$$= d_1 e_{1,3} M e_{2,3} + d_2 e_{2,3} M e_{2,3} + e_{3,4} M e_{2,3}$$

$$\Rightarrow d_1^2 \left[(\phi_3 - \phi_2)^2 e_{1,3} M e_{1,3}^T \right] + d_2^2 \left[(\phi_3 - \phi_2)^2 e_{2,3} M e_{2,3}^T \right].$$

$$+ d_1 d_2 \left[(\phi_3 - \phi_2)^2 \left\{ e_{1,3}^T M e_{2,3} + e_{2,3}^T M e_{1,3} \right\} \right]$$

$$+ d_1 \left[(\phi_3 - \phi_2)^2 e_{1,3}^T M e_{3,4} - e_{1,3}^T M e_{2,3} \right]$$

$$+ d_2 \left[(\phi_3 - \phi_2)^2 e_{2,3}^T M e_{3,4} - e_{2,3}^T M e_{2,3} \right].$$

$$+ \left[(\phi_3 - \phi_2)^2 e_{3,4}^T M e_{3,4} - e_{3,4}^T M e_{2,3} \right] = 0 \longleftarrow (A).$$

$$d_1\left[(\varphi_3-\varphi_2)\, e_{1,3}^T M e_{1,3} - (\varphi_3-\varphi_1)\, e_{1,3}^T M e_{2,3}\right] +$$
$$d_2\left[(\varphi_3-\varphi_2)\, e_{2,3}^T M e_{1,3} - (\varphi_3-\varphi_1)\, e_{2,3}^T M e_{2,3}\right]$$
$$+ \quad (\varphi_3-\varphi_2)\, e_{3,4}^T M e_{1,3} - (\varphi_3-\varphi_1)\, e_{3,4}^T M e_{2,3} = 0. \qquad \text{— } (B)$$

Newton's method :-

$$f = (f^1, f^2) : \mathbb{R}^2 \to \mathbb{R}^2$$

$$J(x_0, y_0) = \begin{pmatrix} f^1_x & f^1_y \\ f^2_x & f^2_y \end{pmatrix}\Bigg|_{(x_0, y_0)}$$

$$v_1 = v_0 - J_{v_0}^{-1} f(v_0)$$

$$N(v) = v - J_v^{-1} f(v)$$

local solver(v_1, v_2, v_3, v_4 phi1, phi2, phi3)

want d_1 & d_2 → Newton's method (v_1, v_2, v_3, v_4, phi1, phi2, phi3)

we check if d_1 & $d_2 \in [0,1]$.

then calculate φ_4.

double f1 (v_1, v_2, v_3, v_4, phi1, phi2, phi3).
double f2 (v_1, v_2, v_3, v_4, ———
double df1-dx (v_1, v_2, v_3, v_4 ———
double df2-dx (v_1, v_2, v_3, v_4 ———).

$$\phi_4 = d_1\phi_1 + d_2\phi_2 + (1-d_1-d_2)\phi_3$$

$$+ \int e^T_{5,4} M e_{5,4}.$$

We have 3D solve

For 2D solver, we want to add 3rd component of the vertex.

$$\phi_3 = d\phi_{1,2} + \phi_1 \|e_{1,3} - de_{1,2}\|.$$

$$\phi_{1,2}^2 \left[(a_1+b_1d)^2 + (a_2+b_2d)^2 + (a_3+b_3d)^2 \right] =$$
$$\left[(a_1+b_1d)b_1 + (a_2+b_2d)b_2 + (a_3+b_3d)b_3 \right]^2$$

$$= (a_1+b_1d)^2 b_1^2 + (a_2+b_2d)^2 b_2^2 + (a_3+b_3d)^2 b_3^2$$
$$+ 2(a_1+b_1d)(a_2+b_2d)b_1b_2 + 2(a_2+b_2d)(a_3+b_3d)b_2b_3$$
$$+ 2(a_3+b_3d)(a_1+b_1d)b_3b_1.$$

