Lecture 22:-

Wednesday, April 19, 2023 1:14 PM

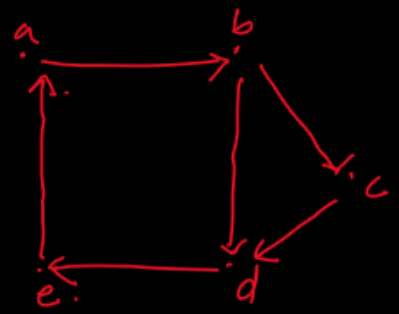
Strongly Connected.

1- Directed Graph.
2- Connected (Connected Components ≥ 2).
3- $\forall a,b \in V$ there must be a path from a to b & b to a.

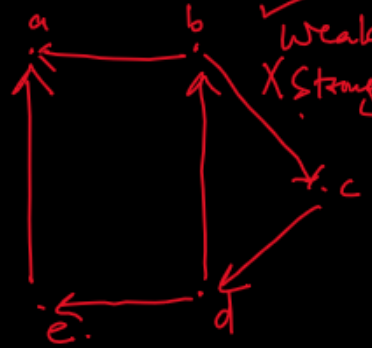
Weakly Connected.

1- Directed Graph.
2- Connected (Connected Components ≥ 2).
3- $\forall a,b \in V$ there should be a path from a to b or b to a.

Ex 11 :-
P565



a to b ∧ b to a
a to c ∧ c to a
a to d ∧ d to a
a to e ∧ e to a
b to c ∧ c to b
b to d ∧ d to b
b to e ∧ e to b
c to d ∧ d to c
c to e ∧ e to c
d to e ∧ e to d.



Weakly Connected.
X Strongly "

a to b ∨ b to a.
a to c ∨ c to a.
a to d ∨ d to a.
a to e ∨ e to a.
b to c ∨ c to b.
⋮

Number of paths :-

$$A^1 = \begin{array}{c} \\ a \\ b \\ c \\ d. \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

a b a b a
a b d b a
a b d c a
a b a c a
a c a c a
a c d c a
a c a b a
a c d b a

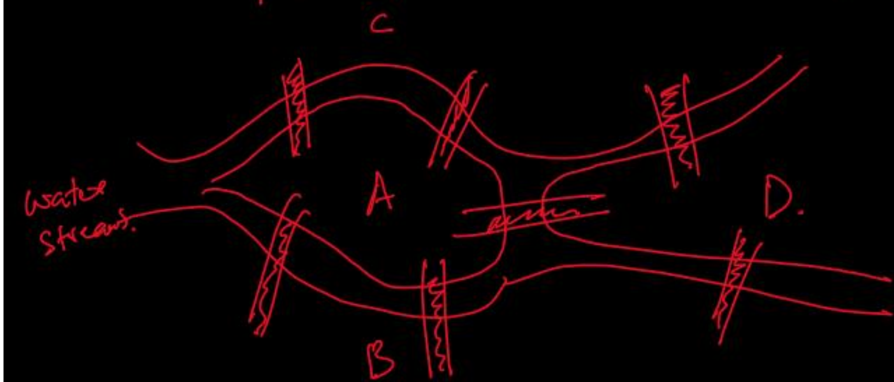
$$A^4 = \begin{array}{c} \\ a \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{c} a \end{array} \\ \begin{bmatrix} 8 & 0 & 0 & 8 \\ 0 & 8 & 8 & 0 \\ 0 & 8 & 8 & 0 \\ 8 & 0 & 0 & 8 \end{bmatrix} \end{array}$$

Try finding the a^{th} path of length 4 from a to a. (you will fail).

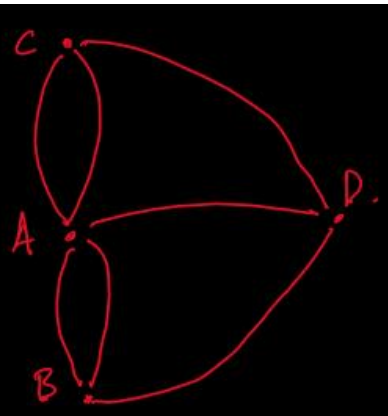
Ex 567–569

Q (1–40);–.

Euler paths & Circuits. Konigsberg. 1776.

