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Master's thesis

Modeling Realistic Human-like Machine Decision Maker

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Modelowanie realistycznego zachowania symulowanych decydentów

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Abstract

Nowadays, we observe a continuous increase in the demand for systems that automate, optimize processes, support decisions, and even assist in everyday tasks - generally understood Artificial Intelligence (AI). These surround us, support us in everyday situations, and co-create a significant part of our environment. Nevertheless, in order to operate efficiently, they require interaction with a human being, whose decisions, due to his various ailments, limitations and imperfections, are often difficult to predict or justify from a rational point of view. In this thesis, we present theoretical decision models based on discoveries from the background of cognitive science, behavioral economics, and psychology. We describe decision systems, heuristics, and biases, which significantly influence the decisions taken. The whole was also implemented in the form of an extensible model that allows modeling the Real Decision Maker's behavior and the use of, e.g., in the optimization of processes that require interaction with a Decision Maker. It was also presented on two pictorial examples using its specific properties and capabilities.

Streszczenie

W dzisiejszych czasach obserwujemy ciągły wzrost zapotrzebowania na systemy automatyzujące, optymalizujące procesy, wspomagające decyzje, a nawet asystujące w codziennych zadaniach – ogólnie pojętą Sztuczną Inteligencję (*ang. Artifical Inteligence (AI)*). Te otaczają nas, wspierają w codziennych sytuacjach i współtworzą znaczną część naszego środowiska. Niemniej jednak, do sprawnego działania wymagają interakcji z człowiekiem, którego decyzje ze względu na jego różne przypadłości, ograniczenia oraz niedoskonałości, są często ciężkie do przewidzenia czy uzasadnienia z racjonalnego punktu widzenia. W pracy przedstawiamy teoretyczne modele decyzyjne bazujące na odkryciach z podłoża kognitywistki, ekonomii behawioralnej oraz psychologii, opisujemy systemy decyzyjne, heurystyki, a także skrzywienia (*ang. biases*), które w znacznym stopniu wpływają na podjęte decyzje. Całość została również zaimplementowana w postaci rozszerzalnego modelu pozwalającego na zamodelowanie zachowań Rzeczywistego Decydenta oraz wykorzystanie m.in. w optymalizacji procesów wymagających interakcji z decydentem, a także zaprezentowana na dwóch obrazowych przykładach wykorzystujących jego określone właściwości i możliwości.

Contents

1 Introduction

Decision making is an obligatory and inevitable part of nearly every conscious man's life, at every level. We live in times when decision process effectiveness is a crucial aspect in every well prospecting enterprise, the pace of decision-making, and its relevance is often determining the success of the investment. In contrast, one ill-considered decision can destroy years of achievements.

However, decision making is present not only in life matters processes. Every non-trivial process at some level requires a decision, for example:

- how to organize the order of our actions and plans for short or long term perspective,
- what flowers to buy for our mom/sister/wife/partner as a gift? How many, what color, maybe some combination with other flowers?
- where to go on vacation, how much to spend on it, whether to buy a trip or organize it yourself,
- whether to invest some funds with high risk and some promise of possible extra profit in the future, or to invest with low risk and low, but certain income?
- whether the current offer is a bargain or a trick?
- whether to cut losses and wrong investments or count on the very unlikely return of circumstances and possible income?

We would like to know the future and the consequences of our decisions to make the best one from our point of view. Unfortunately, we probably never will, so at least we would like to make the best possible decision basing on our current knowledge, experience, and intuition. We are also not alone in this problem;

- there are multiple tools which support our decisions,
- we can ask others for help,
- we can pay someone to decide for us.

Nevertheless, every solution has a weak point – a human, even in the first case, because we need to pass correct data to the system, execute the process, and finally well interpret the result. In other cases, the field for mistakes is much more significant.

Not without reason, there is the concept of *the human factor* in plane crashes or other mishaps. We are unpredictable, fragile, and volatile machines, which can affect on a great scale. Without a valid preparation, increased responsibility can multiply the risk of failure in a non-linear, if not exponential, scale. Just an awareness of this responsibility and uncertainty may impact on our decisions, making them unreasonable and unjustified.

We stated that we have something in common with machines. Every machine (at least for now) needs some program. Every program *have* some bugs, and so do we – we are not perfect. Most of us are easy to manipulate, have limited awareness, and very narrow knowledge. Our mental resources are also precious. Our life experience and the current material situation make every one of us different, not only in terms of behavior and appearance but also in our decisions.

Machines have many advances. One of them is that their procedures are (or at least ought to be) repeatable and predictable, depending on the input parameters. Second, their time is cheap, and yet they work fast. This is very different from most of us; we value our time, yet we often work slowly, not mentioning that we also need breaks. Our decisions can differ without reason. We know that from experience, but is it really obvious for us?

We expect various systems to support us in certain areas. However, we hardly ever let them decide without our control, particularly when the high cost (e.g., human life) is considered, even when they have better accuracy than humans. Nevertheless, we live in times when the share of Artificial Intelligence (AI) is increasing to the not negligible factor. Those systems are theoretically possible to evaluate, explain, and predict – research in this area is also continuously progressing.

To work well and cooperate, these systems need to be aware of others' moves and decisions. They need to control and understand the environment and decide in advance in real-time. Now, we are getting to the point where the key problem is that this environment contains one important factor we need to consider. One, but on a vast scale, in great number, always changing, hardly ever predictable and rational – human.

1.1 Motivations

As stated earlier, a human becomes the weakest part of the modern constantly evolving world of technique, computerization, automation, and AI – because of our weaknesses and biases. Many of them were already named, but we still do not know what more to expect. During years of researches, specialized branches of science were extracted, like the behavioral economy or cognitive science. These are trying to understand the mentioned issues, categorize, and prevent them. The problem is important to the extent that there are surveys focused on certain biases in a given area, to name just a few:

- **Cognitive Biases and Their Impact on Strategic Planning** [BJ84] describes issues during managers and planners everyday work,
- **Cognitive Biases and Heuristics in Medical Decision Making: A Critical Review Using a Systematic Search Strategy** [BBK15] where the authors study cognitive bias and heuristics in both patients' and healthcare professionals' decisions,
- **Cognitive Bias Modification: Past Perspectives, Current Findings, and Future Applications** [HM11] which describes how employing emotions and biases can impact on attention, memory and learning capabilities.

One of the most known scientists in this area is Daniel Kahneman, a recipient of the Nobel Prize in Economic Sciences. In 2011 he wrote the world bestseller *Thinking, Fast and Slow* [Kah11] – one of the most significant positions in this area, where he presents two systems, which describe our mental capabilities:

- **System 1**, which is responsible for immediate reactions, does not require much thinking and focus (so it is cheap), and at the same time is responsible for most of our vital activities,

- **System 2**, which allows us to analyze complex things, learn and innovate – but at the high cost of time and mental fatigue.

As we can see, System 1 is both our savior and bane because it allows us to react, make several things at once (like talking, walking, and scratching our head), but this is also a source of our unpredictability and fragility. We need it to respond and decide fast but at the cost of shallow analysis and often irrational choices.

System 2 is a source of our thought through decisions and often important life choices but at a high cost. It is also easy to say when we use System 2. For example, many of us, when starting to use it, look upright, have enlarged pupils, or a higher heart rate. This part of us is a precious and very limited resource, and therefore when we work for a long time – the question arises, whether we work effectively. This and other topics were breathy and accessibly described by Linda Rising during her talk at GOTO 2019 conference [Ris19], but also in several articles [JEK16, Loc12].

This knowledge is getting to a broader audience. We can often hear about cognitive biases at business, marketing, political, or even IT conferences. However, most of them are affected superficially. Many speakers are trying to give the impression of experts and give the audience a feeling that they are becoming them too because of the subject's simplicity. It is good to raise people's awareness, but the problem lies in the way they are doing it; at the same time, they are employing another of our cognitive bias – overconfidence and illusion of understanding. Even Kahneman said that even though he examined many biases and wrote about them, he does not know them all, and still cannot prevent them.

Since these biases are crucial in our decisions, social interactions, and life choices, we would like to predict them somehow or at least provide a model capable of presenting the impact caused by some of these biases at given conditions, and at the same time, being accessible to the user. However, there is a problem, which we stated earlier – these are hard to predict because of potential lack of rationality without broader context knowledge, like the person fatigue, unexpressed preferences, or mental limits.

At this point, we see that popular conventional decision-supporting tools are not sufficient – they ought to support us in the decision-making process. However, we have our own choices and reasons, which we are often unaware of, so we cannot express them even with the most sincere desire. Things are getting worse when we want to interact with the Decision Maker, e.g. to predict his choices for marketing purposes, to optimize the queering his preference for both the best process efficiency and his experience, or to integrate this knowledge with AI solutions operating in the real-world environment.

1.2 Existing solutions and ideas

The topic of utilization, consideration or prediction of human behavior and biases impact in decisions is rather a niche, nevertheless continuously evolves since 50' first cognitive biases researches, so obviously, some work was already done in both theoretical and practical bases and appliance. To list just a few of them from recent years:

- **Toward a Synthesis of Cognitive Biases: How Noisy Information Processing Can Bias Human Decision Making** [Hil11] that, among others things, focuses on noisy information processing and its impact on decision making.
- **Modeling Behavior-Realistic Artificial Decision-Makers to Test Preference-Based Multiple Objective Optimization Methods** [JBS15] is worth mentioning theoretical research, where several biases are recognized as key factors in the real human decision-making process from a mathematical point of view.
- **Towards Automatic Testing of Reference Point Based Interactive Methods** [VOM16], where authors propose Artificial Decision Maker focused on preference information as a reference point.
- **Interactive Multiobjective Optimization: A Review of the State-of-the-Art** [XCC+18] which focuses on comparing existing interactive optimization methods in decision-making processes.

The problem of Artificial Real-Decision Maker utilization in the process occurs in various processes, especially in optimization [LIK15, SGR15, KCT17, TK19] – therefore, we need a model which allows us to simulate the decision-making process with a true real-world Decision Maker, especially:

- with all his biases and weaknesses about we know and we do not – thus it should be possible to enable or disable them, or add new ones in the future;
- everyone is prone to certain factors in different to varying degrees, so this model should be configurable taking this into account;
- each of us has a different value system, rules, and needs, so this model should also reflect this;
- finally, we would like to know which bias influenced the decision the most.

Results of work [VOM16] at the first look seem promising, but this solution, unfortunately, is quite limited (fixed biases approach and its processing), not extensible, and a kind of a black-box, so it does not meet our requirements. At this moment, we are not aware of any more novel approach, which [XCC+18] also confirms. Therefore, we decided to propose our own solution able to fulfill our current needs and extensible to the extent of allowing further research with the less possible effort in implementation and customization.

1.3 Thesis organization

This thesis is organized as follows. Section 2 presents some of the possible algorithm implementations and the chosen one. In Section 3, we introduce and describe the components of the algorithm and its possible variants. Section 4 provides examples of use cases for the prepared algorithm. Finally, in Section 5, we summarize the goals of the thesis and its achievements.

2 Model structure

We want our decisions made by the model to be reliable, somehow argued, and possible. In general, applying biases and heuristics in our decision-making process can be done in two ways:

- **before decision**, so we should somehow manipulate entry data to influence the final result;
- **after decision**, so we assume some verdict and then try to change it taking into account some conditions.

Clearly, we can also assume a third mixed solution, but first, we ought to describe the mentioned ones. For this consideration, let us assume that we have some rational and repeatable algorithm(s) that, based on the same data and parameters, evaluates them giving results in descending preference order.

