

PCA – przykład

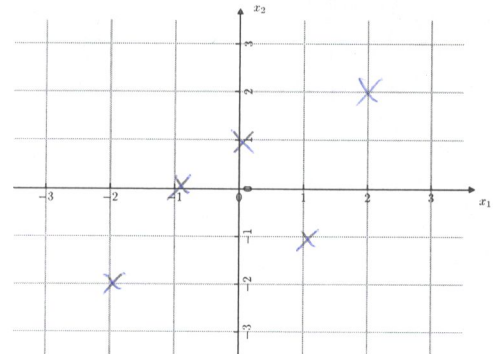
Dane:

	x_1	x_2
	-2	-2
	-1	0
	0	1
	1	-1
	2	2
avg	0	0

Macierz kowariancji:

	x_1	x_2
x_1	2.5	1.75
x_2	1.75	2.5

Wykres:



Miejsce na obliczenia:

$$\text{COV}(x_1, x_1) = \frac{(-2)^2 + (-1)^2 + 1^2 + 2^2}{5-1} = \frac{10}{4} = 2.5 = \text{COV}(x_2, x_2)$$

$$\text{COV}(x_1, x_2) = \frac{-2 \cdot 2 + (-1) \cdot 0 + 0 \cdot 1 + 1 \cdot (-1) + 2 \cdot 2}{4} = \frac{7}{4}$$

EVD

$$\begin{vmatrix} 2.5-\lambda & 1.75 \\ 1.75 & 2.5-\lambda \end{vmatrix} = (2.5-\lambda)^2 - 1.75^2 = 6.25 - 5\lambda + \lambda^2 - 3.0625 = \lambda^2 - 5\lambda + 3.1875$$

$$\Delta^2 = b^2 - 4ac = 25 - 12.75 = 12.25$$

$$\Delta = 3.5$$

$$x_2 = \frac{5-3.5}{2} = 0.75 \quad x_1 = \frac{5+3.5}{2} = 4.25$$

$$K_1 = \begin{bmatrix} -1.75 & 1.75 \\ 1.75 & -1.75 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 1.75 & 1.75 \\ 1.75 & 1.75 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix}$$

Posortowane malejąco wartości i wektory własne (znormalizowane):

$$\lambda_1 = 4.25$$

$$\lambda_2 = 0.75$$

$$k_1 = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$k_2 = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix}$$

$$K = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$$

Dane po PCA:

$$Y = XK =$$

	y_1	y_2
	$-\sqrt{2}$	0
	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
	0	$\sqrt{2}$
	$\sqrt{2}$	0

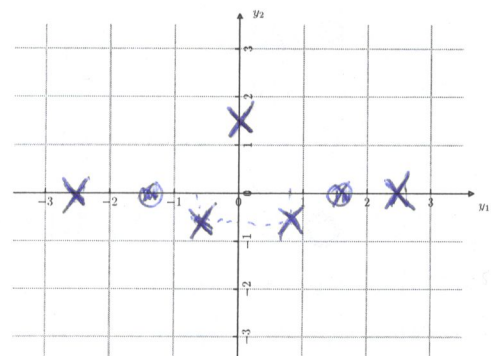
Macierz kowariancji:

	y_1	y_2
y_1	4.25	0
y_2	0	0.75

$$\text{COV}(y_1, y_1) = \frac{8 + \frac{1}{2} + \frac{1}{2} + 8}{4} = \frac{17}{4}$$

$$\text{COV}(y_2, y_2) = \frac{\frac{1}{2} + \frac{1}{2} + 2 + 4}{4} = \frac{3}{4}$$

Wykres:



$$y_1(x_1, x_2) = \langle k_1, x \rangle = \langle k_1, x_1 \rangle x_1 + \langle k_1, x_2 \rangle x_2$$

$$y_2(x_1, x_2) = \langle k_2, x \rangle = \langle k_2, x_1 \rangle x_1 + \langle k_2, x_2 \rangle x_2$$