



Problem of the Week

Problem E and Solution

A Simple System of Equations

Problem

If $2^{3x} = 16^{y+1}$ and $2x = 5y - 17$, determine the value of $x + y$. Try to see if doing this problem without a calculator affects how you think about the problem.

Solution

Solution 1

For this solution we will attempt to answer the question only. We will not determine the values of x and y that generate the sum.

$$2^{3x} = 16^{y+1}$$

$$2^{3x} = (2^4)^{y+1}$$

$$2^{3x} = 2^{4y+4}$$

$$\therefore 3x = 4y + 4 \quad (1)$$

$$\text{But } 2x = 5y - 17 \quad (2)$$

$$(1) - (2) \quad x = -y + 21$$

$$\text{Rearranging } x + y = 21$$

$\therefore x + y = 21$. Notice that the problem only asks for $x + y$, it is not necessary to find values for x and y .

Solution 2

This solution carries on from equations (1) and (2) in solution 1 to find the values of x and y , and then determines the sum.

$$(1) \times 5 \quad 15x = 20y + 20 \quad (3)$$

$$(2) \times 4 \quad 8x = 20y - 68 \quad (4)$$

$$(3) - (4) \quad 7x = 88$$

$$x = \frac{88}{7}$$

$$\text{Substitute in (2)} \quad 2\left(\frac{88}{7}\right) = 5y - 17$$

$$\text{Multiply by 7} \quad 176 = 35y - 119$$

$$295 = 35y$$

$$y = \frac{295}{35} = \frac{59}{7}$$

$$\therefore x + y = \frac{88}{7} + \frac{59}{7} = \frac{147}{7} = 21$$

As before (but with much more work) $x + y = 21$.