With the **before** variant (Equation (1)) we can receive *always* rational decision because the evaluating algorithm has this ability – we only change the conditions on which it operates, therefore it is also repeatable for the same input data. There is also much less space for uncertainty – because of input parameters change, we can possibly receive a different decision. We think it is also more realistic – we decide based on some knowledge, assumptions, or justification. Of course, we often change our mind, but this process needs to be somehow initialized, so probably conditions have changed or the gap between variants was so narrow that we truly did not decide before, due to not finished evaluation – due to the lack of some information.

$$Input \ \rightarrow Biasing \rightarrow Rational \ Decision \tag{1}$$

In the second **after** variant (Equation (2)), we are assuming modification of given rational decision, which is based on not modified client data. At this point, we need to notice that we lose a dimension of alternatives in the process, but instead, we gain final evaluation, which however differs depending on the evaluation method, and is not necessarily expressed as a number value, but for example as some dependency graph. Even if we would like to transfer alternatives data further or differences between pairwise evaluations on them, it would be hard to reevaluate it further in each case to keep it consistent.

$$Input \ \rightarrow Rational \ Decision \rightarrow Biasing \tag{2}$$

After these considerations, it looks obvious that we chose the first option, whereas the second still can be applied in some conditions and possibly give better results. However, it would also complicate the solution architecture, and we did not find it necessary.

2.1 Biases appliance

One of our goals was a transparent, configurable, and extendable solution. Every human is different and thinks in different ways. Therefore, we came to the solution of *biases pipeline* – multiple biases can be applied, where order matters and the result of the preceding one is the input for the next one. Each of them has his own parameters and additionally returns his own results that present the changes made on the input set and other helpful information.

2.2 Model parameters

Our model needs to know the following parameters:

- **all known alternatives** that form the context of the decision. As an example, we can imagine shopping in a store where we see every possible option.
- **alternatives under consideration**, which are identifiers of alternatives between we should make a choice or rank them. In our previous example, these would be our favorite products or the ones we can afford.
- a list of **criteria** we are considering with their types (*gain* or *cost*) and optionally a known values range. In our example, it would be a taste and smell, which are of type *gain*, and price as a *cost*, which we know that can be between \$1 and \$5.
- a list of **biases** to apply in the process – for example, we can specify that our Decision Maker is somehow anchored to the *Coca-Cola*, is so tired that he cannot distinguish apple and orange juice, and also did not tell us that he has a fourth criterion, which is a type of wrapping.
- name of the **preference function** for the final decision, for example *OWA* or *weighted sum*.
- **parameters for the preference function** which are dependant on the chosen function, so for example weights for every criterion or thresholds.

2.3 Convention

In this work we will use following naming convention:

- A – a finite set of alternatives,
- a_i – alternative with index i,
- G – a finite set of criteria,
- g_j – criterion with index j,
- n – a number of alternatives,
- m – a number of criteria,
- $\hat{a}/\hat{g}$ – a modified or generated alternative/criterion,
- $U(a_i)$ – the utility of alternative with index i,
- $I(g_j)$ – the importance (explained later) of criterion with index j.

2.4 Code base

The implementation is open-sourced and can be found in the GitHub repository [Kup] along with a detailed description and documentation. There are also examples for each method/heuristic with some biases applied.

3 Implemented preference functions, heuristics and biases

In this section, we will:

- briefly describe preference methods that are implemented,
- focus on some decision-making heuristics which are replacements for rational well-known preference functions,
- describe popular biases,
- depict how we approached and implemented biases simulation.

However, we will describe here neither implementation details nor parameters – these are only mentioned and more precisely described in the documentation in the code repository.

3.1 Preference methods

In this section, we briefly describe four provided well-known preference methods, sorted in ascending order of complexity.

3.1.1 OWA

Ordered Weighted Averaging aggregation operator [OWA, Ful03] allows arbitrary operator to be used to obtain the utility of a given alternative. We provided *max*, which zips the weights with criteria values, descending like presented in Equation (3).

$$\begin{gathered} OWA(a_i) = \sum_{j=1}^{m} w_j \cdot a_i(g_j) \\ w_x \in W,\ \|W\| = \|G\| \\ \forall_{j,k} j < k \implies w_j \geq w_k,\ a_i(g_j) \geq a_i(g_k) \end{gathered} \tag{3}$$

The alternatives are ranked in order of decreasing result value. An example is presented in Equation (4).

$$\begin{aligned} weights &: (descending)\ w_1 : 5, w_2 : 3, w_3 : 2 \\ \\ a_1 &: 10(g_1), 15(g_2), 3(g_3) \\ U(a_1) &: a_1(g_2) \cdot w_1 + a_1(g_1) \cdot w_2 + a_1(g_3) \cdot w_3 = 75 + 30 + 6 = 111 \\ \\ a_2 &: 15(g_1), 5(g_2), 8(g_3) \\ U(a_2) &: a_2(g_1) \cdot w_1 + a_2(g_3) \cdot w_2 + a_3(g_2) \cdot w_3 = 75 + 24 + 10 = 109 \\ \\ & a_1 > a_2 \end{aligned} \tag{4}$$

3.1.2 Weighted Sum

Weighted Sum [Fis67] generates a single value utility for each alternative based on the sum of the products of the criterion value and its weight, like in Equation (5).

$$WS(a_i) = \sum_{g_j \in G} w_j \cdot a_i(g_j) \tag{5}$$

where w_j is a weight matching criterion g_j. The final ranking is created based on descending utility values. Equation (6) presents the algorithm for the same as in OWA alternatives.

$$\begin{aligned}
weights &: w(g_1) : 3, w(g_2) : 2, w(g_3) : 5 \\
\\
a_1 &: 10(g_1), 15(g_2), 3(g_3) \\
U(a_1) &: a_1(g_1) \cdot w(g_1) + g_2 \cdot w(g_2) + g_3 \cdot w(g_3) = \\
& 30 + 30 + 15 = 75 \\
\\
a_2 &: 15(g_1), 5(g)2), 8(g_3) \\
U(a_2) &: a_2(g_1) \cdot w(g_1) + a_2(g_2) \cdot w(g_2) + a_2(g_3) \cdot w(g_3) = \\
& 45 + 10 + 40 = 95 \\
\\
& a_2 > a_1
\end{aligned} \tag{6}$$

Both Weights Sum and OWA are naturally used in simple decision-making processes.

3.1.3 Choquet integral

Choquet integral [Cho54] tries to simulate interactions of criteria based on the idea that a union of criteria can have higher (or lower) utility than all of them independently. Each criteria subset has its own weight $\in [0, 1]$, also its values should have similar ranges; given subset matches only for the difference of the minimum range of its criteria values and the more general subset. The algorithm is presented in Equation (7); in Equation (8) we present an example.

$$\begin{gathered}
CI(a_i) = a_i\hat{g}_m \cdot w(\hat{g}_m) + \sum_{j=1}^{m} ((a_i(\hat{g}_{j-1}) - a_i(\hat{g}_j)) \cdot w(\hat{g}_{j-1})) \\
\forall_{j \in [1..m]} \|\hat{g}_j\| = j \wedge a_i(\hat{g}_j) = \min_{g_k \in \hat{g}_j}(a(g_k)) \\
\forall_{j,k \in [1..m]} j < k \implies a_i(\hat{g}_j) \geq a_i(\hat{g}_k)
\end{gathered} \tag{7}$$

$$
\begin{aligned}
weights: \; & w(g_1): 0.1, w(g_2): 0.2, w(g_3): 0.5, \\
& w(g_1, g_2): 0.4, w(g_1, g_3): 0.6, w(g_2, g_3): 0.4, \\
& w(g_1, g_2, g_3): 0.3
\end{aligned}
$$

$$
\begin{aligned}
a_1: \; & 10(g_1), 15(g_2), 3(g_3) \\
U(a_1): \; & min(a_1(g_1), a_1(g_2), a_1(g_3)) \cdot w(g_1, g_2, g_3) + \\
& (min(a_1(g_1), a_1(g_2)) - min(a_1(g_1), a_1(g_2), a_1(g_3))) \cdot w(g_1, g_2) + \\
& (a_1(g_2) - min(a_1(g_1), a_1(g_2))) \cdot w(g_3) = \\
& 3 \cdot 0.3 + (10 - 3) \cdot 0.4 + (15 - 10) \cdot 0.2 = 4.7
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
a_2: \; & 15(g_1), 5(g_2), 8(g_3) \\
U(a_2): \; & min(a_2(g_1), a_2(g_2), a_2(g_3)) \cdot w(g_1, g_2, g_3) + \\
& (min(a_2(g_1), a_2(g_3)) - min(a_2(g_1), a_2(g_2), a_2(g_3))) \cdot w(g_1, g_3) + \\
& (a_2(g_1) - min(a_2(g_1), a_2(g_3))) \cdot w(g_1) = \\
& 5 \cdot 0.3 + (8 - 5) \cdot 0.6 + (15 - 8) \cdot 0.1 = 4
\end{aligned}
$$

$$
a_1 > a_2
$$

This method was also used by Tversky and Kahneman in their formulation of cumulative prospect theory [TK92].

3.1.4 Electre III

The method was introduced by B. Roy [Roy91] and is one of the methods which allows us to obtain incomparably. It is based on four coefficients for each criterion:

- **indifference** threshold $q_i(a)$,
- **preference** threshold $p_i(a)$,
- **veto** threshold $v_i(a)$,
- **weight**

Thresholds should be in relation $0 \leq q_i(a) \leq p_i(a) < v_i(a)$ and can be extended to linear functions with given alternative's criterion value as a parameter.

These coefficients are used to extract, for each directed alternatives pair a_i, a_k, concordance index $C_{q_k}(a_i, a_j)$ for the thesis that a_i is in outranking relation $a_i S_{g_k} a_j$ to a_j on criterion q_k. Received values are later aggregated for each directed pair, based on Equation (9), which stands for *concordance test*.

$$
C(a_i, a_j) = \frac{\sum_{x=1}^{m} k_n C_x(a_i, a_j)}{\sum_{x=1}^{m} k_x} \tag{9}
$$

Further, we need to proceed *discordance test* which verifies that there is no criterion critically worse on a_i than on a_j. As the result, we receive discordance indices $D_{q_k}(a_i, a_j)$, later utilized with the *credibility coefficient of the outranking relationship* $\sigma(a_i, a_j)$ calculated like in Equation (10).

$$\begin{aligned} \sigma(a_i, a_j) &= C(a_i, a_j) \prod_{x \in F} \frac{1 - D_x(a_i, a_j)}{1 - C(a_i, a_j)} \\ F &= \{x : D_x(a_i, a_j) > C_x(a_i, a_j)\} \end{aligned} \tag{10}$$

Finally, we need to create two partial preorders that utilize $\sigma(:)$ matrix: *ascending* and *descending* with specialized for each distillation (exploitation) method. Having these, we can receive *median preorder*, where alternative position bases on the rank from the partial preorders when these matches, and where they not, by the differences in both partial preorders. The result is presented as a Hasse diagram [HAS] due to possible incomparabilities – retrieved ranking might create other than linear structures because of the conflicts between partial preorders.

For the more detailed method description and examples please refer to the method overview, which co-author is also the method's creator [FGRS13].

3.2 Bounded rationality heuristics

Before we dive deeply into heuristics description, let us first confirm their existence, find their source and why do they exist.

First, there are two conflicting key concepts:

- **Hommo economus**, which says that human is always rational and aims to increase earnings from the economic point of view. It is also assumed that human has full knowledge about his environment, his value system is well organized, monotonic, and consistent. Finally, he can evaluate the risk and return of investment of each choice in a structured way.

- **Bounded rationality** states that human *does not* aim at optimal solutions. Instead, he strives for *good enough* choices, which means there is a trade-off between environment complexity and his aspirations. Therefore, his choices are often not consistent with the mathematical value model because of the length of the process, the multiplicity of choice, or criteria complexity. It also turns out that Decision Makers often do not have a declared value system – this can change even during the process, or they can decide ad hoc, which sometimes leads to random choices.

As we can see, *bounded rationality* looks more realistic when we consider our or others' behavior in a scenario of typical life choices. To give a simple example, let us bring back a situation with juice selection in the shop. Which one will we choose? The one with a nice wrapper or a little bit cheaper, but with a much worse look? Do we look at the capacity or just the size of the packing? Are we looking at nutritional facts? How do we select the best one? Making a full comparison, or one by one in a previously limited set?

