Directional change of fluid particles in two-dimensional turbulence and of football players

Benjamin Kadoch,[1] Wouter J. T. Bos,[2] and Kai Schneider[3]
[1]*Aix-Marseille Univ., CNRS, IUSTI, Marseille, France*
[2]*LMFA, CNRS, Ecole Centrale de Lyon, Université de Lyon, Ecully, France*
[3]*Aix-Marseille Univ., CNRS, Centrale Marseille, I2M, Marseille, France*
(Received 4 October 2016; published 9 June 2017)

Multiscale directional statistics are investigated in two-dimensional incompressible turbulence. It is shown that the short-time behavior of the mean angle of directional change of fluid particles is linearly dependent on the time lag and that no inertial range behavior is observed in the directional change associated with the enstrophy-cascade range. In simulations of the inverse-cascade range, the directional change shows a power law behavior at inertial range time scales. By comparing the directional change in space-periodic and wall-bounded flow, it is shown that the probability density function of the directional change at long times carries the signature of the confinement. The geometrical origin of this effect is validated by Monte Carlo simulations. The same effect is also observed in the directional statistics computed from the trajectories of football players (soccer players in American English).

DOI: 10.1103/PhysRevFluids.2.064604

I. INTRODUCTION

In the absence of a successful theory derived from the Navier-Stokes equations, the understanding of turbulence is based on phenomenology and statistical characterization. One of the most universal features of turbulence is perhaps its energy distribution among scales, which in a wide variety of flows is well approximated by Kolmogorov's description of locally isotropic turbulence [1,2]. A complementary description of turbulent flows is its geometrical characterization. In the Eulerian frame, such a description could involve the statistics of vorticity patches and filaments, or regions of significant strain. In the Lagrangian frame, i.e., following fluid particles, an obvious geometrical characterization would involve the tortuosity or curvature of the turbulent trajectories. Indeed, it is the spiraling motion of the fluid particles that characterizes the turbulent nature of the flow in the Lagrangian setting.

The curvature in turbulent flows was studied in references [3–8]. The extension to a time scale–dependent curvature-related statistical measure was only recently proposed in Ref. [9], and we used this measure to characterize the directional change of trajectories in isotropic three-dimensional (3D) turbulence [10]. It was shown that three distinct regimes could be observed in the directional change: First, there is a ballistic regime in which the particle changes its direction as expected in a spatially or temporally smooth flow. Then, for larger time lags we showed the existence of an inertial, self-similar regime where the link could be made with Kolmogorov's inertial range phenomenology. Finally, for time scales larger than the Lagrangian correlation time, an uncorrelated regime was observed, in which the angle characterizing the directional change is randomly distributed. One single type of turbulence was considered, the academic case of space-periodic isotropic 3D incompressible turbulence.

The present work can be seen as a logical continuation of the investigation reported in Ref. [10]. We will address two additional questions. First, how do the statistics change if we are in two space dimensions? Indeed, the flow structure and Lagrangian correlations are different in the two-dimensional (2D) case, and we expect the directional change to reflect this difference. Second, what is the influence of the finite size of the domain on the statistics? This is an important question, since in a wide range of practical flows, solid boundaries modify the properties of the turbulence. To address these questions we consider statistically stationary incompressible turbulence in two dimensions. We will consider two different geometries: a spatially periodic case, i.e., an unbounded domain, and a wall-bounded circular domain. Using Monte Carlo simulations we will in particular show which features of the statistics are flow-dependent and which are due to geometrical constraints. The statistical features which are geometry-dependent but flow-independent should be observable in other physical systems. We have therefore also considered the trajectories of football players (soccer players in American English) and show that this system has a certain number of features in common with the turbulent motion of fluid particles.

After a description of the setup in the next section, we will discuss the results on large-scale forced turbulence, small-scale forced turbulence, and football-player trajectories in Secs. III, IV, and V, respectively.

II. DEFINITIONS AND SETUP OF THE SIMULATIONS

A. Preliminary analysis and definitions

The directional change of a fluid particle is characterized by considering the angle between two subsequent position increments on a Lagrangian trajectory, defined by $\boldsymbol{X}(\boldsymbol{x}_0,t)$, where $\boldsymbol{x}_0 = \boldsymbol{X}(\boldsymbol{x}_0,t=t_0)$ is the initial position. The position increment is defined by

$$\delta\boldsymbol{X}(\boldsymbol{x}_0,t,\tau) = \boldsymbol{X}(\boldsymbol{x}_0,t) - \boldsymbol{X}(\boldsymbol{x}_0,t-\tau), \tag{1}$$

and the cosine of the angle characterizing the directional change is

$$\cos(\Theta(t,\tau)) = \frac{\delta\boldsymbol{X}(\boldsymbol{x}_0,t,\tau)\cdot\delta\boldsymbol{X}(\boldsymbol{x}_0,t+\tau,\tau)}{|\delta\boldsymbol{X}(\boldsymbol{x}_0,t,\tau)||\delta\boldsymbol{X}(\boldsymbol{x}_0,t+\tau,\tau)|}. \tag{2}$$

As in our recent investigation [10], the main quantities we will analyze are the averaged modulus of the angle,

$$\theta(\tau) \equiv \langle|\Theta(t,\tau)|\rangle, \tag{3}$$

and the probability distribution functions of $\Theta(t,\tau)$. Since we consider statistically stationary turbulent flow, we perform ensemble and time averaging, denoted by $\langle\cdot\rangle$. It was shown by a second-order Taylor-expansion of the position vector that at short times the angle $\theta(\tau)$ could be estimated by the relation

$$\theta(\tau) \approx 2\tau\langle a_\perp(t)/u(t)\rangle, \tag{4}$$

where $u(t)$ is the norm of the Lagrangian velocity and $a_\perp(t)$ the norm of the acceleration perpendicular to the trajectory. Irrespective of the type of flow, it is thus expected that at short times the directional change θ, is linearly proportional to τ. Physically this corresponds to the range in which the flow can be considered smooth. The smoothness of the trajectories in turbulent flows which allows such an expansion has been shown in Ref. [11]. In 2D turbulence, the velocity scales in the forward-enstrophy cascade range can be considered smooth, since the energy distribution falls off with a power law $k^{-\alpha}$, with $\alpha \geqslant 3$. What this implies for the Lagrangian statistics and in particular whether an inertial range scaling is expected for $\theta(\tau)$ at intermediate values of τ is not clear from the outset.

