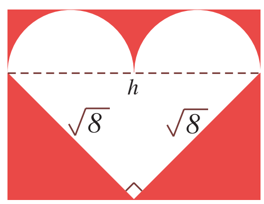
Problem of the Week
Problem D and Solution
Forever Mine Valentine

Problem

A heart is constructed by attaching two white semi-circles to the hypotenuse of an isosceles right triangle whose equal sides measure $\sqrt{8}$ cm. The new figure is then mounted onto a rectangular piece of red construction paper as shown below. (The dashed line, the side measurement and the right angle symbol will not actually be on the finished card.) You are going to write your valentine a message in red ink on the white region of the card. Determine the total amount of area available for your special valentine greeting.

Solution

Let h represent the length of the hypotenuse. Let r represent the radius of the semi-circles. Since the two semi-circles lie along the hypotenuse, $h = 4r$ or $r = \frac{h}{4}$.



Since the triangle is isosceles right, we can find h using Pythagoras' Theorem, $h^2 = (\sqrt{8})^2 + \sqrt{8})^2 = 8 + 8 = 16$ and $h = 4$ cm follows.

Then $r = \frac{h}{4} = \frac{4}{4} = 1$ cm. Since there are two semi-circles of radius 1 cm, the total area of the two semi-circles is the same as the area of a full circle of radius 1 cm. The area of the two semi-circles is $\pi r^2 = \pi(1)^2 = \pi$ cm^2.

The triangle is isosceles right so we can use the lengths of the two equal sides as the base and height in the calculation of the area of the triangle. The area of the triangle is $\frac{1}{2}bh = \frac{1}{2}(\sqrt{8})(\sqrt{8}) = 4$ cm^2.

The total area for writing the message is $(\pi + 4)$ cm^2. This area is approximately 7.1 cm^2. Hopefully you can write that special message in a very limited space. Happy Valentine's Day.