

Lecture 10:-

Anti Symmetric:- $\forall a, b \in A$ if $(a, b) \in R \wedge (b, a) \in R \rightarrow a = b$.

Ex 12:- $R = \{(a, b) \mid a \text{ divides } b\}$. $A = \mathbb{Z}^+$
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Symmetric: $\forall a, b \in A$ if $(a, b) \in R \rightarrow (b, a) \in R$.

$\forall a, b \in \mathbb{Z}^+$ if $a \text{ divides } b \rightarrow b \text{ divides } a$.

$(3, 6) \in R \rightarrow (6, 3) \notin R$.

\therefore Not Symmetric.

Anti Symmetric.

$\forall a, b \in A$ if $(a, b) \in R \wedge (b, a) \in R \rightarrow a = b$.

$(a \geq b \wedge b \geq a) \rightarrow a = b$.

$\forall a, b \in \mathbb{Z}^+$ if $a \text{ divides } b \wedge b \text{ divides } a \rightarrow a = b$.

It is Anti Symmetric.

$R = \{(a, b) \mid a \geq b\}$

$a \geq b$
 $a \geq b$

Ans.

$a \leq b$
 $a \leq b$

Symmetric.
Anti Symmetric

Reflexive.

$A = \mathbb{Z}$

Transitive:- $\forall a, b, c \in A$ if $(a, b) \in R \wedge (b, c) \in R \rightarrow (a, c) \in R$.

Ex 7:- $A = \{1, 2, 3, 4\}$

$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$. $(3, 1) \notin R$.

$$R_2 = \{ \}$$

$$R_2 = \{ (1,2) \}$$

$$A_2 = \{1,2\}$$

$$A \times A_2 = \{ (1,1), (1,2), (2,1), (2,2) \}$$

HW.

Total

Reflexive -

Symmetric.

Anti Symmetric.

Transitive

$$\bar{R} = \{ (a,b) \mid (a,b) \notin R \} = A \times A - R \quad \checkmark$$

$\cup, \cap, -, \text{Complement}$

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$$A_2 = \{1,2,3\}$$

$$B_2 = \{1,2,3,4\}$$

$$| \text{Pos}(A \times B) | = 2 \quad | A \times B | = 2^{12}$$

$$R_1 = \{ \underline{(1,1)}, \underline{(2,2)}, \underline{(3,3)} \}$$

$$R_2 = \{ \underline{(1,1)}, \underline{(1,2)}, \underline{(1,3)}, \underline{(1,4)} \}$$

$$R_1 \cup R_2 = \{ (1,1), (2,2), (3,3), (1,2), (1,3), (1,4) \}$$

$$R_1 \cap R_2 = \{ \}$$

$$R_1 - R_2 = \{ (2,2), (3,3) \}$$

$$R_2 - R_1 = \{ (1,2), (1,3), (1,4) \}$$

Ex 19:

$$R_1 = \{ (a,b) \mid a > b \}$$

$$R_2 = \{ (a,b) \mid a < b \}$$

$$R_1 \cup R_2 = \{ (a,b) \mid a > b \vee a < b \}$$

$$A = \mathbb{R}$$

$$= \{ (a,b) \mid a \neq b \}$$

$$R_1 \cap R_2 = \{ (a,b) \mid a > b \wedge a < b \}$$

$$= \emptyset$$

$$R_1 - R_2 = \{ (a,b) \mid a > b \wedge \neg(a < b) \}$$

$$\neg(\neg) = \leq$$

$$\neg(<) = \geq$$

$$= \{ (a,b) \mid a \geq b \wedge a \geq b \}$$

$$= \{ (a,b) \mid a \geq b \}$$

$$R_2 - R_1 = ?$$

$R_2 = \{ (a,b) \mid a \text{ divides } b \}$. How to expand formula.

$$A = \{1, 2, 3\}$$

$$\forall x P(x), \forall a, b$$

$$\forall a, b \in A \quad \text{if } (a,b) \in R \rightarrow (b,a) \in R.$$

$$\begin{array}{l} 1 \text{ divides } 1 \rightarrow 1 \text{ divides } 1 \quad \wedge \quad 1 \text{ divides } 2 \rightarrow 2 \text{ divides } 1. \\ 1 \text{ divides } 3 \rightarrow 3 \text{ divides } 1 \quad \wedge \end{array}$$

$$\begin{array}{l} 2 \text{ divides } 1 \rightarrow 1 \text{ divides } 2 \quad \wedge \quad 2 \text{ divides } 2 \rightarrow 2 \text{ divides } 2. \\ 2 \text{ divides } 3 \rightarrow 3 \text{ divides } 2 \quad \wedge \end{array}$$

Complete