In the scale range associated to the backward cascade of energy of 2D turbulence, the energy spectrum approximately displays the same $k^{-5/3}$ power law as for 3D turbulence in the inertial range. It is anticipated that the results observed in Ref. [10] will also be valid for this case. We will address this subject in Sec. IV.

B. Setup

The flow database we analyzed is described in Ref. [12]. The flow is statistically stationary, forced 2D turbulence with a Reynolds number, based on the domain size and the RMS velocity fluctuations, of the order of 10^4. A forcing term is added to the Navier-Stokes equations, acting in

TABLE I. Parameters for the simulations of the turbulent flows forced at the small wave numbers [12]. Radius of the circle $R = 2.8$.

Resolution $[N_x N_y]$	[512,512]
Size of domain $[x,y]$	$[2\pi,2\pi]$
Viscosity ν	5×10^{-4}
Timestep dt	10^{-4}
Forcing wave number k	8
Amplitude of forcing	6
Penalization parameter η	10^{-4}
Periodic: $\tau_{k_p} = 1/\sqrt{2Z}$	0.06
Kinetic energy E_p	4.39
Circle: $\tau_{k_c} = 1/\sqrt{2Z}$	0.05
Kinetic energy E_c	4.8

a narrow wave number shell around $k = 8$. The Reynolds number based on the Taylor microscale $\lambda = \sqrt{E/Z}$ is $R_\lambda = \lambda\sqrt{E}/\nu$. For the periodic geometry we have $R_\lambda = 751$ and for the circular geometry $R_\lambda = 633$. Two types of physical domains were considered: a square double-periodic domain and a circular domain with no-slip boundary conditions. Numerically, all computations are carried out on a double-periodic domain using a standard pseudospectral solver. In the nonperiodic case the circular boundaries are imposed by a volume penalization method [13,14]. The penalization technique introduces a numerical parameter η which is chosen proportional to Δx^2 (with Δx the grid size), to minimize the error [15]. Thereby the modeling error of the no-slip conditions is of the same order as the discretization error. The quantitative influence of this error on the statistics is in the present case negligible. All statistics are computed in a statistically stationary state, where 10^4 fluid particles are tracked for about 10^3 eddy-turnover times. The sensitivity to the number of particles is assessed and the error between the statistics obtained in the periodic domain, using either 5×10^3 or 10^4 particles, is less than 1% for all time lags. The parameters of the simulations are given in Table I. For further details on the method and the parameters we refer to Ref. [12].

In addition to these simulations, we also consider a 2D flow forced at small scales, which allows for the development of an inverse cascade of energy. The flow is similar to the one described in Ref. [16]. The domain is again a square periodic box discretized by 1024^2 gridpoints. The forcing is a narrow band random energy input at $k = 210$. At the large scales, energy is evacuated by a Rayleigh friction term, $-\alpha\psi$ on the right-hand side of the Navier-Stokes equations, with ψ the stream function and α the friction constant. At large wave numbers the flow is damped by a fourth order hyperviscosity (in the viscous term, the Laplacian Δ is replaced by Δ^4). The parameters of this simulation are summarized in Table II.

To illustrate the universality of geometric confinement on the statistics, we also consider a different physical system: football. The football-player statistics considered in Sec. V correspond to a training match of two football teams of eight players each. The data used were collected by the real-time locating system deployed on a football field of the Nuremberg Stadium in Germany. Data originate from sensors, sampled with 200 Hz frequency, located near the players' shoes one sensor per leg). Considering the smallest reaction time of a human to be of the order of 0.1 s, this frequency is largely sufficient to capture the smallest relevant time scale of the movement of the players. However, at such small time lags the angles which we calculate are most certainly influenced by the measurement error in the position of the football players. In particular, when the football players are nearly at rest, the angle $\Theta(\tau,t)$ is hard to determine, and such events are more frequent for football players (free kicks, penalties, etc.) than in incompressible turbulence, where due to incompressibility particles are rarely immobile for long time intervals. For the smallest time lags, the results are therefore expected to be less accurate. The positions of the players were monitored during the full duration of the game.

TABLE II. Parameters for the simulation of the inverse cascade regime.

Resolution $[N_x N_y]$	[1024,1024]
Size of domain $[x, y]$	$[2\pi, 2\pi]$
Viscosity ν	10^{-38}
Time step dt	10^{-4}
Forcing wave number k	210
Amplitude of forcing	20
Hyperviscosity	4th order
Rayleigh Friction strength α	0.25
$\tau_k = 1/\sqrt{2Z}$	0.0788
Kinetic energy E	0.0589

The Lagrangian position vectors were subsequently analyzed exactly as the turbulence data. The trajectories of the goal keepers were omitted. Details on the experiment can be found in Ref. [17].