All of these are examples of behaviors most of us are familiar with. Some of them are unique for each of us, but there are also some common, well-known parts. These have been examined, categorized, and named. These are also fast, practical, and somehow rational due to experience, knowledge, or some higher rules contained in them, but unfortunately often do not provide optimal solutions. We call them *heuristics* [Mar02, GB09, GG11].

We decided to provide three the most popular heuristics, which we later describe in detail:

- Satisfaction heuristic,
- Aspect elimination heuristic,
- Majority heuristic.

Because of heuristics' characters and roles, these are provided as a replacement for rational preference functions. There are also researches [ZP10, LH12] which tries to make the use of multiple heuristics used in the single process somehow connected (chaining, weighting, etc.). With our solution, even these can be satisfied by a new heuristic or a biases integration – for example, we can name some new aggregating function e.g. *custom*, and as its parameters pass components required for each of the selected heuristics.

3.2.1 Satisfaction heuristic

Satisfaction heuristic was introduced by Herbert A. Simon in 1956 [Sim56]. This heuristic can be described with the following rules:

- Alternatives order is not said, but it matters;
- The Decision Maker has predefined thresholds for each criterion, which certain alternative have to satisfy;
- If a given alternative has worse value on any criterion than the reference criterion value, it is rejected;
- First alternative, which is good enough on every criterion, is considered as a choice.

In Figure 3.1 we can see a basic example for these rules. Since the search order matters (left to right), the first checked alternative is a *circle* based – it is below the threshold on g_2, and therefore is discarded. The next one is a *square* based, which has criteria values a little bit above the threshold – this is enough for it to be selected. In the base scenario we even do not consider the third, *triangle* based alternative, which is much better than *square* based; in the ranked version, it is placed in the second place.

As we can see, this heuristic is single alternative-centric – it evaluates just a single one at the time. Therefore, there can be no conflicts between alternatives; the first one is always the winner. We can also extend this heuristic to provide the final ranking of alternatives, but this requires some additional thresholds for those alternatives that did

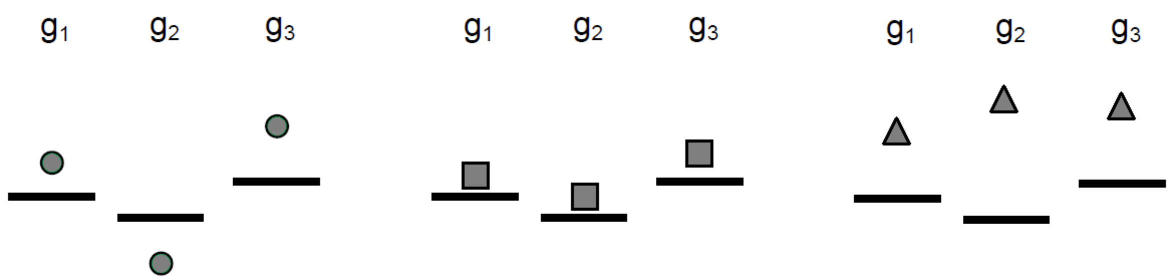

Figure 3.1: Example for *Satisfaction heuristic* decision-making process.

not meet initial thresholds. These are in later positions, depending on the threshold they meet and other alternatives.

Mentioned thresholds can be either defined by the Decision Maker or generated in some rules, but these should – in general, have *decreasing* character, because it should reflect limiting of our expectations.

In our implementation we provided three types of thresholds:

- *provided*,
- generated by the *ratio multiplication*,
- generated by the *ratio subtraction*,

where *ratio* means some coefficient in the range [0, 1] which is used to extract new thresholds based on the criterion values range, using the difference between its upper and lower bounds, with respect to additionally passed *min* and *max* value. In Equation (11) we can see examples for both ratio multiplication and subtraction calculation.

$$\begin{aligned} criterion\ values\ range &: [1, 2] \\ min\ value &: 0.4 \\ max\ value &: 0.9 \\ \\ coefficient &: 0.8 \\ multiplication &: 1.9, 1.72, 1.576, 1.4608 \\ \\ coefficient &: 0.2 \\ subtraction &: 1.9, 1.7, 1.5 \end{aligned} \tag{11}$$

3.2.2 Aspect elimination heuristic

This heuristic was firstly described by Amos Tversky in 1972 [Tve72]. It is in direct opposition to the previously described in Section 3.2.1 *Satisfaction heuristic*:

- In this heuristic we eliminate alternatives till the single one remains;
- Alternatives are examined criterion-wise – each criterion has assigned a weight which specifies its ordering;

- If the given alternative does not meet certain criterion requirements, it is rejected;
- When all alternatives meet the given criterion, the next in order criterion is considered;
- When all criteria were examined and still there is more than one alternative, aspirations need to be increased.

As in *Aspect elimination heuristic*, we are exploring alternatives criterion-wise and eliminates these alternatives, which do not meet any of our conditions. It is possible that some criteria are not considered at all. This approach violates the normative theories because only partial alternative information is taken into account during the decision-making process, which is also shown in Figure 3.2. In this example, g_3 is such a criterion. Here, we have three alternatives: *circle*, *square* and *triangle*. Nearby every figure, there is a signed evaluation order, rejected alternatives are crossed out at the moment and after rejection. First, we examine g_1, where *circle* has not sufficient value, even though we see that it is much better than others on the g_2 – it is rejected anyway. The next criterion is g_2, where we no longer consider *circle* as an option, and *triangle* is below our expectations. Therefore, the only alternative that meets our conditions is *square*.

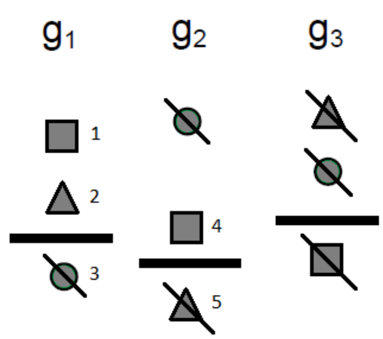

Figure 3.2: Example for *Aspect elimination heuristic*.

Same as in *Satisfaction heuristic*, this one is also alternative-oriented. However, the thresholds mentioned before should have *increasing* character – we exaggerate our expectations, because these were too easy to meet. Similarly to the *Satisfaction Heuristic*, we provided three types of thresholds: *provided*, increasing by the *ratio multiplication* and increasing by the *ratio addition*. Equation (12) presents example for the last two of them.

$$\begin{aligned}
criterion\ values\ range &: [1,2] \\
min\ value &: 0.4 \\
max\ value &: 0.9 \\
\\
coefficient &: 0.2 \\
multiplication &: 1.4, 1.68 \\
\\
coefficient &: 0.2 \\
addition &: 1.4, 1.6, 1.8
\end{aligned} \tag{12}$$

Worth to notify is that the coefficient for *multiplication* is indeed $1 + x$, where x is a passed value – we aim to increase the next value.

3.2.3 Majority heuristic

Majority of confirming dimensions heuristic (full name) which is, from our point of view, the most commonly used heuristic, was originally introduced by J. E. Russo and B. A. Dosher in 1983 [RD83], and it is very similar to the already described in Section 3.1 preference function – Weighted Sum:

- Alternatives are compared pair-wise criterion by criterion;
- The alternative which has better value for a given criterion is considered to be better – on a given criterion; the difference in the value of the criteria is not considered;
- Additionally, each criterion can have a weight assigned – ultimately not every criteria are equal, even from our limited point of view. We might want this weight to change the result in case of a draft, or to model that the better evaluation on one criterion is much more important than in two others taken together. We can also imagine that criteria weights change based on the current criterion level. As an example let us consider criterion value $\in [0, 1]$. For Decision Maker comparison of 0.1 and 0.2 could have a different impact than comparing 0.4 with 0.5 or 0.8 with 0.9, even when their differences are equal. This can be a worth considering extension to the original idea for the further development
- Winning alternative is considered with the next one in the same manner, so ordering matters. Therefore, if we are willing to create a ranking, we can estimate the final position basing on the *last in removal order* – the winner is not removed, the second was removed recently, the third before the second, etc.

In Table 3.1 we present an example that shows that the *Majority heuristic* violates preference transitivity because of comparing order. Let us consider three alternatives $a_{1..3}$ and three equally weighted criteria of type gain $q_{1..3}$. Comparing a_1 with a_2 – a_2

wins on two criteria. Then, comparing a_2 with a_3 – a_3 is better on two criteria also. Therefore, we would assume that a_3 is the best option, and the same result is returned by the *Majority heuristic*. However, when we compare a_1 with a_3 it turns out that a_1 **is better on 2 criteria**, so finally we have $a_1 < a_2 < a_3 < a_1$.

Table 3.1: Example for the *Majority heuristic*, which presents how comparison order matters.

	g_1	g_2	g_3
a_1	1	2	3
a_2	2	3	1
a_3	3	1	2

3.3 Popular biases

So far we described rules, which finalize our decision-making processes:

- rational, predictive, based on mathematics, well-known but also often complicated preference functions,
- much more common and more or less consciously used heuristics, which are also more difficult to predict because of various volatile variables and simplifications.

This section focuses on factors that influence our decisions – biases. Till now, researchers recognized and named many of them, but a lot more is still unknown. Not negligible is that each of us is also different and thus may have his own specific biases.

Some biases are very specific, but others can be generalized or split apart to more general. We focused on well-known biases described in *Modeling Behavior-Realistic Artificial Decision-Makers to Test Preference-Based Multiple Objective Optimization Methods* [JBS15], which are:

- Omission of criteria,
- Mixing of criteria,
- Mental fatigue,
- Anchoring,
- Loss aversion.

This article also contains *bounded rationality* section, which focuses on some of already implemented and described in Section 3.2 heuristics – *Aspect elimination* and *Satisfaction* heuristics. Therefore, we will not cover these here again.

Listed above biases can be described by an algorithm, rather just expressed as a property addition like *representatives* or *availability*, which also can be generalised to *anchoring* (we have some reference point) or mentioned *omission of criteria* (we are *concealing* our additional criterion, which is *similarity*).

We analyzed these biases and then focused on how to make them more general or break them down into components that can be combined rather than strictly implement them and other known biases. But first, let us describe them.

3.3.1 Omission of criteria

This bias assumes an information gap between an algorithm (which is some final decision-making process) and the Decision Maker (or the things he expressed). This gap is caused by a lack of criteria' information and can occur on both sides, which means that

- the Decision Maker can declare some criteria, but does not consider them because of the complexity of the environment, in fact, lower importance or other criteria suppression,
$$G_{DM} \subset G_{Algorithm} \tag{13}$$
- some criteria can be internally considered by the Decision Maker, but unknown to the algorithm – he is *rationally* not aware of them.

$$G_{Algorithm} \subset G_{DM} \tag{14}$$

Now the problem arises: which criteria we can *forget* about, or do not express them? In the first case, possible solutions are the ones which are not essential for us, but there are also other options. In the other case, we can assume that the Decision Maker does not want to sabotage the process, so these criteria omission can have the same reasons.

Therefore, we can assume that each criterion can be somehow omitted with a non-zero probability. However, this probability is not equal for all – the more important criterion is, the lower probability is. This importance can often be expressed as some weight assigned to the criterion. However, probably is also somehow modeled via possible impact on the final decision, because weights in some algorithms depend on criterion values – we would expect much lower weights when comparing the variant with the same *utility measure* when we consider the criterion in the values range [0, 10.000] than we consider the range [0, 1].

3.3.2 Mixing of criteria

The motivation for the Choquet integral was to model some dependencies between criteria by influencing their preferences, when their values are intersecting, because the union of some criteria may result in increased or decreased final preference. Its usefulness was proved in the experiments of Tversky and Kahneman for modeling behaviors [TK92]. These criteria even do not necessarily be connected. Moreover, we can even say that Choquet integral aims to create or remove these connections.

Although we do not make any claims to appreciate or avoid interactions between criteria, we often consider them when making decisions. We do it even not consciously and in both ways, which creates some kind of *bonus* or *penalty* for some values and criteria interactions.

