Factor_Analysis

2023-03-03

Question 1. Decide how many Factors are ideal for your dataset ANS: In this question, two factors are ideal for the dataset. That is because from the scree plot there are significant decrease of the line before the factor is 2. After factor = 2, the change of the line is not significant. And after factor = 2, the data point is under the eigenvalue line. Additionally, from the chart of Very Simple Structure, factor = 2 line has good performance in fit.

Question 2. Explain the output for your factor model ANS: In component analysis, the factor loading between PC1 and area, rooms, bathroom, fire insurance, parking spaces are 0.9, 0.9, 0.8, 0.8, 0.8. The factor loading between PC2 and property tax, hoa are 0.9, 0.8.

Question 3. Show the columns that go into each factor ANS: The columns of each factors are as follows. Click to factors columns

Question 4. Perform some visualizations using the factors ANS: Here is the plot of variance of PCA factors (click). The plot of component analysis (click) is also a kind of visualization of factors and variables.

```
# Factor Analysis
library(psych)
library(readr)
house_data <- read_csv("Dataset/Rent_House_random_200.csv")
```

```
## Rows: 200 Columns: 11
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## chr (3): floor, animal, furniture
## dbl (8): area, rooms, bathroom, parking spaces, hoa, rent amount, property t...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
house_data <- house_data[, c(1:4, 6:11)]
house_vars <- house_data[,-c(5,6,8)]
str(house_vars)
```

```
## tibble [200 x 7] (S3: tbl_df/tbl/data.frame)
##  $ area          : num [1:200] 120 45 50 35 204 177 15 70 180 180 ...
##  $ rooms         : num [1:200] 3 1 2 1 4 3 1 2 3 4 ...
##  $ bathroom      : num [1:200] 4 1 1 1 4 3 1 2 3 4 ...
##  $ parking spaces: num [1:200] 3 1 1 0 2 4 0 1 2 2 ...
##  $ hoa           : num [1:200] 1350 3000 226 260 0 2700 0 1800 700 2600 ...
##  $ property tax  : num [1:200] 560 0 0 0 100 509 0 250 175 584 ...
##  $ fire insurance: num [1:200] 71 70 10 18 62 89 16 55 40 26 ...
```

```
#house_data[-1]
```

```
fit.pc <- principal(house_vars, nfactors=4, rotate="varimax")
fit.pc
```

```
## Principal Components Analysis
## Call: principal(r = house_vars, nfactors = 4, rotate = "varimax")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                  RC1  RC4  RC3   RC2   h2    u2 com
## area            0.74 0.47 0.20  0.24 0.87 0.127 2.1
## rooms           0.33 0.91 0.14  0.10 0.96 0.038 1.3
## bathroom        0.62 0.57 0.24  0.24 0.82 0.183 2.6
## parking spaces 0.83 0.23 0.08  0.32 0.85 0.147 1.5
## hoa             0.20 0.15 0.91  0.28 0.96 0.042 1.4
## property tax    0.22 0.13 0.23  0.92 0.96 0.038 1.3
## fire insurance 0.73 0.29 0.50 -0.05 0.87 0.128 2.1
##
##                        RC1  RC4  RC3  RC2
## SS loadings           2.36 1.54 1.24 1.15
## Proportion Var        0.34 0.22 0.18 0.16
## Cumulative Var        0.34 0.56 0.73 0.90
## Proportion Explained  0.37 0.25 0.20 0.18
## Cumulative Proportion 0.37 0.62 0.82 1.00
##
## Mean item complexity =  1.8
## Test of the hypothesis that 4 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.05
##  with the empirical chi square  18.85  with prob <  NA
##
## Fit based upon off diagonal values = 0.99
```

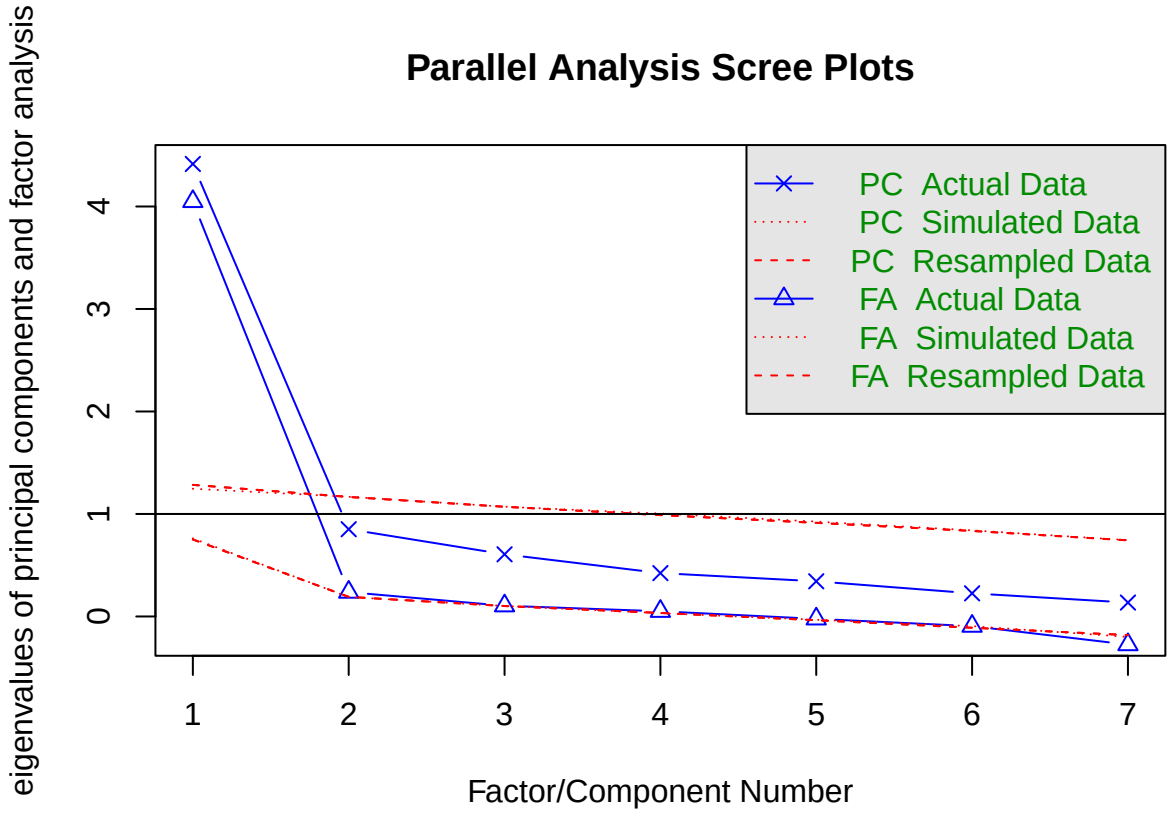
```
round(fit.pc$values, 3)
```

```
## [1] 4.415 0.851 0.606 0.423 0.343 0.226 0.136
```

```
fit.pc$loadings
```