III. DIRECTIONAL STATISTICS IN 2D TURBULENCE FORCED AT THE LARGE SCALES

A. Average absolute angle

In Fig. 1 the mean absolute angle $\theta(\tau)$ is shown as a function of the time lag for 2D turbulence. The time τ is normalized by the Kolmogorov time scale τ_K, defined as $\tau_K = (2Z)^{-1/2}$, where Z is the enstrophy. We find that at short times the evolution is approximately linear in τ, which is expected when τ becomes smaller than all physical time scales of the flow, and expression (4) is valid. At large times an asymptotic value is reached. In the case of a periodic domain this value is $\pi/2$, as expected for the angle between three random points in an infinite domain. For confined flow this asymptotic value is larger, approximately equal to $2\pi/3$. The effect is geometric as we will demonstrate in the following. It is further observed, if we compare with our previous study [10], in three dimensions, that no power law is present for intermediate times. Indeed, in three dimensions we have $\theta(\tau) \sim \tau^{1/2}$ for intermediate time lags, and the exponent can be related to Kolmogorov scaling of the pressure gradient spectrum. In the present flow, where the turbulence is stirred at large scales, no Kolmogorov scaling is observed. See, for instance, the spectra in Fig. 4(b) of Ref. [16],

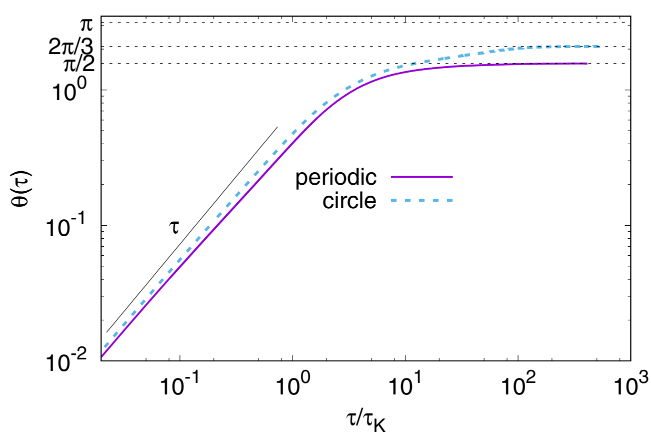
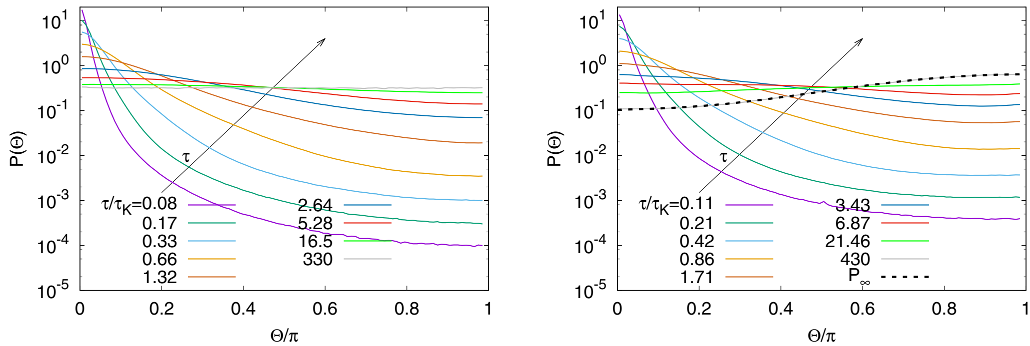


FIG. 1. Mean absolute angle of directional change as a function of the time lag τ, in statistically stationary 2D turbulence, forced at large scales, in periodic and circular-wall-bounded geometry.



(a) Periodic (b) Circle

FIG. 2. Probability density functions of the directional change of fluid particles in 2D turbulent flows. (a) Periodic boundary conditions. An equipartition of the angles is expected at long times. (b) Circular no-slip walls. The asymptotic shape of the PDF, corresponding to the angle between two line segments connected by three randomly placed points in a circle, is represented by a thick black dashed line.

where we used a similar numerical methodology to generate a 2D turbulent flow. It is observed there that a power law scaling approximately proportional to k^{-3} is present in the spectrum. Indeed, the forward-enstrophy range does not show the same scaling as Kolmogorov's spectrum in three dimensions. We will show that these results will change in 2D flow in the inverse cascade range, where Kolmogorov scaling is observed in general. There we expect to obtain results similar to those presented in Ref. [10].

B. Probability density functions

In Fig. 2 the probability density functions (PDFs) of the instantaneous directional change $\Theta(\tau,t)$ are shown. In the following all PDFs are estimated by histograms with 100 equidistant bins. It is observed that, starting from a peaked distribution around $\Theta = 0$ for short times, the PDFs change shape, to gradually approach the long-time asymptotic shape of the distribution. For the PDF of the angle this shape is an equidistribution, since all angles are equally probable for three decorrelated, random points in an infinitely large 2D domain. In three dimensions, it is not the angle, but its cosine that tends to an equidistribution for long times [10]. The reason for this is that in three space dimensions we have for the measure of surface integrals in spherical coordinates $\cos(\theta)d\theta$, which becomes $d\theta$ in two space dimensions.

Following the reasoning in Ref. [10], we can try to fit these PDFs by a known distribution, by assuming both the velocity and the acceleration to be near Gaussian and independent. If this is so, their squares are χ^2-distributed and the ratio of two χ^2-distributed quantities follows a Fischer distribution. For small values of Θ, the cosine of the directional change can be approximated by $\cos(\Theta) \approx 1 - \Theta^2/2$, so that the PDF of $1 - \cos(\Theta)$ should, under these assumptions and when properly normalized, obey an F-distribution. As shown in Fig. 3, the shape of the PDFs can indeed be reasonably well approximated by an $F_{1,2}$ Fischer distribution. The parameters 1,2 correspond to the dimensions of the perpendicular acceleration and local velocity, respectively. Indeed, in a 2D velocity field the acceleration perpendicular to a trajectory is a one-dimensional (1D) quantity. The collapse is not perfect, and this is not expected either, since the derivation of the shape of the PDFs leading to the F-distribution assumes Gaussianity of both the velocity and the acceleration, which is only partially true in general. Apparently, the Fischer-distribution reasonably well describes the distribution of the ratio of two random variables, even when these random variables are not precisely Gaussian distributed. The PDFs for the circularly confined case are similar, except for the long-time asymptotic shape, where no equidistribution is observed. We will now discuss the influence of confinement.

