vmatrix}$

$|A| = 1(6(0)-5(0)) - 2(6(0)-4(0)) + 3(5(0)-4(0))$

$|A| = 0.$

3. **Repitition Property:**

If any two rows or columns of a matrix are same, the determinant will be zero

Example:

$$\begin{vmatrix} 4 & 6 & 4 \\ 4 & 6 & 4 \\ 9 & 6 & 4 \end{vmatrix} = 0$$

4. **Scalar Multiple Property:**

If the elements of a row or column of a determinant are multiplied by any non-zero constant

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then the determinant also gets multiplied by the same constant

Example:

$$\begin{vmatrix} 102 & 18 & 36 \\ 13 & 4 & 1 \\ 17 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 6(17) & 6(3) & 6(6) \\ 13 & 4 & 1 \\ 17 & 3 & 6 \end{vmatrix}$$

$$= 6 \times \begin{vmatrix} 17 & 3 & 6 \\ 13 & 4 & 1 \\ 17 & 3 & 6 \end{vmatrix}$$

5- Triangle Property:

If the elements in determinant below and above ~~a~~ main diagnol are zero then determinant is the product of diagonal elements

Example:

$$A = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{vmatrix}, \text{ So } |A| = 0$$

6- Sum Property:

In a determinant each element is any row or column consists of sum of two factors, then determinant can be expressed as sum of two determinants of same order

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Example

$$\begin{vmatrix} 2+2 & 5 \\ 3+1 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} + \begin{vmatrix} 2 & 5 \\ 1 & 6 \end{vmatrix}$$

$$\begin{vmatrix} 4 & 5 \\ 4 & 6 \end{vmatrix} = [(2)(6)-(3)(5)] + [(2)(6)-(1)(5)]$$

$$(4)(6)-(4)(5) = (12-15)+(12-5)$$

$$24-20 = -3+7$$

$$4 = 4$$

7- **Determinant of co-factor matrix:**

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \Delta_1 = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{23} & C_{32} & C_{33} \end{vmatrix}$$

where C_{ij} denotes the cofactor of elements a_{ij} in Δ

8- **Identity Matrix:**

The determinant of Identity matrix is always zero

Example:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, |A| = (1)(1) - (0)(0)$$
$$= 1$$

9. Determinant of Inverse:

Let A be an $m \times n$ matrix. Then A is inversible iff $\det(A) \neq 0$

Example:

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}, \quad |A| = (2)(1) - (5)(3) = -13$$

Hence A is inversible

10. Multiple-terms Property:

If elements of any row/column consists of two or more terms, then the determinant can be expressed as sum of two or more determinants

Example

$$A = \begin{vmatrix} x & y & z \\ 2x+2a & 2y+2b & 2z+2c \\ a & b & c \end{vmatrix}$$

$$= \begin{vmatrix} x & y & c \\ 2x & 2y & 2z \\ a & b & c \end{vmatrix} + \begin{vmatrix} x & y & z \\ 2a & 2b & 2c \\ a & b & c \end{vmatrix}$$

$$= 0 + 0$$

$$|A| = 0$$

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