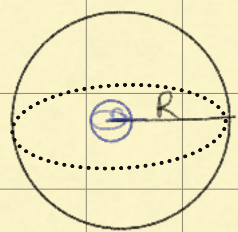
HW1

1.1) a.) Any axcess charge placed on a conductor must lie entirely on its surface.

$$\oint_{\delta V} E\, dA = Q_{enc}/\epsilon_0 \qquad E = 0$$

Case 1: Charge inside $R > r$



$$\frac{Q_{enc}}{\epsilon_0} = 0$$

$\oint_s \vec{E}\cdot\hat{n}\, dA = 4\pi \int_v \rho(x)\, d^3x$ - Gauss' Law

use the difference thm: LOOK UP FORMAL Eq / explaination

$$\int_v (\vec{\nabla}\cdot\vec{E} - 4\pi\rho(\vec{x}))\, d^3x' = 0$$

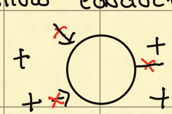
$$\Rightarrow \vec{\nabla}\cdot\vec{E} = 4\pi\rho$$

Case 2: $R < r$ where $r << 1$

$$\oint_V E\, dA = Q_{enc}/\epsilon$$

there is on

b.) closed, hollow conductor shields its interior due to



$$\oint_{dv} E\, dA = Q_{enc}/\epsilon_0$$

① shields its interior from fields due to charges outside of it.

$$\oint_{dv} E\, dA = Q_{enc}/\epsilon_0$$

$$\oint_{dv} 0\, dA = 0/\epsilon_0 = 0$$

② does not shield its exterior from the fields due to charges placed inside of it.

$$\oint_{dv} E\, dA = Q_{enc}/\epsilon$$

arbitrary point x

$$\vec{E}(\vec{x}) = \frac{c\, q_1\, (\vec{x} - x_1)}{|\vec{x}_1 - \vec{x}_1|^3}$$

$\Rightarrow$ q_1 does not cancel out which means there is an $\vec{E}$

C.) $\vec{E}$ @ the surface of a conductor is normal to the surface ^ has a magnitud σ / ϵ_0
Where σ is the charge density / unit Area

$$\sigma = q / dA$$

$$\sigma \, dA = q_s$$

$$E(r) = q\left(\frac{\vec{r}}{r^3}\right) = q\left(\frac{\hat{r}}{r^2}\right)$$

$$\vec{E} \cdot dA = \hat{E} \cdot \hat{N} \, dA = q\left(\frac{1}{r^2}\right) \cdot \hat{r} \cdot \hat{N} \, dA$$

normal

$$\sigma\left(\frac{\hat{r}\hat{N}}{r^2}\right) dA$$

$$\frac{E\,dA}{\hat{E}\,dA} = \vec{N} \qquad \hat{E} = \frac{E}{|E|}$$

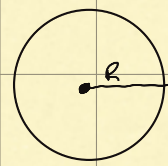
$$\frac{E}{\frac{E}{|E|}} = \vec{N} = |E|$$

$$E = \sigma / \epsilon_0$$

$$\sigma = q / dA$$

$$\boxed{E = \frac{q \, \epsilon_0}{dA}}$$

C.) cylindrical -- a charge Q spred uniformly over a flat circular disk of R



$R = b$

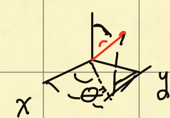
$$p(r, \theta, z)$$

$$p(r, 2\pi, z)$$

$$p(R, 2\pi, z) = Q \cdot 2\pi R \, \delta(R - b)$$

d.) spherical coord. (θ, ϕ, r)

$$p(\theta, \phi, r) =$$



$$r \sin\phi = x$$

$$\delta(r \sin\theta - x)$$

$$p(\phi) \quad \frac{A}{2\pi x} \int_0^{\pi} \int_0^{2\pi} \int_0^{r} \delta(r\sin\phi - x) \cdot \frac{1}{r\sin\phi}$$

$$\frac{1}{r\sin\theta}$$

1.3.) Use Delta functions to express charge distributions (3D) $p(x)$

a.) Spherical coordinats (r, θ, ϕ), a charge uniformly distributed over a spherical shell (R)

$$\oint_V r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\frac{1}{r^2} \delta(x_r - x'_r)$$

$$\frac{1}{\sin\theta} \delta(x_\theta - x'_\theta)$$

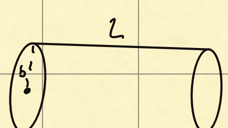
IF I have a volume $\oint_V$ $\quad \delta(x_\phi - x_1)$

$$p(\vec{x}) = \sum_{i=1}^{N} \cdot \frac{1}{r^2} \delta(x - x_1) \frac{1}{\sin\theta} \delta(x - x_1) \, \delta(x_\phi - x'_\phi)$$

$$\frac{Q_{enc}}{\epsilon_0} = \frac{4\pi r^2}{3} p(x)$$

$$Q_{enc} = \int_V p(x)$$

b.) In cylindrical a charge λ / L of radius b (r, ϕ, z)



$$V = 2\pi b$$

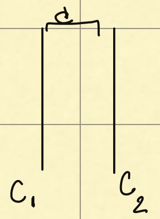
$$\oint E \, dA = \frac{Q_{enc}}{\epsilon}$$

$$\sigma = \frac{\lambda}{2\pi b}$$

$$p(r, \theta, z) = \frac{\lambda}{2\pi b} \delta(r - b)$$

1.6

a.) Use Gauses law to calculate the capacitence of 2 large, flat sheets sep. distance d.