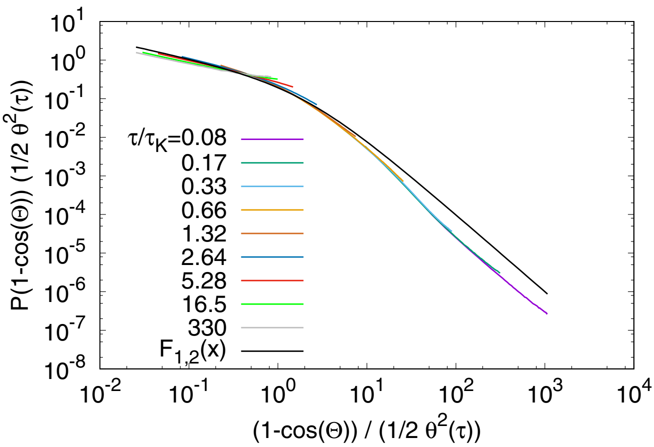


FIG. 3. Probability density functions of $1 - \cos(\Theta)$, where Θ corresponds to the directional change of fluid particles in 2D turbulent flow (forward cascade) in a periodic domain. Rescaled and fitted by a theoretical model.

C. The influence of geometrical confinement on the statistics

We have seen in Figs. 1 and 2 that the difference in the directional change induced by solid boundaries manifests itself only at long times. This is expected, since most of the velocity trajectories are at short times situated far away from the boundaries, so the statistics should be determined by these trajectories for which the influence of the wall is small. Indeed, it was shown in Ref. [12], by position-dependent measures, that the solid boundaries did not strongly affect the statistics in the considered flows away from the wall. At long times the statistics are however significantly affected, when compared to the periodic domain. In particular, the long-time asymptote of $\theta(\tau)$ changes from $\pi/2$ to a value of $\theta(\tau) \approx 2$. What we will show now is that this effect is not due to the modified properties of the velocity field, but due to the effect of geometrical confinement, irrespective of the flow, as long as the Lagrangian velocity correlation of fluid particles decays to zero at long times.

It can be understood that at long times, when the separation of the fluid particles on a trajectory becomes of the order of the domain size, the angles might not be equidistributed. For instance considering the sketch in Fig. 4, it can easily appreciated that for a given distance between the first and the second of three random points, and if all positions within the circle are equiprobable, the chance to have a configuration I, with a large value of Θ is larger than the chance to have a

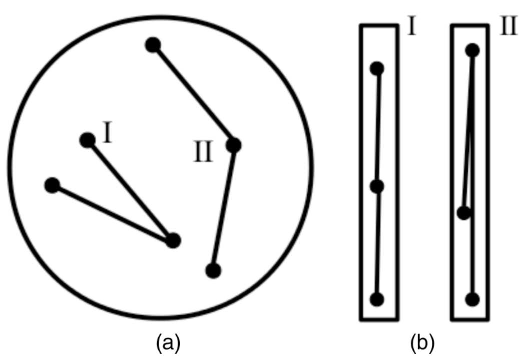


FIG. 4. Sketch illustrating the effect of confinement on the directional change at long times in (a) a circular domain and (b) rectangular domains with a large aspect ratio.

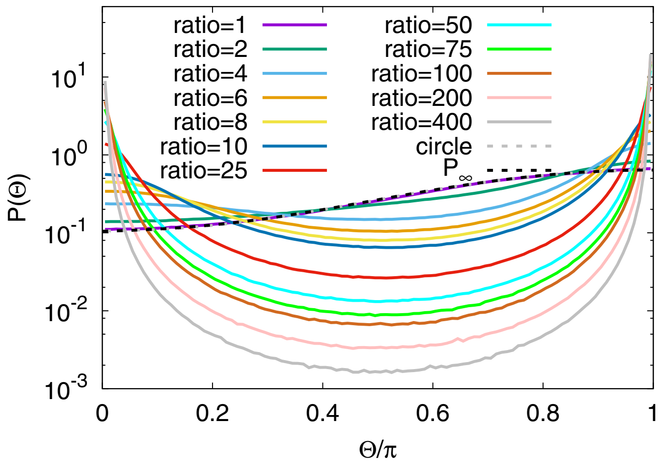


FIG. 5. PDFs for the angle Θ, between two connected line segments defined by three randomly positioned points in a rectangular domain. The aspect ratio of the domain is varied between 1 and 400.

configuration II. Larger values of Θ become thus more probable when the distance between the three position points $\boldsymbol{X}(\boldsymbol{x}_0,t)$ becomes comparable to the domain size.

