


Problem of the Week
Problem D and Solution
A Problem for the Ages


Problem

Grandmother has four grandchildren, two boys and two girls. When the ages of the grandchildren are multiplied together, the product of their ages is 67 184. Only one of the grandchildren is a teenager and the oldest grandchild is under 40. The difference in ages between the oldest and youngest grandchild is thirty years. Determine the ages of Grandmother's grandchildren.

Solution

The first task is to factor 67 184. Since the number is even, we can divide out powers of 2 such that $67\,184 = 2^4 \times 4199$. After some trial we discover that $67\,184 = 1 \times 2^4 \times 13 \times 17 \times 19$. Notice the inclusion of the number 1 as one of the factors. (One of the grandchildren could be 1.)

We can rule out 1 as a possible age since no matter how we combine the other factors to create the three remaining ages, at least one of the ages will be greater than 40.

It should also be noted that multiplying any of the factors 13, 17, 19 together, in any combination, will produce a number greater than 40. Since all of the grandchildren are under 40, we can eliminate these possibilities.

We are left with combining various powers of 2 with the other factors to form the four ages.

If 2^4 is one of the ages, the only possibility for the ages is {13,16,17,19}. This is not a valid combination because we know only one grandchild is a teenager.

If 2^3 is one of the ages, we can multiply the fourth factor of 2 by one of the other ages to produce three possible sets of ages: {8,26,17,19}, {8,13,34,19}, and {8,13,17,38}. In each of these cases there are two teenagers so none of these sets produces valid ages.

If 2^2 is one of the ages, we could multiply one of the ages by 4 or multiply two out of three of the other ages by 2. Multiplying either 13, 17 or 19 by 4 produces a product greater than 40 so this is not a possibility. Multiplying two out of three ages 13, 17, 19 by 2 produces the following results: {4,26,34,19}, {4,26,17,38}, and {4,13,34,38}. In each of these cases exactly one of the grandchildren is a teenager. But only the first possibility also satisfies the condition that the difference in ages between the oldest and the youngest is 30. Therefore, {4,26,34,19} is a possibility for the ages.

There is one final possibility to consider. If one of the grandchildren is 2, we could double each of the remaining ages to create the set {2,26,34,38}. This possibility is quickly dismissed since none of the ages is in the range 13 to 19. That is, there are no teenagers in this list.

Therefore the only possible ages for Grandmother's grandchildren are 4, 19, 26 and 34.