Finans 1 – Föreläsningsanteckningar

Företagsekonomi I (Stockholms Universitet)

Finans 1 – Föreläsningsanteckningar

Föreläsning 2 – Time value of money – Chapter 4

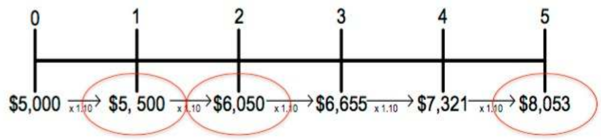
Road Map of this lecture

- Present value and future value over more than one year: one cash-flow
- Present value and future value: a stream of cash flows.
- Special cases:
  → Annuities
  → Perpetuities

The second rule of time travel: Example

- Suppose you have a choice between receiving 5,000 today or 10,000 in five years. You believe you can earn 10% yearly on the 5,000 and you want to know what the 5,000 will be worth in five years.

- The calculations go as follows:



- In five years, the $5,000 will grow to:
  $\$5{,}000 \times (1.10)^5 = \$8{,}053$
- The future value of $5,000 at 10% for five years is $8,053.
- You would be better off forgoing the gift of $5,000 today and taking the $10,000 in five years.

The third rule of Time travel: Another Example

- Suppose you are offered an investment that pays 10,000 SEK in five years. If you expect to earn a 10% return, what is the value of this investment today?

- Solution:

$$X \times (1+0.1)^5 = 10000$$

$$X = \frac{10000}{(1+0.1)^5} = 6{,}209$$

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- Consider the following alternatives:
  1. 132$ received in two years
  2. 160$ received in five years
  3. 200$ received in eight years
  4. 220$ received in ten years
- What would be the ranking of the four alternatives from most valuable to least valuable if the interest rate is 7% per year?
  A. 1,2,3,4
  B. 4,3,2,1
  C. 3,4,2,1
  D. 3,1,2,4

D

The present value of a stream of cash flows: Example

- Would you be willing to pay 5,000 for the following stream of cash flows if the discount rate is 7%?

- The present value of this stream of cash flows is:

$$\frac{3000}{(1+0.07)}+\frac{2000}{(1+0.07)^2}+\frac{1000}{(1+0.07)^3}=5366.91$$

Yes!!!!!

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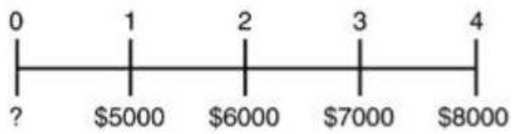
- Consider following timeline detailing a stream of cash flows:



- If the interest rate is 8%, what is the present value of this stream of cash flows?
  A. 26,000$
  B. 21,211$
  C. 22,871$
  D. 24,074$

B

The present value of a stream of cash flows

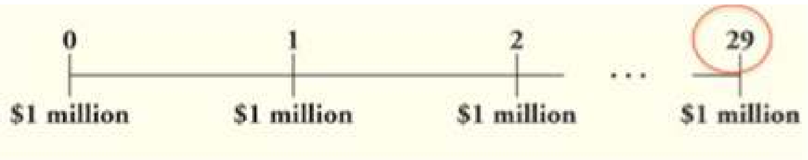
- There are special cases where things are much simpler

  <u>Perpetuity</u>
  → Constant
  → Growing
  <u>Annuity</u>
  → Constant
  → Growing

Annuities: Example

- Suppose you are the lucky winner of a lottery and you can choose to receive the prize amount of 15 million today or instead be given 30 payments of 1 million a year <u>starting today</u>.

Interest rate/Discount code: 8%

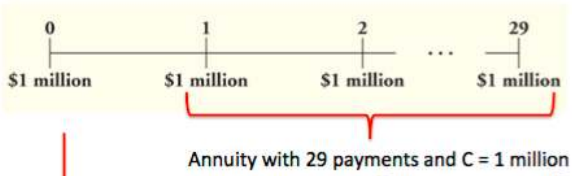


$$PV\,(Annuity) = C\frac{1}{r}\left[1 - \frac{1}{(1+r)^N}\right]$$

$r = 0.08$

$N = 29$

$C = 1{,}000{,}000$



$$1{,}000{,}000 + \quad 1,000,000 * \frac{1}{0.08}\left[1 - \frac{1}{(1+0.08)^{29}}\right]$$

- PV(cash flows) = 12.16 million

○A. $9,818

○B. $18,519

○C. $20,000

○D. $45,761

Question code 30 88 72 at menti.com

- If the interest rate is 8%, then the present value of an investment that pays 1000 per year and lasts 20 years is closest to:

A

Question code 52 48 72

- An investment requires an initial payment of 475 million SEK. It also provides a benefit at the end of every year of 100 million SEK for 5 years. Suppose the interest rate is 0%. What is its NPV?

100 million x 5 years= 500 million.
500-475= 25 million.

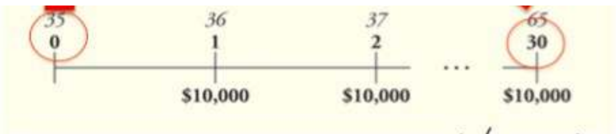
The present value of annuity when r=0

- The full expression of the present value is

$$PV(annuity) = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+r)^N} = \quad \boxed{r=0}$$

$$= \frac{C}{1} + \frac{C}{1^2} + \cdots + \frac{C}{1^N} =$$

$$= C + C + \cdots + C = C \times N$$



$$FV(annuity) = PV(annuity)\,(1+r)^N = C\frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)(1+r)^N =$$

$$= 10{,}000\frac{1}{0.1}\left(1 - \frac{1}{1.1^{30}}\right)(1+0.1)^{30} = 1.645 \text{ million}$$

Future Value of an Annuity: Example

Ellen is 35 years old, and she has decided it is time to plan seriously for her retirement. At the end each year until she is 65, she will save 10,000 in a retirement account. If the account earns 10% per year, how much will Ellen have saved at age 65?

Growing Annuities: Example

- Ellen considered saving 10,000 per year for her retirement. However, she expects her salary to increase each year so she will be able to increase her saving by 5% per year. With this plan, if she earns 10% per year on her savings how much will Ellen have saved at age 65?

- The full expression of the present value is

$$PV(annuity) = \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + \cdots + \frac{C(1+g)^{N-1}}{(1+r)^N} = \boxed{g=r}$$

$$= \frac{C}{1+r} + \frac{C\cancel{(1+r)}}{(1+r)^{\cancel{2}}} + \cdots + \frac{C\cancel{(1+r)^{N-1}}}{(1+r)^{\cancel{N}}} =$$

$$= \frac{C}{1+r} + \frac{C}{1+r} + \cdots + \frac{C}{1+r} = N\frac{C}{1+r}$$

Question code 88 97 33

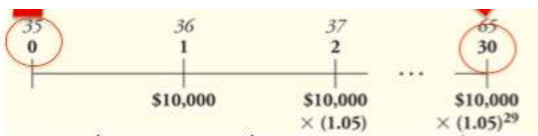
- An investment requires an initial payment of 475 million SEK. It also provides a benefit at the end of the first year of 100 million SEK which grows at a 5% rate during subsequent 4 years (that is, life of the project is 5 years). Suppose the interest rate is 5%. What is its NPV?

The present value of a growing annuity when g=r

$$PV(\text{Perpetuity}) = \frac{C}{r} = \frac{\$100{,}000}{.04} = \$2{,}500{,}000$$

Perpetuities: Example

- You are a wealthy tycoon who wants to donate money to the Nobel Prize Foundation. You would like to donate an amount which could represent a funding of 100,000 per year. If the risk-free rate is 4% annually, how much will you need to donate?



$$PV = C\frac{1}{(r-g)}\left(1-\left(\frac{1+g}{(1+r)}\right)^N\right) = 10{,}000\frac{1}{0.10-0.05}\left(1-\left(\frac{1.05}{1.10}\right)^{30}\right) = 150{,}463$$

$$FV = 150{,}463 * (1+0.10)^{30} = 2.625 \text{ million}$$

Growing Perpetuities: Example

You are again the wealthy tycoon who wants to donate money to the Nobel Prize Foundation. You would like to donate an amount which could represent a funding of $100,000 per year. The Foundation has asked you to increase the donation to account for the effect of inflation, which is expected to be 2% per year. How much will you need to donate to satisfy that request? (r=4%)

The timeline of the cash flows looks like this:

Question code 67 71 28 at menti.com

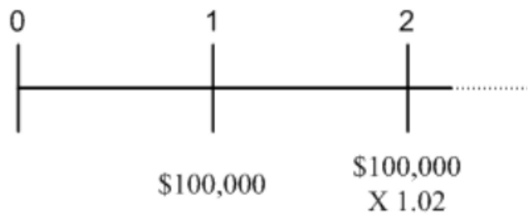
- You are thinking about investing in a mine that will produce 10,000 worth of ore in the first year. The value of the ore that you mine will decline at a rate of 8% per year forever. If the appropriate interest rate is 6%, then the value of this mining operation is closest to:

Question code 63 09 58

- You are considering buying a new home. You will need to borrow 250,000 to purchase the home. A mortgage company offers you a 15-year fixed rate at 0.75% per month. If you borrow the money from this mortgage company, your monthly payment will be closest to:

Road map

- Present value and future value over more than one year: one cash-flow
- Present value and future value: a stream af cash flows
- Special cases:
  → Annuities
  → Perpetuities

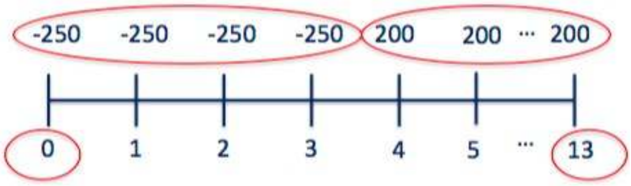


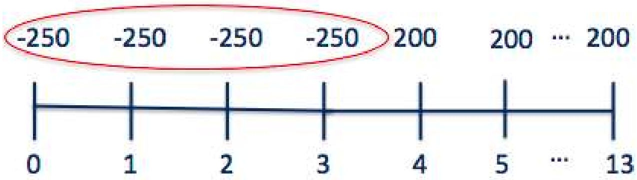
$$PV = \frac{C}{r-g} = \frac{\$100{,}000}{.04-.02} = \$5{,}000{,}000$$

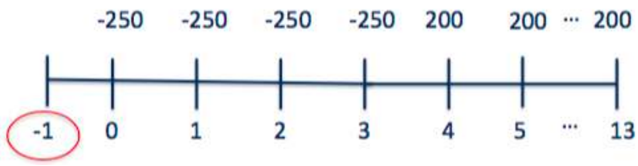
Lecture 3 – Investment Decision Rules

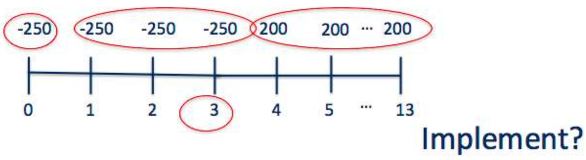
The NPV of a stand-Alone project:
Example:

- Boulderado has come up with a new composite snowboard. Development will take four years and cost 250,000 per year, with the first of the four equal investment payable today upon acceptance of the project. Once production (starting at the end of year four) the snowboard is expected to produce annual cash flows of 200,000 each year for 10 yeas. The company´s discount rate is 10%. What is the NPV?









$$NPV = -250 - 250\frac{1}{0.1}\left[1 - \frac{1}{(1+0.1)^3}\right]$$

$$+ 200\frac{1}{0.1}\left[1 - \frac{1}{(1+0.1)^{10}}\right]\frac{1}{(1+0.1)^3} = 51.59$$

Question code 10 22 67

- The IRR of the snowboard project is closest to:
- A: 15.1%
- B: 10.0%
- C: 11.0%
- D: 12.0 %

Put all the possible r in to the formula above, but put NPV=0 and take away the r above. Then compare, the one who's closest to 0 is the "winner".

Question code 24 05 52

- Recall that the estimated cost of capital of the project is 10%. What is the maximum deviation upwards allowable in the estimate of the cost of capital of the snowboard project that leaves the investment decision uncharged?

