Ф/3

(21)

$sh\,x = \dfrac{e^x - e^{-x}}{2}$

$ch\,x = \dfrac{e^x + e^{-x}}{2}$

$\begin{cases} sh\,y\,\dfrac{\partial S}{\partial x} + ch\,x\,\dfrac{\partial S}{\partial y} = 0 \\ S\big|_{x=0} = ch\,y \end{cases}$

$\gamma:\{x=0\}\,,\qquad \begin{cases} x=0 \\ y=\xi \end{cases}$

$\begin{cases} \dfrac{dX}{d\tau} = sh\,Y\,,\quad X\big|_{\tau=0}=0 \\ \dfrac{dY}{d\tau} = ch\,X\,,\quad Y\big|_{\tau=0}=\xi \end{cases}$

$ch\,X\,\dfrac{dX}{d\tau} = sh\,Y\,\dfrac{dY}{d\tau}$

$\dfrac{d(sh\,X)}{d\tau} = \dfrac{d}{d\tau}(ch\,Y) \iff sh\,X + const = ch\,Y,$

$sh\,X - ch\,Y \equiv const\,,\qquad \tau=0\quad -ch\,\xi \equiv const$

$sh\,x - ch\,y$ — 1-й интеграл

$S(x,y) = F(sh\,x - ch\,y)$

$S\big|_{x=0} = F(-ch\,y) = ch\,y\,,\qquad z = -ch\,y$

$\Rightarrow\qquad F(z) = -z$

$S(x,y) = -(sh\,x - ch\,y) = -sh\,x + ch\,y$

(24)

$$\begin{cases} \dfrac{\partial u}{\partial t} + u \dfrac{\partial u}{\partial x} = 0 \\ u|_{t=0} = x \end{cases}$$

$$\begin{cases} \dfrac{dX}{dt} = U \\ X|_{t=0} = x^0 \\ \dfrac{dU}{dt} = 0 \\ U|_{t=0} = x^0 \end{cases}$$

$\Rightarrow X = x^0 t + x^0$

$\Rightarrow U = x^0$

$$x = x^0 t + x^0 \Rightarrow x^0 = \dfrac{x}{t+1}$$

Ответ:

$$u(t, x) = \dfrac{x}{t+1}$$