Course: Digital Signal and Data Processing

Chebyshev Filters

Lecture 6

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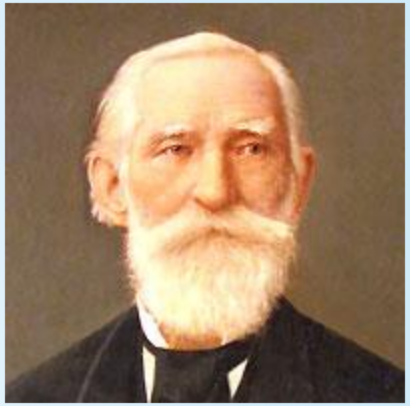
The Chebyshev Responses



The Chebyshev response is a mathematical strategy for achieving a faster ***rolloff*** by allowing ***ripple*** in the frequency response. Analog and digital filters that use this approach are called ***Chebyshev filters***. For instance, analog Chebyshev filters are used for analog-to-digital and digital-to-analog conversion. These filters are named from their use of the ***Chebyshev polynomials***, developed by the Russian mathematician Pafnuti Chebyshev (1821-1894).

The Chebyshev and Butterworth Responses

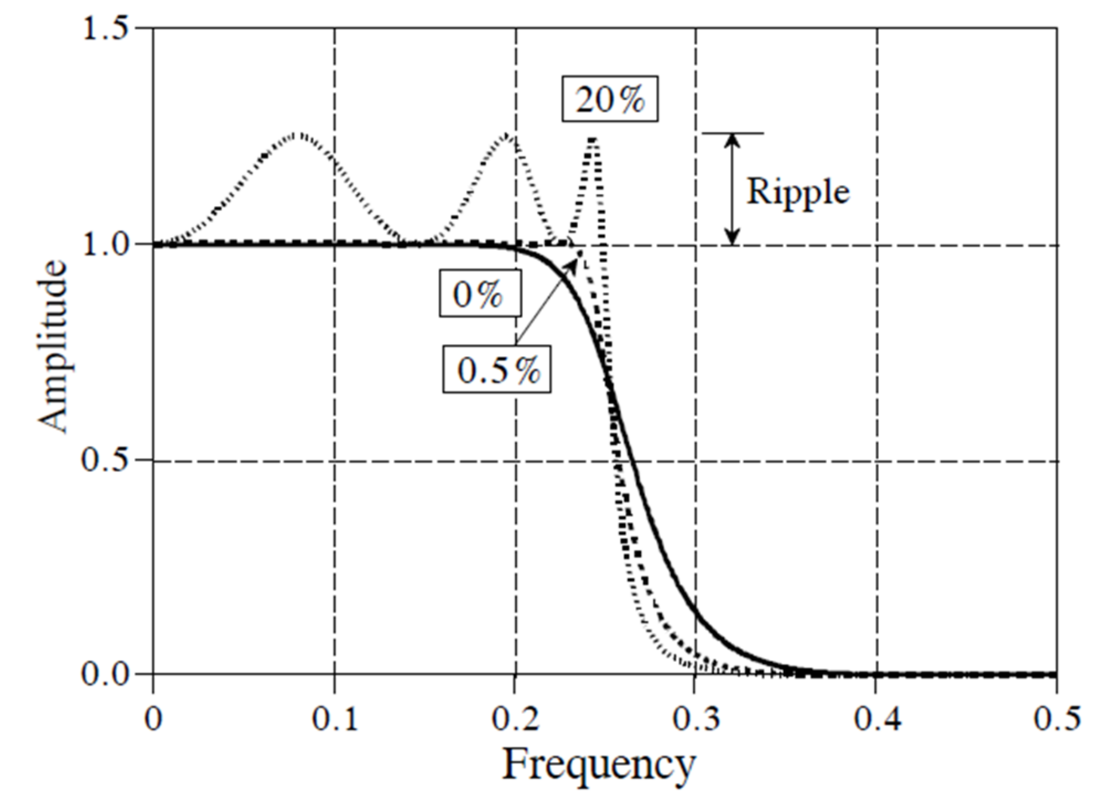


Figure shows the frequency response of low-pass Chebyshev filters with passband ripples of: 0%, 0.5% and 20%. As the **ripple increases** (bad), the **roll-off** becomes **sharper** (good). The Chebyshev response is an **optimal tradeoff** between these two parameters. When the ripple is set to 0%, the filter is called a **maximally flat** or **Butterworth filter** (after S. Butterworth, a British engineer who described this response in 1930). A ripple of 0.5% is a often good choice for digital filters. This matches the typical precision and accuracy of the analog electronics that the signal has passed through.

The Chebyshev and Butterworth Responses

The Chebyshev filters discussed here are called **type 1 filters**, meaning that the ripple is only allowed in the ***passband*****.** In comparison, **type 2 Chebyshev filters** have ripple only in the *stopband*. Type 2 filters are seldom used.

There is, however, an important design called the **elliptic filter**, which has ripple in *both* the passband and the stopband. Elliptic filters provide the fastest roll-off for a given number of poles, but are much harder to design. This filters is frequently the first choice of professional filter designers, both in analog electronics and DSP.

Designing the Chebysher Filter

You must select four parameters to design a Chebyshev filter:

(1) a high-pass or low-pass response;

(2) the cutoff frequency;

(3) the percent ripple in the passband;

(4) the number of poles.