To confirm this idea we carried out Monte Carlo simulations using 6×10^7 randomly distributed points in a circular domain and rectangular domains of different aspect ratio. Computing the PDF for the angle between two connected line segments, defined by three randomly distributed points in a circle, the PDF has exactly the shape of the long-time asymptotic PDF in Fig. 5, with an average value $\theta \approx 2$, as observed in Fig. 1. The functional form that we have fitted for this PDF is

$$P_\infty(\Theta) = a + b\exp\{-[(1-\Theta/\pi)/c]^2\} \tag{5}$$

with $a = 0.10$, $b = 0.54$, $c = 0.46$. The chosen functional form is, however, not better justified than by the fact that it accurately fits the data points. Using the same algorithm, computing the angle between randomly placed points, we have further determined the influence of confinement on angular statistics for rectangular domains with different aspect ratios. The results are also shown in Fig. 5. It is observed that the shape of the PDFs changes as a function of the aspect ratio. When the aspect ratio becomes large, as sketched in the right side of Fig. 4, the problem becomes close to 1D, and the PDF can easily be determined analytically.

In this 1D limit when the rectangle approaches a line, the PDF should tend to the form

$$P(\Theta) = \tfrac{1}{3}\delta(\Theta) + \tfrac{2}{3}\delta(\Theta-\pi). \tag{6}$$

This is qualitatively confirmed in Fig. 5 for the maximum aspect ratio of 400, where two distinct peaks of different height are found. The mean angle corresponds to the first moment of the PDF, and thus we get $\int_0^\pi \Theta P(\Theta)\,d\Theta = \frac{2\pi}{3}$.

The PDF in the 1D limit case can be justified theoretically. Consider three random points (r_1,r_2,r_3) on a line segment (of length L) having a uniform probability distribution U. Using conditional probability we find that the probability that $r_3 > r_2$, given that $r_2 > r_1$, has a probability of 1/3, while the probability that $r_3 < r_2$, given that $r_2 > r_1$, has a probability of 2/3. For the corresponding angles we have only two values, 0 for $r_3 > r_2 > r_1$ and π for $r_3 < r_2 > r_1$. This justifies the above PDF.

For rectangles with finite aspect ratio we consider three random vectors $\vec{r}_i = (x_i, y_i)$ for $i = 1,2,3$. Each component has a uniform probability distribution, i.e., $\vec{r}_i \in U(-L_x/2, L_x/2)\times U(0, L_y)$. Without loss of generality we suppose $L_y > L_x$ and we consider the y component first. Again we can show that we have a probability of 1/3 for $y_3 > y_2$, given that $y_2 > y_1$ (or $y_3 < y_2$, given

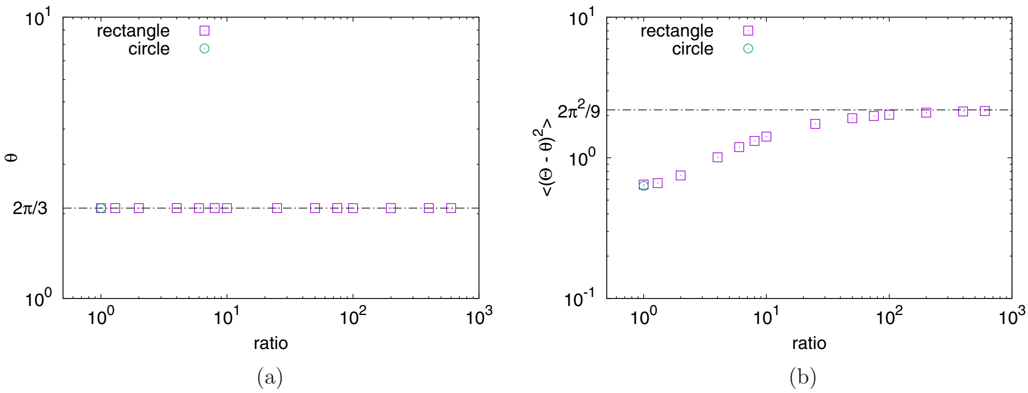


FIG. 6. Monte Carlo results for (a) the mean-absolute angle and (b) the variance (second centered moment), in a circular domain, and in rectangular domains with different aspect ratios.

that $y_2 < y_1$). The x component is uniformly distributed between $-L_x/2, L_x/2$, and thus we obtain values of the angle in $-\pi/2 < \theta < \pi/2$. The symmetry of the domain with respect to the center line $x = 0$ implies a mean value of 0. The probability for $y_3 < y_2$, given that $y_2 > y_1$, is larger and equal to 2/3. This implies values for the angle in $\pi/2 < \theta < 3\pi/2$. Again by symmetry arguments of the domain we find a mean value of π. The mean value of the angle for all realizations thus corresponds to $2\pi/3$, which is indeed observed in our Monte Carlo simulations for rectangles of aspect ratios ranging from 1 to 400 in Fig. 6(a).

The above arguments are also valid for other confined geometries which possess a reflectional symmetry with respect to an axis, like the circular case. Furthermore these arguments also hold for confined geometries in higher dimensions, e.g., in three dimensions. These results concern the mean value θ, it seems that its variance is more complicated to determine from symmetry arguments. However, for the infinite aspect ratio PDF (corresponding to the 1D limit) [Eq. (6)], it is directly found that the second centered moment (or variance) is equal to $(2/9)\pi^2$. This asymptotic value is confirmed in Fig. 6(b) by Monte Carlo simulations.

IV. ANGULAR STATISTICS IN THE INVERSE CASCADE RANGE

We anticipated in the previous section that the directional change in the inverse cascade range of 2D turbulence should resemble the statistics of the 3D case. Our reason to believe this is that Kolmogorov scaling is in general even better observed in such flows, and the scaling of the energy spectrum, proportional to $k^{-5/3}$, is often closely approached. In Fig. 7(a) we show the energy spectrum. We observe a power law in the spectrum close to $k^{-5/3}$ for the large scales.

