1 Exercise one: 1D data

1.1 Chirped pulse

Plot real and imaginary parts of a chirped pulse $s(t)$ with form:

$$s(t) = \begin{cases} \exp\left(jt\left[\omega_0 + \pi\alpha_c t\right]\right) & |t| < \frac{\tau_p}{2} \\ 0 & \text{else} \end{cases} \tag{1}$$

where $\omega_0 = 2\pi f_0$ is the central angular frequency, τ_p is the pulse length and α_c the frequency change rate [Hz/s]. Check out its bandwidth, it should be $f_{bw} = \alpha_c \tau_p \ \ [Hz]$.
- What are reasonable values for the variables?
- What is happening if you change the value of α_c?
Hint: Some relations help you to get nice-looking pulses: $\tau_p = N/f_0$ $(N = 10...50)$, $f_{bw} = k \cdot f_0$ $(k = 0...1)$

1.2 Pulse compression by matched filtering

1.2.1 ...in time domain

Compress the pulse in time domain by a convolution with a filter function $h(t)$, build with the nominal chirp parameters. In this case, you simply calculate the autocorrelation function of the generated pulse:

$$g_1(\tau) = \int_{-\infty}^{+\infty} s^*(t)s(t+\tau)\mathrm{d}t = \int s(t')h(\tau - t')\mathrm{d}t' = (s * h)(\tau) \tag{2}$$

Where $t' = t + \tau$ and $h(t) = s^*(-t)$.
- Plot the filter $h(t)$ and the result of the filtering.
- What happen if the α_c used in $h(t)$ is different from the real one (i.e. the one used to generate the chirp $s(t)$)?

Literature: *Chapter 1-2.1.1 in the "Scientific SAR User's Guide" by Coert Olmsted.* IDL functions which may help: *conj, reverse, convol, findgen, complex, complexarr, fft, abs, plot, window, !P.multi, oplot, legend.pro, greek*

1.2.2 ...in frequency-space

Since the convolution is not computationally efficient, it is preferable to perform the match filter in the frequency domain:

$$g_1(\tau) = \mathcal{F}^{-1}\Big\{\mathcal{F}\left((s * h)(\tau)\right)\Big\} = \mathcal{F}^{-1}\Big\{S(f)) \cdot H(f)\Big\} \tag{3}$$

Literature: *Chapter 2.1.2 in the "Scientific SAR User's Guide" by Coert Olmsted.*

Plot both the transformed signals (S and H) and compare them? Do they overlap on the frequency domain? What happen if you shift in frequency one of the two transformed signals? And Why?

1.3 Hamming Window

The hamming window is used to reduce side lobes.

1.3.1 ...in time domain

Use a hamming window ($\alpha = 0.54$) to smooth the sharp edges of the pulse in time domain and comment on the compressed pulse

$$w(t) = (\alpha - 1) \cdot \cos(\frac{2\pi t}{t_p}) + \alpha \tag{4}$$

1.3.2 ...in frequency domain

Use a hamming window to smooth the sharp edges of the frequency spectrum and comment on the compressed pulse. The result is not exactly the same, as the approximation

$$(h * w)(\omega) \approx h(\omega) \cdot w(\omega - \omega_0) \tag{5}$$

is used. For a hamming window in frequency space use

$$w(t) = (\alpha - 1) \cdot \cos(\frac{\omega - \omega_0}{f_{\text{bw}}}). \tag{6}$$

2 Exercise two: 2D data

You will find the formulas required for the range and azimuth chirp rates in the tutorial on epsilon nought (http://epsilon.nought.de/).

2.1 2D Chirp (point target)

Use the information in the file *chirp_2d_test_constants.txt* and *chirp_2d_test.dat* to focus (or compress) a 2-D chirp. .DAT format: two longs (range and azimuth dimensions) followed by complex values (IDL functions that may help: openr, readu, close, free_lun, lonarr, complexarr, shade_surf)

2.1.1 Compress 2D pulse

As for the previous exercise. Just pay attention at building the right reference function given the nominal parameters.

Plot raw data before and after compression in time and frequency domain (what is the best way to represent them?). Plot also the reference functions. Are the spectra overlapping?

2.1.2 Compress 2D pulse with Hamming in azimuth

As for the 1D chirp. Can you notice any difference with the 2D compressed previously? Why?

