

lecture 11:- Composite .

$$\begin{array}{lcl} R & (a,b) \in & A \times B \\ S & (b,c) \in & B \times C \end{array} \quad \begin{array}{l} a \in A \\ b \in B \\ c \in C \end{array}$$

$$\text{if } (a,b) \in R \wedge (b,c) \in S.$$

$$\text{Then } (a,c) \in S \circ R$$

Ex:- 20:-
P465

$$\begin{array}{l} R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\} \quad A \times B. \quad A = \{1,2,3\}. \\ S = \{(1,0), (2,0), (3,1), (3,2), (4,1)\} \quad B \times C. \quad B = \{1,2,3,4\}. \\ C = \{0,1,2\}. \end{array}$$

$$S \circ R = \{(1,0), (1,2), (2,1), (2,2), (3,0), (3,1)\}.$$

$$R \circ S = ?$$

$$S \circ R \neq R \circ S. \quad \text{HW.}$$

$$\begin{array}{l} R \circ R = ? = R^2 \quad \text{HW} \\ R^2 \circ R = R^3 \quad \text{u.} \end{array}$$

⋮

Theorem 2:- A relation R on A is transitive.
P466. iff $R^n \subseteq R \quad n = 1, 2, 3, \dots$

Exercise 1-30 HW.
P466-P468.

N-ary Relations with Application.

$$\begin{array}{l} \text{Ex 1:-} \\ \text{P469} \end{array} \quad \begin{array}{l} R = \{(a,b,c) \mid a < b < c\} \quad N \times N \times N. \\ (1,2,3) \in R \quad = T \\ (2,4,3) \in R \quad = F. \end{array}$$

Ex2 : $R = \{(a, b, c) \mid b = a + k \wedge c = a + 2k\}$.
 $469 \quad \exists k \in \mathbb{Z} \quad \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$

$(2, 5, 9) \notin R$.
 $\downarrow \downarrow \downarrow$
 $a \quad b \quad c$
 $5 = 2 + k$
 $k = 3$
 $9 = 2 + 2 \cdot 3$
 $9 = 2 + 6$
 $9 \neq 8$

$(4, 3, 5) \in R$? H.W.

$|A| = n \quad |B| = m \quad |C| = r$
How many Relations $2^{n \times m \times r}$

Ex3
 $R = \{(a, b, m) \mid a \equiv b \pmod{m}\}$

$4 \overline{17}^{-5} ?$
 $\begin{array}{r} -5 \\ 4 \overline{17} \\ \underline{+20} \\ 3 \end{array}$

$(-1, 9, 5) \in R$ ✓

$\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}^+$

$(8, 2, 3) \in R$ ✓

$(4, 0, 7) \in R$ ✗

Ex4:- S-ary Relation.

$A \times N \times S \times D \times T$

(PIA, PK SIS, PEW, ISB, 2:45)

A = Set of Airlines
N = " Flight Numbers.
S = " Starting Points.
D = " Destinations.
T = " Departure Time.

Relationship Database.

Airline Flight Numbers Starting Point Destination Time.

Airline	Flight Numbers	Start to Panel	Destinat Pa	T.
PIA	PK515	PEW	ISB	2:45pm

Representant mij M_R Relatons

$A = \{a_1, a_2, \dots, a_m\}$
 $B = \{b_1, b_2, \dots, b_n\}$

$$m_{ij}^{12} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R. \end{cases}$$

MR	Size
Rows 2	A

$$\begin{array}{cc|c} \text{Col} & z & | B) \\ \hline b_1 & b_2 & \end{array}$$

Exd. :-
P476

$$A = \begin{pmatrix} a_1 & a_2 & a_3 \\ 1 & 2 & 3 \\ a, b, c \end{pmatrix}$$

$B_2 \ni 1, 2$
e, f.

$$K_2 = \{(2,1), (3,1), (3,2)\}$$
 $A \times B$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$