Designing the Filter: poles

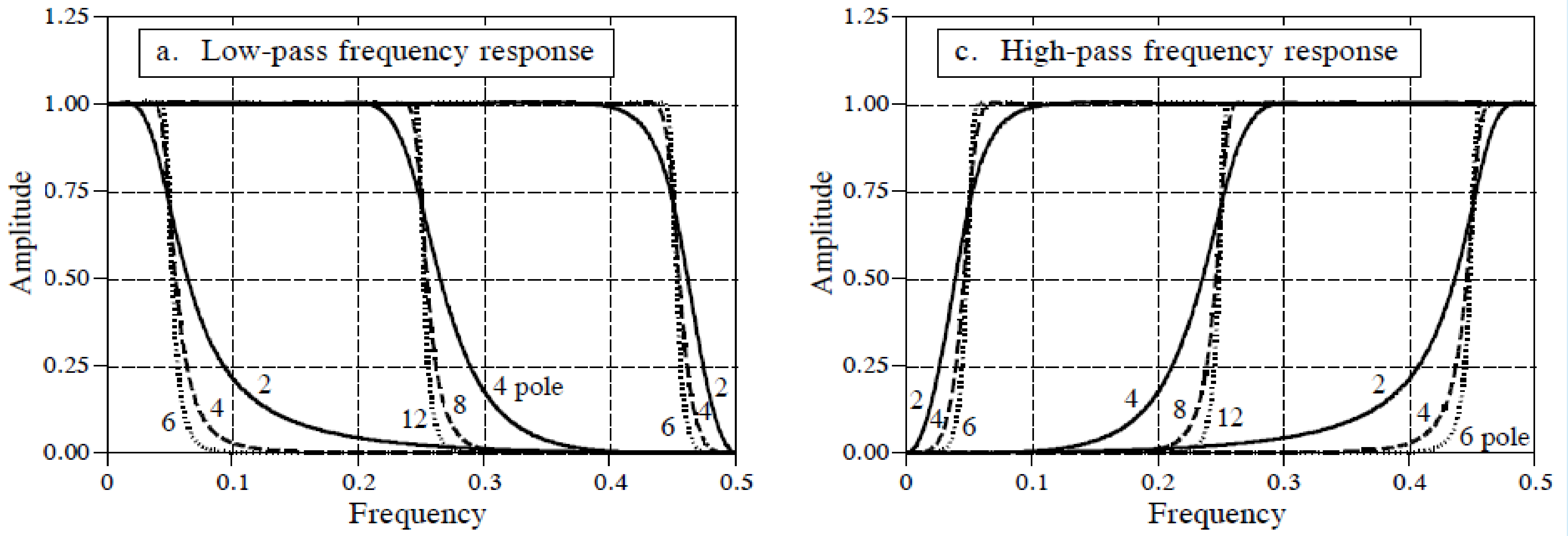
The Laplace transform and z-transform are mathematical ways of breaking an impulse response into sinusoids and decaying exponentials. This is done by expressing the system's characteristics as one complex polynomial divided by another complex polynomial. The roots of the numerator are called ***zeros***, while the roots of the denominator are called ***poles***. Since poles and zeros can be complex numbers, it is common to say they have a "location" in the complex plane. Elaborate systems have more poles and zeros than simple ones.

Recursive filters are designed by first selecting the location of the poles and zeros, and then finding the appropriate recursion coefficients (or analog components). For example, Butterworth filters have poles that lie on a *circle* in the complex plane, while in a Chebyshev filter they lie on an **ellipse**.

The more poles in a filter, the better the filter works!

Designing the Filter

Figure below shows the frequency response of several Chebyshev filters with 0.5% ripple. For the method used here, the number of poles must be ***even***. The cutoff frequency of each filter is measured where the amplitude crosses 0.707 (-3dB). Filters with a cutoff frequency near 0 or 0.5 have a sharper roll-off than filters in the center of the frequency range. For example, a two pole filter at $f_c = 0.05$ has about the same roll-off as a four pole filter at $f_c = 0.25$. Near 0 and 0.5 you may use **fewer poles** because of round-off noise.



Designing the Filter

There are two ways of finding the recursion coefficients without using the z-transform.

1. Simple way: **use a table**. Tables provide the recursion coefficients for low-pass and high-pass filters with 0.5% passband ripple. If you only need a quick and dirty design, copy the appropriate coefficients into your program, and you're done.

Low-pass Chebyshev filters (0.5% ripple)