- A: 2.5%
- B: 11.0%
- C: 1.0%
- D: 0.0%

Another example of a "well-behaved" project

- Consider a project that requires an initial investment f 250 million SEK and from the first year will have a positive cash-flow equal to 35 forever.
- The NPV of this project will be:

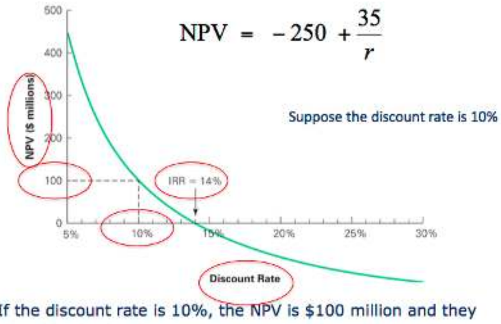
$$\text{NPV} = -250 + \frac{35}{r}$$

- The IRR can be obtained by solving for *r*

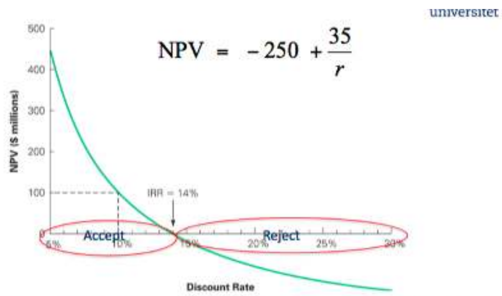
$$0 = -250 + \frac{35}{r} \qquad 250 = \frac{35}{r}$$

$$r = \frac{35}{250} = 0.14$$

Example: a <<well-behaved>> project



- If the discount rate is 10%, the NPV is $100 million and they should undertake the investment.



Whenever the discount rate (cost of capital) is below the IRR of 14%, the project has a positive NPV and you should undertake the investment.

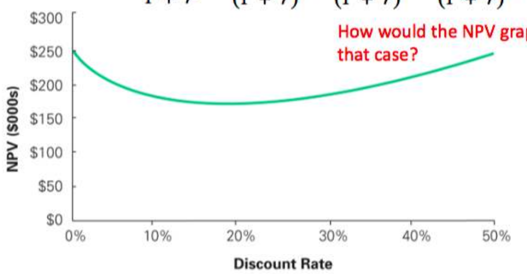
Another project

$$NPV = -500 + \frac{850}{1+r} + \frac{125}{(1+r)^2} - \frac{500}{(1+r)^3}$$

And one more project

$$NPV = 750{,}000 - \frac{500{,}000}{1+r} - \frac{500{,}000}{(1+r)^2} - \frac{500{,}000}{(1+r)^3} + \frac{1{,}000{,}000}{(1+r)^4} = 0$$

How would the NPV graph be in that case?



No IRR exists because the NPV is positive for all values of the discount rate. Thus the IRR rule cannot be used.

Question code 75 87 71

Which statement is **FALSE**?

– A: The IRR will always identify the correct investment decision
– B: The IRR investment rule states that you should turn down any investment opportunity where the IRR is less than the appropriate discount rate
– C: The IRR investment rule states that you should take any investment opportunity where IRR exceeds the appropriate discount rate
– D: There are situations in which multiple IRR' s exists
– E: There are situations in which the IRR does not exist

The payback rule: Example

- Project 1,2 and 3 each have an expected life of 5 years.

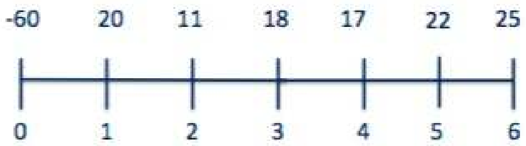
	1	2	3
Cost	$80	$120	$150
Cash Flow	$25	$30	$35

- *What is the payback period for each project?*

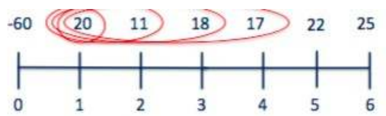
- Payback 1
  - $80 ÷ $25 = 3.2 years
- Project 2
  - $120 ÷ $30 = 4.0 years
- Project 3
  - $150 ÷ $35 = 4.29 years
- If the required payback period is 4 years or less, what project/s are to be rejected?

Payback period: Uneven cash flows

- Consider a project under consideration which has an initial outlay and future cash flows in accordance with the following timeline (in million SEK):



- *What is the payback period of this project?*



4 years?

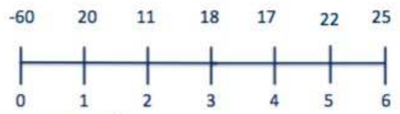
- We need to recover 60 million SEK

1 year	2 years	3 years	4 years
20	20 + 11 = 31	20 + 11 + 18 = 49	20 + 11 + 18 + 17 = 66

5/8/19 Iñaki Rodríguez School of Business

NO!

Payback period: Uneven cash flows - Summery

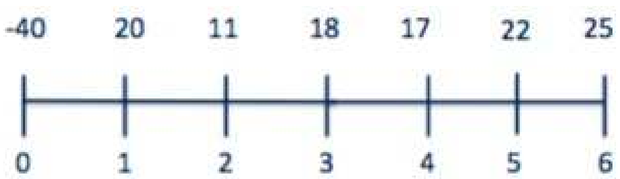


- Let us summarize
  - We need to recover 60 million SEK
  - In 3 years we reach 49
  - We only need 11 million more
  - During year 4 we get 17
  - Payback period must be between 3 and 4 years
- What fraction of a year will give us the 11 we need if we are getting 17 over the entire 4th year?

$$\frac{11}{17} = 0.65 \qquad \text{Payback period} = 3 + 0.65 = 3.65 \text{ years}$$

Question code 11 41 64

- Consider a project under consideration which has an initial outlay and future cash flows in accordance with the following timeline (in million SEK):



- What is the payback period of this project?
  A: 3.00
  B: 2.00
  C: 3.65
  D: 2.50

D…

Equivalent Annual Annuity (EEA): Example:

- Suppose r=8%

Project	Project A	Project B
N	5	7
NPV	11.38	12.38

$$EAA = \frac{NPV}{\frac{1}{r}\left[1 - \frac{1}{(1+r)^N}\right]}$$

$$\underbrace{\frac{1}{r}\left[1 - \frac{1}{(1+r)^N}\right]}_{\text{PV of an annuity with C = 1}} = \frac{1}{0.08}\left[1 - \frac{1}{1.08^5}\right] = 3.99272$$

- Suppose r=8%

Project	Project A	Project B
N	5	7
NPV	11.38	12.38
Denominator	3.99272	

$$EAA = \frac{NPV}{\frac{1}{r}\left[1 - \frac{1}{(1+r)^N}\right]}$$

$$EEA = \frac{11.38}{3.99272} = 2.8502$$

Question code 19 09 23

- The Equivalent Annual Annuity of Project B is closest to:
  A: 2.38
  B:3.24
  C: 1.27
  D:4.54

Question code 51 48 6

- A company is about to choose between investment A and B. The investments are mutually exclusive, i.e. only one investment can be implemented. The expected cash flows of the two investments are shown below.

	Investment 1	Investment 2
– Initial outlay	SEK 15,000	SEK 22,000
– Net cash flow year 1	SEK 5,000	SEK 10,000
– Net cash flow year 2	SEK 6,000	SEK 5,000
– Net cash flow year 3	SEK 10,000	SEK 12,000

- If the discount rate is 10%, what project should be chosen and why?
  - A: 2 because it has the highest EAA
  - B: 2 because it has the highest payback period
  - C: 1 because it has the highest NPV and it is positive
  - D: None of them because both NPV´s are negative

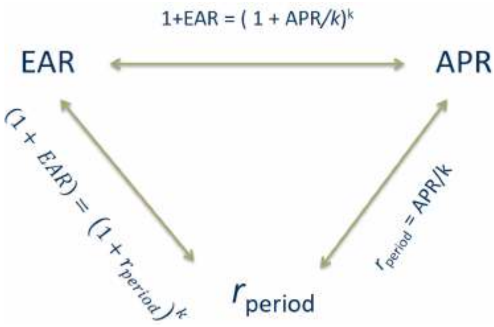
Road Map

- NPV rule and its alternatives:
  → Internal rate of return (IRR) and the IRR rule
  → Payback period and the payback rule

- Choosing between mutually exclusive projects
- The Equivalent Annual Annuity (EAA)

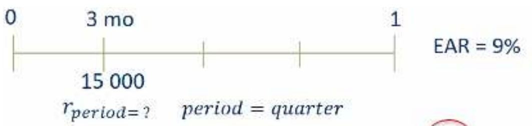
Föreläsning 4 - Interest Rates

Road Map
- Effective Annual Rate (EAR) and Annual Percentage Rate (APR)
- Nominal Rates and Real Rates
- Taxes and interest rates
- Risk and interest rates



The link between the rates
- r_{period}
- EAR & r_{period} —> $(1 + EAR) = 1 + r_{period})^k$
- APR & r_{period} —> $APR / k = r_{period}$

The EAR: Example
- Suppose we are receive a cash-flow of 15 000 SEK in three months. The EAR is equal to 9%. What is the present value of this cash-flow?



$$(1+EAR) = (1+r_{period})^{k} \qquad k=4$$

$$(1\ +\ \text{EAR}) = (1+r_{quarter})^{4}$$

$$(1\ +\ 0.09)^{1/4} = 1+r_{quarter}$$

$$\frac{15{,}000}{1\ +\ r_{quarter}} = \frac{15{,}000}{(1\ +\ 0.09)^{1/4}} = 14{,}680.29$$

$$(1+EAR) = (1+r_{period})^{k}$$

$$(1\ +\ \text{EAR}) = (1+r_{quarter})^{4}$$

$$(1\ +\ 0.09)^{1/4} = 1+r_{quarter}$$

$$r_{quarter} = (1\ +\ 0.09)^{1/4} - 1 \quad = 0.021778$$

$$\frac{15{,}000}{1\ +\ r_{quarter}} = \frac{15{,}000}{1\ +\ 0.021778} = 14{,}680.29$$

Question 1
- you are expecting two payoffs of 15 000 SEK and 30,000 to be received in 6 months and one year, respectively. What is the present value of your payoffs if the monthly discount rate is 0.5%? What is the EAR?

A. 44 627,61 and 3.04%
B. 42 814.93 ND 6.17%

$$PV = \frac{15000}{(1+0.005)^{6}} + \frac{30000}{(1+0.005)^{12}} = 42814.93$$

$$(1\ +\ \text{EAR}) = (1+r_{period})^{n} \qquad period = month \qquad n=12$$

$$(1\ +\ \text{EAR}) = (1+0.005)^{12} \Rightarrow \text{EAR} = (1+0.005)^{12} - 1 \quad = 0.0617$$

$$PV = \frac{15000}{(1+0.0617)^{1/2}} + \frac{30000}{(1+0.0617)^{1}} = 42814.93$$

C. 40 744.15 and 6.17%
D. 45 000 and 0.5%

—> yearly

The EAR: Example

Suppose that the EAR is 9%. If you have no money in the bank today, how much will you need to save at the end of **each quarter** to accumulate 25,000 SEK in 5 years?

$$FV(Annuity) = \frac{C}{r}\left((1+r)^N - 1\right)$$

$$FV(Annuity) = \frac{C}{r_{quarter}}\left((1+r_{quarter})^N - 1\right)$$

$$25000 = \frac{C}{r_{quarter}}\left((1+r_{quarter})^N - 1\right)$$

$$C = \frac{25{,}000}{\frac{1}{0.021778}[1.021778^{20} - 1]} = 1{,}010.82$$

N = 5 years * 4 = 20 quarters/periods
$r_{quarter} = 0.021778$ (2.18%)

Question 2
Suppose an investment requires an initial outlay of 30,000 SEK and then produces payoffs equivalent to an annuity with quarterly payoffs of 2000 SEK over 5 years. The Effective Annual Rate is equal to 7%. What is the NPV of the project?
A. - 8 811.97
E. - 21 799.61
F. 33 650.49
G. 3 650.49

project? $(1 + EAR) = (1 + r_{period})^n$

Period = Quarter (Three Months) $n = 4$

$$1+0.07 = (1+r_{quarter})^4$$

$$\left[1+0.07\right]^{1/4} = \left[(1+r_{quarter})^4\right]^{1/4}$$

$$\left[1+0.07\right]^{1/4} = 1 + r_{quarter}$$

$$r_{quarter} = \left[1+0.07\right]^{1/4} - 1 = 0.01706 \quad (1.706\%)$$

– What is the NPV of the project? $r_{quarter} = 0.01706$

$$PV(Annuity) = C\frac{1}{r}\left[1 - \frac{1}{(1+r)^N}\right]$$

$N = 4 \times 5 = 20$

$$PV(Annuity) = 2000\frac{1}{0.01706}\left(1 - \frac{1}{(1+0.01706)^{20}}\right)$$

NPV = - 30 000 + 33 650.49 = 3 650.49

The Annual Percentage Rate (APR): An Example with a loan

- Suppose that we would like to buy a car whose price is 30,000. We can pay the car in 60 equal monthly payments computed using a 6.75% **APR** with **monthly compounding**

$$PV\ (Annuity) = C\frac{1}{r}\left[1 - \frac{1}{(1+r)^N}\right]$$

- What is the monthly payment?
- We need to fins the C of an annuity whose present value is 30,000

- We need the equivalent monthly discount rate.
  - We obtain it from the APR as 6.75% / 12 = 0.5625%

$$PV\ (Annuity) = C\frac{1}{r}\left[1 - \frac{1}{(1+r)^N}\right]$$

The APR: An Example with a loan

- Once again, the present value of the corresponding annuity

$$30{,}000 = C\,\frac{1}{0.005625}\quad 1 \quad \frac{1}{(1\ +\ 0.005625)^{60}} \div$$

$$C\ =\ \frac{30{,}000}{\frac{1}{0.005625}\quad 1 \quad \frac{1}{(1\ +\ 0.005625)^{60}} \div}\ =\ 590.50$$

- The equivalent monthly discount rate is 0.5625 % and N = 60
- We want to find C

Amount	Adm. Cost	Cost of bill.	Inter. payment	Total
1,000	*350*	*45*	*81*	*1,476*

Many companies offers loan through sending an SMS - Example
An example of such an offer: returning customers - 30 -day loan
The ”30-day loan” implies that you should pay all costs after 30 days (at least in this example). “Total” is the sum of repaying the loan and all costs that you need to pay for.

