

# Multiscale Modeling of Kinetics and Dynamics of Heterogenous Catalysis and Material Degradation

**Azeez G. Akinyemi, Richard H. West**

Department of Chemical Engineering

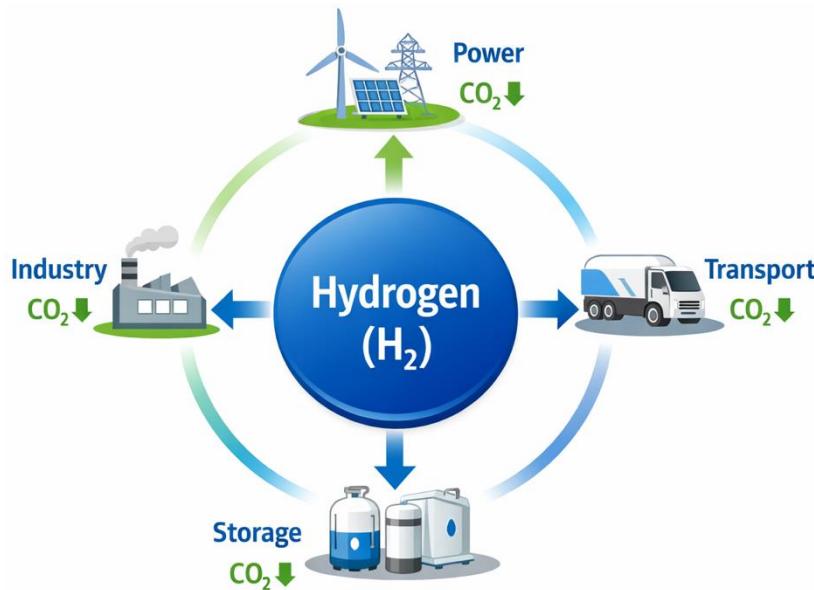
Northeastern University, Boston, MA

January 27, 2026



# The Hydrogen Economy's \$11 Trillion Promise – and Peril

- ▶ Hydrogen will supply **10-20% of global energy** demand by 2050
- ▶ Global hydrogen economy: **\$2.5-11 trillion** by mid-century
- ▶ **700,000+ km** of existing natural gas pipelines could be repurposed for hydrogen
- ▶ But infrastructure must safely operate at **70-100 bar** for 30-50 years
- ▶ A single high-profile failure could delay adoption by decades



**1998-2019: Four major incidents, 1,300+ fatalities**

# Why Hydrogen is Different: The Invisible Threat

## The Permeation Problem

- ▶ **Hydrogen is the smallest atom** → diffuses directly through solid metals
- ▶ Traditional seals, welds, and gaskets **don't stop permeation**
- ▶ Accumulation causes **embrittlement and sudden fracture**
- ▶ **Damage is invisible** until catastrophic failure

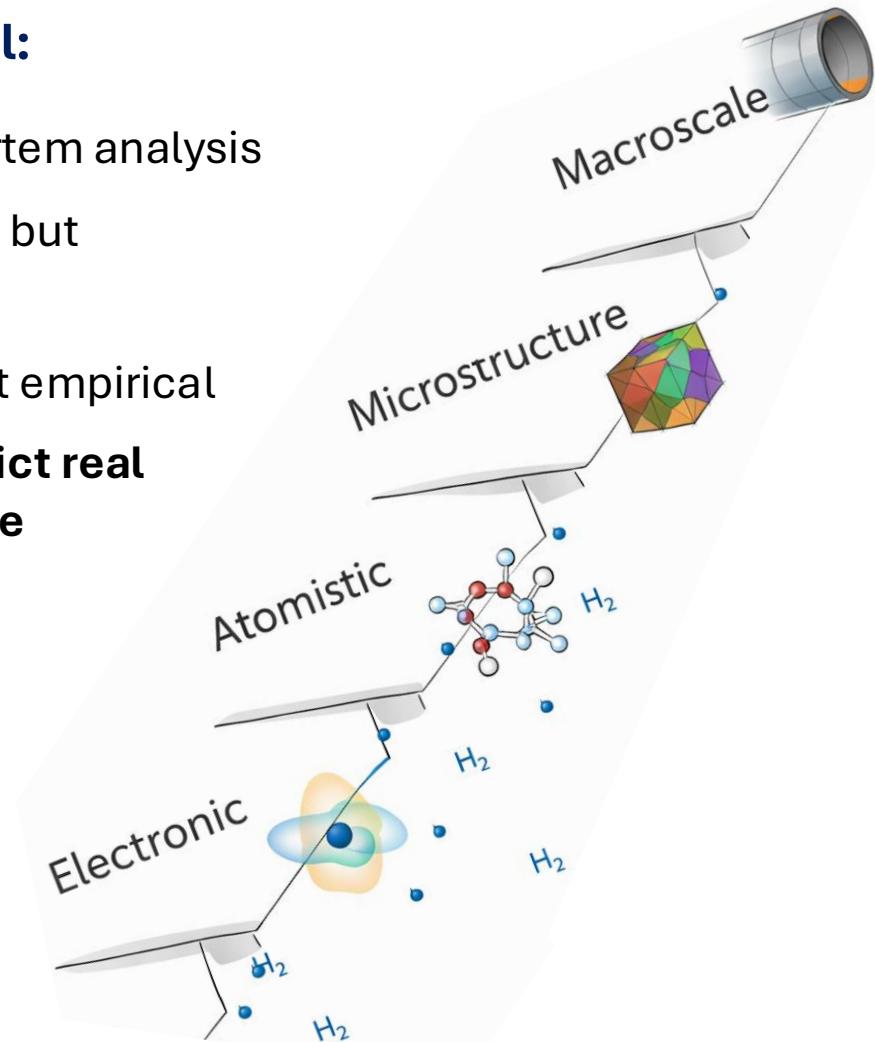
## The Scale Gap Crisis

Despite 150 years of hydrogen research, we cannot reliably predict infrastructure performance

- ▶ The scale gap: Ångströms to meters, picoseconds to decades

## Why existing approaches fail:

- ▶ Experiments: only post-mortem analysis
- ▶ Quantum models: accurate but microscopic
- ▶ Continuum models: fast but empirical
- ▶ **Result: Can't reliably predict real infrastructure performance**



# Cracking the Hydrogen Bottleneck: Predictive, Multi-Scale Modeling

Hydrogen permeation is controlled by coupled processes across all scales

*gas phase → oxide layer → metal–oxide interface → bulk metal*

For the first time: Physically meaningful parameters across all scales, not empirical fitting

## Our Strategy:

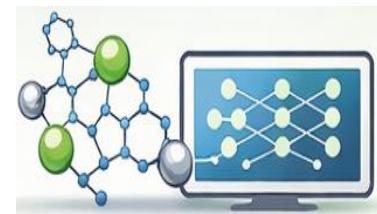
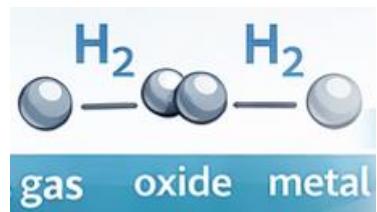
- ▶ Aim 1: Develop a multi-phase continuum model spanning gas to bulk metal
- ▶ Aim 2: Use sensitivity analysis to identify dominant transport mechanisms
- ▶ Aim 3: Compute critical parameters using quantum-accurate Machine-Learned-enabled Molecular Dynamics
- ▶ Aim 4: Incorporate surface reconstruction as dynamic boundary conditions

Continuum Model

Sensitivity Analysis

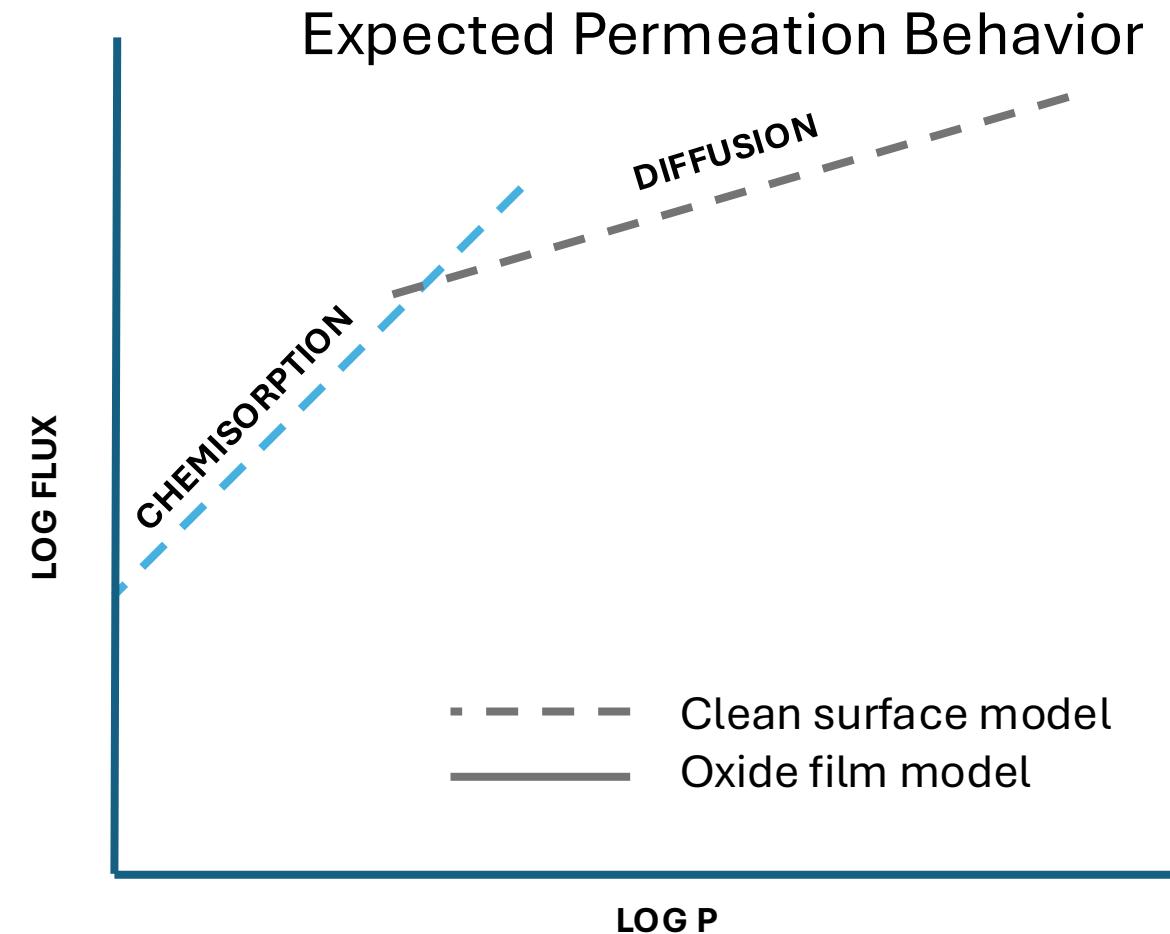
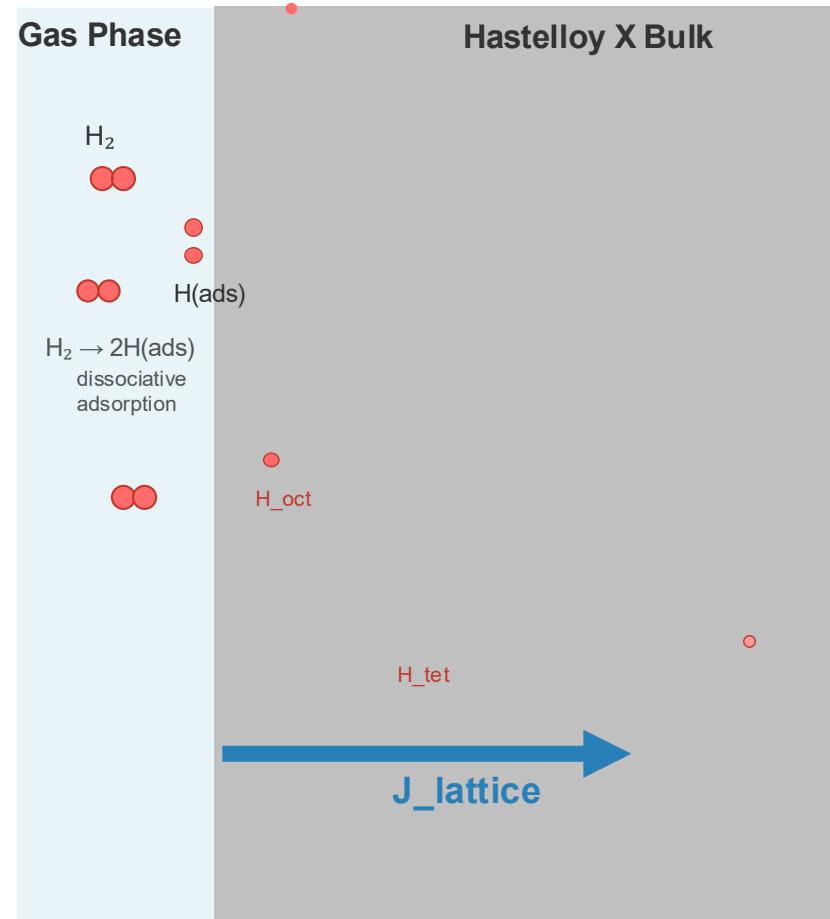
MLMD

Validation + Iteration

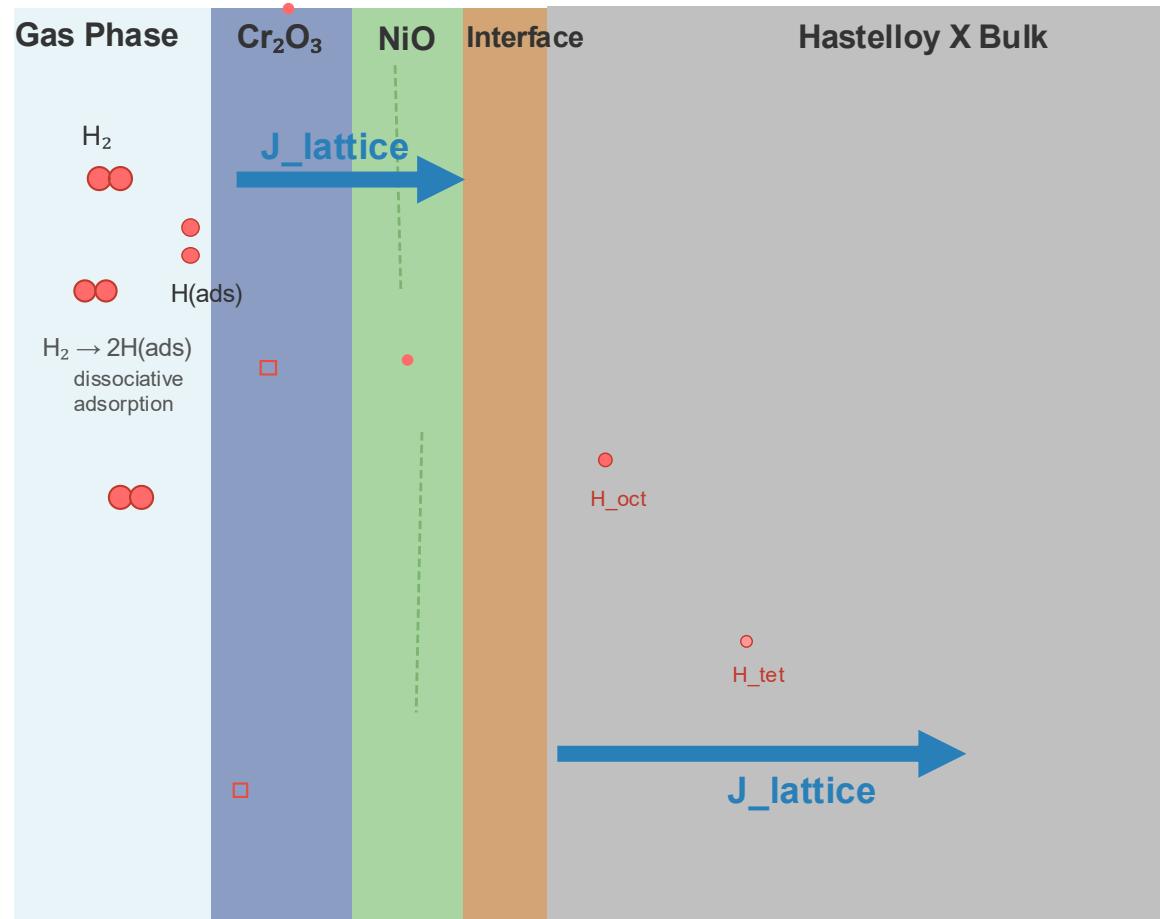


Accurate Permeation Prediction

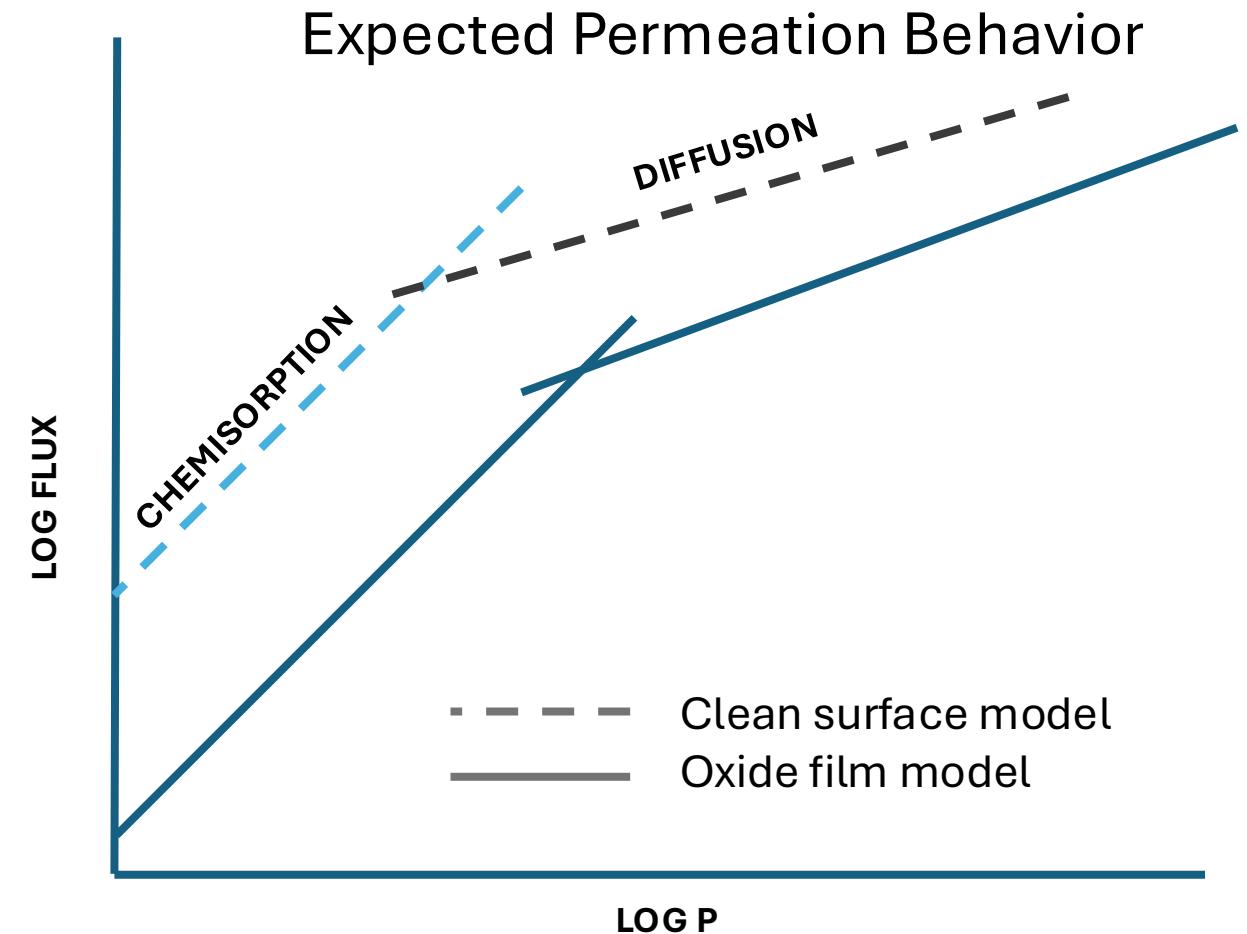
# Development of Hydrogen Permeation Continuum Model



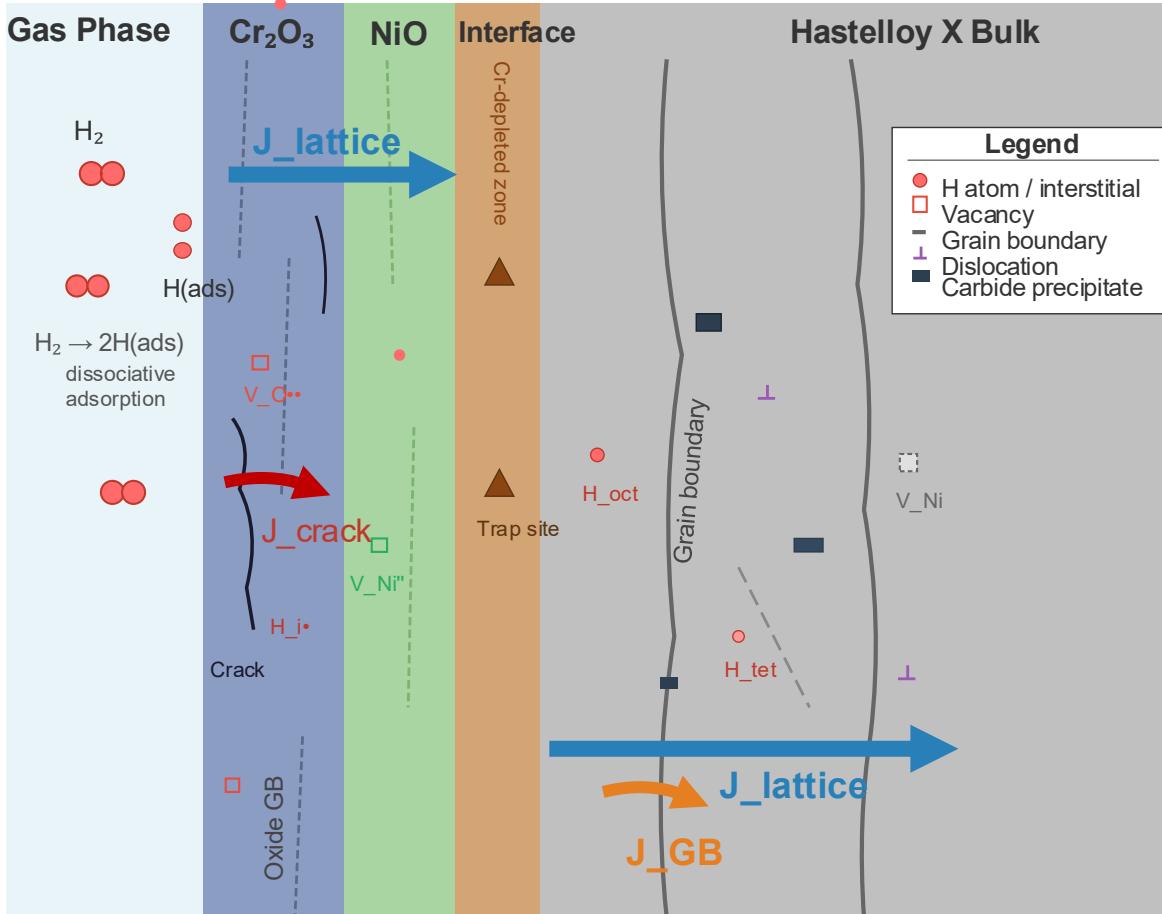
# Development of Hydrogen Permeation Continuum Model



**H<sub>2</sub> through oxide layer into Perfect Metal**

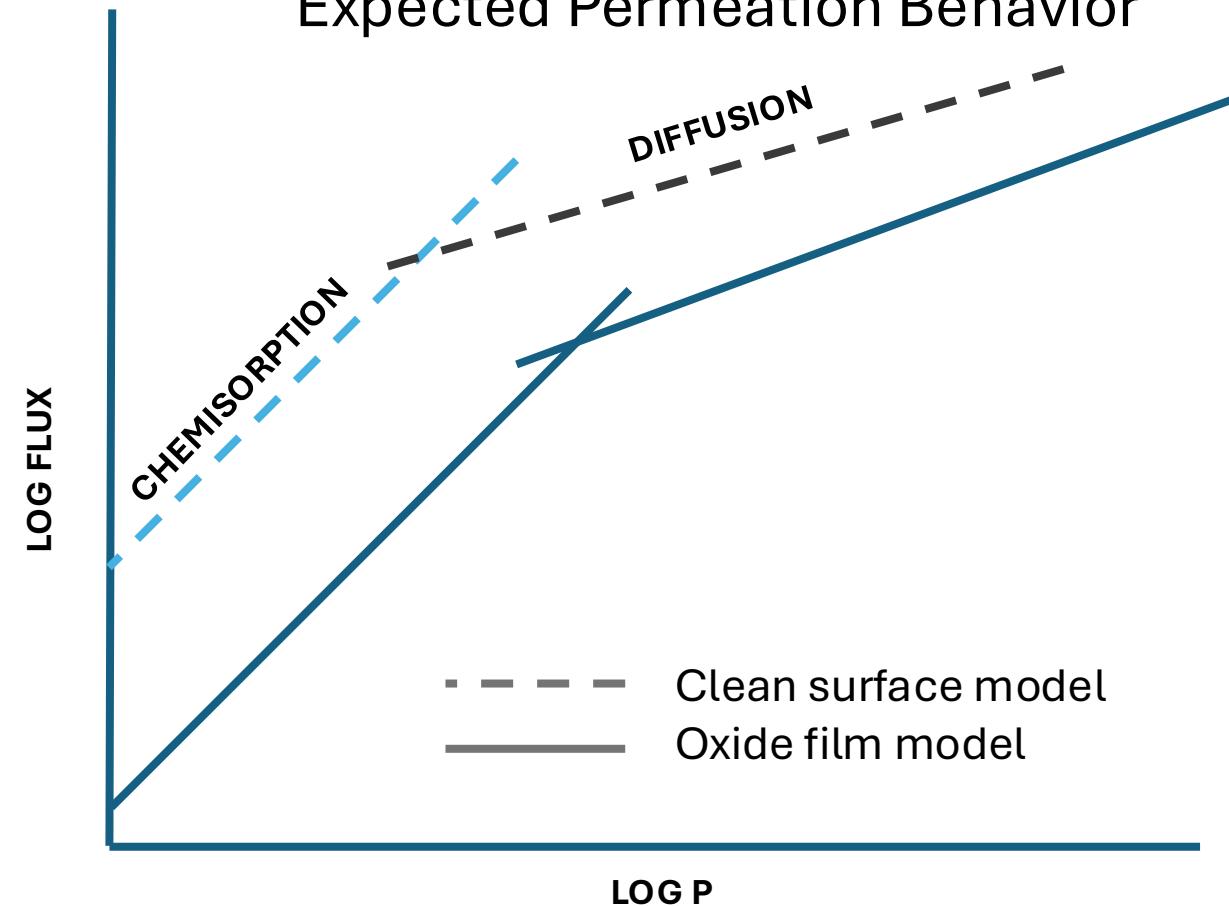


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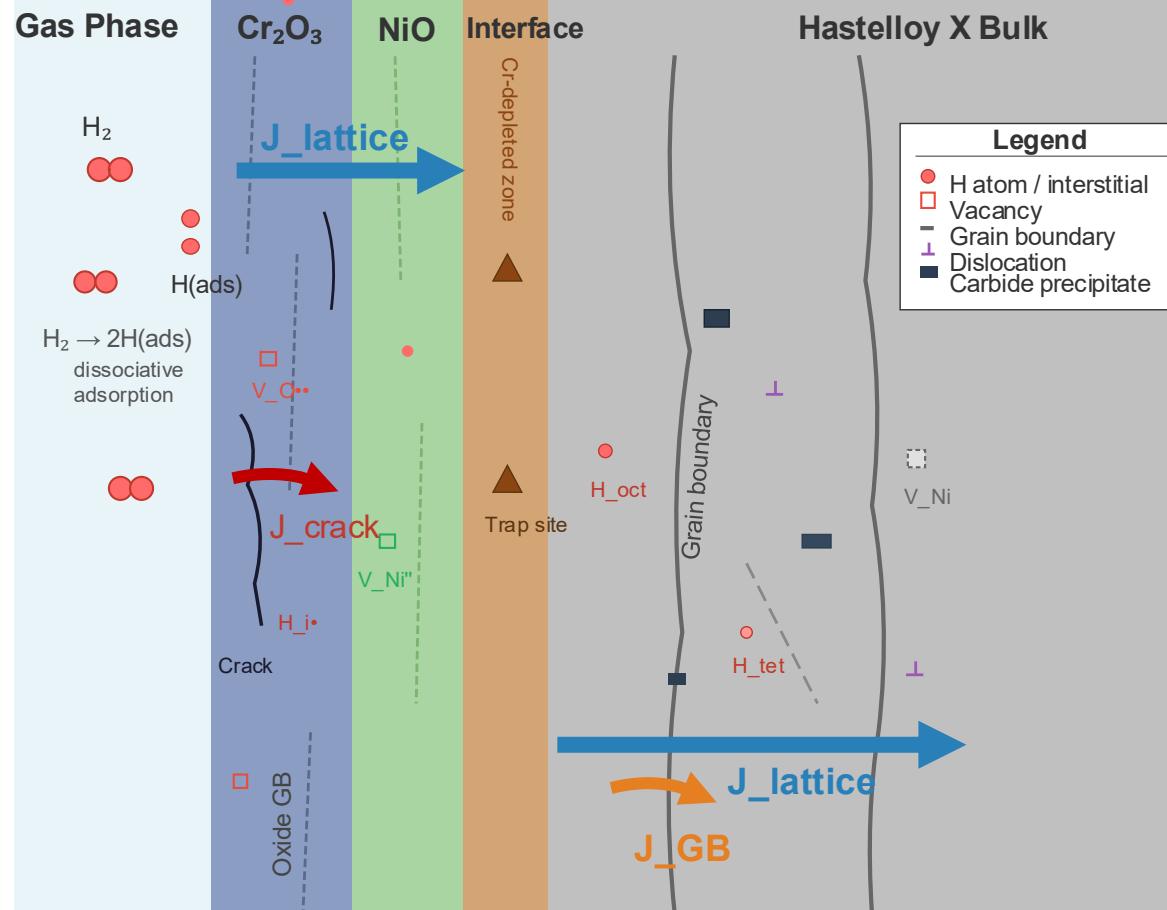


H<sub>2</sub> through Defective Oxide layer and Metal

## Expected Permeation Behavior

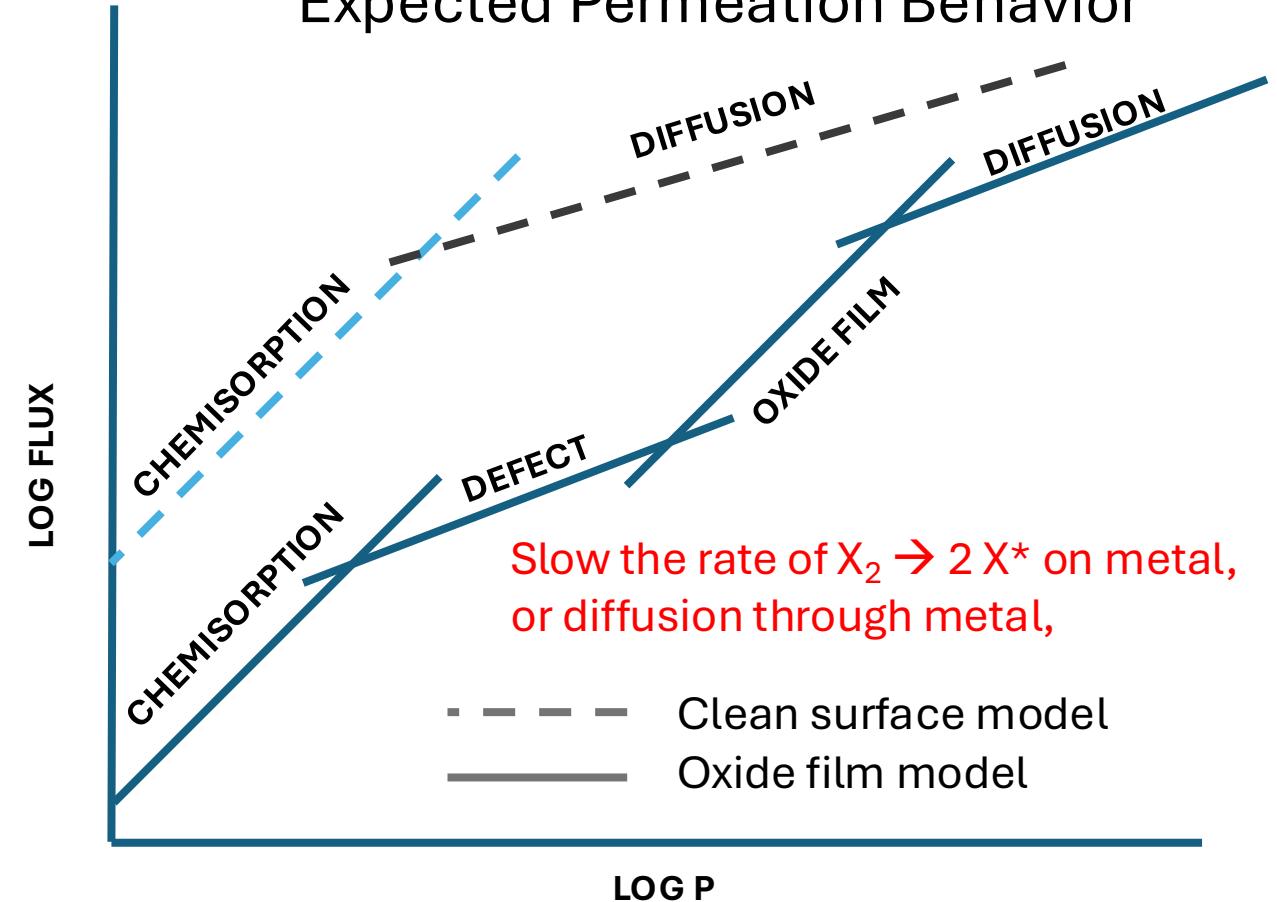


# Development of Hydrogen Permeation Continuum Model



H<sub>2</sub> through Defective Oxide layer and Metal

## Expected Permeation Behavior



# Aim 1

Perfect Metal  
Atomic Diffusion  
(Level 1)

Perfect Oxide  
Molecular Diffusion  
(Level 2a)

Perfect Oxide  
+ Perfect Metal  
Series Resistance  
(Level 2b)

Defective Oxide  
+ Perfect Metal  
The Parallel Path Model  
(Level 3)

Defective Metal  
(Level 4)

Defective Oxide  
+ Defective Metal  
(Level 5)

# Aim 1

**Perfect Metal  
Atomic Diffusion  
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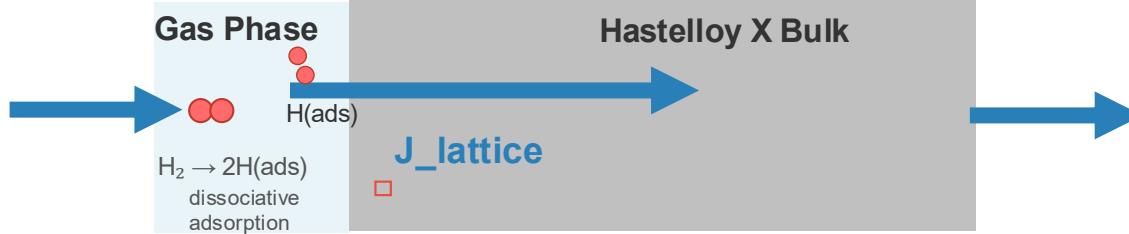
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# Perfect Metal Continuum Model (Baseline)



## Physics: Sieverts' Law + Fick's Law

- Sievert's Law (surface equilibrium)

