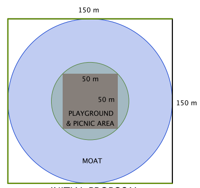
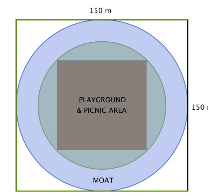


INITIAL PROPOSAL

Problem of the Week
Problem E and Solution
Playground, Picnic and Paddle Park



REVISED PROPOSAL

Problem

A park is square with dimensions 150 m by 150 m. An initial proposal has been submitted to the municipality to construct a playground and picnic area surrounded by a moat for paddle boats. A square playground and picnic area with dimensions 50 m by 50 m is to be built in the centre of the park with its sides parallel to the outer sides of the park. A moat of constant depth is to be constructed around the central area. The moat looks like a donut when viewed from above. Its outer ring touches the midpoints of the outer sides of the park and its inner ring passes through the vertices of the central square. The proposal is reviewed by the municipality and rejected. The amount of water required for the moat in the initial proposal is excessive. A revised proposal will be approved if the moat is constructed in the same way as in the initial proposal but uses 65% less water than the amount that would have been used in the initial proposal. The new moat will have the same depth as the moat in the original proposal. As a result of decreasing the size of the moat the central area dimensions will increase. What are the dimensions of the square playground & picnic area in the revised proposal? Round your final answer to the nearest tenth of a metre.

Solution

Let D represent the diameter of the larger circle and R represent the radius of the larger circle.

Let d represent the diameter of the smaller circle in the initial proposal and r represent the radius of the same circle.

Let d_{new} represent the diameter of the smaller circle in the revised proposal and r_{new} represent the radius of the same circle.

Let A represent the surface area of the moat in the initial proposal.

Let A_{new} represent the surface area of the moat in the revised proposal.

The diameter of the larger circle is the width of the park. Therefore, $D = 150$ m and $R = 75$ m.

In the initial proposal, the length of the diagonal of the centre square is the diameter of the inner circle. Using the Pythagorean Theorem, $d^2 = 50^2 + 50^2 = 5000$. It follows that $d = 50\sqrt{2}$ m and $r = 25\sqrt{2}$ m.

The surface area of the moat is the difference in the area of the two circles.

$$\begin{aligned} A &= \pi R^2 - \pi r^2 \\ &= \pi(75)^2 - \pi(25\sqrt{2})^2 \\ &= 5625\pi - 1250\pi \\ &= 4375\pi \text{ m}^2 \end{aligned}$$

The volume of the moat can be calculated by multiplying the surface area by the depth. Since the depth of the two moats is the same, we need only be concerned with reducing the surface area by 65% which is the same as finding 35% of the initial surface area.

The surface area of the moat in the revised proposal, is the difference in the area of the original outer circle and the new inner circle.

$$\begin{aligned}
A_{new} &= \pi R^2 - \pi(r_{new})^2 \\
0.35A &= \pi(75)^2 - \pi(r_{new})^2 \\
\frac{7}{20}(4375\pi) &= \pi(75)^2 - \pi(r_{new})^2 \\
\text{Dividing by } \pi: \quad \frac{7}{20}(4375) &= (75)^2 - (r_{new})^2 \\
\frac{6125}{4} &= 5625 - (r_{new})^2 \\
(r_{new})^2 &= 5625 - \frac{6125}{4} \\
(r_{new})^2 &= \frac{16375}{4} \\
r_{new} &= \frac{\sqrt{16375}}{2} \text{ m}, \ r_{new} > 0 \\
\therefore \ d_{new} &= \sqrt{16375} \text{ m}
\end{aligned}$$

In addition to d_{new} being the diameter of the inner circle in the revised plan, d_{new} is also the length of the diagonal of the new central square.

Let x represent the side length of the new central square. Using the Pythagorean Theorem,

$$\begin{aligned}
x^2 + x^2 &= (d_{new})^2 \\
x^2 + x^2 &= (\sqrt{16375})^2 \\
2x^2 &= 16375 \\
x^2 &= \frac{16375}{2} \\
x &= \sqrt{\left(\frac{16375}{2}\right)}, \ x > 0 \\
&= \sqrt{\left(\frac{32750}{4}\right)} \\
&= \frac{5}{2}\sqrt{1310} \\
&\approx 90.5 \text{ m}
\end{aligned}$$

The new central square has dimensions 90.5 m by 90.5 m. (There is a great deal of room for a playground and other things!)