f_C	2 Pole	4 Pole	6 Pole
0.01	a0= 8.663387E-04 a1= 1.732678E-03 b1= 1.919129E+00 a2= 8.663387E-04 b2= -9.225943E-01	a0= 4.149425E-07 (!! Unstable !!) a1= 1.659770E-06 b1= 3.893453E+00 a2= 2.489655E-06 b2= -5.688233E+00 a3= 1.659770E-06 b3= 3.695783E+00 a4= 4.149425E-07 b4= -9.010106E-01	a0= 1.391351E-10 (!! Unstable !!) a1= 8.348109E-10 b1= 5.883343E+00 a2= 2.087027E-09 b2= -1.442798E+01 a3= 2.782703E-09 b3= 1.887786E+01 a4= 2.087027E-09 b4= -1.389914E+01 a5= 8.348109E-10 b5= 5.459909E+00 a6= 1.391351E-10 b6= -8.939932E-01
0.025	a0= 5.112374E-03 a1= 1.022475E-02 b1= 1.797154E+00 a2= 5.112374E-03 b2= -8.176033E-01	a0= 1.504626E-05 a1= 6.018503E-05 b1= 3.725385E+00 a2= 9.027754E-05 b2= -5.226004E+00 a3= 6.018503E-05 b3= 3.270902E+00 a4= 1.504626E-05 b4= -7.705239E-01	a0= 3.136210E-08 (!! Unstable !!) a1= 1.881726E-07 b1= 5.691653E+00 a2= 4.704314E-07 b2= -1.353172E+01 a3= 6.272419E-07 b3= 1.719986E+01 a4= 4.704314E-07 b4= -1.232689E+01 a5= 1.881726E-07 b5= 4.722721E+00 a6= 3.136210E-08 b6= -7.556340E-01
0.05	a0= 1.868823E-02 a1= 3.737647E-02 b1= 1.593937E+00 a2= 1.868823E-02 b2= -6.686903E-01	a0= 2.141509E-04 a1= 8.566037E-04 b1= 3.425455E+00 a2= 1.284906E-03 b2= -4.479272E+00 a3= 8.566037E-04 b3= 2.643718E+00 a4= 2.141509E-04 b4= -5.933269E-01	a0= 1.771089E-06 a1= 1.062654E-05 b1= 5.330512E+00 a2= 2.656634E-05 b2= -1.196611E+01 a3= 3.542179E-05 b3= 1.447067E+01 a4= 2.656634E-05 b4= -9.937710E+00 a5= 1.062654E-05 b5= 3.673283E+00 a6= 1.771089E-06 b6= -5.707561E-01
0.075	a0= 3.869430E-02 a1= 7.738860E-02 b1= 1.392667E+00 a2= 3.869430E-02 b2= -5.474446E-01	a0= 9.726342E-04 a1= 3.890537E-03 b1= 3.103944E+00 a2= 5.835806E-03 b2= -3.774453E+00 a3= 3.890537E-03 b3= 2.111238E+00 a4= 9.726342E-04 b4= -4.562908E-01	a0= 1.797538E-05 a1= 1.078523E-04 b1= 4.921746E+00 a2= 2.696307E-04 b2= -1.035734E+01 a3= 3.595076E-04 b3= 1.189764E+01 a4= 2.696307E-04 b4= -7.854533E+00 a5= 1.078523E-04 b5= 2.822109E+00 a6= 1.797538E-05 b6= -4.307710E-01
0.1	a0= 6.372802E-02 a1= 1.274560E-01 b1= 1.194365E+00 a2= 6.372802E-02 b2= -4.492774E-01	a0= 2.780755E-03 a1= 1.112302E-02 b1= 2.764031E+00 a2= 1.668453E-02 b2= -3.122854E+00 a3= 1.112302E-02 b3= 1.664554E+00 a4= 2.780755E-03 b4= -3.502232E-01	a0= 9.086148E-05 a1= 5.451688E-04 b1= 4.470118E+00 a2= 1.362922E-03 b2= -8.755594E+00 a3= 1.817229E-03 b3= 9.543712E+00 a4= 1.362922E-03 b4= -6.079376E+00 a5= 5.451688E-04 b5= 2.140062E+00 a6= 9.086148E-05 b6= -3.247363E-01
0.15	a0= 1.254285E-01 a1= 2.508570E-01 b1= 8.070778E-01 a2= 1.254285E-01 b2= -3.087918E-01	a0= 1.180009E-02 a1= 4.720034E-02 b1= 2.039039E+00 a2= 7.080051E-02 b2= -2.012961E+00 a3= 4.720034E-02 b3= 9.897915E-01 a4= 1.180009E-02 b4= -2.046700E-01	a0= 8.618665E-04 a1= 5.171199E-03 b1= 3.455239E+00 a2= 1.292800E-02 b2= -5.754735E+00 a3= 1.723733E-02 b3= 5.645387E+00 a4= 1.292800E-02 b4= -3.394902E+00 a5= 5.171199E-03 b5= 1.177469E+00 a6= 8.618665E-04 b6= -1.836195E-01

Low-pass Chebyshev filters (0.5% ripple)

