B A D

Problem of the Week
Problem E and Solution
It's a BAD DAY

D A Y

Problem

The six letters, B, A, D, D, A, and Y are arranged to form six letter "words". When examining the "words", how many are there in which the letters B, A and D are not adjacent to each other and in that order or the letters D, A and Y are not adjacent to each other and in that order?

Solution

We will find the total number of possible "words" and then exclude those "words" which include the word BAD but not the word DAY, the word DAY but not the word BAD, and both words BAD and DAY.

1. Determine the total number of "words" formed using 2 As, 2 Ds, 1 B and 1 Y.

   First, place the B in 6 possible positions. Then, for each of the 6 possible placements of the B, there are 5 ways to place the Y. There are then $6 \times 5 = 30$ ways to place the B and the Y.

   Next, place the 2 As. For each of the 30 ways to place the B and the Y, there are four empty spaces. Looking only at the spaces, there are 6 ways to place the 2 As:

   $\underline{A}\ \underline{A}\ \_\ \_$ $\quad$ $\underline{A}\ \_\ \underline{A}\ \_$ $\quad$ $\underline{A}\ \_\ \_\ \underline{A}$ $\quad$ $\_\ \underline{A}\ \underline{A}\ \_$ $\quad$ $\_\ \underline{A}\ \_\ \underline{A}$ $\quad$ $\_\ \_\ \underline{A}\ \underline{A}$

   Therefore, there are $30 \times 6 = 180$ ways to place the B, Y, and 2 As. For each of the 180 ways to place the B, Y, and 2 As, the 2 Ds must go in the remaining two empty spaces in 1 way. Therefore, there are $180 \times 1 = 180$ ways to place the B, Y, 2 As and 2 Ds.

2. Determine how many of the 180 "words" contain the word BAD but not the word DAY.

   For each placement of the word BAD, there are 6 ways to place the letters of the word DAY in the remaining 3 spaces: ADY, AYD, DAY, DYA, YAD and YDA. We need to exclude any of these which would cause the word DAY to also occur in the "word".

   - $B\ A\ D\ \_\ \_\ \_$

     BAD starts with the B in the first position and the six letter "word" does not contain the word DAY.

     We are not be able to use $D,\ A,\ Y$ in that order or $A,\ Y,\ D$ in that order to fill the three remaining spaces. The resulting "words" would be $BAD\underline{DAY}$ and $BA\underline{DAY}D$. Each of these two "words" contain both BAD and DAY, and are excluded from this case. Therefore, there are 4 "words" containing the word BAD with the B in the first position and not containing the word DAY.

- _ B A D _ _

  BAD starts with the B in the second position and the "word" does not contain the word DAY.

  We are not be able to use D, A, Y in that order to fill the three remaining spaces. The resulting "word" would be $DBA\underline{DAY}$. This "word" contains both BAD and DAY, and is excluded from this case. Therefore, there are 5 "words" containing the word BAD with the B in the second position and not containing the word DAY.

- _ _ B A D _

  BAD starts with the B in the third position and the "word" does not contain the word DAY.

  We are be able to use any of the six arrangements of the D, A and Y to fill the three remaining spaces. Therefore, there are 6 "words" containing the word BAD with the B in the third position and not containing the word DAY.

- _ _ _ B A D

  BAD starts with the B in the fourth position and the "word" does not contain the word DAY.

  We are not be able to use D, A, Y in that order to fill the three remaining spaces. The resulting "word" formed would be $\underline{DAY}BAD$. This "word" contains both BAD and DAY, and must be excluded from this case. Therefore, there are 5 "words" containing the word BAD with the B in the fourth position and not containing the word DAY.

Summing the totals from the above four cases, there are $4 + 5 + 6 + 5 = 20$ "words" which contain the word BAD but not the word DAY.

3. Determine how many of the 180 "words" contain the word DAY but not the word BAD.

   The argument for this case is very similar to that presented in the second case and will not be repeated here. However, after the end of the solution, the full argument will be presented.

   There are also 20 "words" which contain the word DAY but not the word BAD.

4. Determine how many of the 180 "words" contain both the word DAY and the word BAD.

   It is straight forward to list the possibilities in which both words occur. There are 4 possibilities:

   $$BADDAY \qquad DAYBAD \qquad BADAYD \qquad DBADAY$$

5. Determine the number of "words" that can be formed in which the letters B, A and D are not adjacent to each other or the letters D, A and Y are not adjacent to each other.

   Using the total from the first case and subtracting the totals from each of the following cases we obtain $180 - 20 - 20 - 4 = 136$. Therefore, there are 136 "words" that can be formed in which the letters B, A and D are not adjacent to each other or the letters D, A and Y are not adjacent to each other.

Here is the complete argument for the third case.

3. Determine how many of the 180 "words" contain the word DAY but not the word BAD.

   For each placement of the word DAY, there are 6 ways to place the letters of the word BAD in the remaining 3 spaces: ABD, ADB, BAD, BDA, DAB and DBA. We need to exclude any of these which would cause the word BAD to also occur in the "word".

   - $D\ A\ Y\ \_\ \_\ \_$

     DAY starts with the D in the first position and the six letter "word" does not contain the word BAD.

     We are not be able to use $B,\ A,\ D$ in that order to fill the three remaining spaces. The resulting "word" would be $DAY\underline{BAD}$. This "word" contains both DAY and BAD, and is excluded from this case. Therefore, there are 5 "words" containing the word DAY with the D in the first position and not containing the word BAD.

   - $\_\ D\ A\ Y\ \_\ \_$

     DAY starts with the D in the second position and does not contain the word BAD.

     We are be able to use any of the six arrangements of the B, A and D to fill the three remaining spaces. Therefore, there are 6 "words" containing the word DAY with the D in the second position and not containing the word BAD.

   - $\_\ \_\ D\ A\ Y\ \_$

     DAY starts with the D in the third position and does not contain the word BAD.

     We are not be able to use $B,\ A,\ D$ in that order to fill the three remaining spaces. The word formed would be $\underline{BAD}AYD$. This word contains both DAY and BAD, and must be excluded from this case. Therefore, there are 5 "words" containing the word DAY with the D in the third position and not containing the word BAD.

   - $\_\ \_\ \_\ D\ A\ Y$

     DAY starts with the D in the fourth position and does not contain the word BAD.

     We are not be able to use $B,\ A,\ D$ in that order or $D,\ B,\ A$ in that order to fill the three remaining spaces. The resulting "words" would be $\underline{BAD}DAY$ and $D\underline{BAD}AY$. Each of these two "words" contain both DAY and BAD, and are excluded from this case. Therefore, there are 4 "words" containing the word DAY with the D in the fourth position and not containing the word BAD.

Summing the totals from the above four cases, there are $5 + 6 + 5 + 4 = 20$ "words" which contain the word DAY but not the word BAD

One Final Note

The solution presented assumes generally that the solver has no prior counting experience. Our goal here is to present a solution that is understandable by the majority of solvers.