

Quiz 3

TOTAL POINTS 15

1. Following is the derivation of Fourier transform of rect function.

1 point

$$\text{rect}(x) = \begin{cases} 1, & \text{for } |x| < \frac{1}{2} \\ 0, & \text{for } |x| > \frac{1}{2} \end{cases}$$

Q. Fill in the blank.

$$\begin{aligned} FT\{\text{rect}(x)\} &= \int_{-\infty}^{\infty} \text{rect}(x) e^{-j2\pi ux} dx \\ &= \int_{-1/2}^{1/2} e^{-j2\pi ux} dx \\ &= \frac{1}{j2\pi u} e^{-j2\pi ux} \Big|_{-1/2}^{1/2} \\ &= \boxed{} \frac{e^{j\pi u} - e^{-j\pi u}}{2j} \\ &= \frac{\sin(\pi u)}{\pi u} = \text{sinc}(u) \end{aligned}$$

- ☐ πu
☒ $\frac{1}{\pi u}$
☐ $\frac{1}{\pi}$
☐ 2π

2. Consider the complex exponential signal

1 point

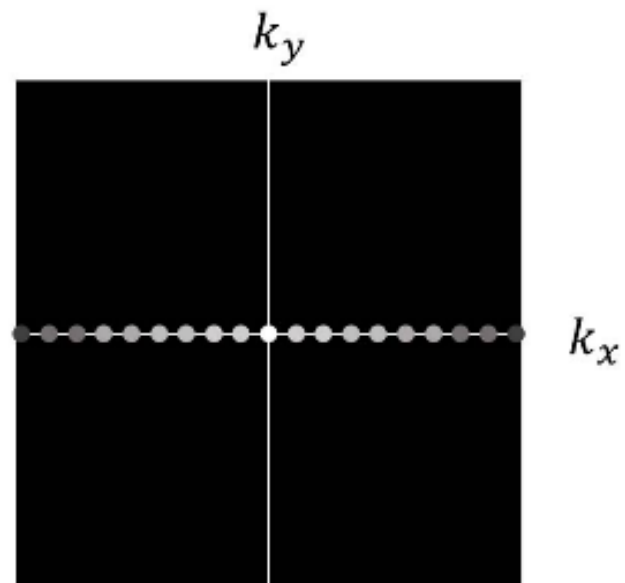
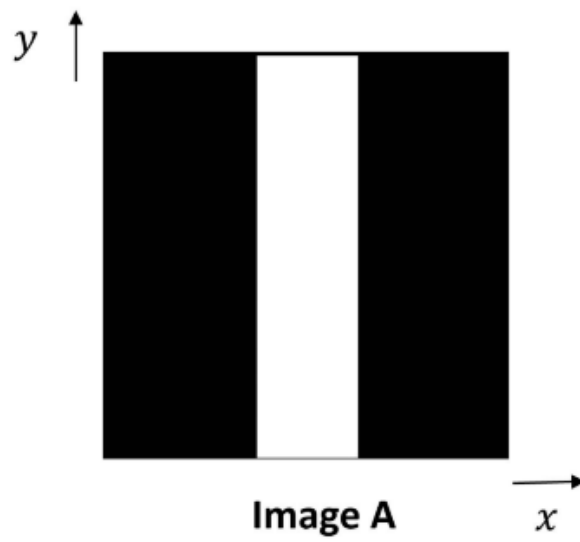
$$f(x) = e^{j2\pi u_0 x}$$

Q. What is its Fourier transform?

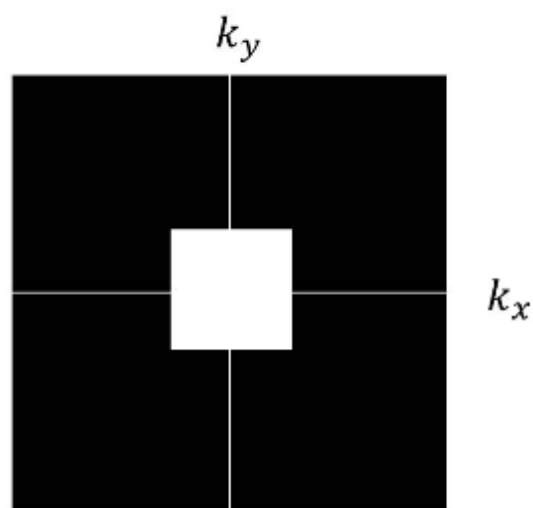
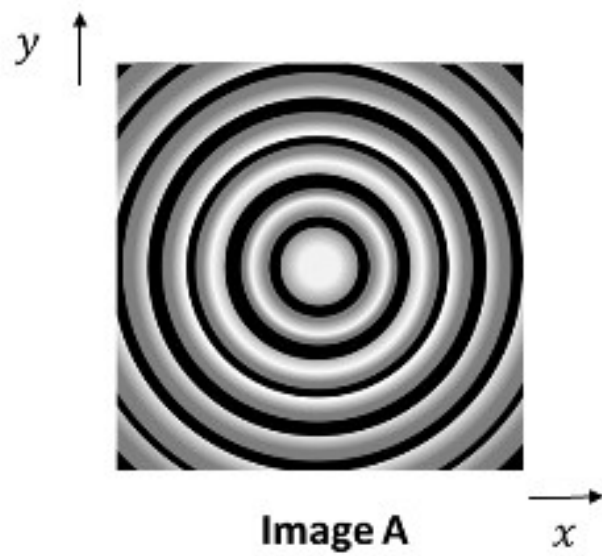
- ☐ $\text{sinc}(u - u_0)$
☒ $\delta(u - u_0)$
☐ $\frac{\sin(u+u_0) + \sin(u-u_0)}{2}$
☐ $\delta(u + u_0)$