Even when modeling problems, we can introduce dependencies that we are not aware of, especially when the values of the criteria are dependent on the Decision Maker (set by him). In these cases Stewart [Ste99] suggests to introduce a criteria mixing in manner presented in Equation (15):

$$\hat{g}_k = (1 - \gamma)g_i + \gamma g_j \tag{15}$$

where $\gamma \in [0, 1]$ is a mixing parameter – we combine some criteria taking into account the share defined for each of them.

3.3.3 Mental fatigue

We stated at the very beginning that our decisions and the way we take them might differ during the process proceeds. It might be because of being overwhelmed by alternatives diversity (or similarity), criteria multiplicity, protracted process, tiredness, or general *fatigue*. Fatigue [HR97, SMMM09, ZSR19] has physical and mental dimensions, which have different sources, but unavoidably interact with each other is some degree. On a great scale, these influence the way we are making decisions or perceive things and limit our abilities, both intellectual and physical. Everyone is different and is susceptible to it to a different extent. Moreover, this changes because of other factors, like continuous lack of sleep or in general incomplete regeneration. However, we can also increase our resistance for it – at least to some degree.

3.3.4 Anchoring

In *Thinking, Fast and Slow* [Kah11] Kahneman describes his and Tversky' experiment, in which they manipulate the roulette to return 10 or 65, ask some group of students to spin it (they did not know about the manipulation), write the retrieved number, and then answer the following questions:

Whether the number of African countries, which are members of the United Nations, is greater or lower than the retrieved number?
What percentage of UN members do you think are African countries?

It turned out that persons, which got 10, answered 25% on average on the second question, whereas those, which got 65, on average answered 45%. Researchers also conducted other experiments, such as estimating Sequoia's average height or Gandhi's death age, and in all cases the results were the same – the substituted value had an effect on the response.

Described phenomenon is called *anchoring* [STH58, TK74], and has fundamentals in both System 1 (framing effect) and System 2 (correction). It describes the tendency to rely on the information offered, even unrelated to the stated problem.

In [JBS15] two types of anchoring are extracted:

- *a psychological or judgement*, which does not consider losses or gains and focuses on some value estimation, like in mentioned above examples with UN or Sequoia height;

- *reference-based* where we are referring to some known state and resist changing it. The reference point can also change in time, even during the single question answering. It can adjust to some value(s) from the past with some coefficients and delays.

3.3.5 Loss aversion

Let us consider the following example, again from [Kah11]:
Decision I: Which do you choose?

- get $900 for sure,
- 90% chance to get $1,000.

Decision II: Which do you choose?

- lose $900 for sure,
- 90% chance to lose $1,000.

Most respondents in the first case choose sure gain, whereas in the second case, they take a risk rather than sure loss. Note that in the first question, the results were the same, even when the expected value of the sure option was lower than in the case of the risk one.

Consider also the following extension:

Decision III: In addition to whatever you own, you have been given $1,000. You are now asked to choose one of these options:

- 50% chance to win $1,000,
- get $500 for sure.

Decision IV: In addition to whatever you own, you have been given $2,000. You are now asked to choose one of these options:

- 50% chance to lose $1,000,
- lose $500 for sure.

In these cases, responses were the same – for Decision III most preferred sure gain, whereas in Decision IV they chose a bet.

We can see that there is some asymmetry – the fact is that most of us would have avoided the risk, but much more we share the aversion to loss [KT79, TK92]. This asymmetry was modeled in [JBS15] by the two conditional linear equations presented in Equation (16),

$$U(\hat{a}_i(\hat{g}_j)) = \begin{cases} U(a_i(g_j)), & \text{if } a_i(g_j) \geq r(g_j) \\ \lambda_j U(a_i(g_j)) - (1-\lambda_j)U(r(g_j)), & \text{otherwise} \end{cases} \tag{16}$$

where λ_j is a coefficient of loss aversion for criterion g_j and r is a reference point. This function is also visualized in Figure 3.3

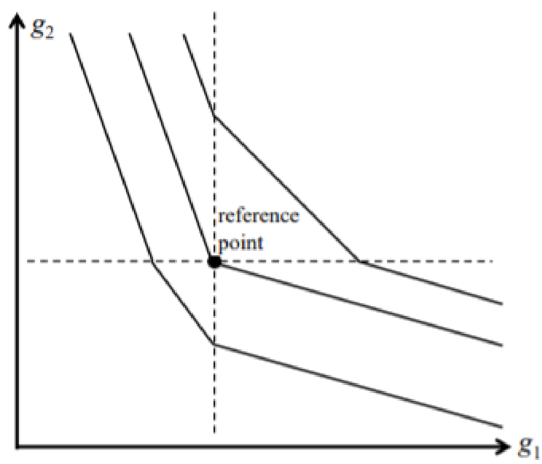

Figure 3.3: Conditionally linear functions proposed in [JBS15] – exemplary indifference curves illustrating loss aversion with respect to the reference point.

Kahneman proposed another approach presented in Figure 3.4, to model these functions as nonlinear, similar to the positively valued side of the logarithmic function. This function is definitely not symmetric – the loss side has a higher slope.

3.4 Biases simulation

This section utilizes observations from the Section 3.3 by providing descriptions, motivations and explanations for created and implemented biases or their substitutions, which are:

- criteria omission,
- criteria concealing,
- criteria mixing,
- preference reversal,
- fatigue,
- anchoring.

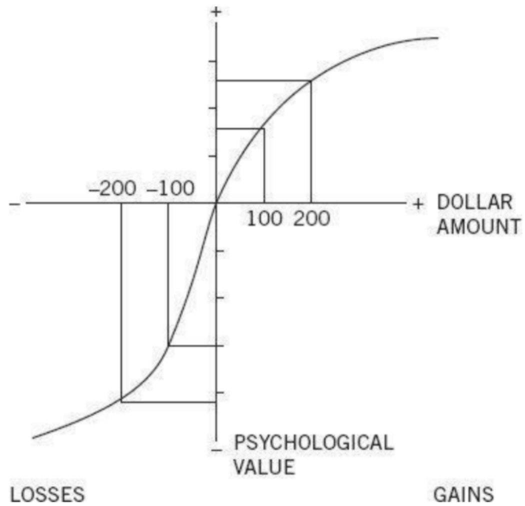

Figure 3.4: Nonlinear model of loss aversion function proposed by Kahaneman; it is much more sloped on the loss side than on the profit side.

In general, there are several things we can do with available information, for example, enrich or impoverish it, change in terms of relative or absolute reference point, create some dependencies inside, etc. – and that is what each of these biases does. Due to the uncertainty of the Decision Maker, for each bias we also provide the probability of activation, and for convenience, the possibility of independently turning off some biases without changing the entire structure.

3.4.1 Criteria omission

The first and the easiest to simulate bias is to remove some information from available data, as a forgiving effect for example. We have also considered removing some alternatives or certain alternative partial information cleansing, but the first one might be too devastating and exaggerated, and the second one can be simulated in another way.

The problem arises – which criteria should we remove and how many? We want to provide some kind of a model, so the answer for the second option is quite easy; it is enough to provide a *ratio* $\in [0, 1]$, which express the relation between the number of removed criteria and its original (before removal) count. This value can be also randomized, but then the question arises – where is the limit; we certainly do not want to remove all criteria. Sometimes, we would also like to remove at least m_{min} criteria, and at most m_{max} criteria.

The first question remains unsolved; in the [JBS15] authors propose to sort criteria ascending by their uniformly generated importance and remove x of m criteria with the inverse proportional probability to its weight. Random weights assigning can be legitimate, but it was probably just an example – for sure it is more probable to forget about less important things, but we express this importance somehow. Also, removal order can be strict, not based on the randomness, which is some simplification, but may be useful.

Therefore we provided five possible removal strategies – each of them is presented based on example set in Table 3.2, assuming Weighted Sum as a preference function. All criteria are of type *gain*, and their importance is calculated in Equation (17) as a product of the weight of the criterion and the sum of its values for all alternatives sequentially.

Table 3.2: Example input set for removal strategies presentation.

	g_1	g_2	g_3
$w(g_i)$	3	2	5
a_1	9	5	1
a_2	7	4	3
a_3	9	3	2

$$\begin{aligned} &w(g_i)\ a_1 \quad a_2 \quad a_3 \quad I(g_i) \\ g_1 &: 3 \cdot (9 + 7 + 9) = 75 \\ g_2 &: 2 \cdot (5 + 4 + 3) = 24 \\ g_3 &: 5 \cdot (1 + 3 + 2) = 30 \end{aligned} \tag{17}$$

- by **ascending importance**; the weakest criteria are first to remove: g_2, g_3, g_1,
- by **descending importance**; the strongest criteria are first to remove: g_1, g_3, g_2,
- by **importance probability**; the greater the importance, the greater the probability that the criterion will be removed next, i.e. g_3, g_1, g_2,
- by **inverse importance probability**; similar to [JBS15], the lower criterion weight, the higher probability for the criterion removal in the next step, i.e. g_2, g_3, g_1 – same as in the ascending importance, but can be also g_3, g_2, g_1, etc.
- random.

Equation (18) presents that removing the weakest criterion g_2 changes the final preference from $a_2 > a_3 > a_1$ to $a_3 > a_2 > a_1$.

$$\begin{aligned} &before \quad g_1 \qquad g_2 \qquad g_3 \qquad U(a_i) \\ a_1 &: 9 \cdot 3 + 5 \cdot 2 + 1 \cdot 5 = 42 \\ a_2 &: 7 \cdot 3 + 4 \cdot 2 + 3 \cdot 5 = 44 \\ a_3 &: 9 \cdot 3 + 3 \cdot 2 + 2 \cdot 5 = 43 \\ &after \quad g_1 \qquad g_3 \qquad U(\hat{a}_i) \\ \hat{a}_1 &: 3 \cdot 9 + 1 \cdot 5 = 32 \\ \hat{a}_2 &: 7 \cdot 3 + 3 \cdot 5 = 35 \\ \hat{a}_3 &: 9 \cdot 3 + 2 \cdot 5 = 37 \end{aligned} \tag{18}$$

3.4.2 Criteria concealing

In contrary to the previous bias, we sometimes have some information which we consider, but we do not share it with the others, therefore we *concealing it*. Simply, we have a set of additional not expressed criteria.

Because of multiple preference functions possible and in general new data addition, the problem arises – how these additional criteria should look like? The easiest and the most reliable approach is to create a new criterion basing on some already existing criterion and its characteristics, with different values for each alternative.

If we apply this approach, the question is which criterion should be the basis. At first, we selected the least influencing the final result – we wanted to make some *blur effect* and avoid much and unexpected change in the final result. However, it turned out that this approach is too strict and limiting, therefore we provided others that allow us to select *reference criterion*. These are:

- based on *the cumulated ascending importance/impact ratio* in context of all criteria, where the ratio is a parameter,
- random choose with uniform probability,
- weighted random choose – the bigger importance, the lower weight and thus lower probability.

The first one is worth explanation – we can consider it employing alternatives from Table 3.2 and importances calculated in Equation (17). For the importance ratio, we need to calculate the sum of all importances, normalize each criterion impact by it, and then for the given ratio find out which criterion matches its range (Equation (19)).

$$\begin{aligned} \textit{sorted criteria}: & \; g_2 \quad g_3 \quad g_1 \\ \textit{total}: & \; 24 + 30 + 75 = 129 \\ \textit{normalized}: & \frac{24}{129} \quad \frac{30}{129} \quad \frac{75}{129} \\ \textit{cumulative}: & \frac{24}{129} \quad \frac{59}{129} \quad \frac{129}{129} \\ \textit{requested ratio}: & \; 0.4 \\ \textit{result}: & \; g_3 \; (0.186, 4574] \end{aligned} \tag{19}$$

The values for this new criterion are randomly generated within the reference criterion values range. However, to add some flexibility, we also provided a way to change retrieved reference criterion values range scaling, to trim them when they exceed scaled criterion values range (this limitation can be scaled up and down) or to fix them when they have negative values – to zeros.