$$\frac{|Q_{c_1}|}{\Phi}$$

$\sigma = Q/A$

$A = L^2$

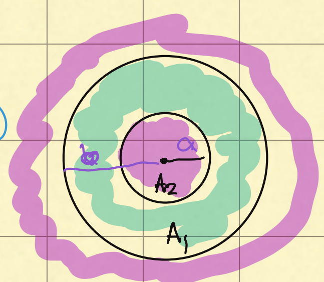
$E = \sigma / \epsilon$

$$E = \frac{Q/A}{\Phi}$$

$\Phi = E \cdot d$ $\qquad C = Q/\underline{\ \ I\ \ }$

$$\Phi = \frac{Q}{A} d \qquad C = \frac{QA}{-Q\ d} \quad // = A/d \cdot \epsilon$$

b.)



$C = Q/V$ $\qquad E = \frac{1}{4\pi\epsilon} \frac{1}{r^2} Q$

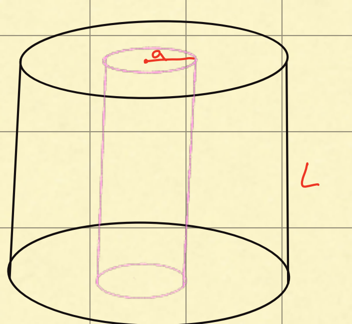
$$\frac{Q}{4\pi\epsilon} \int_a^b \frac{1}{r^2} dr = r^{-2}$$

$$- \frac{1}{r}\Big|_{a,b} = \frac{1}{b} - \frac{1}{a}$$

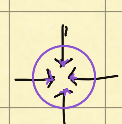
$$V = \left(\frac{1}{b} - \frac{1}{a}\right) \cdot \frac{Q}{4\pi\epsilon}$$

$$C = \frac{4\pi\epsilon}{\left(\frac{1}{b} - \frac{1}{a}\right)}$$

c.) two concentric cylinders compared to their L



$$\frac{Q_{enc}}{\epsilon_0} = E\,dA$$



$$Q_{enc} = \epsilon_0 \cdot (E\,dA)$$

$A =$

$$E = \frac{\lambda}{2\pi\epsilon} \left(\int_a^b 1/x\, dx \right.$$

$$\ln(x)\Big|_{ab} = \ln(b/a)\ \lambda (2\pi\epsilon)^{-1}$$

1.7 i)

$$C \simeq \pi \epsilon_0 \left(\ln \left(\frac{d}{a} \right) \right)^{-1}$$

$$1.2 \times 10^{-11} = C = \pi \epsilon_0 \left(\ln \left(\frac{d}{\sqrt{a_1 a_2}} \right) \right)^{-1}$$

$$a = \sqrt{a_1 a_2} \qquad a = \frac{d}{e^{(2\pi\epsilon_0 / C}}$$

Put it into calculate:

$a(0.5) = 9.55$ mm

$a(1.5) = 28.65$ mm

$a(5.0) = 95.47$ mm

1.8)

$$V = Q/E = \frac{Qd}{\epsilon_0 A}$$

$$\oint_{enc} E = q_{enc}/\epsilon$$

$$C = Q/V$$

energy stored $\rightarrow$ $U = 1/2\, QV = Q^2/2C$

$$C = \frac{\epsilon_0 A}{d}$$

$$C = \frac{4\pi \epsilon}{\left(\frac{1}{b} - \frac{1}{a}\right)}$$ from above

$$U = \frac{1}{2} \left(4\pi\epsilon_0 \left(\frac{1}{b} - \frac{1}{a} \right)^{-1} \right) \left(\frac{Q(b-a)}{4\pi\epsilon ab} \right)^2$$

$$= \left(\frac{1}{2}\right) \frac{Q^2 (b-a)}{4\pi \epsilon ab}$$

$\pi \epsilon_0 \left(\ln \left(\frac{d}{a} \right) \right)^{-1}$ Cylinder cap. from above

Parrallel

$$U = \frac{1}{2} \frac{Q^2 d}{\epsilon A}$$

Sphere

$$U = \frac{1}{2} \frac{Q^2}{4\pi\epsilon \left(\frac{1}{a} - \frac{1}{b}\right)}$$

cyllinder

$$U = \frac{1}{2} \frac{Q^2 \pi \epsilon_0}{\ln(d/a)}$$