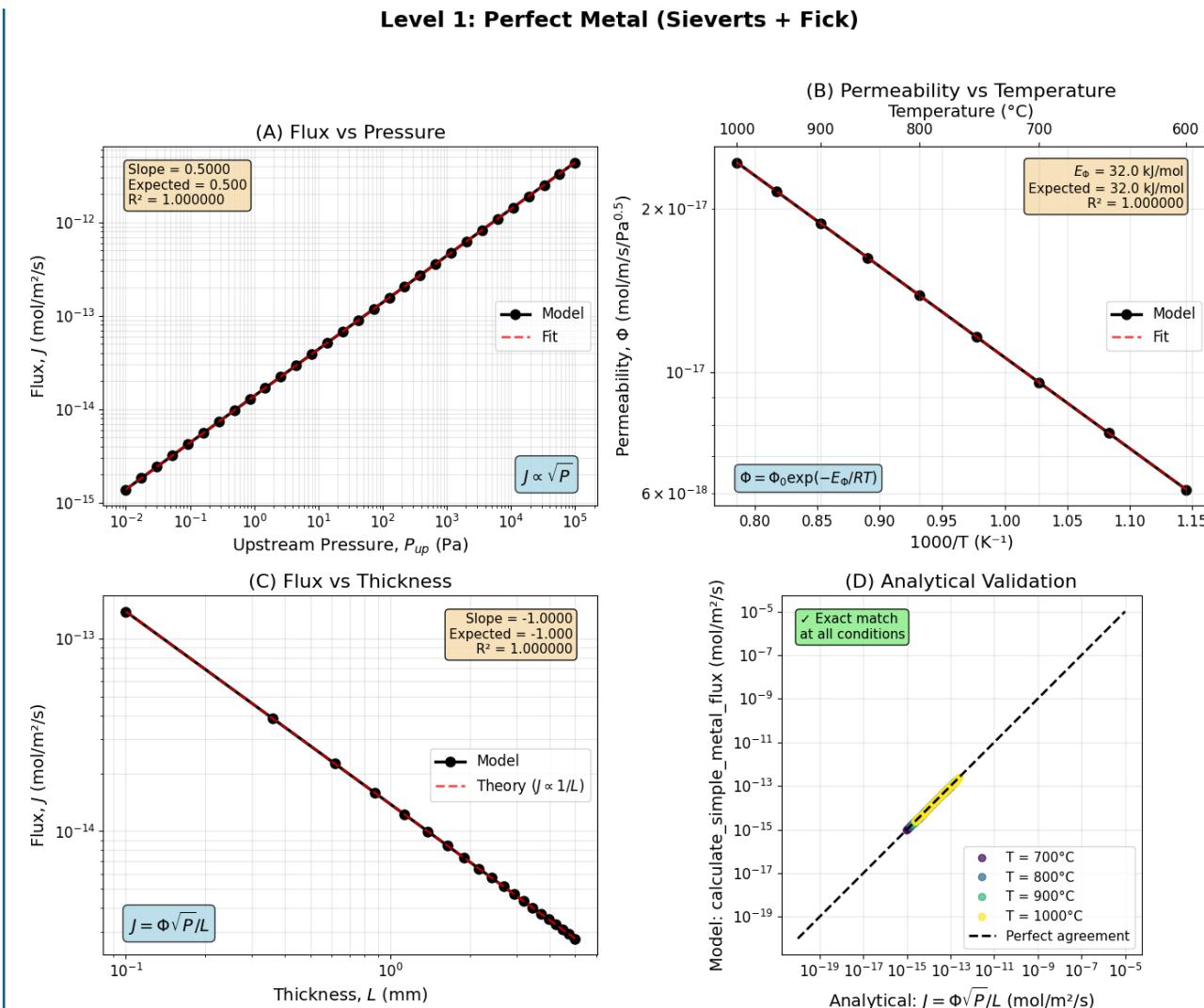
$$C = K_s \cdot \sqrt{P}$$

- Fick's Law (Bulk diffusion)

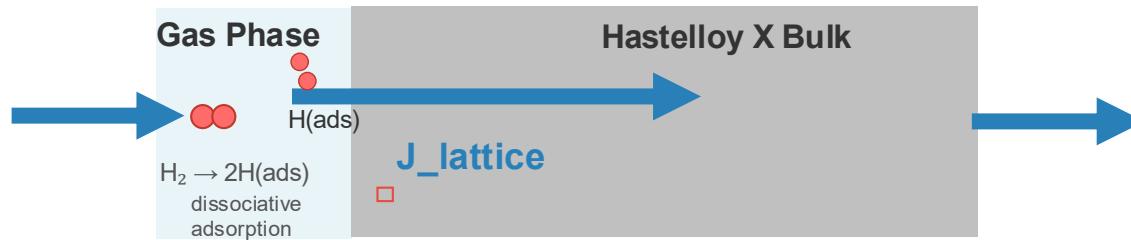
$$\text{Flux } J = \frac{D (C_{up} - C_{down})}{L}$$

- Combined:

$$J = \left( \frac{DK_s}{L} \right) \cdot \left( \sqrt{P_{up}} - \sqrt{P_{down}} \right)$$



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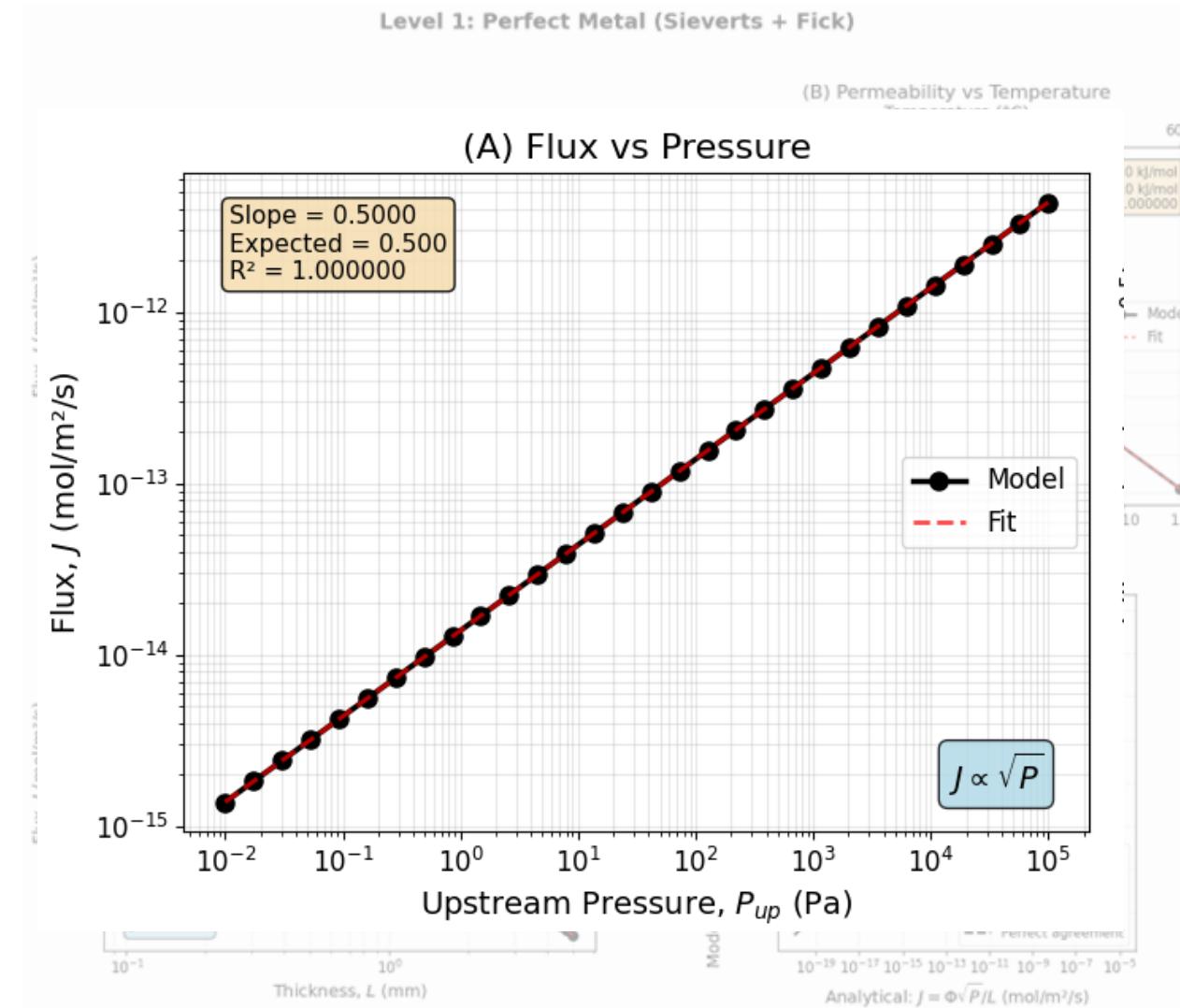
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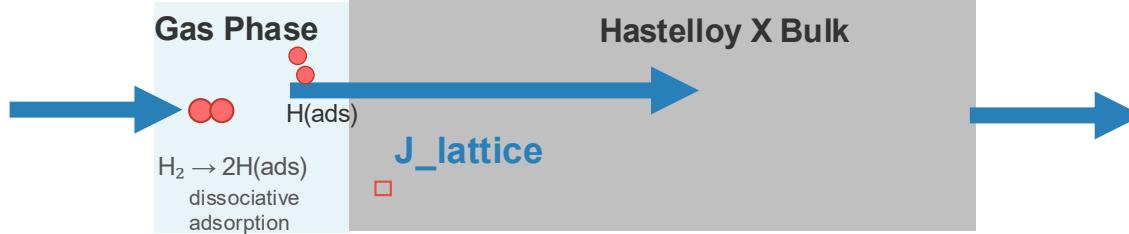
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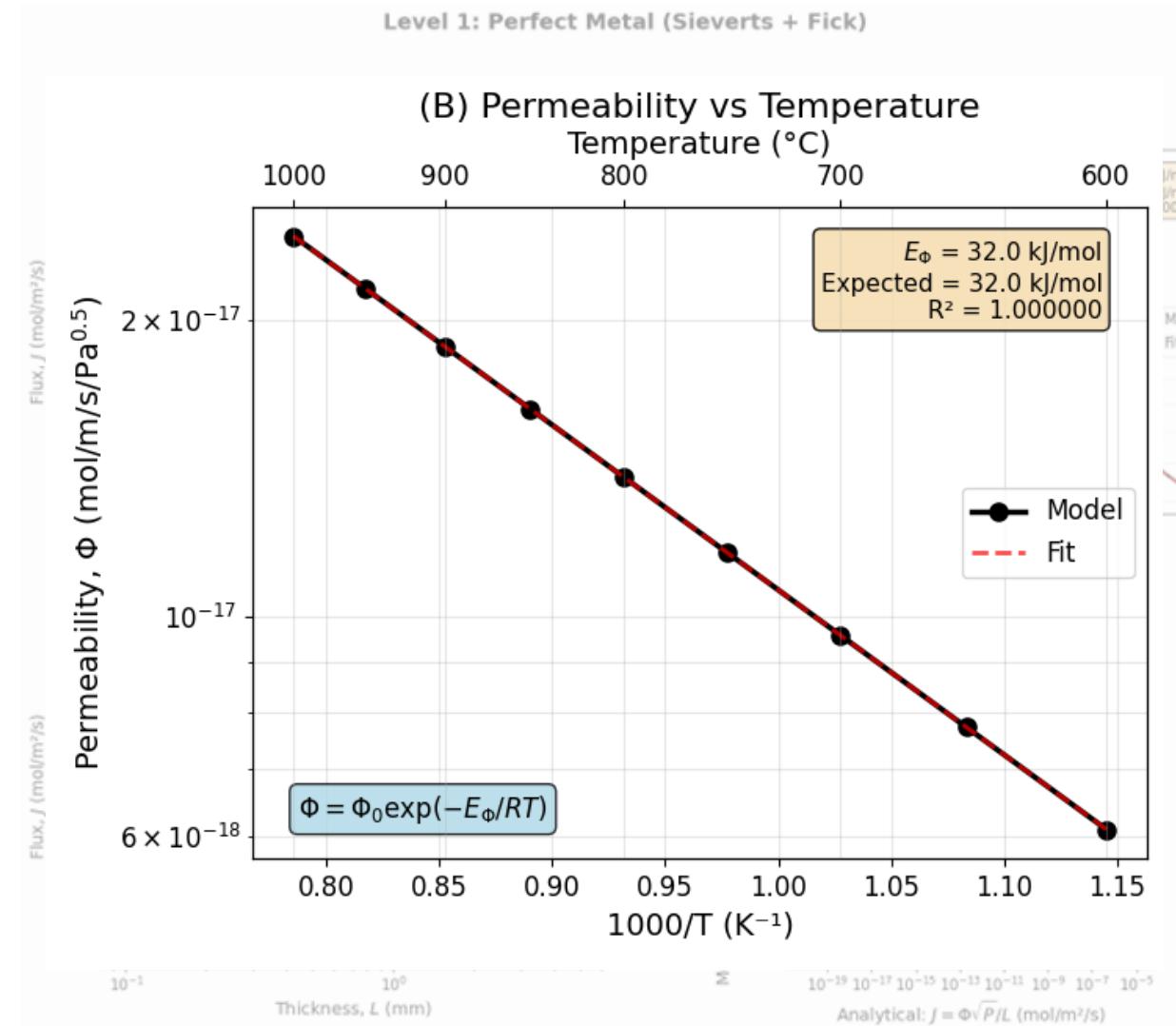
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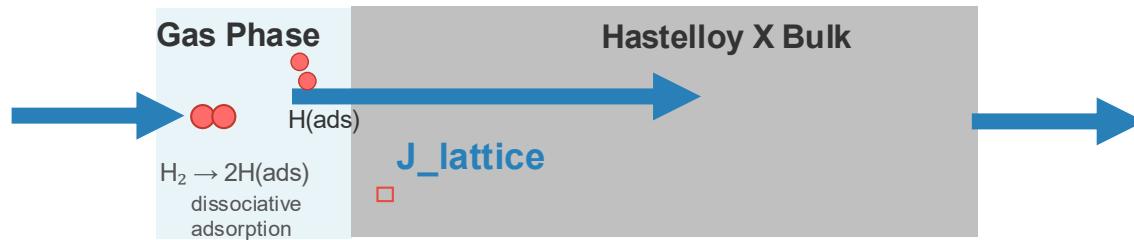
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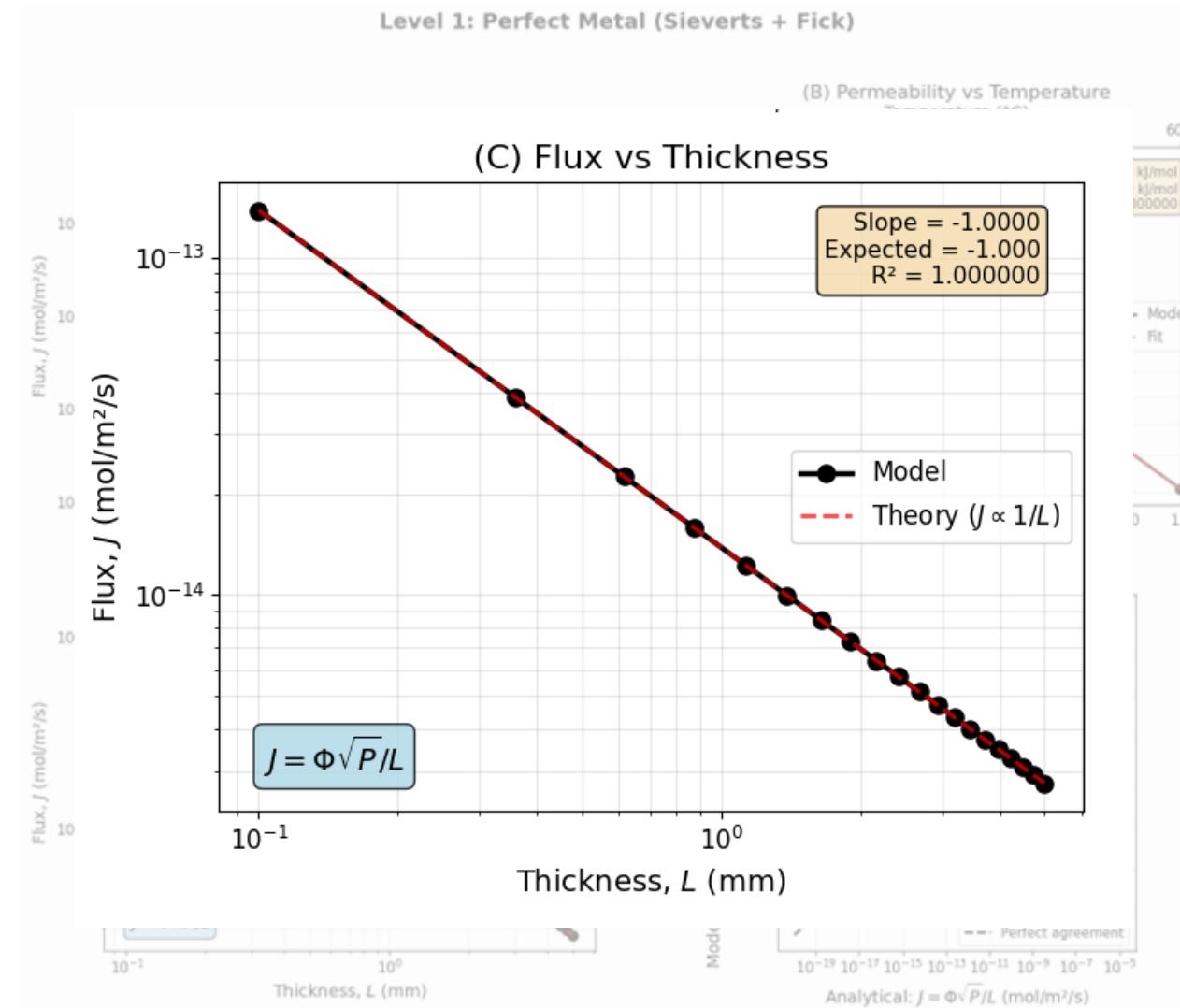
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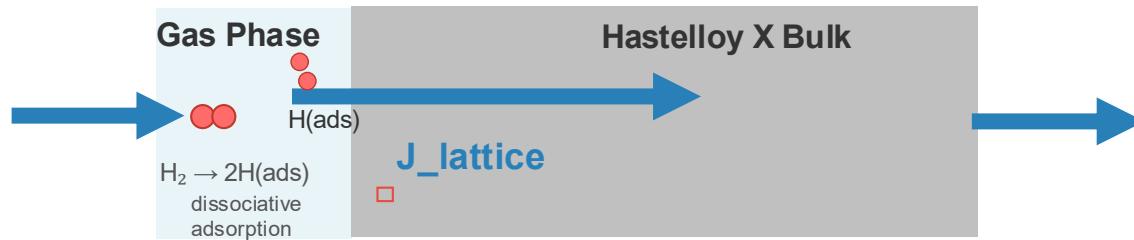
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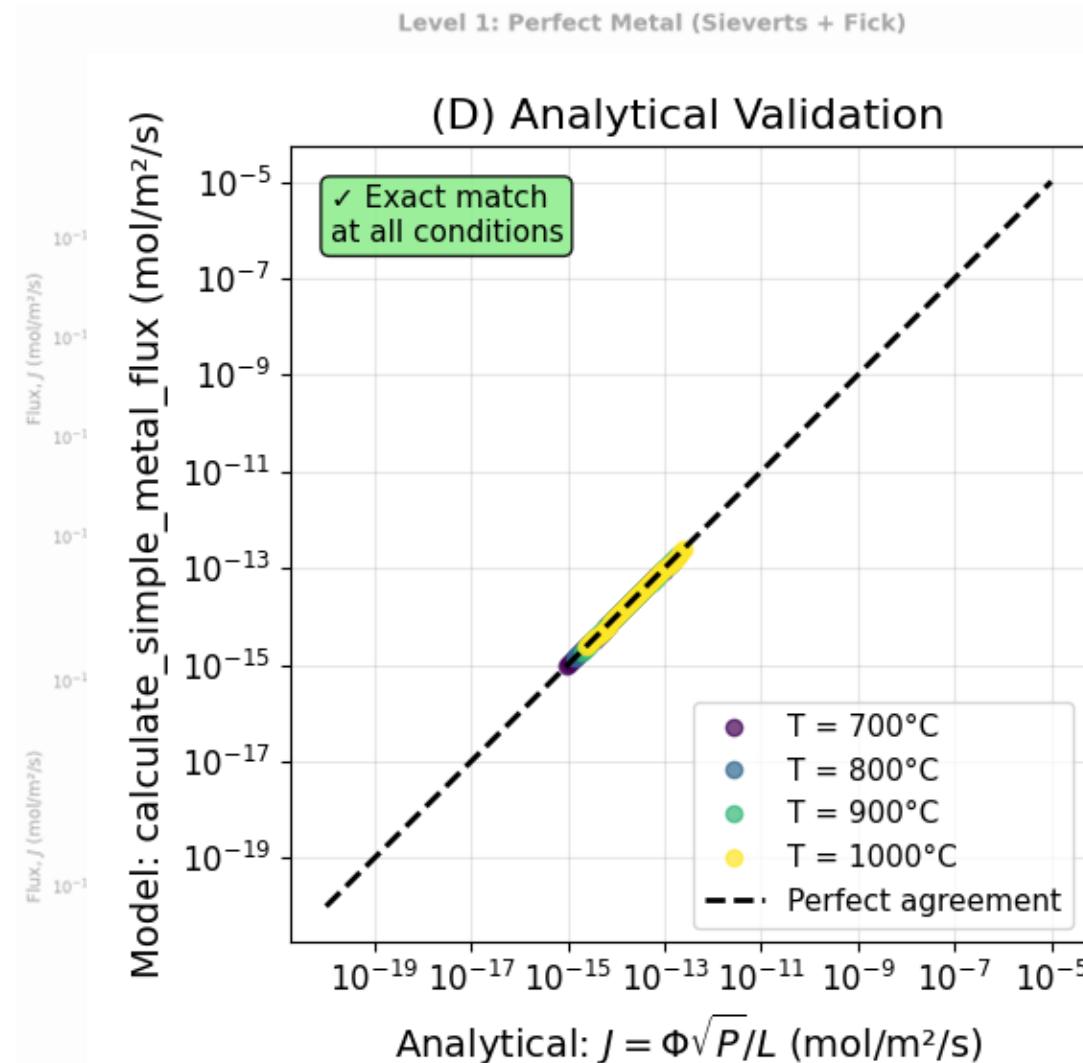
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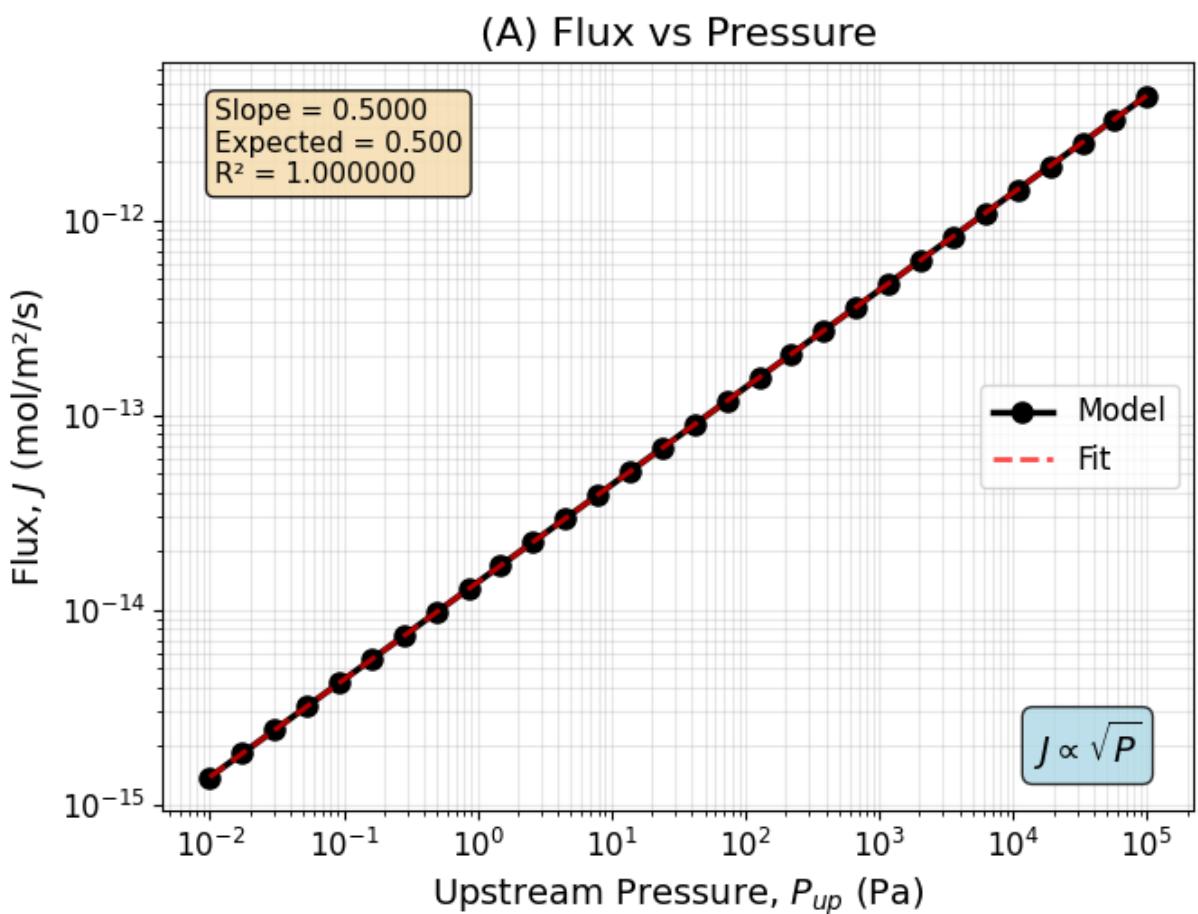
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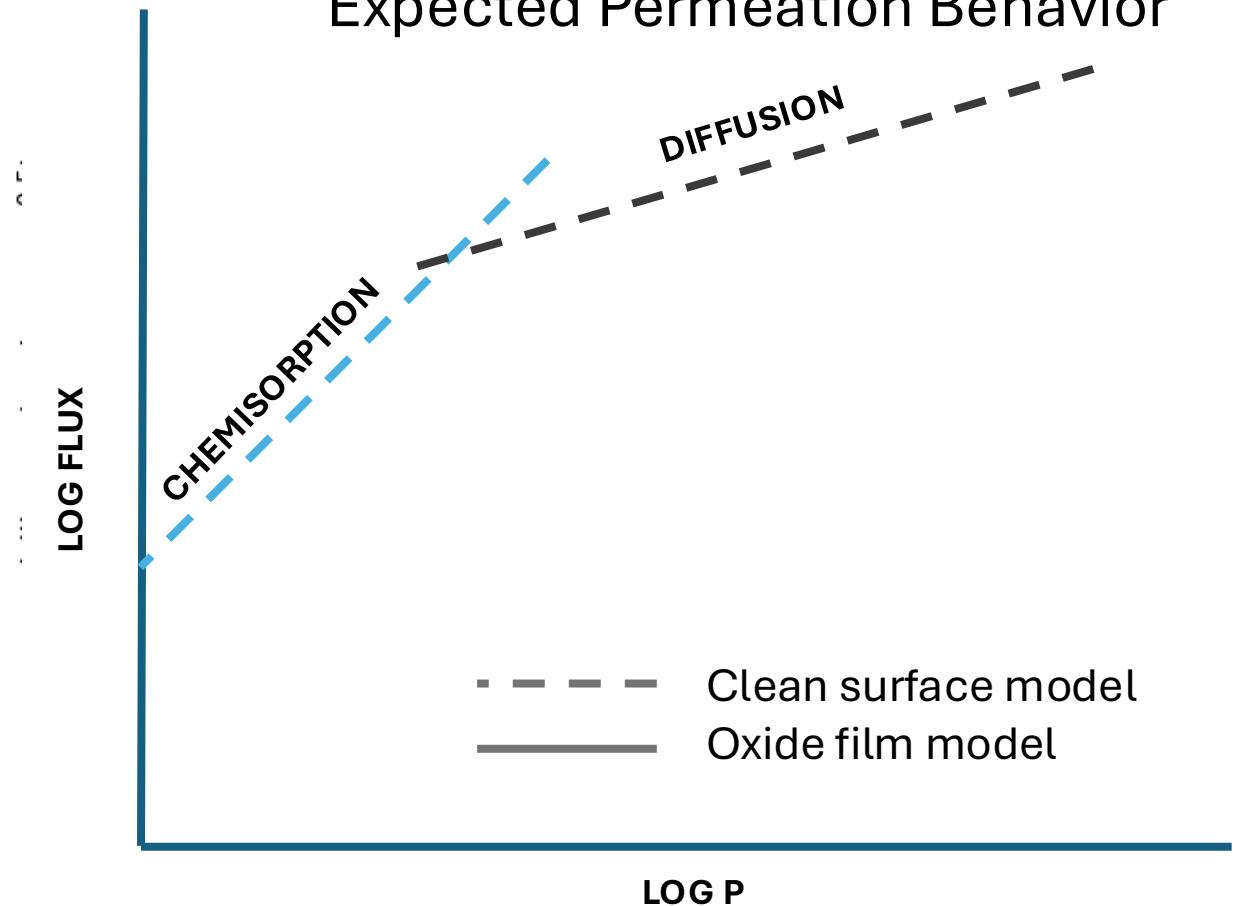
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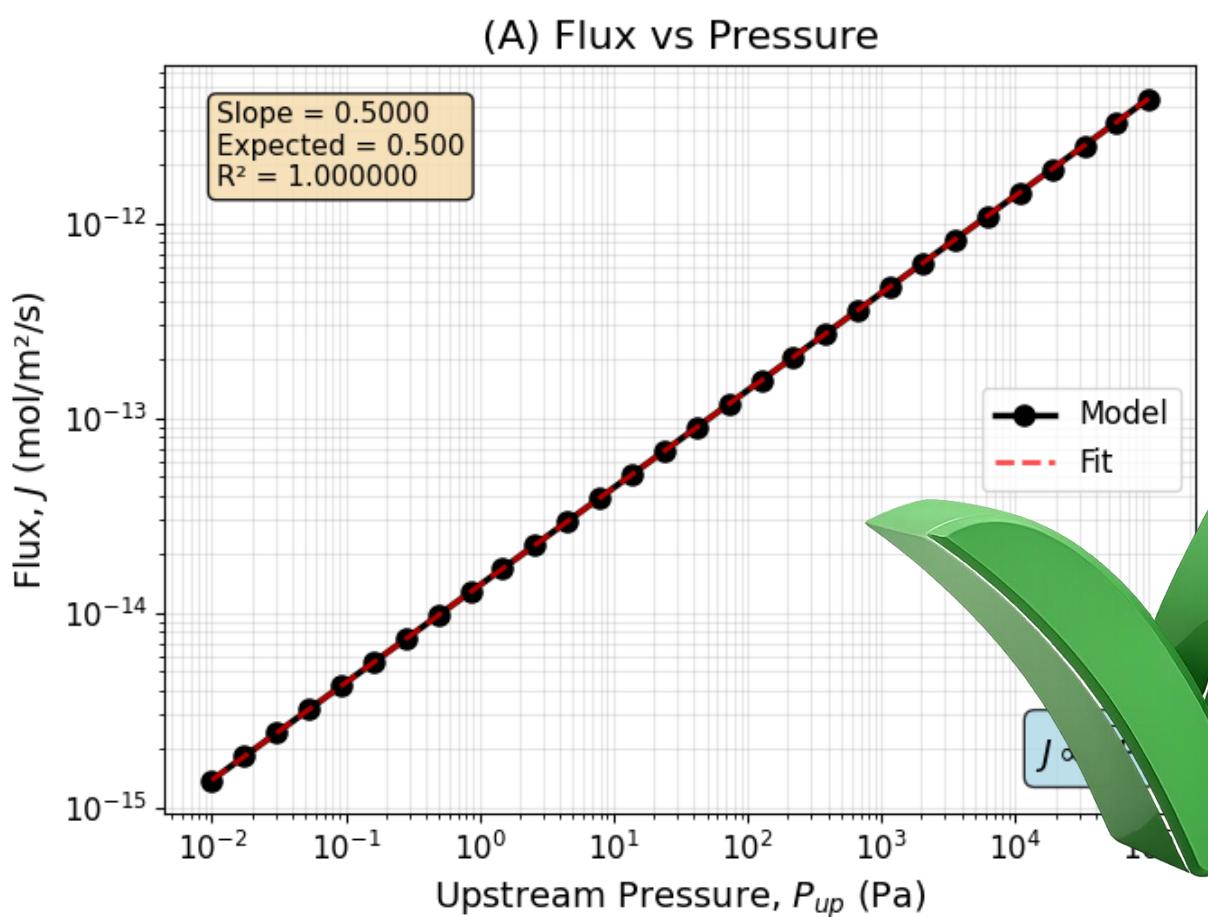
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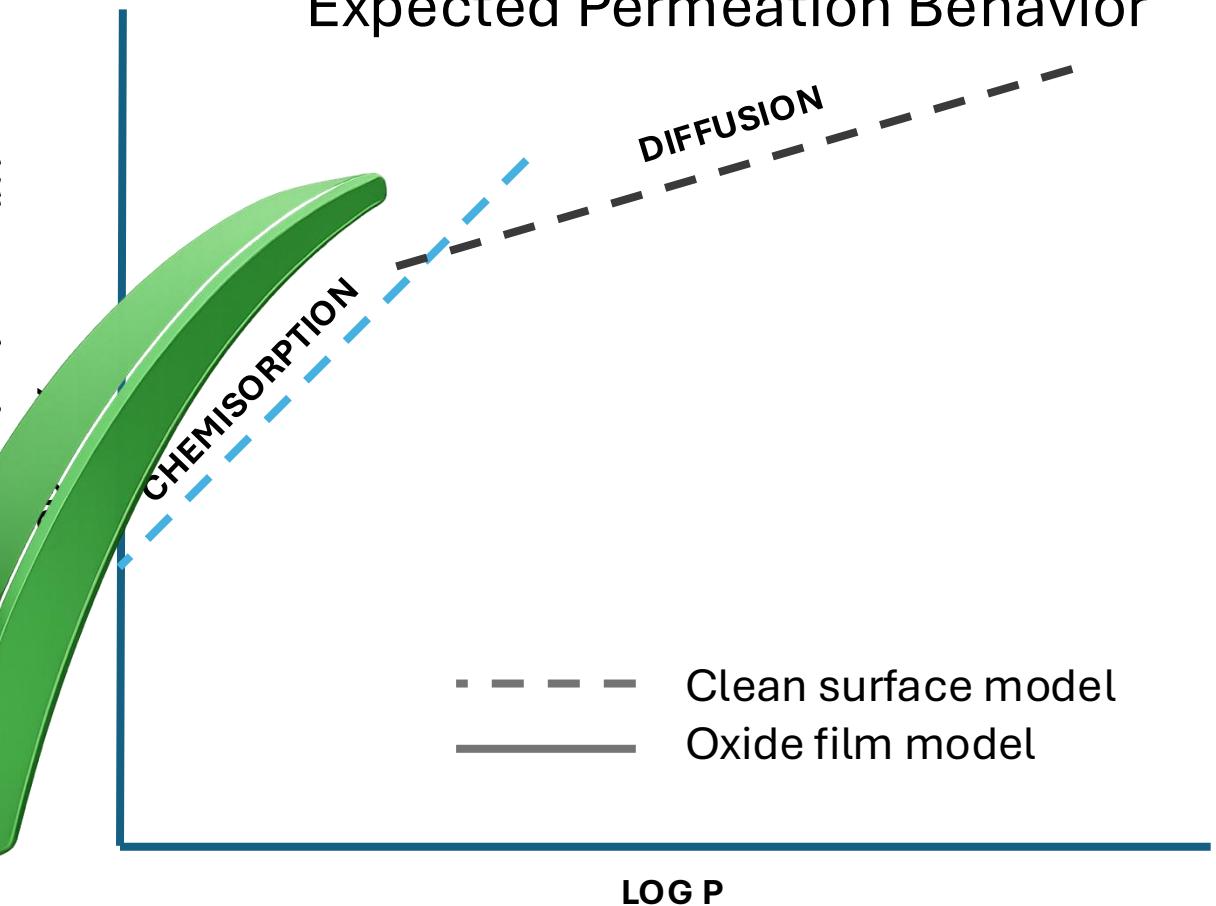
Expected Permeation Behavior



# Perfect Metal Continuum Model (Baseline)



Expected Permeation Behavior



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(Level 1)

Perfect Oxide  
Molecular Diffusion  
(Level 2a)

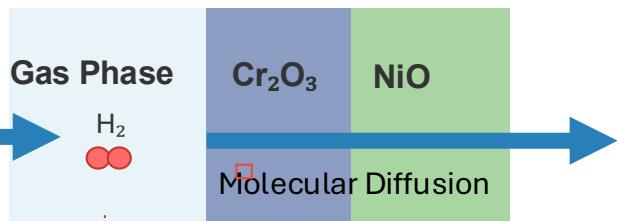
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Defective Oxide  
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# Perfect Oxide Only (Molecular Diffusion)



## Physics: Henry's Law + Fick's Law

- Henry's Law (molecular dissolution)

$$C = K_{ox} \cdot P$$

- Fick's Law (Bulk diffusion)

$$\text{Flux } J = \frac{D_{ox} (C_{up} - C_{down})}{L_{ox}}$$

- Combined:

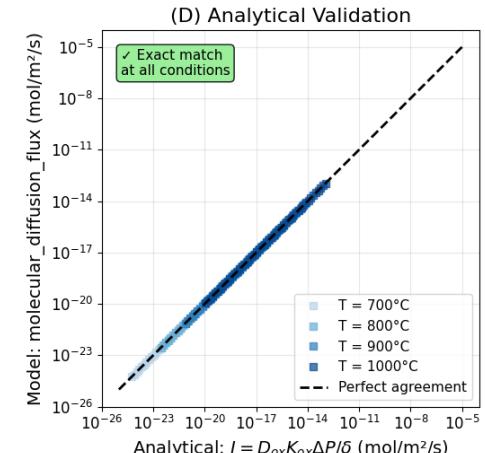
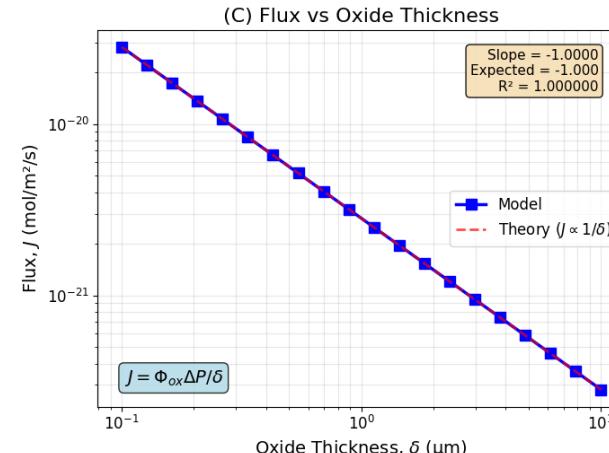
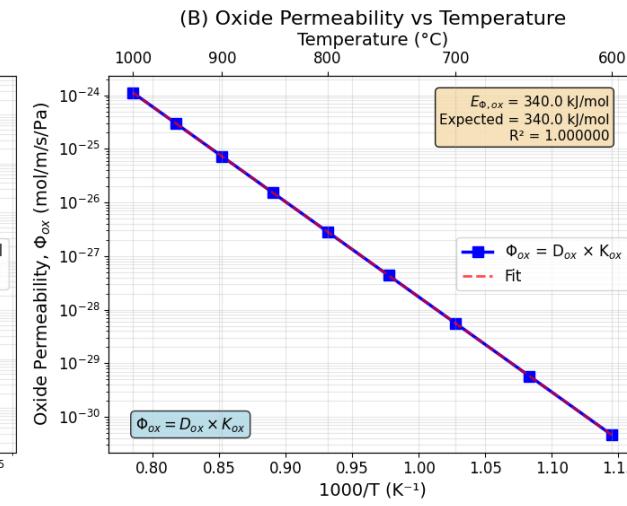
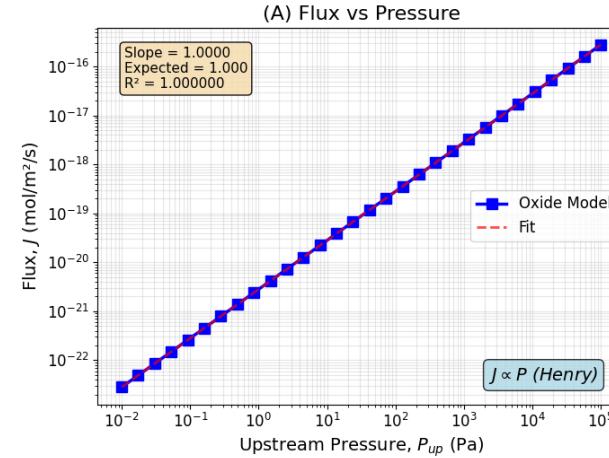
$$J = \left( \frac{D_{ox} K_{ox}}{L_{ox}} \right) \cdot (P_{up} - P_{down}) = \frac{\phi_{ox} \cdot \Delta p}{L_{ox}}$$

