University of Engineering & Management, Kolkata

End Semester Examination, January, 2022

Programme Name: B.Tech in CSE/CST/CSIT/CSBS/CSE(A.I.M.L)/CSE(I.O.T) **Semester: 3rd**

Course Name: Mathematics - III

Course Code: BSC301

Full Marks: 100 **Time: 3 Hours**

GROUP – A (20 marks)

Answer the following questions. Each question is of 2 marks. **10 x 2 = 20**

1. i) Define degree of a partial differential equation with example.

ii) Define non-linear partial differential equation with example.

iii) Define two-dimensional heat equation.

iv) The partial differential equation $5\frac{\partial^2 z}{\partial x^2} + 6\frac{\partial^2 z}{\partial y^2} = \text{xy}$ is classified in which equation?

v) What is random variable?

vi) Define p.d.f. of a random variable.

vii) A random variable X has the following probability distribution. Determine E(X).

X	0	1	2	3	4
P(X)	0	1/5	1/5	2/5	1/5

viii) For a random variable X, var(X)=1, find var(2X+3).

ix) For what value of r the regression coefficient will be zero.

x) Find K so that f(x,y)=K(x+y),0<x<1 and 0<y<1, is a joint probability density function.

GROUP – B (30 marks)

Answer the following questions. Each question is of 5 marks. **6 x 5 = 30**

2. Form partial differential equations from the relationf($x^2 + y^2$, z-xy) = 0.

3. Solve z = $p^2x + q^2\,y$.

4. The probability of a student getting first class, second class and third class at an examination are 1/10,2/5, 1/5 respectively. What is the probability that he fails?

5. A. Solve $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}; -\infty < x < \infty, t > 0$ with $u(x,t) = 0$, $\frac{\partial u}{\partial x} = 0$ as $x \to \pm\infty$ and $u(x,0) = f(x), -\infty < x < \infty$.

OR

B. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$

subject to the conditions $u(0,y) = u(l,y) = u(x,0) = 0, u(x,a) = sin\frac{n\pi}{l}x.$

6. A. The chance that a doctor will diagnose a certain disease correctly is 60%.The chance that a patient will die under his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of the doctor who had the disease died. What is the chance that his disease was diagnosed correctly?

OR

B. If the random variable X has the p.d.f. f(x)=1/4,-2≤x≤2.
Find P(X<1) and P(|X-1|≥1/2).

7. A. Given that X and Y have the joint pdf. (i) Find P(X+Y<3) and (ii) Find P(X<1|Y<3).

$$f(x,y) = \frac{1}{8}(6 - x - y); 0 \le x \le 2; 2 \le y \le 4$$

OR

B. Fit a straight line y=ax+b for following data:

X	1	3	7	8	10
Y	4	7	4	5	8

Answer the following questions. Each question is of 10 marks. **5 x 10 = 50**

8. The probability density function of a random variable X is

$$f(x) = k(x-1)(x-2) \text{ for } 1 \le x \le 2.$$

Determine the following:
(a) the constant k
(b) the distribution function $F(x)$

(c) $P(\frac{5}{4} \le x \le \frac{3}{2})$

9. **i)** Calculate linear regression co-efficient from the following: **7 + 3**

x	1	2	3	4	5	6	7	8
y	3	7	10	12	14	17	20	24

ii) The lines of regression of y on x and x on y are respectively:
y=x+5 and 16x-9y=94. Find the variance of x if the variance of y is 16.

10. A. i) Obtain complete solution of the equation: z = px + qy + sin (x + y). **5 + 5**

ii) Obtain the complete solution of the following equations p^2-q^2 = x – y.

OR

B. i) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x+y}$ **5 + 5**

ii) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for $0 < x < \pi, 0 < y < \pi,$ ith conditions given:

u (0, y) = u (π, y) = u (π, x)=0, u(x, 0) = $sin^2 x$.

11. A. i) Solve the one-dimensional heat flow equation: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$; x≥ $0, t \geq 0$. **5 + 5**

subject to the conditions: **(a)** u = 0, when x = 0, t>0,
(b) u = 1, when 0<x<1,t=0; u =0, when x≥1,t=0
(c) u(x, t) is bounded.

ii) Show that:

$J_{-1/2}$ (x) = $\sqrt{\frac{2}{\pi x}}$ cos x.

OR

B. i) Prove that (2n+1) P_n = (n+1)P_{n+1}+ nP_{n-1} **5 + 5**

ii) Show that (Rodrigue's formula)

$$P_n(x) = \frac{1}{n!\,2^n}\frac{d^n}{dx^n}(x^2 - 1)^n$$

12. A. i) A pair of dice is thrown. Find the probability of getting a sum of 7, when it is known that the digit in the 1st die is greater than that of second. **5 + 5**

ii) Find the probability that at most 5 defective fuses will be found in a box of 200 fuses if experience shows that 2 percent of such fuses are defective.

OR

B. i) A random variable X is exponentially distributed with p.d.f **5 + 5**

$$f(x) = \frac{1}{40}e^{\frac{-x}{40}}, x > 0.$$

Find P(X≤20), P(32<X<48), P(X≥25).

ii) Suppose that the number of arrivals per minute at a toll station at which automobiles arrive at the mean rate of 3 per minute follows Poisson distribution. Find the probability that 5 automobiles arrive in any particular minute.

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