The time lag dependence [Fig. 7(b)] of the directional change contains as expected, as in 3D turbulence, two distinct power laws before the long-time asymptote is reached. The ballistic regime can again be associated with a linear dependence on τ. For inertial range time lags a second power law is indeed observed. In this range, the predicted directional change is proportional to $\tau^{1/2}$. It is observed that the exponent is somewhat smaller. This could be due to the finiteness of the Reynolds number. In general Lagrangian time correlations are rather sensitive to Reynolds number corrections [18]. Furthermore, this is also consistent with our previous results in 3D turbulence, where the scaling of τ was observed to be proportional to a power law of the time lag with an exponent smaller than 1/2.

The PDFs of the angle as a function of τ (Fig. 8) do again evolve from a peak around zero for short times towards an equidistribution for long times. The intermediate evolution of the PDF of $1 - \cos(\Theta)$ can be captured by a Fischer distribution, as in the foregoing. It does seem that the main

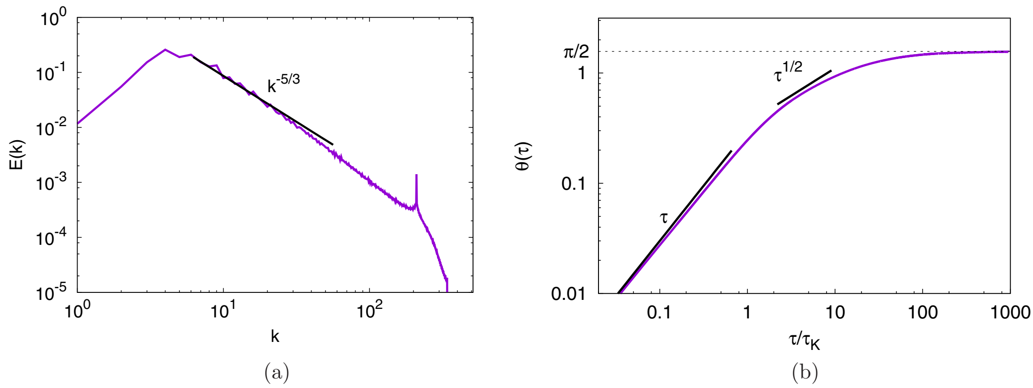


FIG. 7. Statistics in the inverse cascade range of small-scale forced, 2D turbulence. (a) energy spectrum; (b) absolute average directional change as a function of the time lag.

qualitative difference between the directional change in the forward and backward cascades is the behavior of the mean absolute angle θ.

V. ANGULAR STATISTICS OF FOOTBALL PLAYERS

Let us come back here to the influence of confinement on directional change. Since the influence of confinement is purely geometrical at large time lags, it should be universally present for trajectories of quantities in other physical situations than turbulent flows in confined domains, as soon as time lags are considered long compared to the typical correlation time of the moving body. We have therefore studied a totally different system: football. More precisely, we have considered the statistics of a training match of two teams of eight football players. In Fig. 9 we show the trajectories of four of these players during a 5 min interval. For the trajectories of the 14 players (excluding the goal keepers) we will evaluate the same statistics as for the fluid particles in the foregoing sections, in order to evaluate which effects are robust enough to survive when we completely change the physical system.

We have computed the Lagrangian velocity spectrum, which is shown in Fig. 10(a). The data seem valid up to a frequency $f \approx 0.6$, where the spectrum shows an important change of slope, which we do not have a clear interpretation for. For frequencies smaller than $f \approx 0.6$, the spectrum

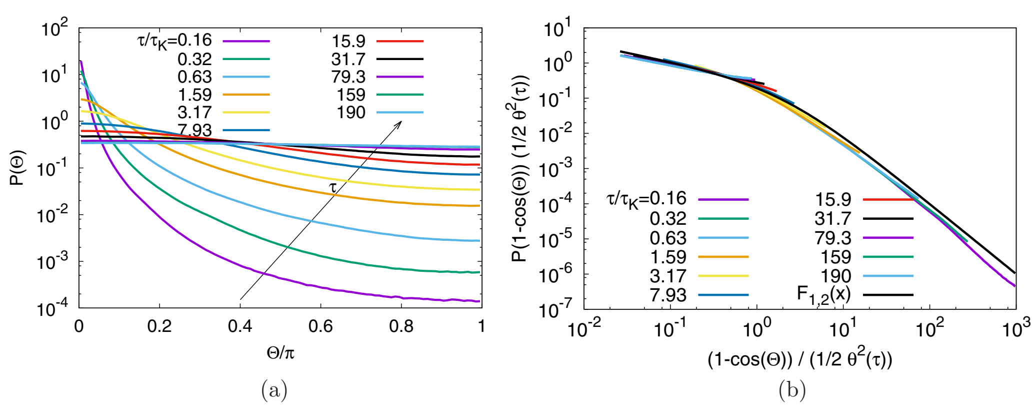


FIG. 8. Statistics in the inverse cascade range of small-scale forced, 2D turbulence: (a) PDF of the instantaneous angle Θ; (b) normalized PDF of $1 - \cos(\Theta)$ and model, assuming Gaussianity and independence.

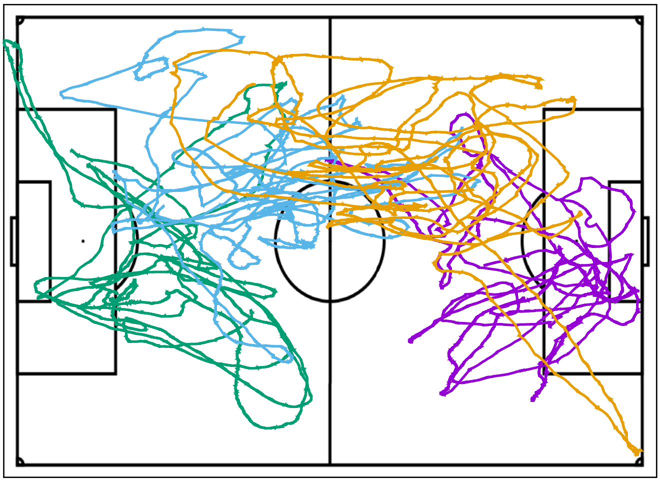


FIG. 9. Trajectories of four players during a 5 min interval of a football match.

shows a clear power law with an exponent close to -2. In the case of 2D flows, this exponent can be expected from Kolmogorov-arguments in the energy-transfer inertial range and is also observed in experiments on 3D turbulence [19].