0.2	a0= 1.997396E-01		a0= 3.224554E-02		a0= 4.187408E-03	
	a1= 3.994792E-01	b1= 4.291048E-01	a1= 1.289821E-01	b1= 1.265912E+00	a1= 2.512445E-02	b1= 2.315806E+00
	a2= 1.997396E-01	b2= -2.280633E-01	a2= 1.934732E-01	b2= -1.203878E+00	a2= 6.281112E-02	b2= -3.293726E+00
			a3= 1.289821E-01	b3= 5.405908E-01	a3= 8.374816E-02	b3= 2.904826E+00
			a4= 3.224554E-02	b4= -1.185538E-01	a4= 6.281112E-02	b4= -1.694128E+00
					a5= 2.512445E-02	b5= 6.021426E-01
					a6= 4.187408E-03	b6= -1.029147E-01
0.25	a0= 2.858110E-01		a0= 7.015301E-02		a0= 1.434449E-02	
	a1= 5.716221E-01	b1= 5.423258E-02	a1= 2.806120E-01	b1= 4.541481E-01	a1= 8.606697E-02	b1= 1.076052E+00
	a2= 2.858110E-01	b2= -1.974768E-01	a2= 4.209180E-01	b2= -7.417536E-01	a2= 2.151674E-01	b2= -1.662847E+00
			a3= 2.806120E-01	b3= 2.361222E-01	a3= 2.868899E-01	b3= 1.191063E+00
			a4= 7.015301E-02	b4= -7.096476E-02	a4= 2.151674E-01	b4= -7.403087E-01
					a5= 8.606697E-02	b5= 2.752158E-01
					a6= 1.434449E-02	b6= -5.722251E-02
0.3	a0= 3.849163E-01		a0= 1.335566E-01		a0= 3.997487E-02	
	a1= 7.698326E-01	b1= -3.249116E-01	a1= 5.342263E-01	b1= -3.904486E-01	a1= 2.398492E-01	b1= -2.441152E-01
	a2= 3.849163E-01	b2= -2.147536E-01	a2= 8.013394E-01	b2= -6.784138E-01	a2= 5.996231E-01	b2= -1.130306E+00
			a3= 5.342263E-01	b3= -1.412021E-02	a3= 7.994975E-01	b3= 1.063167E-01
			a4= 1.335566E-01	b4= -5.392238E-02	a4= 5.996231E-01	b4= -3.463299E-01
					a5= 2.398492E-01	b5= 8.882992E-02
					a6= 3.997487E-02	b6= -3.278741E-02
0.35	a0= 5.001024E-01		a0= 2.340973E-01		a0= 9.792321E-02	
	a1= 1.000205E+00	b1= -7.158993E-01	a1= 9.363892E-01	b1= -1.263672E+00	a1= 5.875393E-01	b1= -1.627573E+00
	a2= 5.001024E-01	b2= -2.845103E-01	a2= 1.404584E+00	b2= -1.080487E+00	a2= 1.468848E+00	b2= -1.955020E+00
			a3= 9.363892E-01	b3= -3.276296E-01	a3= 1.958464E+00	b3= -1.075051E+00
			a4= 2.340973E-01	b4= -7.376791E-02	a4= 1.468848E+00	b4= -5.106501E-01
					a5= 5.875393E-01	b5= -7.239843E-02
					a6= 9.792321E-02	b6= -2.639193E-02
0.40	a0= 6.362308E-01		a0= 3.896966E-01		a0= 2.211834E-01	
	a1= 1.272462E+00	b1= -1.125379E+00	a1= 1.558787E+00	b1= -2.161179E+00	a1= 1.327100E+00	b1= -3.058672E+00
	a2= 6.362308E-01	b2= -4.195441E-01	a2= 2.338180E+00	b2= -2.033992E+00	a2= 3.317751E+00	b2= -4.390465E+00
			a3= 1.558787E+00	b3= -8.789098E-01	a3= 4.423668E+00	b3= -3.523254E+00
			a4= 3.896966E-01	b4= -1.610655E-01	a4= 3.317751E+00	b4= -1.684185E+00
					a5= 1.327100E+00	b5= -4.414881E-01
					a6= 2.211834E-01	b6= -5.767513E-02
0.45	a0= 8.001101E-01		a0= 6.291693E-01		a0= 4.760635E-01	
	a1= 1.600220E+00	b1= -1.556269E+00	a1= 2.516677E+00	b1= -3.077062E+00	a1= 2.856381E+00	b1= -4.522403E+00
	a2= 8.001101E-01	b2= -6.441713E-01	a2= 3.775016E+00	b2= -3.641323E+00	a2= 7.140952E+00	b2= -8.676844E+00
			a3= 2.516677E+00	b3= -1.949229E+00	a3= 9.521270E+00	b3= -9.007512E+00
			a4= 6.291693E-01	b4= -3.990945E-01	a4= 7.140952E+00	b4= -5.328429E+00
					a5= 2.856381E+00	b5= -1.702543E+00
					a6= 4.760635E-01	b6= -2.303303E-01

TABLE 20-1

High-pass Chebyshev filters (0.5% ripple)

