Lecture 15:- Representing Relations Using Graphs.

$\bigcirc$ $\qquad$ V = Set of Vertices.

$(\longrightarrow, \text{——})$ $\qquad$ E = Set of Edges. $\qquad$ $G = (V, E)$.

Syntax. $\qquad$ Semantics.

Let R be a relation on set A.

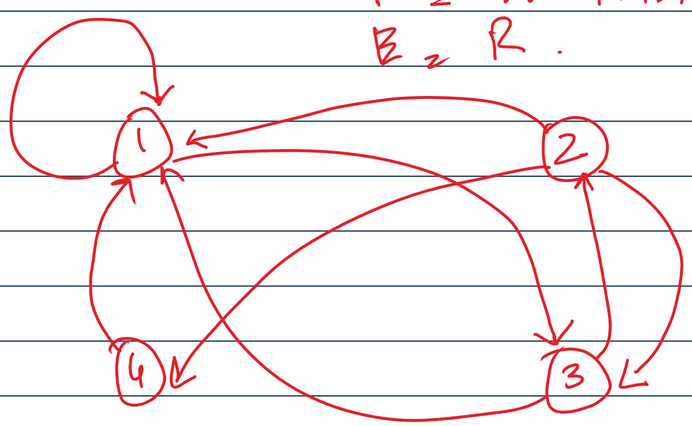
$V = A$

$E = R$.

Ex 8 | 480:- $\quad R = \{(1,1), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (4,1)\}$.

$V = A = \{1,2,3,4\}$.

$E = R$.



HW.
Do the Reverse.

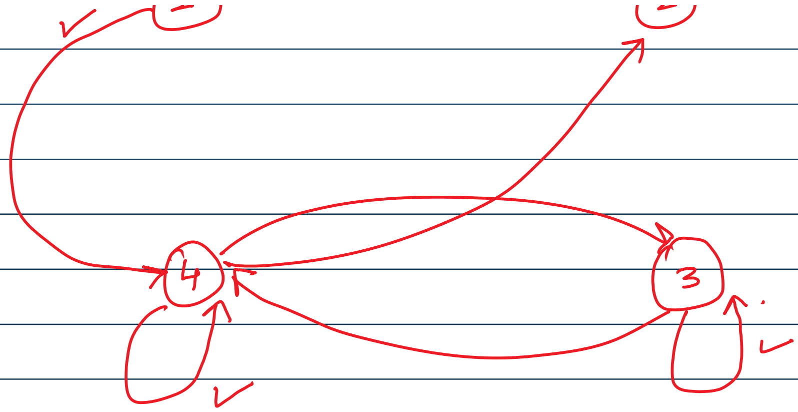
HW.
Represent the Same Using matrix.

Relation in terms of set $\longleftrightarrow$ Relation in terms of matrices

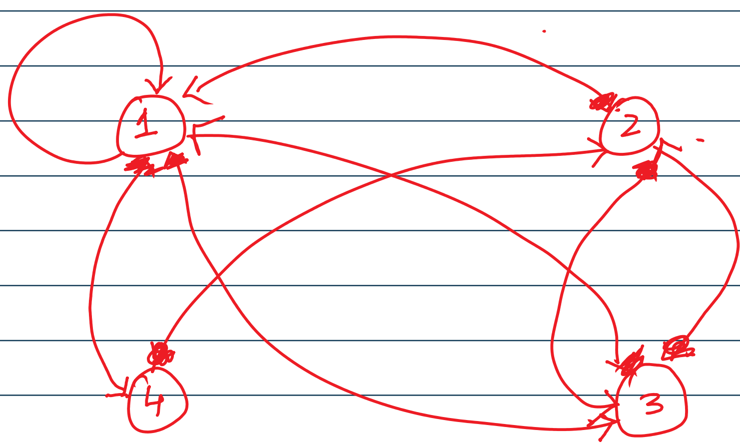
$\longleftrightarrow$ " " " " Graphs.

Find $\quad \overline{R} = \{(a,b) \mid (a,b) \notin R\}$.

