Problem of the Week
Problem E and Solution
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Problem

Cora, Dora and Flora were each in a different mathematics class last semester. The results for each class are summarized in the following table:

	# of Students in Class	Class Average
Cora's Class	22	87
Dora's Class	27	83
Flora's Class	31	81

Cora's mark was 1 mark less than Dora's mark whose mark in turn was 1 less than Flora's mark.

However, each of them discovered an error in the calculation of their final mark. After correcting the errors, Cora, Dora and Flora each had an average of 92% and the combined average for all of the students in the three classes was 83.7%.

Determine each of the student's marks before the error was corrected.

Solution

Let Cora's original mark be c. Then Dora's original mark is $c + 1$ and Flora's original mark is $c + 2$.

The total number of marks for a class can be calculated by multiplying the number of students by the class average.

The total number of marks in Cora's class before the error was discovered was $22 \times 87 = 1914$.
The total number of marks in Dora's class before the error was discovered was $27 \times 83 = 2241$.
The total number of marks in Flora's class before the error was discovered was $31 \times 81 = 2511$.

The total number of students in the three classes combined was $22 + 27 + 31 = 80$. Since the average after correcting the three errors was 83.7%, the total number of marks for the three classes was $80 \times 83.7 = 6696$.

The total mark change can be determined by subtracting the mark total for each class from before the change from the mark total for the three classes after the correction of the three marks.

The total mark change is $6696 - 1914 - 2241 - 2511 = 30$.

But the total mark change can also be calculated by subtracting each of their old marks from 92 and then adding the differences.

Therefore $[92 - c] + [92 - (c + 1)] + [92 - (c + 2)] = 30$.

$$\begin{aligned} 92 - c + 92 - (c + 1) + 92 - (c + 2) &= 30 \\ 276 - c - c - 1 - c - 2 &= 30 \\ 273 - 3c &= 30 \\ -3c &= -243 \\ c &= 81 \end{aligned}$$

Since $c = 81$, $c + 1 = 82$ and $c + 2 = 83$.

The original marks were 81% for Cora, 82% for Dora and 83% for Flora.