3.4.3 Criteria mixing

In general, criteria mixing can have at least two exclusive flavors. In both cases, a new criterion is generated:

- criteria may be mixed by normalization and some ratio share from each of them in the new one. This ratio should be a parameter $\in [0, 1]$. The same approach is presented in [JBS15];

- newly generated criterion can have a value which is the minimum value of both normalized criteria, which would have a similar result to *Choquet integral*.

Obviously, we can imagine also other mixing strategies, like the biggest value or any or any other operator/function utilization, so this should be possible to configure. We can also use more than two criteria, with share assigned to each of them, or assume equal, random or according to any other strategy share within available not used yet share space, for those without assignment – then total should sum to 1 and have positive values. If we assume that we require weights for *all* criteria, we even can remove the sum to 1 requirement. When we decrease minimum requirement of two criteria to one, we could even receive similar to the *criteria concealing* behavior, although then mixing has no point if considered separately. Thus, we can extend Equation (15) to Equation (20):

$$
\begin{gathered}
\hat{g}_k = \sum_{x \in X} \frac{\gamma_x}{\sum_{y \in X} \gamma_y} g_x \\
\forall_{x \in X} \ \gamma_x > 0 \\
2 \leq |X| \leq m
\end{gathered}
\tag{20}
$$

where X is a set of criteria indices contributing to the new criterion.

Next, which criteria we use to mix? The first idea is to use random ones, but we can also use strategies similar to these presented in *criteria omission*; at the end of the day, we need a set of some criteria, but some connections might be more desirable. Also, because we generate a new criterion, we face a similar problems as for *criteria concealing*, and we can utilize the same solution here.

If we would like to model the *interaction* $\psi(\hat{g}_k)$ between these criteria, it can also be a challenge as it can be positive or negative, relative or absolute. We can have many approaches: use differences in contributing criteria values, some weighted average, or any other combination. To extend just two of them, we can model the interaction as follows:

- use interaction in terms of minimum and maximum value of all contributing criteria for given alternative after criteria unification,

$$
\psi(\hat{g}_k) = \min_{x \in X}(\hat{a}_i(\hat{q}_x)) / \max_{x \in X}(\hat{a}_i(\hat{q}_x))
\tag{21}
$$

- take as a reference the maximum unified value of the criterion so the interaction would be expressed as a minimum factor for all contributing unified criteria values and the maximum possible value for the new criterion.

$$
\psi(\hat{g}_k) = \min_{x \in X}(\hat{a}_i(\hat{q}_x)) / \max_{\hat{a} \in \hat{A}}(\hat{a}(\hat{q}_k))
\tag{22}
$$

With the second approach we face the problem that this maximum value can be self dependant if we consider already processed values, which can be also a maximum ($x \rightarrow function(x) \rightarrow \hat{x}$, $\hat{x}$ is a new maximum value). Therefore, we need to consider only raw values – before processing.

We have some value describing the interaction, but how to use it is another matter. Each value can be processed with some function, for example linear or exponential, and used as current criterion value scaling (multiplication) or factor addition based on its values range.

We also need to decide whether to allow to *violate* criterion values boundaries or trim the result to them – same approach as in previous cases can be utilized here.

As an example, we present this process for the alternatives in Table 3.2 and mix criterion g_1 with g_3 with the mixing ratio $\gamma_1 = 0.3$ for g_1 and $\gamma_3 = 0.7$ for g_3. To do this, we use the simple rule presented in Equation (20). We assume that criterion values range for g_1 is [0, 10] and g_3 is [0, 5], so these need to be normalized to the common values range, for example [0, 1]. We also assume that the newly created criterion g_4 has values in range [0, 1], so we do not need to denormalize the result. The weight $w(g_4)$ for this criterion is 4. The process of normalization and mixing is shown in Equation (23).

$$\begin{array}{rl} & \hat{a}_i(\hat{g}_1) \qquad\quad \gamma_1 \qquad \hat{a}_i(\hat{g}_3) \qquad \gamma_3 \quad\; \hat{a}_i(\hat{g}_4) \\ \hat{a}_1(\hat{g}_4): & 9 \cdot \dfrac{1}{10-0} \cdot 0.7 + 1 \cdot \dfrac{1}{5-0} \cdot 0.3 = 0.69 \\ \hat{a}_2(\hat{g}_4): & 7 \cdot \dfrac{1}{10-0} \cdot 0.7 + 3 \cdot \dfrac{1}{5-0} \cdot 0.3 = 0.67 \\ \hat{a}_3(\hat{g}_4): & 9 \cdot \dfrac{1}{10-0} \cdot 0.7 + 2 \cdot \dfrac{1}{5-0} \cdot 0.3 = 0.75 \end{array} \tag{23}$$

For this example we assume local interaction (see Equation (21)). As an interaction function we assume the linear function $a \cdot \hat{a}_i(\hat{g}_4)$ with $a = 0.1$. The calculations are presented in Equation (24).

$$\begin{array}{rl} & a \quad \min(\hat{a}_i(\hat{g}_x)) \;\; \max(\hat{a}_i(\hat{g}_x)) \;\; \psi(\hat{g}_4) \\ \hat{a}_1(\psi(\hat{g}_4)): & 0.1 \cdot \left(\dfrac{1}{5-0} \quad / \quad \dfrac{9}{10-0} \right) \approx 0.02 \\ \hat{a}_2(\psi(\hat{g}_4)): & 0.1 \cdot \left(\dfrac{3}{5-0} \quad / \quad \dfrac{7}{10-0} \right) \approx 0.09 \\ \hat{a}_3(\psi(\hat{g}_4)): & 0.1 \cdot \left(\dfrac{2}{5-0} \quad / \quad \dfrac{9}{10-0} \right) \approx 0.04 \end{array} \tag{24}$$

We apply the results by adding them as a bonus to the original value of the mixed criterion, as in Equation (25).

$$\begin{array}{rlcccc} & \hat{a}_i(\hat{g}_4) & & \hat{a}_i(\psi(\hat{g}_4)) & & \hat{a}_i(g_4)) \\ \hat{a}_1(g_4): & 0.69 & \cdot & 0.02 & = & 0.71 \\ \hat{a}_2(g_4): & 0.67 & \cdot & 0.09 & = & 0.76 \\ \hat{a}_3(g_4): & 0.75 & \cdot & 0.04 & = & 0.79 \end{array} \tag{25}$$

We calculate the final utility according to the Weighted Sum as in Equation (26) – due to the low weight and values of the mixed criterion, it remains unchanged.

$$\begin{array}{rccccccccccccccccc} & \hat{a}_i(g_1) & & w(g_1) & & \hat{a}_i(g_2) & & w(g_2) & & \hat{a}_i(g_3) & & w(g_3) & & \hat{a}_i(g_4) & & w(g_4) & & U(\hat{a}_i) \\ U(\hat{a}_1): & 9 & \cdot & 3 & + & 5 & \cdot & 2 & + & 1 & \cdot & 5 & + & 0.71 & \cdot & 4 & = & 60.08 \\ U(\hat{a}_2): & 7 & \cdot & 3 & + & 4 & \cdot & 2 & + & 3 & \cdot & 5 & + & 0.76 & \cdot & 4 & = & 62.76 \\ U(\hat{a}_3): & 9 & \cdot & 3 & + & 3 & \cdot & 2 & + & 2 & \cdot & 5 & + & 0.79 & \cdot & 4 & = & 61.20 \end{array} \tag{26}$$

3.4.4 Preference reversal

Sometimes we change our preference during the process. As an example, let us consider *price*, which usually is a *cost criterion*. Due to other people, conditions, hidden parameters, or just fatigue we can consider it as a *gain* condition – the more expensive, the better, moreover it is not possible that something cheap can have good quality or is some fraud!

Here we have our criteria already given; the only question is which of them should be considered. We can apply the same parameters as for *criteria omission*, which are criteria ordering, the ratio of all criteria which preference should be reversed, and their minimum and maximum limitations.

Because of preference functions (pure Choquet integral only allows for gain criteria) and other possible complications/required modifications we consider only reversal in terms of given criterion values for all alternatives according to Equation (27):

$$\hat{a}_j(\hat{g}_i) = \max(g_i) - a_j(g_i) + \min(g_i) \tag{27}$$

where $\min(g_i)$ and $\min(g_i)$ values can be also declared given criterion values (so not the true boundaries). As an example we can employ data from Table 3.2 and inverse criterion g_2 assuming its values $\in [1, 5]$; Equation (28) presents that instead of original ranking $a_2 > a_3 > a_1$ we get $a_3 > a_2 > a_1$.

$$\begin{array}{rcccccccc} \hat{g}_2 & \min(g_2) & & g_2 & & \max(g_2) & & \hat{a}_i(\hat{g}_2) \\ a_1: & 5 & - & 5 & + & 1 & = & 1 \\ a_2: & 5 & - & 4 & + & 1 & = & 2 \\ a_3: & 5 & - & 3 & + & 1 & = & 3 \end{array}$$

$$\begin{array}{rl} \textit{utility} & g_1 \quad\quad \hat{g}_2 \quad\quad g_3 \quad\quad U(\hat{a}_i) \\ \hat{a}_1: & 9 \cdot 3 + 1 \cdot 2 + 1 \cdot 5 = 34 \\ \hat{a}_2: & 7 \cdot 3 + 2 \cdot 2 + 3 \cdot 5 = 40 \\ \hat{a}_3: & 9 \cdot 3 + 3 \cdot 2 + 2 \cdot 5 = 43 \end{array} \tag{28}$$

3.4.5 Fatigue

Fatigue can be both the easiest and the hardest to simulate because everyone reacts to it differently and in various ways. It can be easy because every decision change can be justified as fatigue influence. Hard, because there is a soft border between possible and random decision.

Most often it is revealed by laziness (so we do not want to interact, avoid answering), slowed reaction, concepts (criteria) mixing, swapping or fundaments change, preference reversal (gain $\leftrightarrow$ cost), alternatives confusion, or its properties change. However, in common it increases in time.

- **Laziness** might be hard to reflect – we may do not want to answer, tell that *"I do not care"* or *"it is all the same"*. We see that as a direct impact on the result, which however may have foundations reflected as some additional criterion (or their lack) or criteria values modification.

- **Slowed reaction**, from the process perspective, may only increase its duration, and thus Decision Maker tardiness, but also can decrease it because of possible rest in meantime. But also, from the process perspective, time is just a parameter – it can be given, and the process possibly may return its increase basing on other parameters.

- **Concepts or criteria mixing/swapping** is an interesting phenomenon, but we already addressed it – at least from the criteria mixing perspective. Here what could be added to it is the occurrence possibility factor based on the fatigue ratio; however, this is provided for all biases on the higher level.

- **Criterion fundaments change** is similar to criteria mixing, but there we consider some complex criterion given by the Decision Maker and opinionated by him, like quality or look. Due to fatigue, he can forget about (or add a new one) some of its evaluation ingredients, and therefore influence its value change. We can compare it to the *criteria concealing* or *criteria omission* on a lower level.

- **Preference reversal** in some cases seems very possible and realistic, especially in the case of high fatigue. At the first look, it would look strange when the cheapest, much better alternative is treated as much worse. However, later we can imagine that low price may be a bad factor because of quality or product perceiving. The question remains, whether it is valid in given conditions. The problem is already addressed by the previously described bias.

- The last one is **alternatives confusion**, which at first glance seems very different, but in the reality, probably is not so dramatic as we can imagine – it may be a partial process when some criteria values for given alternatives are substituted, or for a greater number of alternatives – some of them in the whole. It can be also generalized to *properties change*, which we can also perceive as blurring them.

