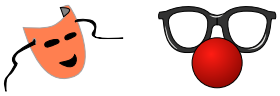

Problem of the Week
Problem D and Solution
Actress or Comedian - Dare to Compare

Problem

Holly Woods is a popular young actress and Joe King is an up and coming young comedian. Joe has an income which is five-eighths of Holly's income. Joe's expenses are one-half those of Holly, and Joe saves 40% of his income. Determine the percentage of her income that Holly Woods saves.

Solution

Solution 1 Using only one variable

Let h represent Holly's income. Then Joe's income is $\frac{5}{8}h$.

Since Joe saves 40% of his income, his expenses are $100\% - 40\% = 60\%$ of his income. Therefore, his expenses are $60\% \times \frac{5}{8}h = \frac{60}{100} \times \frac{5}{8}h = \frac{3}{8}h$.

Joe's expenses are one-half of Holly's expenses so Holly's expenses are twice Joe's expenses. Therefore, Holly's expenses are $2 \times \frac{3}{8}h = \frac{3}{4}h = 0.75h = 75\%$ of h. Since Holly's expenses are 75% of her income, she saves $100\% - 75\% = 25\%$ of her income.

$\therefore$ Holly Woods saves 25% of her income.

Solution 2 Using two variables

Let x represent Holly's income and y represent her expenses.
Then Joe's income is $\frac{5}{8}x$ and his expenses are $\frac{1}{2}y$.

Since Joe saves 40% of his income, his expenses are 60% of his income.

$$\begin{aligned} \frac{1}{2}y &= 0.60 \times \frac{5}{8}x \\ \frac{1}{2}y &= \frac{6}{10} \times \frac{5}{8}x \\ \frac{1}{2}y &= \frac{3}{8}x \\ y &= \frac{3}{4}x \end{aligned}$$

Holly saves whatever is left of her income after expenses. Therefore Holly saves

$$x - y = x - \frac{3}{4}x = \frac{1}{4}x = 0.25x = 25\% \text{ of } x.$$

$\therefore$ Holly Woods saves 25% of her income.

Solution 3 Using two variables a bit differently

Let $8x$ represent Holly's income and $2y$ represent her expenses.
Then Joe's income is $\frac{5}{8}(8x) = 5x$ and his expenses are $\frac{1}{2}(2y) = y$.

Since Joe saves 40% of his income, his expenses are 60% of his income.

$$\begin{aligned} y &= 0.60 \times 5x \\ y &= \frac{6}{10} \times 5x \\ y &= 3x \end{aligned}$$

Holly earns $8x$ and her expenses are $2y$ so her savings are $8x - 2y$. We want the ratio of her savings to her income, $\dfrac{8x - 2y}{8x} = \dfrac{8x - 2(3x)}{8x} = \dfrac{2x}{8x} = \dfrac{1}{4}$ or 25% .

$\therefore$ Holly Woods saves 25% of her income.