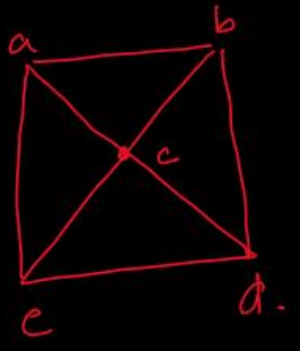


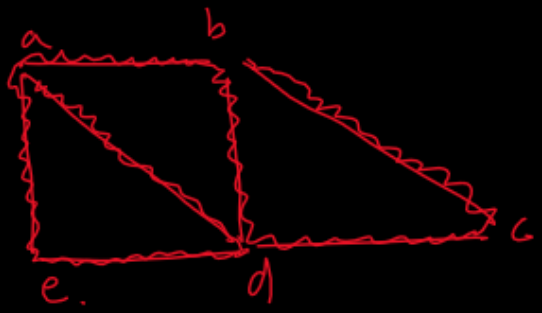
Euler path:-
we traverse each & every edge exactly once.



Euler Circuit.
we start from a point & go through all the edges exactly once & come back to original point.

Ex 2
572

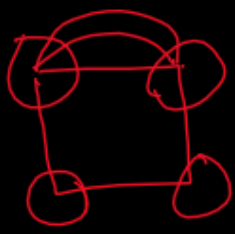




EC ✗

EP ✓

Theorem: A connected multigraph with atleast 2 vertices has an EC iff each of its vertices has even degree.



EC ✓

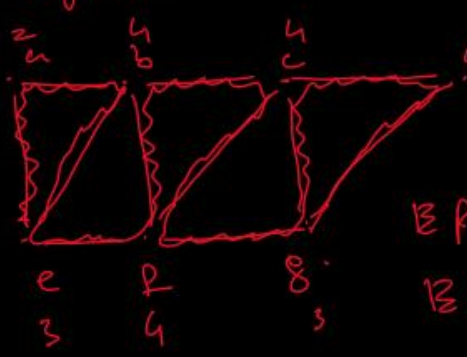
W_3

K_5

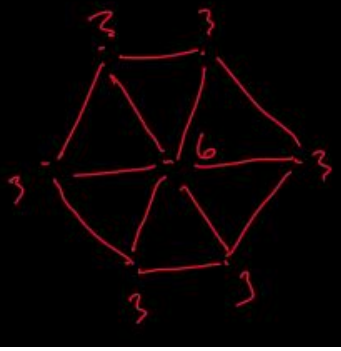
theorem:- A Connected graph has a Euler path but not a Euler Circuit iff it has exactly 2 Vertices of odd degree.

Ex 4
575

ECX
EPL.



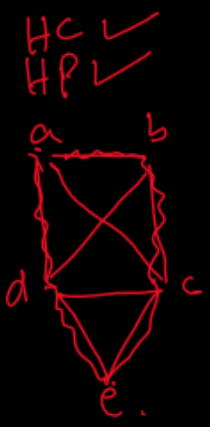
EPL
ECX.



EPX
ECX.

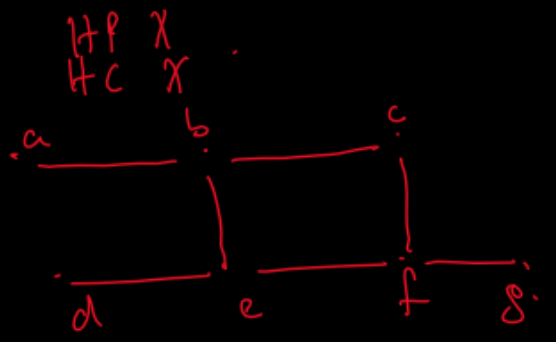
Hamilton paths & Circuits.

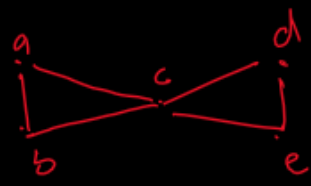
EXS



Simple path:

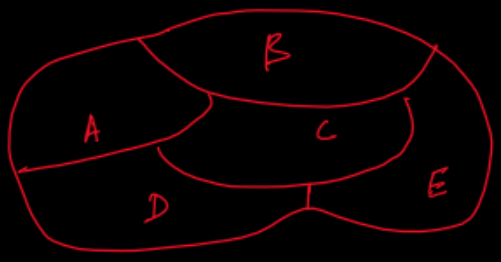
- EP
- EC
- HP
- HC.