## KEY DIFFERENCE FROM METAL

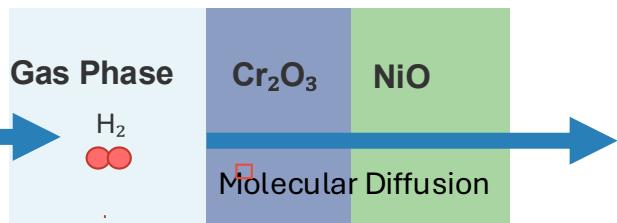
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Oxide:  $C \propto P$  (Henry)  $\rightarrow J \propto P$  (slope = 1.0)

## Level 2a: Perfect Oxide Only (Henry + Fick)



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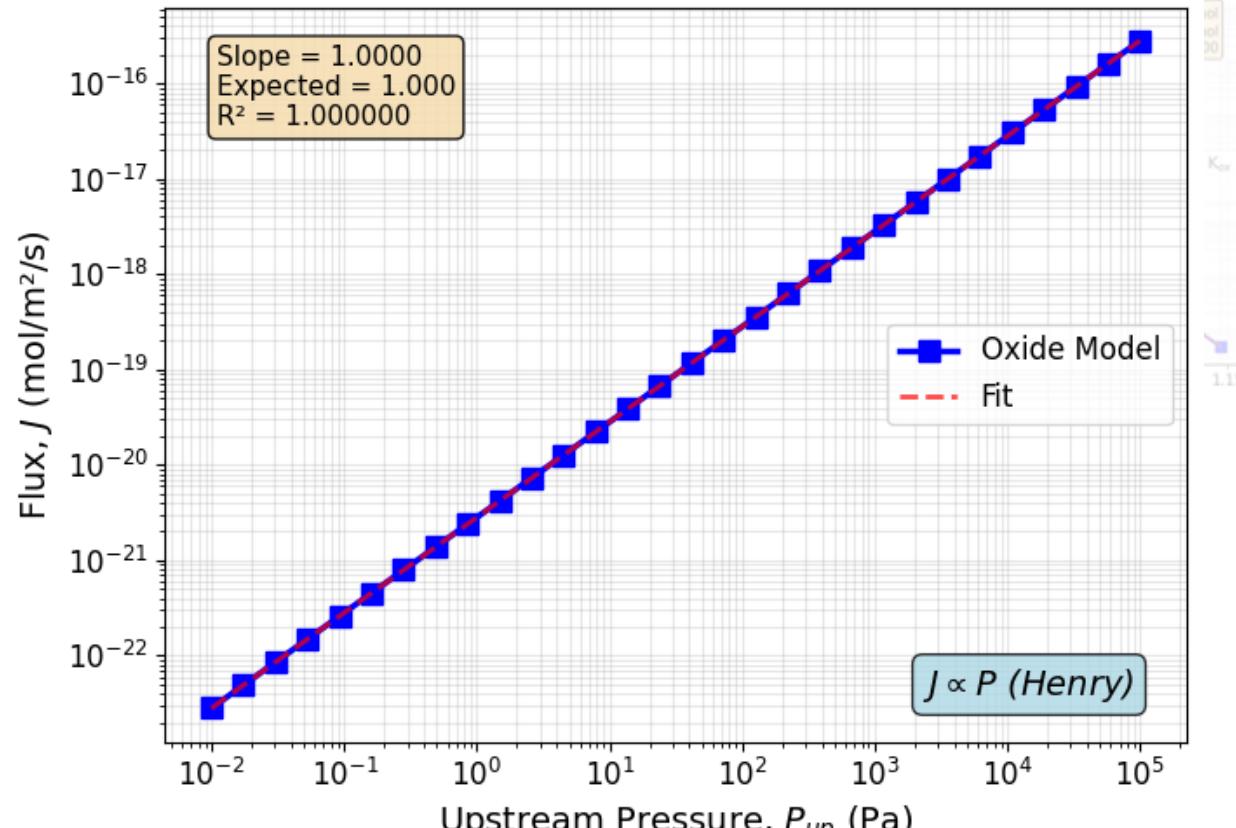
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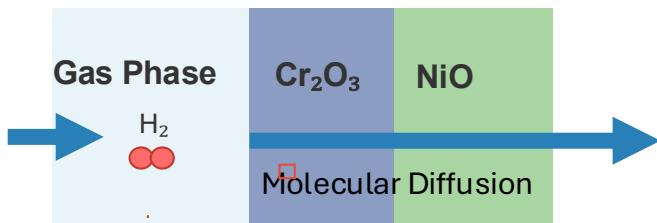
(A) Flux vs Pressure



Oxide Thickness,  $\delta$  ( $\mu\text{m}$ )

Analytical:  $J = D_{ox} K_{ox} \Delta P / \delta$  (mol/m²/s)

# Perfect Oxide Only (Molecular Diffusion)



## Physics: Henry's Law + Fick's Law

- Henry's Law (molecular dissolution)

$$C = K_{ox} \cdot P$$

- Fick's Law (Bulk diffusion)

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## KEY DIFFERENCE FROM METAL

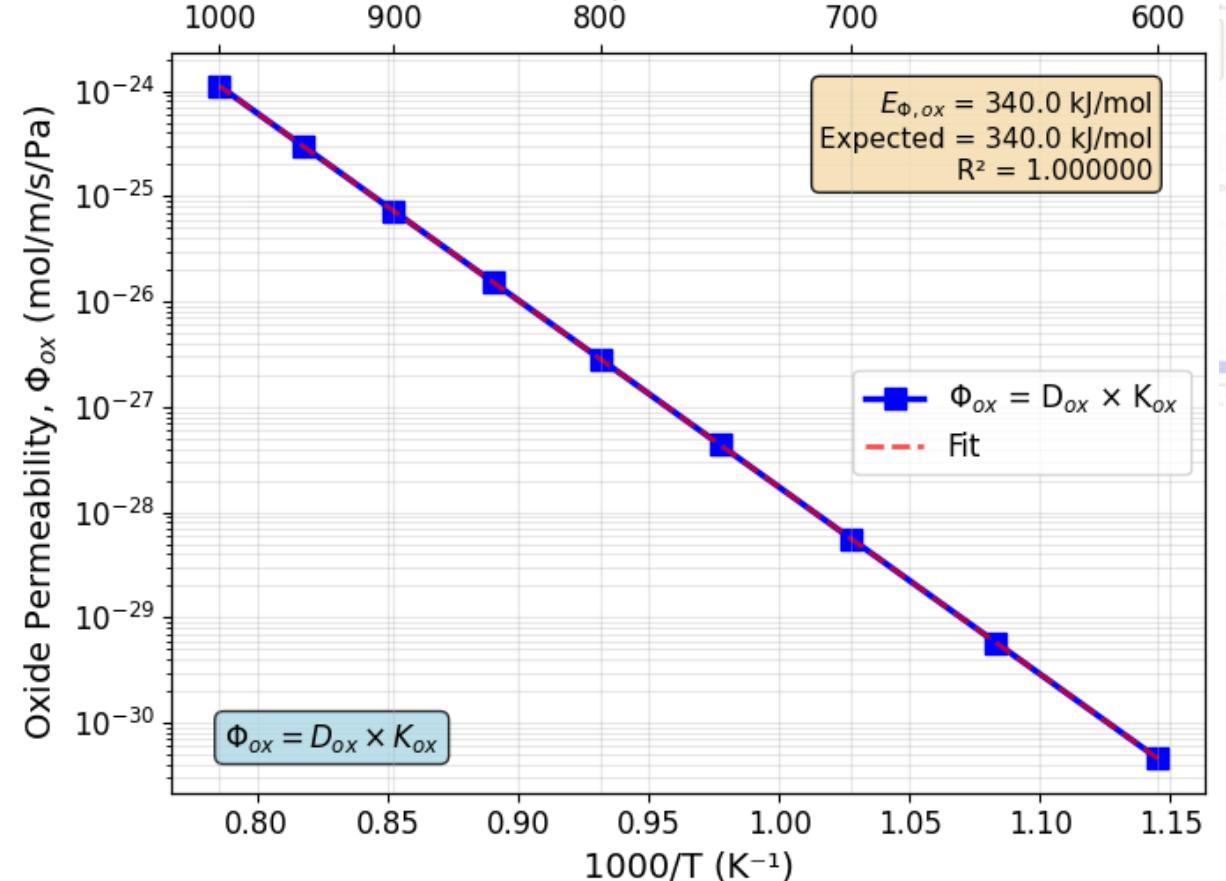
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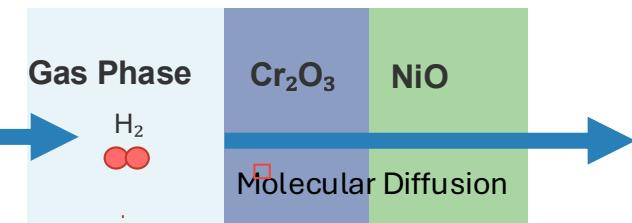
Level 2a: Perfect Oxide Only (Henry + Fick)

### (B) Oxide Permeability vs Temperature

Temperature (°C)



# Perfect Oxide Only (Molecular Diffusion)



## Physics: Henry's Law + Fick's Law

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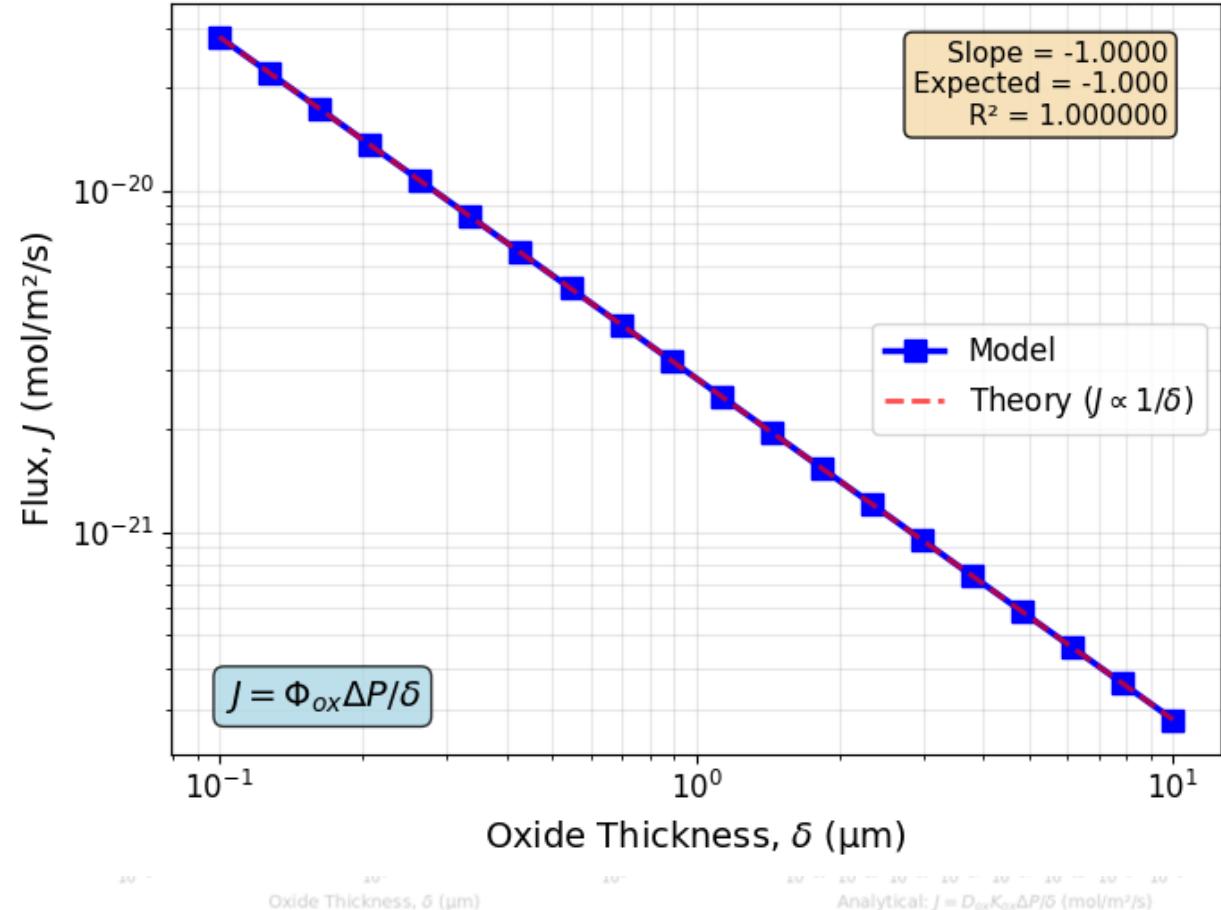
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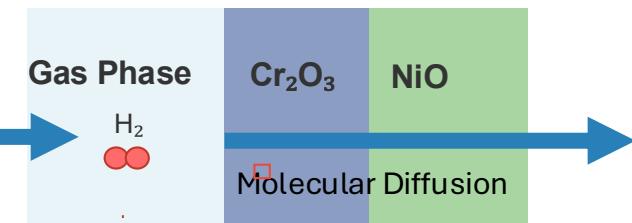
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Level 2a: Perfect Oxide Only (Henry + Fick)

(C) Flux vs Oxide Thickness



# Perfect Oxide Only (Molecular Diffusion)



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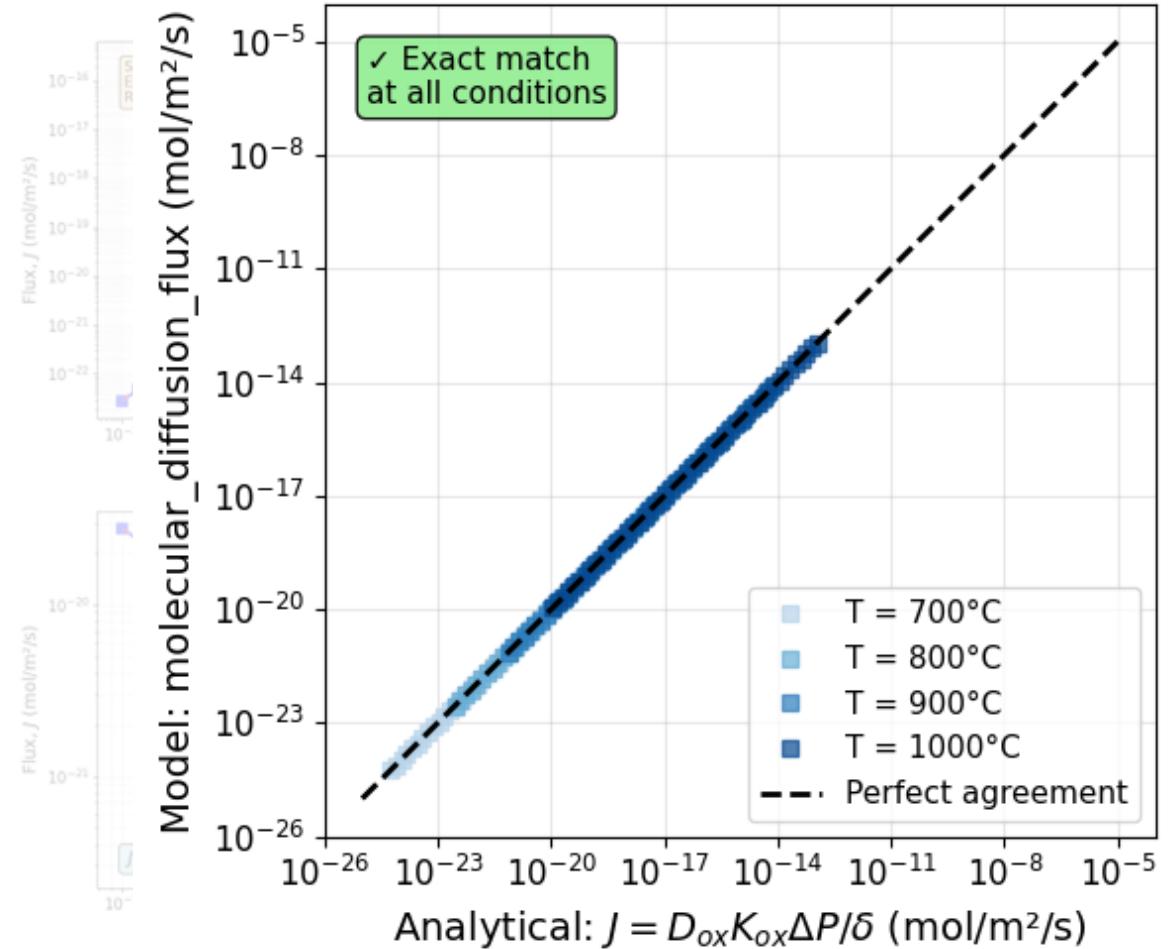
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## (D) Analytical Validation



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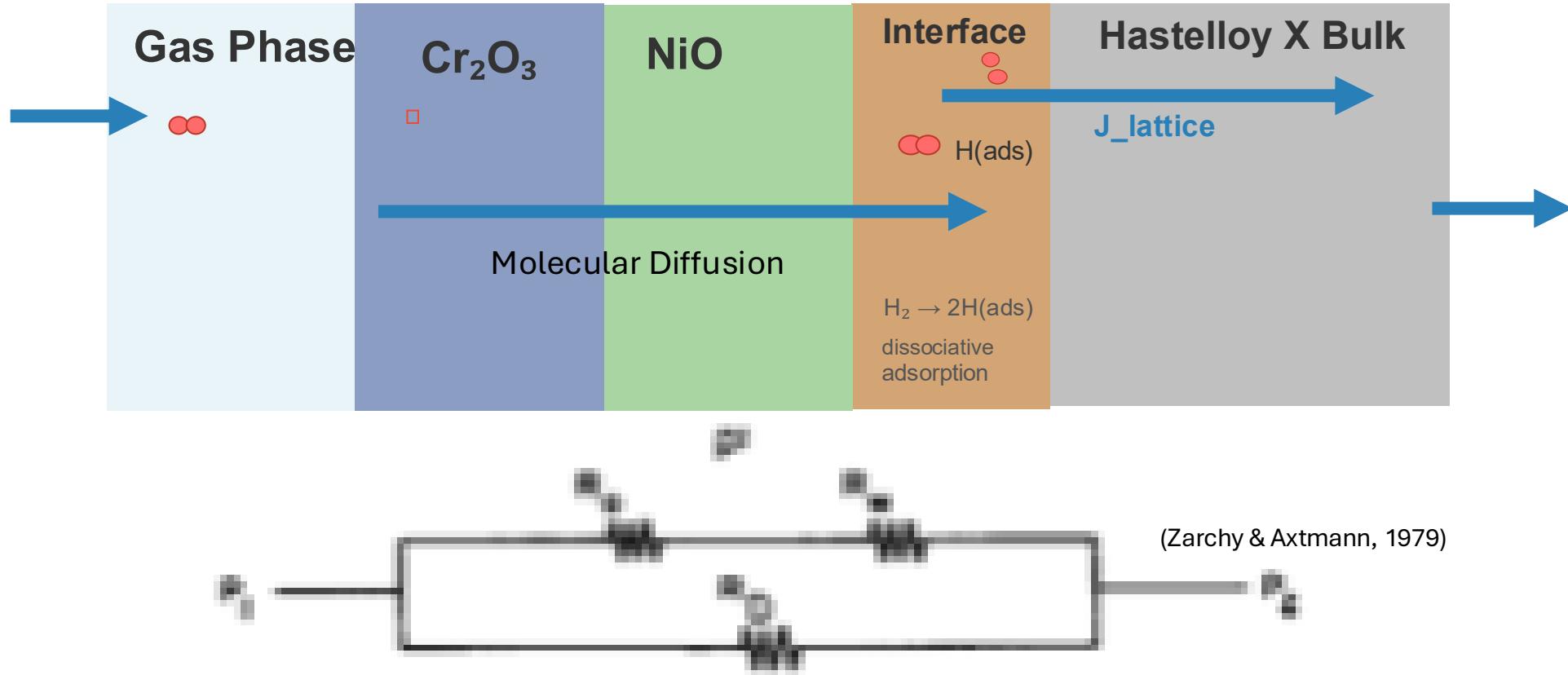
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Defective Oxide  
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(Level 5)

# Perfect Oxide + Perfect Metal (Series Resistance)



# Perfect Oxide + Perfect Metal (Series Resistance)

## Flux Balance Equation

$$\text{Metal Flux} = -J = \frac{DK_s}{x_{metal}} \left( P_{interface}^{1/2} - P_{Downstream}^{1/2} \right)$$

$$\text{Oxide Flux} = -J = \frac{D_{ox}K_{ox}}{x_{oxide}} \left( P_{upstream}^1 - P_{interface}^1 \right)$$

At steady state Metal Flux = Oxide Flux  $\rightarrow$  Solve for  $P_{interface}$

## Solve for $P_{interface}$

$$\text{Let } A = \frac{D_{ox}K_{ox}}{x_{oxide}} \quad B = \frac{DK_s}{x_{metal}} \text{ and } y = P_{interface}^{1/2}$$

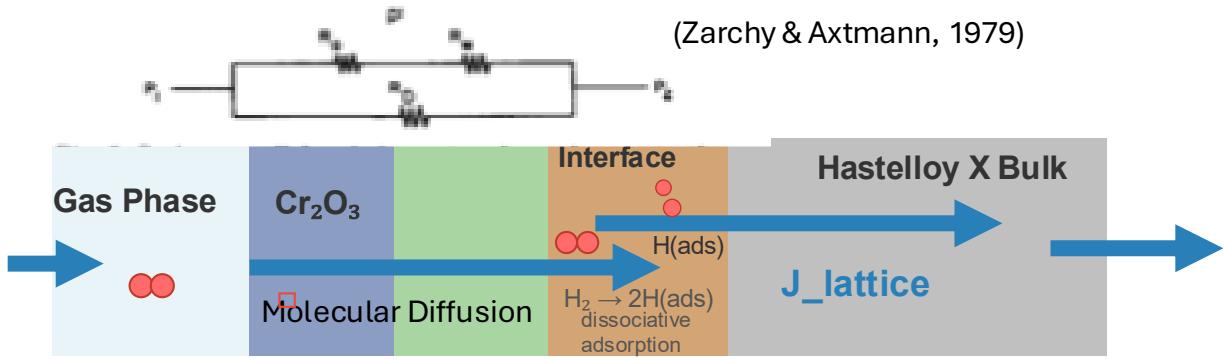
$$Ay^2 + By^2 - (AP_{upstream} - B\sqrt{P_{downstream}}) = 0$$

$$y = \sqrt{p_{interface}} = \frac{-B + \sqrt{(B^2 + 4A(AP_{up} + B\sqrt{P_{down}}))}}{2A}$$

$$p_{interface} = \left\{ \frac{-B + \sqrt{(B^2 + 4A(AP_{up} + B\sqrt{P_{down}}))}}{2A} \right\}^2$$

Metal-Oxide  
Flux matching  
Condition

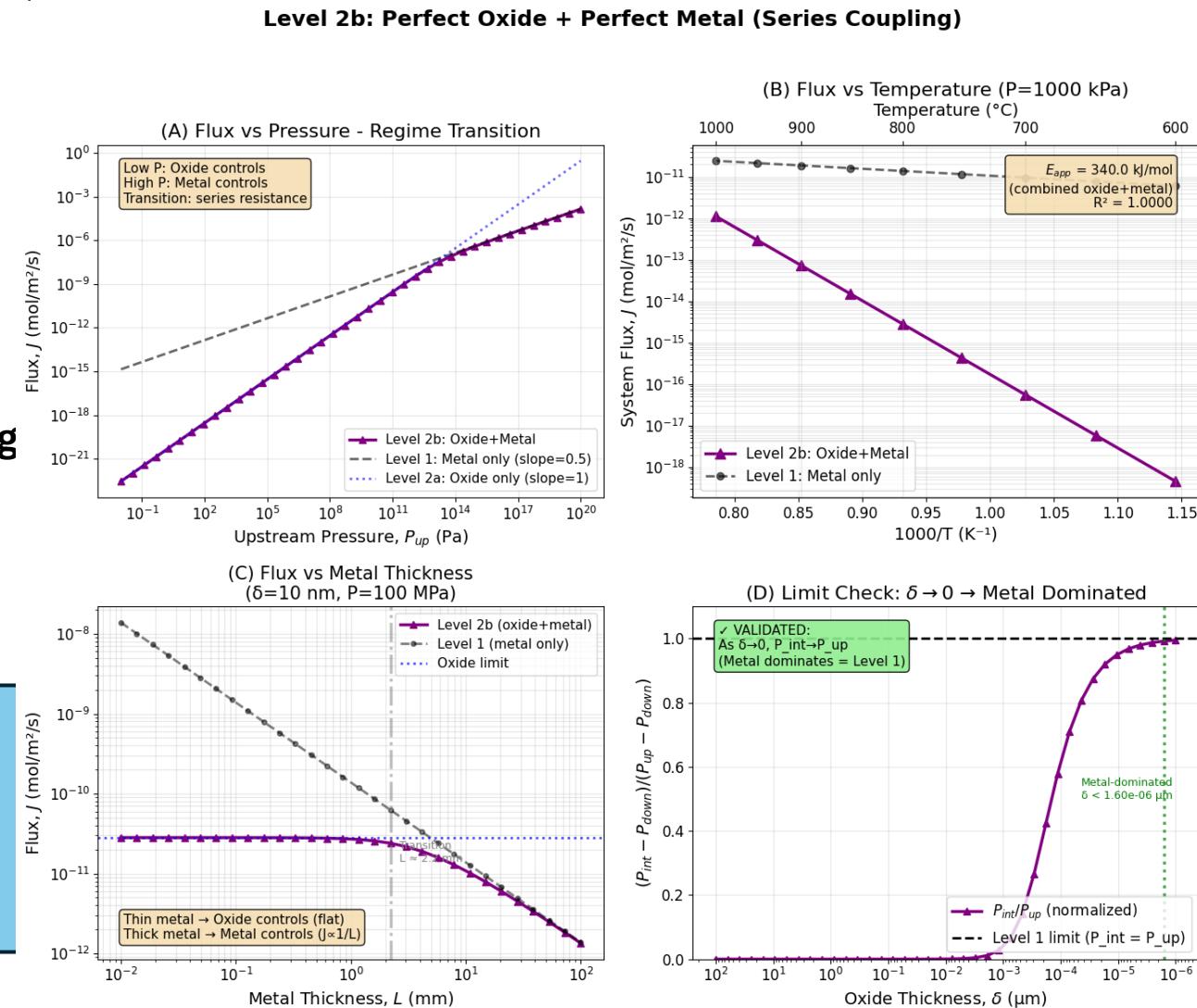
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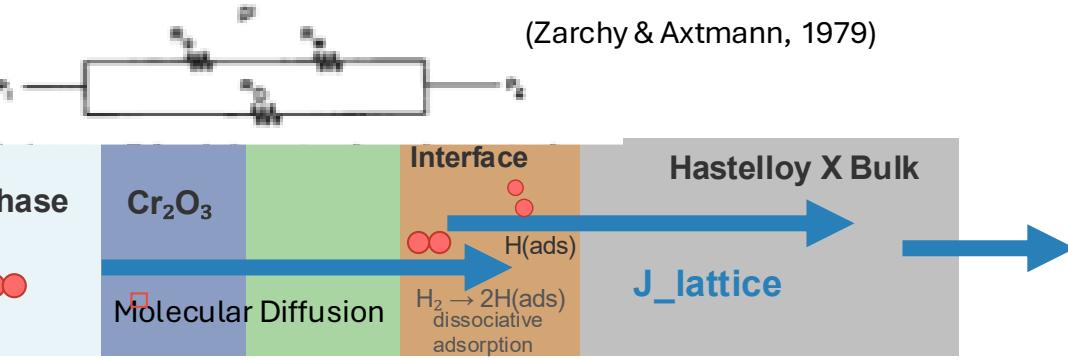
**Physics:** Oxide and Metal in series with interface coupling

At steady state Metal Flux = Oxide Flux  $\rightarrow$  Solve for  
P\_interface

$$p_{\text{interface}} = \left\{ \frac{-B + \sqrt{(B^2 + 4A(AP_{\text{up}} + B\sqrt{P_{\text{down}}}))}}{2A} \right\}^2$$



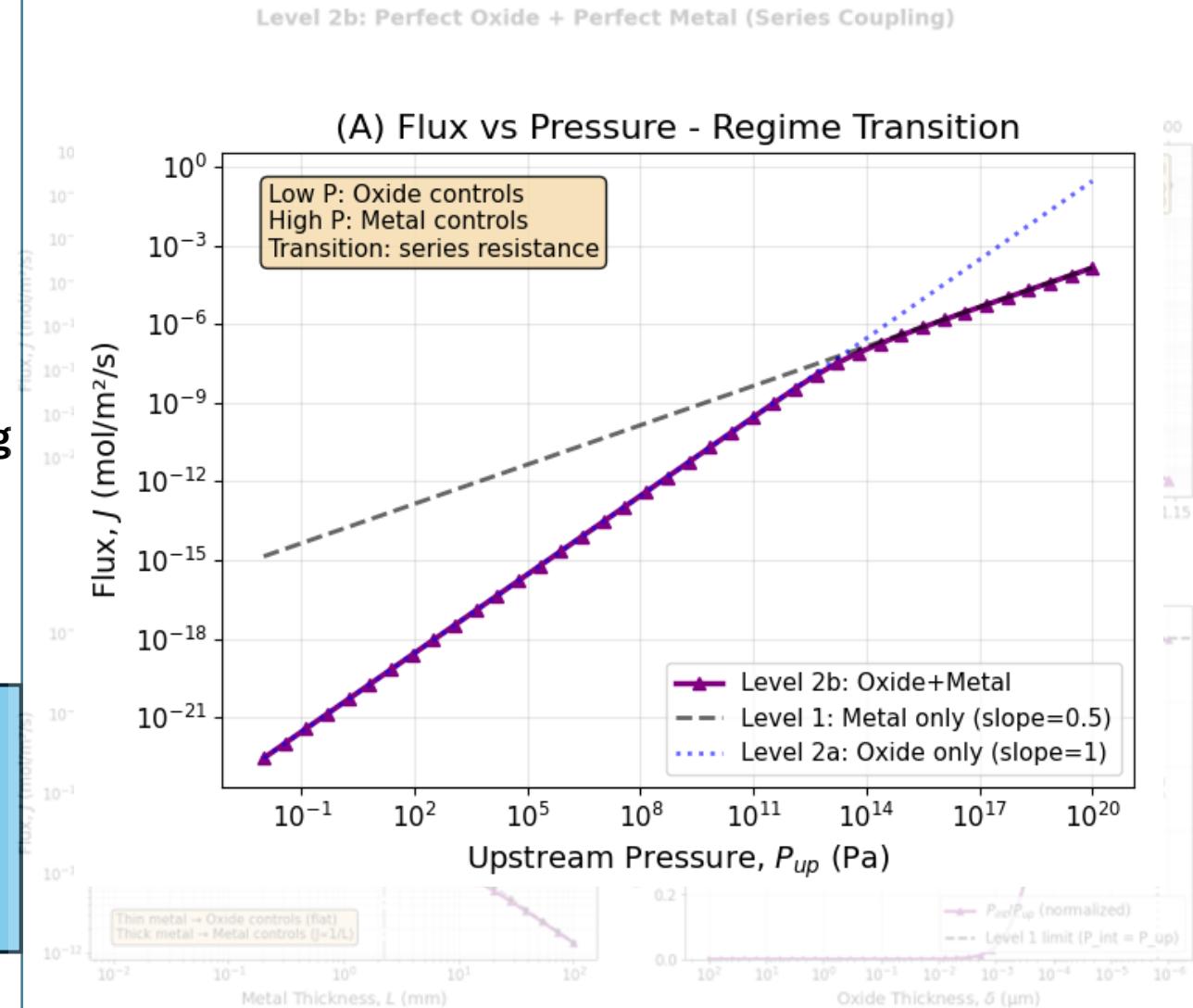
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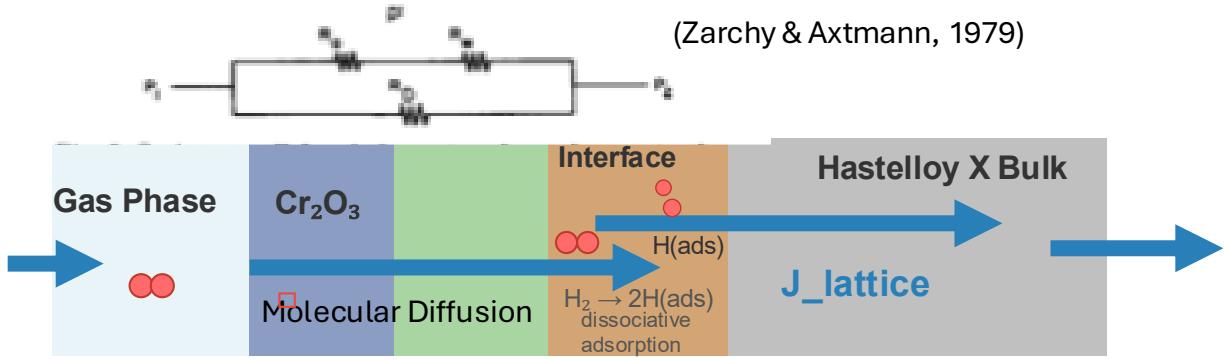
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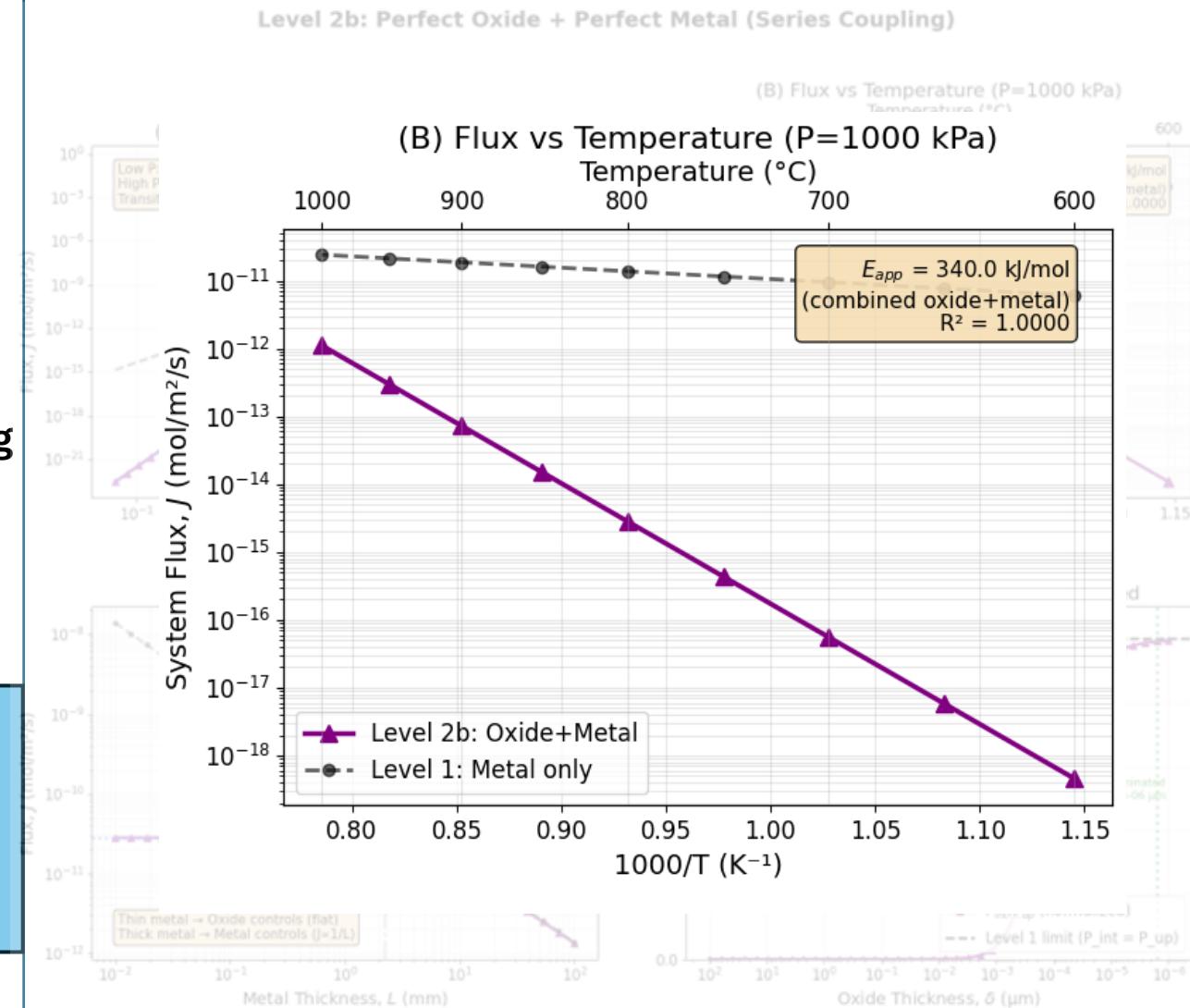
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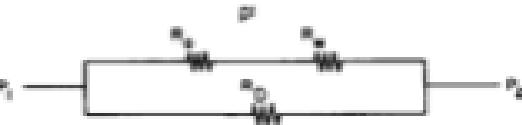
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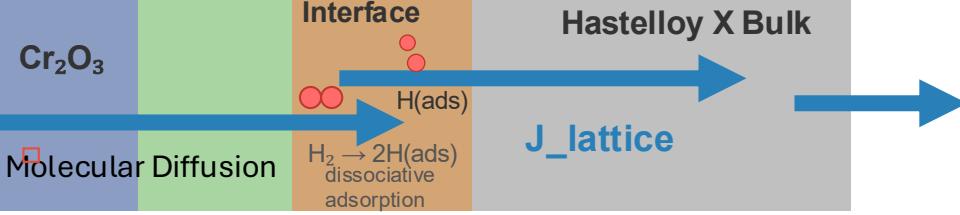


