

## Problem of the Week Problem E and Solution What's Your Angle Anyway III?

## Problem

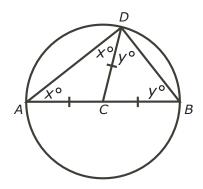
AB is a diameter of a circle with centre C. D is a point on the circumference of the circle other than A and B. Determine the measure of  $\angle ADB$ .

## Solution

Join D to the centre C. Since CA, CB and CD are radii of the circle, CA = CB = CD.

Since CA = CD,  $\triangle CAD$  is isosceles and  $\angle CAD = \angle CDA = x^{\circ}$ . Since CB = CD,  $\triangle CBD$  is isosceles and  $\angle CBD = \angle CDB = y^{\circ}$ .

This new information is marked on the following diagram.



The angles in a triangle add to 180° so in  $\triangle ABD$ 

$$\angle ADB + \angle DAB + \angle DBA = 180^{\circ}$$
$$(x^{\circ} + y^{\circ}) + x^{\circ} + y^{\circ} = 180^{\circ}$$
$$2(x^{\circ} + y^{\circ}) = 180^{\circ}$$
$$x^{\circ} + y^{\circ} = 90^{\circ}$$

But  $\angle ADB = x^{\circ} + y^{\circ}$  so  $\angle ADB = 90^{\circ}$ .

This result is often expressed as a theorem for circles:

An angle  $(\angle ADB)$  inscribed in a circle by the diameter (AB) of a circle is  $90^{\circ}$ .