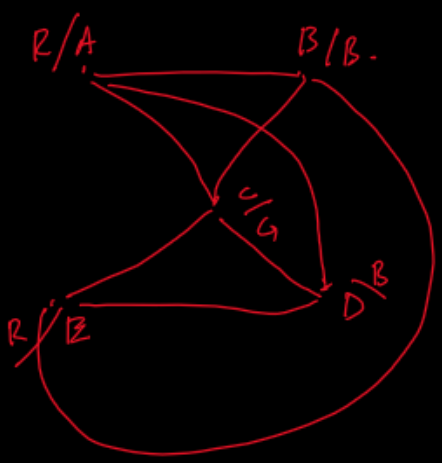




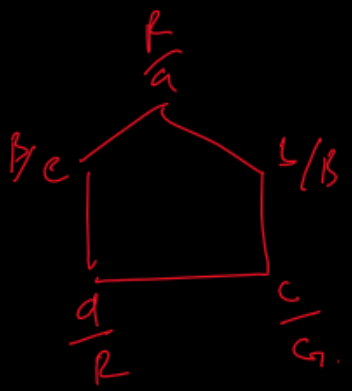
HP ✓
HC ✗.

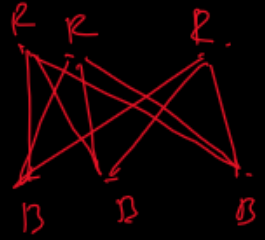
Graph Colouring :- {R, B, G}.

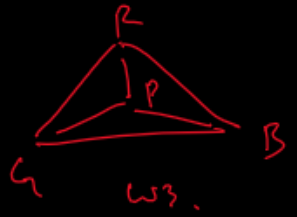


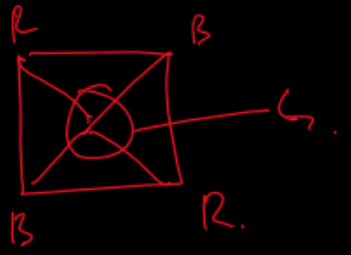


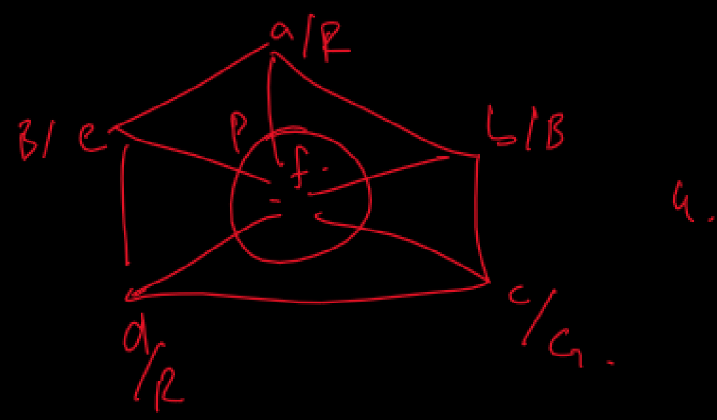
Chromatic Number = 3.

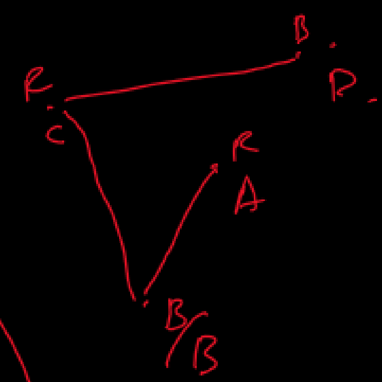


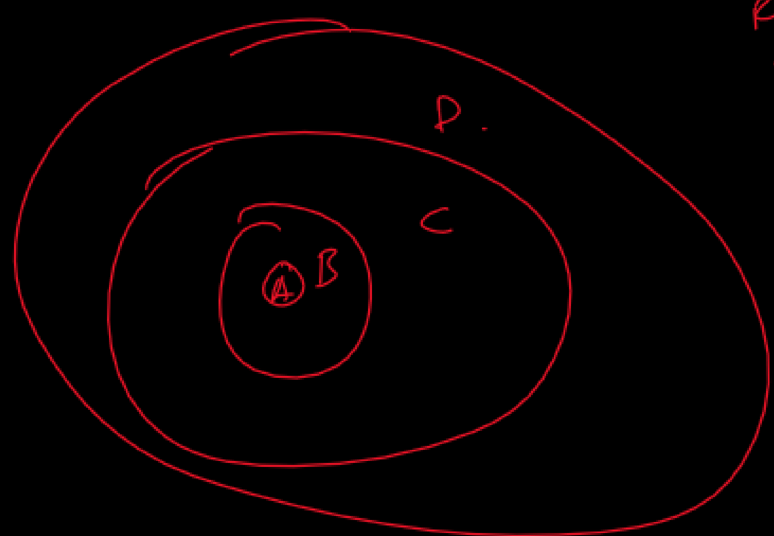












(EK P 612–615
1–45):-