# Perfect Oxide + Perfect Metal (Series Resistance)



(Zarchy & Axtmann, 1979)

Gas Phase



**Physics: Oxide and Metal in series with interface coupling**

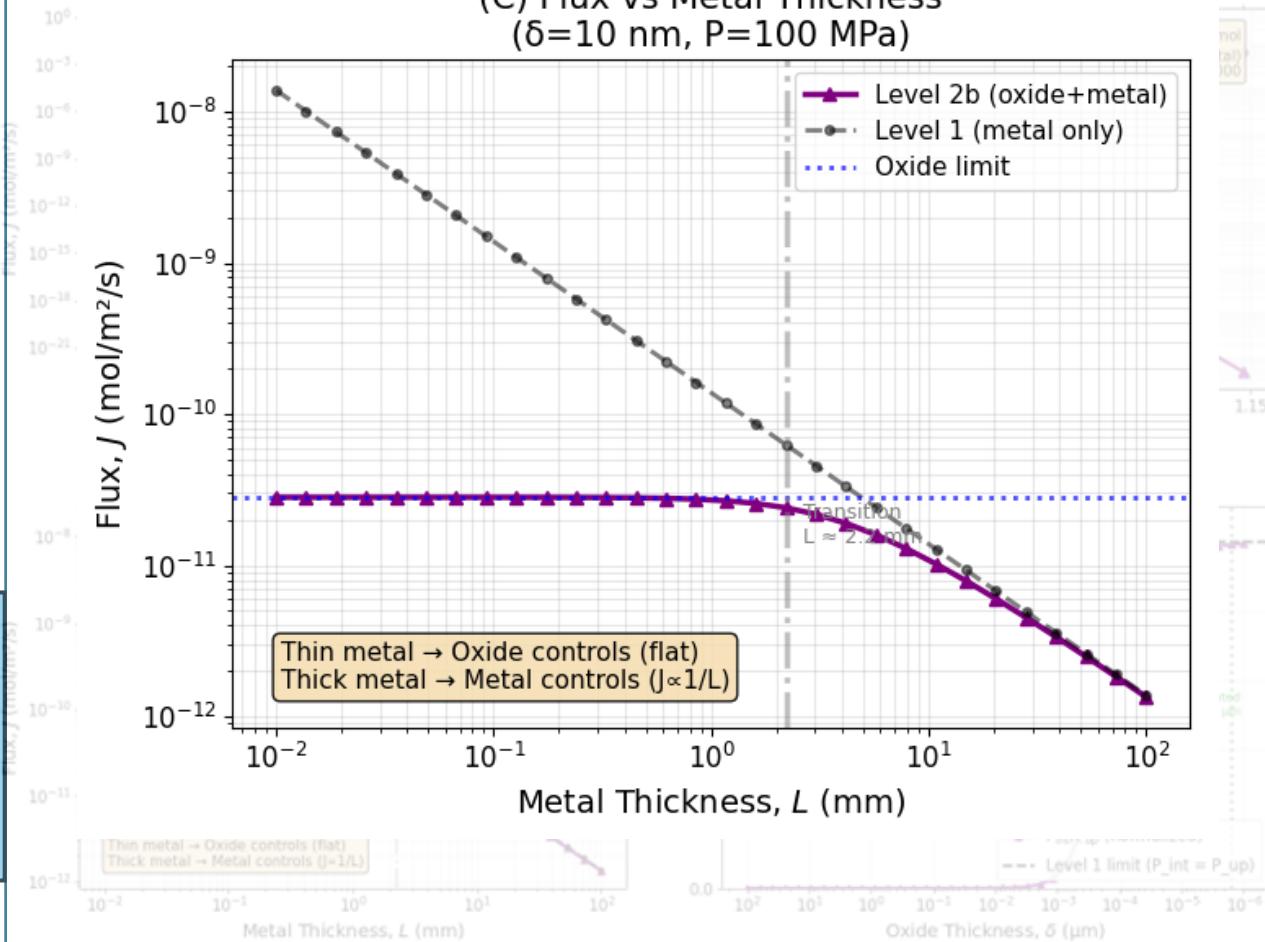
At steady state Metal Flux = Oxide Flux → Solve for  
P\_interface

$$p_{interface} = \frac{\left\{ -B + \sqrt{(B^2 + 4A(AP_{up} + B\sqrt{P_{down}}))} \right\}^2}{2A}$$

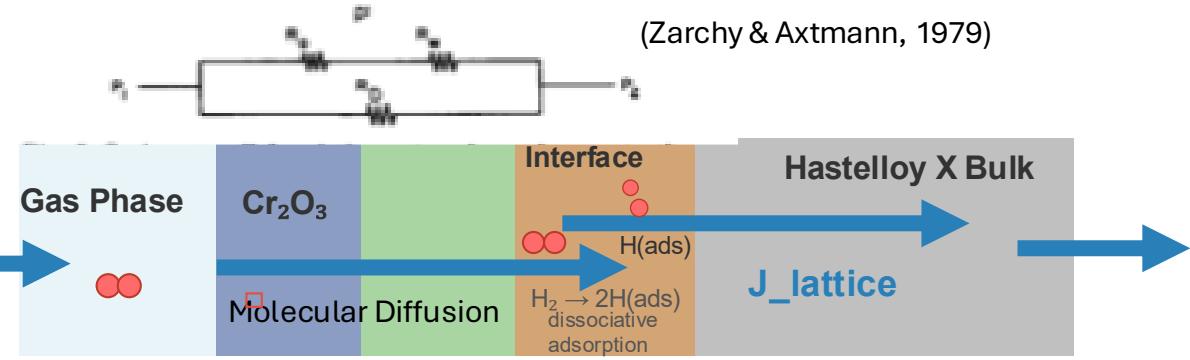
Level 2b: Perfect Oxide + Perfect Metal (Series Coupling)

(B) Flux vs Temperature (P=1000 kPa)

(C) Flux vs Metal Thickness  
(δ=10 nm, P=100 MPa)



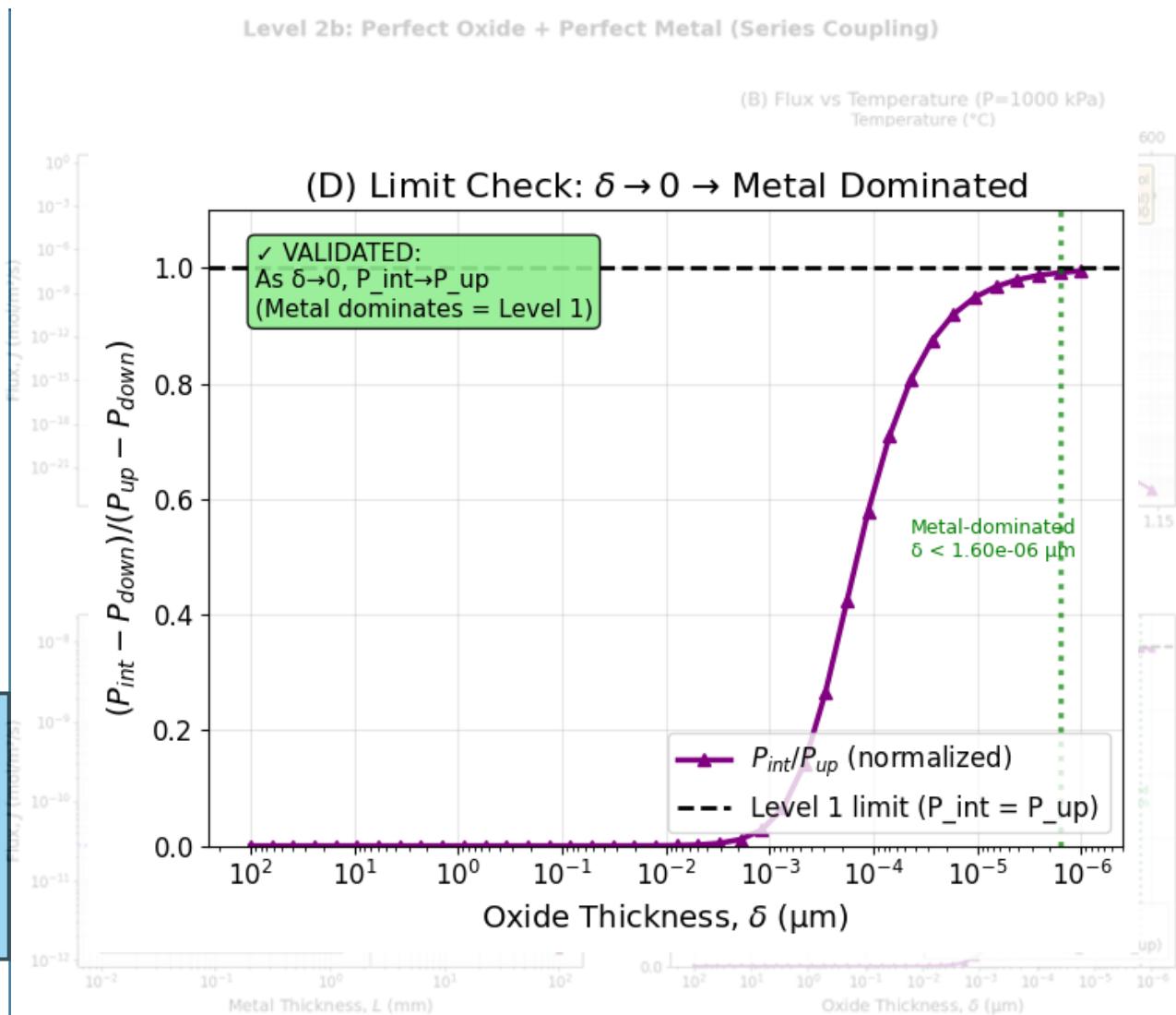
# Perfect Oxide + Perfect Metal (Series Resistance)



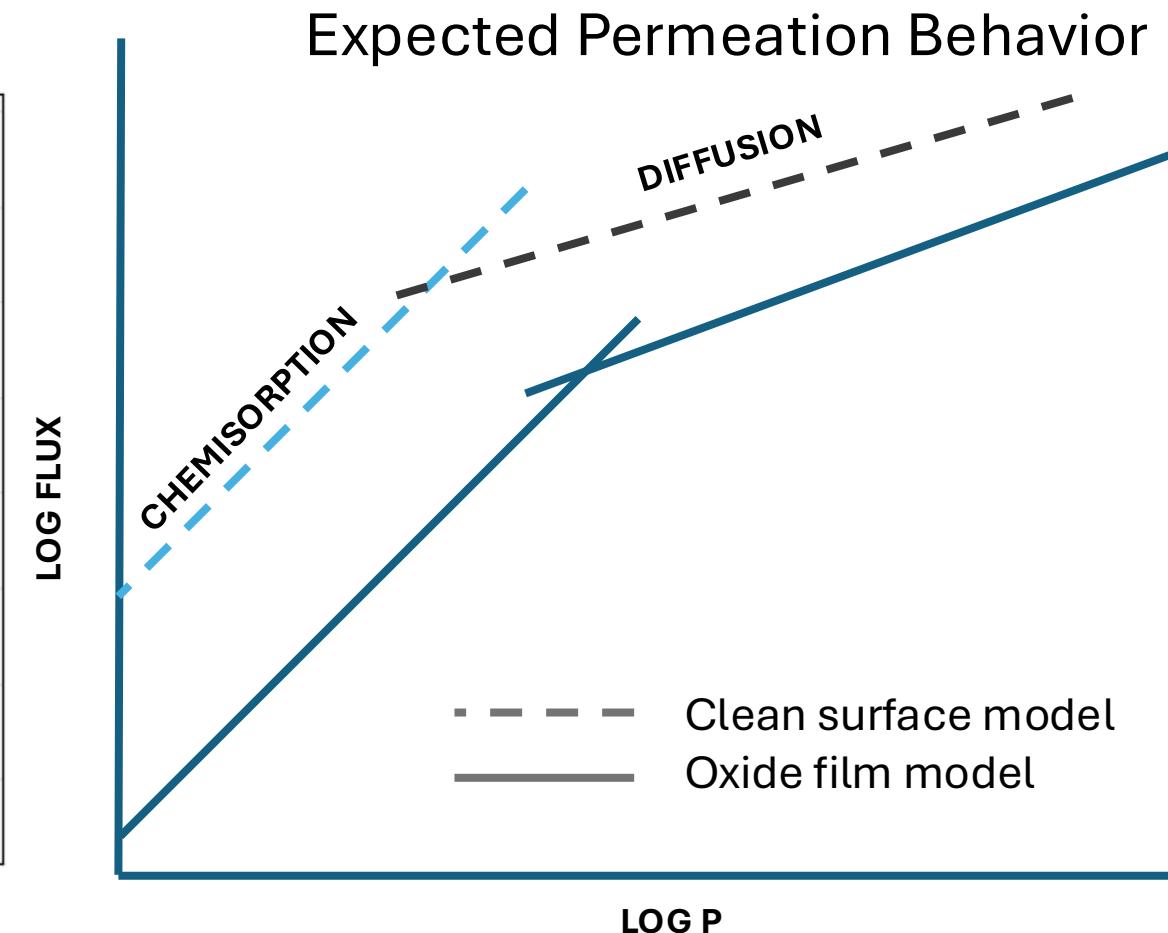
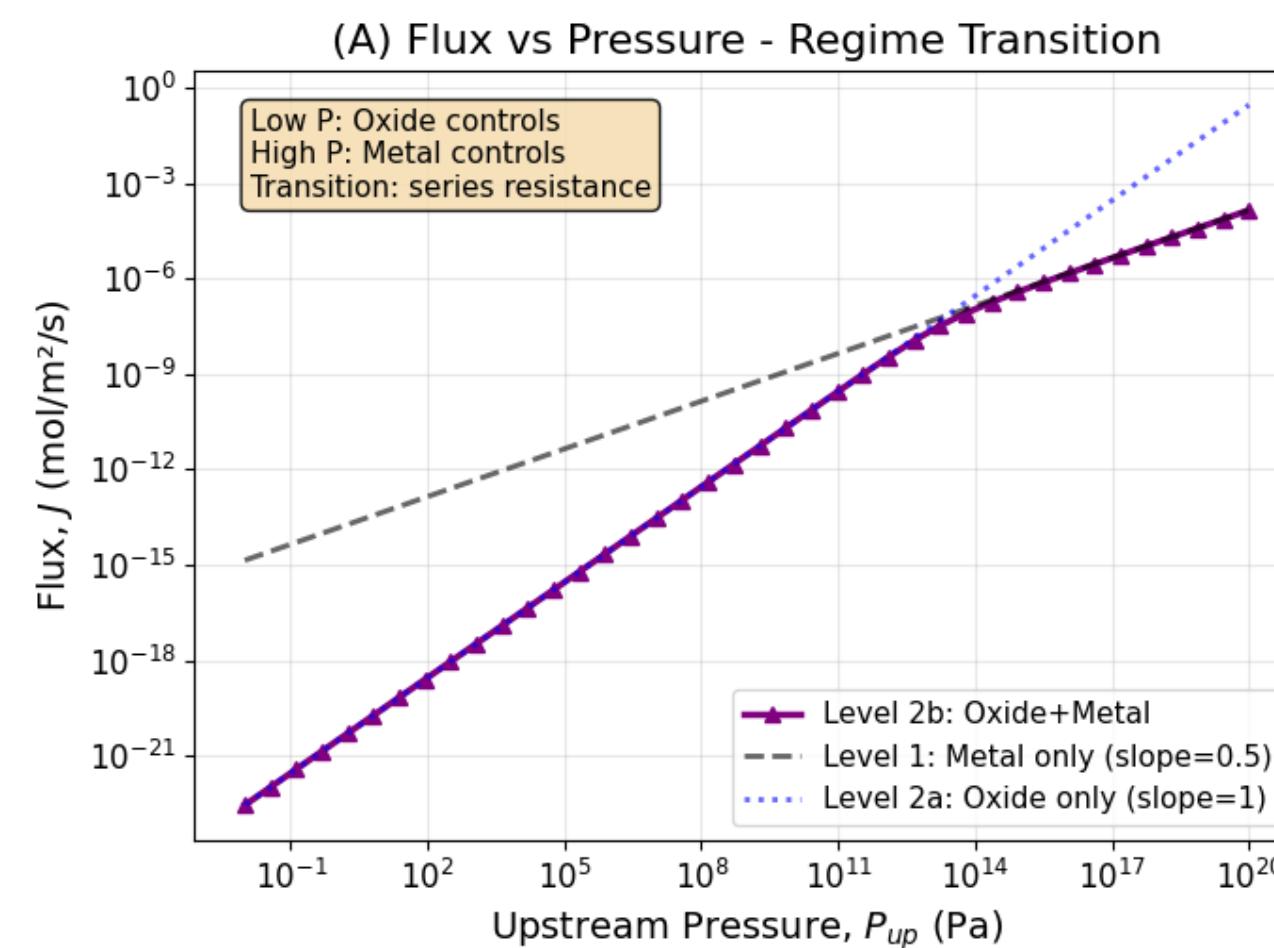
**Physics:** Oxide and Metal in series with interface coupling

At steady state Metal Flux = Oxide Flux → Solve for  
P<sub>interface</sub>

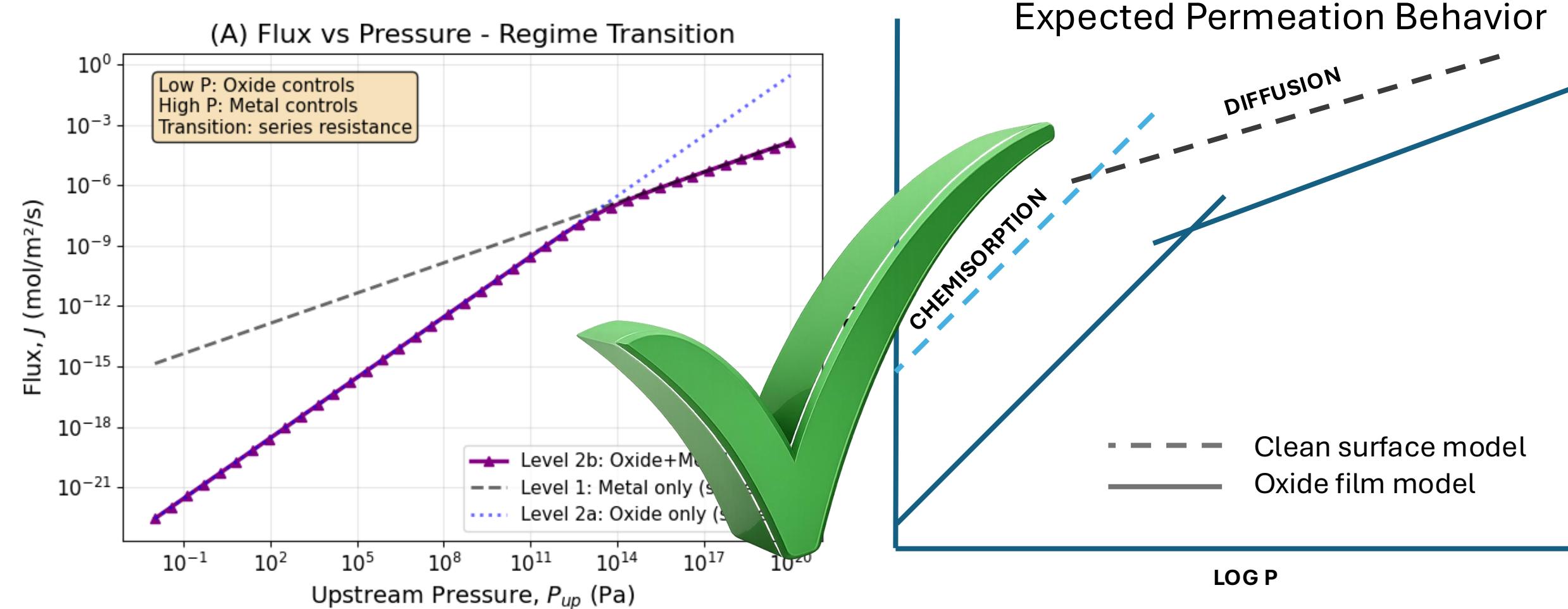
$$p_{\text{interface}} = \frac{\left\{ -B + \sqrt{(B^2 + 4A(AP_{\text{up}} + B\sqrt{P_{\text{down}}}))} \right\}^2}{2A}$$



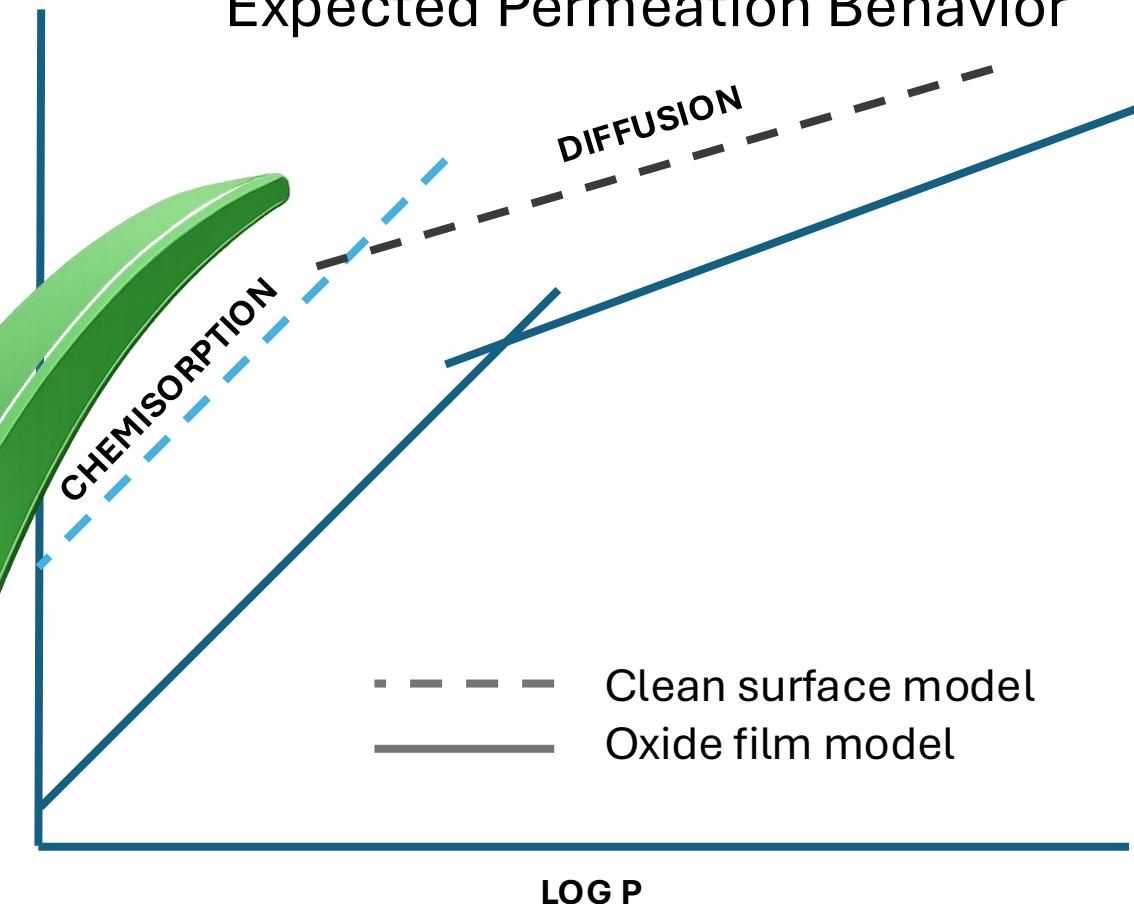
# Perfect Oxide + Perfect Metal (Series Resistance)



# Perfect Oxide + Perfect Metal (Series Resistance)



Expected Permeation Behavior



# Aim 1

Perfect Metal  
Atomic Diffusion  
(Level 1)

Perfect Oxide  
Molecular Diffusion  
(Level 2a)

Perfect Oxide  
+ Perfect Metal  
Series Resistance  
(Level 2b)

Defective Oxide  
+ Perfect Metal  
The Parallel Path Model  
(Level 3)

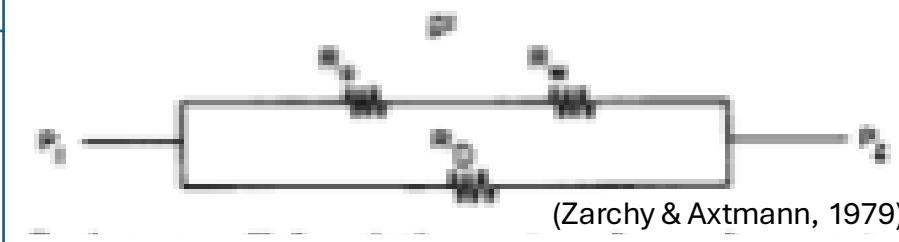
Defective Metal  
(Level 4)

Defective Oxide  
+ Defective Metal  
(Level 5)

# Defective Oxide + Perfect Metal (The Parallel Path Model)

## Classical Modeling Frameworks for Oxide Cracks and Defects

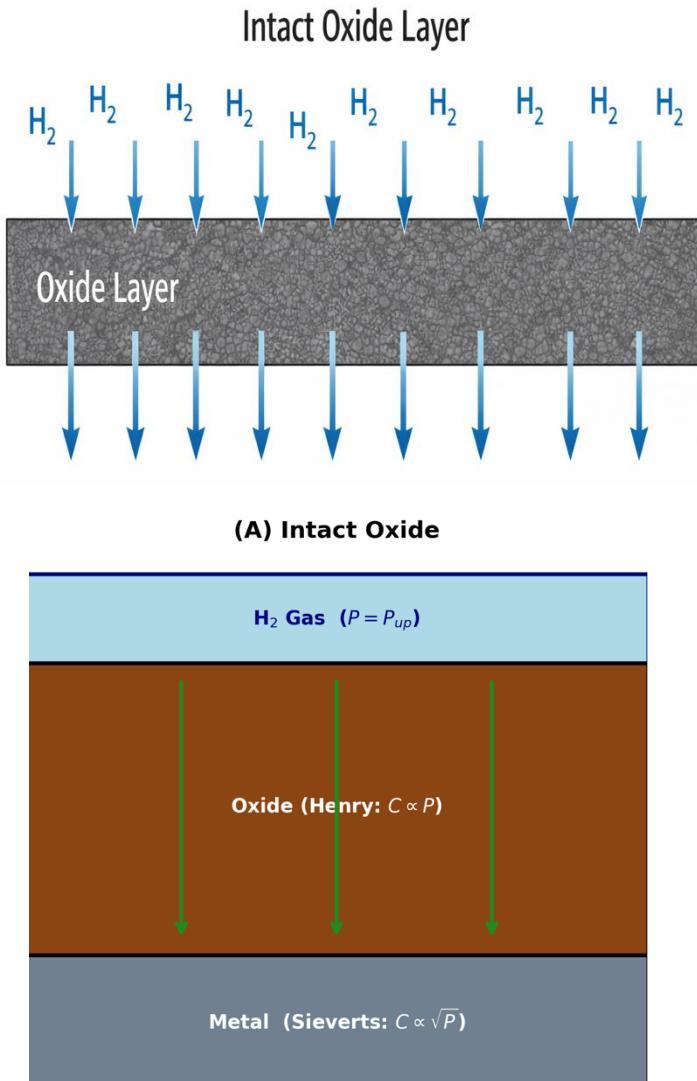
Model Type	Key Features	Mathematical Approach
Parallel Path Model	<ul style="list-style-type: none"><li>• Series-parallel resistor network</li><li>• Intact oxide + defect paths</li><li>• Explains transition regimes</li></ul>	Network resistance: $\frac{1}{R_{total}} = \frac{1}{R_{oxide}} + \frac{1}{R_{defects}}$
Area-Defect Model	<ul style="list-style-type: none"><li>• Defects have effective area</li><li>• Binary (perfect/defective)</li><li>• Simple implementation</li></ul>	$J_{total} = J_{oxide} (1 - f_d) + J_{defects} (1 - f_d)$ where $f_d$ = defect fraction
Permeation Reduction Factor (PRF)	<ul style="list-style-type: none"><li>• Quantifies barrier effectiveness</li><li>• Ratio-based metric</li></ul>	$PRF = \frac{\Phi_{metal}}{\Phi_{oxide\_covered}}$



### Types of Oxide Defects

- Pinholes (complete oxide absence)
- Crack with thin oxide
- Grain Boundaries in oxide

# Defective Oxide + Perfect Metal (The Parallel Path Model)



$$J = \left( \frac{D_{ox} K_{ox}}{L_{ox}} \right) \cdot (P_{up} - P_{down})$$

For Parallel Resistance Analogy:

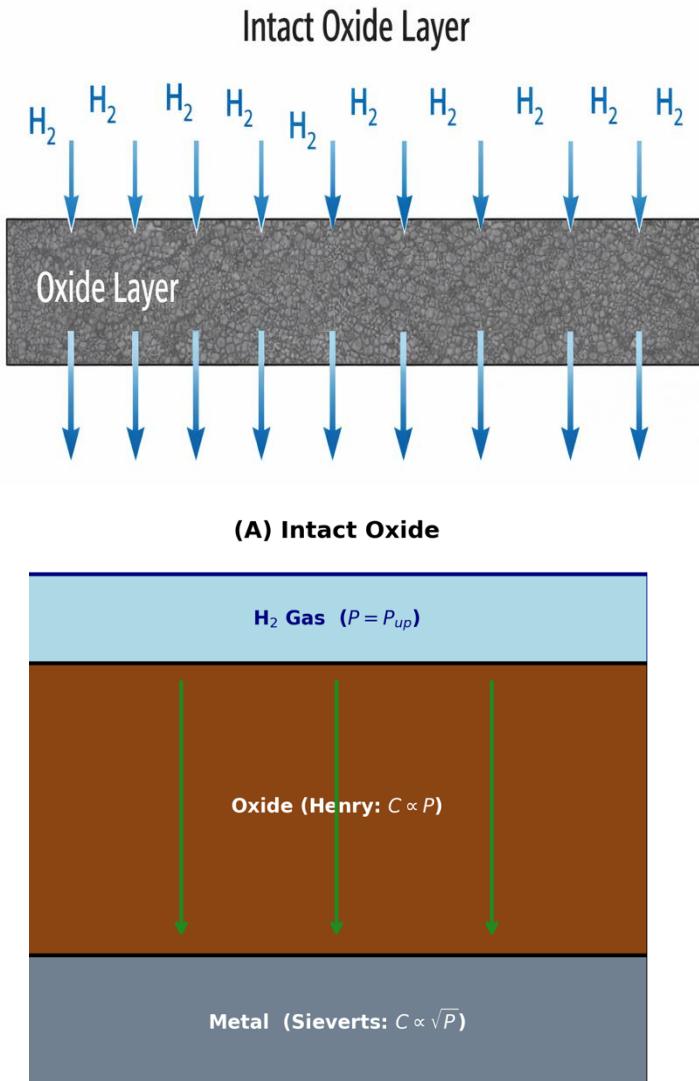
- **Parallel resistors:**  $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$
- **Total Surface Area:**  $A_{total} = A_{intact} + A_{defect}$
- **Area fractions:**  $f_{intact} = \frac{A_{intact}}{A_{total}}$  ;  $f_{defect} = \frac{A_{defect}}{A_{total}}$

$$f_{intact} + f_{defect} = 1$$

- **Flux:**  $J_{total} = J_{intact} * f_{intact} + J_{defect} * f_{defect}$

$$J_{total} = J_{intact} * (1 - f_{defect}) + J_{defect} * f_{defect}$$

# Defective Oxide + Perfect Metal (The Parallel Path Model)



$$J = \left( \frac{D_{ox} K_{ox}}{L_{ox}} \right) \cdot (P_{up} - P_{down})$$

For Parallel Resistance Analogy:

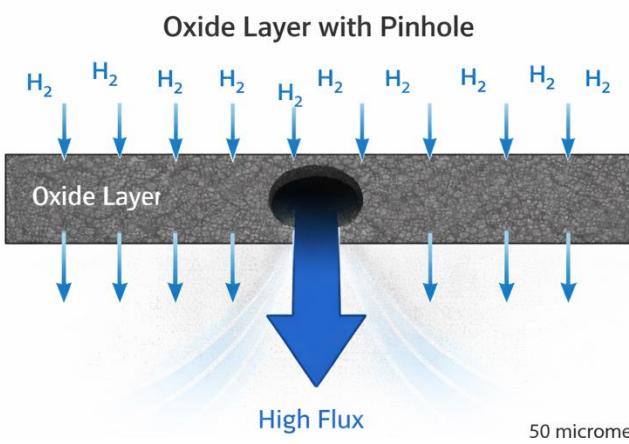
- **Parallel resistors:**  $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$
- **Total Surface Area:**  $A_{total} = A_{intact} + A_{defect}$
- **Area fractions:**  $f_{intact} = \frac{A_{intact}}{A_{total}}$  ;  $f_{defect} = \frac{A_{defect}}{A_{total}}$

$$f_{intact} + f_{defect} = 1$$

- **Flux:**  $J_{total} = J_{intact} * f_{intact} + J_{defect} * f_{defect}$

$$J_{total} = J_{intact} * (1 - f_{defect}) + J_{defect} * f_{defect}$$

# Defective Oxide + Perfect Metal (The Parallel Path Model)

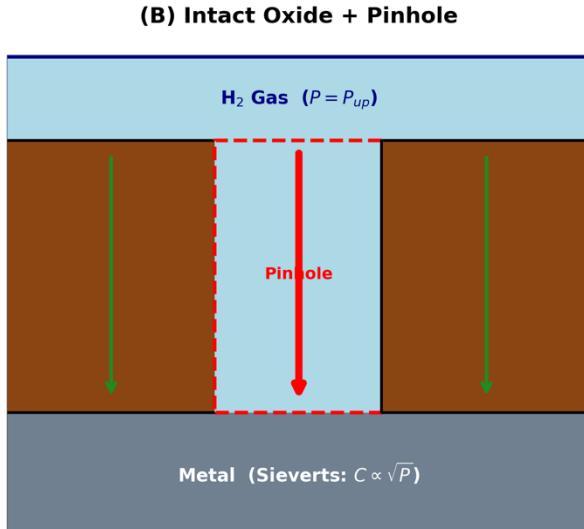


$$J_{total} = J_{intact} * (1 - f_{defect}) + J_{defect} * f_{defect}$$

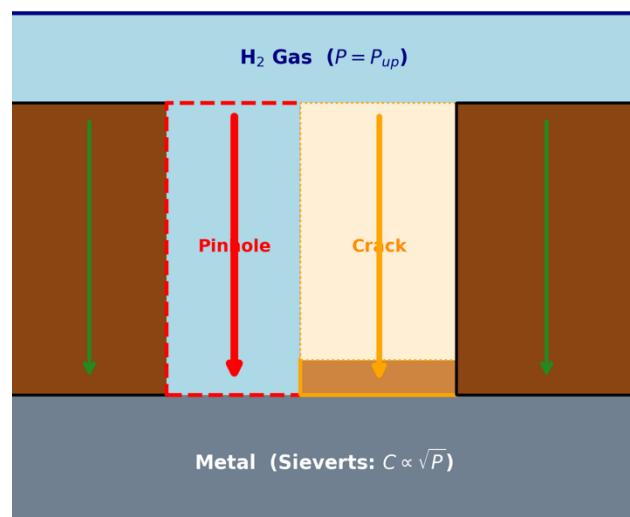
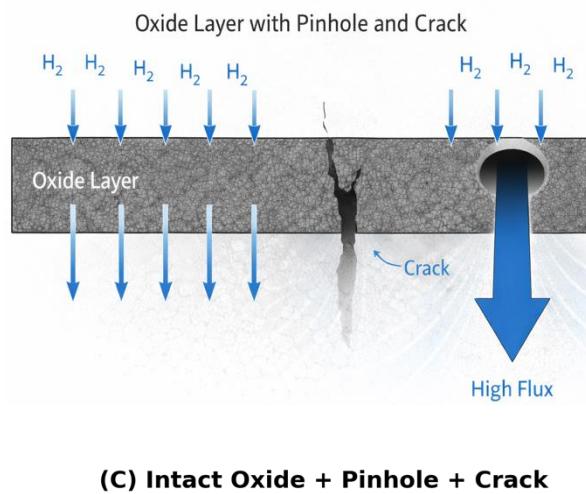
$$J_{intact} = \left( \frac{D_{ox} K_{ox}}{L_{ox}} \right) \cdot (P_{up} - P_{down})$$

$$J_{defect} = \left( \frac{DK_s}{L} \right) \cdot \left( \sqrt{P_{up}} - \sqrt{P_{down}} \right)$$

