



## Problem of the Week

### Problem D and Solution

### License to Ride

#### Problem

A Mathville license plate consists of exactly four digits chosen from the digits 0 to 9. What is the probability that the sum of the digits on any of these license plates is 34 or higher?

#### Solution

First we must determine the total number of possible 4 digit number sequences. There are 10 choices for the first digit. For each of these choices there are 10 choices for the second digit. Therefore, there are  $10 \times 10 = 100$  choices for the first two digits. For each of these possibilities, there are 10 choices for the third digit. Therefore, there are  $100 \times 10 = 1000$  possibilities for the first three digits. And finally, for each of these 1000 choices for the first three digits there are 10 choices for the fourth digit. Therefore there are  $1000 \times 10 = 10\,000$  possibilities for license plates with four digits in Mathville.

Now we must determine the number of license plates with a digit sum of 34 or higher. We will consider cases.

1. **The license contains four nines.** If the digits are all nines, the sum is 36 which is acceptable. There is only one way to use all nines for digits.
2. **The license contains three nines and one other digit.** If three nines are used the sum is 27. In order to get 34 or higher the fourth digit must be 7 or 8. There are two choices for the fourth digit. Once the digit is chosen there are four places to put the digit. Once the digit is placed the remaining spots must be nines. Therefore there are  $2 \times 4 = 8$  license plates containing three nines. (It is possible to list them: 7999, 8999, 9799, 9899, 9979, 9989, 9997, 9998.)
3. **The license contains two nines.** If two nines are used the sum is 18. In order to get to 34 or higher, we need a sum of  $34 - 18 = 16$  or higher from the two remaining digits. The only way to do this, since we cannot use more nines, is to use two eights. The two eights can be placed in six ways and then the nines must go in the remaining spots. Therefore there are six license plates containing two nines. (It is possible to list them: 8899, 8989, 8998, 9889, 9898, 9988.)
4. **The license contains one nine.** If one nine is used the sum is 9. In order to get to 34 or higher, we need a sum of  $34 - 9 = 25$  or higher from the remaining three digits. But this sum would be made from three digits chosen from the digits 0 to 8. The maximum possible sum would be 24 if three eights were used. We need 25 or higher. Therefore no license plate containing only one 9 will produce a sum of 34 or higher. It should be noted that a license with no nines would not produce a sum of 34 or higher either.

Therefore, the number of license plates with a digit sum of 34 or higher is  $1 + 8 + 6 = 15$ . To calculate the probability we divide the number of licenses with a digit sum of 34 or more by the number of possible license plates. The probability of getting a license plate in Mathville with a digit sum of 34 or higher is  $\frac{15}{10\,000} = \frac{3}{2\,000}$ . Another way of looking at this result is out of every 2 000 cars with Mathville plates you could expect, on average, to find 3 with a digit sum of 34 or higher.