We have computed the average absolute angle as a function of the time lag, and the results are shown in Fig. 10(b). It seems that at small times the accuracy is not sufficient to show a power law behavior corresponding to a ballistic regime. At such small time lags the angles are most certainly influenced by the measurement error in the position of the football players, and the Lagrangian energy spectrum also showed an unexplained increasing behavior for these time lags. However, for larger times, the mean absolute angle shows a behavior corresponding to a Kolmogorov-like inertial range, where θ is proportional to $\tau^{1/2}$. If we would push the analogy further, we would be tempted to say that the trajectories of football players are similar to those of fluid particles in the inverse cascade range of 2D turbulence, but at high enough Reynolds number to observe asymptotic scaling. In this same range, an f^{-2} power law for the Lagrangian velocity spectrum is observed [Fig. 10(a)], analogous to the inertial range scaling of isotropic turbulence, thereby corroborating our conjecture that these two ranges are associated with the same dynamics.

The long-time behavior observed in the turbulent wall-bounded flow is shown to be remarkably robust. Indeed, even though a football match does not seem to be ergodic, and some may argue that the trajectories of football players are not completely random, the long-time PDF of the directional change converged to a shape close to the one for fluid particles, as shown in Fig. 10(c). Moreover, the mean-absolute angle for long times converges to a value $\theta \approx 2\pi/3$, exactly as for confined 2D turbulence. It seems that the fact that in a football match the movement of the players is strongly anisotropic does not prevent the PDF from converging to a universal shape corresponding to random positions in a closed domain. The anisotropy might only show up in more global statistics, such as the global velocity correlations, in a sense similar as the return to isotropy of the smallest scales in complex turbulent flows. Furthermore, again the PDFs of $1 - \cos(\Theta)$ are close to an $F_{1,2}$ distribution [Fig. 10(d)].

VI. CONCLUSION

We have investigated geometrical multiscale statistics by analyzing the directional change of fluid particles in 2D turbulence, and we have put a particular emphasis on the distinction between flow-dependent and geometry-dependent features.

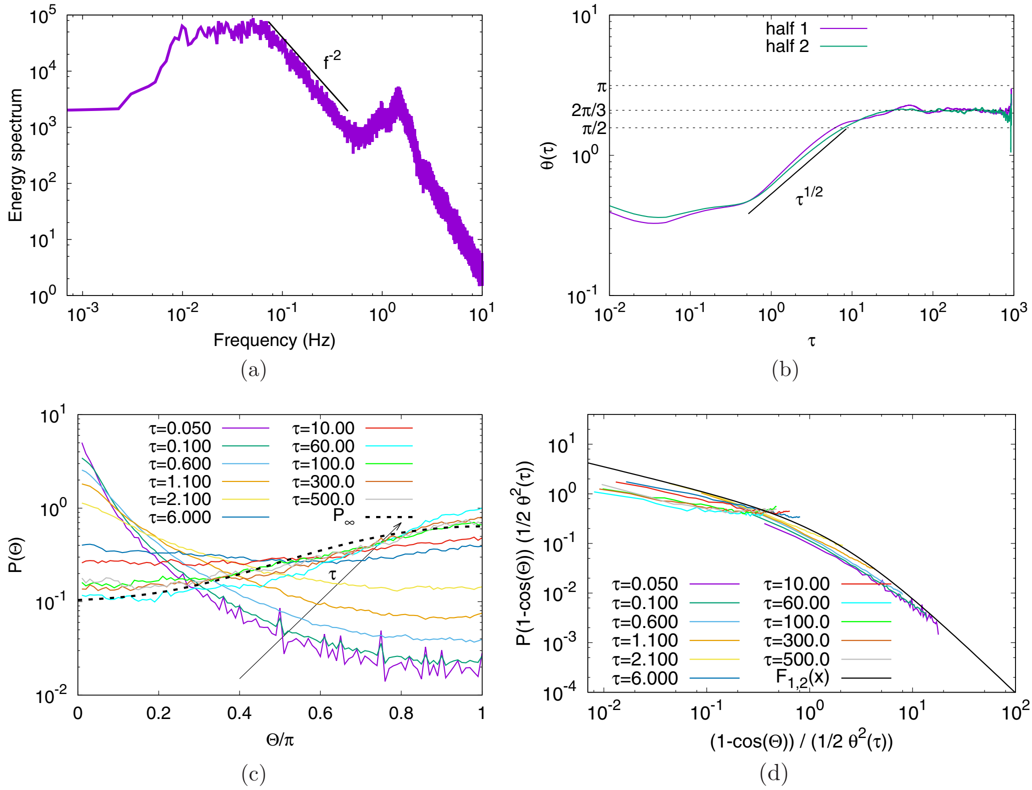


FIG. 10. Angular statistics of football players: (a) Lagrangian velocity spectrum; (b) mean absolute angle for the two halftimes of the match; (c) PDFs of the instantaneous angle $\Theta(\tau,t)$; (d) normalized PDFs.