Pinholes



# Defective Oxide + Perfect Metal (The Parallel Path Model)



$$J_{total} = J_{intact} * (1 - f_{defect}) + J_{defect} * f_{defect}$$

$$J_{intact} = \left( \frac{D_{ox} K_{ox}}{L_{ox}} \right) \cdot (P_{up} - P_{down})$$

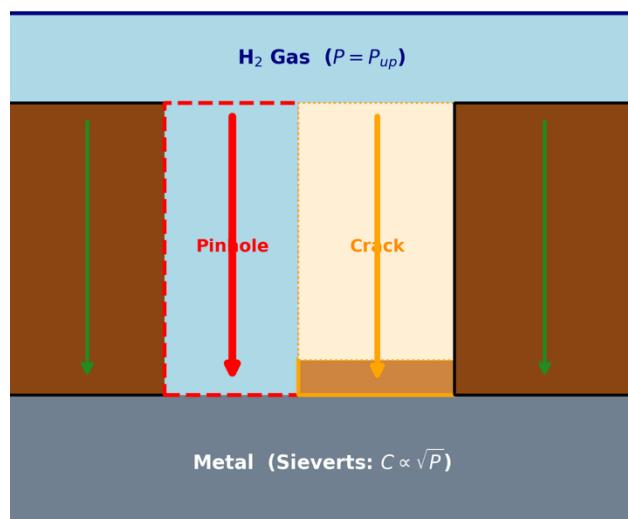
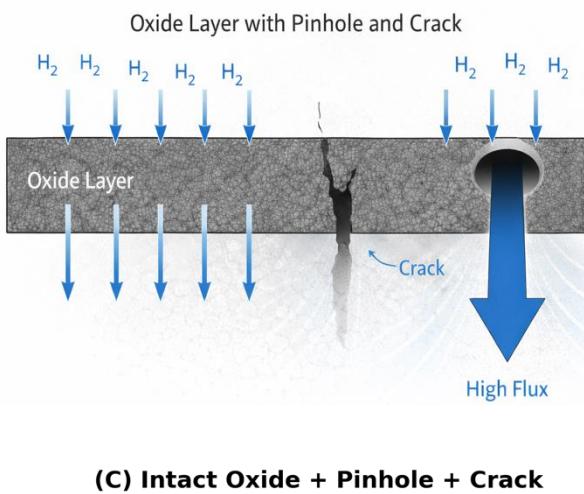
$$J_{defect} = f_{pinhole} * J_{pinhole} + f_{crack} * J_{crack}$$

$$J_{pinhole} = \left( \frac{DK_s}{L} \right) \cdot \left( \sqrt{P_{up}} - \sqrt{P_{down}} \right)$$

$$J_{crack} = \left( \frac{D_{ox} K_{ox}}{L_{ox}} \right) \cdot (P_{up} - P_{down}) \quad \bullet \quad L_{crack} = \alpha * L_{oxide}; \alpha < 1$$

$$J_{total} = J_{intact} * (1 - f_{defect}) + f_{pinhole} * J_{pinhole} + f_{crack} * J_{crack}$$

# Defective Oxide + Perfect Metal (The Parallel Path Model)



$$J_{total} = J_{intact} * (1 - f_{defect}) + J_{defect} * f_{defect}$$

$$J_{intact} = \left( \frac{D_{ox} K_{ox}}{L_{ox}} \right) \cdot (P_{up} - P_{down})$$

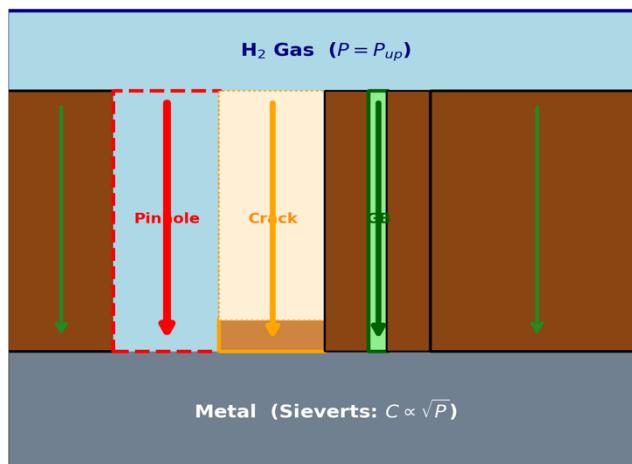
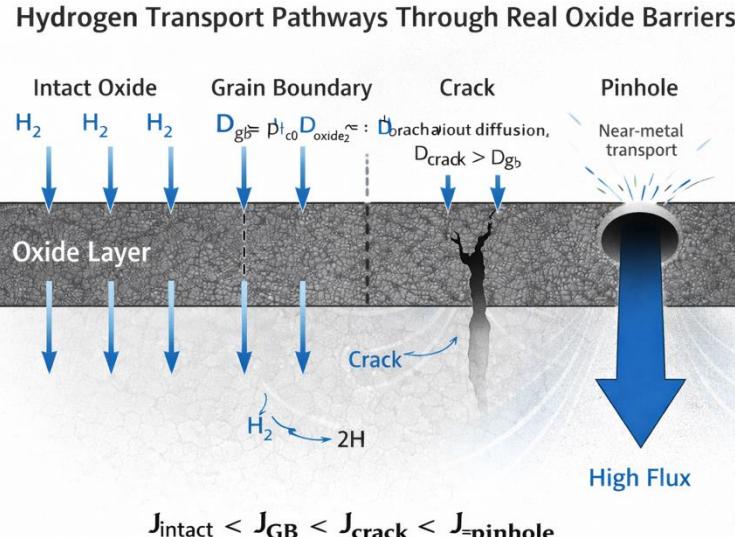
$$J_{defect} = f_{pinhole} * J_{pinhole} + f_{crack} * J_{crack}$$

$$J_{pinhole} = \left( \frac{DK_s}{L} \right) \cdot \left( \sqrt{P_{up}} - \sqrt{P_{down}} \right)$$

$$J_{crack} = \left( \frac{D_{ox} K_{ox}}{L_{ox}} \right) \cdot (P_{up} - P_{down}) \quad \bullet \quad L_{crack} = \alpha * L_{oxide}; \alpha < 1$$

$$J_{total} = J_{intact} * (1 - f_{defect}) + f_{pinhole} * J_{pinhole} + f_{crack} * J_{crack}$$

# Defective Oxide + Perfect Metal (The Parallel Path Model)



$$J_{total} = J_{intact} * (1 - f_{defect}) + f_{pinhole} * J_{pinhole} + f_{crack} * J_{crack} + f_{GB} * J_{GB}$$

$$J_{intact} = \left( \frac{D_{ox} K_{ox}}{L_{ox}} \right) \cdot (P_{up} - P_{down})$$

$$J_{defect} = f_{pinhole} * J_{pinhole} + f_{crack} * J_{crack} + f_{GB} * J_{GB}$$

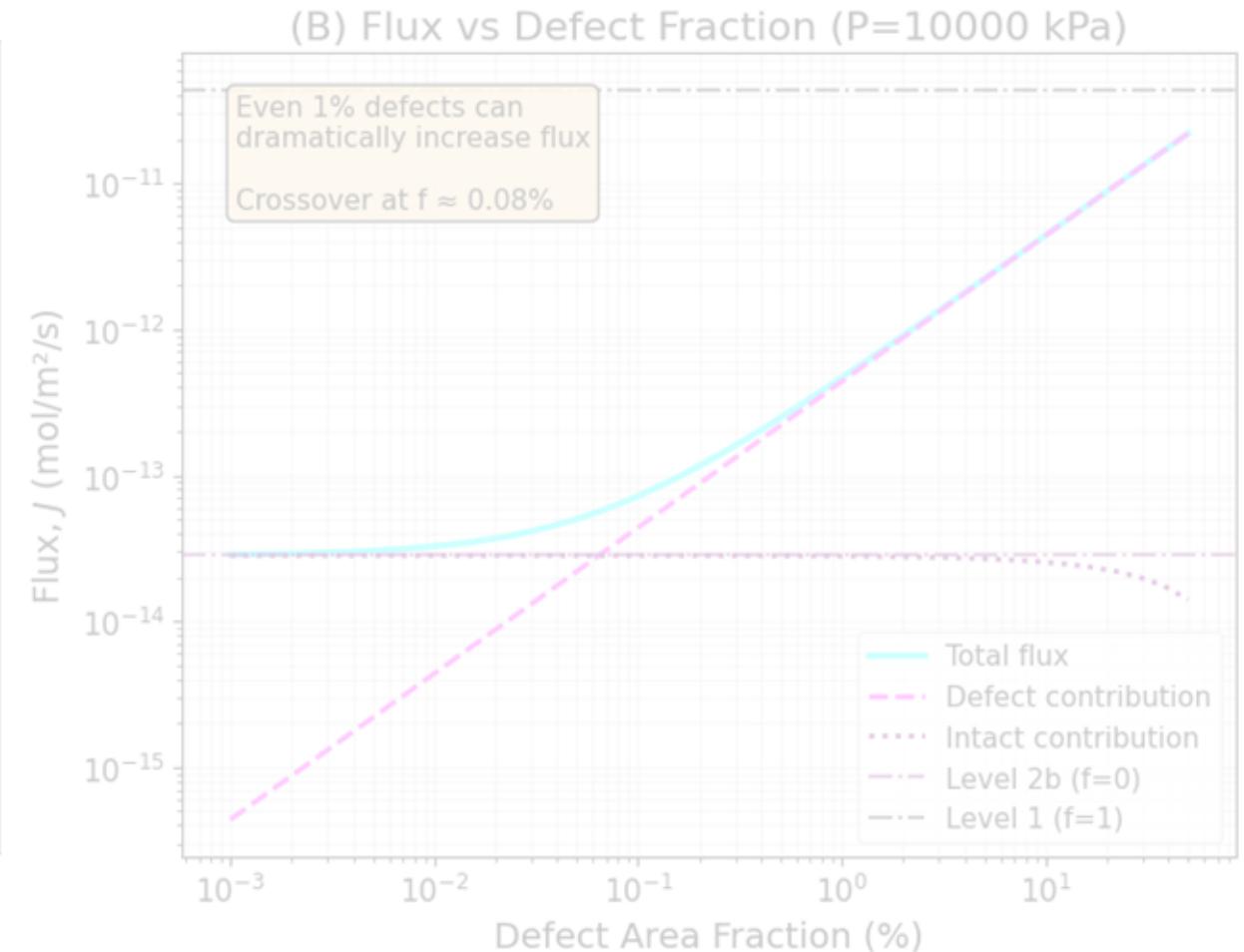
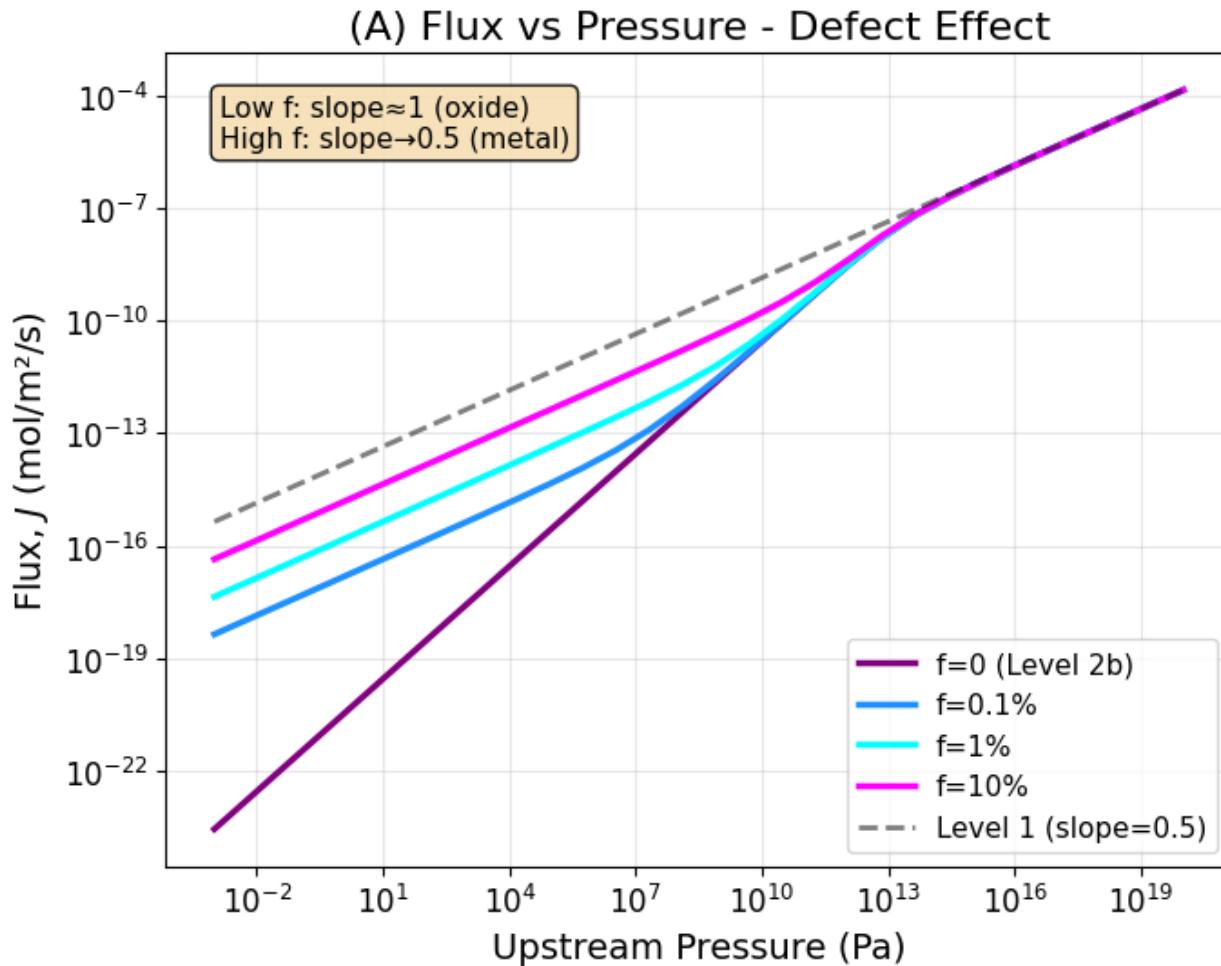
$$J_{pinhole} = \left( \frac{DK_s}{L} \right) \cdot \left( \sqrt{P_{up}} - \sqrt{P_{down}} \right)$$

$$J_{crack} = \left( \frac{D_{ox} K_{ox}}{L_{ox}} \right) \cdot (P_{up} - P_{down}) \quad \bullet \quad L_{crack} = \alpha * L_{oxide}; \alpha < 1$$

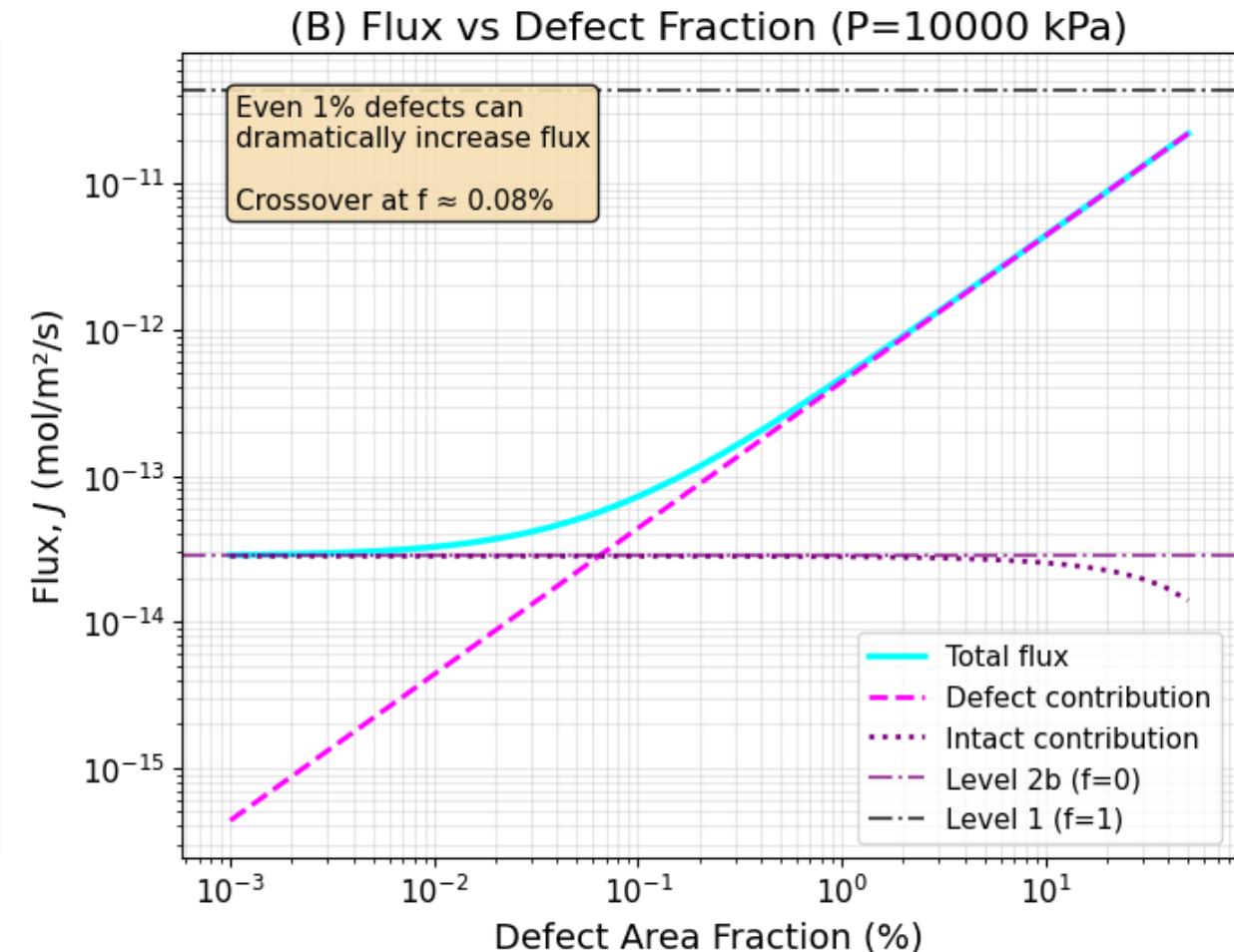
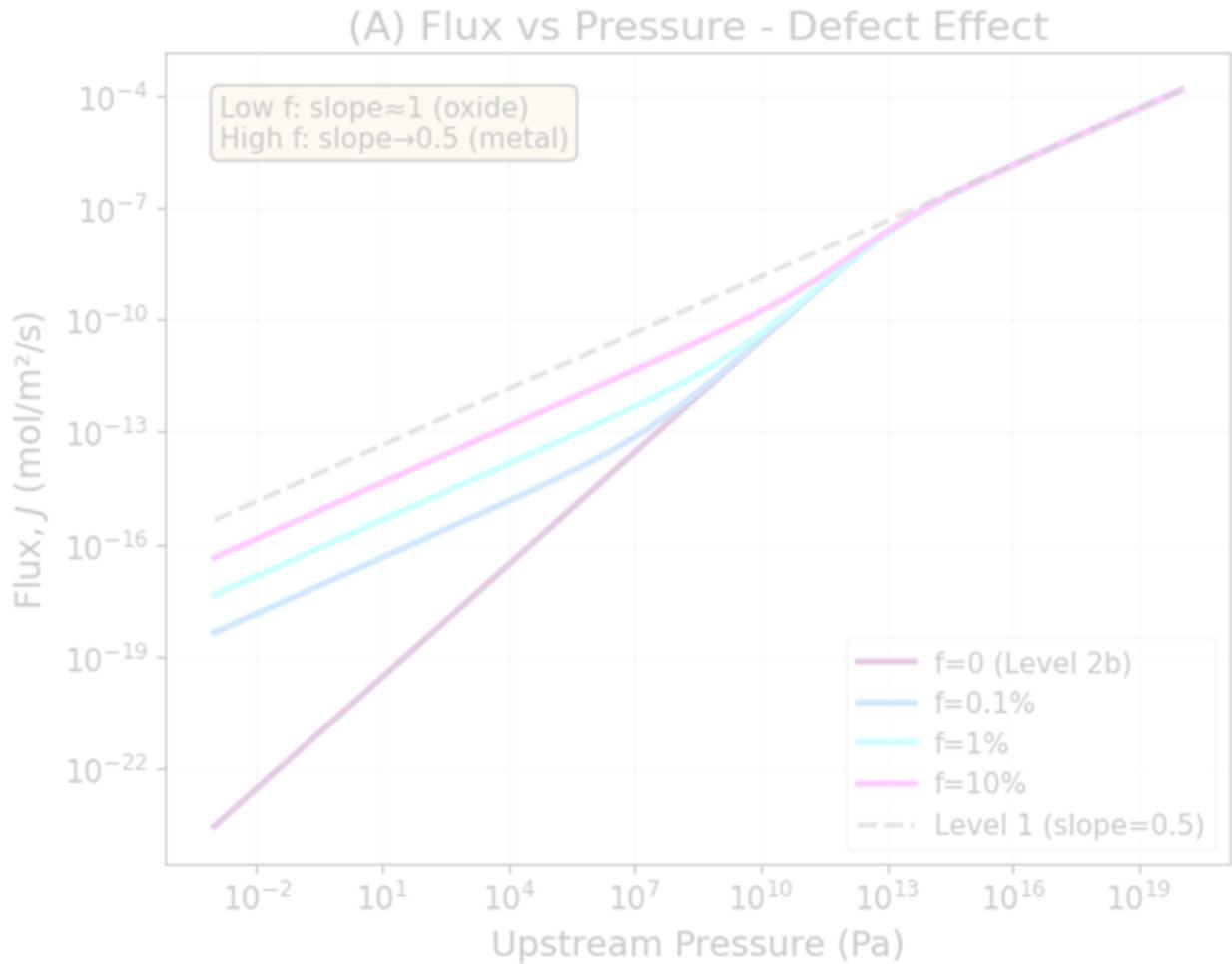
$$J_{GB} = \left( \frac{D_{ox} K_{ox}}{L_{ox}} \right) \cdot (P_{up} - P_{down}) \quad \bullet \quad D_{gb} = \beta * D_{ox}; \beta > 1$$

$$J_{total} = J_{intact} * (1 - f_{defect}) + f_{pinhole} * J_{pinhole} + f_{crack} * J_{crack} + f_{GB} * J_{GB}$$

# Defective Oxide + Perfect Metal (The Parallel Path Model)

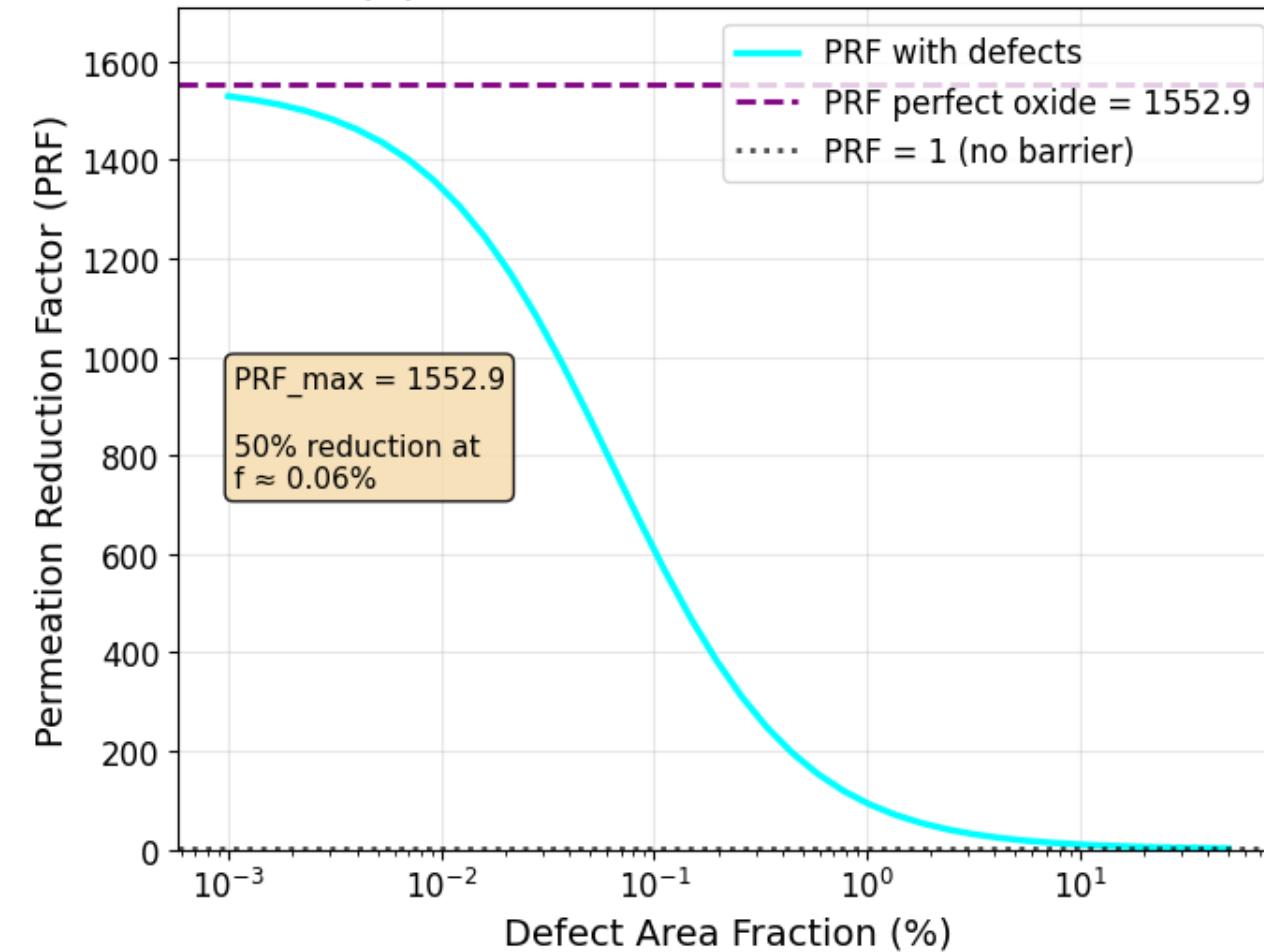


# Defective Oxide + Perfect Metal (The Parallel Path Model)

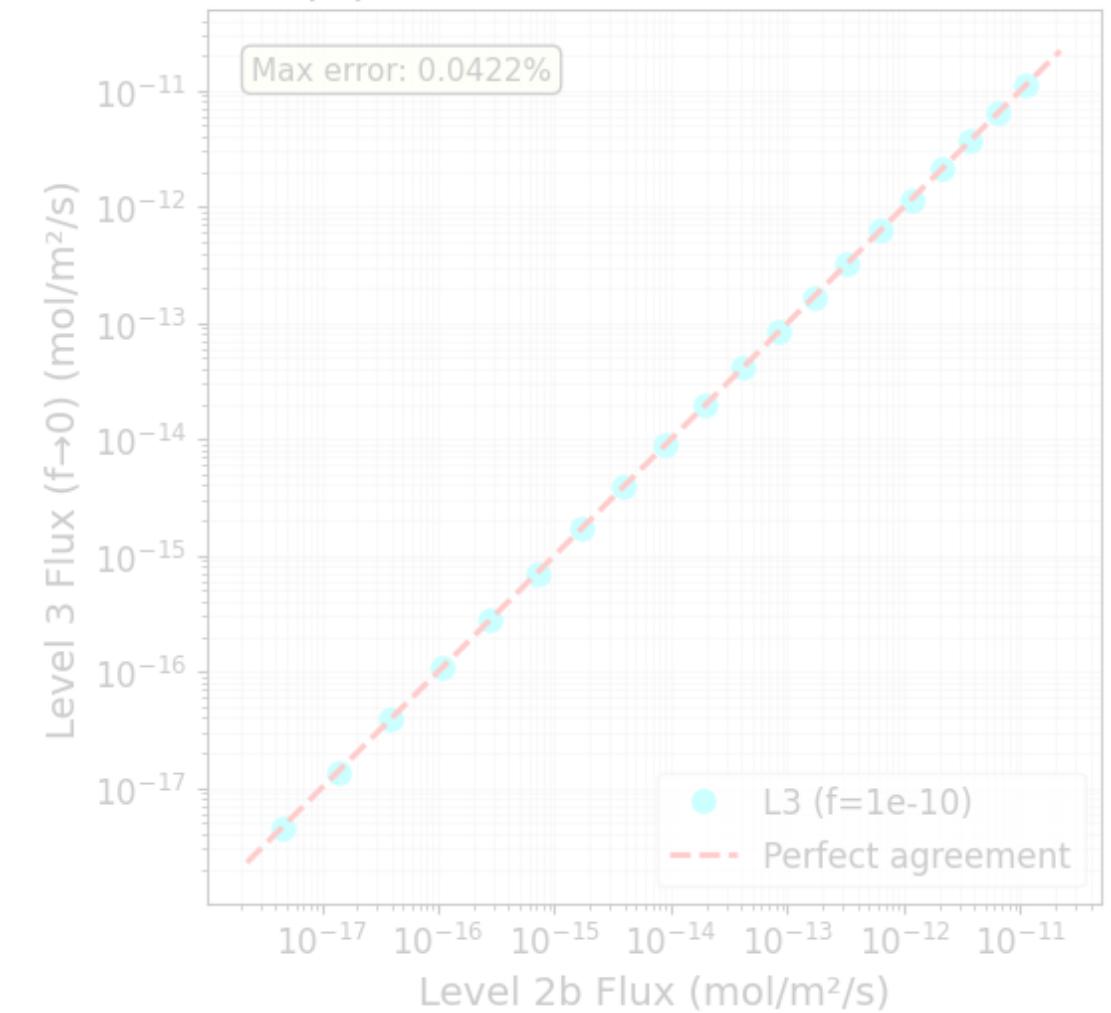


# Defective Oxide + Perfect Metal (The Parallel Path Model)

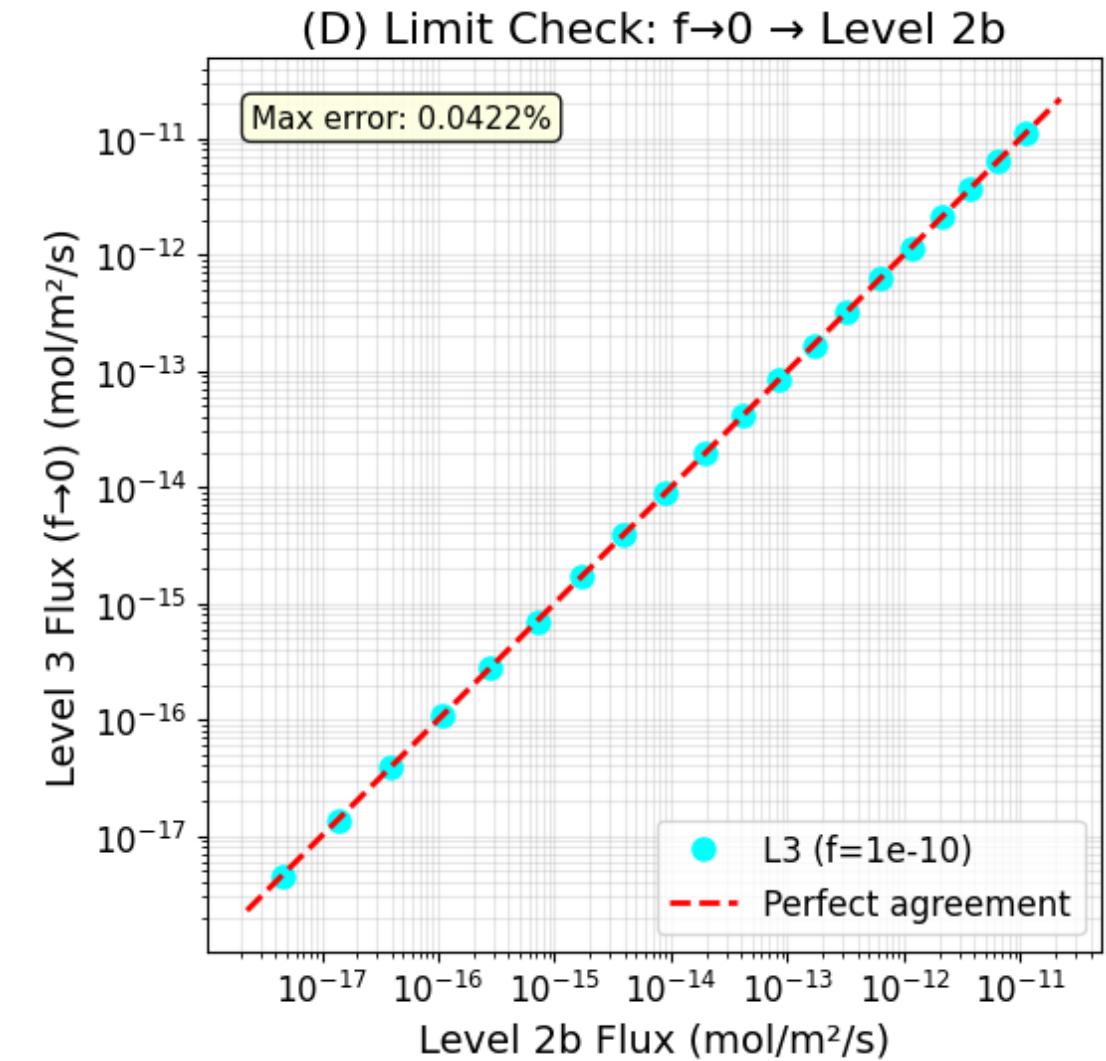
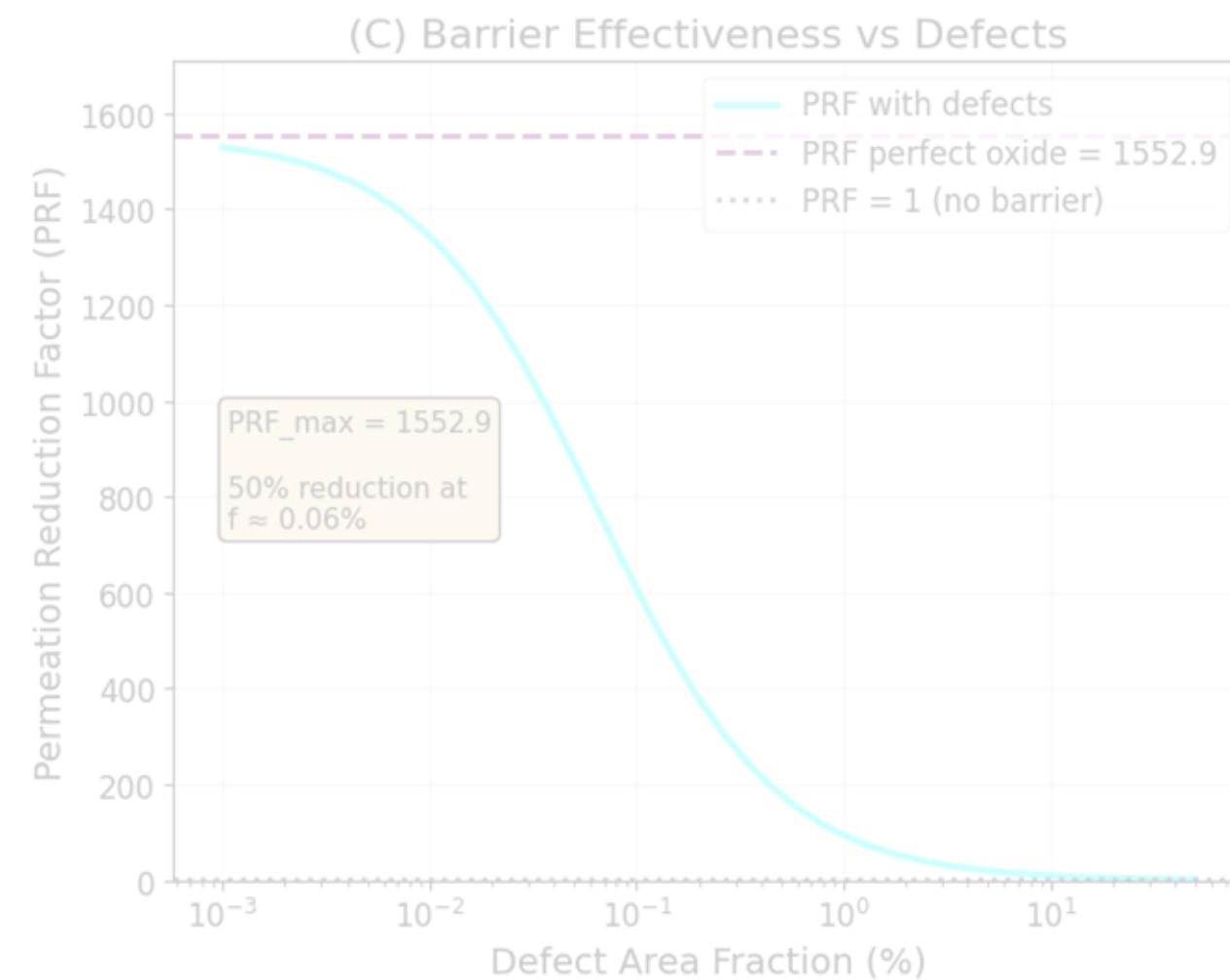
(C) Barrier Effectiveness vs Defects