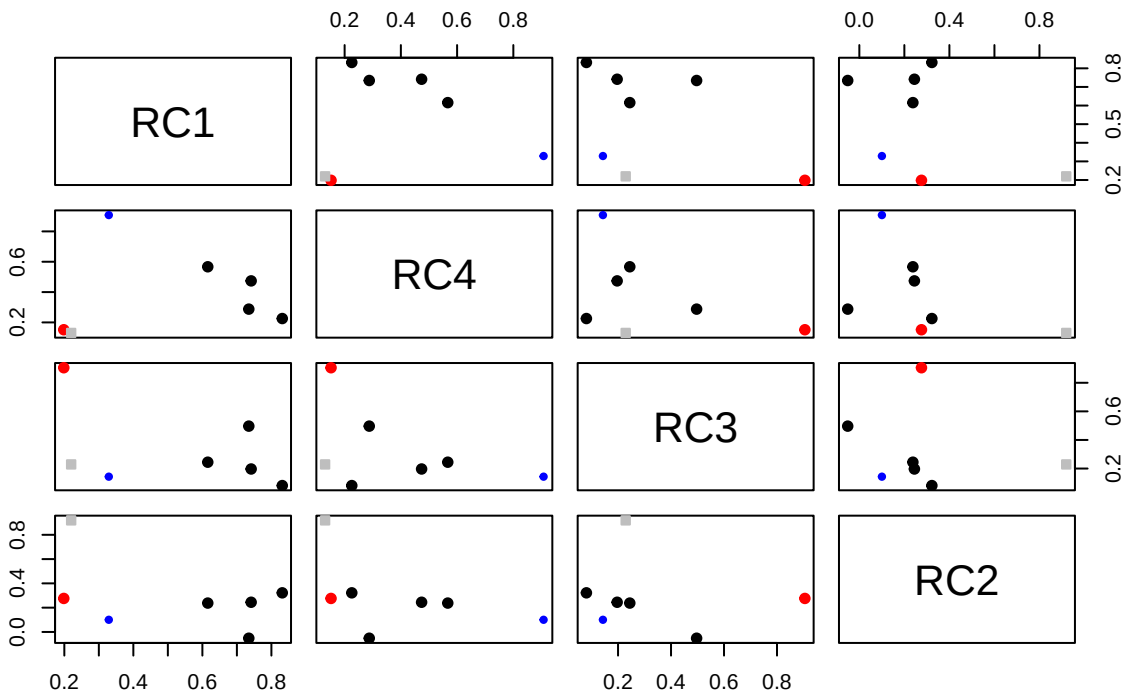
```
##
## Loadings:
##                 RC1    RC4    RC3    RC2
## area             0.741  0.474  0.197  0.245
## rooms            0.329  0.908  0.143  0.100
## bathroom         0.616  0.567  0.244  0.238
## parking spaces   0.832  0.226         0.322
## hoa              0.198  0.152  0.905  0.276
## property tax     0.220  0.131  0.229  0.919
## fire insurance   0.735  0.288  0.497
##
##                   RC1   RC4   RC3   RC2
## SS loadings     2.355 1.543 1.244 1.153
## Proportion Var 0.336 0.220 0.178 0.165
## Cumulative Var 0.336 0.557 0.735 0.899
```

```
fa.parallel(house_vars)
```



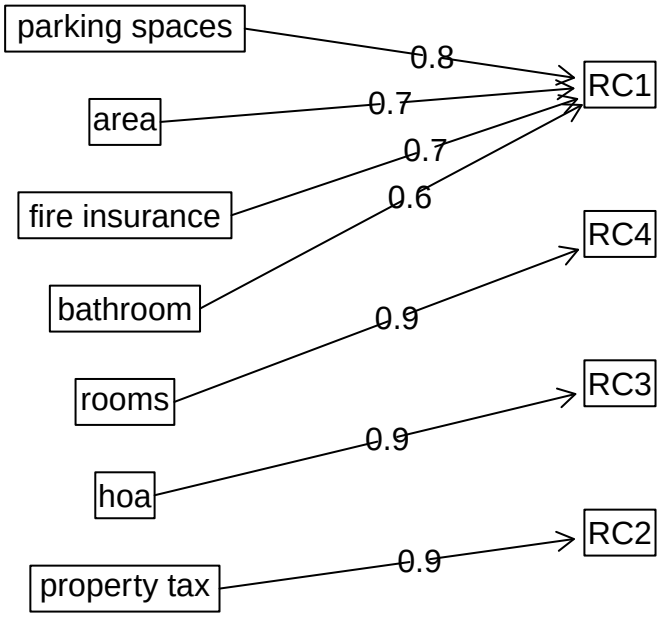

```
## Parallel analysis suggests that the number of factors =  2  and the number of components =  1
```

```
fa.plot(fit.pc)
```




```
fa.diagram(fit.pc)
```

Components Analysis

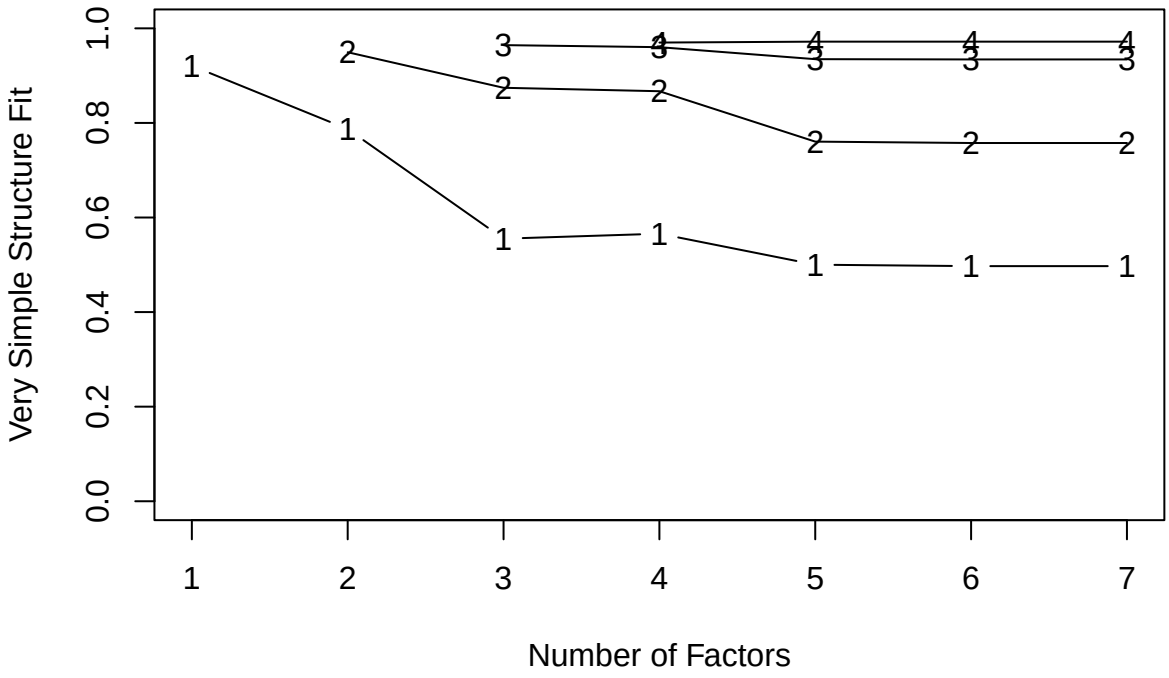



```
vss(house_vars)
```

```
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect.  Try a
## different factor score estimation method.
```

```
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected.  Examine the results carefully
```

```
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected.  Examine the results carefully
```




```
##
## Very Simple Structure
## Call: vss(x = house_vars)
## VSS complexity 1 achieves a maximimum of 0.92  with  1  factors
## VSS complexity 2 achieves a maximimum of 0.95  with  2  factors
##
## The Velicer MAP achieves a minimum of 0.05  with  1  factors
## BIC achieves a minimum of  7.67  with  3  factors
## Sample Size adjusted BIC achieves a minimum of  17.18  with  3  factors
##
## Statistics by number of factors
##   vss1 vss2   map dof   chisq    prob sqresid  fit RMSEA  BIC SABIC complex
## 1 0.92 0.00 0.053  14 8.6e+01 2.0e-12    1.66 0.92  0.16 12.0    56     1.0
## 2 0.79 0.95 0.102   8 5.3e+01 1.0e-08    1.06 0.95  0.17 10.8    36     1.4
## 3 0.56 0.87 0.179   3 2.4e+01 3.1e-05    0.74 0.96  0.19  7.7    17     1.7
## 4 0.57 0.87 0.306  -1 1.8e-01      NA    0.63 0.97    NA   NA    NA     1.8
## 5 0.50 0.76 0.425  -4 1.7e-10      NA    0.50 0.98    NA   NA    NA     2.1
## 6 0.50 0.76 1.000  -6 2.5e-11      NA    0.50 0.98    NA   NA    NA     2.1
## 7 0.50 0.76    NA  -7 2.5e-11      NA    0.50 0.98    NA   NA    NA     2.1
##    eChisq    SRMR eCRMS eBIC
## 1 3.1e+01 6.1e-02 0.074  -43
## 2 1.1e+01 3.6e-02 0.058  -32
## 3 3.4e+00 2.0e-02 0.053  -13
## 4 2.3e-02 1.6e-03    NA   NA
## 5 1.4e-11 4.1e-08    NA   NA
## 6 2.4e-12 1.7e-08    NA   NA
```

```
## 7 2.4e-12 1.7e-08    NA   NA
```

