University of Engineering & Management, Kolkata

Term - I Examination, August - September, 2021

Programme Name: B.Tech in Computer Science   **Semester: 3rd**

Course Name: Mathematics-III

Course Code: BSC301

Full Marks: 100   **Time: 3 hours**

GROUP A (20 Marks)

Answer the following questions. Each question is of 2 marks.

Question No.		Questions	Marks Distribution
1.	**i)**	If X is a constant random variable then find Var(X).	2
	ii)	Find the value of $E(X^2)$ when $X \sim N(0,1)$.	2
	iii)	Find the mean of the binomial distribution B(5,2/5).	2
	iv)	Find the value of k such that the following function is a p.d.f. $$f(x) = \begin{cases} k, & -2 < x < 2 \\ 0, & otherwise \end{cases}$$	2
	v)	If Y=3+5X then find var(Y) where var(X)=2.	2
	vi)	Find the regression line y on x where mean of x is 62.33 , mean of y is 65.32, $$s_x^2 = 303.30$$ $$s_y^2 = 186.83, Cov(x,y) = 174.57.$$	2
	vii)	Obtain $\bar{x}$ and $\bar{y}$ from the two regression lines $$3x - 5y = 1$$ $$4x - 7y = 3.$$	2
	viii)	Find mean of the function $$f(x) = \begin{cases} \dfrac{1}{4}e^{-\frac{x}{4}}, & x > 0 \\ 0 & elsewhere \end{cases}$$	2
	ix)	The mean and SD of a binomial distribution are respectively 4 and $\sqrt{8/3}$ Find the values of n and p.	2
	x)	Find the value of k such that the following function is a p.d.f. $$f(x) = \begin{cases} k, & -2 < x < 2 \\ 0, & otherwise \end{cases}$$	2

GROUP B (30 Marks)

Answer the following questions. Each question is of 5 marks.

2.   If a continuous random variable X has exponential distribution with parameter, μ then show that the variance is $\frac{1}{\mu^2}$ **5**

3.   The probability density of a continuous distribution is given by the following pdf. Compute mean and variance?   **5**

$$f(x) = \frac{3x(2-x)}{4} \quad 0 < x < 2.$$

4.   If the random variable X has the following p.d.f, find P(X<1) and
$P(|X - 1| \geq \frac{1}{2})$
$f(x) = \frac{1}{4}, -2 \leq x \leq 2$ **5**

5.   **A.**   A and B play a game in which their chances of winning are in the ratio 3:2. Find A's chance of winning at least three games out of the five games played.   **5**

OR

     B.   There are 500 misprints in a book of 500 pages. What is the probability of the given page will contain at most 3 misprints.   **5**

6.   **A.**   Consider the following set of points: {(2 , 1) , (1 , 3) , (3 , 7)}. Find the least square regression line for the given data points.   **5**

OR

     B.   Fit a straight line for the given pairs of (x,y) which are (1,5), (2,7), (3,9), (4,10), (5,11).   **5**

7.   **A.**   A random variable has a Poisson distribution such that P(1)=P(2), find (i) mean of distribution?   **5**

OR

     B.   Define Poisson distribution and find out the expression for mean and variance of it.   **5**

GROUP C (50 Marks)

Answer the following questions. Each question is of 10 marks.

8.   The probability density function of a random variable X is
$f(x) = k(x - 1)(x - 2) \quad 1 \leq x \leq 2$
(i) Determine the value of k
(ii) the distribution function $F(x)$
(iii) Find $P(\frac{5}{4} \leq x \leq \frac{3}{2})$ **10**

9.   A random variable X has the following probability distribution

a) Find the value of k.

b) Find P(X<6).

c) Find cumulative distribution function of X.

d) The smallest value of 'x' for which $P(X \leq x) > \frac{1}{2}$

10

X	0	1	2	3	4	5	6	7
PX)	0	K	2k	2k	3k	K^2	$2 K^2$	$7K^2+k$

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10. A.   The length of bolts produced by a machine is normally distributed with mean 4 and s.d. 0.5. A bolt is defective if its length does not lie in the interval (3.8, 4.3). Find the percentage of defective bolts produced by the machine. Given that

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.6} e^{\frac{-t^2}{2}} dt = 0.7257 \text{ and}$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.4} e^{\frac{-t^2}{2}} dt = 0.6554$$

OR

B.   The mean weight of 500 students at a certain college is 150 lbs and the standard deviation is 15lbs. Assuming that the weight is normally distributed find how many students weight

(i)   between 120 and 155 lbs

(ii)more than 155 lbs

Given that

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{2} e^{\frac{-t^2}{2}} dt = 0.9772, \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.33} e^{\frac{-t^2}{2}} dt = 0.6293$$

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11. A.   The values of x and their corresponding values of y are shown in the table below

a) Find the least square regression line y = a x + b.

b) Estimate the value of y when x = 12.

x	2	7	9	8	2
y	3	1	10	7	9

OR

B.

In a study of the relationship between expenditure (X) and annual sales volume (Y), a sample of 10 firms yielded the coefficient of correlation r= 0.93. Can we conclude on the basis of this data that X and Y are linearly related?

12. **A.** 91. The sales of a company (in million dollars) for each year are shown in the table below. Find the least square regression line $y = a \, x + b$.

x (year)	2005	2006	2007	2008	2009
y (sales)	12	19	29	37	45

OR

B. By the method of least square, find the straight line that best fits the following data:

x		1	2	3	4	5
Y		14	27	40	55	68

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