note

- [x] means `Yes` / `True` option
- [ ] means `No` / `False` option

since github don't support latex symbol, here have a png version.

q1

(a)

- [x] i
- [x] ii
- [ ] iii
- [ ] iv
- [x] v
- [x] vi

(b)

No.

Because only one instance here.

> When saying one functional dependencies that **hold** on schema $r(R)$, it means that in every legal instance of $r(R)$ it satisfies the functional dependency.

q2

(a)

iv

(b)

- [x] i ($Q \rightarrow U \rightarrow V$)
- [ ] ii
- [x] iii ($Q \rightarrow U \Rightarrow QS \rightarrow SU \rightarrow T$)
- [x] iv ($QS \rightarrow SU \rightarrow R \rightarrow W$)
- [x] v ($(PQ \rightarrow S) \wedge (Q \rightarrow U) \Rightarrow PQ \rightarrow SU \rightarrow R \rightarrow W$)
- [ ] vi

Here, "$\wedge$" means logical conjunction "and".

(c)

- [x] .

(d)

- [ ] .

q3

(a)

First, $AB \rightarrow A,\ \ AB \rightarrow B$

$(AB \rightarrow B) \wedge (B \rightarrow CD) \Rightarrow AB \rightarrow BCD$

Therefore, $AB \rightarrow ABCD$

(b)

Let $F = \{A \rightarrow B,\ B \rightarrow CD,\ E \rightarrow F\}$

- [x] i

$AB \cap BCD = B$ and $B \rightarrow BCD\ \in F^+\ \Rightarrow AB\ and\ BCD$ is lossless

Furthermore, $\ AB \cap EF,\ BCD \cap EF = \varnothing$

- [x] ii

$F_1 = \{A \rightarrow B\}, F_2 = \{A \rightarrow B\}, F_3 = \{A \rightarrow B\}$

And thus, $F' = F_1 \cup F_2 \cup F_3 = F$,which means, of course, $F'^+ = F^+$.

(c)

- [ ] i
- [x] ii

(d)

- [x] i
- [ ] ii

q4

(a)

$candidate\ keys = \{RS\}, \{PQ\}$

(b)

- [ ]

(c)

- [ ] i
- [x] ii
- [x] iii
- [ ] iv
- [ ] v

(d)

- [x]

(e)

- [x] i
- [x] ii
- [x] iii
- [x] iv
- [x] v

(f)

$\mathcal{E}$ itself is 3NF.

(g)

$R \to P$ violates BCNF, thus we break $\mathcal{E}$ in two parts:

- $\mathcal{E}_1(RP)$, corresponding functional dependencies: $F_1 = \{R \to P\}$. Now, $\mathcal{E}_1$ is in BCNF.
- $\mathcal{E}_2(RQS)$, $F_2 = \{S \to Q\}$, F_2 violates BCNF

Break $\mathcal{E}_2$ into $\mathcal{E}_3(SQ)$ and $\mathcal{E}_4(SR)$

$\mathcal{E}_3$ is in BCNF but $\mathcal{E}_4$ does not, so we continue break $\mathcal{E}_4$ into: $\mathcal{E}_5(S),\ \mathcal{E}_6(R)$

Finally got one decomposition: $\mathcal{E}_1(RP),\ \mathcal{E}_3(SQ),\ \mathcal{E}_5(S),\ \mathcal{E}_6(R)$

To show this decomposition has as few tables as possible, one could see that the rest FDs which violates BCNF is $S \to Q$.

And the decomposition from $S \to Q$ leads to similar results except the difference in letters(R/S, P/Q). Thus both decomposition have the same number of tables .