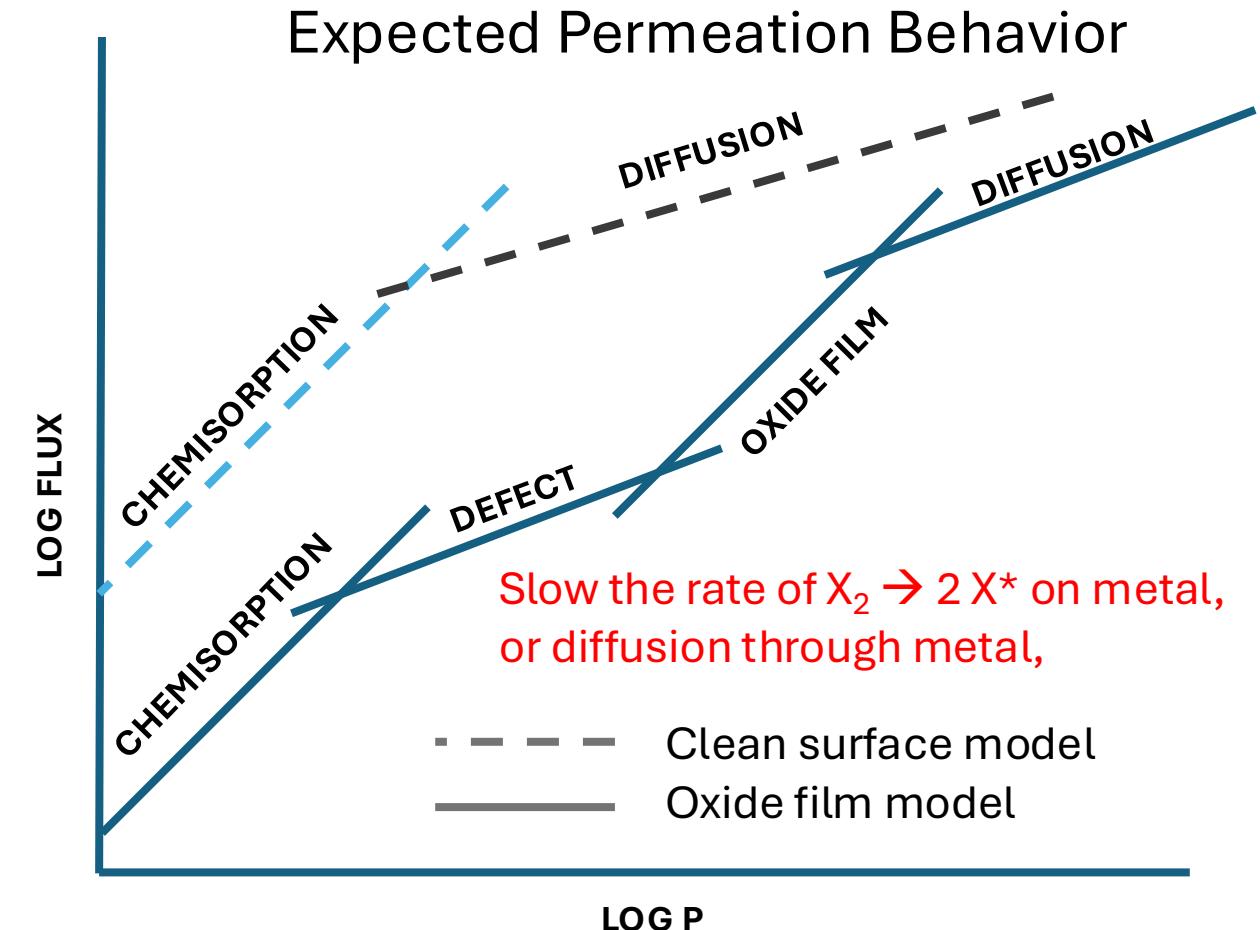
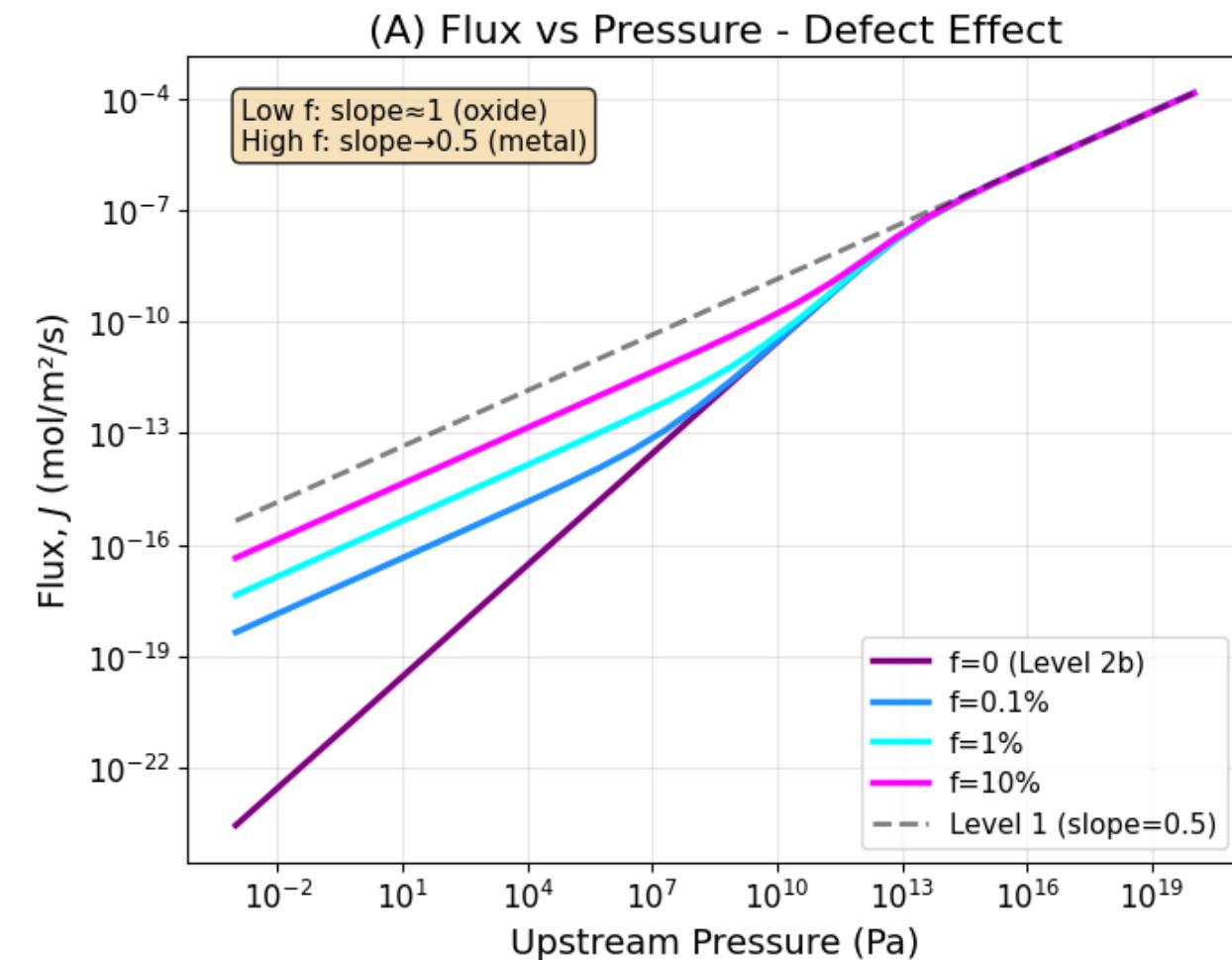
(D) Limit Check:  $f \rightarrow 0 \rightarrow$  Level 2b



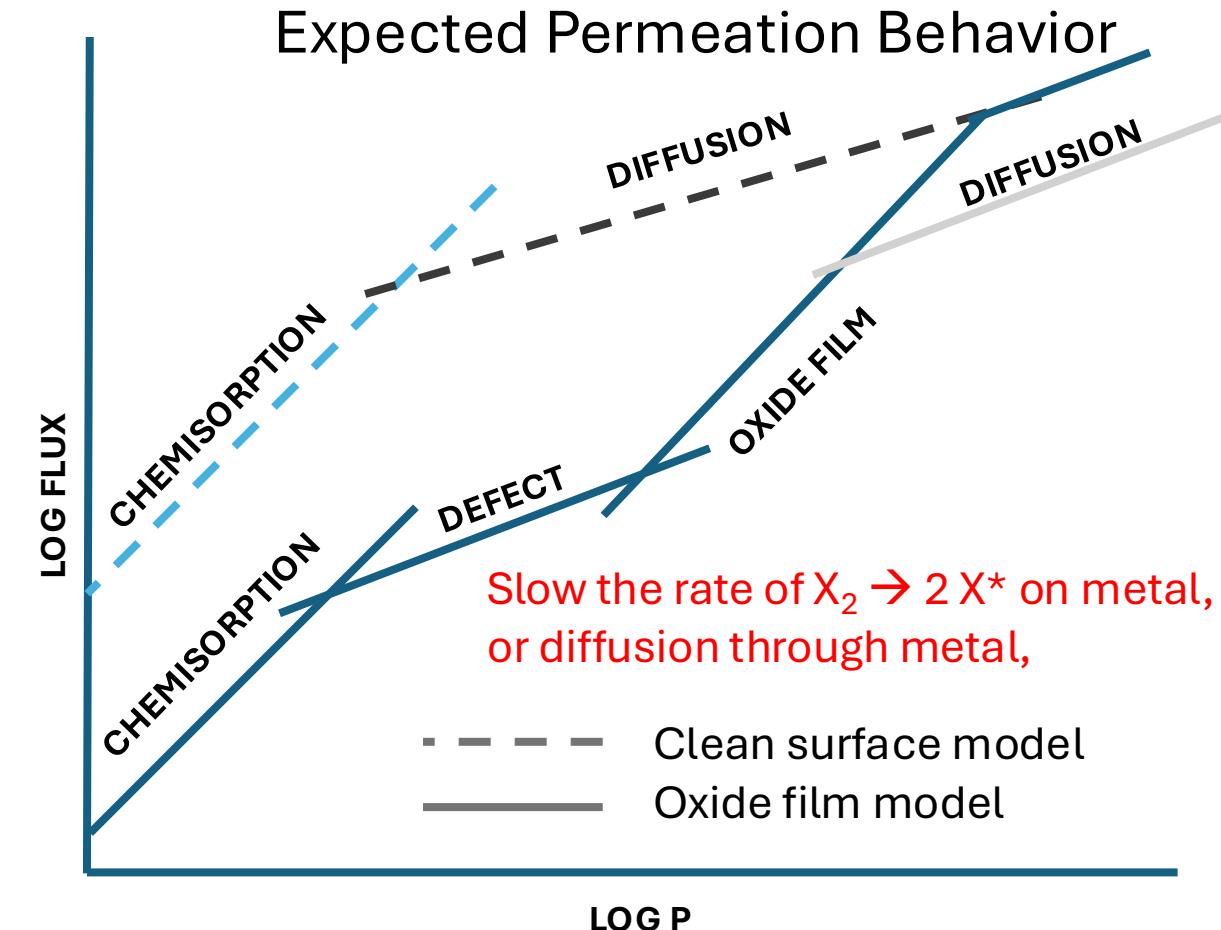
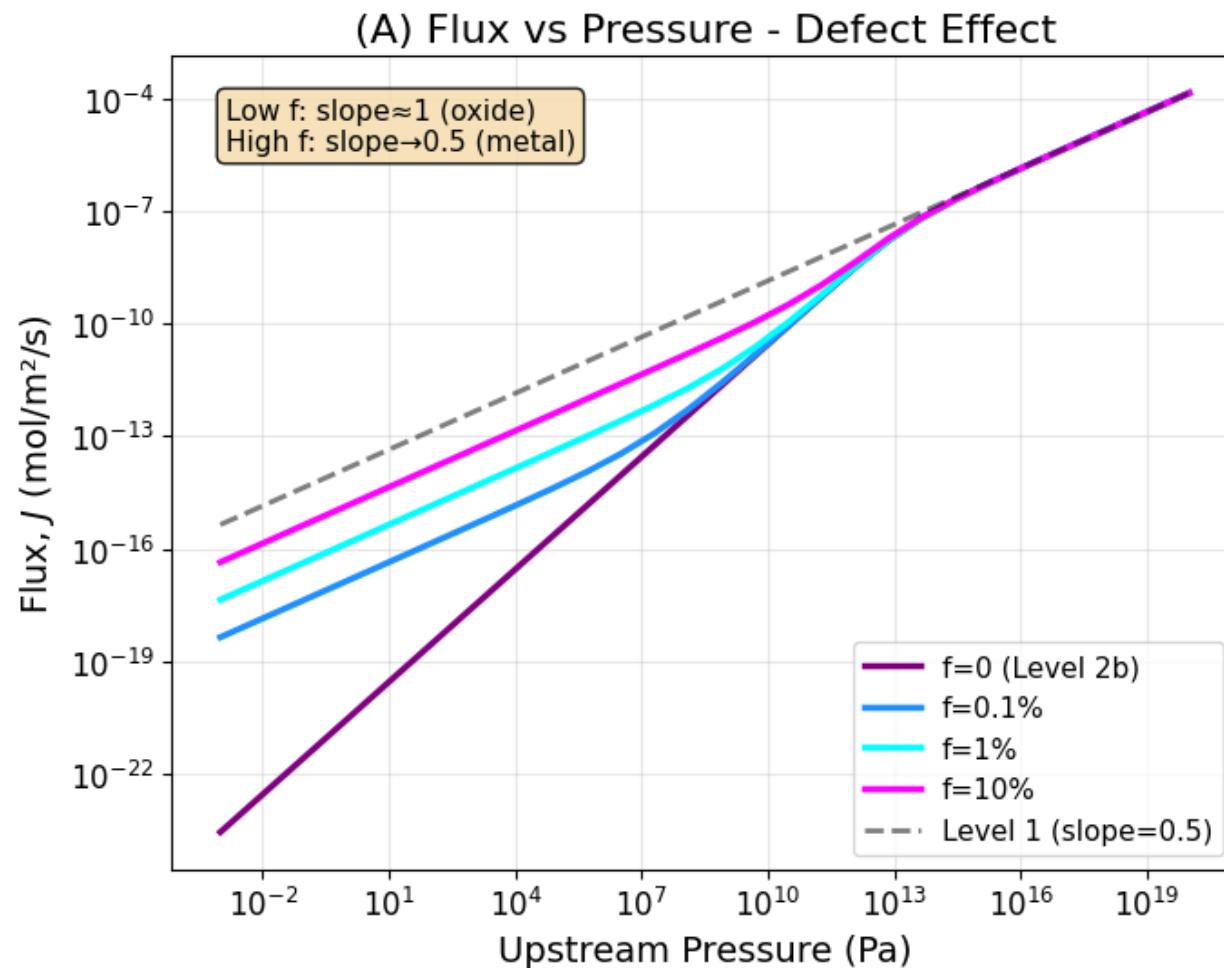
# Defective Oxide + Perfect Metal (The Parallel Path Model)



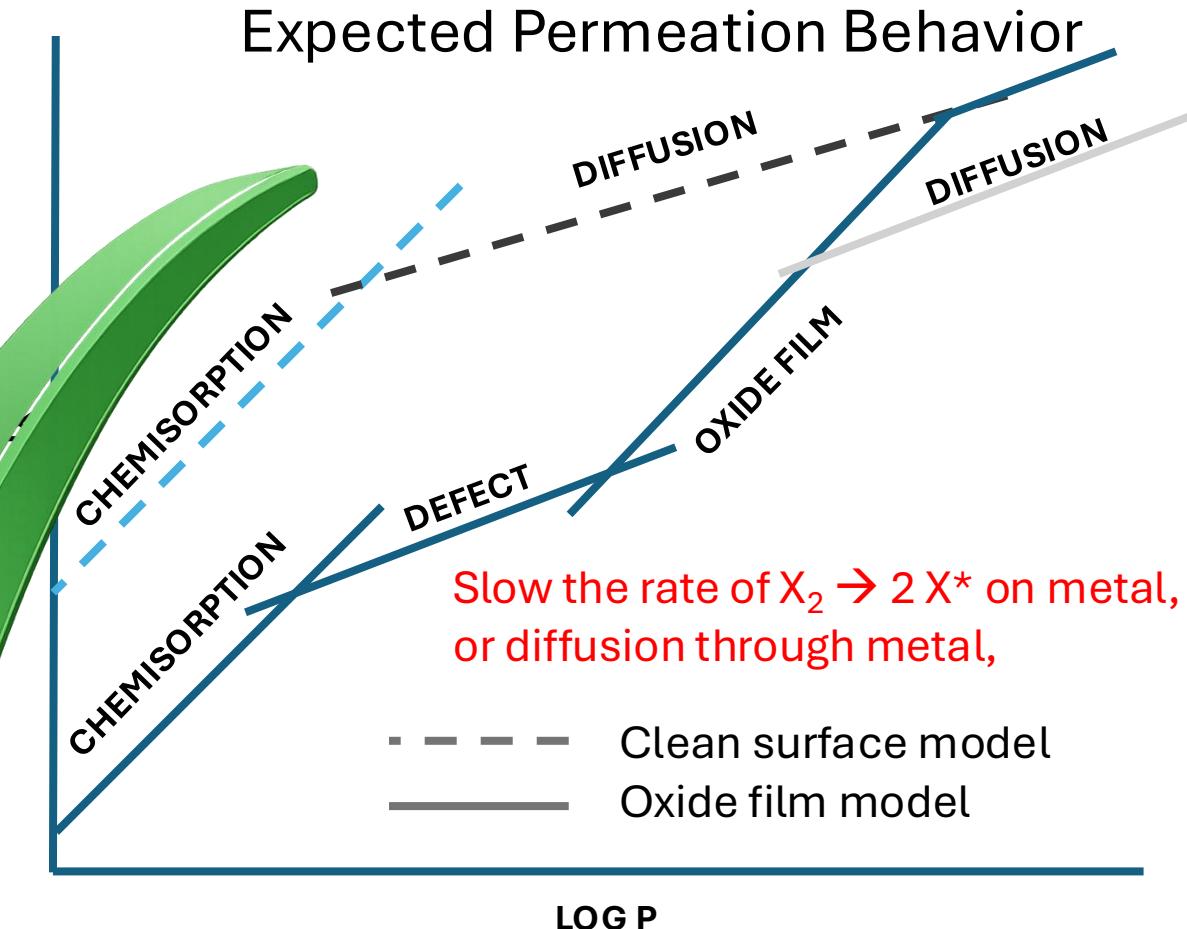
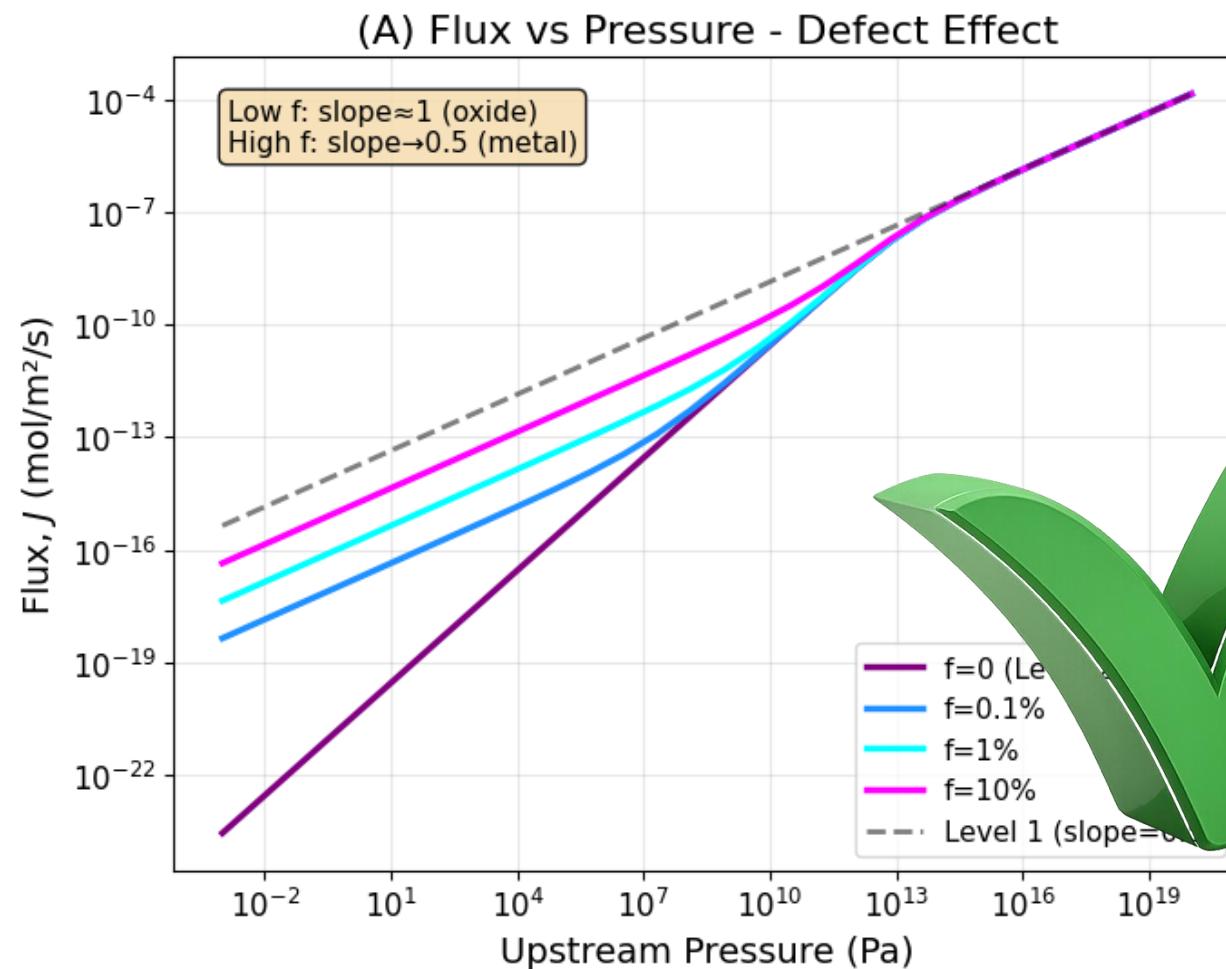
# Defective Oxide + Perfect Metal (The Parallel Path Model)



# Defective Oxide + Perfect Metal (The Parallel Path Model)



# Defective Oxide + Perfect Metal (The Parallel Path Model)



# Aim 1

Perfect Metal  
Atomic Diffusion  
(Level 1)

Perfect Oxide  
Molecular Diffusion  
(Level 2a)

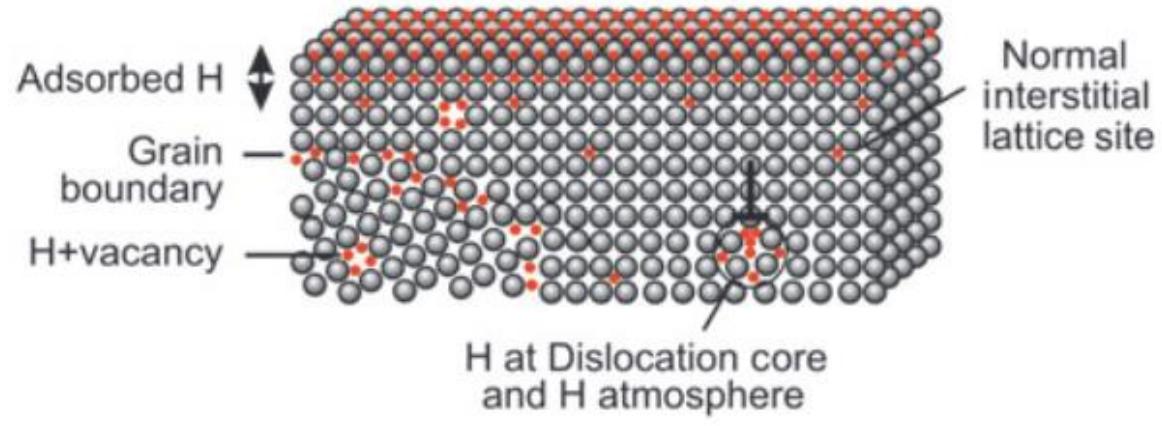
Perfect Oxide  
+ Perfect Metal  
Series Resistance  
(Level 2b)

Defective Oxide  
+ Perfect Metal  
The Parallel Path Model  
(Level 3)

Defective Metal  
(Level 4)

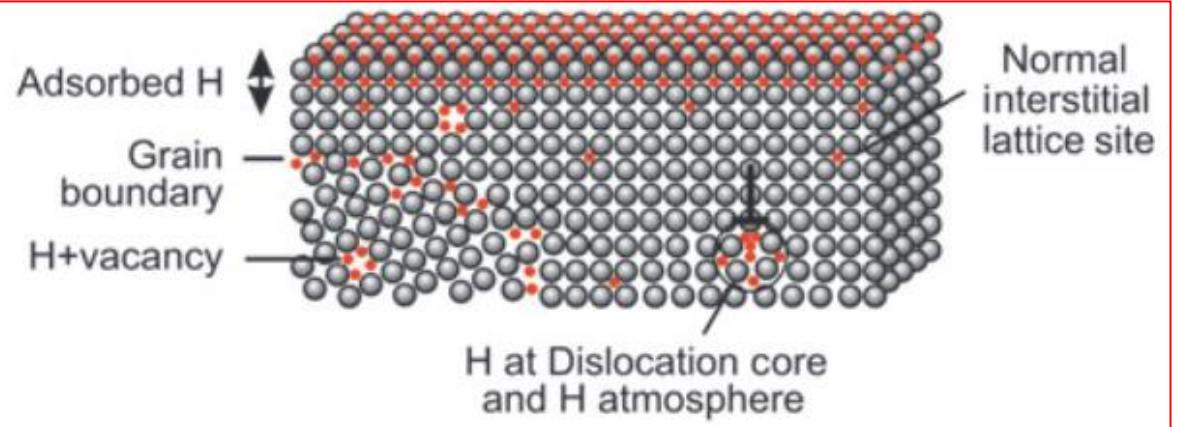
Defective Oxide  
+ Defective Metal  
(Level 5)

# Defective Metal Modeling



|

# Defective Metal Modeling



## Defective Metal

Lattice

Grain Boundaries

Traps

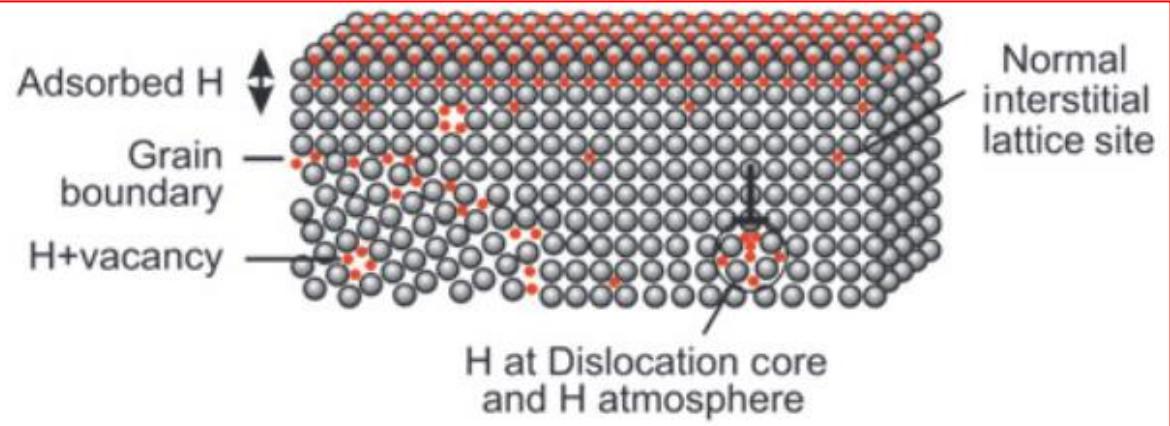
Vacancies

Dislocations

Grain Boundaries

Precipitates

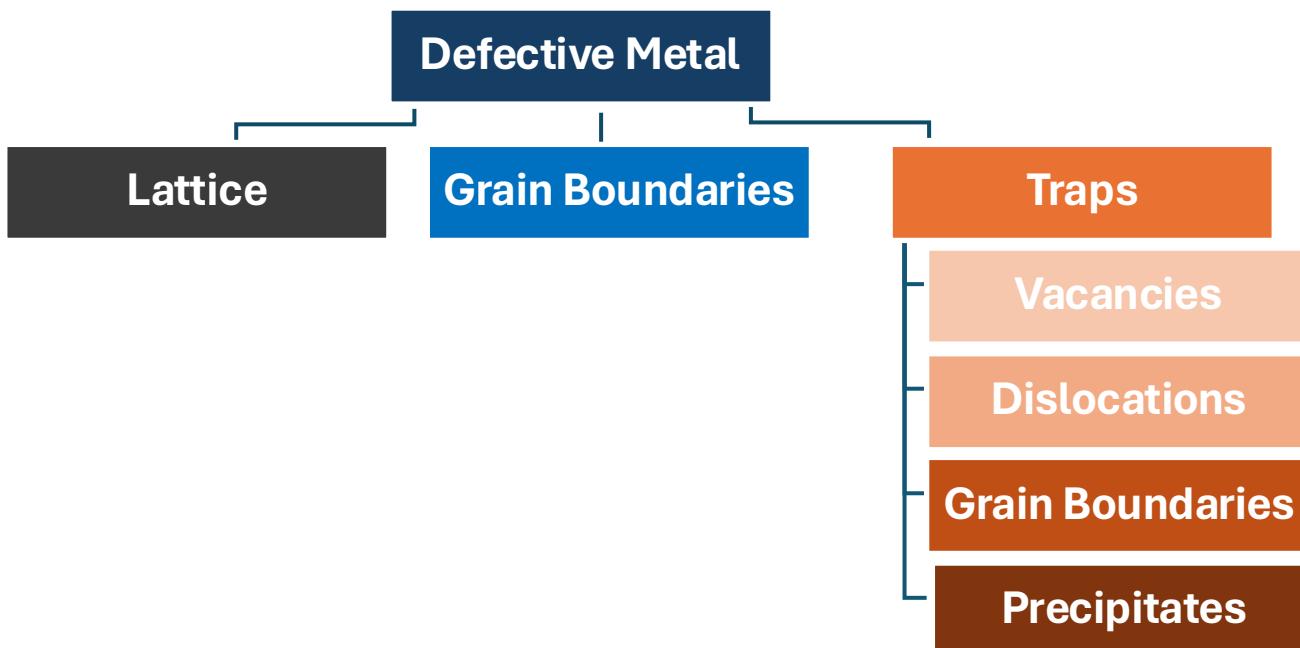
# Defective Metal Modeling



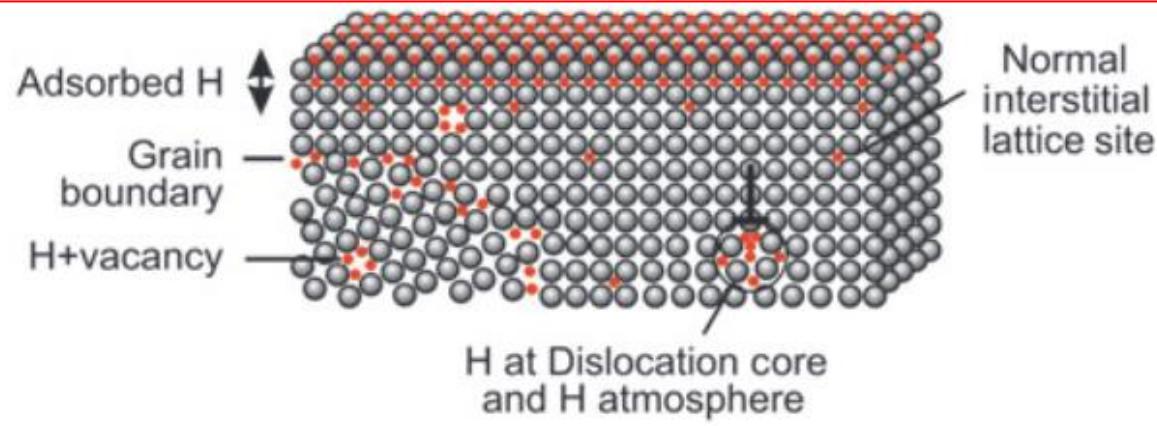
Lattice

Fick Diffusion

$$\text{Diffusivity} = D_{\text{Lattice}}$$



# Defective Metal Modeling



Lattice

Fick Diffusion

$$\text{Diffusivity} = D_{\text{Lattice}}$$

Grain Boundaries

Parallel Path Model (Hart's Equ)

$$D_{\text{eff}} = D_{\text{Lattice}}(1 - f_{\text{GB}}) + D_{\text{GB}}f_{\text{GB}}$$

$$D_{\text{GB}} \gg D_{\text{lattice}} \quad f_{\text{GB}} = \frac{3 * \text{GB thickness}}{\text{grain size}}$$

Defective Metal

Lattice

Grain Boundaries

Traps

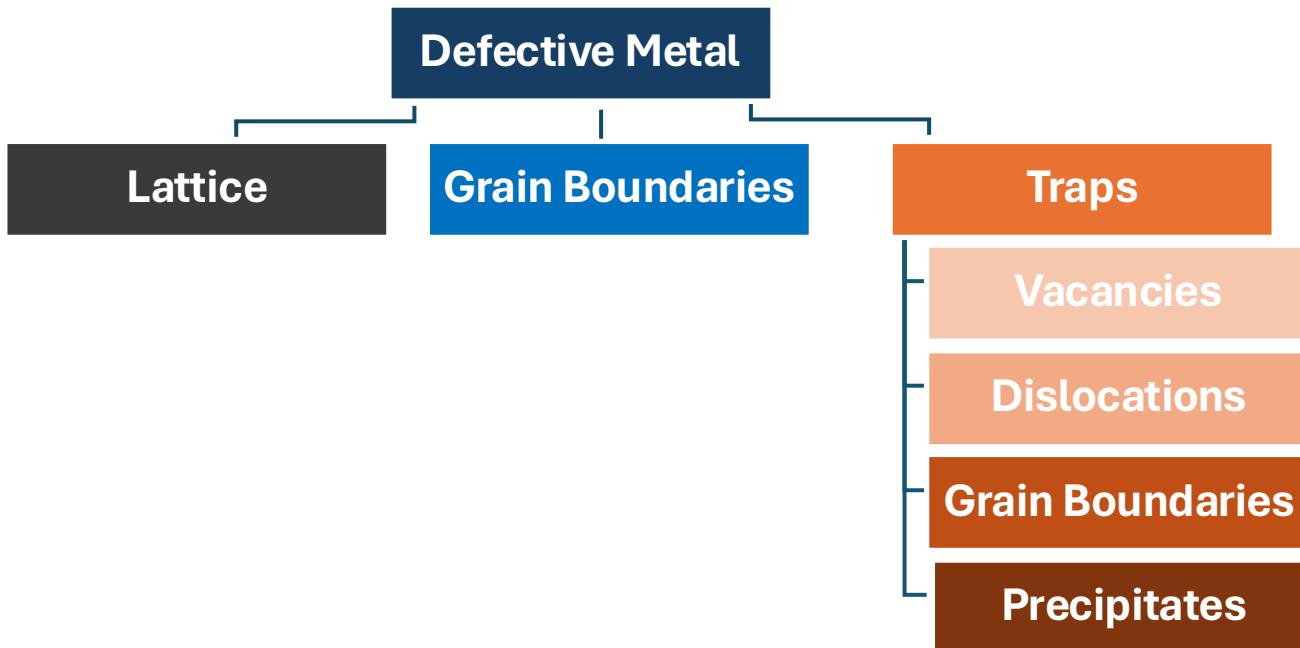
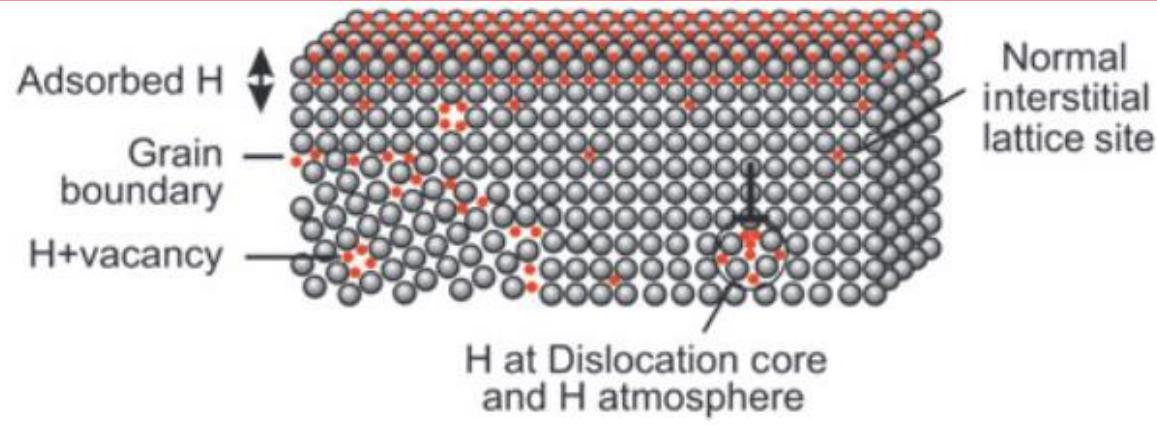
Vacancies

Dislocations

Grain Boundaries

Precipitates

# Defective Metal Modeling



Lattice

Grain Boundaries

Traps

## Fick Diffusion

$$\text{Diffusivity} = D_{\text{Lattice}}$$

## Parallel Path Model (Hart's Equ)

$$D_{\text{eff}} = D_{\text{Lattice}}(1 - f_{\text{GB}}) + D_{\text{GB}}f_{\text{GB}}$$

$$D_{\text{GB}} \gg D_{\text{lattice}} \quad f_{\text{GB}} = \frac{3 * \text{GB thickness}}{\text{grain size}}$$

Traps

$$D_{\text{eff}} = \frac{D_{\text{lattice}}}{1 + \sum \frac{N_{T\_i} K_{eq\_i}}{N_L}}$$

