



Problem of the Week

Problem D

Formidable Fractions

People from the town of Formidable like to pose problems involving fractions. Here is one of their problems.

There are some positive integers a and c such that

$$\frac{\left(\frac{a}{c} + \frac{a}{2} + 1\right)}{\left(\frac{2}{a} + \frac{2}{c} + 1\right)} = 18.$$

For example, when $a = 36$ and $c = 5$, the value of the numerator is

$$\frac{a}{c} + \frac{a}{2} + 1 = \frac{36}{5} + \frac{36}{2} + 1 = \frac{72}{10} + \frac{180}{10} + \frac{10}{10} = \frac{262}{10} = \frac{131}{5},$$

the value of the denominator is

$$\frac{2}{a} + \frac{2}{c} + 1 = \frac{2}{36} + \frac{2}{5} + 1 = \frac{1}{18} + \frac{2}{5} + 1 = \frac{5}{90} + \frac{36}{90} + \frac{90}{90} = \frac{131}{90},$$

and the left side of the equation simplifies to

$$\frac{\left(\frac{a}{c} + \frac{a}{2} + 1\right)}{\left(\frac{2}{a} + \frac{2}{c} + 1\right)} = \frac{131}{5} \div \frac{131}{90} = \frac{131}{5} \times \frac{90}{131} = 18.$$

Since 18 is also the value of the right side of the equation, then the ordered pair $(a, c) = (36, 5)$ satisfies the given equation.

Many ordered pairs satisfy the given equation so we will add a restriction.

Determine the total number of ordered pairs (a, c) that satisfy the equation such that $a + 3c \leq 99$.

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