- Compute the EAR and the APR

Monthly rate (without extra costs): 81 / 1000 = 0.081 —> 8.1% —> 1000(1 + 0.081) = 1081
APR = 8.1% * 12 = 97.20 %
EAR: 1 + EAR = $(1 + 0.081)^{12}$ = 2.5463 —> EAR = 2.5463 —> 1 = 1.5463 —> 154.63%

Amount	Adm. Cost	Cost of bill.	Inter. payment	Total
1,000	*350*	*45*	*81*	*1,476*

Question 3

- Compute the APR and EAR <u>if you treat all costs as interest</u> payments

H. 47.6% and 105.914 %
I. 571.2% and 1059.14%
J. 476% and 1059.14%
K. 571.2% and 10 591.4%

Interest payment is: 81 + 350 + 45 = 476
Equivalent monthly rate: 476 / 1000 = 0.476 (47.6%) —> APR = 47.6% * 12 = 571.2%
EAR: 1 + EAR = $(1 + 0.476)^{12}$ = 106.914 —> EAR = 106.914 - 1 = 105.914 (10 591.4%)

Real and Nominal interest rates

$$\frac{1+r}{1+i} - 1 = r_r \Rightarrow \frac{1+r}{1+i} - \frac{1+i}{1+i} = r_r \Rightarrow \frac{\cancel{1}+r-(\cancel{1}+i)}{1+i} = r_r$$

The Real Interest Rate:

$$\boxed{r_r = \frac{r-i}{1+i} \approx r-i}$$

- Nominal interest rate: Growth in *money* if invested for a certain period
- Real interest rates: Growth in *purchasing power* after adjusting for inflation
- Growth in

Purchasing Power = Growth of Money / Growth of Prices = 1 + r / 1 + I = 1 + r_r

Real and Nominal interest rates: Example

$$1 + r_r = \frac{1+r}{1 + i} \Longrightarrow r_r = \frac{1+r}{1 + i} - 1 = \frac{r \quad i}{1 + i}$$

- If the nominal interest rate and inflation rate in 2008 had been 1.82% and 0.28% respectively, **what would have been the real interest rate in 2008?**

$$r_r = \frac{r \quad i}{1 + i} = \frac{0.0182 \quad 0.0028}{1 + 0.0028} = 0.0154 \quad (1.54\%)$$

Question 4

- In 1975, interest rates were 7.88% and the rate of inflation was 12.29% in US
- **What was the real interest rate in US in 1975?**

A. 12.29%
L. - 4.2%
M. 7.88%
N. - 3.93%

Taxes and interest rates

If you earn interest on savings (or pay interest on borrowing) taxes reduces the amount that you can keep (or have to pay).

After –Tax Interest Rate

$$r - (\tau \times r) = r(1 - \tau)$$

□ = □□□□□□□□ □□□□ (□□□ □□□□)
□ = □□□ □□□□

Taxes and Interest rates: Example

- Suppose that the nominal EAR before taxes is 8%. If inflation is equal to 2% and the tax rate is 40%, what is the nominal rate and the real rate after tax?
- The nominal rate after tax is:

$$r(1 - \quad) = 0.08(1 - 0.4) = 0.048 \quad (4.8\%)$$

$$1 + r_r = \frac{1+r}{1 + i} \Longrightarrow r_r = \frac{1+r}{1 + i} - 1 = \frac{r \quad i}{1 + i}$$

- The real rate (after tax) is:

$$r_r = \frac{r \quad i}{1+i} = \frac{0.048 - 0.02}{1 + 0.02} = 0.027 \quad (2.7\%)$$

Question 5

You are enrolling in an MBA program. To pay for your tuition, you can either take a standard loan with EAR 5.5% or you can take a tax-deductible home equity loan with an APR (monthly compounding) of 6%. You anticipate being in a very low tax bracket, so your tax rate will only be 15%. What is the after-tax EAR of the home-equity loan? Which loan should you choose?

A. Either because the rates are equal
O. The home equity loan because its after tax rate is 5.243%
P. The home equity loan because its after tax rate is 4.925%
Q. The standard loan because the after tax rate of the home equity loan is 6.168%

- The home equity loan has an **APR** of 6% with **monthly** compounding
- The before tax equivalent monthly discount rate is: 0.06 / 12 = 0.005 (0.5%)
- The **before-tax** EAR is: EAR = $(1 + r_{month})^{12} - 1 = (1 + 0.005)^{12} - 1 = 0.06168$ (6.618%)
- And finally the **after-tax** EAR is: EAR(1- □) = 0.06168 * (1 - 0.15) = 0.05242 (5.243%)

Question 6

Suppose that you would like to go on a world trip in 12 years with current cost of 80,000 SEK. Assuming that you will need the same purchasing power in 12 years, how much money do you need to put in a bank account today if you are promised an annual interest rate of 5.2 % and the inflation is expected to be 1.9 % annually? Assume that the tax rate is 30 %.

The after-tax nominal rate is

$$r(1-\tau) = 0.052\times(1-0.3) = 0.0364$$

The nominal amount we need to pay in 12 years

$$80{,}000\times(1+0.019)^{12} = 100272.12$$

The amount to be deposited is

$$PV(100{,}272.12 \text{ in } 12 \text{ years}) = \frac{100{,}272.12}{(1+0.0364)^{12}} = 65{,}290.70$$

The after-tax nominal rate is

$$r(1-\tau) = 0.052\times(1-0.3) = 0.0364$$

The real after-tax rate is

$$r_r = \frac{r-i}{1+i} = \frac{0.0364-0.019}{1+0.019} = 0.01708$$

The amount we need to deposit

$$PV(80{,}000 \text{ in } 12 \text{ years}) = \frac{80{,}000}{(1+0.01708)^{12}} = 65{,}290.70$$

A. 43,541.27
B. 52,090.81
C. 65,290.70
D. 54,574.69

$$PV = \frac{80\,000\,(1+i)^{12}}{(1+r)^{12}}$$

$$PV = \frac{80\,000}{(1+r_r)^{12}} \qquad 1 + r_r = \frac{1+r}{1+i}$$

Comparing the use of nominal and real rates

Nominal values and nominal rates:

$$PV = \boxed{\frac{80\,000\,(1+i)^{12}}{(1+r)^{12}}}$$

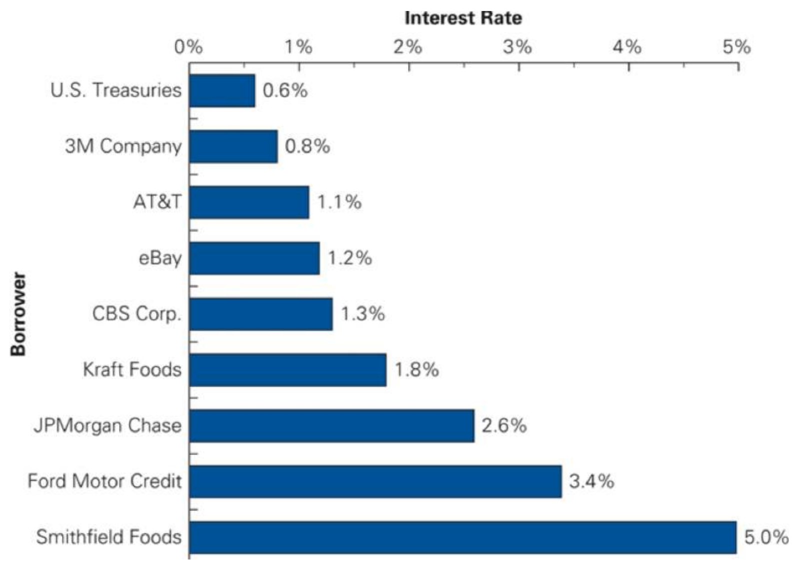
Real values and real rates:

$$PV = \frac{80\,000}{\left(\frac{1+r}{1+i}\right)^{12}} = \frac{80\,000}{\frac{(1+r)^{12}}{(1+i)^{12}}} = \boxed{\frac{80\,000\,(1+i)^{12}}{(1+r)^{12}}}$$

Risk and Interest Rates

- Interest rates are affected by default risk
  - The possibility that the borrower will nit be able pay back the loan

- Hence, investors will demand a higher interest rate, the higher the default risk



Figur 5.4 Interest Rates on Five-Year Loans for Various Borrowers, July 2012

Suppose the US government owed your firm \$1000, to be paid in five years. Based on the interest rates in Figur 5.4, what is the present value of this cash flow? Suppose instead Smithfield Foods owes your firm \$1000. Estimate the present value in this case.

PV (US Treasuries) = $1000 / (1 + 0.006)^5 = 970.53$
PV (Smithfiels Foods) = $1000 / (1 + 0.05)^5 = 783.53$

The opportunity Cost of Capital

- With so many rates to choose from, what is the the discount rate to be used in evaluating cash flows: **Investor's Opportunity Cost of Capital**
- The best best available expected return offered in the market on an investment of comparable risk and term to the cash flow being discounted
- Also referred to as **Cost of Capital**

Lecture 5 – Bonds

The prize of a zero-coupon bond: Example

- Consider a zero-coupon with 20 years to maturity. What is the price this bond will trade at if the YTM is 6 % and its face value is 1,000?

$$P = \frac{FV}{(1+YTM)^N} = \frac{1000}{(1+0.06)^{20}} = 311.8$$

N = number of years to maturity

Question 99 85 60

- Consider a zero-coupon bond with a 1,000 face value and 10 years left to maturity. If the YTM of this bond is 10.4%, then the price of this bond is closest to:

– A: 1,000

– B: 602

– C: 372 C

– D: 1,040

The YTM of a zero-coupon bond: Example

- A three-month treasury bill with a face value of 100 sold for a price of 99.31119998. What is the YTM of the zero-coupon bond?
- YTM=IRR of investment in zero-coupon bond expresses as an EAR

$$P = \frac{FV}{(1+YTM)^N}$$

N = number of years to maturity

N = 0.25

The YTM of a zero-coupon bond: Example

- A three-month treasury bill with a face value of 100 sold for a price of 99.3111998. What is the YTM of this bond expresses as an EAR?

$$P=\frac{FV}{(1+YTM)^N} \Rightarrow (1+YTM)^N=\frac{FV}{P}$$

$$YTM=\left(\frac{FV}{P}\right)^{\frac{1}{N}}-1=\left(\frac{100}{99.311998}\right)^{\frac{1}{0.25}}-1$$

$$=0.028 \quad (2.8\%)$$

Question code 65 56 70

- A zero-coupon bond with maturity in two years and face value 1000 SEK has a price equal to 990 SEK. What is its yield-to-maturity?

A. 1.01%
B. 2.02%
C. 0.50%
D. 3.00%
E. It cannot be computed

$$P=\frac{FV}{(1+YTM)^N}$$

C

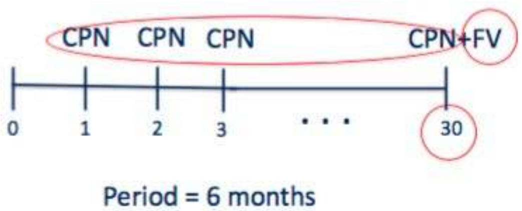
Question 93 51 00

- A certain company has issued a bond with a face value of 1,000 that reaches maturity in 15 years. The bond certificate indicates that stated coupon rate for this bond is 8% and that the coupon payments are to be made semiannually.
- How much will each semiannual coupon payment be?

– A: 80
– B: 40
– C: 120
– D: 60

The price of a coupon bond: Example

- A certain company has issued a bond with a face value of 1,000 that reaches maturity in 15 years. The bond certificate indicates that the stated coupon rate for this bond is 8% and tht the coupon payments are to be made semiannually.
- What is the price of the bond if YTM=9% (expresses as an APR with semiannual compounding)?