For the average absolute angle $\theta(\tau)$, characterizing the directional change as a function of the time lag τ, it was shown that in the cases we considered, a short-time regime could be identified where the angle scales linearly, $\theta(\tau) \sim \tau$. This linear regime corresponds to smooth trajectories, and its existence is roughly independent of the type of flow, or the confining geometry.

For intermediate time lags, unlike 3D turbulence, no inertial range scaling was observed in a flow dominated by forward enstrophy transfer, which is due to the fact that such flow is statistically smooth in space. Different results are observed for the inverse-cascade range, which is shown to behave similar to 3D turbulence with respect to scaling. Indeed, an inertial range was observed for the mean angle of directional change, but its exponent deviates from the prediction based on Kolmogorov scaling. The presence or absence of an inertial range scaling in the angular statistics is thus clearly flow-dependent, at least in two space dimensions.

A salient feature of the present study was the investigation of the effect of confinement. It was shown that the size of the domain influences the long-time behavior of the directional change, and the precise behavior can be reproduced by considering the angle between two connected line segments defined by three points, randomly placed in the domain. Monte Carlo simulations allowed to disentangle the geometrically dependent features from the flow-dependent features. Indeed, the effect of confinement seems independent of the flow properties, as long as the flow can be considered ergodic, which should be the case for time lags much larger than the Lagrangian correlation time.

In all considered cases, for short times the shape of the PDFs for different time lags is reasonably well described by a Fischer distribution. At long times, the PDF is entirely determined by the shape of the geometry and becomes independent of the flow properties. Even for the case of football players, who are, in general, not randomly spaced on a field, the shape of the PDF of the directional change is close to these behaviors, both at short and at long times.

ACKNOWLEDGMENTS

Dr. A. B. H. Bos is thanked for his help with the understanding of the statistical influence of geometric confinement. C. Mutschler is thanked for providing us the football-player data. This work was granted access to the HPC resources of Aix-Marseille Université financed by the project Equip@Meso (ANR-10-EQPX-29-01) of the program "Investissements d'Avenir" supervised by the Agence Nationale de la Recherche.

---

[1] A. N. Kolmogorov, The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers, Dokl. Akad. Nauk SSSR **30**, 301 (1941).

[2] U. Frisch, *Turbulence, the Legacy of A. N. Kolmogorov* (Cambridge University Press, Cambridge, 1995).

[3] W. Braun, F. De Lillo, and B. Eckhardt, Geometry of particle paths in turbulent flows, J. Turbul. **7**, N62 (2006).

[4] H. Xu, N. T. Ouellette, and E. Bodenschatz, Curvature of Lagrangian Trajectories in Turbulence, Phys. Rev. Lett. **98**, 050201 (2007).

[5] M. Wilczek, O. Kamps, and R. Friedrich, Lagrangian investigation of two-dimensional decaying turbulence, Physica D **237**, 2090 (2008).

[6] B. Kadoch, D. Del Castillo Negrete, W. J. T. Bos, and K. Schneider, Lagrangian statistics and flow topology in forced two-dimensional turbulence, Phys. Rev. E **83**, 036314 (2011).

[7] A. Scagliarini, Geometric properties of particle trajectories in turbulent flows, J. Turbul. **12**, N25 (2011).

[8] P. Schaefer, Curvature statistics of streamlines in various turbulent flows, J. Turbul. **13**, N28 (2012).

[9] S. Burov, S. M. A. Tabei, T. Huynh, M. P. Murrell, L. H. Philipson, S. A. Rice, M. L. Gardel, N. F. Scherer, and A. R. Dinner, Distribution of directional change as a signature of complex dynamics, Proc. Natl. Acad. Sci. USA **110**, 19689 (2013).

[10] W. J. T. Bos, B. Kadoch, and K. Schneider, Angular Statistics of Lagrangian Trajectories in Turbulence, Phys. Rev. Lett. **114**, 214502 (2015).

[11] P. Constantin, V. Vicol, and J. Wu, Analyticity of Lagrangian trajectories for well posed inviscid incompressible fluid models, Adv. Math. **285**, 352 (2015).

[12] B. Kadoch, W. J. T. Bos, and K. Schneider, The influence of walls on Lagrangian statistics in two-dimensional turbulence, Phys. Fluids **23**, 085111 (2011).

[13] P. Angot, C. H. Bruneau, and P. Fabrie, A penalization method to take into account obstacles in viscous flows, Num. Math. **81**, 497 (1999).

[14] K. Schneider, Numerical simulation of the transient flow behavior in chemical reactors using a penalization method, Comput. Fluids **34**, 1223 (2005).

[15] R. Nguyen van yen, D. Kolomenskiy, and K. Schneider, Approximation of the Laplace and Stokes operators with Dirichlet boundary conditions through volume penalization: A spectral viewpoint, Num. Math. **128**, 301 (2014).

[16] W. J. T. Bos, B. Kadoch, K. Schneider, and J.-P. Bertoglio, Inertial range scaling of the scalar flux spectrum in two-dimensional turbulence, Phys. Fluids **21**, 115105 (2009).

[17] C. Mutschler, H. Ziekow, and Z. Jerzak, The DEBS 2013 grand challenge, in *Proceedings of the 7th ACM International Conference on Distributed Event-Based Systems* (2013), pp. 289–294.

[18] P. K. Yeung, S. B. Pope, and B. L. Sawford, Reynolds number dependence of Lagrangian statistics in large numerical simulations of isotropic turbulence, J. Turbul. **7**, N58 (2006).

[19] N. Mordant, E. Lévêque, and J.-F. Pinton, Experimental and numerical study of the Lagrangian dynamics of high Reynolds turbulence, New J. Phys. **6**, 116 (2004).