Lecture 17:- Equivalence Relation.

- Reflexive
- Symmetric
- Transitive.

Ex2:- $R = \{(a,b) \mid a-b \in \mathbb{Z}\}$. $A = \mathbb{R}$.
494.

Reflexive:- $\forall a \in A \quad (a,a) \in R$.
$\forall a \in \mathbb{R} \quad a-a \in \mathbb{Z}$.

Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \rightarrow (a,b) \in R$.
$\forall a,b \in \mathbb{R}$ if $(a-b) \in \mathbb{Z} \rightarrow (b-a) \in \mathbb{Z}$.

Transitive:- $\forall a,b,c \in A$ if $(a,b) \in R \land (b,c) \in R \rightarrow (a,c) \in R$.
$\forall a,b,c \in \mathbb{R}$ if $(a-b) \in \mathbb{Z} \land (b-c) \in \mathbb{Z} \rightarrow (a-c) \in \mathbb{Z}$. ✓

EQUIVALENCE ✓

Ex3. Congruence Modulo.
494.

$R = \{(a,b) \mid a \equiv b \bmod m\}$. $A = \mathbb{Z}$.

Reflexive:- $\forall a \in A \quad (a,a) \in R$
$\forall a \in \mathbb{Z} \quad a \equiv a \bmod m$.

Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \rightarrow (a,b) \in R$.
$\forall a,b \in \mathbb{Z}$ if $\exists_{k \in \mathbb{Z}}\ (a-b) = km \rightarrow \exists_{k \in \mathbb{Z}}\ (b-a) = km$..

Transitive :- $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in \mathbb{Z}$ if $\exists_{k \in \mathbb{Z}}\ a-b = km \wedge \exists_{l \in \mathbb{Z}}\ b-c = lm \rightarrow \exists_{l+k \in \mathbb{Z}}\ a-c = (k+l)m$

EQUIVALENCE . $\checkmark$

Ex 6 4a5 $\quad R = \{(a,b) \mid a \div b\}$. $\quad A = \mathbb{Z}^+$.

Reflexive: $\forall a \in A \quad (a,a) \in R$

$\forall a \in \mathbb{Z}^+ \quad a \div a \quad \checkmark$

Symmetric: $\forall a,b \in A$ if $(a,b) \in R \rightarrow (a,b) \in R$.

$\forall a,b \in \mathbb{Z}^+$ if $a \div b \rightarrow b \div a$. X

Transitive :- $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

X.

EQUIVALENCE X.

Ex 7 4a5 $\quad R = \{(x,y) \mid |x-y| < 1\}$. $\quad A = \mathbb{R}$.

Reflexive: $\forall a \in A \quad (a,a) \in R$

$\forall a \in \mathbb{R} \quad |a-a| < 1$.

Symmetric: $\forall a,b \in A$ if $(a,b) \in R \rightarrow (a,b) \in R$.

$\forall a,b \in \mathbb{R}$ if $|a-b| < 1 \rightarrow |b-a| < 1$.

Transitive :- $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$

$\forall a,b,c \in \mathbb{R}$ if $|a-b| < 1 \wedge |b-c| < 1 \rightarrow |a-c| < 1$.

$|1.5 - 1.2| < 1 \wedge |1.2 - 0.3| < 1 \rightarrow |1.5 - 0.3|$

$\forall a,b,c \in \mathbb{R}$ if $|a-b|<1 \wedge |b-c|<1 \rightarrow |a-c|<1$.

$|1.5-1.2|<1 \wedge |1.2-0.3|<1 \rightarrow |1.5-0.3|$

$1.2 \not< 1$

EQUIVALENCE R.

EQUIVALENCE CLASS.

$[a] = \{ s \mid (a,s) \in R \}$

Ex8 :- $R = \{(a,b) \mid a=b \vee a=-b\}$. $A = \mathbb{Z}$.

496 $\quad 7=7$

$[7] = \{7, -7\}$.

$(7,7)$.

$(7,-7)$.

$1 \equiv b \mod 4$

Ex9 :- $R = \{(a,b) \mid a \equiv b \mod m\}$. $A = \mathbb{Z}$.

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$[0] = \{0, \pm 4, \pm 8, \pm 12, \pm 16 \dots\}$. $\quad 0 \equiv b \mod 4$.

$(0,0)$ $\quad (0,-4)$

$(0,4)$ $\quad (0,-8)$

$(0,8)$ $\quad \vdots$

$[1] = \{1, 5, 9, 13, \dots, -3, -7, -11, -15 \dots\}$. $\quad 1 \equiv b \mod 4$.