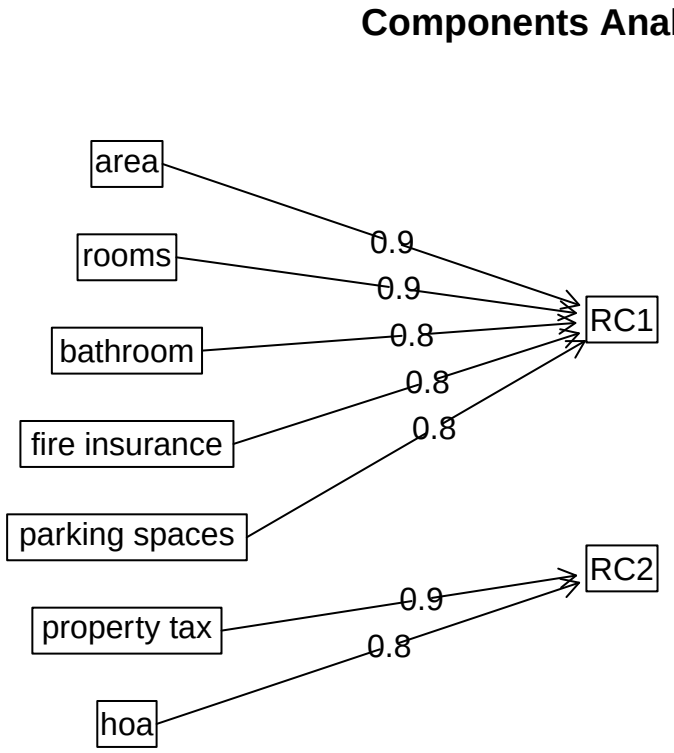
```
fit.pc1 <- principal(house_vars, nfactors=2, rotate="varimax")
round(fit.pc$values, 3)
```

```
## [1] 4.415 0.851 0.606 0.423 0.343 0.226 0.136
```

components_analysis

Components Analysis

```
fa.diagram(fit.pc1)
```



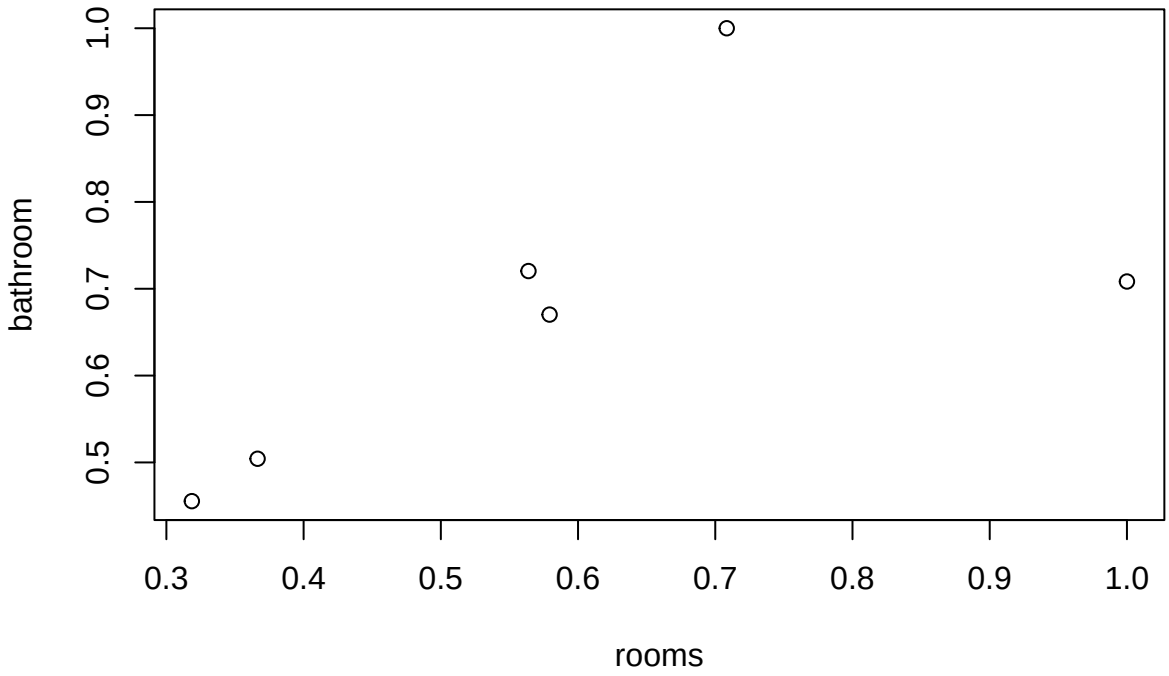
variance

```
corrm.emp <- cor(house_vars[-1])
corrm.emp
```

```
##                    rooms  bathroom parking spaces       hoa property tax
## rooms          1.0000000 0.7083450      0.5638594 0.3664647    0.3185592
```

```
## bathroom       0.7083450 1.0000000      0.7204068 0.5042408    0.4554111
## parking spaces 0.5638594 0.7204068      1.0000000 0.4270628    0.4716517
## hoa            0.3664647 0.5042408      0.4270628 1.0000000    0.4855746
## property tax   0.3185592 0.4554111      0.4716517 0.4855746    1.0000000
## fire insurance 0.5792533 0.6702586      0.6088245 0.5550327    0.3346348
##                fire insurance
## rooms               0.5792533
## bathroom            0.6702586
## parking spaces      0.6088245
## hoa                 0.5550327
## property tax        0.3346348
## fire insurance      1.0000000
```

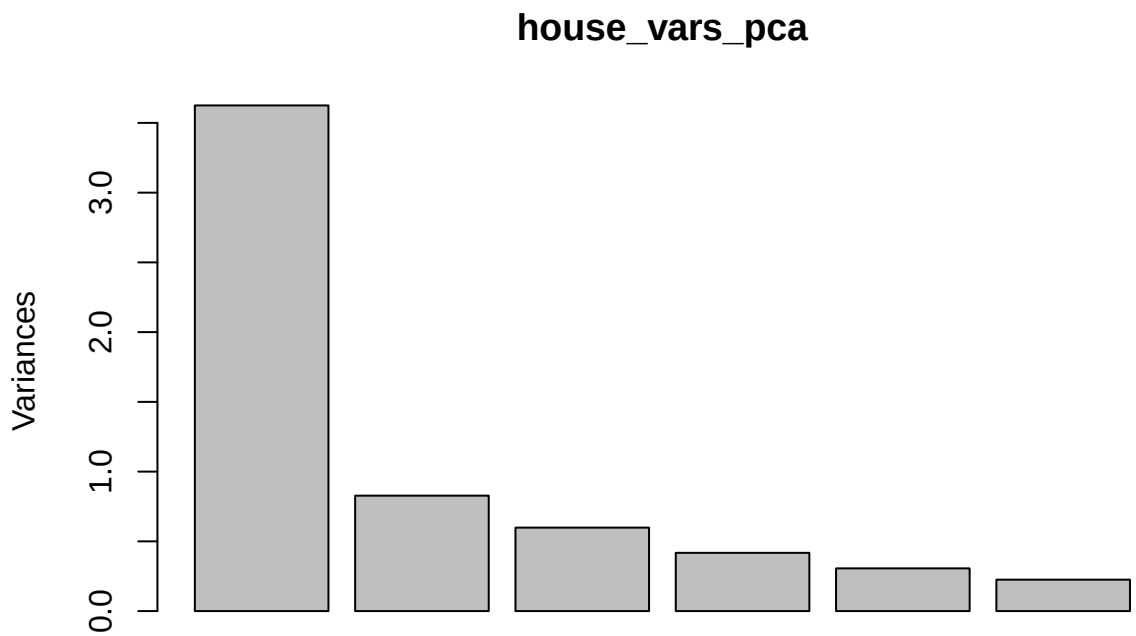
```
plot(corrm.emp)
```




```
house_vars_pca <- prcomp(house_vars[-1], scale=TRUE)
summary(house_vars_pca)
```

```
## Importance of components:
##                           PC1    PC2     PC3    PC4    PC5     PC6
## Standard deviation     1.9040 0.9096 0.77360 0.6462 0.5532 0.47458
## Proportion of Variance 0.6042 0.1379 0.09974 0.0696 0.0510 0.03754
## Cumulative Proportion  0.6042 0.7421 0.84186 0.9115 0.9625 1.00000
```

```r
plot(house_vars_pca)
```



