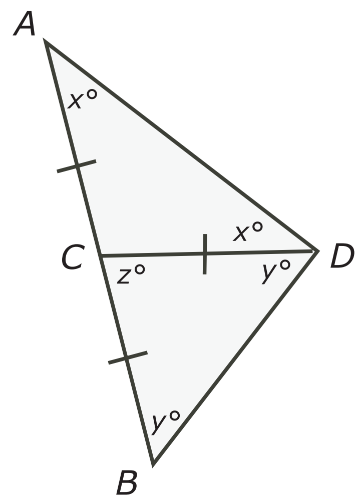
Problem of the Week

Problem D and Solution

What's Your Angle Anyway II?



Problem

In $\triangle ABD$, C is on AB such that $CA = CB = CD$ and $\angle BCD = z^\circ$.

Determine the measure of $\angle ADB$.

Solution

Solution 1

Since ACB is a straight line, $\angle ACD + \angle DCB = 180^\circ$ but $\angle BCD = z^\circ$ so $\angle ACD = 180^\circ - z^\circ$.

In $\triangle ACD$, since $CA = CD$, $\triangle ACD$ is isosceles and $\angle CAD = \angle CDA = x^\circ$. The angles in a triangle sum to 180° so in $\triangle ACD$

$$\begin{aligned}
\angle CAD + \angle CDA + \angle ACD &= 180^\circ \\
x^\circ + x^\circ + 180^\circ - z^\circ &= 180^\circ \\
2x^\circ &= z^\circ \\
x^\circ &= \frac{z^\circ}{2}
\end{aligned}$$

Similarly, in $\triangle BCD$, since $CB = CD$, $\triangle CBD$ is isosceles and $\angle CBD = \angle CDB = y^\circ$.

The angles in a triangle sum to 180° so in $\triangle CBD$

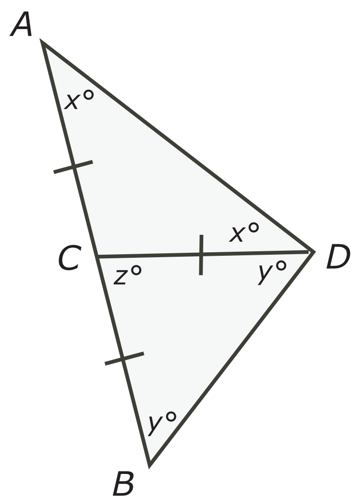
$$\begin{aligned}
\angle CBD + \angle CDB + \angle BCD &= 180^\circ \\
y^\circ + y^\circ + z^\circ &= 180^\circ \\
2y^\circ &= 180^\circ - z^\circ \\
y^\circ &= \frac{180^\circ - z^\circ}{2}
\end{aligned}$$

Then $\angle ADB = \angle CDA + \angle CDB = x^\circ + y^\circ = \frac{z^\circ}{2} + \frac{180^\circ - z^\circ}{2} = \frac{180^\circ}{2} = 90^\circ$.

$\therefore$ the measure of $\angle ADB$ is 90°.

See Solution 2 for a more general approach to the solution.

It turns out that it is not necessary to find expressions for x and y in terms of z in the problem.



Solution 2

Here is a second solution to the problem.

In $\triangle CAD$, since $CA = CD$, $\triangle CAD$ is isosceles and $\angle CAD = \angle CDA = x^\circ$.

In $\triangle CBD$, since $CB = CD$, $\triangle CBD$ is isosceles and $\angle CBD = \angle CDB = y^\circ$.

The angles in a triangle sum to 180° so in $\triangle ABD$

$$\begin{aligned} \angle BAD + \angle ADB + \angle ABD &= 180^\circ \\ x^\circ + (x^\circ + y^\circ) + y^\circ &= 180^\circ \\ 2x^\circ + 2y^\circ &= 180^\circ \\ x^\circ + y^\circ &= 90^\circ \end{aligned}$$

But $\angle ADB = \angle ADC + \angle CDB = x^\circ + y^\circ = 90^\circ$.

$\therefore$ the measure of $\angle ADC$ is 90°.