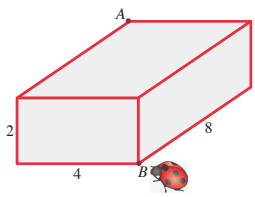


Problem of the Week

Problem E and Solution

Bugged

Problem

A ladybug wishes to travel from B to A on the surface of a wooden block with dimensions $2 \times 4 \times 8$ as shown in the diagram. Determine the shortest distance for the ladybug to walk.

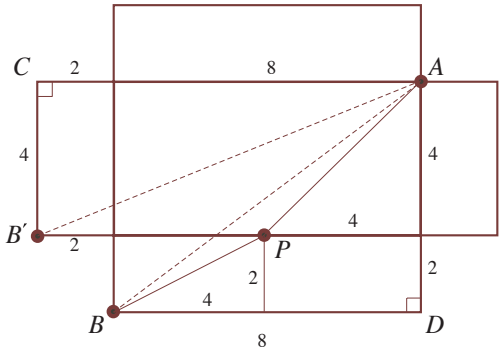
Solution

Many strategies could be attempted. Perhaps the bug walks along the edges and travels $8 + 2 + 4 = 14$ units. Perhaps the bug travels across the right side of the block from B to the midpoint of the top edge (marked P on the diagram below) and then across the top of the box to A. Referring to the diagram below, it can be shown that this distance is

$$BP + PA = \sqrt{BP^2} + \sqrt{PA^2} = \sqrt{4^2 + 2^2} + \sqrt{4^2 + 4^2} = \sqrt{20} + \sqrt{32} \doteq 10.13 \text{ units.}$$

But is this the shortest distance?

To visualize the possible routes fold out the sides of the box so that they are laying on the same plane as the top of the box. Label the diagram as shown below. Note that as a result of folding out the sides, corner B appears twice. The second corner is labelled B'.



The shortest distance for the ladybug to travel is a straight line from B to A or B' to A. So both cases must be considered.

BA is the hypotenuse of right-angled triangle ABD. Using Pythagoras' Theorem,

$$BA^2 = BD^2 + DA^2 = 8^2 + 6^2 = 100 \text{ and } BA = 10 \text{ follows.}$$

$B'A$ is the hypotenuse of right-angled triangle $AB'C$. Using Pythagoras' Theorem,

$$(B'A)^2 = (B'C)^2 + CA^2 = 4^2 + 10^2 = 116 \text{ and } B'A = 2\sqrt{29} \doteq 10.77 \text{ follows.}$$

Since $BA < B'A$, the shortest distance for the ladybug to travel is 10 units on the surface of the block in a straight line from B to A.

This problem is quite straight forward once the three-dimensional nature of the problem is removed.