f_C	2 Pole		4 Pole		6 Pole	
0.01	a0= 9.567529E-01		a0= 9.121579E-01	(!! Unstable !!)	a0= 8.630195E-01	(!! Unstable !!)
	a1= -1.913506E+00	b1= 1.911437E+00	a1= -3.648632E+00	b1= 3.815952E+00	a1= -5.178118E+00	b1= 5.705102E+00
	a2= 9.567529E-01	b2= -9.155749E-01	a2= 5.472947E+00	b2= -5.465026E+00	a2= 1.294529E+01	b2= -1.356935E+01
			a3= -3.648632E+00	b3= 3.481295E+00	a3= -1.726039E+01	b3= 1.722231E+01
			a4= 9.121579E-01	b4= -8.322529E-01	a4= 1.294529E+01	b4= -1.230214E+01
					a5= -5.178118E+00	b5= 4.689218E+00
					a6= 8.630195E-01	b6= -7.451429E-01
0.025	a0= 8.950355E-01		a0= 7.941874E-01		a0= 6.912863E-01	(!! Unstable !!)
	a1= -1.790071E+00	b1= 1.777932E+00	a1= -3.176750E+00	b1= 3.538919E+00	a1= -4.147718E+00	b1= 5.261399E+00
	a2= 8.950355E-01	b2= -8.022106E-01	a2= 4.765125E+00	b2= -4.722213E+00	a2= 1.036929E+01	b2= -1.157800E+01
			a3= -3.176750E+00	b3= 2.814036E+00	a3= -1.382573E+01	b3= 1.363599E+01
			a4= 7.941874E-01	b4= -6.318300E-01	a4= 1.036929E+01	b4= -9.063840E+00
					a5= -4.147718E+00	b5= 3.223738E+00
					a6= 6.912863E-01	b6= -4.793541E-01
0.05	a0= 8.001102E-01		a0= 6.291694E-01		a0= 4.760636E-01	
	a1= -1.600220E+00	b1= 1.556269E+00	a1= -2.516678E+00	b1= 3.077062E+00	a1= -2.856382E+00	b1= 4.522403E+00
	a2= 8.001102E-01	b2= -6.441715E-01	a2= 3.775016E+00	b2= -3.641324E+00	a2= 7.140954E+00	b2= -8.676846E+00
			a3= -2.516678E+00	b3= 1.949230E+00	a3= -9.521272E+00	b3= 9.007515E+00
			a4= 6.291694E-01	b4= -3.990947E-01	a4= 7.140954E+00	b4= -5.328431E+00
					a5= -2.856382E+00	b5= 1.702544E+00
					a6= 4.760636E-01	b6= -2.303304E-01
0.075	a0= 7.142028E-01		a0= 4.965350E-01		a0= 3.259100E-01	
	a1= -1.428406E+00	b1= 1.338264E+00	a1= -1.986140E+00	b1= 2.617304E+00	a1= -1.955460E+00	b1= 3.787397E+00
	a2= 7.142028E-01	b2= -5.185469E-01	a2= 2.979210E+00	b2= -2.749252E+00	a2= 4.888651E+00	b2= -6.288362E+00
			a3= -1.986140E+00	b3= 1.325548E+00	a3= -6.518201E+00	b3= 5.747801E+00
			a4= 4.965350E-01	b4= -2.524546E-01	a4= 4.888651E+00	b4= -3.041570E+00
					a5= -1.955460E+00	b5= 8.808669E-01
					a6= 3.259100E-01	b6= -1.122464E-01
0.1	a0= 6.362307E-01		a0= 3.896966E-01		a0= 2.211833E-01	
	a1= -1.272461E+00	b1= 1.125379E+00	a1= -1.558786E+00	b1= 2.161179E+00	a1= -1.327100E+00	b1= 3.058671E+00
	a2= 6.362307E-01	b2= -4.195440E-01	a2= 2.338179E+00	b2= -2.033991E+00	a2= 3.317750E+00	b2= -4.390464E+00
			a3= -1.558786E+00	b3= 8.789094E-01	a3= -4.423667E+00	b3= 3.523252E+00
			a4= 3.896966E-01	b4= -1.610655E-01	a4= 3.317750E+00	b4= -1.684184E+00
					a5= -1.327100E+00	b5= 4.414878E-01
					a6= 2.211833E-01	b6= -5.767508E-02
0.15	a0= 5.001024E-01		a0= 2.340973E-01		a0= 9.792321E-02	
	a1= -1.000205E+00	b1= 7.158993E-01	a1= -9.363892E-01	b1= 1.263672E+00	a1= -5.875393E-01	b1= 1.627573E+00
	a2= 5.001024E-01	b2= -2.845103E-01	a2= 1.404584E+00	b2= -1.080487E+00	a2= 1.468848E+00	b2= -1.955020E+00
			a3= -9.363892E-01	b3= 3.276296E-01	a3= -1.958464E+00	b3= 1.075051E+00
			a4= 2.340973E-01	b4= -7.376791E-02	a4= 1.468848E+00	b4= -5.106501E-01
					a5= -5.875393E-01	b5= 7.239843E-02
					a6= 9.792321E-02	b6= -2.639193E-02

High-pass Chebyshev filters (0.5% ripple)

