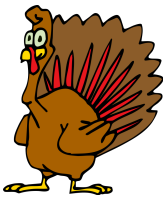

Problem of the Week
Problem D and Solution
Turkey Time

Problem

Among grandfather's papers a faded old piece of paper was found with the following written on it:

72 turkeys \$ ■67.9■

The first and the last digits of the number that represented the total price of the turkeys have been replaced here with black squares as the digits had faded and were no longer legible. Each black square represents one digit. Determine the missing digits.

Solution

Let the total price of 72 turkeys be $A679B$ cents.

We know the total value of 72 turkeys. We could find the value of 1 turkey by dividing the total value by 72. Since the total value of the turkeys is divisible by 72, it is also divisible by the divisors of 72, namely 1,2,3,4,6,8,9,12,18,24,36 and 72.

If a number is divisible by 4, the last two digits of the number are divisible by 4. Therefore $9B$ is divisible by 4. The only two digit numbers beginning with 9 that are divisible by 4 are 92 and 96. So the only possible values for B are 2 and 6. The value of the 72 turkeys is either $A6792$ or $A6796$. But the number must also be divisible by 8. To be divisible by 8, the last three digits of the number must be divisible by 8. Of the two possible numbers, 792 and 796, only 792 is divisible by 8. Therefore the last digit of the price is 2 and we now know that 72 turkeys cost $A6792$ cents.

If a number is divisible by 9, the sum of the digits of the number is divisible by 9. So $A + 6 + 7 + 9 + 2 = A + 24$ must be divisible by 9. A is a single digit from 0 to 9. The sum $A + 24$ is therefore an integer from 24 to 33. The only number in this range divisible by 9 is 27. It follows that $A + 24 = 27$ and $A = 3$.

$\therefore$ 72 turkeys cost \$367.92 and the missing digits are 3 and 2. Each turkey costs \$367.92 $\div$ 72 or \$5.11.

Note: This approach is very efficient but the solver must be careful. The number 4 divides 72 and any number that is divisible by 72. It does not follow that a number divisible by 4 is also divisible by 72. For example, 76 is divisible by 4 but not 72. In this solution we found a number divisible by both 8 and 9. Since 8 and 9 have no common factors, a number divisible by 8 and 9 is also divisible by 72.

It is possible to approach this question systematically using multiplication. In essence you can check possible values for B by performing the multiplication. In this case, since $B = 2$, the solution can be found reasonably quickly. However, in general, this would not be an effective strategy.