$\boldsymbol{Artifact\ substats\ optimization\ of\ \%ATK, \%Crit\ Rate, and\ \%Crit\ DMG}$

$n = number\ of\ artifact\ substats\ rolls;$

$a_{bo} = Character\ \%ATK\ bonus\ (other\ than\ artifact\ substats);$

$r_{bo} = Character\ \%Crit\ Rate\ bonus\ (other\ than\ artifact\ substats);$

$x_{bo} = Character\ \%Crit\ DMG\ bonus\ (other\ than\ artifact\ substats);$

$f_a = \%ATK\ substats\ roll\ factor$

$f_r = \%Crit\ Rate\ substats\ roll\ factor$

$f_x = \%Crit\ DMG\ substats\ roll\ factor$

$A_b = Character\ base\ ATK$

$A_f = Character\ total\ flat\ ATK\ (+ATK)$

$d_{bo} = Character\ total\ damage\ bonus\ (\%DMG\ bonus)$

$Input: n, a_{bo}, r_{bo}, x_{bo}, f_a, f_r, f_x, A_f, A_b, d_{bo}$

$\Phi_a = 0.041 + 0.017 f_a;\ \Phi_r = 0.027 + 0.012 f_r;\ \Phi_x = 0.054 + 0.024 f_x;$

$$V = n + \frac{x_{bo}}{\Phi_x} + \frac{r_{bo}}{\Phi_r} + \frac{1 + a_{bo} + \frac{A_f}{A_b}}{\Phi_a}$$

$$V_{min} = \sqrt{\frac{12}{\Phi_r \Phi_x}}$$

$if\ V < V_{min}\ \{$

$print$ your base stats is too small;

$return;\}$

$$n_r = \frac{1}{6}\left(V + \sqrt{V^2 - \frac{12}{\Phi_r \Phi_x}}\right) - \frac{r_{bo}}{\Phi_r}$$

$$r = n_r \Phi_r + r_{bo}$$

$if\ r > 1\ \{$

$r = 1$

$$n_r = \frac{1 - r_{bo}}{\Phi_r}$$

$$n_a = \frac{1}{2}\left(n + \frac{r_{bo} - 1}{\Phi_r} + \frac{x_{bo} - 1}{\Phi_x} - \frac{a_{bo} + \frac{A_f}{A_b} - 1}{\Phi_a} \right)$$

$$n_x = \frac{1}{2}\left(n + \frac{r_{bo} - 1}{\Phi_r} - \frac{x_{bo} - 1}{\Phi_x} + \frac{a_{bo} + \frac{A_f}{A_b} - 1}{\Phi_a} \right)$$

$\}\ else\ \{$

$$n_a = \frac{1}{3}\left(2V - \sqrt{V^2 - \frac{12}{\Phi_r \Phi_x}} \right) - \frac{1 + a_{bo} + \frac{A_f}{A_b}}{\Phi_a}$$

$$n_x = \frac{1}{6}\left(V + \sqrt{V^2 - \frac{12}{\Phi_r \Phi_x}} \right) - \frac{x_{bo}}{\Phi_x}$$

$a = n_a \Phi_a + a_{bo}$

$x = n_x \Phi_x + x_{bo}$

$D = (1 + d_{bo})\left((1 + a) A_b + A_f \right)(1 + rx)$

$output: a, r, x, D$