Problem of the Week
Problem D and Solution
Walking is Good Exercise

Problem

Ali, Bill and Carl are lined up such that Ali is 100 m west of Bill and Carl is 160 m east of Bill. At noon, Carl begins to walk north at a constant rate of 41 $\frac{\text{m}}{\text{min}}$ and Ali walks south at a constant rate of 20 $\frac{\text{m}}{\text{min}}$. (Bill does not move.) At what time will the distance between Carl and Bill be the twice the distance between Ali and Bill?

Solution

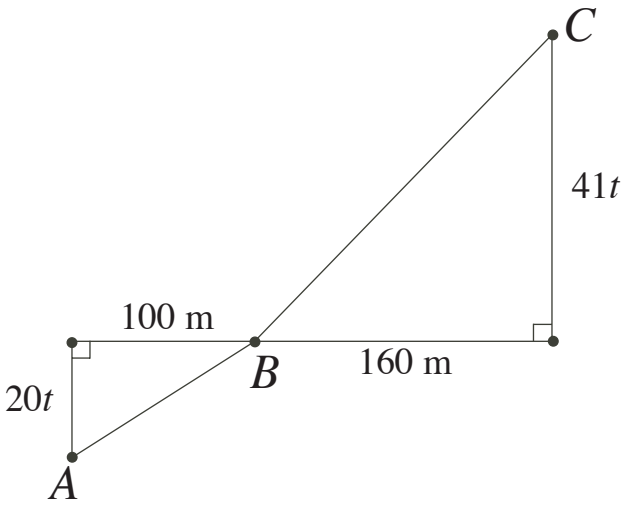
Solution 1

Let t represent the number of minutes until Carl's distance to Bill is twice that of Ali's distance to Bill.

In t minutes Ali will walk $20t$ m and Carl will walk $41t$ m. The following diagram contains the information showing Ali's position at A and Carl's position at C at time t.

In the diagram both triangles are right triangles and we can use the Pythagorean Theorem to set up an equation.

$$\begin{aligned}
BC &= 2AB \\
(BC)^2 &= (2AB)^2 \\
(BC)^2 &= 4\,(AB)^2 \\
(41t)^2 + (160)^2 &= 4\left[(20t)^2 + (100)^2\right] \\
1681t^2 + 25600 &= 4\left[400t^2 + 10000\right] \\
1681t^2 + 25600 &= 1600t^2 + 40000 \\
81t^2 &= 14400 \\
t^2 &= \frac{14400}{81} \\
t &= \frac{120}{9} \text{ since } t > 0 \\
t &= \frac{40}{3} \text{ min}
\end{aligned}$$



$\therefore$ in $13\frac{1}{3}$ minutes (13 minutes 20 seconds), Carl's distance to Bill will be twice that of Ali's distance to Bill.

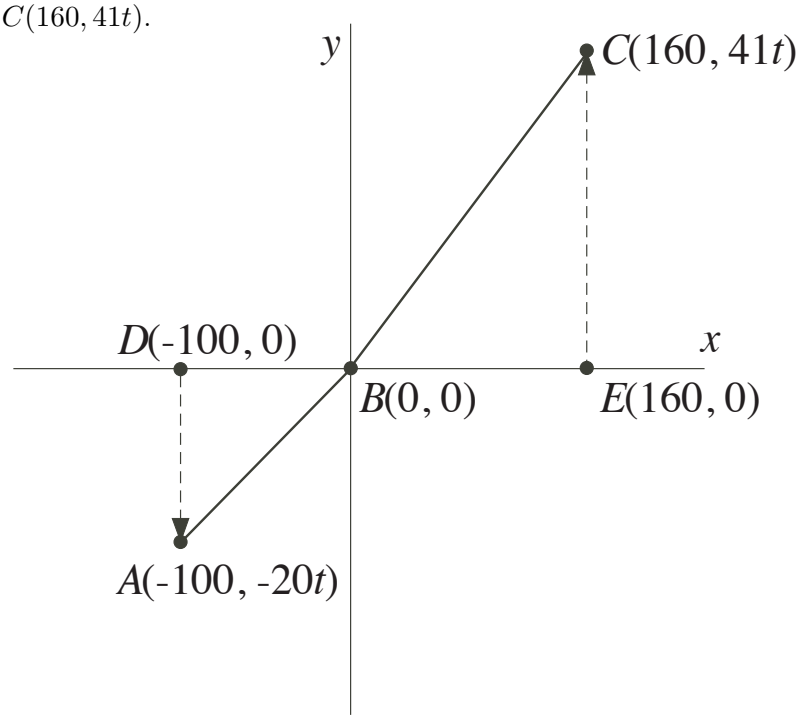
In the second solution coordinate geometry will be used to solve the problem.

Solution 2

Represent Ali, Bill and Carl's respective positions at noon as points on the x-axis so that Bill is positioned at the origin $B(0,0)$, Ali is positioned 100 units left of Bill at $D(-100,0)$ and Carl is positioned 160 units right of Bill at $E(160,0)$.

Let t represent the number of minutes until Carl's distance to Bill is twice that of Ali's distance to Bill.

In t minutes Ali will walk south $20t$ m to the point $A(-100,-20t)$. In t minutes Carl will walk north $41t$ m to the point $C(160,41t)$.



The distance from a point $P(x,y)$ to the origin can be found using the formula $d=\sqrt{x^2+y^2}$.

Then $AB=\sqrt{(-100)^2+(-20t)^2}=\sqrt{10000+400t^2}$ and $CB=\sqrt{(160)^2+(41t)^2}=\sqrt{25600+1681t^2}$.

$$\begin{aligned} CB &= 2AB \\ \sqrt{25600+1681t^2} &= 2\sqrt{10000+400t^2} \\ \text{Squaring both sides,}\quad 25600+1681t^2 &= 4(10000+400t^2) \\ 25600+1681t^2 &= 40000+1600t^2 \\ 81t^2 &= 14400 \\ t^2 &= \frac{14400}{81} \\ t &= \frac{120}{9} \text{ since } t>0 \\ t &= \frac{40}{3} \text{ min} \end{aligned}$$

$\therefore$ in $13\frac{1}{3}$ minutes (13 minutes 20 seconds), Carl's distance to Bill will be twice that of Ali's distance to Bill.