Homework 2

Problem 1:

1.) $\int_0^\infty x^3 e^{(-x^2/2\sigma^2)}$

$u = x^2/2\sigma^2$ $\qquad x^3 = x(x^2)$

$u\,2\sigma^2 = x^2 \quad \Rightarrow \quad x(2u\sigma^2)$

$x = \sigma\sqrt{2u}$

$dx = \sigma^2/\sqrt{u}\ du$

$$\Rightarrow \int_0^\infty \cancel{\sqrt{2\sigma^2 u}}\,(2\sigma^2 u)\, e^{-u} \frac{\sigma^2}{\cancel{\sqrt{\sigma^2 2u}}}\, du$$

$$\int_0^\infty \frac{2\sigma^4 u}{e^u}\, du$$

$$2\sigma^4 \int_0^\infty \frac{u}{e^u}\, du = -(u+1)e^{-u} = -\left(\frac{x^2}{\sigma}+1\right)e^{(x^2/2\sigma^2)}\Big|_0^\infty = 1$$

$\Rightarrow 2\sigma^4 = EQ1$ analytic value

2) Gausian $= f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}$ should equal $\int_{-\infty}^\infty f(x) = 1$

#Shift to $[0,\infty]$ s.t the integra equal 1 instead of 0.5.

$$f_{half} = \frac{1}{\sqrt{2\pi\sigma}} e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)} = 1/2$$

$$\frac{2}{\sqrt{2\pi\sigma}} e^{(-x^2/2\sigma^2)} \quad \forall x \geq 0$$

3.) $EQ1 = 2\sigma^4$

$$\int_0^\infty x^3 e^{(-x^2/2\sigma^2)} = 2\sigma^4$$

to renormalize $\quad f(x) = \frac{x^3 e^{\left(-\frac{x^2}{2\sigma^2}\right)}}{2\sigma^4}$

From $[0,\infty]$

Monte Carlo integration

$\Rightarrow$ For this part I utilized Chat GPT

Gives posSamp that are drawn from the Gaussian

$\sum_{i=1}^{N}$ posSamp^3 $\cdot \sqrt{2\pi}$ from eq

⇒ compared against the analytic value

⇒ when increasing the number of samples I used chat GPT to explore what this is doing visually.

eg

⇒ For a sample set w/ 10^3 points, the probability of

sample = gauss.rvs(N) pos. samples

posSamp = samp [samp > 0]

batch R = np.mean (Eq) • analytic val

→ the results show the sample at each samp size w/ error bars

Plot:

INTERPRETATION

as we increase the sample size the max & min values get closer

Problem 2

1.) from scipy.stats

$N = 10^5$

$\boxed{gaus}$ = scipy.stats(150, 15)

↳ probability that $X \leq x$

2.) CDF ~ cummulative distribution function → gives the prob. that a random $\underline{x} \leq x$

→ ---.pdf(x) ⇒ at each 1000 x points, what is the probability that x, to within $\boxed{gaus}$

xrangeN, xrangeP = (mu - 4 (sig), mu + 4 * (sig))

nsamp = np. linspace (xrangeN, xrangeP, 1000)

gausPPF = gaus.ppf(nsamp)

gaus CDF = gaus. cdf(nsamp)
plt.plot (nsamp, gausPPF)
plt.plot (nsamp, gaus CDF)

Generate 10 sample ~

draws = gaus.rvs (size=10)

inverse transform sampling ~ Inverse CDF

10^5 points

unisamp = np. random, uniform(0,1 , N)

probability of being between 0 &1

is = gaus.ppf(unisamp)

plt. hist (i. s)

plt. show()

3.) Fraction of people IQ > 150

accept = 150
accepted = if gaus pdf > accept

4.) What IQ corresponds to 1 in a million.

1 in a million = 10^{-6}

1mill = 10 * -6

where is th probability $\underbrace{\text{BIGGer}}$ than $1/10^6$

IQ = gaus. ppf (1 - 1mill)

Problem 3.

1.) data = np.array [[Deaths, # of Groups, μ]]

count = data[: 0]

freq = data[: 2]

2.) Poisson eq = $\sigma = \frac{\Sigma N \mu}{\Sigma \mu}$

$$\mu = \frac{\Sigma \mu \, (N - \sigma)^2}{\Sigma \mu}$$

Standard Dev = $\sqrt{\sigma}$

muVal = [σ-1, σ+1]

x = np.arange(0, 5)

```
For i in muVal:
    pp = poisson.pmf(x, i) * np.sum(mu)
    plt.plot(x, pp).
```