$(1,1), (1,5), (1,9) \dots$

-1

$4 \overline{)\,-3}$

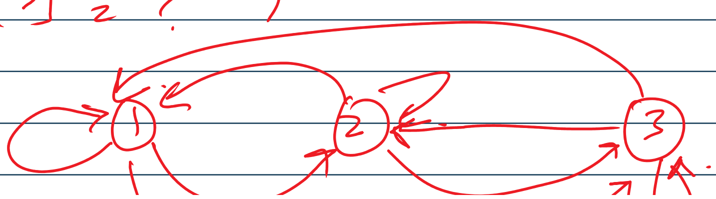
-4

$+$

1

$[2] = ?$

$[3] = ?$ $\quad$ HW.



$[1] = \{1, 2, 3\}$

$[2] = [1] = \{1, 2, 3\}$.
$= [3]$.

$$\begin{array}{c|ccc} & a & b & c \\ a & 1 & 0 & 1 \\ b & 0 & 1 & 0 \\ c & 1 & 0 & 1 \end{array}$$

$[a] = \{a, c\}. = [c].$
$[b] = \{b\}.$

PARTITION. Let S be Set. A partition $P = \{P_1, P_2, \dots P_n\}$ where $P_i \subseteq S$.
if.

i) $\forall i \; P_i \neq \phi$. $= P_1 \neq \phi \wedge P_2 \neq \phi \wedge \dots \wedge P_n \neq \phi$.

ii) $\forall i, j \quad P_i \cap P_j = \phi \quad i \neq j$.

iii). $\bigcup_{i=1}^{n} P_i = S$.

Ex12
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$S = \{1, 2, 3, 4, 5, 6\}$.

$P_1 = \{1, 2, 3\} \qquad P_2 = \{4, 5\}, \qquad P_3 = \{6\}$.

Check (i) $\quad P_1 \neq \phi \wedge P_2 \neq \phi \wedge P_3 \neq \phi$
$\quad T \wedge T \wedge T = T$.

Check (ii) $\quad P_1 \cap P_2 = \phi \wedge P_1 \cap P_3 = \phi \wedge P_2 \cap P_3 = \phi$.
$\quad T \wedge T \wedge T = T$.

Check (iii) $\quad P_1 \cup P_2 \cup P_3 = S$.

Hence a Partition.

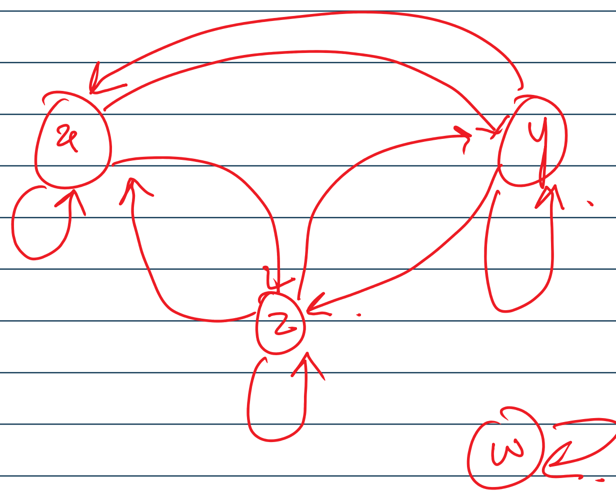
Theorem: Equivalence classes creates a partition of the set based on which the relation is defined.

$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 0 & 1 \\ b & 0 & 1 & 0 \\ c & 1 & 0 & 1 \end{array}$$

$[c] = [a] = \{a, c\}.$

$[b] = \{b\}.$

$P = \{\underbrace{\{a, c\}}, \underbrace{\{b\}}\}.$



$[x] = [y] = [z] = \{x, y, z\}.$

$[w] = \{w\}.$

$P = \{\{x, y, z\}, \{w\}\}.$

$A = \{x, y, z, w\}.$

$R = \{(x,x), (x,y), (x,z), (y,x), (y,y), (y,z), (z,x), (z,y), (z,z), (w,w)\}.$

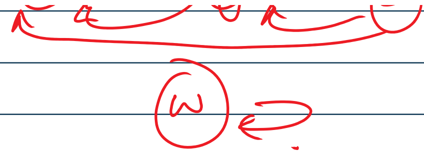
M..

G

ER $\rightarrow$ EC $\rightarrow$ P

ER $\leftarrow$ EC $\leftarrow$ P

$$\begin{array}{c|cccc} & x & y & z & w \\ \hline x & 1 & 1 & 1 & 0 \\ y & 1 & 1 & 1 & 0 \\ z & 1 & 1 & 1 & 0 \\ w & 0 & 0 & 0 & 1 \end{array}$$



Sessional #2 Uptil This Point.