Problem of the Week

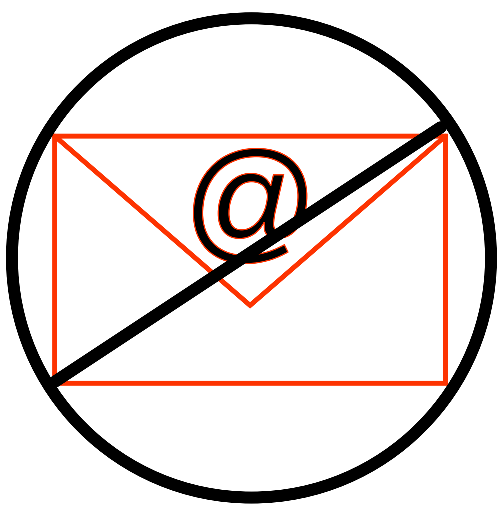
Problem E

No More Emails!

Jesse decided to stop answering emails indefinitely. However, he would look each day and count the number of emails but leave all of the emails unread. At the end of the first day there were 2 unread emails. At the end of the second day there were 5 unread emails, an increase of 3 emails from the previous day. At the end of the third day there were 12 unread emails, an increase of 7 emails from the previous day. At the end of the fourth day there were 23 unread emails, an increase of 11 emails from the previous day.

Jesse noticed that the increases from day to day so far formed an arithmetic sequence. (An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, 3,5,7,9 is an arithmetic sequence with four terms and constant difference 2.)

Jesse's email provider allows him to have a maximum of 6330 unread emails in his inbox. Once this limit is reached his account is will no longer be able to receive emails. Supposing that the pattern of the daily increases continues, for how many days can Jesse receive emails before new emails are no longer allowed?



The information on the next page *may* be helpful in solving the problem.

3,5,7,9 is an arithmetic sequence with four terms and constant difference 2. Each number in the sequence is called a term. For example, we say that $t_1 = 3$. The subscript 1 is the position of the term in the sequence and 3 is the value of the term. The term in position n is denoted t_n.

The general term of an arithmetic sequence is $t_n = a + (n-1)d$, where a is the first term, d is the constant difference and n is the number of terms.

The sum, S_n, of the first n terms of an arithmetic sequence can be found using either $S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = n\left(\frac{t_1 + t_n}{2}\right)$, where t_1 is the first term of the sequence and t_n is the n^{th} term of the sequence.

The following example is provided to verify the accuracy of the formulas and to illustrate their use.

For the arithmetic sequence 3,5,7,9, $a = t_1 = 3$, $d = 2$, $n = 4$ and $t_n = t_4 = 9$.

$$S_n = 3 + 5 + 7 + 9 = 24$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{4}{2}[2(3) + (3)2] = 2[12] = 24$$

$$S_n = n\left(\frac{t_1 + t_n}{2}\right) = 4\left(\frac{3+9}{2}\right) = 4(6) = 24$$