Problem of the Week

Problem E and Solution

Minimum Multiplier

Problem

The number 1867 is multiplied by a positive integer k. The last four digits of the product are 1992. Determine the minimum value of k.

$$\begin{array}{ccccccc} & & & 1 & 8 & 6 & 7 \\ \times & ? & ? & ? & ? & ? & ? \\ \hline & . & . & . & 1 & 9 & 9 \; 2 \end{array}$$

Solution

To begin with we will show that k has four digits or less. A number with five digits, $pqrst$ for example, can be written $p \times 10^4 + qrst = p0000 + qrst$. The digit p in the multiplier cannot affect the final four digits in the product. Therefore the minimum k is a number with four or fewer digits.

Let the multiplier be $abcd$ such that $1867 \times abcd$ is a number whose last four digits are 1992.

Then multiplying 7, the units digit of 1867, by d, the units digit in $abcd$, produces a number ending in 2. The only possible value for d is 6 since $7 \times 6 = 42$. (Note the possible last digits when 7 multiplies a single digit number: $7 \times 0 = 0,\ 7 \times 1 = 7,\ 7 \times 2 = 14,\ 7 \times 3 = 21,\ 7 \times 4 = 28,$ $7 \times 5 = 35,\ 7 \times 6 = 42,\ 7 \times 7 = 49,\ 7 \times 8 = 56,\ 7 \times 9 = 63.$) Therefore the multiplier is $abc6$.

The second last digit in the product 1992 is 9. This digit is produced by multiplying 67 from 1867 with $c6$ from $abc6$.

$$\begin{array}{ccc} & 6 & 7 \\ \times & c & 6 \\ \hline 4 & 0 & 2 \\ . & 7c & \\ \hline . & 9 & 2 \end{array}$$

So $0 + 7c$ is a number that ends in 9. The only possible value for c is 7. (Refer back to the product list given above.) Therefore the multiplier is a number of the form $ab76$. Is $k = 76$? The product, $1867 \times 76 = 141\ 892$, does not end in 1992. So k is at least a three digit number.

The third last digit in the product 1992 is 9. This digit is produced by multiplying 867 from 1867 with $b76$ from $ab76$.

$$\begin{array}{ccccc} & & 8 & 6 & 7 \\ \times & & b & 7 & 6 \\ \hline & 5 & 2 & 0 & 2 \\ 6 & 0 & 6 & 9 & \\ . & . & 7b & & \\ \hline . & . & 9 & 9 & 2 \end{array}$$

So $2 + 6 + 7b$ is a number that ends in 9 and it follows that $7b$ is a number that ends in 1. The only possible value for b is 3. (Refer back to the product list given above.) Therefore the multiplier is a number of the form $a376$. Is $k = 376$? The product $1867 \times 376 = 701\ 992$ does end in 1992. So 376 multiplied by 1867 produces a number ending in 1992. $\therefore$ the smallest value of k is 376.