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## Problem of the Week

### Problem E and Solution

### Partly Salted

#### Problem

When one kilogram of salt is added to a solution of salt and water, the solution becomes  $\frac{1}{3}$  salt by mass. One kilogram of water is then added to the new solution resulting in a solution that is  $\frac{3}{10}$  salt by mass. What fraction of the original solution was salt?

#### Solution

Let  $S$  represent the mass of salt in the original solution.

Let  $W$  represent the mass of water in the original solution.

Then  $S + W$  represents the total mass of the original solution.

After adding 1 kg of salt to the original solution, there is  $(S + 1)$  kg of salt and  $(S + W + 1)$  kg of solution. Since this new solution is one third salt by mass,

$$\begin{aligned}\frac{S + 1}{S + W + 1} &= \frac{1}{3} \\ 3(S + 1) &= 1(S + W + 1) \\ 3S + 3 &= S + W + 1 \\ 2S + 2 &= W \quad (1)\end{aligned}$$

After adding 1 kg of water to the new solution, there is still  $(S + 1)$  kg of salt in  $(S + W + 2)$  kg of solution. Since this new solution is three tenths salt by mass,

$$\begin{aligned}\frac{S + 1}{S + W + 2} &= \frac{3}{10} \\ 10(S + 1) &= 3(S + W + 2) \\ 10S + 10 &= 3S + 3W + 6 \\ 7S - 3W &= -4 \quad (2)\end{aligned}$$

Substituting (1) into (2),

$$\begin{aligned}7S - 3(2S + 2) &= -4 \\ 7S - 6S - 6 &= -4 \\ S &= 2 \text{ kg}\end{aligned}$$

Substituting for  $S$  in (1),  $W = 6$  kg. Then the original solution was  $S + W = 8$  kg. The fraction of the original solution that was salt was  $\frac{2}{8} = \frac{1}{4}$ .

$\therefore$  one quarter of the original solution was salt by mass.