$|R| \cup |\overline{R}| = |A \times A|$

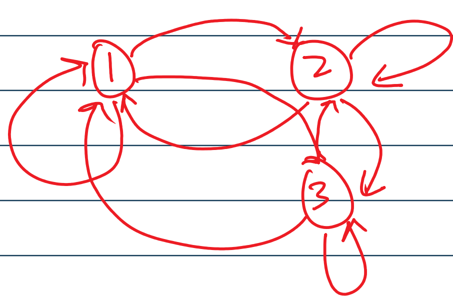


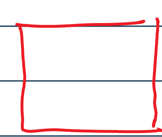
$R^{-1} = \{(b,a) \mid (a,b) \in R\}$.



R^{-1}.

Reflexive :- $\forall a \in A \quad (a,a) \in R$.





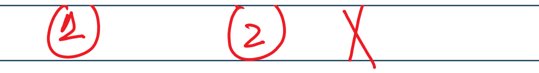
$V = A = \{1,2\}$.

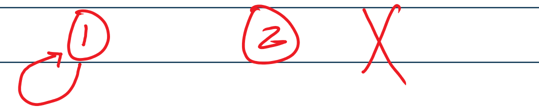
$|A \times A| = 4$.

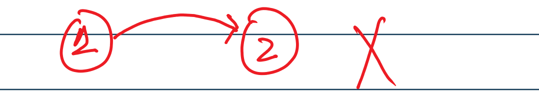
1- 1 2 X

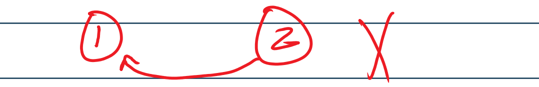
$|A \times A| = 4.$

$2^4 = 16.$

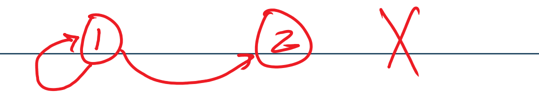
1- 

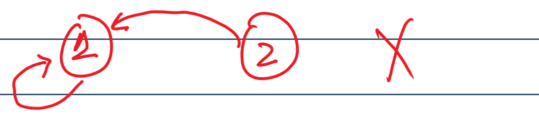
2- 

3- 

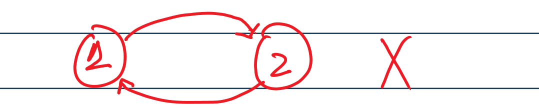
4- 

5- 

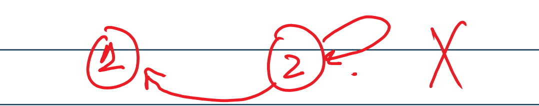
6- 

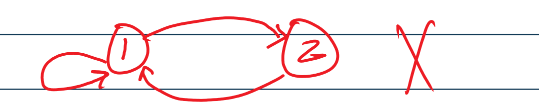
7- 

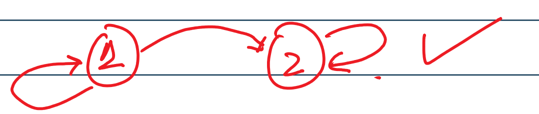
8- 

9- 

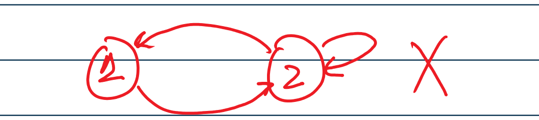
10 

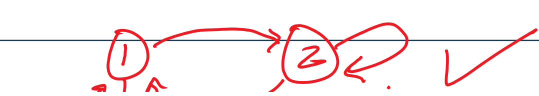
11- 

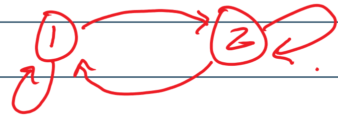
12- 

13 - 

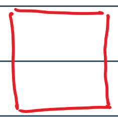
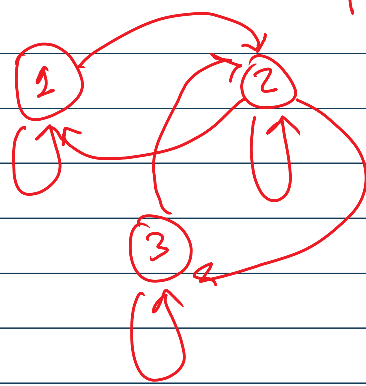
14 - 

15 - 

16 - 

16 -  ✓

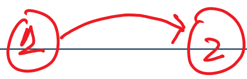
Symmetric. $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

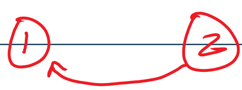
  $R = \{ \}$.