$$(\phi_{1,2}^2 - b_1^2)(a_1+b_1d)^2 + (\phi_{1,2}^2 - b_2^2)(a_2+b_2d)^2$$
$$+ (\phi_{1,2}^2 - b_3^2)(a_3+b_3d)^2 = d^2(2b_1^2b_2^2 + 2b_2^2b_3^2 + 2b_1^2b_3^2)$$
$$+ d(2a_1b_1b_2^2 + 2a_2b_1^2b_2) + d(2a_2b_2b_3^2 + 2a_3b_2^2b_3)$$
$$+ d(2a_3b_1^2b_3 + 2a_1b_1b_3^2) + (2a_1a_2b_1b_2 + 2a_2a_3b_2b_3$$
$$+ 2a_1a_3b_1b_3).$$

$$(\varphi_{1,2}^2 - b_1^2)(a_1^2 + b_1^2 d^2 + 2a_1 b_1 d) + (\varphi_{1,2}^2 - b_2^2)$$

$$(a_2^2 + b_2^2 d^2 + 2a_2 b_2 d) + (\varphi_{1,2}^2 - b_3^2)(a_3^2 + b_3^2 d^2 + 2a_3 b_3 d)$$

$$= d^2\left[(\varphi_{1,2}^2 - b_1^2) b_1^2 + (\varphi_{1,2}^2 - b_2^2) b_2^2 + (\varphi_{1,2}^2 - b_3^2) b_3^2\right].$$

$$+ \; d\left[(\varphi_{1,2}^2 - b_1^2)\cdot 2a_1 b_1 + (\varphi_{1,2}^2 - b_2^2)\cdot 2a_2 b_2 + (\varphi_{1,2}^2 - b_3^2)\cdot 2a_3 b_3\right] + \left[(\varphi_{1,2}^2 - b_1^2) a_1^2 + (\varphi_{1,2}^2 - b_2^2) a_2^2 + (\varphi_{1,2}^2 - b_3^2) a_3^2\right].$$

So, we have a quadratic equation to solve.

$$d^2\left[(\varphi_{1,2}^2 - b_1^2) b_1^2 + (\varphi_{1,2}^2 - b_2^2) b_2^2 + (\varphi_{1,2}^2 - b_3^2) b_3^2\right.$$

$$d^2 \left[(\phi_{1,2}^2 - b_1^2) b_1^2 + (\phi_{1,2}^2 - b_2^2) b_2^2 + (\phi_{1,2}^2 - b_3^2) b_3^2 - 2b_1^2 b_2^2 - 2b_2^2 b_3^2 - 2b_1^2 b_3^2 \right]$$

$$+$$

$$d \left[(\phi_{1,2}^2 - b_1^2) \cdot 2a_1 b_1 + (\phi_{1,2}^2 - b_2^2) \cdot 2a_2 b_2 + (\phi_{1,2}^2 - b_3^2) \cdot 2a_3 b_3 - 2a_2 b_2 b_3^2 - 2a_3 b_2^2 b_3 - 2a_3 b_1^2 b_3 - 2a_1 b_1 b_3^2 - 2a_1 b_1 b_2^2 - 2a_2 b_1^2 b_2 \right].$$

$$+ \left[(\phi_{1,2}^2 - b_1^2) a_1^2 + (\phi_{1,2}^2 - b_2^2) a_2^2 + (\phi_{1,2}^2 - b_3^2) a_3^2 - 2a_1 a_2 b_1 b_2 - 2a_2 a_3 b_2 b_3 - 2a_1 a_3 b_1 b_3 \right].$$

$$= 0.$$

$$d^2 A + dB + C = 0$$

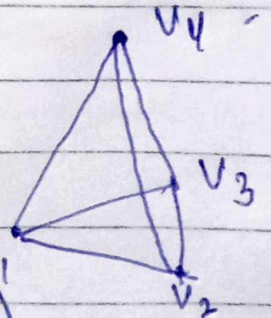
$$\phi_3 = d(\phi_2 - \phi_1) + \phi_1 + \| e_{1,3} - d\, e_{1,2} \|.$$

$$d = 0 \Rightarrow \phi_3 = \phi_1 + \| e_{1,3} \|$$

$$d = 1 \Rightarrow \phi_3 = \phi_2 + \| e_{2,3} \|.$$

Apply this to triangles.

$\Delta_{1,2,4}$, $\Delta_{1,3,4}$, $\Delta_{2,3,4}$.



we get solution at v_4.
(3 different solutions at v_4 for 3 Δs).

Take $\min \{ \phi_4^{\Delta 1}, \phi_4^{\Delta 2}, \phi_4^{\Delta 3} \}$.