Problem of the Week
## Problem E
Double Check

Debit and credit cards contain account numbers which consist of many digits. Often, when purchasing items online you are asked to type in your account number. Because there are so many digits it is easy to type the number incorrectly. The last digit of the number is a "check digit" which can be used to quickly verify the validity of the number. A common algorithm used for verifying that numbers have been entered correctly is called the Luhn Algorithm. A series of operations are performed on the number and a final result is produced. If the final result ends in zero, the number is valid. Otherwise, the number is invalid.

The steps performed in the Luhn Algorithm are outlined in the flowchart below. Two examples are provided.

Luhn Algorithm
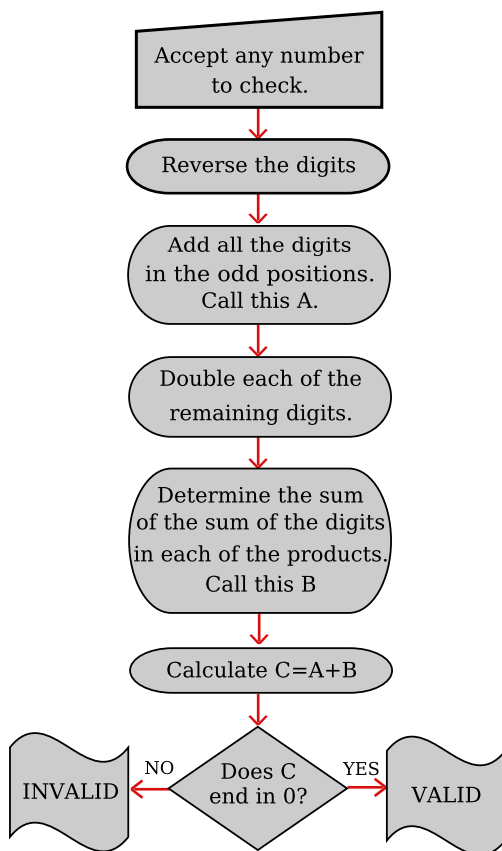
Example 1

Number: 135792

Reversal: 297531

$A = 2 + 7 + 3$
  $= 12$

$2 \times 9 = 18$
$2 \times 5 = 10$
$2 \times 1 = 2$

$B = (1+8)+(1+0)+2$
  $= 9 + 1 + 2$
  $= 12$

$C = 12 + 12$
  $= 24$

C does not end in zero.
The number is not valid.

Accept any number to check.

Reverse the digits

Add all the digits in the odd positions. Call this A.

Double each of the remaining digits.

Determine the sum of the sum of the digits in each of the products. Call this B

Calculate C=A+B

NO ← Does C end in 0? → YES

INVALID   VALID

Example 2

Number: 1357987

Reversal: 7897531

$A = 7 + 9 + 5 + 1$
  $= 22$

$2 \times 8 = 16$
$2 \times 7 = 14$
$2 \times 3 = 6$

$B = (1+6)+(1+4)+6$
  $= 7 + 5 + 6$
  $= 18$

$C = 22 + 18$
  $= 40$

C ends in zero.

The number is valid.

Suppose the number 2853 0QPP 362 is a valid number when verified by the Luhn Algorithm. P and Q are each integers from 0 to 9 such that $P \le Q$. Determine all possible values of P and Q.