0.2	a0= 3.849163E-01 a1= -7.698326E-01 b1= 3.249116E-01 a2= 3.849163E-01 b2= -2.147536E-01	a0= 1.335566E-01 a1= -5.342262E-01 b1= 3.904484E-01 a2= 8.013393E-01 b2= -6.784138E-01 a3= -5.342262E-01 b3= 1.412016E-02 a4= 1.335566E-01 b4= -5.392238E-02	a0= 3.997486E-02 a1= -2.398492E-01 b1= 2.441149E-01 a2= 5.996230E-01 b2= -1.130306E+00 a3= -7.994973E-01 b3= -1.063169E-01 a4= 5.996230E-01 b4= -3.463299E-01 a5= -2.398492E-01 b5= -8.882996E-02 a6= 3.997486E-02 b6= -3.278741E-02
0.25	a0= 2.858111E-01 a1= -5.716222E-01 b1= -5.423243E-02 a2= 2.858111E-01 b2= -1.974768E-01	a0= 7.015302E-02 a1= -2.806121E-01 b1= -4.541478E-01 a2= 4.209182E-01 b2= -7.417535E-01 a3= -2.806121E-01 b3= -2.361221E-01 a4= 7.015302E-02 b4= -7.096475E-02	a0= 1.434450E-02 a1= -8.606701E-02 b1= -1.076051E+00 a2= 2.151675E-01 b2= -1.662847E+00 a3= -2.868900E-01 b3= -1.191062E+00 a4= 2.151675E-01 b4= -7.403085E-01 a5= -8.606701E-02 b5= -2.752156E-01 a6= 1.434450E-02 b6= -5.722250E-02
0.3	a0= 1.997396E-01 a1= -3.994792E-01 b1= -4.291049E-01 a2= 1.997396E-01 b2= -2.280633E-01	a0= 3.224553E-02 a1= -1.289821E-01 b1= -1.265912E+00 a2= 1.934732E-01 b2= -1.203878E+00 a3= -1.289821E-01 b3= -5.405908E-01 a4= 3.224553E-02 b4= -1.185538E-01	a0= 4.187407E-03 a1= -2.512444E-02 b1= -2.315806E+00 a2= 6.281111E-02 b2= -3.293726E+00 a3= -8.374815E-02 b3= -2.904827E+00 a4= 6.281111E-02 b4= -1.694129E+00 a5= -2.512444E-02 b5= -6.021426E-01 a6= 4.187407E-03 b6= -1.029147E-01
0.35	a0= 1.254285E-01 a1= -2.508570E-01 b1= -8.070777E-01 a2= 1.254285E-01 b2= -3.087918E-01	a0= 1.180009E-02 a1= -4.720035E-02 b1= -2.039039E+00 a2= 7.080051E-02 b2= -2.012961E+00 a3= -4.720035E-02 b3= -9.897915E-01 a4= 1.180009E-02 b4= -2.046700E-01	a0= 8.618665E-04 a1= -5.171200E-03 b1= -3.455239E+00 a2= 1.292800E-02 b2= -5.754734E+00 a3= -1.723733E-02 b3= -5.645387E+00 a4= 1.292800E-02 b4= -3.394902E+00 a5= -5.171200E-03 b5= -1.177469E+00 a6= 8.618665E-04 b6= -1.836195E-01
0.40	a0= 6.372801E-02 a1= -1.274560E-01 b1= -1.194365E+00 a2= 6.372801E-02 b2= -4.492774E-01	a0= 2.780754E-03 a1= -1.112302E-02 b1= -2.764031E+00 a2= 1.668453E-02 b2= -3.122854E+00 a3= -1.112302E-02 b3= -1.664554E+00 a4= 2.780754E-03 b4= -3.502233E-01	a0= 9.086141E-05 a1= -5.451685E-04 b1= -4.470118E+00 a2= 1.362921E-03 b2= -8.755595E+00 a3= -1.817228E-03 b3= -9.543712E+00 a4= 1.362921E-03 b4= -6.079377E+00 a5= -5.451685E-04 b5= -2.140062E+00 a6= 9.086141E-05 b6= -3.247363E-01
0.45	a0= 1.868823E-02 a1= -3.737647E-02 b1= -1.593937E+00 a2= 1.868823E-02 b2= -6.686903E-01	a0= 2.141509E-04 a1= -8.566037E-04 b1= -3.425455E+00 a2= 1.284906E-03 b2= -4.479272E+00 a3= -8.566037E-04 b3= -2.643718E+00 a4= 2.141509E-04 b4= -5.933269E-01	a0= 1.771089E-06 a1= -1.062654E-05 b1= -5.330512E+00 a2= 2.656634E-05 b2= -1.196611E+01 a3= -3.542179E-05 b3= -1.447067E+01 a4= 2.656634E-05 b4= -9.937710E+00 a5= -1.062654E-05 b5= -3.673283E+00 a6= 1.771089E-06 b6= -5.707561E-01

TABLE 20-2

Designing the Filter

There are two problems with using tables to design digital filters.

1. Tables have a limited choice of parameters. For instance, Table only provides 12 different cutoff frequencies, a maximum of 6 poles per filter, and ***no choice of passband ripple***. Without the ability to select parameters from a continuous range of values, the filter design cannot be ***optimized***.
2. The coefficients must be manually transferred from the table into the program. This is very time consuming and will discourage you from trying alternative values.

Designing the Filter

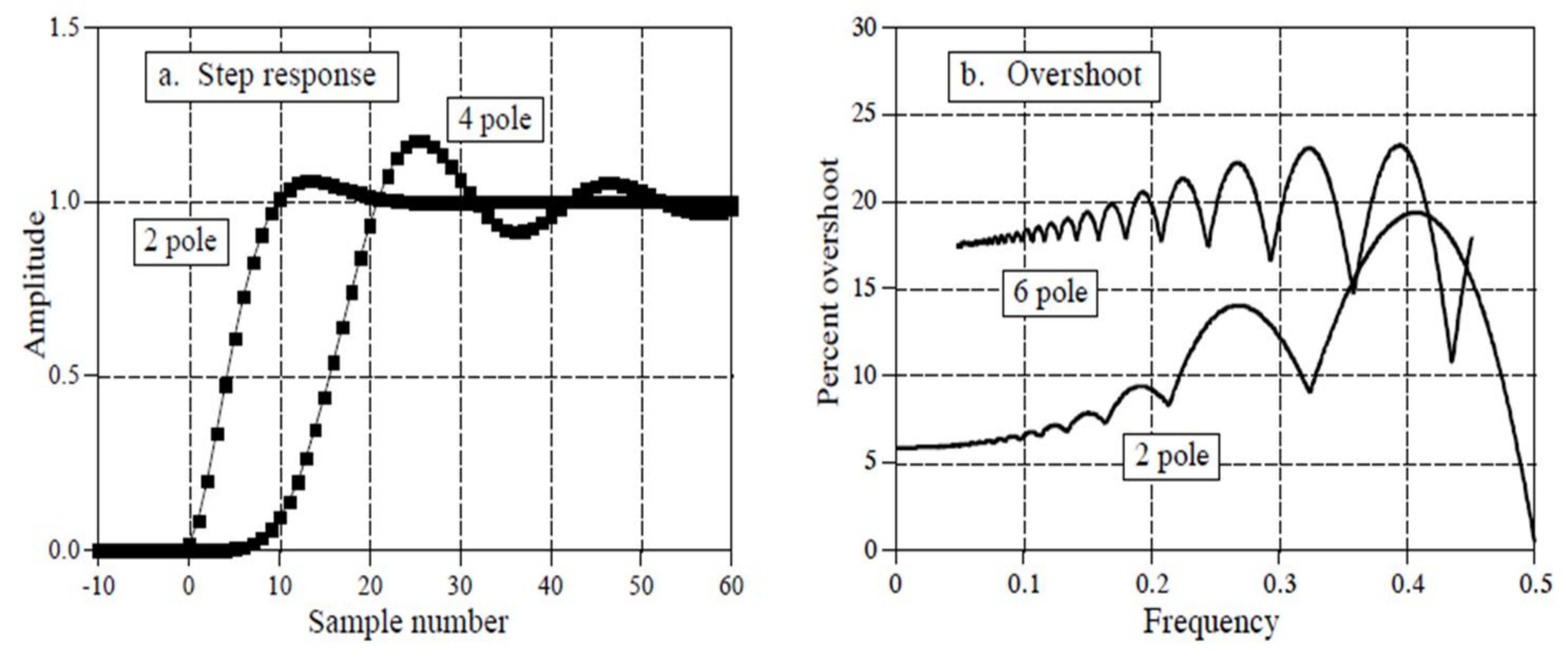
Instead of using tabulated values, include a function (block) in your program that **calculates** the coefficients.

