Sample Question from the CEMC Mental Math Bowl

936
x11
???

Problem of the Week
Problem E and Solution
Mathletes in Action

Problem

Four mathletes, Jen, John, JP and Judith, competed in the finals of the CEMC Mental Math Bowl. Prizes were awarded for the top three competitors as follows: first place received an anniversary Math Faculty Pink Tie, second place received a book of Brain Teasers, and third place received an "I Love Math" T-shirt. Three staff members of the CEMC predicted how the prizes would be awarded.

- Dean predicted that Judith would win the pink tie, John would win the book and JP would win the T-shirt.
- Ian predicted that Jen would win the pink tie, Judith would win the book and JP would win the T-shirt.
- Sandy predicted that John would win the pink tie, Jen would win the book and Judith would win the T-shirt.

It turns out that each staff member predicted exactly one prize winner correctly. Determine precisely how the prizes were awarded.

Solution

Let's look at Dean's prize predictions. Assume he is correct that Judith won the pink tie. This leads to the following six possibilities.

First - Pink Tie	Second - Brain Book	Third - T-shirt	Fourth - No Prize	
Judith	Jen	John	JP	(1)
Judith	Jen	JP	John	(2)
Judith	John	Jen	JP	(3)
Judith	John	JP	Jen	(4)
Judith	JP	Jen	John	(5)
Judith	JP	John	Jen	(6)

Since the other two parts of Dean's prize prediction are not correct, John cannot win the book prize so we can rule out (3) and (4), and JP cannot win the T - shirt so we can also rule out (2). This leaves (1), (5) and (6) as the only possibilities for Dean.

First - Pink Tie	Second - Brain Book	Third - T-shirt	Fourth - No Prize	
Judith	Jen	John	JP	(1)
Judith	JP	Jen	John	(5)
Judith	JP	John	Jen	(6)

Now look at Ian's prize predictions in light of Dean's three valid possibilities. Ian makes exactly one true prize prediction. None of Dean's possibilities would ever make one of Ian's prize predictions true. Therefore, our original assumption that Dean correctly predicted Judith to win the pink tie was incorrect.

So now we assume that Dean correctly predicts that John would win the book prize. This leads to the following six possibilities.

First - Pink Tie	Second - Brain Book	Third - T-shirt	Fourth - No Prize	
Jen	John	JP	Judith	(7)
Jen	John	Judith	JP	(8)
JP	John	Jen	Judith	(9)
JP	John	Judith	Jen	(10)
Judith	John	Jen	JP	(11)
Judith	John	JP	Jen	(12)

Since the other two parts of Dean's prize prediction are not correct, Judith cannot win the pink tie so we can rule out (11) and (12), and JP cannot win the T - shirt so we can also rule out (7). This leaves (8), (9) and (10) as the only possibilities for Dean.

First - Pink Tie	Second - Brain Book	Third - T-shirt	Fourth - No Prize	
Jen	John	Judith	JP	(8)
JP	John	Jen	Judith	(9)
JP	John	Judith	Jen	(10)

Now look at Ian's prize predictions in light of Dean's three valid possibilities. Ian makes exactly one true prize prediction. Possibility (8) is the only one of the possibilities that works for Ian.

Now we must check Sandy's prize prediction to see if it is still valid with (8). Sandy predicted that John would win the pink tie, Jen would win the book and Judith would win the T-shirt. This prediction works since exactly one of Sandy's prize predictions, that Judith wins the T-shirt, is true. The other two predictions are false.

We should check to see that this is the only correct solution. We do this by assuming Dean's third prediction, JP wins the T-shirt, was his only correct prediction. This leads to the following six possibilities.

First - Pink Tie	Second - Brain Book	Third - T-shirt	Fourth - No Prize	
Jen	John	JP	Judith	(13)
Jen	Judith	JP	John	(14)
John	Jen	JP	Judith	(15)
John	Judith	JP	Jen	(16)
Judith	Jen	JP	John	(17)
Judith	John	JP	Jen	(18)

Since the other two parts of Dean's prize prediction are not correct, Judith cannot win the pink tie so we can rule out (17) and (18), and John cannot win the Brain Teaser Book so we can also rule out (13). This leaves (14), (15) and (16) as the only possibilities for Dean.

First - Pink Tie	Second - Brain Book	Third - T-shirt	Fourth - No Prize	
Jen	Judith	JP	John	(14)
John	Jen	JP	Judith	(15)
John	Judith	JP	Jen	(16)

Now look at Ian's prize predictions in light of Dean's three valid possibilities. Ian makes exactly one true prize prediction. Possibility (15) is the only one of the possibilities that works for Ian. In both (14) and (16) Ian would make at least two correct prize predictions.

Now we must check Sandy's prize prediction to see if it is still valid with (15). Sandy predicted that John would win the pink tie, Jen would win the book and Judith would win the T-shirt. Since two of Sandy's prize predictions would be true in (15) but Sandy only made one correct prize prediction, (15) does not work for Sandy.

∴ the only possibility is that Jen finishes first and wins the pink tie, John finishes second and wins the Brain Teaser book, and Judith finishes third and gets the T-shirt. JP finishes fourth and leaves with the satisfaction of making it to the finals of the Mental Math Bowl.

By the way, the correct answer to the Sample Question from the CEMC Mental Math Bowl was 10 296. Remember, no calculators are used to perform mental math.