Problem of the Week

Problem D

Formidable Fractions

People from the town of Formidable like to pose problems involving fractions. Here is one of their problems.

There are some positive integers a and c such that

$$\frac{\left(\frac{a}{c}+\frac{a}{2}+1\right)}{\left(\frac{2}{a}+\frac{2}{c}+1\right)}=18.$$

For example, when $a=36$ and $c=5$, the value of the numerator is

$$\frac{a}{c}+\frac{a}{2}+1=\frac{36}{5}+\frac{36}{2}+1=\frac{72}{10}+\frac{180}{10}+\frac{10}{10}=\frac{262}{10}=\frac{131}{5},$$

the value of the denominator is

$$\frac{2}{a}+\frac{2}{c}+1=\frac{2}{36}+\frac{2}{5}+1=\frac{1}{18}+\frac{2}{5}+1=\frac{5}{90}+\frac{36}{90}+\frac{90}{90}=\frac{131}{90},$$

and the left side of the equation simplifies to

$$\frac{\left(\frac{a}{c}+\frac{a}{2}+1\right)}{\left(\frac{2}{a}+\frac{2}{c}+1\right)}=\frac{131}{5}\div\frac{131}{90}=\frac{131}{5}\times\frac{90}{131}=18.$$

Since 18 is also the value of the right side of the equation, then the ordered pair $(a,c)=(36,5)$ satisfies the given equation.

Many ordered pairs satisfy the given equation so we will add a restriction.

Determine the total number of ordered pairs (a,c) that satisfy the equation such that $a+3c\leq 99$.