The program is relatively simple in structure. After the four filter parameters are entered, the program returns the "$\boldsymbol{a}$" and "$\boldsymbol{b}$" coefficients in the arrays A[ ] and B[ ]. The program calls the another function (block). Six variables enter the function (routine), five variables leave the routine, and fifteen temporary variables (plus indexes) are used within.

P.S. Program has been discussed in the book (Smith, chapter 20, pp.340-341)

Step Response Overshoot

Butterworth and Chebyshev filters have an overshoot of 5 to 30% in their step responses, becoming larger as the number of poles is increased. Figure (a) shows the step response for two example Chebyshev filters. Figure (b) shows something that is unique to digital filters: the amount of overshoot in the step response depends to a small degree on the cutoff frequency of the filter.



Stability

The main limitation of digital filters carried out by convolution is ***execution time***. It is possible to achieve nearly any filter response, provided you are willing to wait for the result.

Recursive filters are just the **opposite**. They are fast, but they are **limited in performance**. For example, consider a 6 pole, 0.5% ripple, low-pass filter with a 0.01 cutoff frequency. The recursion coefficients for this filter can be obtained from Table:

Stability

a0=	1.391351E-10		
a1=	8.348109E-10	b1=	5.883343E+00
a2=	2.087027E-09	b2=	-1.442798E+01
a3=	2.782703E-09	b3=	1.887786E+01
a4=	2.087027E-09	b4=	-1.389914E+01
a5=	8.348109E-10	b5=	5.459909E+00
a6=	1.391351E-10	b6=	-8.939932E-01

Look carefully at these coefficients. The "b" coefficients have an absolute value of about ***ten***. Using single precision, the round-off noise on each of these numbers is about one ten-millionth of the value, i.e., 10^{-6} . Now look at the "a" coefficients, with a value of about 10^{-9}. Something is obviously wrong here. The contribution from the input signal (via the "a" coefficients) will be 1000 times smaller than the ***noise*** from the previously calculated output signal (via the "b" coefficients). **This filter won't work!** In short, round-off noise limits the number of poles that can be used in a filter.

The actual number will depend slightly on the ripple and if it is a high or low-pass filter. The maximum numbers of poles for single precision are:

Cutoff frequency	0.02	0.05	0.10	0.25	0.40	0.45	0.48
Maximum poles	4	6	10	20	10	6	4

Stability

The filter's performance will start **to degrade** as this limit is approached; the step response will show more overshoot, the stopband attenuation will be poor, and the frequency response will have excessive ripple.

If the filter is pushed too far, or there is an error in the coefficients, the output will probably oscillate until an overflow occurs.

Stability

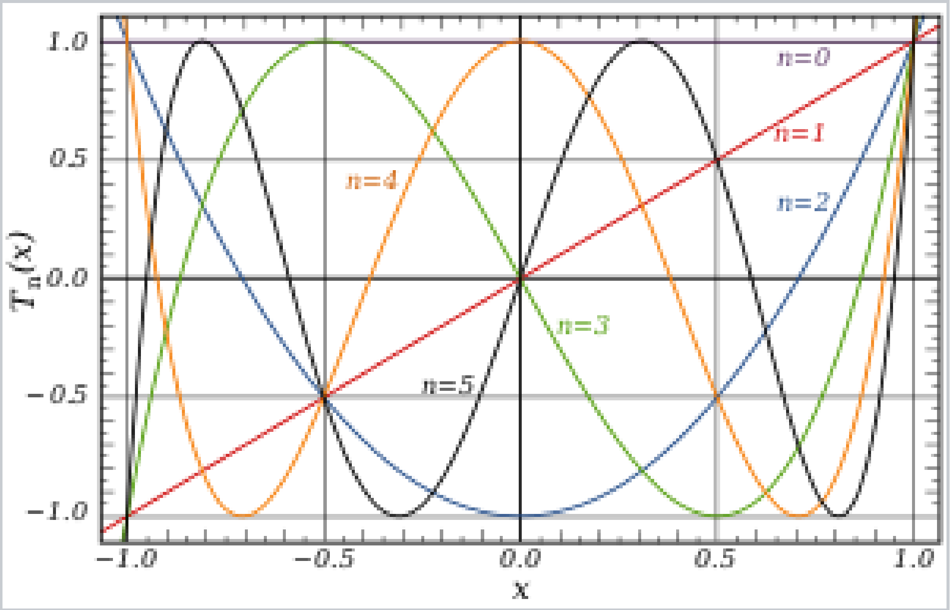
There are two ways of **extending the maximum number of poles** that can be used.

