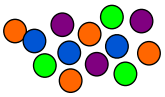
Problem of the Week
Problem D and Solution
Tiddlywinks Anyone?



Problem

The game of Tiddlywinks dates back to 1955. Rumour has it that the game has made a comeback with students in recent years. Last year, our local TPL, Tiddlywink Premier League, launched its first season. Each team in the league played each of the other teams in the league the same number of times. In its second season the TPL has grown. The total number of matches played this season will be twice the number of matches played in the first season and there will be 40% more teams than in season one. The number of games that each team plays each other team stays the same from season to season. How many teams were in the TPL in season one?

Solution

Let t represent the number of teams in the league in season one.
Then, the number of teams in the league in the second season is $1.4t$.
Note that both t and $1.4t$ must be positive integers.

We need to first establish how many games are played. If there were 4 teams, A, B, C, D, and each team played every other team once, then there would be 6 games played: AB, AC, AD, BC, BD, CD. Often, when counting something like this we "double-count". That is, there are 4 teams and each plays the 3 other teams so there are $4 \times 3 = 12$ games. But each game is counted twice. We need to divide the result by 2. So in a 4 team league, with each team playing each other team once, there are $\frac{4\times 3}{2} = 6$ games played.

In general, if there are t teams and each team plays every other team once, there would be $\frac{t(t-1)}{2}$ games played. If each of t teams plays every other team n times, there would be $n\left(\frac{t(t-1)}{2}\right)$ games played. If each of $1.4t$ teams plays every other team n times, there would be $n\left(\frac{1.4t(1.4t-1)}{2}\right)$ games played.

We know that the number of games played in season two is twice the number of games played in season one. So,

$$n\left(\frac{1.4t(1.4t-1)}{2}\right) = 2\left[n\left(\frac{t(t-1)}{2}\right)\right]$$

Dividing both sides by $\frac{n}{2}$, $n \neq 0$, this simplifies to

$$1.4t(1.4t-1) = 2t(t-1)$$

Dividing both sides by t, $t \neq 0$, this simplifies to

$$\begin{aligned} 1.4(1.4t-1) &= 2(t-1) \\ 1.96t - 1.4 &= 2t - 2 \\ 0.6 &= 0.04t \\ 15 &= t \end{aligned}$$

Therefore, in season one there were 15 teams in the TPL.