factors_columns

```r
(eigen_house_vars <- round(house_vars_pca$sdev^2,3))
```

```
## [1] 3.625 0.827 0.598 0.418 0.306 0.225
```

```r
names(eigen_house_vars) <- paste("PC",1:6,sep="")
sumlambdas <- sum(eigen_house_vars)
propvar <- round(eigen_house_vars/sumlambdas,2)
cumvar_house_vars <- cumsum(propvar)
matlambdas <- rbind(eigen_house_vars,propvar,cumvar_house_vars)
rownames(matlambdas) <- c("Eigenvalues","Prop. variance","Cum. prop. variance")
eigvec.emp <- house_vars_pca$rotation
pcafactors.emp <- eigvec.emp[,1:2]
unrot.fact.emp <- sweep(pcafactors.emp,MARGIN=2,house_vars_pca$sdev[1:2],`*`)
communalities.emp <- rowSums(unrot.fact.emp^2)
communalities.emp
```

```
##          rooms       bathroom parking spaces            hoa   property tax
##      0.7578867      0.8239901      0.6942779      0.6835603      0.7962199
```

```
## fire insurance
##      0.6967861
```

```
rot.fact.emp <- varimax(unrot.fact.emp)
rot.fact.emp
```

```
## $loadings
##
## Loadings:
##                PC1    PC2
## rooms          -0.866
## bathroom       -0.844  0.335
## parking spaces -0.747  0.369
## hoa            -0.343  0.752
## property tax   -0.175  0.875
## fire insurance -0.774  0.312
##
##                  PC1   PC2
## SS loadings    2.767 1.686
## Proportion Var 0.461 0.281
## Cumulative Var 0.461 0.742
##
## $rotmat
##           [,1]       [,2]
## [1,] 0.8324983 -0.5540276
## [2,] 0.5540276  0.8324983
```

```
fact.load.emp <- rot.fact.emp$loadings[1:6,1:2]
fact.load.emp
```

```
##                       PC1       PC2
## rooms          -0.8656407 0.0924822
## bathroom       -0.8437599 0.3347525
## parking spaces -0.7469370 0.3692736
## hoa            -0.3425851 0.7524598
## property tax   -0.1746465 0.8750534
## fire insurance -0.7742501 0.3119660
```

```
scale.emp <- scale(house_vars[-1])
scale.emp
```

```
##             rooms   bathroom parking spaces         hoa property tax
##   [1,]  0.5433617  1.3639634      1.1410493  0.38772522  0.212165314
##   [2,] -1.1546437 -0.8720422     -0.2676535  1.80601197 -0.357884228
##   [3,] -0.3056410 -0.8720422     -0.2676535 -0.57842891 -0.357884228
##   [4,] -1.1546437 -0.8720422     -0.9720049 -0.54920361 -0.357884228
##   [5,]  1.3923644  1.3639634      0.4366979 -0.77269122 -0.256089667
##   [6,]  0.5433617  0.6186282      1.8454007  1.54814165  0.160250088
##   [7,] -1.1546437 -0.8720422     -0.9720049 -0.77269122 -0.357884228
##   [8,] -0.3056410 -0.1267070     -0.2676535  0.77453070 -0.103397825
##   [9,]  0.5433617  0.6186282      0.4366979 -0.17099381 -0.179743746
##  [10,]  1.3923644  1.3639634      0.4366979  1.46218488  0.236596008
```

```
##  [11,] -0.3056410 -0.1267070     -0.2676535 -0.21397220 -0.312076675
##  [12,]  0.5433617  1.3639634      1.1410493  0.77453070  0.863650504
##  [13,] -0.3056410 -0.1267070     -0.9720049 -0.77269122 -0.078967131
##  [14,] -1.1546437 -0.8720422     -0.9720049 -0.32657557 -0.357884228
##  [15,] -0.3056410 -0.8720422     -0.2676535 -0.46324684 -0.342615044
##  [16,]  0.5433617 -0.1267070     -0.9720049 -0.07214352 -0.275430633
##  [17,] -1.1546437 -0.1267070     -0.2676535 -0.21397220 -0.201120604
##  [18,] -1.1546437 -0.8720422     -0.9720049  0.07398299 -0.217407734
##  [19,]  0.5433617  0.6186282     -0.2676535  0.25879006 -0.049446708
##  [20,] -1.1546437 -0.8720422     -0.9720049 -0.47184251 -0.299861328
##  [21,] -0.3056410 -0.8720422     -0.2676535 -0.77269122 -0.357884228
##  [22,]  0.5433617 -0.1267070      0.4366979 -0.30766508 -0.213335951
##  [23,]  1.3923644  2.1092986      1.8454007 -0.77269122  0.405574980
##  [24,]  4.7883752  1.3639634     -0.9720049 -0.38588574 -0.001603264
##  [25,]  0.5433617 -0.1267070      0.4366979 -0.64375606 -0.327345859
##  [26,] -1.1546437 -0.8720422     -0.9720049 -0.17099381 -0.328363805
##  [27,] -1.1546437 -0.8720422     -0.9720049 -0.54920361 -0.342615044
##  [28,] -1.1546437 -0.8720422     -0.2676535 -0.77269122 -0.357884228
##  [29,] -0.3056410 -0.1267070     -0.2676535 -0.17099381 -0.205192386
##  [30,]  0.5433617  1.3639634      0.4366979 -0.77183165  0.278331778
##  [31,] -1.1546437 -0.8720422     -0.9720049 -0.17099381 -0.357884228
##  [32,]  1.3923644 -0.1267070      0.4366979  0.79172205  0.321085494
##  [33,]  0.5433617  0.6186282      1.1410493  0.34474683  0.293600962
##  [34,] -0.3056410 -0.8720422     -0.2676535 -0.42886413 -0.357884228
##  [35,]  1.3923644  2.1092986      1.1410493  0.70576528  0.176537217
##  [36,]  0.5433617  2.1092986      1.1410493  1.02466491  0.610182047
##  [37,]  1.3923644  2.8546338      1.8454007  6.21817314  4.243229928
##  [38,]  0.5433617 -0.1267070     -0.2676535 -0.77269122 -0.332435587
##  [39,] -1.1546437 -0.8720422     -0.9720049 -0.29992897 -0.357884228
##  [40,]  1.3923644 -0.1267070     -0.2676535 -0.25695058 -0.217407734
##  [41,] -1.1546437 -0.8720422     -0.9720049 -0.77269122 -0.323274077
##  [42,]  1.3923644  1.3639634      1.8454007  1.06678372  0.239649845
##  [43,]  0.5433617  0.6186282      0.4366979  0.25879006 -0.009746829
##  [44,]  0.5433617  0.6186282      0.4366979 -0.38588574 -0.213335951
##  [45,]  1.3923644  0.6186282     -0.2676535  0.57683012  0.157196251
##  [46,]  0.5433617  0.6186282      1.8454007 -0.77269122 -0.195012930
##  [47,]  1.3923644  1.3639634      1.8454007  1.46218488  0.914547784
##  [48,]  0.5433617 -0.8720422     -0.9720049 -0.49762955 -0.311058730
##  [49,]  0.5433617  1.3639634      1.8454007  3.12974628 -0.357884228
##  [50,] -1.1546437 -0.8720422     -0.9720049 -0.60077767 -0.345668880
##  [51,]  0.5433617 -0.1267070     -0.2676535 -0.03346297 -0.286628035
##  [52,] -0.3056410 -0.1267070     -0.2676535 -0.42886413 -0.274412688
##  [53,]  0.5433617 -0.1267070     -0.2676535 -0.33259254 -0.220461570
##  [54,]  0.5433617  0.6186282      1.1410493  1.89196875  1.848003908
##  [55,]  1.3923644  2.1092986      2.5497521  2.23579584 10.666466725
##  [56,] -0.3056410 -0.8720422     -0.9720049 -0.25695058 -0.357884228
##  [57,] -1.1546437 -0.8720422     -0.9720049 -0.54060793 -0.357884228
##  [58,] -1.1546437 -0.8720422     -0.9720049 -0.77269122 -0.357884228
##  [59,] -0.3056410 -0.8720422     -0.2676535 -0.14606634 -0.233694863
##  [60,] -0.3056410 -0.8720422     -0.2676535 -0.77269122 -0.312076675
##  [61,] -1.1546437 -0.8720422     -0.9720049 -0.50708479 -0.329381751
##  [62,] -0.3056410  0.6186282      0.4366979  0.56393660  0.390305795
##  [63,]  0.5433617  0.6186282     -0.9720049 -0.06784568 -0.207228277
##  [64,] -1.1546437 -0.8720422     -0.9720049 -0.77269122 -0.286628035
```