## Oriani Equilibrium Model

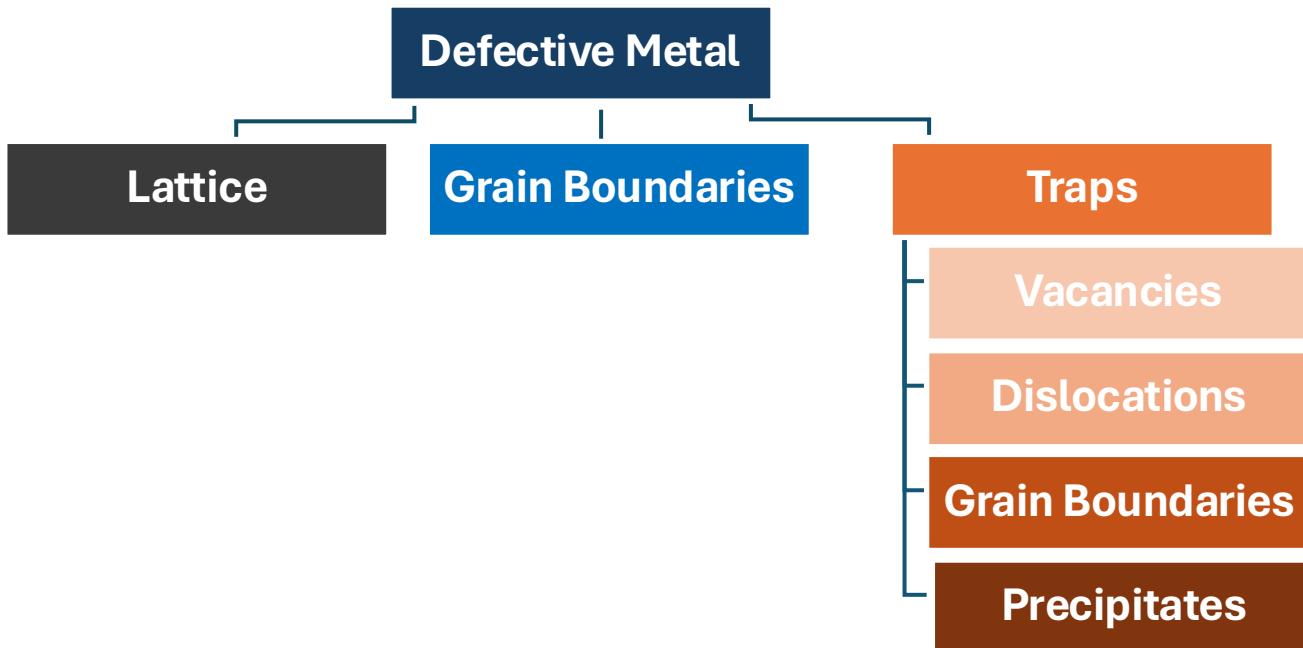
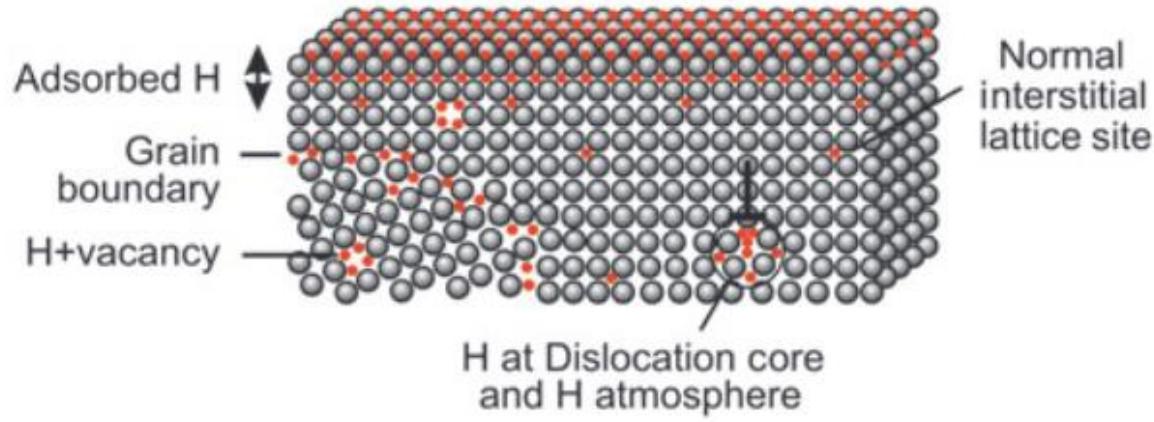
$$K_{eq\_i} = \exp\left(\frac{E_b}{RT}\right)$$

$K_{eq\_i}$  = Equilibrium constant for trap type I

$N_{T\_i}$  = Density of trap type I (trap/m<sup>3</sup>)

$N_L$  = Lattice density,  $E_b$  = Binding Energy

# Defective Metal Modeling



Lattice

Grain Boundaries

Traps

## Fick Diffusion

$$\text{Diffusivity} = D_{\text{Lattice}}$$

Parallel Path Model (Hart's Equ)

$$D_{\text{eff}} = D_{\text{Lattice}}(1 - f_{\text{GB}}) + D_{\text{GB}}f_{\text{GB}}$$

$$D_{\text{GB}} \gg D_{\text{lattice}} \quad f_{\text{GB}} = \frac{3 * \text{GB thickness}}{\text{grain size}}$$

Oriani Equilibrium Model

$$D_{\text{eff}} = \frac{D_{\text{lattice}}}{1 + \sum \frac{N_{T\_i} K_{eq\_i}}{N_L}}$$

$K_{eq\_i}$  = Equilibrium constant for trap type I

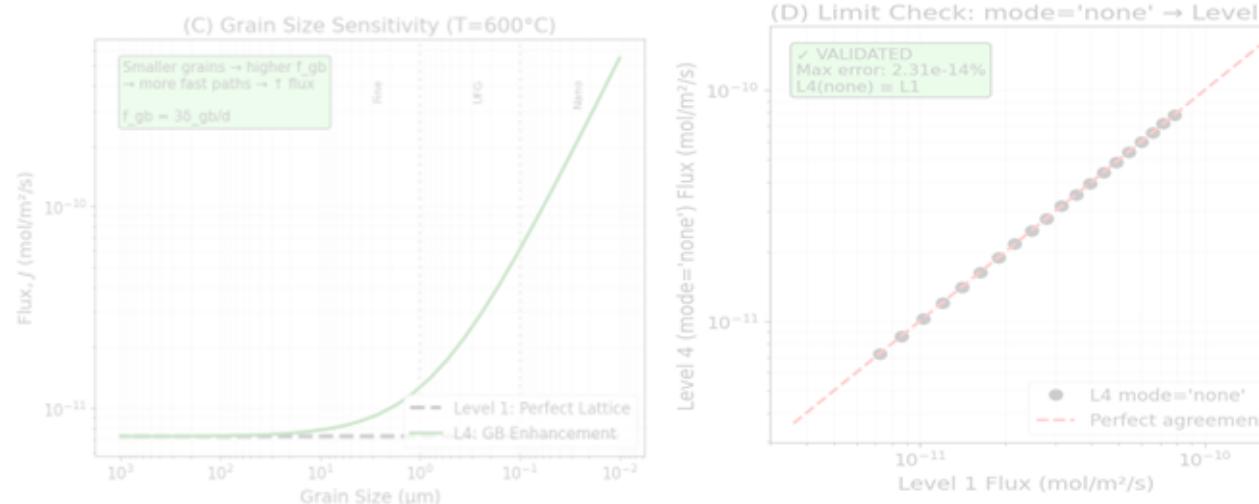
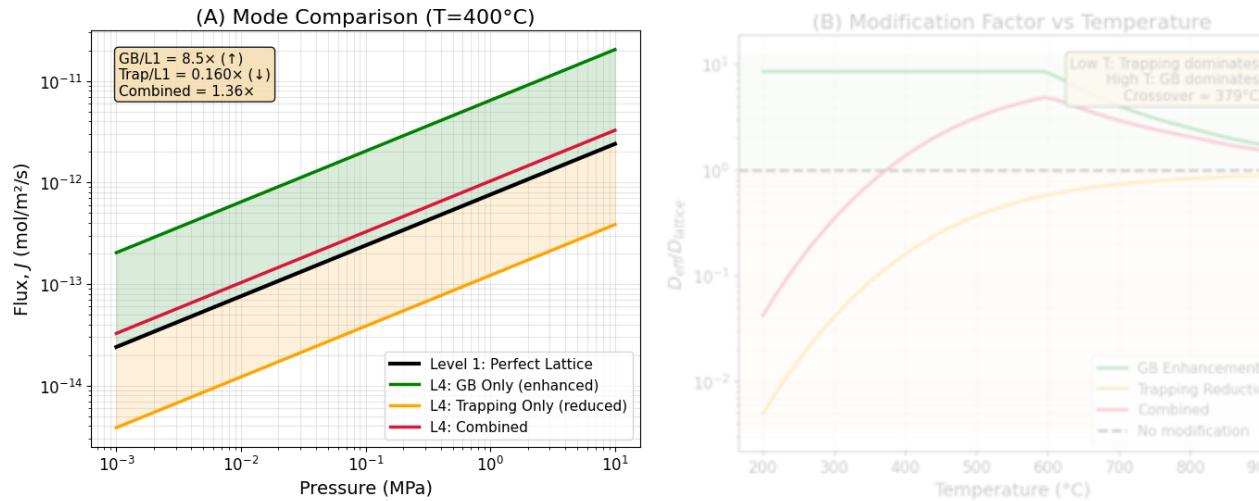
$N_{T\_i}$  = Density of trap type I (trap/m<sup>3</sup>)

$N_L$  = Lattice density,  $E_b$  = Binding Energy

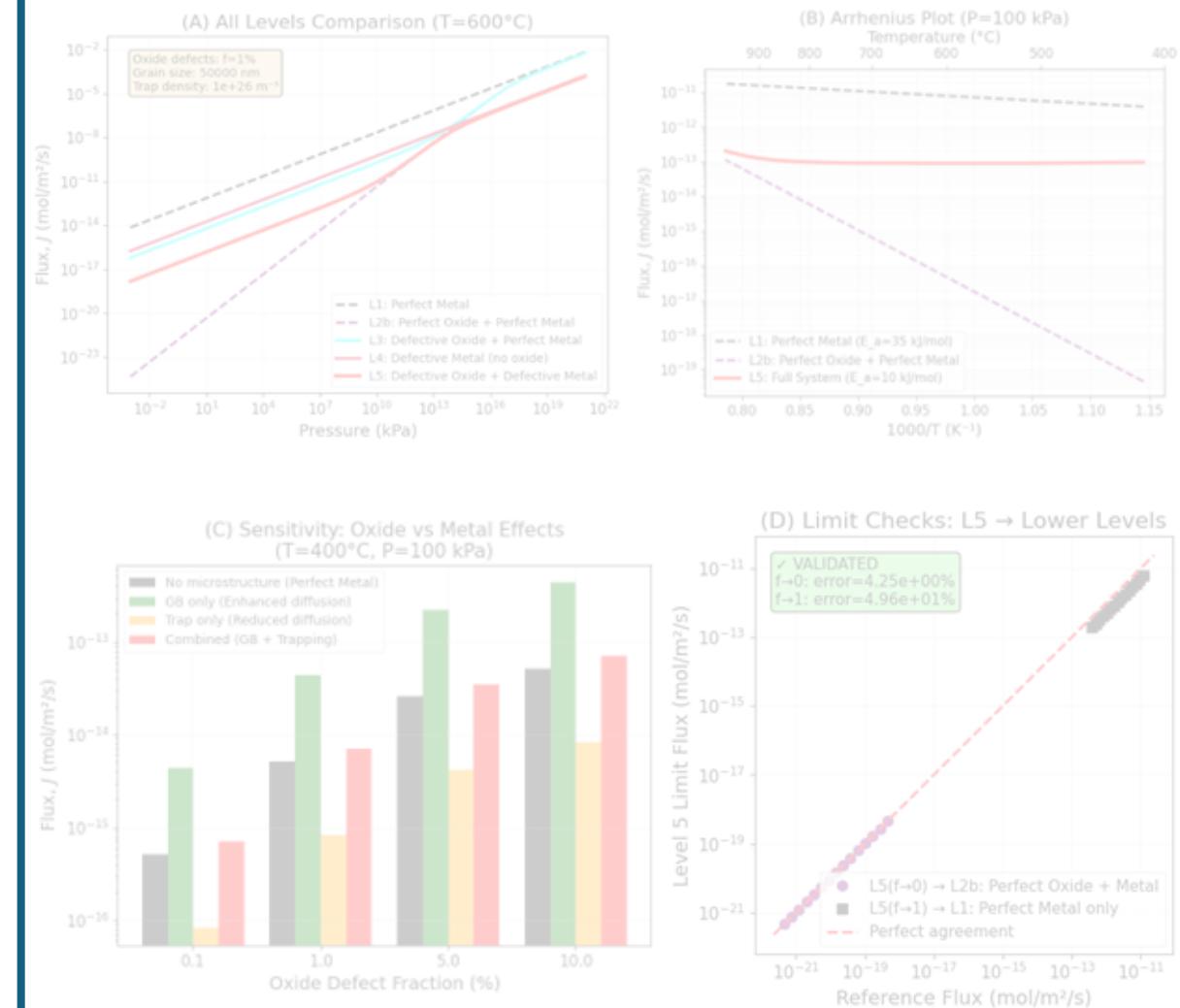
Combined:  $D_{\text{eff}} = \frac{D_{\text{lattice}} * \text{GB enhancement factor}}{1 + \text{trapping Factor}}$

# Defective Metal Shows Competing Effect of Grain Boundaries and Trappings

## Level 4: Defective Metal (GB Enhancement + Trapping)

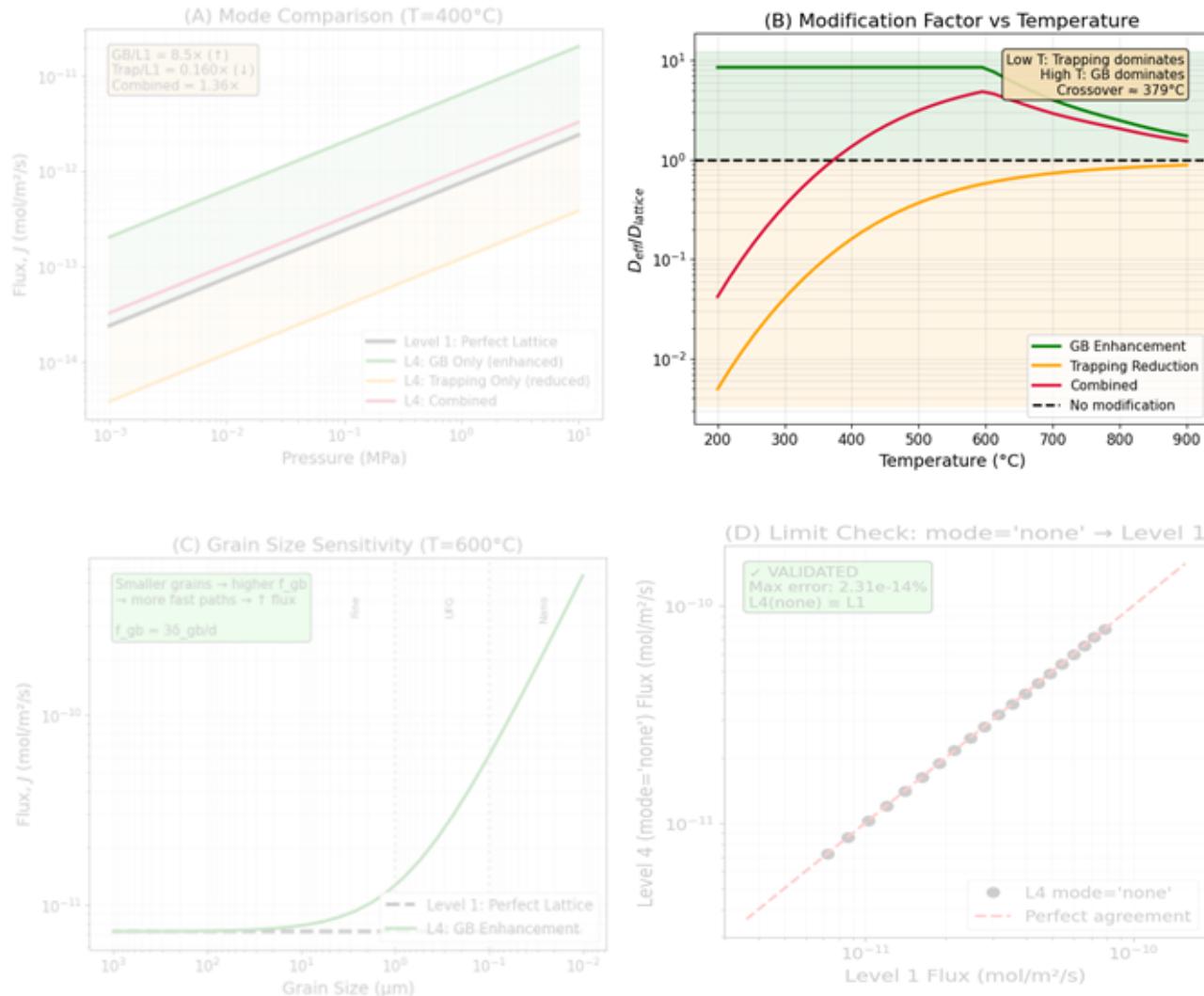


## Level 5: Full System (Defective Oxide + Defective Metal)

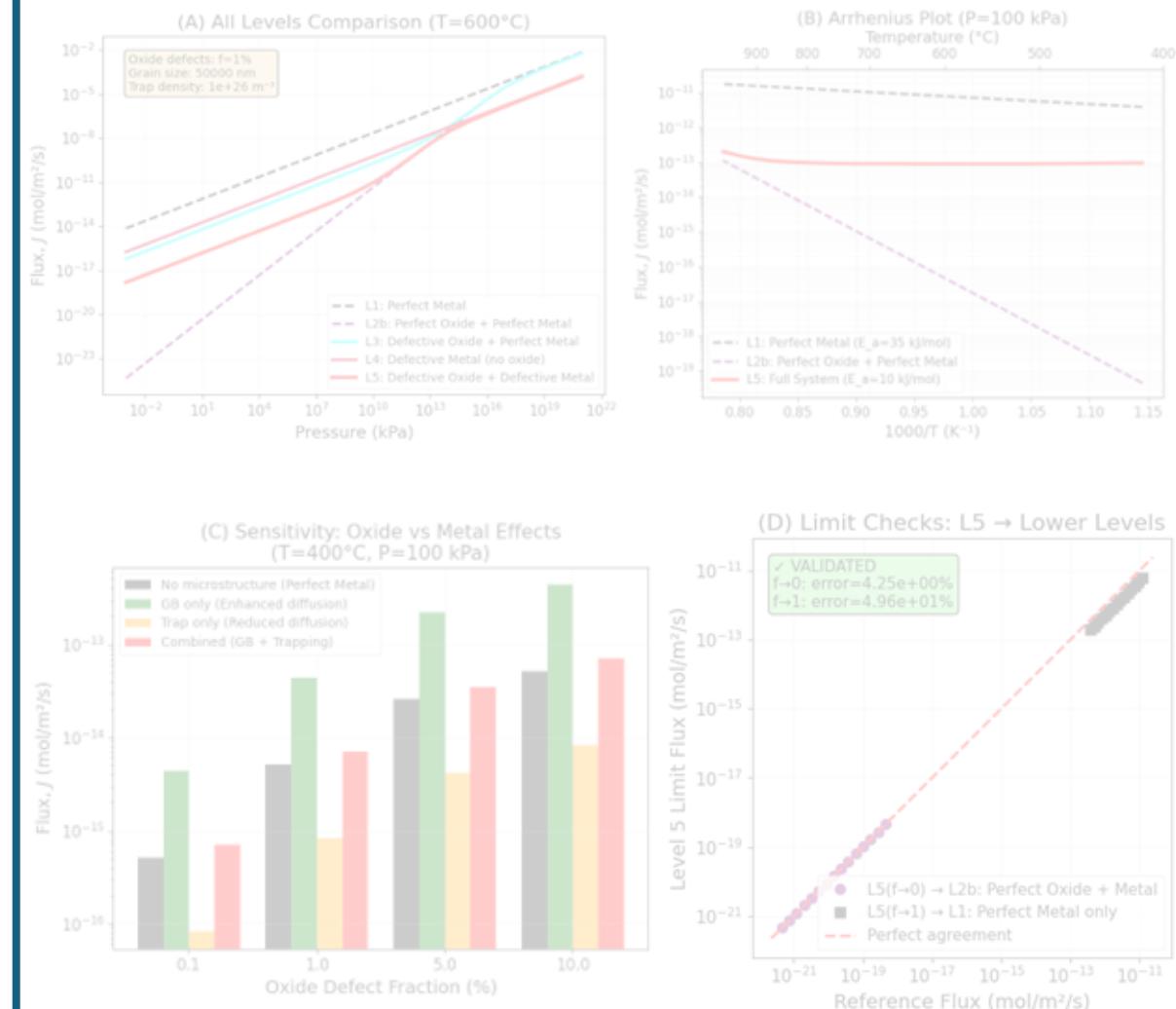


# Defective Metal Shows Competing Effect of Grain Boundaries and Trappings

## Level 4: Defective Metal (GB Enhancement + Trapping)

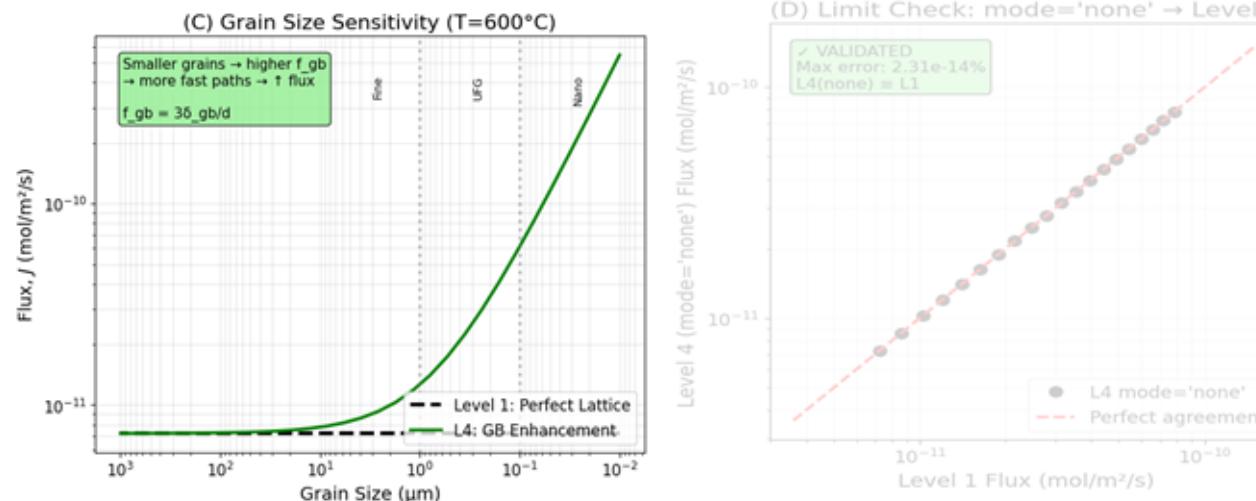
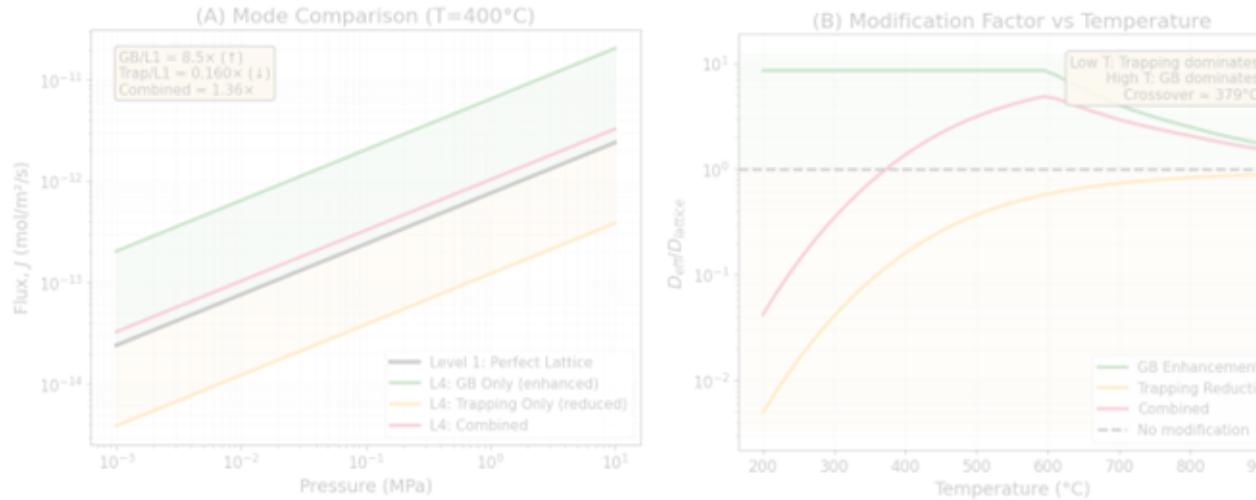


## Level 5: Full System (Defective Oxide + Defective Metal)

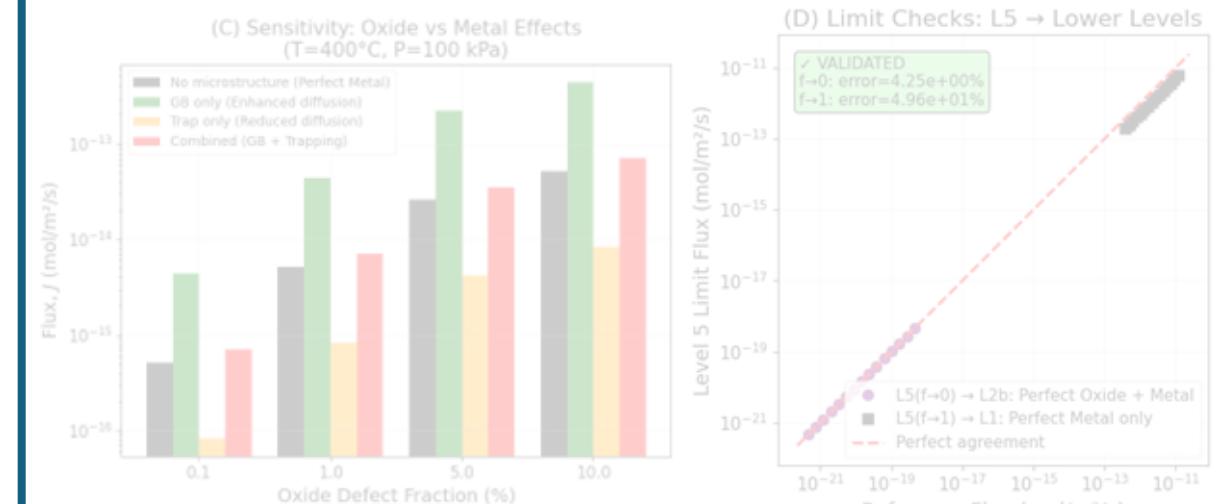
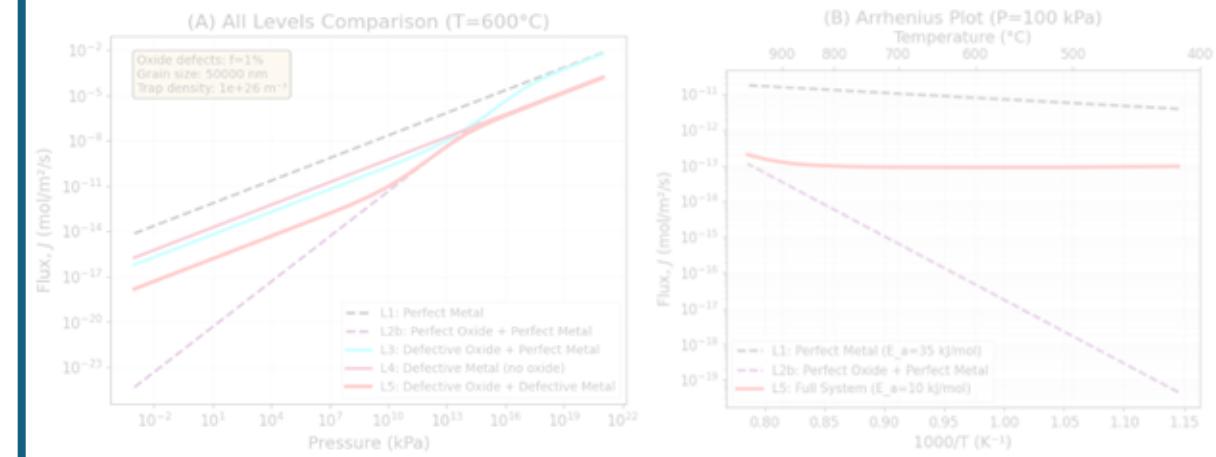