After this analysis we decided to provide a bias which focus on criteria values change depending on the fatigue ratio. This ratio can be a result of a function that takes a time or something else as a parameter. We provided an exponential function that starts at 0 (Equation (29)),

$$f_{exp}(t) = a(\exp(\alpha t) - 1) \tag{29}$$

and a constant value fatigue that defines fatigue factor. Result of this function defines how much the current criterion value *can* change, up and down, by some delta like in Equation (30):

$$\hat{a}_i(\hat{q}_j) = a_i(q_j) + f(t) \cdot a_i(q_j) \cdot random \cdot sign \tag{30}$$

where

- $f(t)$ is an exemplary fatigue function with takes time as a parameter,
- $sign$ is a random value $\in \{-1, 1\}$ that describes whether to add a value or to subtract it,
- $random$ is a random value $\in [0, 1)$ that scales the result for diversity.

Because *fatigue ratio* is not limited and it scales the current criterion value (not its values range), the problem is whether to allow exceeding criteria values range or let these values to be negative. This is problem and values specific, so we exposed it as the bias parameters.

Applying this bias to example from Table 3.2 assuming exponential function (Equation (29)), we can receive similar results (random values occurrence) utilizing Weighted Sum as a preference function to presented in Equation (31). Preference changed noticeably from $a_3 > a_2 > a_1$ to $a_1 > a_3 > a_2$, but it can be justified by the high value of time $t = 20$.

$$\begin{aligned}
&params: \ a = 1,\ \alpha = 0.02,\ t = 20\\
&f(t) = \exp(0.02 \cdot 20) - 1 \approx 0.49\\
&\qquad a_i(g_i) \qquad f(t) \ \ rand \ \ sign \quad \hat{a}_i(\hat{g}_j)\\
a_1 \mid\ &\hat{g}_1 : 9 \cdot (1 + 0.49 \cdot 0.5 \cdot -1) = 6.795\\
&\hat{g}_2 : 5 \cdot (1 + 0.49 \cdot 0.25 \cdot \ \ 1) = 5.6125\\
&\hat{g}_3 : 1 \cdot (1 + 0.49 \cdot 0.75 \cdot \ \ 1) = 1.3675\\
\\
a_2 \mid\ &\hat{g}_1 : 7 \cdot (1 + 0.49 \cdot 0.35 \cdot \ \ 1) = 8.2005\\
&\hat{g}_2 : 4 \cdot (1 + 0.49 \cdot 0.9 \cdot -1) = 2.236\\
&\hat{g}_3 : 3 \cdot (1 + 0.49 \cdot 0.2 \cdot -1) = 2.706\\
\\
a_3 \mid\ &\hat{g}_1 : 9 \cdot (1 + 0.49 \cdot 0.1 \cdot -1) = 8.559\\
&\hat{g}_2 : 3 \cdot (1 + 0.49 \cdot 0.2 \cdot -1) = 2.706\\
&\hat{g}_3 : 2 \cdot (1 + 0.49 \cdot 0.15 \cdot \ \ 1) = 2.147\\
\\
&\qquad \hat{g}_1 \qquad\quad \hat{g}_2 \qquad\quad \hat{g}_3 \qquad U(\hat{a}_i)\\
U(\hat{a}_1) :\ &6.7950 + 5.6125 + 1.3675 = 13.7750\\
U(\hat{a}_2) :\ &8.2005 + 2.2360 + 2.7060 = 13.1425\\
U(\hat{a}_3) :\ &8.5590 + 2.7060 + 2.1470 = 13.4120
\end{aligned} \tag{31}$$

3.4.6 Anchoring

In Section 3.3.4 we noticed that anchoring has two faces:

- **based on objective estimation**, when we have no emotional/valuable link to the estimated stuff,
- **contextual**, when we have a reference point and a need to change it for something else or choose between two or more scenarios.

Because we are considering biases in terms of alternatives ranking, the first option has no use for us – therefore, we focused on the second variant.

Now, let us expand this subject. When we consider some choose with the reference point, we cannot avoid gains or loses; in the very specific scenario we even can choose a variant with the same theoretical utility, but for us it has *lower value* than the previous one, because of the *ownership* effect – the similar case was presented in [Kah11] (Figure 3.5) in terms of choose between higher salary (A) or leisure days per year (B). We assume that both scenarios have theoretically equal utility.

In this experiment there are two twins, which start at *1*. Then, one of them selects scenario A, and the other scenario B. After a year they are asked to swap their scenario; from economic point of view it should not make a difference, but perspective theory proves that both definitely prefer to stay with their current scenario.

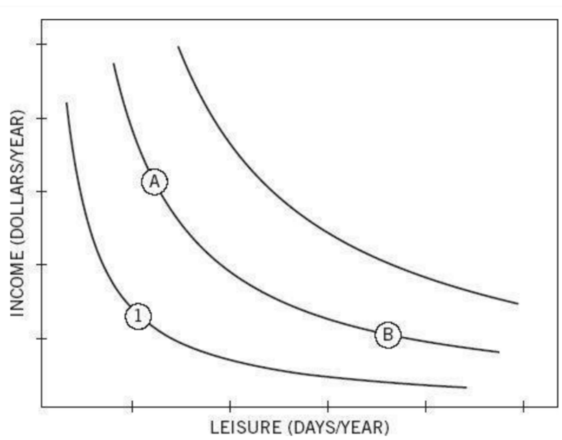

Figure 3.5: Example of ownership effect presented in [Kah11].

In this kind of choice, we can choose only one variant, which results in loss of the other, potentially the one we own – now, we consider it also in terms of *loss aversion.*

If we analyze these two biases, it seems that they are closely related:

- in anchoring we always would choose objectively the best variant, but sometimes we have higher expectations for some criteria or just owning effect,
- in loss aversion, we have some reference point which makes some kind of a loss possibility, even when considered variant is better (has higher utility) than the owned one.

There are multiple options to model such a utility function that consider loss aversion. We can have a different preference in gain purposes, for example, one does not care for \$10 or \$100 income when only above \$1000 is noticeable and \$10000 meaningful (exponential function) whereas the other considers \$10 or \$100 as a big income and \$1000 or higher as something which he cannot imagine. Of course, he knows that \$10000 is more than \$1000, but it is just too much in terms of practical utility calculation, which makes it more like a logarithmic function. So in both cases we allow the consideration of a separate function for gain or lose – each is considered in terms of given normalized criterion, so input values are absolute depending on the situation.

Another topic is an anchoring point – one or many – because we may have a couple; these can accumulate, have a different impact in time or specified meaning. For this reason, we allow multiple anchoring variants $\vec{a}_x \in \vec{A}$, each of which has a relative weighting factor $\vec{w}_x$. These can be later aggregated to some other – one or many – reference point(s) $r_x \in R$. For simplicity we now provide *ideal* and *nadir* reference point, but some median or approach similar to TOPSIS [TH11, TOP] method (the aim is to have the lowest distance from the ideal alternative and the highest from the nadir) can be also applied.

Therefore, we can calculate the difference to the reference point on certain criterion $\Delta_{a_i,r_k}(g_j)$ with Equation (32):

$$
\begin{aligned}
\Delta_{a_i,r_k}(g_j) &= \begin{cases} f_{gain}(a_i(\bar{g}_j) - r_k(\bar{g}_j)), & \text{if } a_i(g_j) \geq r_k(g_j) \\ -f_{loss}(r_k(\bar{g}_j) - a_i(\bar{g}_j)), & \text{otherwise} \end{cases} \\
a_i(\bar{g}_j) &= \frac{a_i(g_j)}{\max_{a\in A}(a(g_j)) - \min_{a\in A}(a(g_j))}
\end{aligned}
\tag{32}
$$

The last not resolved problem is how to apply the retrieved anchoring result for each reference point. In general, there are two possibilities:

- apply anchoring as a separate criterion for each reference point,
- include aggregated gain or loss value for each criterion in their values.

The first approach looks cleaner and might be interesting from a heuristic perspective – even it looks realistic when we consider it with the aspect elimination heuristic. However, we face the problem with further processing; we would like to apply a new criterion *only on the considered alternatives*, but we also need to keep the whole environment consistent – all alternatives must have the same criteria (same count, types, names). Recall that some of the not considered alternatives are part of the reference points.

We do not have any neutral value. Potentially, the center of the value range is a good proposition, which is also a base for this criterion; in case of gain, we add to it, in case of loss, we subtract. Nevertheless, any further bias application would stack and have earlier value included with converging to the common value – maybe we would like to use anchoring many times on different criteria set. We can also use the highest or the lowest value depending on the reference point, but this would be incorrect when we have multiple points, not mentioning other not considered alternatives. Taking this into account, the less harmful solution is just to add a new criterion on all alternatives, with their relation to the reference points, in the same manner as for the considered ones. We can sum up this approach with an Equation (33),

$$\begin{gathered}\hat{a}_i(\hat{g}_{r_i}) = \min_{a \in A}(a(g_{ref})) + [g_{ref}] \cdot (1 + \sum_{g_j \in G} \bar{I}(g_j) \cdot \Delta_{a_i,r_k}(g_j) \cdot a_i(g_j)) \\ [g_{ref}] = \frac{\max_{a \in A}(a(g_{ref})) - \min_{a \in A}(a(g_{ref}))}{2} \quad \bar{I}(g_j) = \frac{I(g_j)}{\sum_{g \in G} I(g)}\end{gathered} \tag{33}$$

where g_{ref} is a *reference criterion* that provides base parameters for a newly created one.

The second approach, which also was the initial one, does not have the mentioned issue, because we only change criteria values. However, we always can apply it also on the not considered variants, as it is parameterized. The idea is based on the observation that we can price lower things that are worse than any other variant – most of the preference functions bases on provided values. In this approach, we are inlining other function into it, and the difference between variants losses proportional linear relation – for example, $2 - 1$ might be no longer 1, but something between 0 and 1 in terms of our true preference with rational preference function appliance. This approach is presented in Equation (34).

$$\hat{a}_i(\hat{g}_j) = a_i(g_j) + \frac{\sum_{r_k \in R} \Delta_{a_i,r_k}(g_j)}{\|R\|} \cdot \left(\max_{a \in A}(a(g_j)) - \min_{a \in A}(a(g_j)) \right) \tag{34}$$

In both cases we have the same problem like in *criteria concealment* and *fatigue* – values can be negative or exceed some values range. It is possible to configure it in the same manner as for the mentioned ones.

To give a better insight on the described algorithm, let us consider two anchoring alternatives in Table 3.3 – each of them have also a *weight* used for calculating an aggregated reference point, which we take as an ideal alternative. As a set of alternatives, we assume the previously presented in Table 3.2.

Table 3.3: Anchoring alternatives used in example.

	$\vec{w}_x$	g_1	g_2	g_3
$\vec{a_1}$	2	8	3	3
$\vec{a_2}$	1.5	10	6	4

The calculation of the ideal reference point with explanations is presented in Equation (35).

$$\begin{array}{l} \quad \vec{a}_1(g_j)\ \vec{w}_1 \quad \vec{a}_2(g_j)\ \vec{w}_2 \quad r(g_j) \\ g_1: \ 8 \ \cdot \ 2 \ \ | \ \ 10 \cdot 1.5 \rightarrow 8\ (\vec{a_1} - product\ greater) \\ g_2: \ 3 \ \cdot \ 2 \ \ | \ \ 6 \cdot 1.5 \rightarrow 6\ (\vec{a_2} - product\ greater) \\ g_3: \ 3 \ \cdot \ 2 \ \ | \ \ 4 \cdot 1.5 \rightarrow 4\ (\vec{a_2} - products\ equal,\ value\ greater) \end{array} \tag{35}$$

All criteria in this example are of type *gain*, so to explain the behavior for *cost* type – to choose the better value, at the same time we are comparing only two anchoring alternatives, and the one that is better is passed further. Therefore, we can utilize this and as an *utility* of a given criterion value multiply its value and *the opposite* anchoring alternative weight, as shown in Equation (36). In this case, the lower value is preferred and the calculated utility is only valid for this comparison. For the *nadir* reference point, both cases are inverted.

$$\begin{aligned} U(\vec{a}_1(g_j)) &= \vec{a}_1(g_j) \cdot \vec{w}_2 \\ U(\vec{a}_2(g_j)) &= \vec{a}_2(g_j) \cdot \vec{w}_1 \end{aligned} \tag{36}$$

After we have obtained reference point(s), we need to compute differences for all alternatives for all reference points – in this case we have only one. Calculated differences $\delta a_i(g_j) = a_i(g_j) - r(g_j)$ are presented in Table 3.4.

