Problem of the Week

Problem E

A Skinny Quadrilateral

In the diagram, $\triangle QP_1R_1$ is right-angled with $QP_1 = 2$ and $QR_1 = 5$. Lines QP_1 and QR_1 are extended and many more points are labelled at intervals of 1 unit, so that

$$P_1P_2 = P_2P_3 = P_3P_4 = P_4P_5 = \cdots = 1, \text{ and}$$

$$R_1R_2 = R_2R_3 = R_3R_4 = R_4R_5 = \cdots = 1.$$

In fact, $P_1P_j = j - 1$ and $R_1R_k = k - 1$ for any positive integers j and k.

For example, $P_1P_5 = 5 - 1 = 4$ and $R_1R_4 = 4 - 1 = 3$.

Determine the value of n so that the area of quadrilateral $P_nP_{n+1}R_{n+1}R_n$ is 2016. That is, determine the value of n so that the area of the quadrilateral with vertices P_n, P_{n+1}, R_{n+1}, and R_n is 2016.