# Homework 4 write Up:

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1.)

a.) → read in data; np.load
get voltage; data[:,0]
get uncertain; " [:,1

→ mean(volage); np.mean
variance(uncert); np.std

b.) $\ln(L) = L(\theta | x) = \sum \log(f_i \theta | \theta)$

↑ log of the Likelihood of θ Given θ.

c.) Bayesian posterior probability density
- $\mu = [3.5, 6.5]$

$$P(\theta | x) = \frac{P(x|\theta)P(\theta)}{P(x)}$$

$P(\theta|x)$ ~ posterior prob
$P(x|\theta)$ ~ likely hood
$P(\theta)$ ~ prior
$P(x)$ ~ next point

⟹ step 1: Log Likelyhood
step 2: Prior ~ we will assume Gaussian
step 3: Compute Post. using Bayes Rules

$$\text{Prior} = p(\mu) = (2\pi\gamma_0^2)^{-1/2} e^{(- \quad )}$$

mu0 = muVol
sig0 = sigmaVol
mu = rang
sig

$p(\mu|x) \propto p(x|\mu)\, p(\mu)$

$$p(x|\mu) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\{\frac{(x_i-\mu)^2}{2\sigma^2}\}}$$

$$p(\mu) = \frac{1}{\sqrt{2\pi\gamma}} e^{\{-\frac{(\mu-M)^2}{2\gamma^2}\}}$$

γ = variation of μ around M
M = mean Guess
$\mu = \{3.5 - 6.5\}$

⟹ posterior:

$$\frac{1}{2}\left[\frac{\mu^2 - 2\mu M_\mu^2}{\gamma^2} + \frac{\sum x^2 - 2N\bar{x}\mu + N\mu^2}{\sigma^2}\right]$$

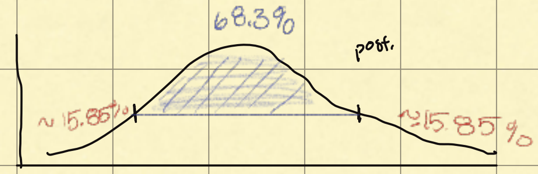
...

return

mean $\dfrac{\sigma^2 M + n r^2 \bar{x}}{N\gamma^2 + \sigma^2}$

variance: $\dfrac{\sigma^2 \gamma^2}{N\gamma^2 + \sigma^2}$

4.) Draw samples from posterior Find the equal tailed 68.3% credible region for the mean; Compare the upper & lower boundaries to and Find the MAP value of the mean.




upper ~ @ 15.85
lower ~ @ 84.15

5.) Repeat 3 & 4 for μ [4.5, 5.5]
; make it zero ∀ ≠ 4.5, 5

6.) Repeat 3 & 4 with a mean of 6.1 & std of .25 ✓✓

7.) Plot pdfs.

normalization
↓

$$\log L\left(0.5 \cdot \sum \left(\frac{(x_i-\mu)}{\sigma^2} + \log(2\pi\sigma^2)\right)\right.$$

Log(theta, data, model = poly fit)
x = data[:,0]
y = data[:,1]
sigma = data[:,2]
y_fit = model(theta, x)

\# model(theta, x)
→ return theta * x times a polynomial

LogL ( Σ (

## Problem 2

1. read in data
   data = np.read[ ]
   x, y, sigma_y = [:0], [:1], [:2]

2. y-intercept fixed @ -0.23

the model used: $y = mx + b$;
$b = -0.23$
$m = ?$

x - independent variable,
y = dependent
sigma_y = $\sigma(y)$

posterior PDF:

$p(m|data) \propto P(data|m)\,P(m)$

P(data|m) → likelihood
p(m) → prior

P(data|m):

to get the p(data|m)

$\log(L) = -\sum_i \left( \frac{(y_i - (mx+b))^2}{2\sigma_{y_i}^2} \right)$

P(m):

to get the prior, assume a uniform prior:
stat. uniform
[0.5, 1.5]
∀ else [0]

→ take m = np.arange()
LogL = P(data|m)
prior = P(m)
post

→

[theta is a vector]
[$\theta_0$,

mu_values = np.linspace(0, 5, 100)
mu_values = np.linspace(0.5, 1.5, 100)
bays prior(mu, mumin, mu max)
array, array[0], array[len[max-1]]

return np.where(m >= mumin & m <= mumax, # 6
m >= 0.05, m <= 1.5
1 / [m >= mumin & m <= mumax] # 16

#returns array w/ mu is in range, cov, ^ ?)

bay post (mu values, voltage, sigma)
x, y
[x, x, x]

( mu_valus , x )

sum(scipy.norm.logpdf)
for args in zip(

$x_y \wedge \sigma_y$

Consider one point

$\sum P_i = N_{pix}$

$\ln(L_i) = \frac{1}{2}\ln(2\pi\sigma_i^2) - \frac{1}{2}\frac{(y_{data}^i - y_{model}^i)}{\sigma_i^2}$

$y_{data}$ = measurments
$y_{model}$ = the fit

and is also the same as sum

model Y for a polynomial of degree on

$y_{lin} = \theta_0 + \theta_1 x$

$y_{quad} = \theta_0 + \theta_1 x + \dots + \theta_N x^N$

x  f(x)  sigma