Table 3.4: Per criterion differences to the reference point for each alternative presented in example.

	g_1	g_2	g_3
δa_1	1	-1	-3
δa_2	-1	-2	-1
δa_3	1	-3	-2

We still need to define the functions for the *gain* and *loss* cases compared to the reference point. For simplicity, we assume linear functions $a \cdot \delta a_i(\bar{g}_j)$ for both cases – for the *gain* $a = 0.2$ and *loss* $a = 0.5$. For convenience, we assumed that all differences should be normalized within the possible values of a given criterion; we assume $a_i(g_j) \in [0, 10]$ for all criteria. Note that in Equation (32) this process (normalizing and getting differences) is inverted – we chose this method for readability reasons. Nevertheless, for the sake of simplicity, we could omit it entirely – here we have same criterion values range and transform function is also simple. The normalization and application of the function is shown in Equation (37).

$$
\begin{aligned}
&& a \qquad && \delta a_i(\bar{g}_j) \qquad && f(a_i(\bar{g}_j)) \\
a_1 \mid g_1 &:0.2 \cdot (\ \ 1 \cdot \frac{1}{10-0}) &&= \ \ 0.02 \\
g_2 &:0.5 \cdot (-1 \cdot \frac{1}{10-0}) &&= -0.05 \\
g_3 &:0.5 \cdot (-3 \cdot \frac{1}{10-0}) &&= -0.15 \\
a_2 \mid g_1 &:0.5 \cdot (-1 \cdot \frac{1}{10-0}) &&= -0.05 \\
g_2 &:0.5 \cdot (-2 \cdot \frac{1}{10-0}) &&= -0.10 \\
g_3 &:0.5 \cdot (-1 \cdot \frac{1}{10-0}) &&= -0.05 \\
a_3 \mid g_1 &:0.2 \cdot (\ \ 1 \cdot \frac{1}{10-0}) &&= \ \ 0.02 \\
g_2 &:0.5 \cdot (-3 \cdot \frac{1}{10-0}) &&= -0.15 \\
g_3 &:0.5 \cdot (-2 \cdot \frac{1}{10-0}) &&= -0.10
\end{aligned}
\tag{37}
$$

Now we have the option of applying these values directly to the criterion value of each alternative or creating a new one from the aggregated values for each alternative. In the first option, the obtained values ($f(a_i(\bar{g}_j))$) have to be rescaled to the original range of the criterion value and added to it, as presented in Equation (38) together with the Weighted Sum utility.

$$
\begin{aligned}
& a_i(g_j) \quad f(a_i(g_j)) && \hat{a}_i(\hat{g}_j) \\
\hat{a}_1 \mid \hat{g}_1 &: 9 + 0.02 \cdot (10-0) &&= \ \ 9.2 \\
\hat{g}_2 &: 5 - 0.05 \cdot (10-0) &&= \ \ 4.5 \\
\hat{g}_3 &: 1 - 0.15 \cdot (10-0) &&= -0.5 \\
\hat{a}_2 \mid \hat{g}_1 &: 7 - 0.05 \cdot (10-0) &&= \ \ 6.5 \\
\hat{g}_2 &: 4 - 0.10 \cdot (10-0) &&= \ \ 3 \\
\hat{g}_3 &: 3 - 0.05 \cdot (10-0) &&= \ \ 2.5 \\
\hat{a}_3 \mid \hat{g}_1 &: 9 + 0.02 \cdot (10-0) &&= \ \ 9.2 \\
\hat{g}_2 &: 3 - 0.15 \cdot (10-0) &&= \ \ 1.5 \\
\hat{g}_3 &: 2 - 0.10 \cdot (10-0) &&= \ \ 1
\end{aligned}
\tag{38}
$$

$$
\begin{array}{lccccccccccccl}
 & \hat{a}_i(\hat{g}_1) & w(g_1) & & \hat{a}_i(\hat{g}_2) & & w(g_2) & & \hat{a}_i(\hat{g}_3) & & w(g_3) & U(\hat{a}_i) \\
U(\hat{a}_1): & 9.2 & \cdot & 3 & + & 4.5 & \cdot & 2 & - & 0.5 & \cdot & 5 & = 34.1 \\
U(\hat{a}_1): & 6.5 & \cdot & 3 & + & 3 & \cdot & 2 & + & 2.5 & \cdot & 5 & = 38 \\
U(\hat{a}_1): & 9.2 & \cdot & 3 & + & 1.5 & \cdot & 2 & + & 1 & \cdot & 5 & = 35.6
\end{array}
$$

In the second variant of creating a new criterion, we create it based on the weighted average for each value based on the importance of each criterion that was calculated earlier for *criteria omission* example, in Equation (17). This process is presented in Equation (39); we assume that the weight w_4 of a new criterion g_4 is 4, and its values $\in [0, 10]$.

$$
\begin{array}{l}
|g_{ref}| = \dfrac{10-0}{2} = 5 \\
\begin{array}{c|cccccccccc}
 & |g_{ref}| & & f(a_i(\bar{g}_1))) & I(g_1) & f(a_i(\bar{g}_2)) & I(g_2) & f(a_i(\bar{g}_j)) & I(g_3) & & \hat{a}_i(g_4) \\
\hat{a}_1(g_4) & 5 \cdot & \Bigg(1+(& 0.02 \cdot & \dfrac{24}{129} & -0.05 \cdot & \dfrac{30}{129} & -0.15 \cdot & \dfrac{75}{129}) & \Bigg) & = 4.52 \\
\hat{a}_2(g_4) & 5 \cdot & \Bigg(1+(& -0.05 \cdot & \dfrac{24}{129} & -0.10 \cdot & \dfrac{30}{129} & -0.05 \cdot & \dfrac{75}{129}) & \Bigg) & = 4.69 \\
\hat{a}_3(g_4) & 5 \cdot & \Bigg(1+(& 0.02 \cdot & \dfrac{24}{129} & -0.15 \cdot & \dfrac{30}{129} & -0.10 \cdot & \dfrac{75}{129}) & \Bigg) & = 4.55
\end{array} \\
\begin{array}{lccccccccc}
 & \hat{a}_i(\hat{g}_1) & w(g_1) & \hat{a}_i(\hat{g}_2) & w(g_2) & \hat{a}_i(\hat{g}_3) & w(g_3) & \hat{a}_i(\hat{g}_4) & w(g_4) & U(\hat{a}_i) \\
U(\hat{a}_1): & 9 \cdot & 3 + & 5 \cdot & 2 + & 1 \cdot & 5 & +4.52 \cdot & 4 & = 60.08 \\
U(\hat{a}_1): & 7 \cdot & 3 + & 4 \cdot & 2 + & 3 \cdot & 5 & +4.69 \cdot & 4 & = 62.76 \\
U(\hat{a}_1): & 9 \cdot & 3 + & 3 \cdot & 2 + & 2 \cdot & 5 & +4.55 \cdot & 4 & = 61.20
\end{array}
\end{array}
\tag{39}
$$

We see that for these parameters the preference remains unchanged in both cases.

4 Illustrative Case Study

To illustrate how we can use our application, we provide two examples:

- the first focuses on heuristics,
- the second shows application in querying a Decision Maker for multiple pairwise comparison.

For brevity, we omit input structure – all examples with the final results can be found in the *httpClient/examples* directory in the code repository [Kup].

4.1 Example 1: Vegetable choice

This example presents **aspect elimination heuristic** with **criteria omission** and **anchoring**. As a running problem, we assume the choice of vegetables for dinner by Josh and his family among *cauliflower, broccoli, carrot, broad bean, kidney bean, cucumber,* and *eggplant*. We evaluate them based on taste, smell, nutritional facts, seasonal availability, and price – the last two criteria are of type *cost* ↓; he highly appreciates something hardly available and does not like to pay a lot for food. The properties of considered alternatives are presented in Table 4.1.

Table 4.1: Alternatives considered in Example 1.

	taste ↑	*smell* ↑	*nut. facts* ↑	*seas. av.* ↓	*price* ↓
cauliflower	4	6	4	5	4
broccoli	6	5	6	4	5
carrot	5	4	7	7	2
broad bean	8	5	10	2	9
kidney bean	9	5	9	6	7
cucumber	4	4	4	7	3
eggplant	2	3	3	5	6

Additionally, Josh very likes *spinach*, but it is not available in his nearest shop. (Table 4.2). It is much harder for him to reconcile with a loss than with an adequate profit. He also tends to perceive things just as less or more worthy by comparing them with others in terms of their properties.

Table 4.2: Properties for anchoring alternative used in Example 1.

	taste ↑	*smell* ↑	*nut. facts* ↑	*seas. av.* ↓	*price* ↓
spinach	4	3	7	7	7

In general, he takes care of himself and his loved ones, so he is very attentive to the nutritional value. His value system for the importance of particular criteria is presented in Table 4.3.

Table 4.3: Criteria weights used in Example 1.

$taste \uparrow$	$smell \uparrow$	$nut.\ facts \uparrow$	$seas.\ av. \downarrow$	$price \downarrow$
3	5	9	4	5

Even though he is very fond of fragrant dishes, unfortunately, he currently has a cold that he has not told us about, so the *smell* loses its importance.

Our Decision Maker needs to decide quickly – his mother is visiting him today for dinner, so his time is limited. Therefore, he decides by elimination. If we pass this information to our algorithm, it processes as follows:

- our first bias is to omit the *smell* criterion,
- as a second bias, we are applying anchoring, as the Decision Maker profile says – by inlining properties,
- we check each criterion ordered as *nut. facts, price, seas. av.*, and *taste* on all alternatives in random order, which effectively is *eggplant, carrot, kidney bean, cauliflower, broad bean, broccoli*, and *cucumber*.

Because of omitting *smell* and anchoring to *spinach*, Josh can see these variants rather like in Table 4.4.

Table 4.4: Biased alternatives in Example 1.

	$taste \uparrow$	$nut.\ facts \uparrow$	$seas.\ av. \downarrow$	$price \downarrow$
$cauliflower$	4	3.94	6.1	5.6
$broccoli$	7.1	5.98	6.1	5.6
$carrot$	5.6	7	7	4.6
$broad\ bean$	10.1	11.6	2	9
$kidney\ bean$	11.6	10.1	6.6	7
$cucumber$	4	3.94	7	5.1
$eggplant$	1.96	2.92	6.1	6.6

He decides by elimination, so he decided to increase the demands continuously when it is necessary; his minimum requirement is at least 5 on all criteria, and if it is not sufficient to get a result, 7 for gain and 3 for cost criteria. Based on this $Aspect$ *elimination heuristic* processing looks like follows:

- we start with criterion $nut.\ facts \uparrow$, and as the first alternative to process is *eggplant*, which is rejected because of its too low value on this criterion (2.92 vs 5);
- the second is *carrot*, which has this criterion value above the assumed threshold; therefore we need to go further;

- next, we are looking at *kidney bean*, which has both $taste \uparrow$ and $nut.\ facts \uparrow$ even above the boundaries of the criterion value;
- then, we check *cauliflower* which does not meet the required threshold (3.94 vs 5);
- further, we validate *broad bean* for which the situation is similar to *kidney bean*;
- as the sixth alternative, we check *broccoli*, which also meets Josh's requirements;
- finally, we need to verify *cucumber* – it has too low value on the current criterion (3.94 vs 5);
- so far we rejected three alternatives, therefore we need to change criterion to $price \downarrow$ – *carrot* matches it, but *kidney bean*, *broad bean* and *broccoli* do not, so these need to be rejected, and as a result *carrot* is the only remaining alternative, and thus is selected.

