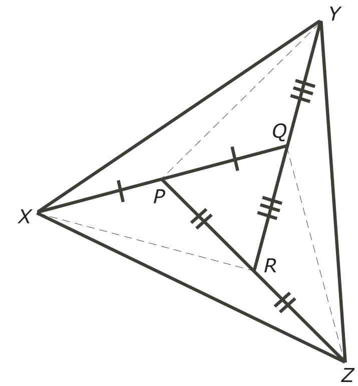
Problem of the Week
Problem E and Solution
So Many Triangles



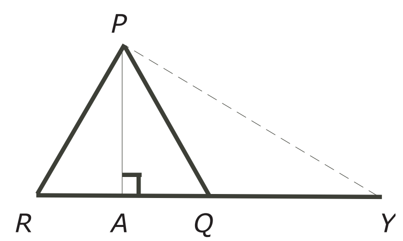
Problem

$\triangle PQR$ has side QP extended to X so that $QP = PX$, PR extended to Z so that $PR = RZ$, and RQ extended to Y so that $RQ = QY$. If the area of $\triangle XYZ = 1176$ cm^2, determine the area of $\triangle PQR$.

Solution

On the above diagram the lengths of the equal sides, $QP = PX$, $PR = RZ$, and $RQ = QY$, have been marked. Join P to Y, Q to Z, and R to X.

$\triangle PQR$ and $\triangle PQY$ have a common altitude drawn from vertex P to the line segment RY, meeting it at A. The triangles have equal base lengths, $RQ = QY$.
$\therefore$ area $\triangle PQR =$ area $\triangle PQY = x$.



At this point we can proceed to look at various other triangles with equal areas.

$\triangle PQY$ and $\triangle PXY$ have the same height and equal base lengths.
$\therefore$ area $\triangle PXY =$ area $\triangle PQY = x$.

$\triangle PXR$ and $\triangle PQR$ have the same height and equal base lengths.
$\therefore$ area $\triangle PXR =$ area $\triangle PQR = x$.

$\triangle PXR$ and $\triangle RXZ$ have the same height and equal base lengths.
$\therefore$ area $\triangle RXZ =$ area $\triangle PXR = x$.

$\triangle PQR$ and $\triangle QRZ$ have the same height and equal base lengths.
$\therefore$ area $\triangle QRZ =$ area $\triangle PQR = x$.

$\triangle QRZ$ and $\triangle QYZ$ have the same height and equal base lengths.
$\therefore$ area $\triangle QYZ =$ area $\triangle QRZ = x$.

Then the area of $\triangle XYZ$
$=$ area $\triangle PXY +$area $\triangle PQY +$area $\triangle PQR+$area $\triangle PXR+$area $\triangle RXZ+$area $\triangle QRZ+$area $\triangle QYZ$
$= x + x + x + x + x + x + x$

$\therefore 7x = 1176$ and $x = 168$ cm^2.

$\therefore$ the area of $\triangle PQR$ is 168 cm^2.