

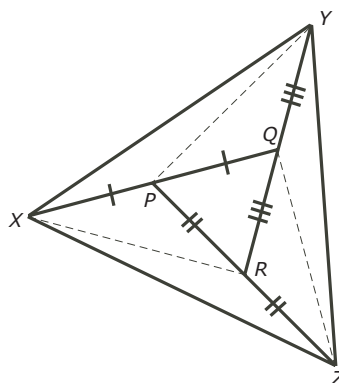
Problem of the Week

Problem E and Solution

So Many Triangles

Problem

$\triangle PQR$ has side QP extended to X so that $QP = PX$, PR extended to Z so that $PR = RZ$, and RQ extended to Y so that $RQ = QY$. If the area of $\triangle XYZ = 1176 \text{ cm}^2$, determine the area of $\triangle PQR$.

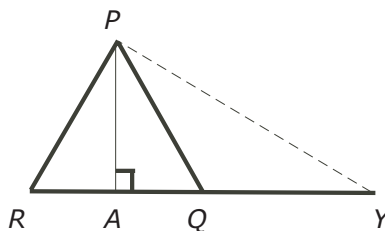


Solution

On the above diagram the lengths of the equal sides, $QP = PX$, $PR = RZ$, and $RQ = QY$, have been marked. Join P to Y , Q to Z , and R to X .

$\triangle PQR$ and $\triangle PQY$ have a common altitude drawn from vertex P to the line segment RY , meeting it at A . The triangles have equal base lengths, $RQ = QY$.

$\therefore \text{area } \triangle PQR = \text{area } \triangle PQY = x$.



At this point we can proceed to look at various other triangles with equal areas.

$\triangle PQY$ and $\triangle PXY$ have the same height and equal base lengths.

$\therefore \text{area } \triangle PXY = \text{area } \triangle PQY = x$.

$\triangle PXR$ and $\triangle PQR$ have the same height and equal base lengths.

$\therefore \text{area } \triangle PXR = \text{area } \triangle PQR = x$.

$\triangle PXR$ and $\triangle RXZ$ have the same height and equal base lengths.

$\therefore \text{area } \triangle RXZ = \text{area } \triangle PXR = x$.

$\triangle PQR$ and $\triangle QRZ$ have the same height and equal base lengths.

$\therefore \text{area } \triangle QRZ = \text{area } \triangle PQR = x$.

$\triangle QRZ$ and $\triangle QYZ$ have the same height and equal base lengths.

$\therefore \text{area } \triangle QYZ = \text{area } \triangle QRZ = x$.

Then the area of $\triangle XYZ$

$= \text{area } \triangle PXY + \text{area } \triangle PQY + \text{area } \triangle PQR + \text{area } \triangle PXR + \text{area } \triangle RXZ + \text{area } \triangle QRZ + \text{area } \triangle QYZ$
 $= x + x + x + x + x + x + x$

$\therefore 7x = 1176$ and $x = 168 \text{ cm}^2$.

\therefore the area of $\triangle PQR$ is 168 cm^2 .