The price of a coupon bond: Example

- A certain company has issued a bond with a face value of 1,000 that reaches maturity in 15 years. The bond certificate indicates that the stated coupon rate for this bond is 8% and tht the coupon payments are to be made semiannually.
- What is the price of the bond if YTM=9% (expresses as an APR with semiannual compounding)?

$$P = CPN \frac{1}{y} \; 1 \;\; \frac{1}{(1+y)^N} + \frac{FV}{(1+y)^N}$$

$$CPN = \frac{1000 \quad 0.08}{2} = 40 \qquad \text{Period} = 6 \text{ months} \quad N = 30$$

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$$y = \frac{0.09}{2} = 0.045$$

$$P = 40 \frac{1}{0.045} \; 1 \;\; \frac{1}{(1+0.045)^{30}} + \frac{1000}{(1+0.045)^{30}}$$

$$= 918.56$$

Question 53 55 24

- A certain company has issued a bond with a face value of 1,000 that reaches maturity in 30 years. The bond certificate indicates that the stated coupon rate for this bond is 4% and that the coupon payments are to be made annually.
- What is the price of the bond if the YTM is 3.5%?

- A: 927
- B: 1000
- C: 1092
- D: 953

C

Question 28 63 60

- A certain company has issued a bond with a face value of 1,000 that reaches maturity in 30 years. The bond certificate indicates that the stated coupon rate for this bond is 4% and that the coupon payments are to be made annually.
- Assuming that this bond trades for 1,112, then the YTM for this bond is closest to;

- A: 2.4%
- B: 4.0%
- C: 3.4%
- D: 4.6%

Lecture 6 – Stock Valuation

A one-year investor: Example

- Assume a company has a current stock price of 45 and will pay 1.85 as dividend in one year; its equity cost of capital is 12%.
- What price must you expect the stock to sell for immediately after the firm pays the dividend in one year to justify its current price?

$$P_0 = \frac{Div_1 + P_1}{1 + r_E} \Rightarrow P_1 = P_0(1 + r_E) - Div_1$$

?

$$P_0 = \frac{Div_1 + P_1}{1 + r_E} \Rightarrow P_1 = P_0(1 + r_E) - Div_1$$
$$= 45 \times (1 + 0.12) - 1.85$$
$$= 48.55$$

Question 43 25 5

- Assume a company has a current stock price of 50 and will pay 2 as dividend in one year; its equity cost of capital is 15%.
- What price must you expect the stock to sell for immediately after the firm pays the dividend in one year to justify its current price?

– A: 50

C

– B: 57.5

– C: 55.5

– D: 48

Dividend Yield and Capital Gains: Example

- The stock of a given firm has current price of 12 and it is expected to pay a dividend of 1 in one year. Also, its price after paying that dividend is expected to be 35.

- What is the stock´s dividend yield?

$$P_0 = \frac{Div_1 + P_1}{1 + r_E} \Rightarrow 1 + r_E = \frac{Div_1 + P_1}{P_0}$$

$$r_E = \frac{Div_1 + P_1}{P_0} - 1 = \frac{Div_1 + P_1 - P_0}{P_0} = \frac{Div_1}{P_0} + \frac{P_1 - P_0}{P_0}$$

Dividend Yield and capital Gains: Example

- The stock of a given firm has current price of 12 and it is expected to pay a dividend of 1 in one year. Also, its price after paying that dividend is expected to be 35.
- What is the stock´s dividend?

$$r_E = \underbrace{\frac{Div_1}{P_0}}_{\text{Dividend Yield}} + \underbrace{\frac{P_1 - P_0}{P_0}}_{\text{Capital Gain Rate}} \qquad \frac{Div_1}{P_0} = 0.0833 \ (8.33\%)$$

Dividend Yield and capital gains: Example

- The stock of a given firm has current price of 12 and it is expected to pay a dividend of 1 in one year. Also, its price after paying that dividend is expected to be 35.
- What is the capital gain rate?

$$r_E = \underbrace{\frac{Div_1}{P_0}}_{\text{Dividend Yield}} + \underbrace{\frac{P_1 - P_0}{P_0}}_{\text{Capital Gain Rate}} \qquad \frac{35 - 12}{12} = 1.9167(191.97\%)$$

Dividend Yield and Capital Gains: Example

- The stock of a given firm has current price of 12 and it is expected to pay a dividend of 1 in one year. Also, its price after paying that dividend is expected to be 35.
- What is the stock´s return? / Equity cost of capital?

$$r_E = \underbrace{\frac{Div_1}{P_0}}_{\text{Dividend Yield}} + \underbrace{\frac{P_1 - P_0}{P_0}}_{\text{Capital Gain Rate}} = 0.0833 + 1.9197$$

$$= 2.00 \ (200\%)$$

Question 41 85 57

- Krell industries has a share price of 20 today. If Krell is expected to pay a dividend of 1.1 this year, and its stock price is expected to grow to 25 at the end of the year
- What is Krell´s dividend yield and equity cost of capital?

– A: 4.4% and 29.4%
– B: 5.5% and -14.5%
– C: 5.5% and 30.5%
– D: 4.4% and 29.4%

C

A multiyear investor: Example

- Suppose a given company will pay a dividend of 2.50 per share at the end of this year, and 3 per share next year. You expect its stock price to be 54 in two years. If its equity cost of capital is 9.5%.
- What price would you be willing to pay for a share of this company today, if you planned to hold the stock for two years?

$$P_0 = \frac{2.50}{1+0.095} + \frac{57}{(1+0.095)^2} = 49.821$$

Question 10 60 79

- Suppose a given Company will pay a dividend of 2.5 per share at the end of this year, and 3 per share next year. You expect its stock price to be 54 in two years. If its equity cost of capital is 9.5%,
- What price would you expect to be able to sell a share of the company for in one year (after the dividend has been paid)? Given this answer, what price would you be willing to pay for a share of the company today, if you planned to hold the stock for one year?

B

– A: 52.054 and 97.360
– B: 52.054 and 49.821
– C: 55.054 and 49.821
– D: 2.720 and 2.283

Applying the Dividend-Discount Model: Example

- NoGrowth Corporation currently pays a dividend of 3.88 per year, and it will continue to pay this dividend forever.

- What is the price per share if its equity cost of capital is 11% per year?

Year		1	2	3	4	5
FCF	In millions	53.2	67.5	78.9	74.6	80.3

Question 78 57 54

- A company has a dividend yield of 2.1%. Its equity cost of capital is 8.1%, and its dividends are expected to grow at a constant rate.
- What is its dividend growth?

– A: 8.1%
– B: 2.1%
– C: 6.0%
– D: 12.0%

Question 34 54 03

- A company has a dividend yield of 2.1%. Its equity cost of capital is 8.1%, and its dividends are expected to grow at a constant rate.
- What would the capital gain rate be?

C

– A: 8.1%
– B: 2.1%
– C: 6.0%
– D: 12.0%

$$r_E = \frac{Div_1}{P_0} + \frac{P_1 - P_0}{P_0}$$

Lecture 7 – The discounted free cash flow model

Example 1

- A firm is expected to generate the following free cash flows over the next five years:

Year	In millions	1	2	3	4
Sales	433	468	516	547	574,3
EBIT		53,8	59,6	62,1	65,2
Less: Income tax at 40%		21,5	23,8	24,8	26,1
Plus: Depreciation		7	7,5	9	9,5
Less: Capital Expenditures		7,7	10	9,9	10,4
Less: Increases in NWC		6,3	8,6	5,6	4,9
Free Cash Flow		25,3	24,6	30,8	33,3

- After that, the free cash flows are expected to grow ar the industry average of 4.3% per year. Using the DFC model and a weighted average cost of capital of 13.6%.
- What is the enterprise value of the firm?

$$V_0 = PV(FCF) = \frac{53.2}{1.136} + \frac{67.5}{1.136^2} + \frac{78.9}{1.136^3} + \frac{74.6}{1.136^4} + \frac{80.3}{0.136 - 0.043} \times \frac{1}{1.136^4}$$

$$PV(\text{growing perpetuity}) = \frac{C}{r-g} \qquad C = 80.3 \quad r = 0.136 \quad g = 0.043$$

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Value of firm (V0) = 716,22

- A firm is expected to generate the following free cash flows over the next five years:

Year	1	2	3	4	5
FCF In millions	53.2	67.5	78.9	74.6	80.3

- After that, the free cash flows are expected to grow at the industry average of 4.3% per year. Using the DCF model and a weighted average cost of capital of 13.6%,
- what is the enterprise value of the firm?

$$V_0 = PV(FCF) = \frac{53.2}{1.136} + \frac{67.5}{1.136^2} + \frac{78.9}{1.136^3} + \frac{74.6}{1.136^4} + \frac{80.3}{0.136 - 0.043} \times \frac{1}{1.136^4}$$

$$= 716.22$$

Example 1 – 2.0

- *If the firm has no cash, debt of 288 and 39 million shares outstanding, what is* ***its share price?***

- The share price is by definition

$$P_0 = \frac{Market\ Value\ of\ Equity_0}{\#\ \text{Shares outstanding}}$$

- The firm´s enterprise value is

$$V_0 = Market\ Value\ of\ Equity_0 + Debt_0 - Cash_0$$

- The Market Value of Equity is

$$Market\ Value\ of\ Equity_0 = V_0 - Debt_0 + Cash_0$$

- The share price is thus

$$P_0 = \frac{V_0 - Debt_0 + Cash_0}{\#\ \text{Shares outstanding}} = \frac{716.22 - 288}{39} = 10.98$$

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$V_0 = 716.22$
Debt = 288
No excess cash
Shares = 39

Question 71 42 06

Year	In millions	1	2	3	4
Sales	433	468	516	547	574,3
EBIT		53,8	59,6	62,1	65,2
Less: Income tax at 40%		21,5	23,8	24,8	26,1
Plus: Depreciation		7	7,5	9	9,5
Less: Capital Expenditures		7,7	10	9,9	10,4
Less: Increases in NWC		6,3	8,6	5,6	4,9
Free Cash Flow		25,3	24,6	30,8	33,3

- The weighted average cost of capital is equal to 9% and FCF is expected to grow at 3.5 rate beyond year 4
- What is the estimate of the **firm's enterprise value?**

- A: 535.22
- B: 100.34
- C: 623.23
- D: 712.13

A!!

Question 22 55 51

- The firm has 63 million shares, 127 million in debt and 58 million in cash.
- What is the estimate of the firm´s **share price?**

- A: 14.22
- B: 7.40
- C: 5.23
- D: 6.42

Example 2

- In the table above of projected DCF, net working capital needs are estimated to be 18% of sales (which is their current level in year 0). Suppose this requirement can be reduced yo 12% of sales starting in year 1.
- **What is the estimate of the firm´s enterprise value?**

Example 2

$r_{wacc} = 0.09$
g beyond year 4 = 0.035

Year	0	1	2	3	4
Sales	433	468	516	547	574,3

- Value of NWC is set equal to 18% of sales

NWC	77,94	84,24	92,88	98,46	103,37

- Hence, increases in NWC are according to first forecast

Increases in NWC	6,3	8,6	5,6	4,9

- If value of NWC reduced from year one to 12%

NWC	77,94	56,16	61,92	65,64	68,916

- And increases in NWC become

Increases in NWC	-21,78	5,76	3,72	3,276

Example 2

$r_{wacc} = 0.09$
g beyond year 4 = 0.035

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Sales	433	468	516	547	574,3
EBIT		53,8	59,6	62,1	65,2
Less: Income tax at 40%		21,5	23,8	24,8	26,1
Plus: Depreciation		7	7,5	9	9,5
Less: Capital Expenditures		7,7	10	9,9	10,4
Less: Increases in NWC		6,3	8,6	5,6	4,9
Free Cash Flow		25,3	24,6	30,8	33,3

Forecasted FCF when NWC is set to 18% of sales
(Original projection)

Example 2

$r_{wacc} = 0.09$
g beyond year 4 = 0.035

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Sales	433	468	516	547	574,3
EBIT		53,8	59,6	62,1	65,2
Less: Income tax at 40%		21,5	23,8	24,8	26,1
Plus: Depreciation		7	7,5	9	9,5
Less: Capital Expenditures		7,7	10	9,9	10,4
Less: Increases in NWC		−21,78	5,76	3,72	3,276
Free Cash Flow		53,38	27,54	32,68	34,924

Forecasted FCF when NWC is set to 12% of sales from year 1
(New projection)

$$V_0 = PV(FCF) = \frac{53.38}{1.09} + \frac{27.54}{1.09^2} + \frac{32.68}{1.09^3} + \frac{34.924}{0.09 - 0.035} \times \frac{1}{1.09^3}$$
$$= 587.71$$

