lecture 26 :- Transitive Closure.

P484 $R = \{(1,3), (1,4), (2,1), (3,2)\}$ $A = \{1,2,3,4\}$.

$R_m = \{(1,2), (2,3). (2,4), (3,1)\}$.

$R \cup R_m = \{(1,3), (1,4), (2,1), (3,2), (1,2), (2,3), (2,4), (3,1)\}$.

(3,4) further missing.

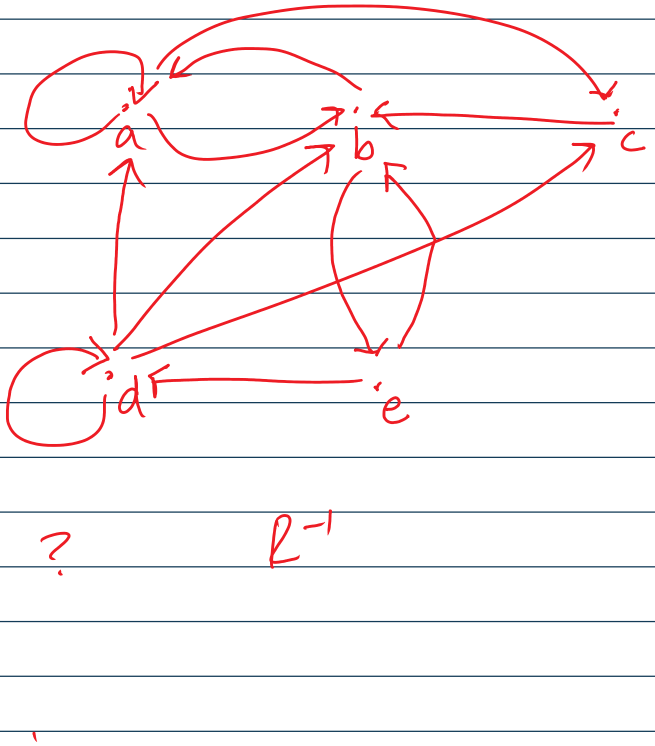
PATHS iN DiRECTED GRAPH.

($\rightarrow$).

A path from 'a' to 'b' in a directed Graph G when ∃ a Sequence of edges such that

$(a, x_1), (x_1, x_2), \ldots (x_{n-1}, x_n), (x_n, b)$.

Ex3
P484.



"a" to "e"

(a,b) (b,e). 2

a b e 3-1=2

(a,c),(c,b),(b,e) 3

a c b e 4-1=3.

R_2 ? R^{-1} (a,a) 1.

a,a 2-1=1.

Theorem :-

Let R be a relation on A. there is a path of length $n \in Z^+$ from a to b if

$(a,b) \in R^n$.

HW. $R^2 = R \circ R$.

Graph.

Confirm Yourself.

R^3

Connectivity Relation: R^*.

Let R be a relation on A. The Connectivity Relation R^* Consists of pairs (a,b) Such That

$\exists$ a path from (a,b) in R.

$$R^* = \bigcup_{i=1}^{\infty} R^i = R^1 \cup R^2 \cup R^3 \cup \dots \cup R^{\infty}.$$

Exh:- $R = \{(a,b) \mid a \text{ has met } b\}$ $A =$ Set of all people in world.

What is R^*.

$R^2 = R \circ R$.

So R (a,b) R $A \times B$.

(b,c) S $B \times C$.

$(a,c) \in S \circ R$ if $(a,b) \in R \wedge (b,c) \in S$.

$R \circ R$ $(a, \overset{x_1}{\cancel{b}})$, R $A \times A$

$(\underset{x_1}{\cancel{b}}, \underset{b}{\cancel{x}})$. R $A \times A$.

1 person - $(a,b) \in R \circ R$ if $(a,x_1) \in R \wedge (x_1,b) \in R$.

if a has met $x_1 \wedge x_1$ has met b.

2 persons. $(a,b) \in R^3$ if $(a,x_1) \in R \wedge (x_1,x_2) \in R \wedge (x_2,b) \in R$.

if a has met $x_1 \wedge x_1$ has met $x_2 \wedge$
x_2 has met b.

$\vdots$

n-1 persons $(a,b) \in R^n$. - - - - - - -
- - - - - - -

$\vdots$

∞ -1 persons. $(a,b) \in R^\infty$. - - - - - -

$$R^* = \bigcup_{i=1}^{\infty} R^i = R^1 \cup R^2 \cup R^3 \cup \dots \cup R^\infty.$$

When $\exists$ any Number of persons btw
a & b. who has met in a sequence.

Ex6:- $R = \{(a,b) \mid$ a and b has Common border$\}$
A = Set of States of US.

H.W.

Theorem: the transitive closure of a relation
Equals The Connectivity Relation. R^*.

EQUIVALENCE RELATION.

1- Reflexive.

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1- Reflexive.
2- Symmtric
3- Transitive.

Ex1 :- $R = \{(a,b) \mid a = b \text{ or } a = -b\}$. $A = \mathbb{Z}$.

Phay

Reflexive:- $\forall a \in A \quad (a,a) \in R$.

$\forall a \in \mathbb{Z} \quad (a = a \lor a = -a)$ ✓.

Symmetric:- $\forall a,b \in A \quad \text{if } (\overset{1\ -1}{a,b}) \in R \rightarrow (b,a) \in R$.

$\forall a,b \in \mathbb{Z} \quad \text{if } a = b \lor a = -b \rightarrow b = a \lor b = -a$.

✓

Transitive:- $\forall a,b,c \in A \quad \text{if } (a,b) \in R \land (b,c) \in R \rightarrow (a,c) \in R$

$\forall a,b,c \in \mathbb{Z} \quad \text{if } (a = b \lor a = -b) \land (b = c \lor b = -c) \rightarrow a = c \lor a = -c$.

✓.

☐ ⓐ X. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓.

$[1]$ $[0]$ X