1. Use **double precision**. This requires using double precision in the coefficient calculation as well (including the value for *pi*).
2. The second method is to implement the **filter in stages**. For example, a **six pole** filter starts out as a cascade of three stages of two poles each. Combine these three stages into a single set of recursion coefficients for easier programming. This requires knowing the "$\boldsymbol{a}$" and "$\boldsymbol{b}$" coefficients for each of the stages. These can be obtained programly. The function is called once for each stage in the cascade. For example, it is called three times for a six pole filter. At the end of the function, five variables are return to the main program: $A0, A1, A2, B1$, and $B2$. These are the recursion coefficients for the **two pole stage** being worked on, and can be used to implement the filter in stages.

Chebyshev Polynomials

Chebyshev Polynomials have the property to remain in the range $-1 < T_n(x) < 1$ for an input in the range −1 < x < 1 and then rapidly grow outside this range. This characteristic is a good prerequisite for devising transfer functions with limited oscillations in a given frequency range and steep roll-offs at its borders.

The Chebyshev polynomials of the first kind are defined by the recurrence relation:



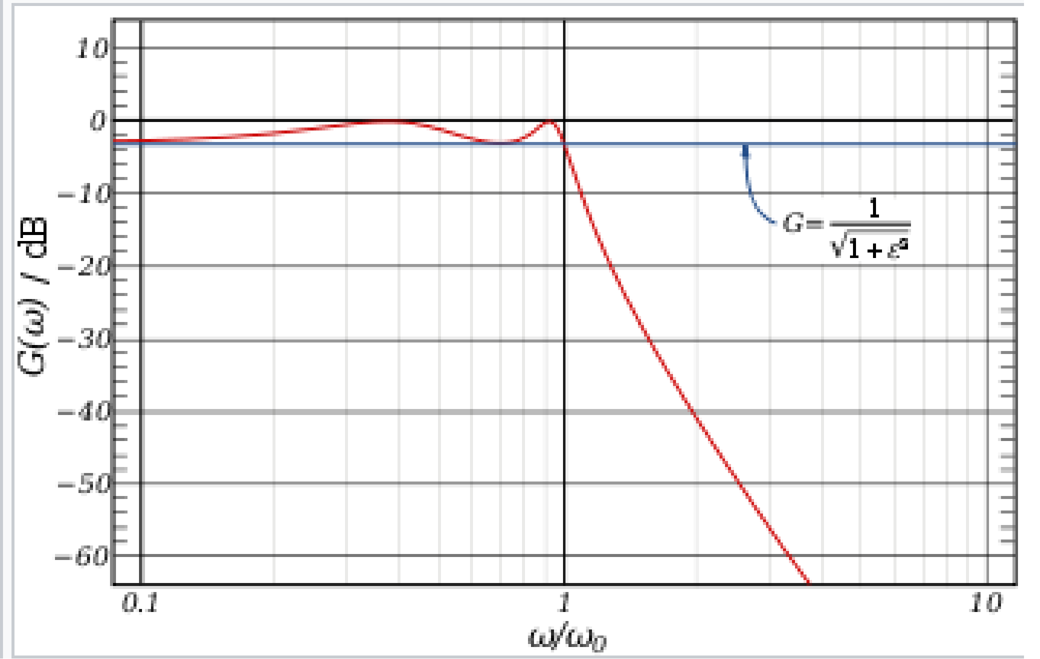
$$T_0(x) = 1$$
$$T_1(x) = x$$
$$T_{n+1}(x) = 2x \cdot T_n(x) - T_{n-1}(x)$$

$$T_0(x) = 1$$
$$T_1(x) = x$$
$$T_2(x) = 2x^2 - 1$$
$$T_3(x) = 4x^3 - 3x$$
$$T_4(x) = 8x^4 - 8x^2 + 1$$

Chebyshev Type 1 Filter Design

Chebyshev type I filters show a ripple in the passband.
The **amplitude response** as a function of angular frequency ω of the *n*th-order low-pass filter is:

$$G_n(\omega) = |H_n(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2 T_n^2\left(\frac{\omega}{\omega_0}\right)}}$$



where ε is the **ripple factor**, ω_0 is the **cutoff frequency** and T_n is a **Chebyshev polynomial** of order n.

Chebyshev Type 1 Filter Design

In the passband, the Chebyshev polynomial alternates between 0 and 1 so the filter gain will alternate between maxima at G = 1 and minima at $G = 1/\sqrt{1+\varepsilon^2}$. At the cutoff frequency ω_0 the gain again has the value $1/\sqrt{1+\varepsilon^2}$ but continues to drop into the stop band as the frequency increases. This behavior is shown in the diagram on the right. The common practice of defining the cutoff frequency at −3dB is usually not applied to Chebyshev filters; instead the cutoff is taken as the point at which the gain falls to the value of the ripple for the final time.