Example 2

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Sales	433	468	516	547	574,3
EBIT		53,8	59,6	62,1	65,2
Less: Income tax at 40%		21,5	23,8	24,8	26,1
Plus: Depreciation		7	7,5	9	9,5
Less: Capital Expenditures		7,7	10	9,9	10,4
Less: Increases in NWC		−21,78	5,76	3,72	3,276
Free Cash Flow		53,38	27,54	32,68	34,924

- Recall that the firm has 63 million shares, 127 million in debt and 58 million in cash.
- What is the new estimate of the firm´s share price? $V_0 = 587.71$

$$P_0 = \frac{V_0 - Debt_0 + Cash_0}{\#\text{ Shares outstanding}} = \frac{587.71 - 127 + 58}{63} = 8.233$$

Question 13 97 65

Question 13 97 65

r_{wacc} = 0.09
g beyond year 4 = 0.035
Debt = 127 million
Excess cash= 58 million
shares = 63 million

Stockholms universitet

Sales	433	468	516	547	574,3
EBIT		53,8	59,6	62,1	65,2
Less: Income tax at 40%		21,5	23,8	24,8	26,1
Plus: Depreciation		7	7,5	9	9,5
Less: Capital Expenditures		7,7	10	9,9	10,4
Less: Increases in NWC		6,3	8,6	5,6	4,9
Free Cash Flow		25,3	24,6	30,8	33,3

- Suppose that the projection is equal to the original one except for an increase of 2 million in the projected capital expenditures every year from year 1 to 4.
- What is the estimate of the **firm´s enterprise value?** What is the estimate of the **share price?**

- A: 502.08 and 9.06
- B: 487.97 and 6.65
- C: 465.80 and 6.30
- D: 502.08 and 6.87

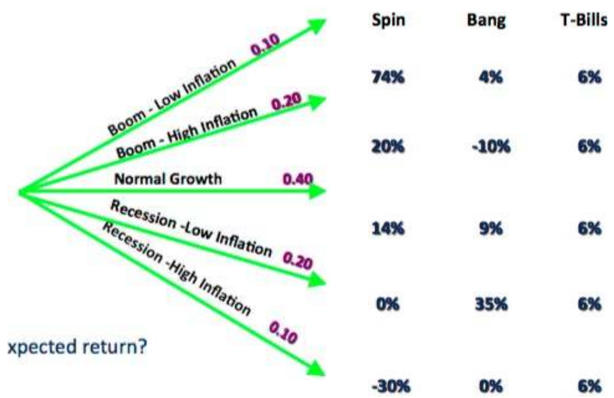
D!!

Lecture 8 – Risk and Return

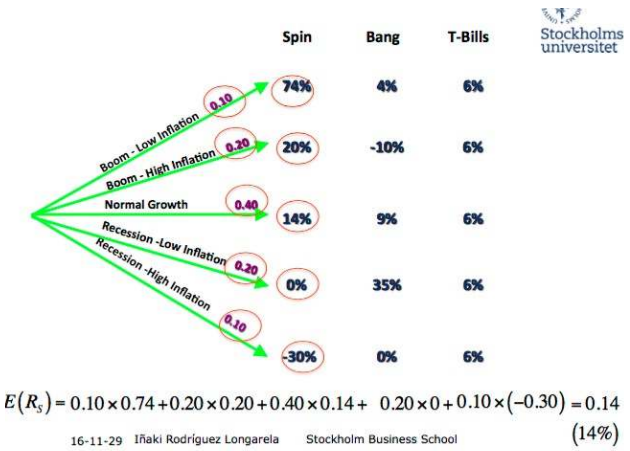
Expected returns: Example

- Suppose that the economy prospects in one year are reduced to five possible scenarios with the following probabilities.
- Your investment opportunities are reduced to investments in the stock of two different firms (Spin and Bang) and a risk-free asset.

RETURNS



- What is Spin´s expected return?



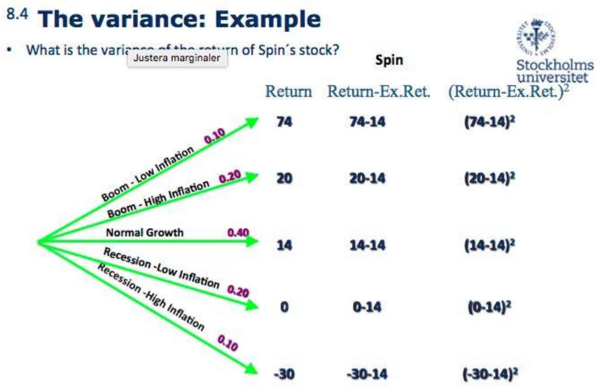
Question code 22 34 80

C

Question code 17 13 07

B

The variance: Example



$$\sigma_S^2 = 0.1\times(0.74-0.14)^2+0.2\times(0.2-0.14)^2+0.4\times(0.14-0.14)^2+0.2\times(0-0.14)^2+0.1\times(-0.3-0.14)^2=0.06$$

The standard deviation: Example

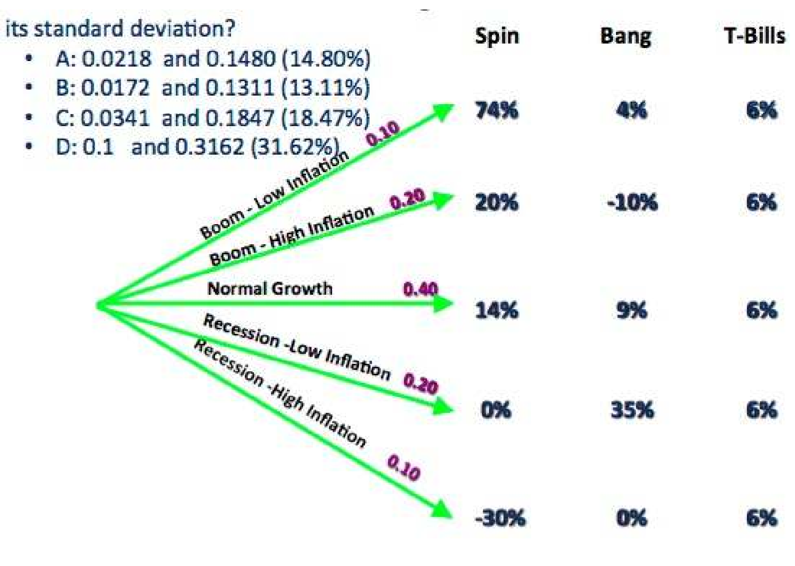
- What is the standard deviation of the return of Spin´s stock?

$$\sigma_S^2 = 0.06$$

$$\sigma_S = \sqrt{0.06} = 0.2449 \quad (24.49\%)$$

Question code 46 91 20

- What is the variance of return of Bang´s stock? What is its standard deviation?



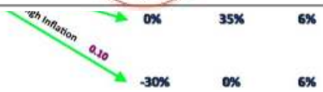
A

Question code 96 79 87

- What is the variance and standard deviation of return of the risk-free asset?

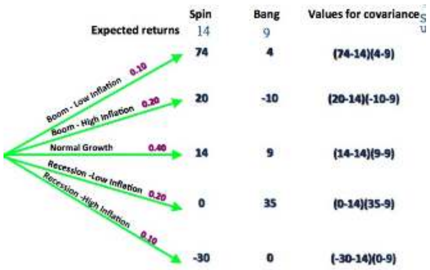
Year End	Index Realized Return	Stock A Realized Return
2003	30,50%	86,90%
2004	9,00%	23,10%
2005	-2,00%	2,00%
2006	-17,30%	-3,20%

B



The covariance: Example

- What is the covariance between the returns of Spin and Bang?



$$\sigma_{S,B} = 0.1\times(0.74-0.14)(0.04-0.09)+0.2\times(0.2-0.14)(-0.1-0.09)$$
$$+0.4\times(0.14-0.14)(0.09-0.09)+0.2\times(0-0.14)(0.35-0.09)+$$
$$0.1\times(-0.-14)(0-0.09)=-0.0086$$

Question code 14 48 41

	Spin	Bang
Expected Value	14%	9%
Variance	0.06	0.0219
Standard Deviation	24.5%	14.8%
Covariance	-0.0086	

- The table above presents a summary of all the values that we have computed so far. What is the correlation between Spin and Bang?

C

- A: -0.4725
- B: 0
- C: 0.2372
- D: -0.2372

Estimating the expected return: Example

- What was the average return of the index over the period 2003-2006?

$$\bar{R}_I = \frac{0.305+0.09-0.02-0.173}{4} = 0.0505 \ (5.05\%)$$

Question code 14 22 32

Year End	Index Realized Return	Stock A Realized Return
2003	30,50%	86,90%
2004	9,00%	23,10%
2005	-2,00%	2,00%
2006	-17,30%	-3,20%

- What was the average return of Stock A over the period 2003 – 2006?

- A: 0%
- B: 27.2%
- C: 5.05%
- D: 23.1%

B

Estimating the variance of the return: Example

universitet

Year End	Index Realized Return	Stock A Realized Return
2003	30,50%	86,90%
2004	9,00%	23,10%
2005	-2,00%	2,00%
2006	-17,30%	-3,20%
Average	5,05%	27,20%

- What was the variance of the return of the Index over the period 2003-2006?

$$\sigma_I^2 = \frac{(0.305-0.0505)^2+(0.09-0.0505)^2+(-0.02-0.0505)^2+(-0.173-0.0505)^2}{} = 0.0404$$

Question code 94 18 03

Year End	Index Realized Return	Stock A Realized Return
2003	30,50%	86,90%
2004	9,00%	23,10%
2005	-2,00%	2,00%
2006	-17,30%	-3,20%
Average	5,05%	27,20%

- A: 0.1997
- B: 0.2342
- C: 1.2737
- D: 0.1713

- What was the variance of return of the stock over the period 2003-2006?

The standard deviations: Example

Year End	Index Realized Return	Stock A Realized Return
2003	30,50%	86,90%
2004	9,00%	23,10%
2005	-2,00%	2,00%
2006	-17,30%	-3,20%
Average	5,05%	27,20%
Variance	0,0404	0,1713

- What was the standard deviations of returns of the index and the stock over the period 2003-2006?

$$\sigma_I = \sqrt{0.0404} = 0.201 \ (20.1\%) \qquad \sigma_A = \sqrt{0.1713} = 0.4139 \ (41.39\%)$$

Estimating the covariance: Example

Year End	Index Realized Return	Stock A Realized Return
2003	30,50%	86,90%
2004	9,00%	23,10%
2005	-2,00%	2,00%
2006	-17,30%	-3,20%
Average	5,05%	27,20%
Variance	0,0404	0,1713
Standard Dev.	20,10%	41,39%

- What was the covariance between the return of index and the stock over the period 2003-2006?

$$\sigma_{I,A} = \frac{1}{3}\Big[(0.305-0.0505)\times(0.869-0.272)+(0.09-0.0505)\times(0.231-0.272)+ +(-0.02-0.0505)\times(0.02-0.272)+(-0.173-0.0505)\times(-0.032-0.272)\Big] = 0.0787$$

Estimating the correlation: Example

Year End	Index Realized Return	Stock A Realized Return
2003	30,50%	86,90%
2004	9,00%	23,10%
2005	-2,00%	2,00%
2006	-17,30%	-3,20%
Average	5,05%	27,20%
Variance	0,0404	0,1713
Standard Dev.	20,10%	41,39%

$\sigma_{I,A} = 0.0787$

- What was the correlation between the return of the index and the stock over the period 2003-2006?

Question code 39 74 66

$$\rho_{I,A} = \frac{\sigma_{I,A}}{\sigma_I \sigma_A} = \frac{0.0787}{0.201 \times 0.4139} = 0.946$$

How does the relationship between the average return and the historical volatility of individual stocks differ from the relationship between the average return and the historical volatility of large, well-diversified portfolios?

- **A:** There is a clear relationship for individual stocks since higher average returns are associated with higher standard deviations in this case.
- **B:** There are no clear relationship in any case.
- **C:** Large portfolios with higher average returns have higher standard deviations. For individual stock no such relationship exists.
- **D:** Lower average return are always associated with low standard deviations.

C

Question code 23 76 46

Which ones of the following risks are diversifiable?

1. Your main production plan may be shut down due to a tornado

2. The economy may slow, decreasing demand for your firm´s products.

3. Your best employee may be hired away.

4. The next product you expect from the R&D division may not materialize.

- **A:** 1 and 2
- **B:** 1, 2 and 4
- **C:** 2 and 3
- **D:** 1, 3 and 4.

D

Question code 43 49 78

Which one of the following statements is false?

- **A:** The risk premium of a stock is determined by its systematic risk and does not depend on its diversifiable risk.
- **B:** Fluctuations of a stock´s return that are due to firm-specific news are market risk.
- **C:** The standard deviation of the return of a portfolio declines as we add more stocks until only the systematic risk remains.
- **D:** When we combine many stocks in a portfolio, the firm-specific risk of each averages out and is diversified.

