$10^{2016} - 2016$

Problem of the Week
Problem D and Solution
This Difference is Some Sum

Problem

Determine the sum of the digits in the difference when $10^{2016} - 2016$ is evaluated.

Solution

Solution 1

When the number 10^{2016} is written out there is a one followed by 2016 zeroes, a total of 2017 digits. Let's look at what happens in our effort to subtract.

$$\begin{array}{cccccccccccc} 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & - & 2 & 0 & 1 & 6 \\ \hline \end{array}$$

Using the standard subtraction algorithm, we start with the rightmost digits. In this case we need to borrow. But the borrowing creates a chain reaction. The result after the borrowing is complete is shown below.

$$\begin{array}{cccccccccccc} \cancel{1} & \cancel{0}^{9} & \cancel{0}^{9} & \cancel{0}^{9} & \cancel{0}^{9} & \cdots & \cancel{0}^{9} & \cancel{0}^{9} & \cancel{0}^{9} & \cancel{0}^{9} & \cancel{0}^{9} & {}^{1}0 \\ & & & & & & & - & 2 & 0 & 1 & 6 \\ \hline & 9 & 9 & 9 & 9 & \cdots & 9 & 9 & 7 & 9 & 8 & 4 \end{array}$$

The four rightmost digits in the difference are 7, 9, 8 and 4. To the left of these digits every digit is a 9. But how many nines are there? The difference has one less digit than 10^{2016} and therefore has 2016 digits. We have accounted for the four rightmost digits. So to the left of 7984 there are $2016 - 4 = 2012$ nines.

The digit sum is now straightforward to calculate. The digit sum is

$$2012 \times 9 + (7 + 9 + 8 + 4) = 18\,108 + 28 = 18\,136.$$

Note: If you were able to solve this problem, consider attempting level E problem.

Solution 2

The expression $10^{2016} - 2016$ has the same value as $(10^{2016} - 1) - (2016 - 1)$.

As mentioned in Solution 1, when 10^{2016} is written out, there is a one followed by 2016 zeroes, a total of 2017 digits. The number $(10^{2016} - 1)$ is one less than 10^{2016} and therefore is the positive whole number made up of exactly 2016 nines. When 1 is subtracted from 2016, the difference is 2015. The following is the equivalent subtraction question:

$$\begin{array}{r} 9\ 9\ 9\ 9\ \cdots 9\ 9\ 9\ 9\ 9\ 9 \\ -\ 2\ 0\ 1\ 5 \\ \hline 9\ 9\ 9\ 9\ \cdots 9\ 9\ 7\ 9\ 8\ 4 \end{array}$$

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