```
##  [65,]  0.5433617  0.6186282      0.4366979 -0.77269122 -0.145133595
##  [66,]  0.5433617 -0.1267070     -0.2676535 -0.51482090 -0.206210332
##  [67,]  0.5433617  0.6186282      0.4366979  0.21581167 -0.272376796
##  [68,] -1.1546437 -0.8720422     -0.2676535 -0.43745981 -0.282556253
##  [69,] -1.1546437 -0.8720422     -0.9720049 -0.42886413 -0.306986947
##  [70,] -1.1546437 -0.8720422     -0.9720049 -0.54060793 -0.351776554
##  [71,] -0.3056410 -0.8720422     -0.9720049 -0.47184251 -0.239802537
##  [72,]  0.5433617 -0.1267070      0.4366979 -0.77269122  0.107316916
##  [73,]  1.3923644 -0.1267070     -0.2676535 -0.77269122 -0.145133595
##  [74,] -1.1546437 -0.8720422     -0.9720049 -0.77269122 -0.322256131
##  [75,]  0.5433617 -0.1267070      0.4366979  0.08687651 -0.027051905
##  [76,] -1.1546437 -0.8720422     -0.9720049 -0.77269122 -0.246928156
##  [77,] -0.3056410 -0.1267070     -0.2676535 -0.59218200 -0.357884228
##  [78,] -1.1546437 -0.8720422     -0.2676535 -0.57928848 -0.291717763
##  [79,] -1.1546437 -0.8720422     -0.9720049 -0.40393666 -0.320220240
##  [80,]  1.3923644 -0.1267070      0.4366979  0.39374219 -0.143097704
##  [81,]  0.5433617 -0.1267070     -0.9720049 -0.77269122 -0.347704772
##  [82,] -0.3056410 -0.1267070     -0.2676535 -0.52771442 -0.297825437
##  [83,]  0.5433617 -0.1267070      1.1410493 -0.77269122 -0.319202295
##  [84,] -1.1546437 -0.8720422      1.8454007 -0.77269122 -0.281538307
##  [85,]  0.5433617 -0.1267070      0.4366979 -0.64375606 -0.210282114
##  [86,] -0.3056410 -0.8720422     -0.2676535 -0.63687952 -0.357884228
##  [87,] -1.1546437 -0.1267070      0.4366979 -0.77269122 -0.357884228
##  [88,] -1.1546437 -0.8720422     -0.9720049 -0.77269122 -0.357884228
##  [89,] -0.3056410  0.6186282     -0.9720049 -0.77269122 -0.343632989
##  [90,]  1.3923644  0.6186282      0.4366979  0.84845352  0.048276071
##  [91,]  1.3923644 -0.1267070      0.4366979 -0.25695058 -0.357884228
##  [92,] -1.1546437 -0.1267070     -0.9720049 -0.77269122 -0.226569244
##  [93,]  0.5433617  2.1092986      4.6628062 -0.77269122  3.535757729
##  [94,] -1.1546437 -0.8720422     -0.9720049 -0.38588574 -0.241838428
##  [95,]  0.5433617  0.6186282     -0.2676535  0.24159870 -0.140043867
##  [96,] -0.3056410  3.5999689      0.4366979 -0.55178231 -0.357884228
##  [97,] -0.3056410  0.6186282      1.8454007  1.46218488  0.023845376
##  [98,] -0.3056410 -0.1267070     -0.2676535 -0.31540119 -0.308004893
##  [99,] -0.3056410 -0.1267070     -0.2676535 -0.39448142 -0.319202295
## [100,] -1.1546437 -0.1267070     -0.9720049  2.94064138 -0.357884228
## [101,]  1.3923644  2.1092986     -0.9720049  2.78420005  1.652558351
## [102,] -1.1546437 -0.8720422     -0.9720049  0.02498763 -0.295789546
## [103,] -0.3056410 -0.8720422     -0.9720049 -0.26296756  0.049294016
## [104,] -1.1546437 -0.8720422     -0.9720049 -0.34290736 -0.357884228
## [105,] -1.1546437 -0.8720422     -0.2676535 -0.36869439 -0.267287068
## [106,]  1.3923644  1.3639634      1.8454007  2.07849494  0.557248875
## [107,]  0.5433617  1.3639634      1.1410493  1.72005520 -0.230641027
## [108,] -0.3056410 -0.8720422     -0.9720049 -0.66954309 -0.314112566
## [109,] -1.1546437 -0.8720422     -0.9720049 -0.25523145 -0.225551298
## [110,] -0.3056410 -0.8720422     -0.9720049 -0.41167277 -0.271358851
## [111,] -1.1546437 -0.8720422     -0.9720049 -0.47614035 -0.327345859
## [112,]  1.3923644  2.8546338      3.2541035  2.87703337  1.917224210
## [113,] -1.1546437 -0.8720422     -0.2676535 -0.34290736 -0.295789546
## [114,] -1.1546437 -0.8720422     -0.2676535 -0.29992897 -0.256089667
## [115,] -0.3056410  0.6186282     -0.9720049 -0.06268827 -0.133936194
## [116,]  0.5433617 -0.1267070     -0.2676535  0.48227767 -0.057590273
## [117,]  0.5433617 -0.1267070      0.4366979  1.43897655  0.783232801
## [118,]  0.5433617 -0.1267070     -0.2676535 -0.04205865 -0.357884228
```

```
## [119,]  0.5433617  2.1092986      1.1410493  1.41920649  0.451382532
## [120,]  0.5433617 -0.1267070      0.4366979  0.34474683 -0.103397825
## [121,] -0.3056410 -0.8720422     -0.9720049 -0.60077767 -0.357884228
## [122,] -1.1546437 -0.1267070     -0.2676535 -0.12801542 -0.145133595
## [123,] -0.3056410 -0.8720422     -0.2676535 -0.51482090 -0.287645981
## [124,] -1.1546437 -0.8720422     -0.9720049 -0.77269122 -0.357884228
## [125,] -1.1546437 -0.8720422     -0.9720049 -0.51482090 -0.247946102
## [126,] -1.1546437 -0.8720422     -0.9720049 -0.72971283 -0.272376796
## [127,] -0.3056410 -0.8720422     -0.9720049 -0.77269122 -0.297825437
## [128,]  0.5433617 -0.8720422     -0.2676535 -0.37729006 -0.357884228
## [129,]  0.5433617 -0.1267070     -0.2676535 -0.29992897 -0.289681872
## [130,] -1.1546437 -0.8720422     -0.2676535 -0.32485643 -0.319202295
## [131,] -1.1546437 -0.8720422     -0.9720049 -0.77269122 -0.306986947
## [132,] -0.3056410 -0.8720422     -0.9720049 -0.77269122 -0.315130512
## [133,]  0.5433617 -0.8720422     -0.2676535 -0.33431168 -0.357884228
## [134,] -1.1546437 -0.8720422     -0.2676535  1.80601197 -0.357884228
## [135,]  1.3923644  1.3639634      1.1410493  2.66557971  0.745568813
## [136,] -1.1546437 -0.8720422     -0.2676535 -0.15895986 -0.164474562
## [137,] -1.1546437 -0.8720422     -0.9720049 -0.42972370 -0.315130512
## [138,] -1.1546437 -0.1267070     -0.9720049 -0.56209713 -0.311058730
## [139,] -0.3056410 -0.1267070     -0.2676535 -0.17099381 -0.189923202
## [140,]  1.3923644  0.6186282      1.1410493  0.77453070  0.538925854
## [141,]  1.3923644 -0.1267070      0.4366979  0.17283328 -0.055554382
## [142,] -1.1546437 -0.8720422     -0.9720049 -0.77269122 -0.327345859
## [143,]  0.5433617  1.3639634      0.4366979 -0.77269122 -0.060644110
## [144,] -0.3056410 -0.1267070     -0.2676535 -0.46324684 -0.270340905
## [145,]  0.5433617  1.3639634      0.4366979  0.71436095 -0.046392871
## [146,]  0.5433617 -0.1267070      0.4366979 -0.77269122 -0.285610089
## [147,]  2.2413671  1.3639634      1.8454007 -0.77269122  0.252883138
## [148,] -1.1546437 -0.8720422     -0.9720049 -0.47184251 -0.357884228
## [149,] -0.3056410 -0.1267070      0.4366979 -0.38588574 -0.327345859
## [150,] -1.1546437 -0.8720422     -0.9720049 -0.77269122 -0.357884228
## [151,] -1.1546437 -0.8720422     -0.2676535 -0.00767594 -0.245910211
## [152,] -0.3056410 -0.8720422      1.1410493 -0.77269122 -0.357884228
## [153,] -0.3056410 -0.8720422     -0.2676535 -0.55779929 -0.327345859
## [154,] -0.3056410 -0.1267070      0.4366979 -0.64375606 -0.170582236
## [155,] -1.1546437 -0.8720422     -0.9720049 -0.77269122 -0.309022838
## [156,]  0.5433617  0.6186282      1.1410493  1.80601197 -0.272376796
## [157,]  0.5433617 -0.1267070      0.4366979 -0.55779929 -0.357884228
## [158,]  1.3923644  1.3639634      1.1410493  2.23579584  0.109352807
## [159,]  0.5433617  0.6186282     -0.2676535  0.68857392 -0.357884228
## [160,]  2.2413671  1.3639634     -0.2676535 -0.77269122 -0.197048821
## [161,] -1.1546437 -0.8720422     -0.9720049 -0.55779929 -0.345668880
## [162,] -0.3056410 -0.8720422     -0.2676535 -0.42284715 -0.345668880
## [163,]  0.5433617 -0.1267070     -0.2676535 -0.35408174 -0.246928156
## [164,]  0.5433617  0.6186282      0.4366979  0.86048747  0.151088577
## [165,] -1.1546437 -0.8720422     -0.2676535  0.80891341  0.058455527
## [166,] -0.3056410 -0.8720422     -0.2676535  0.05249380 -0.357884228
## [167,]  0.5433617 -0.1267070      0.4366979 -0.44605548 -0.162438671
## [168,]  1.3923644  2.8546338      1.1410493 -0.77269122  0.796466094
## [169,]  1.3923644  2.1092986      3.2541035 -0.77269122  1.169034187
## [170,] -1.1546437 -0.8720422     -0.2676535 -0.24835490 -0.267287068
## [171,]  1.3923644  2.8546338      1.8454007 -0.77269122  0.702815098
## [172,] -0.3056410  0.6186282      1.1410493  0.60261715 -0.060644110
```

```
## [173,]  0.5433617 -0.1267070     -0.2676535  1.97792552  4.731843820
## [174,]  0.5433617 -0.1267070     -0.2676535  0.19174377 -0.122738792
## [175,] -1.1546437 -0.8720422     -0.2676535 -0.77269122 -0.357884228
## [176,]  0.5433617  0.6186282      0.4366979 -0.77269122 -0.357884228
## [177,] -1.1546437 -0.8720422     -0.9720049 -0.68243661 -0.357884228
## [178,] -1.1546437 -0.8720422     -0.9720049 -0.37986877 -0.357884228
## [179,]  0.5433617 -0.1267070     -0.2676535 -0.16583640 -0.136990030
## [180,] -1.1546437 -0.8720422     -0.9720049 -0.18818516 -0.306986947
## [181,]  0.5433617  0.6186282     -0.9720049  0.07828083 -0.272376796
## [182,]  1.3923644  2.1092986      1.1410493  1.03240102 -0.357884228
## [183,]  1.3923644  0.6186282      2.5497521 -0.77269122 -0.054536436
## [184,]  0.5433617 -0.1267070     -0.2676535  0.34474683 -0.123756737
## [185,] -1.1546437 -0.8720422     -0.9720049 -0.77269122 -0.217407734
## [186,] -1.1546437 -0.8720422     -0.2676535 -0.77269122 -0.357884228
## [187,]  1.3923644  1.3639634      1.8454007  1.89196875  0.632576850
## [188,] -0.3056410  0.6186282      0.4366979  0.08687651 -0.242856374
## [189,]  1.3923644 -0.1267070     -0.9720049  0.17283328 -0.347704772
## [190,] -0.3056410 -0.1267070      0.4366979 -0.24835490 -0.113577281
## [191,]  0.5433617  0.6186282     -0.2676535  0.36623602 -0.004657101
## [192,]  1.3923644 -0.1267070     -0.2676535  0.37912954 -0.340579152
## [193,]  1.3923644  0.6186282      1.1410493  1.67707681  0.354677699
## [194,]  0.5433617  0.6186282      1.1410493  0.52439649  0.179591054
## [195,] -0.3056410 -0.8720422     -0.2676535 -0.77269122 -0.306986947
## [196,] -0.3056410 -0.8720422     -0.2676535 -0.47614035 -0.341597098
## [197,]  1.3923644  2.1092986      1.8454007  2.89336515  1.270828748
## [198,]  0.5433617 -0.1267070     -0.2676535 -0.34290736 -0.220461570
## [199,] -0.3056410 -0.8720422     -0.9720049 -0.77183165 -0.356866282
## [200,] -0.3056410 -0.8720422     -0.2676535 -0.22686571 -0.276448579
##        fire insurance
##   [1,]     0.46427333
##   [2,]     0.44270430
##   [3,]    -0.85143739
##   [4,]    -0.67888516
##   [5,]     0.27015208
##   [6,]     0.85251584
##   [7,]    -0.72202322
##   [8,]     0.11916888
##   [9,]    -0.20436654
##  [10,]    -0.50633294
##  [11,]    -0.78673030
##  [12,]     0.33485916
##  [13,]    -0.24750460
##  [14,]    -0.72202322
##  [15,]    -0.72202322
##  [16,]    -0.82986836
##  [17,]     0.31329013
##  [18,]    -0.42005682
##  [19,]     0.44270430
##  [20,]    -0.76516127
##  [21,]    -0.78673030
##  [22,]     0.27015208
##  [23,]     1.04663709
##  [24,]     2.34077878
##  [25,]    -0.13965946
```

```