B

Lecture 9 – Portfolio Theory 1 (Chapter 11)

Portfolio weights: Example

- Suppose that you are going to invest 20,000 SEK. You will allocate 2,200 SEK to stock Spin, 8,400 to stock Bang and the rest to a risk-free asset.

- What are your portfolio weights?

Year End	Index Realized Return	Stock A Realized Return
2003	30,50%	86,90%
2004	9,00%	23,10%
2005	-2,00%	2,00%
2006	-17,30%	-3,20%
Average	5,05%	27,20%

$$x_S = \frac{2,200}{20,000} = 0.11$$

$$x_B = \frac{8,400}{20,000} = 0.42$$

$$x_f = \frac{20,000 - 2,200 - 8,400}{20,000} = 1 - 0.11 - 0.42 = 0.47$$

Portfolio Returns: Example

- What will be the return of you portfolio if the economy displays a boom with high inflation?

$$0.11 \times 0.2 + 0.42 \times (-0.1) + 0.47 \times 0.06 = 0.0082\ (0.82\%)$$

The expected return of a portfolio: Example

- What is the expected return of your portfolio?

$$0.11 \times 0.14 + 0.42 \times 0.09 + 0.47 \times 0.06 = 0.0814\ (8.14\%)$$

Question 78 34 89

Year End	Index Realized Return	Stock A Realized Return
2003	30,50%	86,90%
2004	9,00%	23,10%
2005	-2,00%	2,00%
2006	-17,30%	-3,20%
Average	5,05%	27,20%
Variance	0,0404	0,1713
Standard Dev.	20,10%	41,39%

$\rho_{I,A} = 0.946$

- An investor held a portfolio with weights in the index and the stock equal to 0.80 and 0.2+, respectively. What was the return in 2004? What was the average return of her portfolio over year 2003-2006?

C

- A: 9.79% and 9.48%
- B: -1.2% and 7.25%
- C: 11.82% and 9.48%
- D: 41.78% and -1.27%.

Average	5,05%	27,20%

- An investor held a portfolio with weights in the Index and the stock equal to 0.80 and 0.20, respectively. What was her return in 2004? What was the average return of her portfolio over the years 2003-2006?
  - A: 9.79% and 9.48%
  - B: -1.2% and 7.25%
  - C: 11.82% and 9.48%
  - D: 41.78% and -1.27%.

$$R_{P2004} = 0.8 \times 0.09 + 0.2 \times 0.231 = 0.1182 \quad (11.82\%)$$

$$\bar{R}_P = 0.8 \times 0.0505 + 0.2 \times 0.272 = 0.0948 \quad (9.48\%)$$

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The variance of a portfolio of two risky assets: Example

	Spin	Bang
Expected Value	14%	9%
Variance	0.06	0.0219
Standard Deviation	24.5%	14.8%
Covariance	-0.0086	

- Suppose that you have some wealth to be invested and you would like to invest it all in the same stocks of Spin and Bang. You will invest 54,000 SEK in Spin and 36,000 SEK in Bang. What is the standard deviation of the return of your portfolio?

$$x_S = \frac{54,000}{54,000 + 36,000} = 0.6$$

$$x_B = 1 - 0.6 = 0.4$$

portfolio? $x_S = 0.6$ $x_B = 0.4$

$$x_S^2 \sigma_S^2 + x_B^2 \sigma_B^2 + 2 x_S x_B \sigma_{S,B} =$$

$$= 0.6^2 \times 0.06 + 0.4^2 \times 0.0219 + 2 \times 0.6 \times 0.4 \times (-0.0086) = 0.021$$

$$\sqrt{0.021} = 0.1448 \quad (14.48\%)$$

Question 92 94 6

- Consider the previous investor and her portfolio with weights 0.8 and 0.2 in the index and stock A, respectively. What is the standard deviation of the return of her portfolio? (Note that you are given the correlation and not the covariance)

- A: 24.06%
- B: 0%
- C: 0.946%
- D: 15.66%

A

9.6

Solution

Year End	Index Realized Return	Stock A Realized Return
2003	30,50%	86,90%
2004	9,00%	23,10%
2005	-2,00%	2,00%
2006	-17,30%	-3,20%
Average	5,05%	27,20%
Variance	0,0404	0,1713
Standard Dev.	20,10%	41,39%

$\rho_{I,A} = 0.946$

- Consider the previous investor and her portfolio with weights 0.8 and 0.2 in the index and stock A, respectively. What is the standard deviation of the return of her portfolio? (Note that you are given the correlation and not the covariance)

$$\rho_{I,A} = \frac{\sigma_{I,A}}{\sigma_I \sigma_A} \Rightarrow \sigma_{I,A} = \sigma_I \sigma_A \rho_{I,A}$$

9.6

Solution

Year End	Index Realized Return	Stock A Realized Return
2003	30,50%	86,90%
2004	9,00%	23,10%
2005	-2,00%	2,00%
2006	-17,30%	-3,20%
Average	5,05%	27,20%
Variance	0,0404	0,1713
Standard Dev.	20,10%	41,39%

$\rho_{I,A} = 0.946 \quad \sigma_{I,A} = 0.0787$

- Consider the previous investor and her portfolio with weights 0.8 and 0.2 in the index and stock A, respectively. What is the standard deviation of the return of her portfolio? (Note that you are given the correlation and not the covariance)

$$x_I^2\sigma_I^2 + x_A^2\sigma_A^2 + 2x_I x_A \sigma_{I,A} =$$
$$= 0.8^2 \times 0.0404 + 0.2^2 \times 0.1713 + 2 \times 0.8 \times 0.2 \times 0.0787 = 0.0579$$

9.6

Solution

Year End	Index Realized Return	Stock A Realized Return
2003	30,50%	86,90%
2004	9,00%	23,10%
2005	-2,00%	2,00%
2006	-17,30%	-3,20%
Average	5,05%	27,20%
Variance	0,0404	0,1713
Standard Dev.	20,10%	41,39%

$\rho_{I,A} = 0.946 \quad \sigma_{I,A} = 0.0787$

- Consider the previous investor and her portfolio with weights 0.8 and 0.2 in the index and stock A, respectively. What is the standard deviation of the return of her portfolio? (Note that you are given the correlation and not the covariance)

$$x_I^2\sigma_I^2 + x_A^2\sigma_A^2 + 2x_I x_A \sigma_{I,A} =$$
$$= 0.8^2 \times 0.0404 + 0.2^2 \times 0.1713 + 2 \times 0.8 \times 0.2 \times 0.0787 = 0.0579$$
$$\sqrt{0.0579} = 0.2406 \quad (24.06\%)$$

Question 54 67 43

	Spin	Bang
Expected Value	14%	9%
Variance	0.06	0.0219
Standard Deviation	24.5%	14.8%
Covariance	-0.0086	

- Suppose that you changed your mind and you will invest instead your 90,000 in portfolio containing the stock Spin and the risk-free asset with a weight in the stock equal to 1.5.
- What is the size of your position in the stock of Spin? What is the standard deviation of the return of your portfolio?

B

- A: 135,000 and 36.74%
- B: 135,000 and 24.5%
- C: 90,000 and 24.5%
- D: 90,000 and 0%

9.7

Solution

	Spin	Bang
Expected Value	14%	9%
Variance	0.06	0.0219
Standard Deviation	24.5%	14.8%
Covariance	-0.0086	

- Suppose that you changed your mind and you will invest instead your 90,000 in a portfolio containing the stock of Spin and the risk-free asset with a weight in the stock equal to 1.5. What is the size of your position in the stock of Spin? What is the standard deviation of the return of your portfolio?

$$1.5 \times 90{,}000 = 135{,}000$$

$$x_S^2\sigma_S^2 = 1.5^2 \times 0.06 = 0.135$$
$$\sqrt{0.135} = 0.3674 \quad (36.74\%)$$

Efficient Portfolios: Example

	Spin	Bang
Expected Value	14%	9%
Variance	0.06	0.0219
Standard Deviation	24.5%	14.8%
Covariance	-0.0086	

- Suppose that you changed your mind again and you prefer to invest in a portfolio containing the stocks of Spin and Bag with a weight in Spin equal to 0.1. What is the standard deviation of the return of your portfolio?

$$x_S = 0.1 \quad x_B = 0.9$$

$$x_S^2\sigma_S^2 + x_B^2\sigma_B^2 + 2x_Sx_B\sigma_{S,B} =$$

$$= 0.1^2 \times 0.06 + 0.9^2 \times 0.0219 + 2 \times 0.1 \times 0.9 \times (-0.0086) = 0.0168$$

$$\sqrt{0.0168} = 0.1296 \quad (12.96\%)$$

Efficient portfolios: Example

	Spin	Bang
Expected Value	14%	9%
Variance	0.06	0.0219
Standard Deviation	24.5%	14.8%
Covariance	-0.0086	

- Suppose that you changed your mind again and you prefer to invest in a portfolio containing the stocks of Spin and Bang with a weight in Spin equal to 0.1.
- What is the expected return of your portfolio?

$$x_S = 0.1 \qquad x_B = 0.9$$

$$0.1 \times 0.14 + 0.9 \times 0.09 = 0.095 \quad (9.5\%)$$

- Would investing all your money in Bang be an efficient portfolio? NO, compare to different alternatives.

Question 28 18 47

	Spin	Bang
Expected Value	14%	9%
Variance	0.06	0.0219
Standard Deviation	24.5%	14.8%
Correlation	-1	

- You stick your portfolio containing the stocks of Spin and Bang with a weight in Spin equal to 0.1.
- However, the correlation between the two stocks changes and it becomes -1. Everything else stays the same.
- **Will the expected return of the portfolio rise or fall?**

- A: Cannot tell from the information provided
- B: Rise
- C: Fall
- D: Remain the same

D

$$x_S E(R_S) + (1 - x_S) E(R_B) =$$
$$= 0.1 \times 0.14 + 0.9 \times 0.09 =$$
$$= 0.095 \qquad (9.5\%)$$

Question 83 16 58

- Same information as above.
- **Will the standard deviation of the return of the portfolio rise or fall?**

- A: Cannot tell from the information provided
- B: Rise
- C: Fall
- D: Remain the same

C

$$x_S^2 \sigma_S^2 + (1 - x_S)^2 \sigma_B^2 + 2 x_S (1 - x_S) \sigma_S \sigma_B \rho_{S,B}$$

$$x_S^2 \sigma_S^2 + (1 - x_S)^2 \sigma_B^2 + 2 x_S (1 - x_S) \sigma_S \sigma_B \rho_{S,B}$$

$0.1^2 x 0.06 + 0.9^2 x 0.0219$ $-2x0.1x0.9x0.245x0.148$

$$x_S^2 \sigma_S^2 + (1 - x_S)^2 \sigma_B^2 + 2 x_S (1 - x_S) \sigma_S \sigma_B \rho_{S,B} = 0.0118$$
$$\sqrt{0.0118} = 0.1087$$

Question code 66 31 75

You are a risk-averse investor considering investing in one of two economies. The expected return and standard deviation of all stocks in both economies is the same. In the first one, all stocks move together – in good times all prices go up and in bad times they all fall together. In the second economy, the stock returns do not move together at all

- **A:** You are indifferent in both cases because you face unpredictable risk.
- **B:** You choose the economy in which stock returns are independent because risk can be diversified away in a large portfolio.
- **C:** You choose the economy in which stock returns are independent because by combining the stocks into a portfolio you can get a higher expected return than in other economy.
- **D:** You have no idea.

B/C

Question 65 89 84

	Spin	Bang Stockho univers
Expected Value	14%	9%
Variance	0.06	0.0219
Standard Deviation	24.5%	14.8%
Correlation	-1	

- What is the weight on Spin of the minimum variance portfolio? What is the variance of such portfolio?

B

- A: 0.3766 and -0.4525
- B: 0.3766 and 0
- C: -0.9040 and 0.2532
- D: 0.50 and 0.0024

$$x_S^* = \frac{\sigma_B^2 - \sigma_S \rho_{S,B} \sigma_B}{\sigma_S^2 - 2\sigma_S \rho_{S,B} \sigma_B + \sigma_B^2}$$

1

$$= \frac{0.0219 + 0.245 \times 0.148}{0.06 + 2 \times 0.245 \times 0.148 + 0.0219} = 0.3766$$

2

$$x_S^2 \sigma_S^2 + (1 - x_S)^2 \sigma_B^2 + 2x_S(1 - x_S)\sigma_S \sigma_B \rho_{S,B} =$$

$$= 0.3766^2 \times 0.06 + (1 - 0.3766)^2 \times 0.0219 + 2 \times 0.3766 \times (1 - 0.3766) \times 0.245 \times (-1) \times 0.148 =$$

$$= 0$$

Lecture 10 – Portfolio Theory 2

Meet the Tangency portfolio

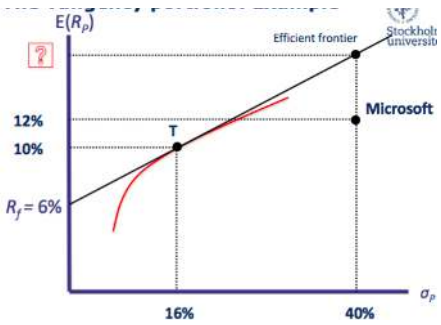
- The Sharpe ratio of a portfolio with name A is given by

$$S_A = \frac{E(R_A) - R_f}{\sigma_A}$$

- Imagine portfolios created out of all available **risky** financial assets in an economy.
- The **tangency portfolio** is the one that has the largest Sharpe Ratio.

