Problem of the Week
Problem E and Solution
Five Solutions - Really?

Problem

There are five values of x that satisfy the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$. Determine these five values of x.

Solution

Let's consider the ways that an expression of the form a^b can be 1:

- The base, a, is 1.

  In this case, the exponent can be any value and we need to solve $x^2 - 5x + 5 = 1$.

  $$x^2 - 5x + 5 = 1$$
  $$x^2 - 5x + 4 = 0$$
  $$(x-4)(x-1) = 0$$

  So $x = 4$ or $x = 1$.

- The exponent, b, is 0.

  In this case, the base can be any number other than 0 and we need to solve $x^2 + 4x - 60 = 0$.

  $$x^2 + 4x - 60 = 0$$
  $$(x-6)(x+10) = 0$$

  So $x = 6$ or $x = -10$.

  When $x = 6$, the base is $6^2 - 5(6) + 5 = 11 \neq 0$. That is, when $x = 6$, the base does not equal 0.
  When $x = -10$, the base is $(-10)^2 - 5(-10) + 5 = 155 \neq 0$. That is, when $x = -10$, the base does not equal 0.

- The base, a, is -1 and the exponent, b, is even.

  We first need to solve $x^2 - 5x + 5 = -1$.

  $$x^2 - 5x + 5 = -1$$
  $$x^2 - 5x + 6 = 0$$
  $$(x-2)(x-3) = 0$$

  So $x = 2$ or $x = 3$.

  When $x = 2$, the exponent is $2^2 + 4(2) - 60 = -48$, which is even.
  Therefore, when $x = 2$, $(x^2 - 5x + 5)^{x^2+4x-60} = 1$.

  When $x = 3$, the exponent is $3^2 + 4(3) - 60 = -39$, which is odd.
  Therefore, when $x = 3$, $(x^2 - 5x + 5)^{x^2+4x-60} = -1$. So $x = 3$ is not a solution.

Therefore, the values of x that satisfy $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ are $x = -10$, $x = 1$, $x = 2$, $x = 4$ and $x = 6$.