##  [26,]    -0.74359225
##  [27,]    -0.59260905
##  [28,]    -0.78673030
##  [29,]    -0.72202322
##  [30,]     1.60743182
##  [31,]    -0.52790196
##  [32,]     0.24858305
##  [33,]     0.18387596
##  [34,]    -0.61417808
##  [35,]    -0.05338334
##  [36,]    -0.65731613
##  [37,]     3.05255671
##  [38,]    -0.74359225
##  [39,]    -0.63574710
##  [40,]    -0.42005682
##  [41,]    -0.67888516
##  [42,]     1.04663709
##  [43,]    -0.09652140
##  [44,]    -0.01024529
##  [45,]    -0.07495237
##  [46,]     0.31329013
##  [47,]     3.05255671
##  [48,]    -0.65731613
##  [49,]     3.87217978
##  [50,]    -0.65731613
##  [51,]    -0.57104002
##  [52,]    -0.48476391
##  [53,]    -0.54947099
##  [54,]     0.44270430
##  [55,]    -0.50633294
##  [56,]    -0.74359225
##  [57,]    -0.70045419
##  [58,]    -0.65731613
##  [59,]    -0.80829933
##  [60,]    -0.39848780
##  [61,]    -0.82986836
##  [62,]    -0.18279751
##  [63,]     0.67996361
##  [64,]    -0.39848780
##  [65,]     2.01724336
##  [66,]    -0.46319488
##  [67,]    -0.31221168
##  [68,]    -0.72202322
##  [69,]    -0.52790196
##  [70,]    -0.87300641
##  [71,]    -0.76516127
##  [72,]     0.14073791
##  [73,]    -0.03181432
##  [74,]    -0.72202322
##  [75,]     0.18387596
##  [76,]    -0.33378071
##  [77,]    -0.80829933
##  [78,]    -0.76516127
##  [79,]    -0.16122849
```

```
##  [80,]     0.01132374
##  [81,]    -0.59260905
##  [82,]    -0.80829933
##  [83,]    -0.37691877
##  [84,]    -0.72202322
##  [85,]    -0.59260905
##  [86,]    -0.72202322
##  [87,]     1.24075834
##  [88,]    -0.85143739
##  [89,]    -0.18279751
##  [90,]     0.52898042
##  [91,]    -0.05338334
##  [92,]    -0.85143739
##  [93,]     2.18979558
##  [94,]    -0.39848780
##  [95,]    -0.24750460
##  [96,]    -0.44162585
##  [97,]    -0.72202322
##  [98,]    -0.54947099
##  [99,]    -0.72202322
## [100,]    -0.20436654
## [101,]     3.24667796
## [102,]    -0.50633294
## [103,]    -0.78673030
## [104,]    -0.63574710
## [105,]    -0.50633294
## [106,]     0.59368750
## [107,]     1.62900085
## [108,]    -0.65731613
## [109,]    -0.57104002
## [110,]    -0.59260905
## [111,]    -0.61417808
## [112,]     0.76623973
## [113,]    -0.63574710
## [114,]    -0.29064265
## [115,]    -0.37691877
## [116,]    -0.07495237
## [117,]     1.67213891
## [118,]    -0.42005682
## [119,]     1.67213891
## [120,]    -0.09652140
## [121,]    -0.78673030
## [122,]     1.06820612
## [123,]    -0.18279751
## [124,]    -0.59260905
## [125,]    -0.82986836
## [126,]    -0.85143739
## [127,]    -0.78673030
## [128,]    -0.20436654
## [129,]    -0.48476391
## [130,]    -0.89457544
## [131,]    -0.78673030
## [132,]    -0.65731613
## [133,]    -0.57104002
```

```
## [134,]     0.44270430
## [135,]     3.11726379
## [136,]     0.20544499
## [137,]    -0.48476391
## [138,]    -0.85143739
## [139,]    -0.18279751
## [140,]     0.65839458
## [141,]     0.09759985
## [142,]    -0.74359225
## [143,]    -0.01024529
## [144,]    -0.59260905
## [145,]     0.33485916
## [146,]     0.01132374
## [147,]     2.12508850
## [148,]    -0.76516127
## [149,]    -0.29064265
## [150,]    -0.78673030
## [151,]     0.07603082
## [152,]     0.24858305
## [153,]    -0.67888516
## [154,]    -0.05338334
## [155,]    -0.76516127
## [156,]     2.49176198
## [157,]    -0.39848780
## [158,]     1.47801765
## [159,]     0.72310167
## [160,]     0.57211847
## [161,]    -0.80829933
## [162,]    -0.59260905
## [163,]    -0.67888516
## [164,]     1.54272474
## [165,]    -0.24750460
## [166,]    -0.35534974
## [167,]    -0.44162585
## [168,]     1.28389640
## [169,]     3.16040185
## [170,]    -0.61417808
## [171,]     2.70745226
## [172,]    -0.20436654
## [173,]     1.26232737
## [174,]    -0.22593557
## [175,]    -0.76516127
## [176,]     2.31920975
## [177,]    -0.70045419
## [178,]    -0.76516127
## [179,]     0.01132374
## [180,]    -0.76516127
## [181,]    -0.09652140
## [182,]     0.05446180
## [183,]     0.20544499
## [184,]    -0.48476391
## [185,]    -0.18279751
## [186,]    -0.65731613
## [187,]     3.05255671
```

```
## [188,]    -0.11809043
## [189,]    -0.50633294
## [190,]    -0.46319488
## [191,]     0.18387596
## [192,]     0.59368750
## [193,]     2.23293364
## [194,]     0.83094681
## [195,]    -0.65731613
## [196,]    -0.82986836
## [197,]     2.36234781
## [198,]    -0.72202322
## [199,]     0.05446180
## [200,]    -0.50633294
## attr(,"scaled:center")
##          rooms       bathroom parking spaces            hoa   property tax
##          2.360          2.170          1.380        898.930        351.575
## fire insurance
##         49.475
## attr(,"scaled:scale")
##          rooms       bathroom parking spaces            hoa   property tax
##       1.177853       1.341678       1.419746    1163.375454     982.370758
## fire insurance
##      46.362775
```

```
as.matrix(scale.emp)%*%fact.load.emp%*%solve(t(fact.load.emp)%*%fact.load.emp)
```

```
##                PC1           PC2
##   [1,] -1.02101847  0.1380849672
##   [2,]  0.87780566  1.0788328277
##   [3,]  0.57532859 -0.4095601191
##   [4,]  1.09042847 -0.1711974262
##   [5,] -1.44057642 -1.0874339308
##   [6,] -0.93782839  0.7707497585
##   [7,]  1.07410649 -0.2913994903
##   [8,]  0.28520741  0.4317580903
##   [9,] -0.57203522 -0.3969285318
##  [10,] -0.77664555  0.5299969078
##  [11,]  0.37020663 -0.2201966280
##  [12,] -0.75003626  0.8322227233
##  [13,]  0.38333590 -0.3929895673
##  [14,]  1.13260938 -0.0475507340
##  [15,]  0.55572510 -0.3412969473
##  [16,]  0.21647984 -0.3712047268
##  [17,]  0.44930583  0.0548693338
##  [18,]  1.13332991  0.2605135371
##  [19,] -0.49035121 -0.1095453392
##  [20,]  1.14269214 -0.0825287932
##  [21,]  0.53037460 -0.5186805949
##  [22,] -0.49596536 -0.4773082683
##  [23,] -2.10407472 -0.6517243610
##  [24,] -3.09209142 -1.7858390674
##  [25,] -0.44840184 -0.7258697080
##  [26,]  1.16772017  0.0600762038
##  [27,]  1.06870602 -0.1639360424
```

```