The tangency portfolio: Example

- Your portfolios consist of a full investment in just one stock, Microsoft. Suppose this stock has an expected return of 12% and volatility of 40%. Suppose further that the tangency portfolio has an expected return of 10 % and a volatility of 16%. Also, assume that the risk-free rate is 6%.
- What is the expected return of the alternative investment that has the highest possible expected return while having the same volatility as your investment?



$$E(R_P) = x_T E(R_T) + (1 - x_T) R_f \;= 2.5 \times 0.1 + (1 - 2.5) \times 0.06 = 0.16$$

$$\sigma_P = \; x_T \sigma_T = 0.40 \Rightarrow x_T = \frac{0.40}{0.16} = 2.5 \qquad (16\%)$$

$$S_T = \frac{E(R_T) - R_f}{\sigma_T} = \frac{0.1 - 0.06}{0.16} = 0.25$$

the same volatility as your investment? $S_T = 0.25$

$$0.25 = \frac{E(R_P) - 0.06}{0.4} \Rightarrow E(R_P) = 0.4 \times 0.25 + 0.06 = 0.16$$

(16%) **Question 1**

Which of the following statement is ***false?***

- **A:** By combining the tangency portfolio with the risk-free asset, an investor will earn the highest possible expected return for any level of volatility she is willing to bear. True
- **B:** All efficient portfolios are combinations of the risk-free asset and the tangency portfolio. True
- **C:** The optimal portfolio of **risky** assets to be combined with the risk-free asset depends on the risk-aversion of the investor. False
- **D:** The tangency portfolio is the portfolio with the highest Sharpe ratio in the economy. True

SVAR: C

Question 2

- Your portfolios consist of a full investment in just one stock, Microsoft. Suppose this stock has an expected return of 12% and volatility of 40%. Suppose further that the tangency portfolio has an expected return of 10% and a volatility of 16%. Also, assume that the risk-free rate is 6%.
- What is the volatility of the alternative investment that has the lowest possible volatility while having the same expected return as your investments?

C

- A: 0%
- B: 16%
- C: 24%
- D: Impossible to compute

$$\sigma_P = x_T \sigma_T = 1.5 \times 0.16 = 0.24 \ (24\%)$$

$$E(R_P) = x_T 0.10 + (1 - x_T) 0.06 = 0.12 \Rightarrow x_T = 1.5$$

n as your investment? $S_T = 0.25$

$$0.25 = \frac{0.12 - 0.06}{\sigma_p} \Rightarrow \sigma_T = \frac{0.12 - 0.06}{0.25} = 0.24$$

Meet the Market Portfolio

- Imagine portfolios created out of all available **risky** financial assets in an economy

- The **Market portfolio** is the one that uses as weights the relative market value of each one of these financial assets.
- According to CAPM, the forces of financial markets will make the Market portfolio and the Tangency portfolio equal.

	Expected Return	Volatility
Value Stocks	12%	11%
Growth Stocks	17%	27%

- Hence, according to CAPM, the Market portfolio should have the highest Sharpe ratio.

Question

- Suppose there are only two firms in the economy. Firm VALUE has 1000 shares and a share price of 50. Firm Growth has 250 shares and a share price of 200.
- Find the weights of the market portfolio

A: 0.8 in Value and 0.2 in Growth

B: 0.55 in Value and 0.45 in Growth

C: 0.5 in Value and 0.5 in Growth

D: 1 in the risk-free asset and nothing in the stock of the firms

C

SOLUTION

- The total market value of Value (AKA market capitalization of Value)
  - 1000 Shares x 50 = 50,000
- The total market value of Growth (AKA market capitalization of Growth)
  - 250 Shares x 200 = 50,000
- Total market value of the financial wealth of the economy: 100,000
- Relative market value of Value: 50,000/100,000 = 0.5
- Relative market value of Growth: 50,000/100,000 = 0.5

The market portfolio

- Suppose we have that

	Expected Return	Volatility
Value Stocks	12%	11%
Growth Stocks	17%	27%

- The correlation of their returns is 0.5 and the risk-free rate is 4%.
- What is the expected return and volatility of the market portfolio?

$$E(R_M) = 0.5 \times 0.12 + 0.5 \times 0.17 = 0.145 \ (14.5\%)$$

$$\sigma_M^2 = x_V^2\sigma_V^2 + x_G^2\sigma_G^2 + 2x_Vx_G\sigma_V\sigma_G\rho_{V,G} = 0.5^2 \times 0.11^2 + 0.5^2 \times 0.27^2 +$$
$$+2 \times 0.5 \times 0.5 \times 0.11 \times 0.27 \times 0.5 = 0.0287$$

$$\sigma_M = \sqrt{0.0287} = 0.1693 \qquad (16.93\%)$$

The market portfolio

- Suppose we have that

- The correlation of their returns is 0.5 and the risk-free rate is 4%.
- Calculate the Sharpe ratios of Value, Growth and the market portfolio.

$$E(R_M) = 14.5\% \quad \sigma_M = 16.93\%$$

$$Sharpe\ Ratio_V = \frac{E(R_V) - R_f}{\sigma_V} = \frac{0.12 - 0.04}{0.11} = 0.73$$

$$Sharpe\ Ratio_G = \frac{E(R_G) - R_f}{\sigma_G} = \frac{0.17 - 0.04}{0.27} = 0.48$$

$$Sharpe\ Ratio_M = \frac{E(R_M) - R_f}{\sigma_M} = \frac{0.145 - 0.04}{0.1693} = 0.62$$

Question 3

- Suppose we have that

	Expected Return	Volatility
Value Stocks	12%	11%
Growth Stocks	17%	27%

- The correlation of their returns is 0.5 and the risk-free rate is 4%.
- Does the CAPM hold in this economy? (Hint: Is the market portfolio efficient?)

– A: Yes
– B: No

B

$$Sharpe\ Ratio_V = \frac{E(R_V) - R_f}{\sigma_V} = \frac{0.12 - 0.04}{0.11} = 0.73$$

$$Sharpe\ Ratio_G = \frac{E(R_G) - R_f}{\sigma_G} = \frac{0.17 - 0.04}{0.27} = 0.48$$

$$Sharpe\ Ratio_M = \frac{E(R_M) - R_f}{\sigma_M} = \frac{0.145 - 0.04}{0.1693} = 0.62$$

Because Sharpe Ratio V is bigger than Sharpe Ratio M (market).

Risk-Free saving and Borrowing: Example

- Assume the risk-free rate is 4%. You are a financial adviser and must choose one of the funds below. Your clients will combine it with the risk-free asset.

	Expected Return	Volatility
Fund A	9%	8%
Fund B	7%	2%
Fund C	9%	3%

- What fund would you recommend?

$$Sharpe\ Ratio_A = \frac{E(R_A)-R_f}{\sigma_A} = \frac{0.09-0.04}{0.08} = 0.625$$

$$Sharpe\ Ratio_B = \frac{E(R_B)-R_f}{\sigma_B} = \frac{0.07-0.04}{0.02} = 1.5$$

$$Sharpe\ Ratio_C = \frac{E(R_C)-R_f}{\sigma_C} = \frac{0.09-0.04}{0.03} = 1.667$$

Question 4

- Suppose that the market portfolio has an expected return of 17% and a volatility of 12%. The risk-free rate is 5%. You would like to maximize your expected return without bearing a standard deviation larger than 10%.
- If CAPM holds, what is the maximum expected return that you may earn?

- A: 5%
- B: 15%
- C: 17%
- D: Impossible to compute

CAPM in practice: Example

- Spin industries is seeking to raise capital from a large group of investors to fund a new project. Suppose that the market portfolio has an expected return of 14% and a volatility of 20%. The new project expected to have a volatility of 40% and correlation with the efficient portfolio of 0.7. The risk-free rate is 4%.
- What is the required return for the new project?

$$\beta_S = \frac{\sigma_{S,M}}{\sigma_M^2} = \frac{\sigma_M \sigma_S \rho_{S,M}}{\sigma_M^2} = \frac{\sigma_S}{\sigma_M}\rho_{S,M} = \frac{0.4}{0.2} \times 0.7 = 1.4$$

$$E(R_S) = R_f + \beta_S\left[E(R_M) - R_f\right] = 0.04 + 1.4 \times [0.14 - 0.04]$$
$$= 0.18 \quad (18\%)$$

Question 5

- Suppose the risk-free return is 3.8% and the market portfolio has an expected return of 8.4% and a volatility of 15.2%. The stock of SEB has a 19.5% volatility and a correlation with the market of 5.055.
- What is SEB´s beta with the market? Under the CAPM assumptions, what is its expected return?

- A: 0.071 and 4.12%
- B: 0.071 and 8.4%
- C: 0.055 and 3.8%
- D: 0.084 and 4.12%

The beta of a portfolio: Example

- Consider a portfolio consisting of the following three stocks:

Stocks:	Portfolio Weight	Volatility	Correlation with the Market Portfolio
HEC Corp	0.26	13%	0.36
Green Midget	0.31	23%	0.68
AliveAndWell	0.43	13%	0.58

- The volatility of the market portfolio is 10% and it has a expected return of 8%. The risk-free rate is 3%.
- **Compute the beta and expected return of each stock.**

$$\beta_H = \frac{\sigma_H}{\sigma_M}\rho_{H,M} = \frac{0.13}{0.10}0.36 = 0.468$$
$$\beta_G = \frac{\sigma_G}{\sigma_M}\rho_{G,M} = \frac{0.23}{0.10}0.68 = 1.564$$
$$\beta_A = \frac{\sigma_A}{\sigma_M}\rho_{A,M} = \frac{0.13}{0.10}0.58 = 0.754$$

Stocks:	Portfolio Weight	Volatility	Correlation with the Market Portfolio
HEC Corp	0.26	13%	0.36
Green Midget	0.31	23%	0.68
AliveAndWell	0.43	13%	0.58

- **Compute the beta and expected return of each stock.**

$$\beta_H = 0.468 \quad \beta_G = 1.564 \quad \beta_A = 0.754$$
$$E(R_H) = R_f + \beta_H\left[E(R_M) - R_f\right] = 0.03 + 0.468\times[0.08-0.03] = 0.0534$$
$$E(R_G) = R_f + \beta_G\left[E(R_M) - R_f\right] = 0.03 + 1.564\times[0.08-0.03] = 0.1082$$
$$E(R_A) = R_f + \beta_A\left[E(R_M) - R_f\right] = 0.03 + 0.754\times[0.08-0.03] = 0.0677$$

- **Using your previous answers, calculate the expected return of the portfolio.**

$$E(R_H) = 5.34\% \quad E(R_G) = 10.82\% \quad E(R_A) = 6.77\%$$
$$E(R_P) = 0.26\times0.0534 + 0.31\times0.1082 + 0.43\times0.0677 = 0.0765$$
$$(7.65\%)$$

- **What is the beta of your portfolio?**

$$\beta_H = 0.468 \quad \beta_G = 1.564 \quad \beta_A = 0.754$$
$$\beta_P = x_H\beta_H + x_G\beta_G + x_A\beta_A = 0.26\times0.468 + 0.31\times1.564 + 0.43\times0.754 =$$
$$= 0.931$$

The beta of portfolio: Example

- Consider a portfolio consisting of the following three stocks:

Stocks:	Portfolio Weight	Volatility	Correlation with the Market Portfolio
HEC Corp	0.26	13%	0.36
Green Midget	0.31	23%	0.68
AliveAndWell	0.43	13%	0.58

- The volatility of the market portfolio is 10% and it has a expected return of 8%. The risk-free rate is 3%.
- Use the beta of the portfolio to calculate its expected and verify that it matches the value for it that you obtained earlier?

earlier? $\beta_P = 0.931$ $E(R_P) = 0.0765$ (7.65%)

$$E(R_P) = R_f + \beta_P[E(R_M) - R_f] = 0.03 + 0.931 \times [0.08 - 0.03] = 0.0765$$

Question 6

- Suppose the stock of Spin has a beta of 2.17, whereas he stock of Bang has a beta of 0.68. If the risk-free rate is 3% and the expected return of the market portfolio is 11.5%-
- What us the expected return of a portfolio that consists of 65% of Spin and 35% of Bang, according to CAPM?