2.1.3 Multi-look compression in azimuth

Literature: *Chapter 2.2 in the "Scientific SAR User's Guide" by Coert Olmsted.*

Use two split synthetic apertures (raw data in azimuth) to obtain two instances of the same compressed image, then average them. What do you need to average (the complex numbers, the intensities, the phases)? And why the variance reduces?

Do the same obtaining the two "independent" realisations considering two splits of the Fourier transform in azimuth. Does the result change? And why?

IDL-functions that you may need: *mean, max, min*

2.2 ERS simulated raw data: range and azimuth compression

Use the information in the file *ers_constants.txt* and *ers_raw_demo.dat* to focus (or compress) an ERS simulated SAR signal. Note, that the transmitted signal is a negative chirp ($Bw = -1.55404d7$).

Perform the same analysis than before. Pay particular attention on signals in both time and frequency. (tip: you might want to scale the amplitudes after Hamming to conserve power) (IDL functions that may help: tv, bytscl, congrid, hanning)
Literature: *Chapter 3.1 in the "Scientific SAR User's Guide" by Coert Olmsted.*

2.3 2D simulated data with range already compressed

Use the information in the file *rdemo040689_cmp_constants.txt* and *rdemo040689_cmp.dat* to focus (or compress) a 2-D chirp.

Same analysis than before.

Is the interpretation of this compressed image easy? Why it is so hard to understand different features in the image? What would you suggest to make the image of easier visual interpretation?

3 Exercise three: InSAR: Interferometry

3.1 Read the data

Interferometric data considers two independent acquisitions, separated by a spacial (and eventually temporal) baseline. You can use the simulated SIR-X data with nominal parameters provided (note, this are simulated data, the reality is a bit more complex!).

Read the two .dat files. They are in xdr format with header (first two long integer). Visualise the intensity (or dB power) of the images. Do they appear fairly similar or very different? Why? Then visualise the phases. Can you see anything in the separated phase images? Why?

3.2 Corregistration

First step is to make the two images overlap each other (why?). We can perform a 2-D cross-correlation to find range/azimuth shifts between Image1 and Image2 (in frequency domain and you may want to use some zero padding). Why the cross-correlation is not centered in (0,0)? Among what do you perform the correlation (complex pixel, magnitude, phase)? Why?

With the information regarding the cross-correlation maximum, shift Image2 to align with Image1.

Can you see a situation when a rigid translation of Image2 fails (it leads to large errors)? How would you solve the problem in such situations?

Let $x(t)$ and $y(t)$ be two signals. The cross-correlation function is defined as

$$R_{xy}(\tau) = \int x(t) \cdot y^*(t+\tau)dt = \mathcal{F}^{-1}\left\{X(f) \cdot Y^*(f)\right\} \tag{7}$$

3.3 Interferogram: first attempt

Obtain the interferogram considering the phase of the scalar product between the corespective pixels (i.e. images cross correlation). Plot absolute value and phase of the product. You are now looking at the fringes!

3.4 Flat Earth Removal

Why all the fringes seem to be aligned along the azimuth direction? Is this related to terrain topography? The removal can be done knowing exactly the orbital parameters, but here we can use a trick since the ”low complexity” of the data allows it. Look for the dominant frequency component of the fringes (with an FFT) and remove it from the spectrum. What do you need to transform? How do you remove the component from the second image?

3.5 Coherence

Compute the coherence as the averaged normalised inner product between the image pixels. The averaging is commonly refereed with filtering. Use a simple box-car filter (i.e. rectangular moving window). Display magnitude and phase of the coherence for different values of the moving window. How do the magnitude and phase change with different filtering?

Compute the coherence before and after the Flat Earth Removal. Is the coherence magnitude changing?

3.6 Range Filtering: Common Band Filter

Perform range filtering on the two coregistered images from Gatelli94 (preserve only overlapping bandwidth between master and slave). To see better the bands you may consider some averaging.

Is the coherence magnitude changing now? Why removing the slices of bandwidth is helping, since they will disappear anyway after the product? Is there some relation between the bands shift and the Flat Earth? Can you try to explain this visualising what happen to the fringes when the bands separate too much, and how this is related to the Flat Earth?

Display histograms of coherences BEFORE and AFTER range filtering.

4 Exercise four: Polarimetry

4.1 Read an visualise polarimetric data

Data set: Real E-SAR data quad-polarimetric in L-band (Alling, Germany 2000). The data can be read as for the previous case. Plot the intensity images for each polarimetric channel. Are the images different and why? Try to visualise the intensity after some smoothing with a boxcar filter. Why the images are different after the smoothing? To know why, take one of the images, select an area that appear homogeneous (e.g. one field) and plot the histograms of the single and averaged intensity (with different averaging windows). Are these histograms resembling some pdf that you already saw in the literature?

