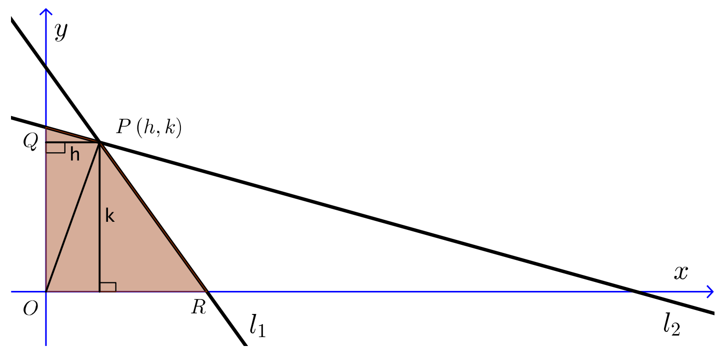
Problem of the Week
Problem D and Solution
A Shady Region

Problem

The shaded region on the diagram is bounded by the lines whose equations are $5x + 2y = 30$, $x + 2y = 22$, $x = 0$, and $y = 0$. Determine the area of the shaded region.



Solution

On the diagram, l_1 represents the line $5x + 2y = 30$ that crosses the x-axis at point R. l_2 represents the line $x + 2y = 22$ which crosses the y-axis at point Q.

Let $P(h, k)$ represent the point of intersection of l_1 and l_2. Then h is the horizontal distance from the y-axis to P and k is the vertical distance from the x-axis to P. Let O represent the origin.

To find the x-intercept of l_1 let $y = 0$ in $5x + 2y = 30$. Therefore the x-intercept is 6 and the coordinates of R are $(6, 0)$.

To find the y-intercept of l_2 let $x = 0$ in $x + 2y = 22$. Therefore the y-intercept is 11 and the coordinates of Q are $(0, 11)$.

To find the intersection of l_1 and l_2, we can use elimination.

$$\begin{array}{rrcl} l_1: & 5x + 2y & = & 30 \\ l_2: & x + 2y & = & 22 \\ \text{Subtracting, we obtain,} & 4x & = & 8 \\ & \therefore x & = & 2 \end{array}$$

Substituting $x = 2$ in l_1, $10 + 2y = 30$ and $y = 10$. The coordinates of P, the point of intersection, are $(2, 10)$. Therefore, $h = 2$ and $k = 10$. To find the shaded area:

$$\begin{aligned} \text{Area } PQOR &= \text{Area } \triangle PQO + \text{Area } \triangle POR \\ &= \frac{1}{2}h \times OQ \quad + \quad \frac{1}{2}k \times OR \\ &= \frac{1}{2}(2)(11) \ + \ \frac{1}{2}(10)(6) \\ &= 11 + 30 \\ &= 41 \end{aligned}$$

Therefore the shaded area is 41 units^2.