Lecture 9:- Properties of Relations

$|A| = 4$

$R \subseteq A \times A$

$PS(A \times A) = 2^{|A| \times |A|}$

$= 2^{4 \times 4} = 2^{16}$

1) Reflexive

$\forall a \in A \quad (a,a) \in R \qquad (1,1) \in R \wedge (2,2) \in R \wedge$

$(3,3) \in R \wedge (4,4) \in R$

Ex 7 : 462

$A = \{1,2,3,4\}$

$R_1 = \{(1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4)\}$. ✗

$R_2 = \{(1,1),(1,2)\}$. ✗

$R_3 = \{(1,1),(2,2)\}$. ✗

$R_4 = \{\}$. ✗

$R_5 = \{(1,1),(2,2),(3,3),(1,2),(4,4)\}$. ✓

Ex: $A = \{\}$. $A \times A = \{(\phi,\phi)\}$.

$PS(A \times A) = \{\phi\}$.

$|PS(A \times A)| = 2^{|A \times A|} = 2^{|A| \times |A|} = 2^{0 \times 0}$

$= 2^0 = 1$.

Reflexive

$\forall a \in A \quad (a,a) \in R \qquad (\phi,\phi) \in R$.

ϕ

Ex $A = \{a\}$. $A \times A = \{(a,a)\}$.

$PS(A \times A) = \{\overset{\times}{\phi}, \{(a,a)\}\overset{\checkmark}{}\}$.

Pdf...

$|PS(A\times A)| = 2^4 = 16$ $\{(a,a),(a,b)\}$.

Reflexive

$\forall a \in A \quad (a,a) \in R \qquad\qquad (a,a) \in R.$

Ex $\quad A = \{a,b\}.$

$A \times A = \{(a,a),(a,b),(b,a),(b,b)\}$

$PS(A\times A) = \{\phi, \{(a,a)\}, \{(a,b)\}, \{(b,a)\}, \{(b,b)\},$

$\{(a,a),(a,b)\}, \{(a,a),(b,a)\}, \{(a,a),(b,b)\}, \{(a,b),(b,a)\},$

$\{(a,b),(b,b)\}, \{(b,a),(b,b)\}, \{(a,a),(a,b),(b,a)\},$

$\{(a,a),(a,b),(b,b)\}, \{(a,b),(b,a),(b,b)\},$

$\{(a,a),(b,a),(b,b)\}. \quad \{(a,a),(a,b),(b,a),(b,b)\}.$

Reflexive

$\forall a \in A \quad (a,a) \in R \qquad\qquad (a,a) \in R \wedge (b,b) \in R.$

Symmetric :- $\quad \forall a,b \in A \quad$ if $(a,b) \in R \rightarrow (b,a) \in R$

$(2,1) \in R \rightarrow (1,2) \in R.$

$A = \{1,2,3,4\}.$

Ex 462 :-

$R = \{(1,1),(1,2)\}.$ X.

$R' = \{\}.$ ✓

$R_3 = \{(1,2),(2,1),(1,1)\}.$ ✓

Ex $\quad A = \{\}.$ Hw.

Ex $\quad A = \{a\}$ Hw.

$A \times A = \{(a,a)\}.$

$PS(A\times A) = \{\phi, \{(a,a)\}\}.$

✓ ✓

✓ ✓

Symmetric :- $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$

Quiz # 4 22- FEB- 2023.

Find all Symmetric Relations on $A \times A$.

Solution :- $A = \{a,b\}$.

$A \times A = \{(a,a),(a,b),(b,a),(b,b)\}$.

$PS(A \times A) = \{\phi, \{(a,a)\}, \{(a,b)\}, \{(b,a)\}, \{(b,b)\},$

$\{(a,a),(a,b)\}, \{(a,a),(b,a)\}, \{(a,a),(b,b)\}, \{(a,b),(b,a)\}.$

$\{(a,b),(b,b)\}, \{(b,a),(b,b)\}, \{(a,a),(a,b),(b,a)\},$

$\{(a,a),(a,b),(b,b)\}, \{(a,b),(b,a),(b,b)\},$

$\{(a,a),(b,a),(b,b)\}. \{(a,a),(a,b),(b,a),(b,b)\}.$