Problem of the Week
Problem D and Solution
A Sour Taste

Problem

A container is filled with a mixture of water and vinegar in the ratio 2:1, by volume. Another container, with twice the volume of the first container, is filled with a mixture of water and vinegar in the ratio 3:1, by volume. The contents of the two containers are emptied into a third container. Determine the ratio of water to vinegar, by volume, in the third container.

Solution

Solution 1

Let V represent the volume of the first container. Then $2V$ represents the volume of the second container.

Since the ratio of water to vinegar, by volume, in the first container is 2:1 then $\frac{2}{3}$ of the volume of the first container is water. That is, the volume of water in the first container is $\frac{2}{3}V$ and the volume of vinegar in the first container is $\frac{1}{3}V$.

Since the ratio of water to vinegar, by volume, in the second container is 3:1 then $\frac{3}{4}$ of the volume of the second container is water. That is, the volume of water in the second container is $\frac{3}{4}(2V) = \frac{3}{2}V$ and the volume of vinegar in the second container is $\frac{1}{4}(2V) = \frac{1}{2}V$.

When the contents of the two containers are combined into the third container, the volume of water is $\frac{2}{3}V + \frac{3}{2}V = \frac{4}{6}V + \frac{9}{6}V = \frac{13}{6}V$ and the volume of vinegar is $\frac{1}{3}V + \frac{1}{2}V = \frac{2}{6}V + \frac{3}{6}V = \frac{5}{6}V$.

The ratio of water to vinegar, by volume, in the third container is $\frac{13}{6}V : \frac{5}{6}V = 13 : 5$.

Solution 2

Let $\frac{1}{6}$ of the contents of the first container be a unit of volume. Since the ratio of water to vinegar, by volume, in the first container is 2:1, then 4 units of volume are water and 2 units of volume are vinegar, a total of 6 units of volume in the first container.

Since the second container has twice the volume of the first container, the second container has 12 units of volume. The ratio of water to vinegar, by volume, in the second container is 3:1 so 9 units of volume are water and 3 units of volume are vinegar.

When the two containers are combined there is a total of $6 + 12 = 18$ units of volume, $4 + 9 = 13$ of which are water and $2 + 3 = 5$ of which are vinegar.

$\therefore$ the ratio of water to vinegar, by volume, in the third container is $13 : 5$.