Problem of the Week
Problem E and Solution
Five Solutions - Really?

Problem

There are five values of x that satisfy the equation $(x^2-5x+5)^{x^2+4x-60}=1$. Determine these five values of x.

Solution

Let's consider the ways that an expression of the form a^b can be 1:

- The base, a, is 1.

  In this case, the exponent can be any value and we need to solve $x^2-5x+5=1$.

  $$\begin{aligned} x^2-5x+5&=1\\ x^2-5x+4&=0\\ (x-4)(x-1)&=0 \end{aligned}$$

  So $x=4$ or $x=1$.

- The exponent, b, is 0.

  In this case, the base can be any number other than 0 and we need to solve $x^2+4x-60=0$.

  $$\begin{aligned} x^2+4x-60&=0\\ (x-6)(x+10)&=0 \end{aligned}$$

  So $x=6$ or $x=-10$.

  When $x=6$, the base is $6^2-5(6)+5=11\neq 0$. That is, when $x=6$, the base does not equal 0.
  When $x=-10$, the base is $(-10)^2-5(-10)+5=155\neq 0$. That is, when $x=-10$, the base does not equal 0.

- The base, a, is -1 and the exponent, b, is even.

  We first need to solve $x^2-5x+5=-1$.

  $$\begin{aligned} x^2-5x+5&=-1\\ x^2-5x+6&=0\\ (x-2)(x-3)&=0 \end{aligned}$$

  So $x=2$ or $x=3$.

  When $x=2$, the exponent is $2^2+4(2)-60=-48$, which is even.
  Therefore, when $x=2$, $(x^2-5x+5)^{x^2+4x-60}=1$.

  When $x=3$, the exponent is $3^2+4(3)-60=-39$, which is odd.
  Therefore, when $x=3$, $(x^2-5x+5)^{x^2+4x-60}=-1$. So $x=3$ is not a solution.

Therefore, the values of x that satisfy $(x^2-5x+5)^{x^2+4x-60}=1$ are $x=-10$, $x=1$, $x=2$, $x=4$ and $x=6$.