✓

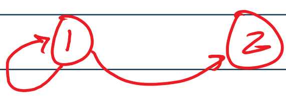
1 -  ✓

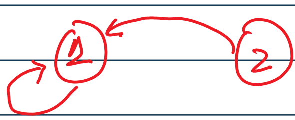
2 -  ✓

3 -  ✗

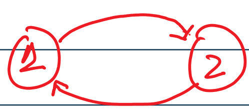
4 -  ✗

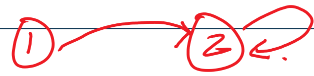
5 -  ✓

6 -  ✗

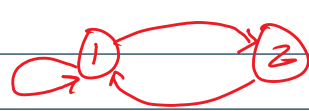
7 -  ✗

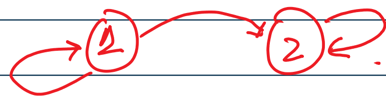
8 -  ✓

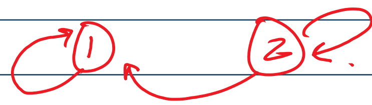
9 -  ✓

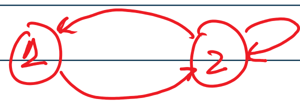
10  X

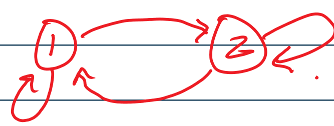
11-  X

12-  ✓

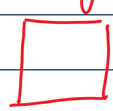
13 -  X

14 -  X

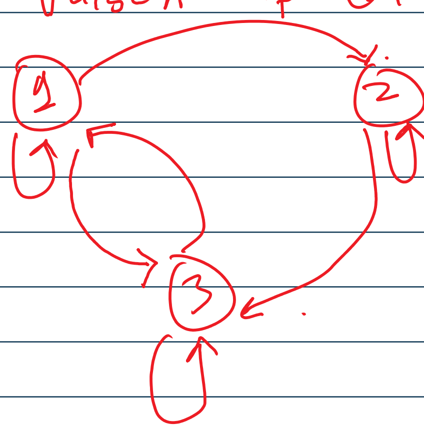
15 -  ✓

16 -  ✓

Anti Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \Rightarrow a=b$.



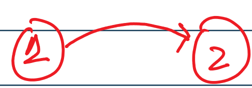
✓



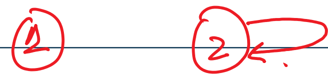
X.

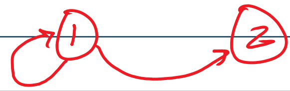
1-  ✓

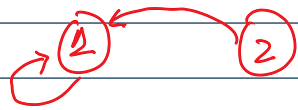
2-  ✓

3-  ✓

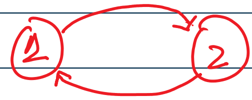
4-  ✓

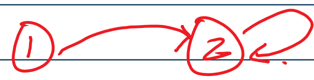
5-  ✓

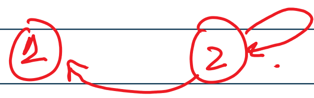
6-  ✓

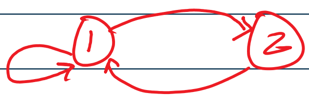
7-  ✓

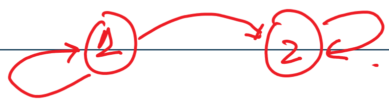
8-  ✓

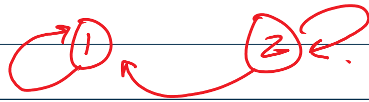
9-  X

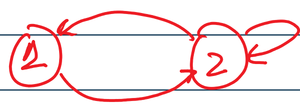
10  ✓

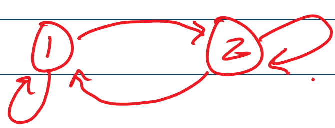
11-  ✓

12-  X

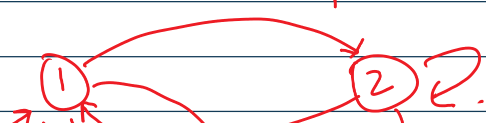
13 -  ✓

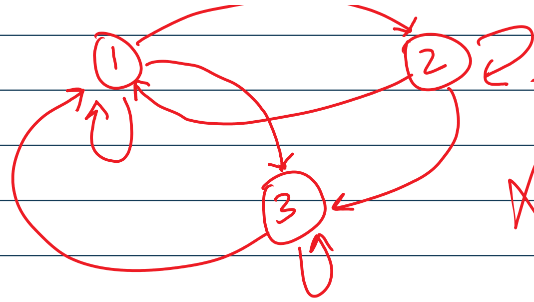
14 -  ✓

15 -  X

16 -  X

Transitive: $\forall a,b,c \in A$ if $(a,b) \in R \land (b,c) \in R \to (a,c) \in R$.