- A: 11.5%
- B: 3%
- C: 17.01%
- D: 15.23%

Question 7
The beta of the risk-free asset is equal to:
- **A:** 1
- **B:** 0
- **C:** Unable to answer without knowing the expected return of the market portfolio
- **D:** Unable to answer without knowing the risk-free rate

Question 8
The beta of the market portfolio is equal to:
- **A:** a
- **B:** 0
- **C:** Unable to answer without knowing the expected return of the market portfolio.
- **D:** Unable to answer without knowing the volatility of the market portfolio.

Options 1 – Lecture 11

Question 1

Which of the following statements is false?

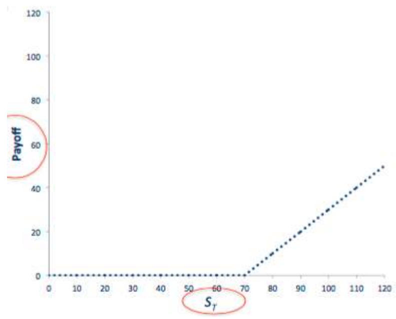
- **A:** A put option gives its owner the right to sell a given asset at a fixed price at some future date.
- **B:** A call option gives its owner the obligation to purchase a given asset at a fixed price at some future date.
- **C:** A call option gives its owner the right to purchase a given asset at a fixed price at some future date.
- **D:** There are two styles of options. American options allow their holders to exercise the right on any date up to and including a final date called expiration date

Call payoffs (holder): Example

- Suppose you have purchased an European call option on the stock of Amazon with strike price K=70. What would be your payoff at maturity for each one of the possible values of the stock at maturity?

S_T	Payoff at maturity
40	0
50	0
60	0
70	0
80	10
90	20
100	30
110	40
120	50

- Long position on a call option on Amazon with strike price K=70.



Call payoffs (holder): Example 2

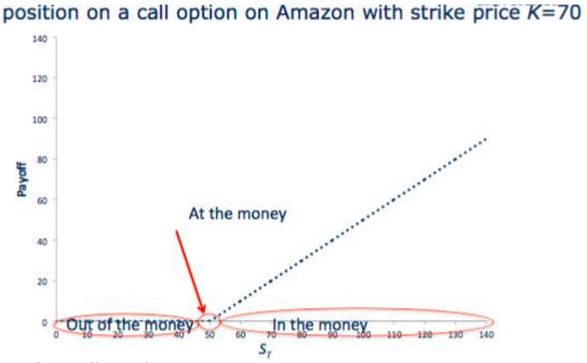
- Suppose you have purchased an European call option on the stock of Amazon with strike price K=50. What would be your payoff at maturity for each one of the possible values of the stock at maturity?

Question 2

- What call option would you prefer? The one with strike price 50 or the one with strike price 70?

S_T	Payoff at maturity
40	0
50	0
60	10
70	20
80	30
90	40
100	50
110	60
120	70

- Long position on a call option on Amazon with strike price K=70



Call option with $K = 50$

S_T	Payoff at maturity
40	0
50	0
60	10
70	20
80	30
90	40
100	50
110	60
120	70

Call option with $K = 70$

S_T	Payoff at maturity
40	0
50	0
60	0
70	0
80	10
90	20
100	30
110	40
120	50

A

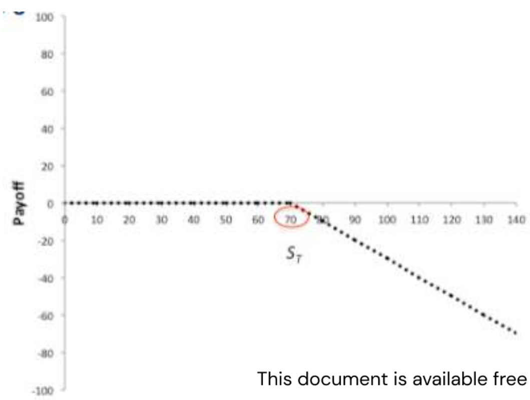
- A: The one with $K = 50$
- B: The one with $K = 70$
- C: I am indifferent

Call payoffs (writer): Example

- Suppose you have written (sold) a European call option on the stock of Amazon with strike price K=70. What would be your payoff at maturity for each one of the possible values of the stock at maturity?

S_T	Payoff at maturity
40	0
50	0
60	0
70	0
80	-10
90	-20
100	-30
110	-40
120	-50

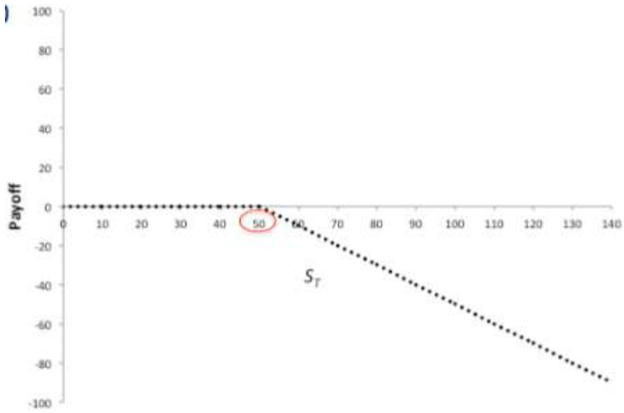
- Short position on a call option on Amazon with strike price K=70.



Call payoffs (writer): Example 2

- Suppose you have written (sold) a European call option on the stock of Amazon with strike price K=50. What would be your payoff at maturity for each one of the possible values of the stock at maturity?

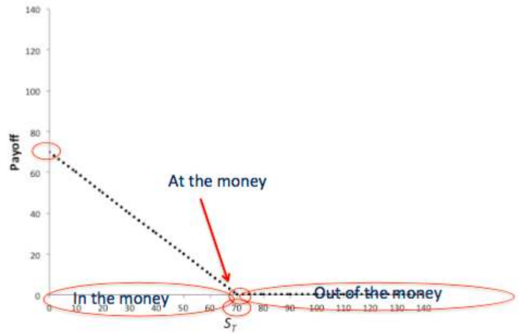
S_T	Payoff at maturity
40	0
50	0
60	-10
70	-20
80	-30
90	-40
100	-50
110	-60
120	-70



Put payoffs (holder): Example

- Suppose you have purchased a European call option on the stock of Amazon with strike price K=70. What would be your payoff at maturity for each one of the possible values of the stock at maturity?

S_T	Payoff at maturity
40	30
50	20
60	10
70	0
80	0
90	0
100	0
110	0
120	0

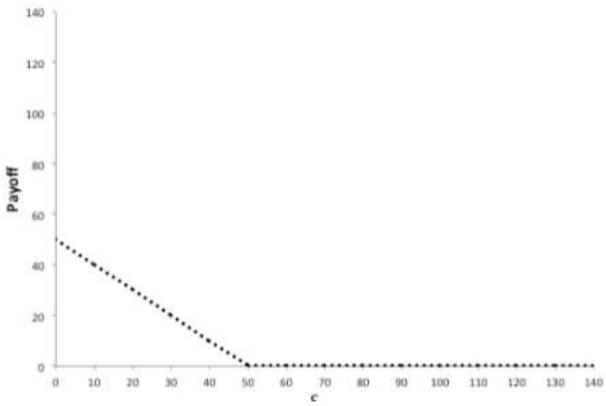


Put payoffs (holder): Example 2

- Suppose you have purchased a European call option on the stock of Amazon with strike price K=50. What would be your payoff at maturity for each one of the possible values of the stock at maturity?

S_T	Payoff at maturity
40	10
50	0
60	0
70	0
80	0
90	0
100	0
110	0
120	0

Question 3



- What put option would you prefer? The one with strike price 50 or the one with strike price 70?

Put option with $K = 50$

S_T	Payoff at maturity
40	10
50	0
60	0
70	0
80	0
90	0
100	0
110	0
120	0

Put option with $K = 70$

S_T	Payoff at maturity
40	30
50	20
60	10
70	0
80	0
90	0
100	0
110	0
120	0

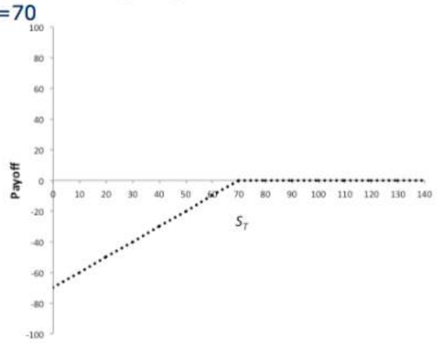
– A: The one with K = 50
– B: The one with K = 70
– C: I am indifferent

Put payoffs (writer): Example

- Suppose you have written (sold) a European call option on the stock of Amazon with strike price K=70. What would be your payoff at maturity for each one of the possible values of the stock at maturity?

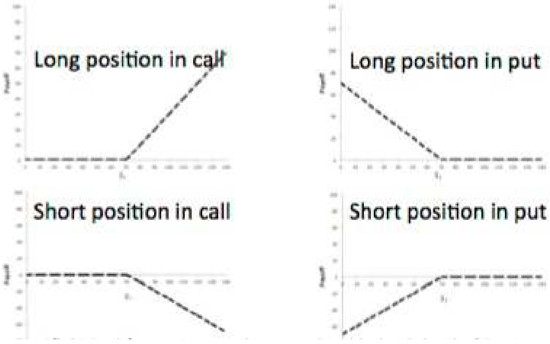
S_T	Payoff at maturity
40	-30
50	-20
60	-10
70	0
80	0
90	0
100	0
110	0
120	0

- Short position on a put option on Amazon with strike price K=70



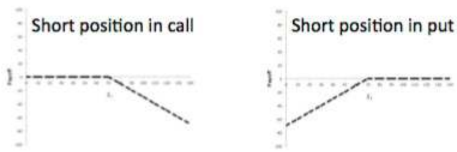
Question 4

Which of the following positions benefits if the price of the underlying increases?
- **A:** Long position in a call
- **B:** Long position in a put
- **C:** Short position in a call
- **D:** Short position in a put
- **E:** Both A and D



Question 5
What position has more downside exposure?
- **A:** A short position in a call
- **B:** A short position in a put
- **C:** The two of them have the same exposure

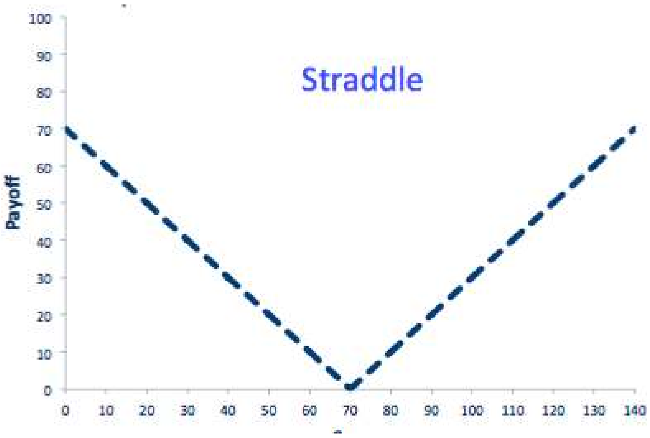
(Downside exposure: losses in the worst-case scenarios)



Combined strategy

- Suppose you have purchased a European call option on the stock of Amazon with strike price K=70 and a put option with the same strike price. What would be your payoff at maturity for each one of the possible values of the stock at maturity?

S_T	Call with K=70 Payoff at maturity	Put with K=70 Payoff at maturity	Total Payoff at maturity
0			
10			
20			
30			
40			
50			
60			
70			
80			
90			
100			
110			
120			



Hedging: Example

- Suppose that you own shares of IBM and its current price is 206. You would like to ensure against a price drop below 200 any time between now and one month from now. The price of a call option with strike price 200 and maturity in one month is 8.4 and the price of a put with the same maturity and strike is 2.
- What would you do? What would be the cost of such insurance as a percentage of your current position in IBM?
- You would buy as many put options as shares you own
- Insurance cost in percentage:

$$\frac{2}{206} = 0.0097 \quad (0.97\%)$$

Question 6

- Suppose that you would like to bet for an increase in the stock of IBM due to some good news for the company that you believe will be released before the end of the month. The price of IBM is currently 206. A call option on IBM with strike 215 and maturity in one month costs 1.1.
- Suppose that your expectations materialize and the stock price in one month takes value 220.
- What would have been your return over one month if you had bought the stock? What would have been your return over one month if you had purchased the call option?

– A: 6.8% and 2.43%
– B: 6.8% and 2.32%
– C: 0% and 2.32%
– D: 6.8% and 354.5%

Question 7