# Defective Metal Shows Competing Effect of Grain Boundaries and Trappings

## Level 4: Defective Metal (GB Enhancement + Trapping)

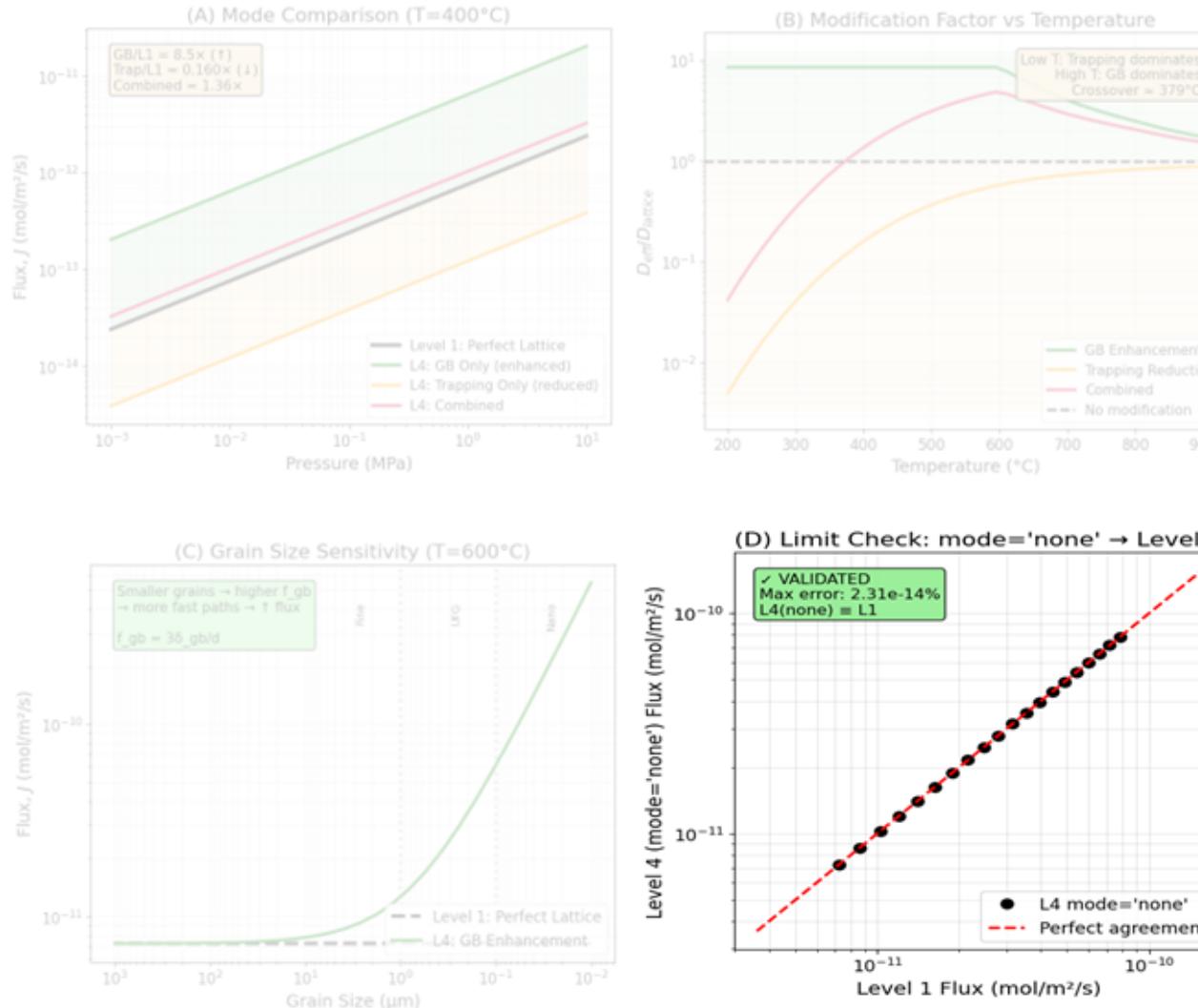


## Level 5: Full System (Defective Oxide + Defective Metal)

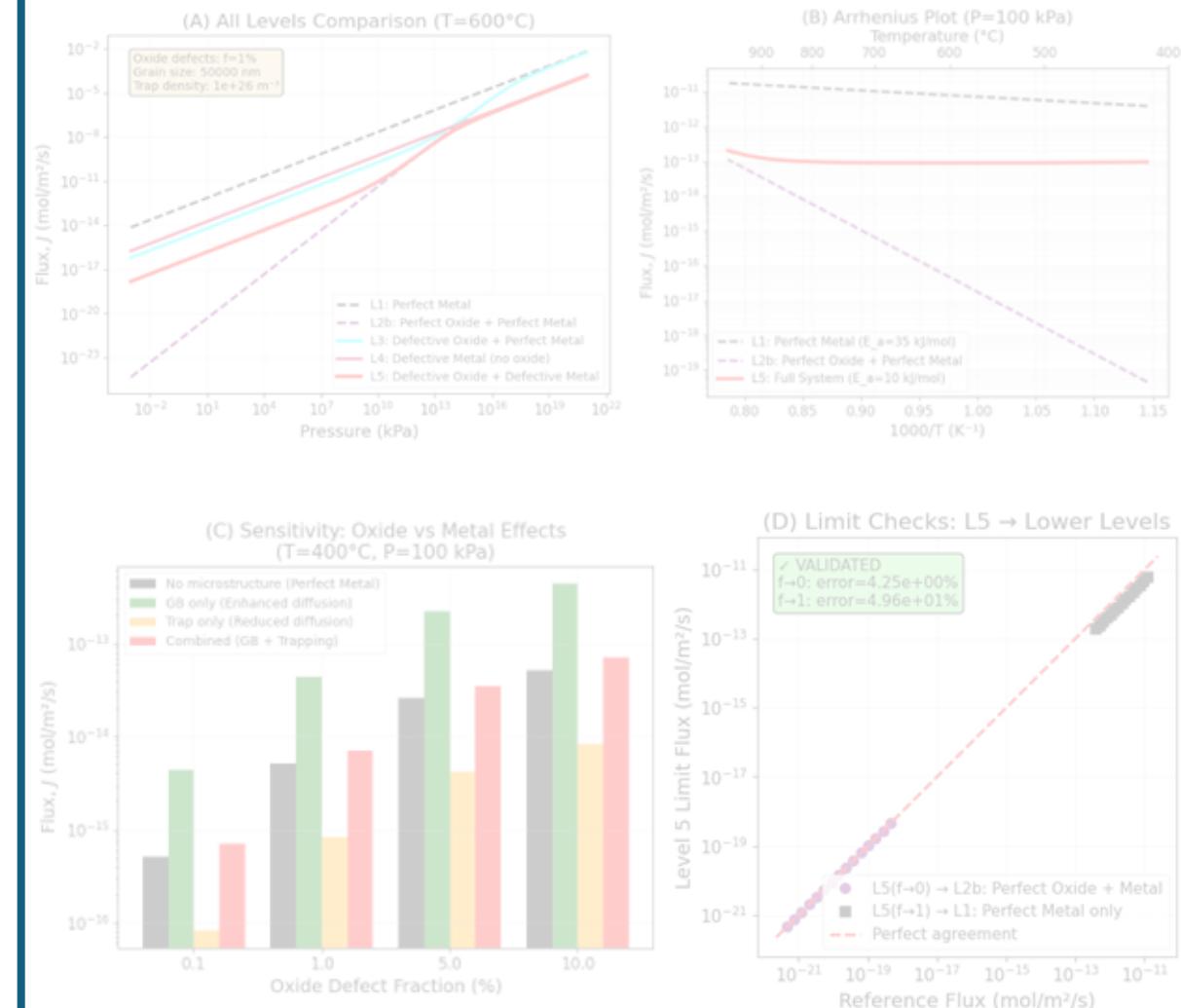


# Defective Metal Shows Competing Effect of Grain Boundaries and Trappings

## Level 4: Defective Metal (GB Enhancement + Trapping)

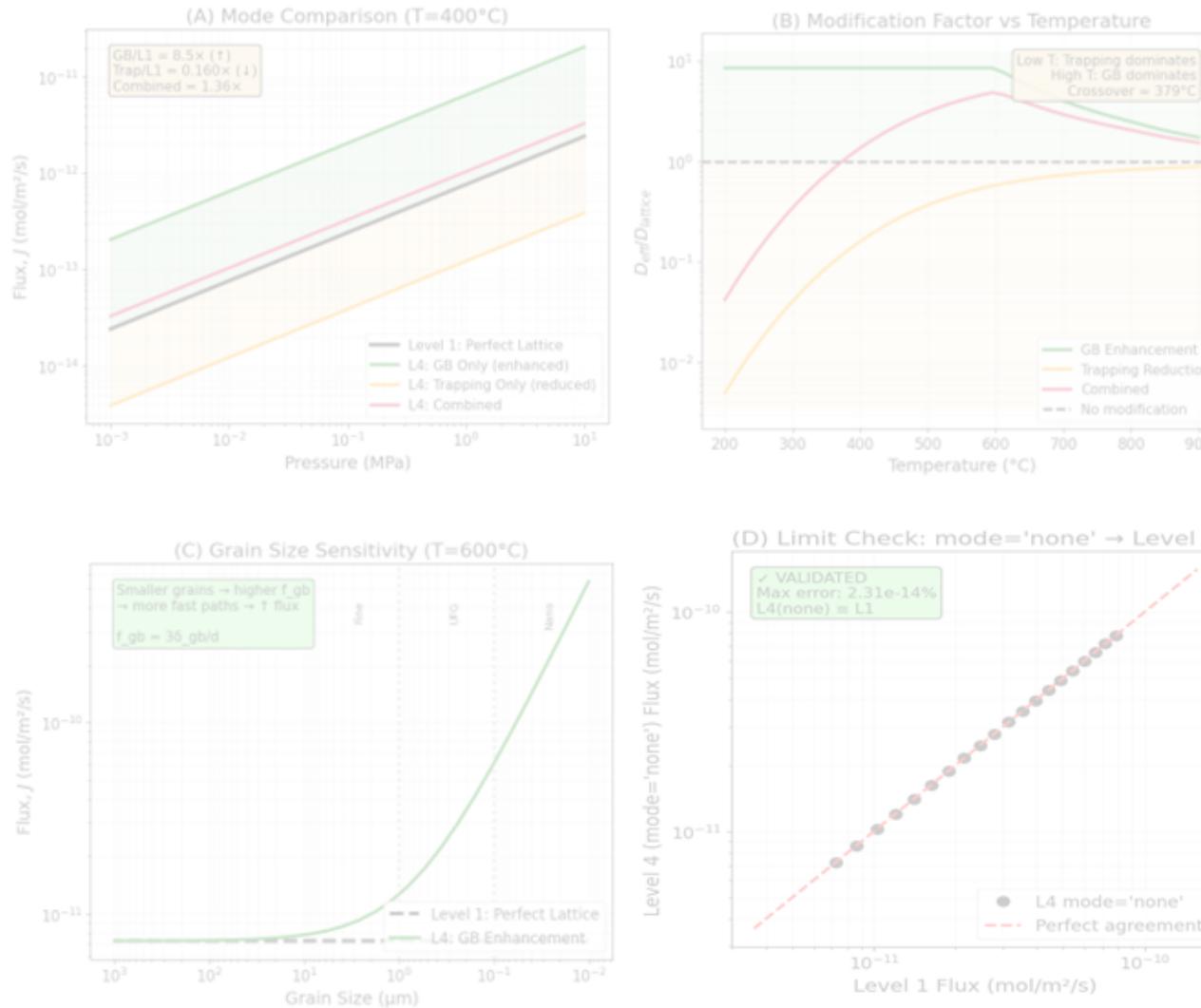


## Level 5: Full System (Defective Oxide + Defective Metal)

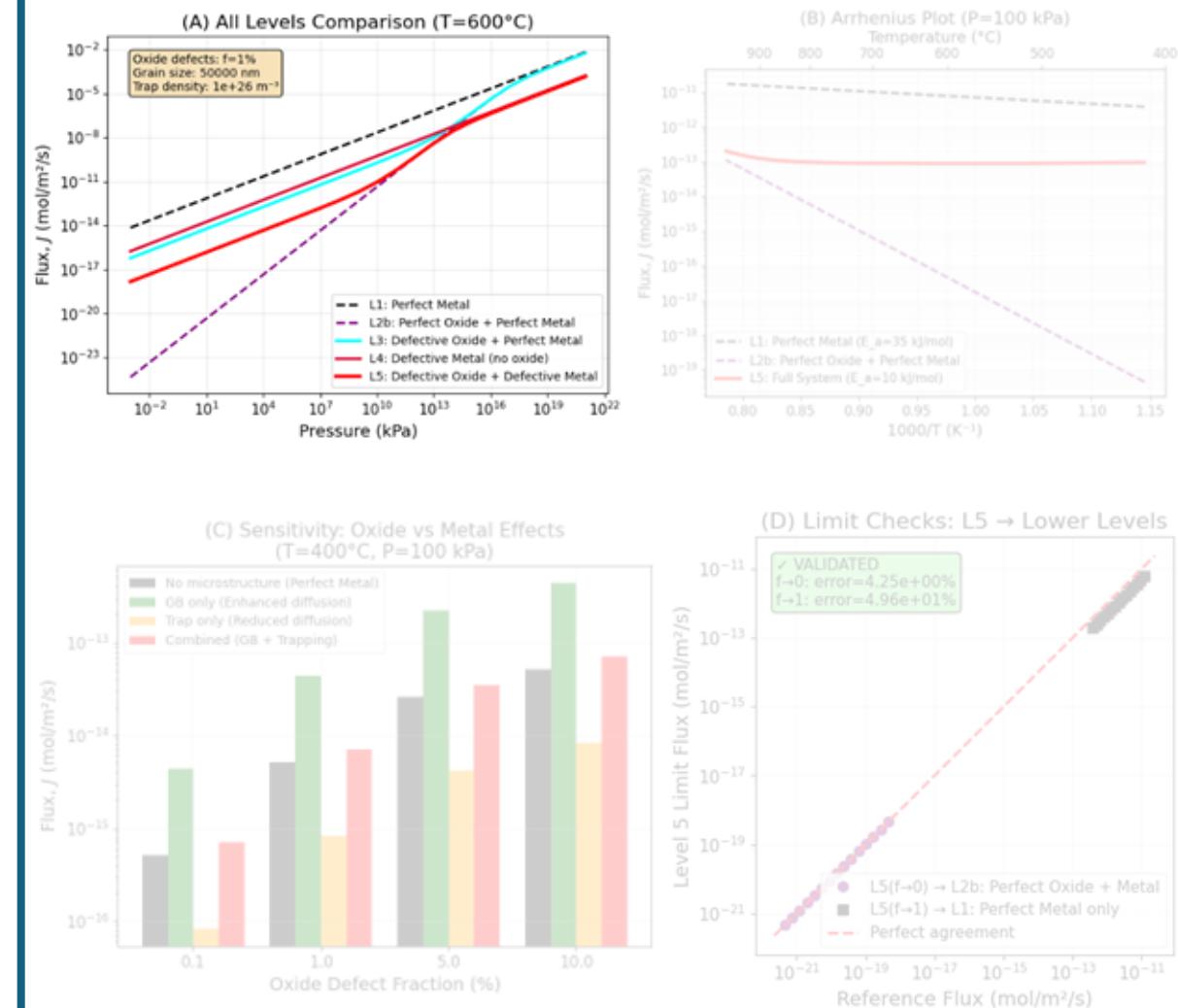


# Full System Validated Effect of Oxides and Microstructure

## Level 4: Defective Metal (GB Enhancement + Trapping)

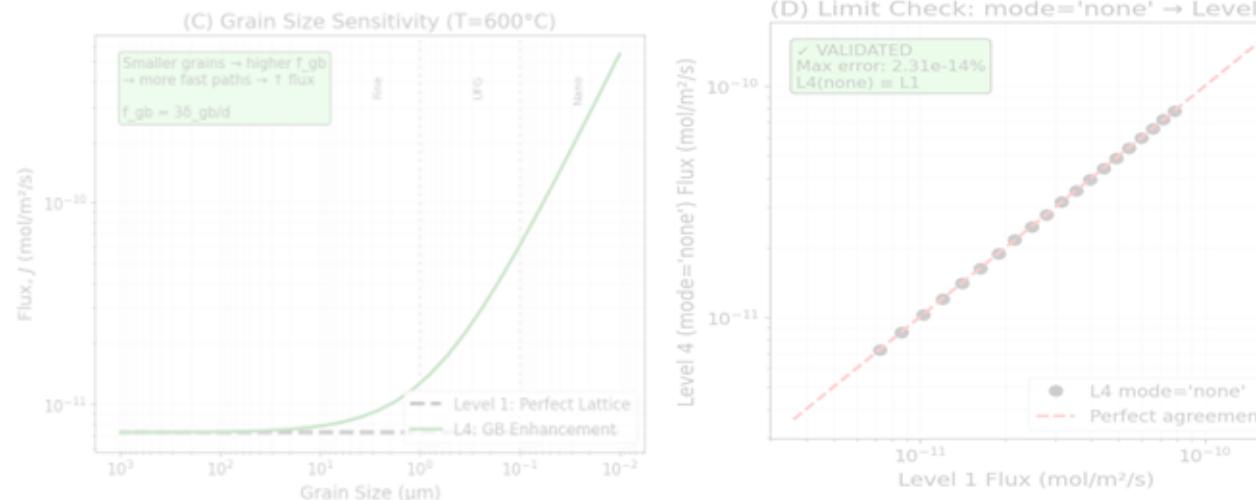
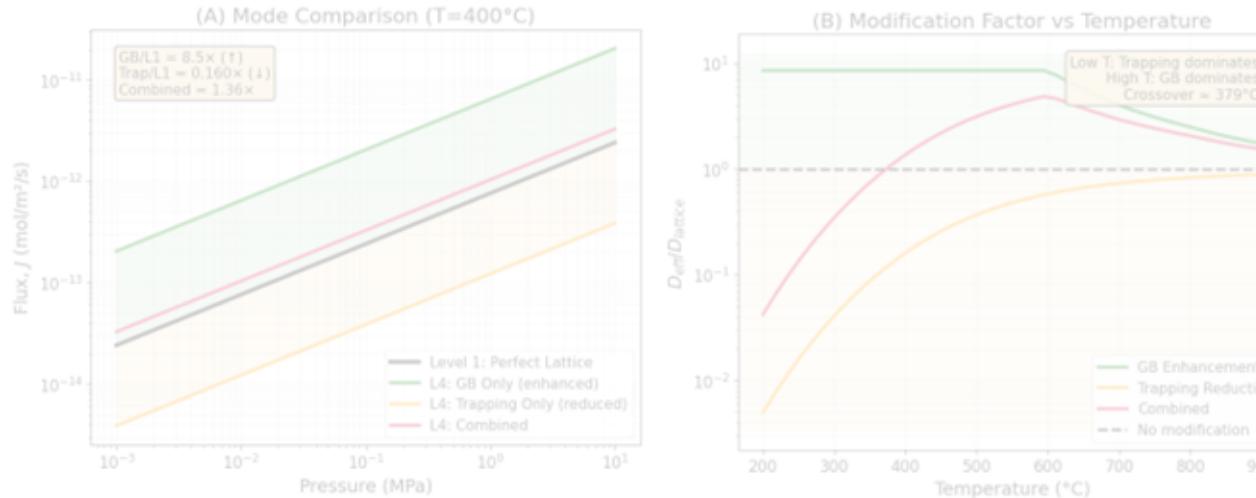


## Level 5: Full System (Defective Oxide + Defective Metal)

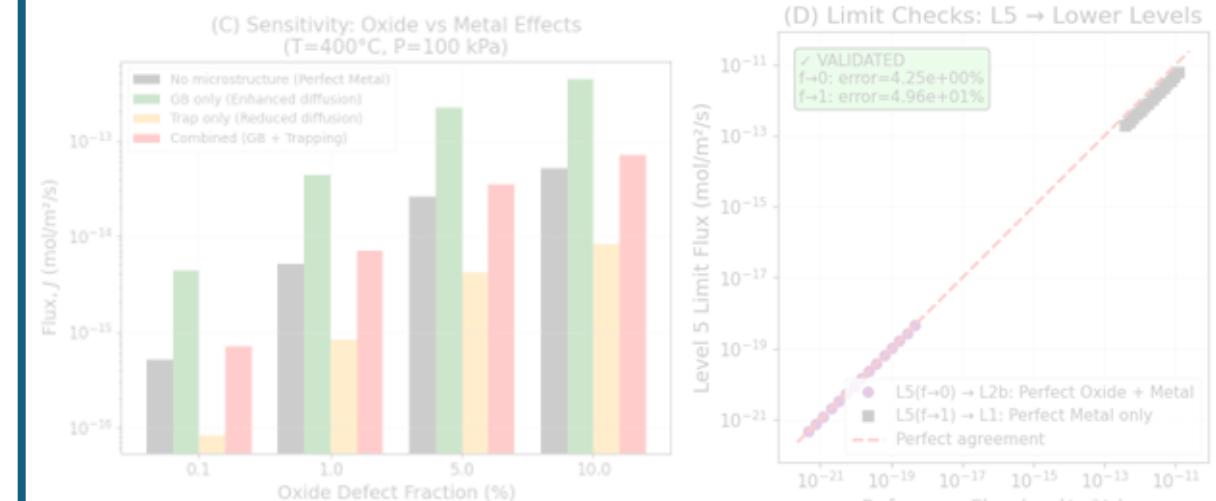
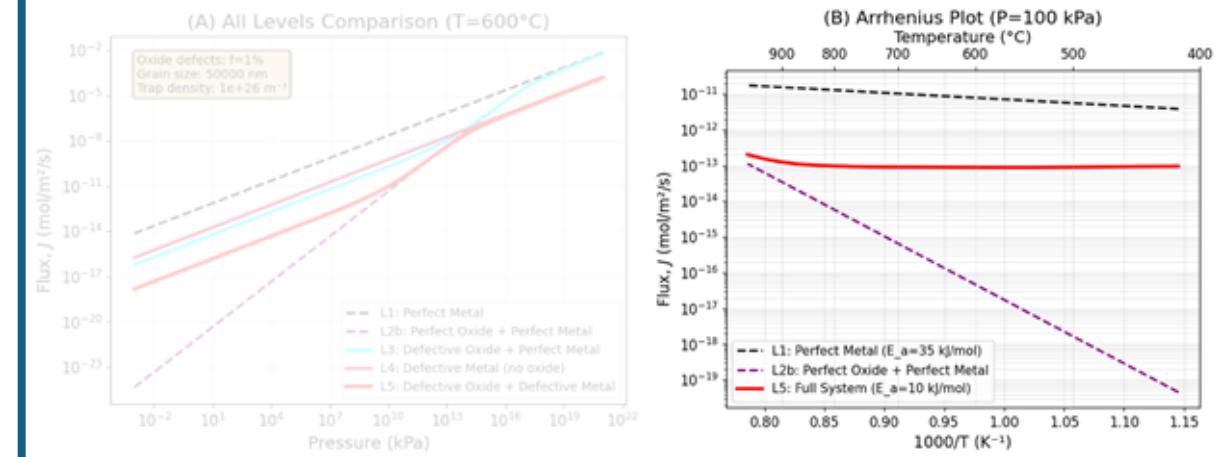


# Full System Validated Effect of Oxides and Microstructure

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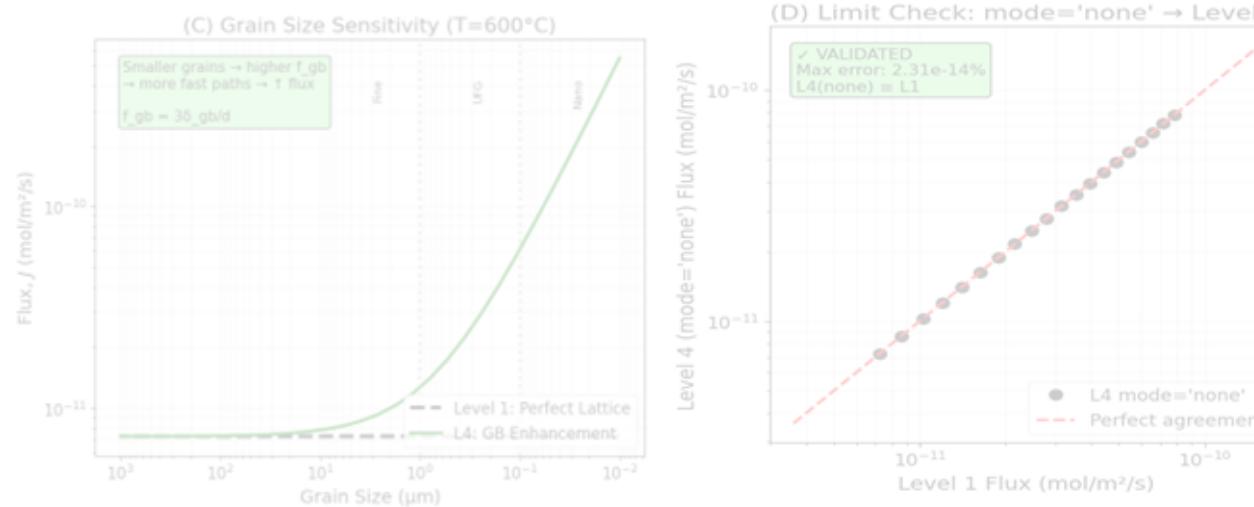
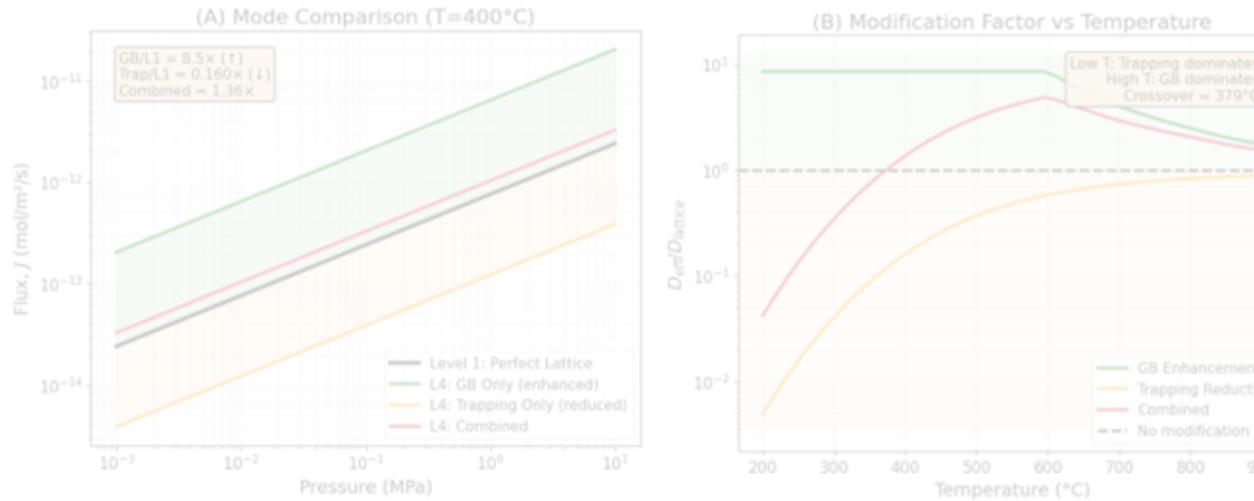


## Level 5: Full System (Defective Oxide + Defective Metal)

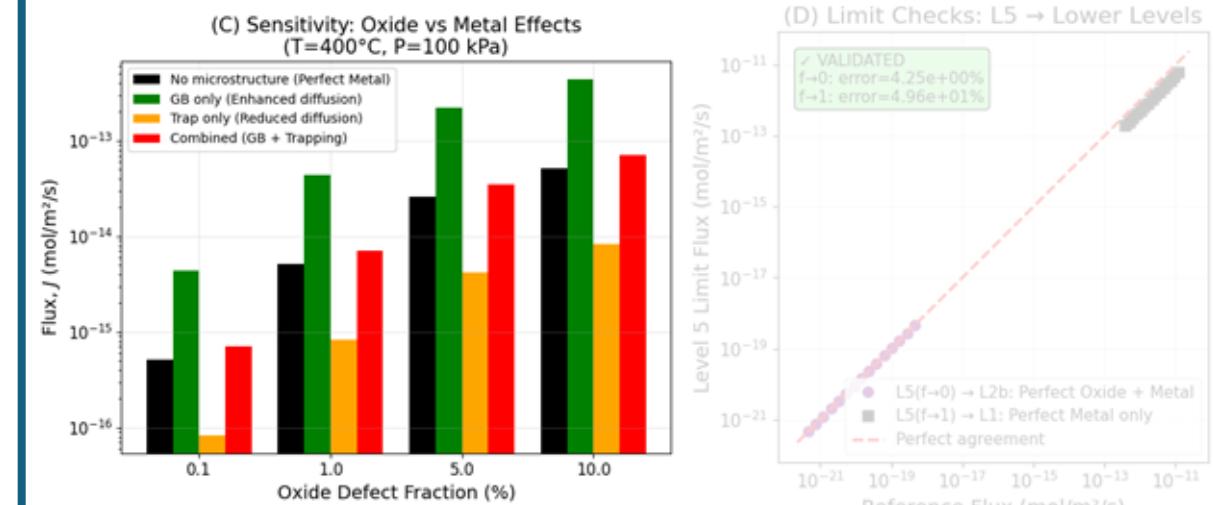
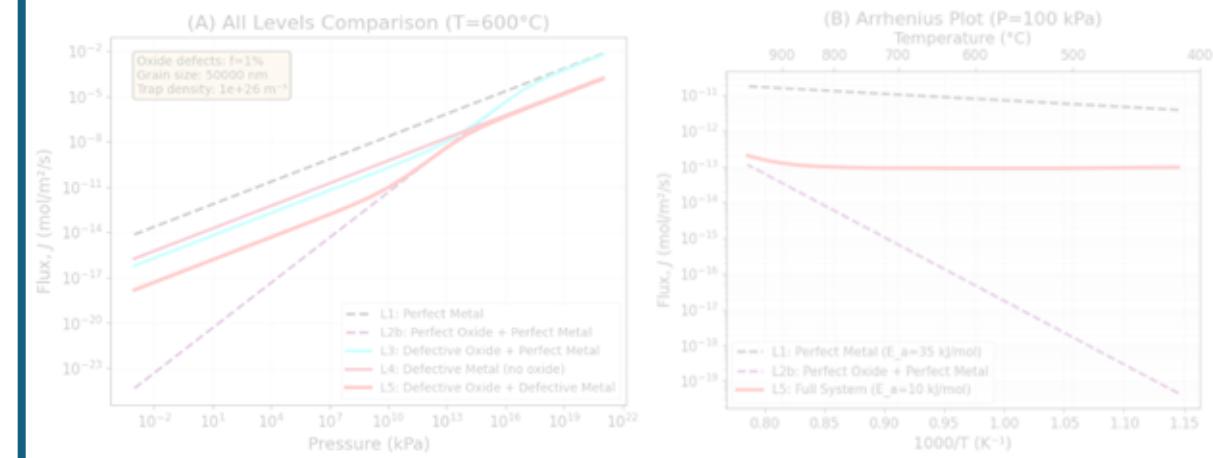


# Full System Validated Effect of Oxides and Microstructure

## Level 4: Defective Metal (GB Enhancement + Trapping)

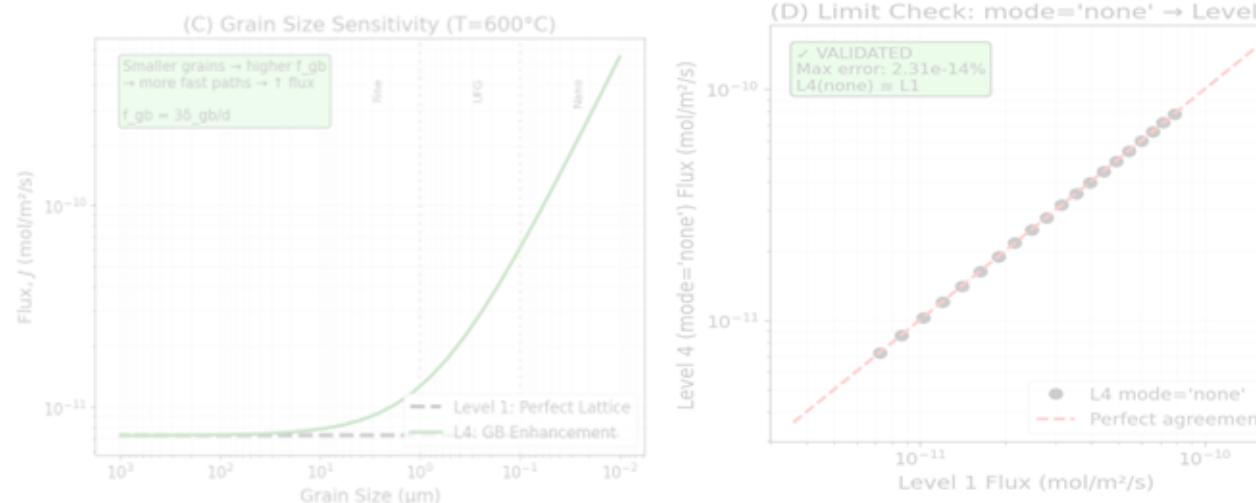
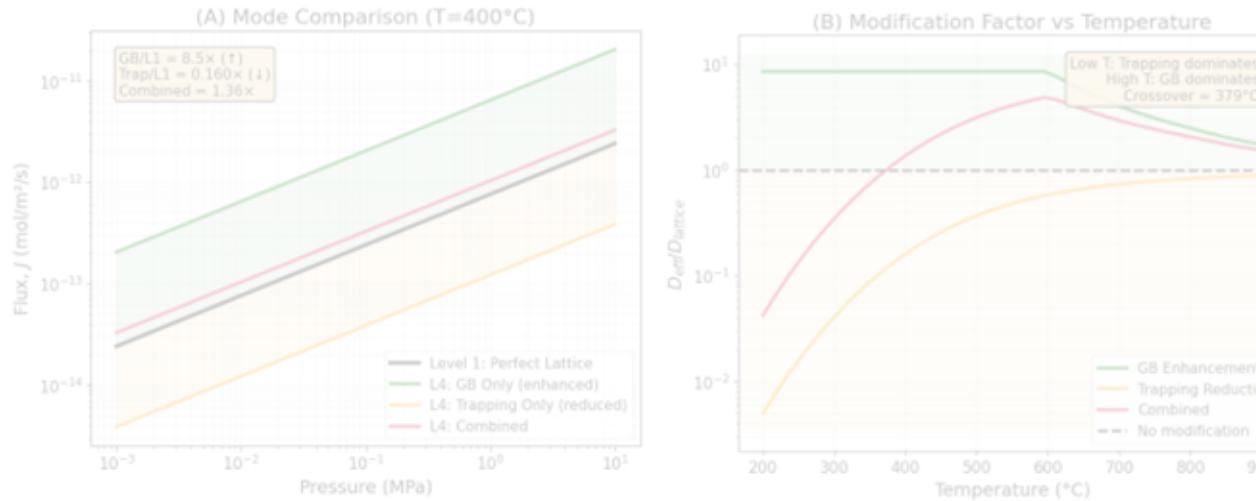


## Level 5: Full System (Defective Oxide + Defective Metal)

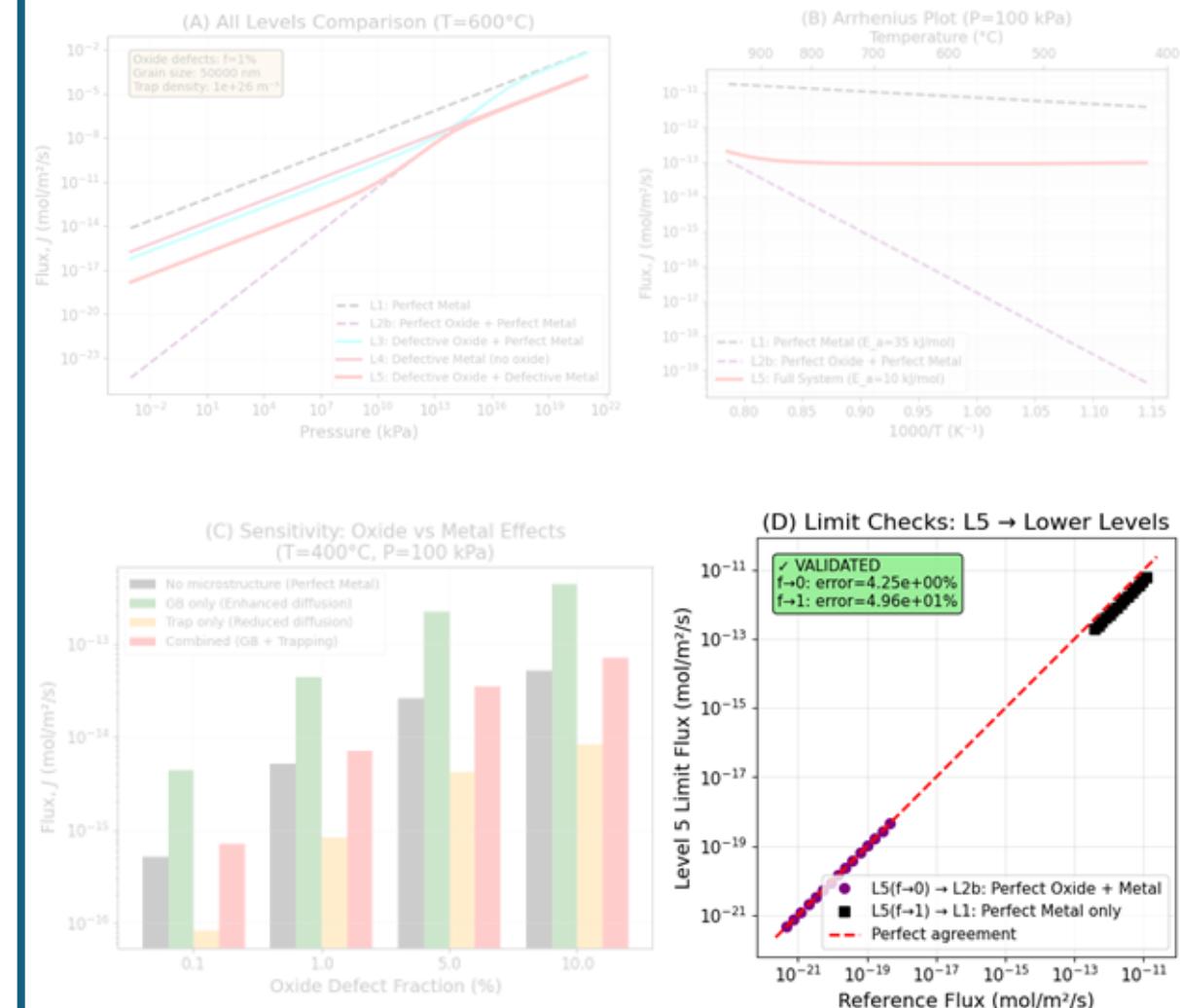


# Full System Validated Effect of Oxides and Microstructure

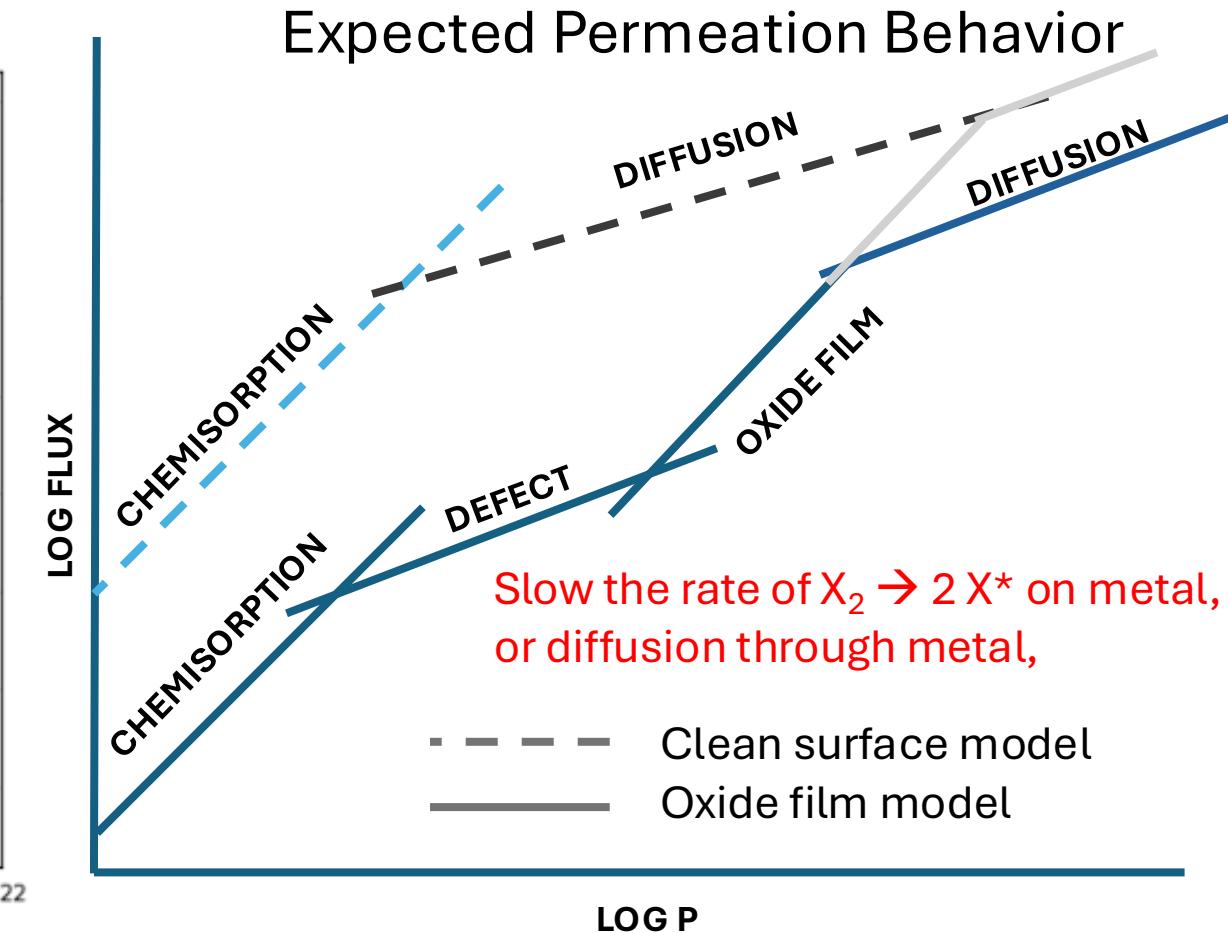
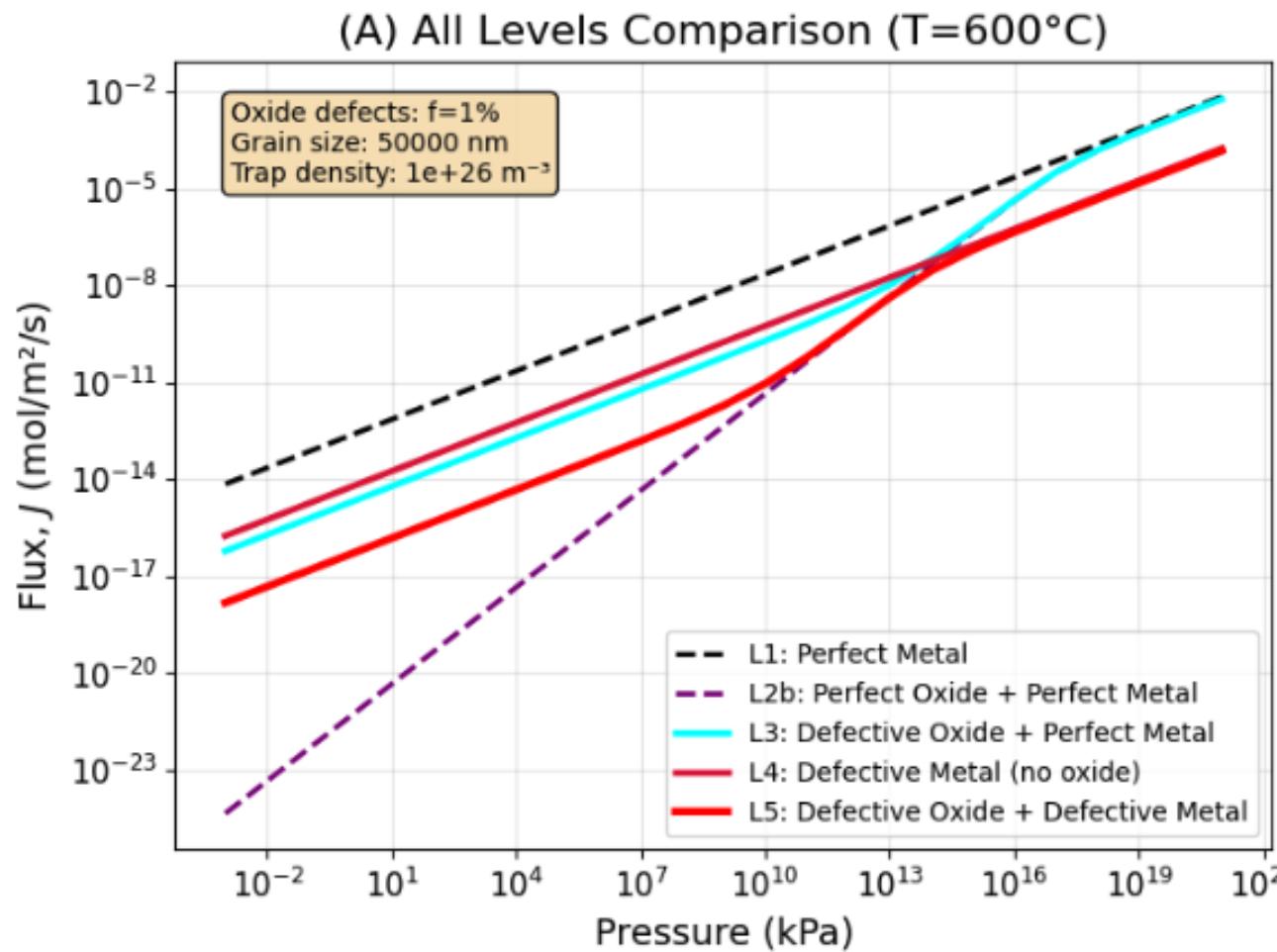
## Level 4: Defective Metal (GB Enhancement + Trapping)



## Level 5: Full System (Defective Oxide + Defective Metal)



# Full System Validated Effect of Oxides and Microstructure



# Full System Validated Effect of Oxides and Microstructure