□ 



X.

1-  ✓

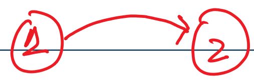
$R = \{\ \}$. $\Delta = \{(1,1),(2,2)\}$

$R \cup \Delta = \{(1,1),(2,2)\}$.

2-  ✓

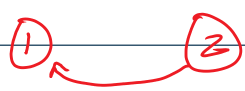
$R = \{(1,1)\}$

$R \cup \Delta = \{(1,1),(2,2)\}$.

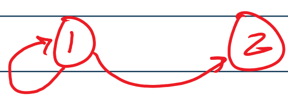
3-  ✓

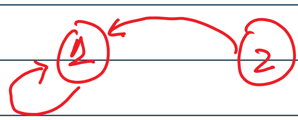
$R = \{(1,2)\}$.

$R \cup \Delta = \{(1,1),(1,2),(2,2)\}$

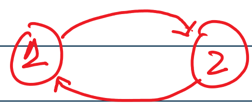
4-  ✓

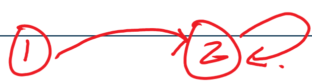
5-  ✓

6-  ✓

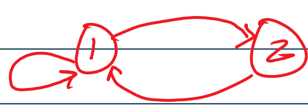
7-  ✓

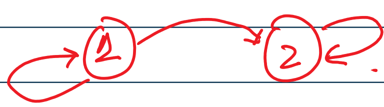
8-  ✓

9-  X.

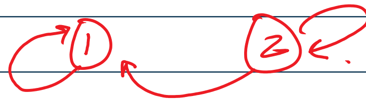
10  ✓

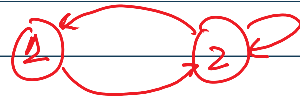
11-  ✓

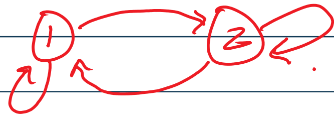
12-  X

13 -  ✓

13 –  ✓

14 –  ✓

15 –  ✗

16 –  ✓

$(a,a) \in A.$

Closure :-

Closure w.r.t Reflexive.

P483:- $R = \{(1,1), (1,2), (2,1), (3,2)\}.$ $A = \{1,2,3\}.$

$\Delta = \{(1,1), (2,2), (3,3)\}. = \{(a,a) \mid \forall a \in A\}.$

Ex1 483 $R = \{(a,b) \mid a<b\}.$ $A = \mathbb{Z}.$

$R = \{(a,b) \mid a<b\}.$ $\Delta = \{(a,a) \mid \forall a \in \mathbb{Z}\}.$

$\Delta = \{(a,a) \mid \forall a \in \mathbb{Z}\}.$

$= \{(a,b) \mid a=b\}.$ $\{(-\infty,-\infty), (-\infty+1, -\infty+1), \ldots (0,0)$
$(1,1), \ldots (+\infty,+\infty)\}.$

$R \cup \Delta = \{(a,b) \mid a<b \lor a=b\}.$

$= \{(a,b) \mid a \leq b\}.$

Symmetric Closure :-

$R \cup R^{-1}$ $R = \{(a,b) \mid \text{———}\}.$

$R^{-1} = \{(b,a) \mid (a,b) \in R\}.$

Ex2:- b a b a

Ex2:-

483

$R^{-1} = \{(b,a) \mid (a,b) \in R\}$.

$R = \{(\overset{b}{a},\overset{a}{b}) \mid \overset{b}{a} > \overset{a}{b}\}$. $b > a$ $a < b$

$R^{-1} = \{(a,b) \mid (b,a) \in R\}$.

$R^{-1} = \{(a,b) \mid a < b\}$.

$R \cup R^{-1} = \{(a,b) \mid a > b \vee a < b\}$

$= \{(a,b) \mid a \neq b\}$.