##  [28,]  0.90973487 -0.2749163366
##  [29,]  0.38610848 -0.1214093217
##  [30,] -1.31494178 -0.5127192027
##  [31,]  1.09457588  0.0286833716
##  [32,] -0.57593848  0.2719908123
##  [33,] -0.67339912  0.2273612364
##  [34,]  0.52352020 -0.3385721534
##  [35,] -1.45894145  0.0251307742
##  [36,] -0.73528197  0.7880025465
##  [37,] -0.96936193  5.8469241541
##  [38,] -0.10107460 -0.7862175119
##  [39,]  1.11013211 -0.0369000582
##  [40,] -0.47818449 -0.6785635186
##  [41,]  1.07075221 -0.2680247874
##  [42,] -1.66283861  0.2727382001
##  [43,] -0.50083075 -0.0424753687
##  [44,] -0.66800030 -0.5477838395
##  [45,] -0.61466827 -0.0047341133
##  [46,] -1.17872075 -0.7333870387
##  [47,] -2.02701234  0.8918275331
##  [48,]  0.34500925 -0.5972413300
##  [49,] -2.02980247  1.0785793543
##  [50,]  1.08057158 -0.1914259675
##  [51,] -0.04332909 -0.3564535126
##  [52,]  0.26160663 -0.3237910337
##  [53,] -0.07063612 -0.4725089488
##  [54,] -0.11584424  2.1990381180
##  [55,]  1.42956281  8.5869102791
##  [56,]  0.76887253 -0.2522790540
##  [57,]  1.09804860 -0.1655203737
##  [58,]  1.05462775 -0.2943353439
##  [59,]  0.65360233 -0.0842891378
##  [60,]  0.42624975 -0.5027681507
##  [61,]  1.14933413 -0.1204631750
##  [62,]  0.05584694  0.6648051977
##  [63,] -0.46466591 -0.4278817887
##  [64,]  0.99654241 -0.2539247671
##  [65,] -1.31007957 -0.8012846566
##  [66,] -0.11653899 -0.5655994357
##  [67,] -0.51462414 -0.2484060737
##  [68,]  0.95518061 -0.0394790211
##  [69,]  1.07492322 -0.0750168158
##  [70,]  1.15169161 -0.1532210886
##  [71,]  0.78004543 -0.2823346873
##  [72,] -0.42875721 -0.4909289129
##  [73,] -0.64257742 -0.9251856686
##  [74,]  1.08402132 -0.2653224947
##  [75,] -0.36641378 -0.1213902140
##  [76,]  0.98811162 -0.2278033970
##  [77,]  0.31435409 -0.4594766987
##  [78,]  0.94701773 -0.1217514696
##  [79,]  0.96412996 -0.0877136005
##  [80,] -0.68588287 -0.2745279137
##  [81,]  0.03307615 -0.8177909923
```

```
##  [82,]  0.33952183 -0.3802800751
##  [83,] -0.57547224 -0.7660736745
##  [84,]  0.35995108 -0.1813310138
##  [85,] -0.27947333 -0.6196371757
##  [86,]  0.52870598 -0.4473811816
##  [87,] -0.13063464 -0.3938009650
##  [88,]  1.11306398 -0.2855277831
##  [89,]  0.04401905 -0.6300820941
##  [90,] -0.97501105  0.0501613191
##  [91,] -0.81150716 -0.7844705908
##  [92,]  0.90342209 -0.2298572763
##  [93,] -1.93315215  1.8854126800
##  [94,]  1.05973181 -0.0097127009
##  [95,] -0.31004436 -0.1539365112
##  [96,] -1.20550341 -0.6426925026
##  [97,] -0.13371715  0.9391311348
##  [98,]  0.28661645 -0.2834227249
##  [99,]  0.32507323 -0.3270150075
## [100,]  1.15905184  1.6743955838
## [101,] -1.21748675  2.0511745046
## [102,]  1.13106378  0.1802775409
## [103,]  0.89438160  0.0444120846
## [104,]  1.10449599 -0.0603922313
## [105,]  0.90351845 -0.0005018677
## [106,] -1.30542963  1.0787527186
## [107,] -1.32014342  0.4893971640
## [108,]  0.70097522 -0.4496809350
## [109,]  1.13334142  0.0814533605
## [110,]  0.72721096 -0.2803713556
## [111,]  1.08902954 -0.1118448751
## [112,] -1.73426179  2.4490154297
## [113,]  0.93792575 -0.0013964531
## [114,]  0.85072317  0.0354950577
## [115,]  0.25391976 -0.0797020888
## [116,] -0.06129458  0.0705821820
## [117,] -0.41162126  1.1432143033
## [118,] -0.10973637 -0.4201562636
## [119,] -1.42896908  0.7817410703
## [120,] -0.26943524 -0.0235944197
## [121,]  0.73676943 -0.4382592027
## [122,]  0.24890646  0.1085801904
## [123,]  0.40193603 -0.3537199227
## [124,]  1.03514900 -0.2972711975
## [125,]  1.17098209 -0.0650872047
## [126,]  1.14249568 -0.1994508205
## [127,]  0.73093853 -0.4882695308
## [128,]  0.02755787 -0.5727395435
## [129,] -0.10509451 -0.5082546281
## [130,]  1.01169245  0.0030773626
## [131,]  1.10774928 -0.2512107858
## [132,]  0.68716526 -0.5068072074
## [133,]  0.14357355 -0.5326108667
## [134,]  0.87780566  1.0788328277
## [135,] -1.75185407  1.4094457599
```

```
## [136,]  0.74536784  0.1570963059
## [137,]  1.05955842 -0.0834043512
## [138,]  0.90752676 -0.1765853608
## [139,]  0.22803481 -0.1346989131
## [140,] -1.07097172  0.3765478568
## [141,] -0.71646202 -0.3351172511
## [142,]  1.08909783 -0.2680691619
## [143,] -0.92241834 -0.6878966928
## [144,]  0.29069543 -0.3347114548
## [145,] -0.82732944  0.1197054070
## [146,] -0.49914609 -0.7726492146
## [147,] -2.60438777 -1.0157362169
## [148,]  1.12654514 -0.1249970432
## [149,]  0.01022554 -0.3343020864
## [150,]  1.09358524 -0.2884636367
## [151,]  0.78150201  0.1860559008
## [152,] -0.14898607 -0.5385596526
## [153,]  0.53458902 -0.3837611084
## [154,] -0.05138799 -0.3712811406
## [155,]  1.10068980 -0.2536795177
## [156,] -1.33401721  0.5071313061
## [157,] -0.36771305 -0.6894936580
## [158,] -1.49180413  0.7832383514
## [159,] -0.60423202 -0.1130979174
## [160,] -1.71055685 -1.3152328322
## [161,]  1.13165810 -0.1610834693
## [162,]  0.52121571 -0.3273211829
## [163,] -0.04186199 -0.4977548105
## [164,] -0.87062833  0.3297591047
## [165,]  1.07068288  0.8698585051
## [166,]  0.50872972 -0.0872032297
## [167,] -0.28568341 -0.4834058249
## [168,] -2.12905173 -0.4303764113
## [169,] -2.89562060 -0.1617415489
## [170,]  0.95176415  0.0701693062
## [171,] -2.76749633 -0.5499631364
## [172,] -0.24193137  0.3704138124
## [173,]  1.06511739  4.3329287651
## [174,] -0.07207429 -0.1290582319
## [175,]  0.90324196 -0.2758949545
## [176,] -1.46018606 -0.9707022235
## [177,]  1.07944942 -0.2430445447
## [178,]  1.13860642 -0.0747237929
## [179,] -0.19435477 -0.3457087065
## [180,]  1.17790754  0.0673041498
## [181,] -0.22988813 -0.3604618062
## [182,] -1.59729394 -0.1923767343
## [183,] -1.67037408 -0.8558930390
## [184,]  0.02562199 -0.0344277384
## [185,]  0.95087635 -0.2130470686
## [186,]  0.87077738 -0.2807880439
## [187,] -2.04911994  0.9203684696
## [188,] -0.20239331 -0.0623192419
## [189,] -0.24826123 -0.5486419149
```

```
## [190,]  0.13969339 -0.0948362160
## [191,] -0.38588159 -0.0062889833
## [192,] -0.73421396 -0.4670262965
## [193,] -1.47786991  0.6635890661
## [194,] -0.87635506  0.2127535976
## [195,]  0.50558114 -0.4872994512
## [196,]  0.58678212 -0.3427064528
## [197,] -1.77859323  1.9257607752
## [198,] -0.02004547 -0.4703181273
## [199,]  0.46139727 -0.5691790914
## [200,]  0.54020783 -0.1734474680
```