Lecture 5 :- Propositional Equivalence.

Tautology :

contradiction :

p	$\neg p$	$p \wedge \neg p$	$p \vee \neg p$
T	F	F	T
F	T	F	T

Contingency:

p	q	$p \wedge q$	$(p \wedge q) \vee p$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
T	T	T				
T	F	F				
F	T	F				
F	F	F				

p and q are said to be logically equivalent if.
$p \leftrightarrow q$ = Tautology. where p, q are compound propositions.

P22 :- $\underline{\neg(p \vee q)}$ and $\underline{\neg p \wedge \neg q}$ are logically equivalent.

Ex2

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

Ex3 P22 HW. Ex4 P23 HW.

P24:- logical Equivalences.

1) $p \wedge T \equiv p$ Identity laws.
$p \vee F \equiv p$

2). $p \vee T \equiv T$ Domination laws.
$p \wedge F \equiv F$

3) $p \vee p \equiv p$ Idempotent "
$p \wedge p \equiv p$

$p \wedge p \equiv p$

4). $p \vee q \equiv q \vee p$. Commutative laws.
$p \wedge q \equiv q \wedge p$.

5). $p \vee (q \vee r) \equiv (p \vee q) \vee r$ Associative.
$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$.

6). $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$. Distributive.
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

8) $\neg (p \wedge q) \equiv \neg p \vee \neg q$
$\neg (p \vee q) \equiv \neg p \wedge \neg q$. De-Morgans.

9). $p \vee (p \wedge q) \equiv p$
$p \wedge (p \vee q) \equiv p$.

10) $p \equiv \neg(\neg p)$.

Predicates and Quantifiers.

$P(x) = x + 3 = 4$. Domain $x = \{0, 1, 2, 3\}$.

Subject. Condition | Predicate.

$P(0) = 0 + 3 = 4. = F$
$P(1) = 1 + 3 = 4. = T$
$P(2) = 2 + 3 = 4 = F$
$P(3) = 3 + 3 = 4. = F$.

Ex) :- P31
$P(x) = x > 3$ $P(2) = ?$ $P(4) = ?$
$P(2) = 2 > 3 = F$
$P(4) = 4 > 3. = T$.

Ex2 HW.
P31.

Ex3 :- P31 $Q(x,y) = \quad x = y+3.$ $\quad Q(3,0) = ?$ HW.

$Q(1,2) = \quad 1 = 2+3 \quad = F.$

Ex4 :- P31 $A(c,n) =$ "Computer c is connected to network n"

c = { Computers on Campus }

n = { Networks " " }.

Computer MATH1 is connected to network CAMPUS2.

A(MATH1, CAMPUS1) = ? F

A(MATH1, CAMPUS2) = ? T

$\Downarrow$ English = HW.

Ex5 HW.
P31

Quantifiers. $\quad x = \{1, 2, 3 \ldots N\}.$

Universal. $\forall$ $\quad \forall x\, P(x) = P(1) \wedge P(2) \wedge \cdots \wedge P(N).$

for all, for every, for any, p33.

$\forall x P(x) = T$ when all $P(i)$'s are true. $i = \{1, 2, \ldots N\}$.

$\forall x P(x) = F$ when one of $P(i)$'s is false.

Existential $\exists$ $\quad \exists x\, P(x) = P(1) \vee P(2) \vee \cdots \vee P(N).$

there exist, for atleast one.
for some.

Ex 10
p 34

$P(x) = x^2 > 0 \qquad x \in \mathbb{Z}.$

$\forall x P(x) = ? \quad = F. \qquad$ Counter Example.

$P(0) = 0^2 > 0 = F.$

E 11 :-
P 34

$P(x) = x^2 < 10. \qquad x = \{1, 2, 3, 4\}.$

$\forall x P(x) = ?$

$\forall x P(x) = P(1) \wedge P(2) \wedge P(3) \wedge P(4).$

$= (1^2 < 10) \wedge (2^2 < 10) \wedge (3^2 < 10) \wedge (4^2 < 10).$

$= T \wedge T \wedge T \wedge F$

$= F.$

Quiz # 3 06 - FEB - 2023 . (Section A).

Let $P(x) = \quad x = x + 1. \qquad x = \{1, 2\}.$

$Q(x) = \quad x > 4.$

Find $(\forall x \neg P(x) \wedge \exists x \; Q(x))$

$\forall x P(x)$
$= P(1) \wedge P(2).$

$\forall x \neg P(x) = \neg P(1) \wedge \neg P(2).$

$\neg(1 = 1 + 1) \wedge \neg(2 = 2 + 1).$

$\neg(1 = 2) \wedge \neg(2 = 3).$

$\neg(F) \wedge \neg(F)$

$T \wedge T$

$= T.$

$\exists x Q(x) = Q(1) \vee Q(2).$

$(1 > 4) \vee (2 > 4).$

$F \vee F = F.$

$(\forall x \neg P(x) \wedge \exists x \; Q(x)) = T \wedge F = F.$