It is worth noting that if we do not use anchoring, the result would be different because we would not reject *broccoli* and *carrot* would not meet the $seas.av. \downarrow$ requirement (7 vs 5). Therefore *broccoli* will be selected. The final rankings obtained in the two considered scenarios are presented in Table 4.5.

Table 4.5: The final ranking obtained using *the Aspect Elimination heuristic* in Example 1.

	with anchoring	*without anchoring*
carrot	**1**	**2**
broccoli	**2**	**1**
broad bean	3	3
kidney bean	4	4
cucumber	5	5
cauliflower	6	6
eggplant	7	7

4.2 Example 2: Continuous querying of the Decision Maker

The second example focuses on asking the Decision Maker, which of the two alternatives described he would choose. However, we **ask similar questions many times** with different alternatives; the alternative more preferred in the previous question may or may not be the subject of the current question. This example illustrates the use of **the Majority heuristic** influenced by **criteria concealing** and **fatigue**.

Imagine Jane, who is a sales assistant in an electronics store. She helps customers to select a new phone. Most of them are not yet decided but have some ideas about what they want and what is important to them. Phones are also publicly available at the stand and possible to use and test in the limited scope.

Jane thinks about herself that she is honest, but her salary is not very high, and she has no added benefit to customer satisfaction. Thus she wants to advise them well, but without much effort. However, her friend, who is working for *Apple*, promised her a little gift for each phone sold. Another friend is working for *Samsung*, and when he found out about the *Apple* deal, he promised her a little bit better offer for advising their products. Actually, she has some additional profit for most of the available devices; these fees are not very significant, but they can make a difference (let us assume that their weight is 1.42). These are a bit complicated, so she has them listed on the same list as in Table 4.6.

Table 4.6: *Additional profit* for each device manufacturer in Example 2.

	Apple	*Samsung*	*Huawei*	*Xiaomi*	*Sony*	*LG*	*OnePlus*
add. profit ↑	5.63	8	4.12	0	4.21	4.63	2.84

Despite being economical in life, she is not a technical geek. To some extent, she knows the specifications of most devices asked by customers. However, she has no personal experience in using them – she only compares the parameters of the device, which are *screen size, storage size, RAM size, main camera resolution*, and *price* – all except *price* are of type *gain* ↑. Table 4.7 presents available mainly asked flagship devices for each manufacturer.

Table 4.7: Alternatives considered in Example 2.

	screen ↑	*storage* ↑	*RAM* ↑	*camera* ↑	*price* ↓
Apple	5	2	3	10	9
Samsung	6	6	8	7	8
Huawei	5	4	6	9	6
Xiaomi	4	3	4	3	4
Sony	5	4	6	4	8
LG	3	3	4	2	5
OnePlus	4	5	6	8	6

However, during this period and conversations with colleagues, she noted that not all parameters are equally important in customer choices. She perceives them more or less as in Table 4.8.

Table 4.8: Criteria weights used in Example 2.

screen ↑	*storage* ↑	*RAM* ↑	*camera* ↑	*price* ↓
6	3	4	5	3

It is worth noting that she has been working in this position for two months, and she does not feel comfortable with it. She is already a little bit tired of it, and each subsequent client makes her feel worse.

Today she has already had about 20 clients, and another ten are taking a look at the phones. She was asked by her manager to advise them.

The first client asked for comparing *Apple* and *Samsung*; she answered that *Samsung* is better, according to Table 4.9 and higher *evaluation* ↑ for this alternative based on Equation (40) – for example, *Samsung* has a better screen, so the weight of *storage* ↑ (3) is assigned to it, while *Apple* has a better *camera* ↑, so we add to it evaluation equal to the weight for this criterion (5). For readability reasons better values are bolded.

Table 4.9: Comparison #1 in Example 2: *Samsung* vs *Apple*.

	screen ↑	*storage* ↑	*RAM* ↑	*cam.* ↑	*price* ↓	*add. pro.* ↑	*eval.* ↑
Samsung	**6.03**	**6.03**	**8.24**	6.48	**7.54**	**8.32**	**17.42**
Apple	5.61	2.06	3.05	**9.18**	9.31	5.67	5

$$
\begin{array}{rcc|cc}
 & \textit{Samsung} & \textit{Apple} & \textit{Samsung eval.} & \textit{Apple eval.} \\
\textit{screen}\uparrow: & \mathbf{6.03} & 5.61 & 6 & 0 \\
\textit{storage}\uparrow: & \mathbf{6.03} & 2.06 & 3 & 0 \\
\textit{RAM}\uparrow: & \mathbf{8.24} & 3.05 & 4 & 0 \\
\textit{camera}\uparrow: & 6.48 & \mathbf{9.18} & 0 & 5 \\
\textit{price}\downarrow: & \mathbf{7.54} & 9.31 & 3 & 0 \\
\textit{additional profit}\uparrow: & \mathbf{8.32} & 5.67 & 1.42 & 0 \\
\textit{total evaluation}\uparrow: & & & \mathbf{17.42} & 5
\end{array} \tag{40}
$$

Another client asked her to compare *Huawei* and *Xiaomi*. In her opinion, the latter was worse (see Table 4.10 and Equation (41), which is based on the same principles as Equation (40) as well as all subsequent comparisons).

Table 4.10: Comparison #2 in Example 2: *Huawei* vs *Xiaomi*.

	screen ↑	*storage* ↑	*RAM* ↑	*cam.* ↑	*price* ↓	*add. pro.* ↑	*eval.* ↑
Huawei	**5.03**	**4.02**	**6.19**	**8.3**	5.64	**4.3**	**19.42**
Xiaomi	3.73	3.1	4.07	2.74	**4.14**	0	3

$$
\begin{array}{rcc|cc}
 & \textit{Huawei} & \textit{Xiaomi} & \textit{Huawei eval.} & \textit{Xiaomi eval.} \\
\textit{screen}\uparrow: & \mathbf{5.03} & 3.73 & 6 & 0 \\
\textit{storage}\uparrow: & \mathbf{4.02} & 3.1 & 3 & 0 \\
\textit{RAM}\uparrow: & \mathbf{6.19} & 4.07 & 4 & 0 \\
\textit{camera}\uparrow: & \mathbf{8.3} & 2.74 & 5 & 0 \\
\textit{price}\downarrow: & 5.64 & \mathbf{4.14} & 0 & 3 \\
\textit{additional profit}\uparrow: & \mathbf{4.3} & 0 & 1.42 & 0 \\
\textit{total evaluation}\uparrow: & & & \mathbf{19.42} & 3
\end{array}
\tag{41}
$$

Next, she was asked to compare *Sony* and *LG* – in her opinion *Sony* was better (see Table 4.11).

Table 4.11: Comparison #3 in Example 2: *Sony* vs *LG*.

	screen ↑	*storage* ↑	*RAM* ↑	*cam.* ↑	*price* ↓	*add. pro.* ↑	*eval.* ↑
Sony	**5.03**	**4.02**	**6.2**	**3.67**	7.49	4.4	**18**
LG	2.79	3.1	4.08	1.82	**5.19**	**4.67**	4.42

Then, this client also wanted to compare *Sony* with *Huawei*; the first of them slightly wins this comparison as presented in Table 4.12 (*Sony* pays a little bit more).

Table 4.12: Comparison #4 in Example 2: *Sony* vs *Huawei*.

	screen ↑	*storage* ↑	*RAM* ↑	*cam.* ↑	*price* ↓	*add. pro.* ↑	*eval.* ↑
Sony	**5.03**	4.02	**6.21**	3.66	7.47	**4.41**	**11.42**
Huawei	4.63	**4.14**	6.12	**8.15**	**6.23**	4.15	11

Another client wanted to compare *Huawei* with *OnePlus*, and it turned out that *Huawei* is much better (Table 4.13, note that *Huawei's* parameters are different from those in Table 4.12).

Table 4.13: Comparison #5 in Example 2: *Huawei* vs *OnePlus*.

	screen ↑	*storage* ↑	*RAM* ↑	*cam.* ↑	*price* ↓	*add. pro.* ↑	*eval.* ↑
Huawei	**5.03**	4.02	**6.22**	**8.2**	5.59	**4.32**	**19.42**
OnePlus	3.69	**5.19**	6.12	7.21	**6.24**	2.86	3

Seeing this, some young man asked her which phone would she advise: *Sony* or *OnePlus*; she replied that she would choose *OnePlus* (Table 4.14), which confused him a bit as she previously said that *Sony is better than Huawei which was also better than OnePlus*. She was still asked about a few other phone comparisons and was getting more and more tired. Then, the first customer asked her to compare *Apple* and *Samsung* again.

Table 4.14: Comparison #6 in Example 2: *OnePlus* vs *Sony*.

	screen ↑	*storage* ↑	*RAM* ↑	*cam.* ↑	*price* ↓	*add. pro.* ↑	*eval.* ↑
OnePlus	4.03	**5.03**	**6.24**	**7.23**	**5.55**	2.99	**15**
Sony	**4.58**	4.16	6.14	3.57	8.35	**4.25**	7.42

She did not remember that she had already answered this question and replied that *"Apple was better"* by making a quick comparison (Table 4.15). This answer helped him a lot as he considered *Apple* because of his son's advice that it was better; Nor did he remember that *Jane had advised choosing Samsung* before.

Table 4.15: Comparison #7 in Example 2: *Apple* vs *Samsung*.

	screen ↑	*storage* ↑	*RAM* ↑	*cam.* ↑	*price* ↓	*add. pro.* ↑	*eval.* ↑
Apple	**6.05**	2.01	3.13	**8.93**	**8.25**	5.96	**14**
Samsung	5.44	**6.27**	**8.2**	6.17	8.39	**8.08**	8.42

5 Conclusions

In this work, we started with a common perspective on the decision-making processes, emphasizing the differences between them and the real choices we do. We have also mentioned consistently increasing requirements for automated systems to integrate with real humans decisions and their utilization in other appliances like decision making processes optimization or product targeting. We signalized that there are a lot of helpful algorithms and systems, which can provide the optimal solution for decisions. However, at the same time, they cannot be used with high reliability to simulate real human choices.

Then we shaped some behaviors, introduced two thinking systems, and distinguished two human characters from an economic point of view: *rational*, which decisions are mathematically justified, and *bounded*, which is often hard to predict, makes simplifications, but still has some logic. For the latter, we described the three commonly used in everyday life of each of us: *Satisfaction, Aspect Elimination,* and *Majority heuristic*, we gave examples of their operation and provided their implementation.

Later, we noticed that there are factors which do not precisely influence *how* we make decisions, but rather *bases* on which we make them – we called them *biases*. Then, we categorized, named, and described a couple of these biases (*criteria omission, criteria mixing, fatigue, anchoring,* and *loss aversion*) with some scientific research references and often real behaviors' examples.

Finally, based on previous research, we described how we approached each of these biases when the relationship was not always one-to-one in practical implementation. We decided to create relatively simple modules allowing for parametrization and composition, which should allow for better adaptation to the current needs and conditions, as well as enable and facilitate the further development of the library and allow us to model many complex behaviors. We also mentioned some problems we encountered, how we solved them, and the assumptions we made. However, this topic is not and will probably never be exhausted, so we have outlined some perspectives and ideas for its further development.

Our solution can be used in many areas, for example in the case of simulation of human behavior, human interaction with artificial intelligence, the process of supporting a real Decision Maker, or optimization of algorithms that focus on it; it allows the validation of their robustness with respect to different decision making policies, accuracy, and performance. It could be also used as a background or a proof of concept for more advanced algorithms in automotive, marketing, or human assistance domains to evaluate opportunities for specific decisions or behaviors.

Since we have implemented it as a library, it can be integrated in many ways – as an example, we provide a fully functioning *http rest server* with a whole set of ready-to-use simple examples focused on specific algorithms and biases, available with the code in the code repository. To ease understanding and illustrate how the algorithms work, we have provided two descriptive real-world examples focused on certain biases that show their impact on the decisions made in these cases. Their purpose is also to provide a better insight into how our system can be utilized to perceive real human choices under certain conditions or during some optimization processes that require real human interaction.

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