



Problem of the Week

Problem D and Solution

Creating Cubes

Problem

A wooden cube is cut into n cubes each of edge length 1 unit. The combined surface area of the n cubes is eight times the surface area of the original uncut cube. Determine the edge length of the original uncut cube.

Solution

Let the edge length of the uncut cube be x units, $x > 0$. After cutting the original cube into n cubes of edge length 1 unit, there will be x^3 cubes of edge length 1 unit. It follows that $n = x^3$.

Each of the 6 sides of the original cube has area x^2 so the total surface area of the original cube is $6x^2$.

Consider one of the smaller cubes. The surface area of one the six faces is 1 unit². So the total surface area of one of these smaller cubes is 6 units². The total surface area of the n smaller cubes is $(6n)$ units².

Since the total surface area of the n cubes is eight times the surface area of the original uncut cube,

$$\text{new surface area} = 8 \times \text{original surface area}$$

$$6n = 8(6x^2)$$

$$\text{Dividing both sides by 6,} \quad n = 8x^2$$

$$\text{But } n = x^3 \text{ so} \quad x^3 = 8x^2$$

$$\text{Dividing by } x^2 \text{ since } x > 0, \quad x = 8$$

Therefore, the edge length of the original uncut cube was 8 units.

Extension:

Note, that if the combined surface area of the n cubes of edge length 1 unit was Q times the surface area of the original uncut cube, then the edge length of the original uncut cube would have been Q units. Can you see why?