3. Choose the correct 2D Fourier transform of the image A.

1 point



4. Choose the correct 2D Fourier transform of the Image A.



5. Following is the proof of one of the properties of Fourier transform.

1 point

Q. Choose one which connects i) the property and ii) corresponding contents in blank correctly.

$$\begin{aligned}
 \text{If } FT\{f(x, y)\} &= F(u, v), \\
 FT\{f(ax, by)\} &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(ax, by) e^{-j2\pi(ux+vy)} dx dy \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(ax, by) e^{-j2\pi\left[\frac{u(ax)}{a} + \frac{v(by)}{b}\right]} dx dy \frac{1}{ab} d(ax) d(by) \\
 &= \frac{1}{|ab|} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(p, q) e^{-j2\pi\left[\frac{u}{a}p + \frac{v}{b}q\right]} dp dq \\
 &= \frac{1}{|ab|} F\left(\frac{u}{a}, \frac{v}{b}\right)
 \end{aligned}$$

- ☐ Translation, $\frac{u}{a}p + \frac{v}{b}q$
☐ Scaling, $(ua)p + (vb)q$
☐ Translation, $(ua)p + (vb)q$
☒ Scaling, $\frac{u}{a}p + \frac{v}{b}q$

6. What is Fourier transform of $x(t) = e^{-at} \cdot u(t)$, where $u(t)$ represents the unit step function and $a > 0$?

1 point

- ☒ $1/(a + j2\pi u)$
☐ $a/(a - j2\pi u)$
☐ $1/(a - j2\pi u)$
☐ $a/(a + j2\pi u)$

7. Choose the one that correctly fills the parentheses.

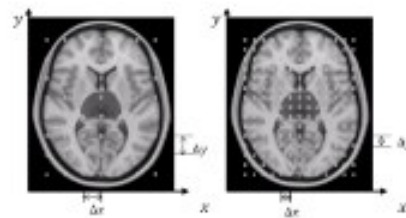
1 point

When the maximum bandwidth of a signal $f(x)$ is 5kHz, the sampling period should be () for a sampled signal of $f(x)$ to be free from aliasing.

- ☒ longer than 0.1 ms
☐ shorter than 0.1 ms
☐ shorter than 0.2 ms
☐ longer than 0.2 ms

8. Q. If we increase the sampling period along x-axis as twice as large, which of the following factors will be increased?

1 point



- ☐ Sampling frequency
- ☐ Image contrast
- ☐ Resolution
- ☒ Partial volume effect

9. Q. When the maximum bandwidths of signals $x(t)$ and $y(t)$ are V and W , respectively, what is the Nyquist sampling frequency of a new signal $x(t)+y(t)$?

1 point

- ☐ $2(V+W)$
- ☐ $2 \cdot \min\{V, W\}$ (minimum of the two, V and W)
- ☐ $2 \cdot V \cdot W$
- ☒ $2 \cdot \max\{V, W\}$ (maximum of the two, V and W)

10. You acquired an MRI image and found an aliasing artifact. Choose the one that avoids the aliasing artifact.

1 point

- ☒ Acquiring the MRI k-space data at a higher sampling rate
- ☐ Adding higher frequency components to the MRI k-space data
- ☐ Making the MRI k-space signals stronger
- ☐ Removing some k-space lines with higher frequency components

11. The peripheral part of k-space decides the contrast of image, while the central part contains the edge information.

1 point

- ☒ False
- ☐ True

12. Select a statement about Fourier transform that is not true.

1 point

- ☐ Reversing the polarity of the time for a time-domain signal reverses the polarity of the frequency of its Fourier transform.
- ☒ If $\hat{f}(u,v)$ and $\hat{f}^*(u,v)$ are Fourier transforms of $f(x,y)$ and $f^*(x,y)$, then $\hat{f}(u,v)$ equals to $\hat{f}^*(u,v)$ for a complex valued $f(x,y)$.
- ☐ If a signal can be represented by product of two functions separable by two independent variables, its Fourier transform can also be represented by product of two functions separable by the two independent variables.
- ☐ Fourier transform is a linear operator.

13.

Which is the Fourier transform pair of $\frac{\sin(\pi t)}{\pi t}$? (i.e., $x(t) \leftrightarrow X(u)$)

1 point

- ☒
- ☐ $\delta(u)$
- ☐ $u(u)$
- ☐ $e^{-j2\pi ut}$

14. Sampling is a kind of a digitization of continuous signals into discrete signals which can be used with computers.

1 point

- ☐ True
- ☒ False

15. Which is the lowest possible sampling frequency for a signal with maximum bandwidth W to be represented accurately?

1 point

- ☐ $0W$
 - ☐ W
 - ☒ $2W$
 - ☐ $0.5W$
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