lecture 8 :- Resolution Principle

literal :- A variable or its negation $p \quad q$
$\neg p \quad \neg q$.

clause :- A disjunction of literals

$\neg p \vee \neg q$

$p \vee \neg q$. $\quad p \vee q \vee r$

$\neg p \vee q$

Step 1 :- For each premise find out corresponding clauses.

P1 :- $p \wedge q$ $\quad$ C1 :- p
C2 :- q

P2 :- $p \rightarrow q$ $\quad$ C1 :- $\neg p \vee q$

Step 2 :- Find the negation of conclusion. and determine clause.

Step 3 :- Repeatedly Apply PR.

$$\begin{array}{l} p \vee \cancel{q} \\ \underline{\cancel{\neg q} \vee r} \\ \therefore\ p \vee r. \end{array}$$

P1 $\quad p$
P2 $\quad \underline{p \rightarrow Q}$
C $\quad \therefore\ Q$.

C1: p ✓
C2 :- $\neg p \vee Q$ ✓
C3 :- $\neg Q$. ✓
C4 :- Q ✓ from C1 & C2
C5 :- $\Box$ " C3 & C4.

Argument is Valid.

Ex 11 $\quad$ P65 $\quad$ P1 $\quad T \rightarrow (M \vee E)$
P2 $\quad S \rightarrow \neg E$
P3 $\quad \underline{T \wedge S}$
C. $\quad \therefore\ M$

C1 :- $\neg T \vee M \vee E$ ✓
C2 :- $\neg S \vee \neg E$ ✓

C2:- $\neg S \vee \neg E$ ✓

C3:- T ✓

C4:- S ✓

C5:- $\neg M$ ✓

C6:- $\neg T \vee M \vee \neg S$ ✓ from C1, C2.

C7:- $M \vee \neg S$ ✓ " C3, C6.

C8:- M ✓ " C4, C7.

C9:- $\square$ " C5, C8.

Ex7:- P62:-

p1	$p \to q$
p2	$\neg p \to r$
p3	$r \to s$
C	$\therefore \neg q \to \neg s$

$\neg(\neg q \to \neg s)$.

$\neg(q \vee s)$.

$\neg q \wedge \neg s$.

C1:- $\neg p \vee q$ ✓

C2:- $p \vee r$ ✓

C3:- $\neg r \vee s$. ✓

C4:- $\neg q$ ✓

C5:- $\neg s$. ✓

C6:- $q \vee r$ ✓ from C1, C2.

C7:- $q \vee s$ ✓ " C3, C6.

C8:- s ✓ " C4, C7.

C9:- $\square$ " C5, C8.

Argument valid.

HW.
P 70 – 72.
Exercise 1–30

Quiz # 5

P1	$L \to A$
p2	$E \to \neg I$
p3	$A \to E$
C.	$\therefore L \to \neg I$.

Prove or disproof.

RELATIONS

→ SET :- A collection of distinct objects.

→ SET : A Collection of distinct objects.

(شكل ظاهري) Syntax. $\{\ \}$.

1- جامع

2- مانع

Semantics Repetition Not allowed.

$A \times B$ $\quad A = \{1, 2, 3\}$. $\quad B = \{a, b\}$.

$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$.

$B \times A = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$.

Cardinality of a Set $\quad |A| = 3$

" " " " $\quad |B| = 2$

$|B \times A| = |A \times B| = |A| \times |B| = 3 \times 2 = 6$.

Subset $\quad A \subseteq B$.

$\{1, 2, 3, 4\}$
$= \{2, 3, 4, 1\}$.

Power Set (A) = All Subsets of A.

$A = \{1, 2, 3\}$.

$PS(A) = \{\Phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.

$|PS(A)| = 2^{|A|} = 2^3 = 8$.

$PS(A \times B) = 2^{|A \times B|} = 2^{3 \times 2} = 2^6 = 64$.

$= \{\Phi, \{(1,a)\}, \{(1,b)\} \cdots \{(1,a), (1,b)\} \cdots\}$.

Relation:- A binary Relation R defined on $A \times B$.

$$R \subseteq A \times B.$$

Question. $|A| = 5 \qquad |B| = 10$.

How many Relations on $A \times B$.

Total Relations = Total Subsets = Powerset.

$$|\text{Power Set}(A \times B)| = 2^{|A \times B|} = 2^{|A| \times |B|} = 2^{10 \times 5} = 2^{50}$$

$$|PS(A \times B)| = 2^{|A \times B|} = 2^{|A| \times |B|} = 2^{0 \times 0} = 2^0 = 1.$$

$A = \{\}$ $\qquad B = \{\}$.

$PS(A \times B) = \{\phi\}$.

<u>Ex3</u>:- $A = \{0, 1, 2\} \qquad B = \{a, b\}$. $\qquad A \times B$.

P460

$R = \{(0, a), (0, b), (1, a), (2, b)\}$.

aRb. $\qquad (a, b) \in R$. $\qquad a \in A \quad b \in B$. $\qquad A \times B$.

$a \not R b$. $\qquad (a, b) \notin R$

Syntax $\quad$ Semantics.

$\{ \quad | \quad \}$.

شکل $\qquad$ کب آئیگا

Ex4:- P461.

$A = \{1, 2, 3, 4\}$.

$R = \{(a, b) \in A \times A \mid a \text{ divides } b\}$.

$R = \{(1, 1), (1, 2), (1, 3), (1, 4),$
$(2, 2), (2, 4), (3, 3), (4, 4)\}$.

$A \times A = \{(1, 1), (1, 2), (1, 3), (1, 4),$
$(2, 1), (2, 2), (2, 3), (2, 4),$
$(3, 1), (3, 2), (3, 3), (3, 4),$
$(4, 1) \; (4, 2), (4, 3), (4, 4)\}$.

Ex:- $R_1 = \{(a, b) \mid a \le b\}$. HW.
$R_2 = \{(a, b) \mid a > b\}$.

Ex:

$R_2 = \{(a,b) \mid a > b\}$.

$R_3 = \{(a,b) \mid a = b\}$.

$R_4 = \{(a,b) \mid a = b+1\}$.

R is defined on A.

Definitions:- $\bar{R} = \{(a,b) \mid (a,b) \notin R\}$.

$\bar{R} = A \times A - R$

$|R| = 9$ $\quad |A| = 5$.

$|\bar{R}| = ?$

$$\bar{R} = |A \times A| - |R| = 5 \times 5 - 9 = 25 - 9 = 16.$$

Definition:- $R^{-1} = \{(b,a) \mid (a,b) \in R\}$.

$=$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$.

$R^{-1} = \{(1,1), (2,1), (3,1), (4,1), (2,2), (4,2), (3,3), (4,4)\}$.

$|R| = |R^{-1}|$.