Problem of the Week
Problem E and Solution
A Simple System of Equations

Problem

If $2^{3x} = 16^{y+1}$ and $2x = 5y - 17$, determine the value of $x + y$. Try to see if doing this problem without a calculator affects how you think about the problem.

Solution

Solution 1

For this solution we will attempt to answer the question only. We will not determine the values of x and y that generate the sum.

$$\begin{aligned}
& & 2^{3x} &= 16^{y+1} & \\
& & 2^{3x} &= \left(2^4\right)^{y+1} & \\
& & 2^{3x} &= 2^{4y+4} & \\
& \therefore & 3x &= 4y + 4 & (1)\\
& \text{But} & 2x &= 5y - 17 & (2)\\
& (1) - (2) & x &= -y + 21 & \\
& \text{Rearranging} & x + y &= 21 &
\end{aligned}$$

$\therefore x + y = 21$. Notice that the problem only asks for $x + y$, it is not necessary to find values for x and y.

Solution 2

This solution carries on from equations (1) and (2) in solution 1 to find the values of x and y, and then determines the sum.

$$\begin{aligned}
(1) \times 5 \qquad & 15x &= 20y + 20 \qquad (3)\\
(2) \times 4 \qquad & 8x &= 20y - 68 \qquad (4)\\
(3) - (4) \qquad & 7x &= 88 & \\
& x &= \frac{88}{7} & \\
\text{Substitute in (2)} \qquad & 2\left(\frac{88}{7}\right) &= 5y - 17 & \\
\text{Multiply by 7} \qquad & 176 &= 35y - 119 & \\
& 295 &= 35y & \\
& y &= \frac{295}{35} = \frac{59}{7} & \\
& \therefore x + y &= \frac{88}{7} + \frac{59}{7} = \frac{147}{7} = 21 &
\end{aligned}$$

As before (but with much more work) $x + y = 21$.