4.2 Polarimetric coherences

As for the interferometric case, it is interesting to evaluate the coherence (normalised cross correlation, or normalised inner product of each pixel) to understand how the polarimetric channels are correlated each other. Calculate the coherences between the co-polarisations (HH/VV), the cross-polarisations (HV/VH), the co and cross polarisations (HH/HV, VV/VH), using two different dimensions of moving window. Comment on the result. What happen when the window is 1x1? Why HV/VH is so high (or so low)? To compare the different coherences you may also plot their histogram on the same figure. Is it worth to plot the phases as well? Take the same area used before for evaluating the histograms and plot the histogram of magnitude and phase of some coherence. What happen when you average?

4.3 Polarimetric Covariance Matrix

I hope at this point it is clear that we need to average the data to extract more reliable information. A way to have a compact and physically meaningful representation is to consider the covariance matrix $[C]$ of the three polarimetric channels over a sorter moving window. $[C]$ contains the second order statistics of the polarimetric channels, in the Gaussian assumption these are necessary and sufficient to characterise our (statistical) scattering process. What properties $[C]$ has? Is it really important that it has these properties or we don't really care? Plot the diagonal and off diagonal elements of $[C]$. Do you find any relationship with what you were plotting in the two previous points? Could you find a mathematical link with the observables in the previous two points?

4.4 Pauli basis

Onece you have the Scattering matrix $[S]$ you have all the polarimetric information (that you can acquire by SAR). So you can simulate any other polarimetric channels or decide the mapping procedure to express $[S]$ in a more easily tractable vector space. A clever way to perform this mapping is using the Pauli spin basis. Calculate the components of the scattering vector in Pauli basis and estimate its covariance matrix (in the community it is preferred to call it Coherency matrix $[T]$). Plot again diagonal and off diagonal terms. Visualise the first three diagonal terms in an RGB image. Can you try to assign some physical interpretation to the colours? Calculate the Span of $[S]$ (Frobenious Norm $\|[S]\|^2$), the squared norm of the scattering vector, the Trace of $[C]$ ($Tr([C])$) and the Trace of $[T]$ ($Tr([T])$). What is their relationship and why?

4.5 Cloude-Pottier decomposition

Why do we "decompose" polarimetric data? What are the basis of the Cloude-Pottier decomposition? Calculate the eigenvalues and eigenvectors of [T]. Plot the eigenvalues, singularly and in an RGB. Calculate the entropy (H), alpha angle (alpha), dominant alpha angle and anisotropy. Visualize entropy-alpha in a 2D-histogram plot and compare with literature. Why there is a boundary region? Make the noise remouval with the 4^{th} eigenvalue and recalculate entropy and anisotropy. Did they change?

4.6 Yamaguchi decomposition

Calculate the model-based Yamaguchi decomposition on the [T]-matrix using [6], calculate and visualise the powers. Is there any problem with this decomposition?

References

[1] Dissertation: Chapter 5 of Irena's Dissertation.

[2] Book: Polarimetric RADAR imaging: From basics to applications (Lee, Pottier) - chapter 2-3 and chapter 6-7

[3] Paper: A review of target decomposition theorems in RADAR polarimetry (Cloude & Pottier)

[4] Paper: An entropy based classification scheme for land applications of polarimetric SAR (Cloude & Pottier)

[5] Paper: Inversion of surface parameters from polarimetric SAR (Hajnsek & Pottier & Cloude)

[6] Paper: A Four-Component Decomposition of PolSAR images based on the coherency matrix (Yamaguchi & Yajima & Yamada)

[7] Paper: Removal of Additive Noise in Polarimetric Eigenvalue Processing (Hajnsek, Papathanassiou, Cloude)

5 Exercise five: PolInSAR

In this last exercise, interferometric and polarimetric data are combined. The dataset is obtained from a POLSARsim simulation (a forest stand over bare ground).

5.1 Read and prepare the data for PolInSAR analysis

Read and plot the data: elements of Coherency matrix for each of the acquisitions. Perform the Flat Earth Removal with the provided phase ramp. Compare the interferometric coherences for different polarimetric channels and the two baselines. Which is the polarimetric channel that seems to have lower coherence (higher de-correlation)? Compare the interferometric phase for different polarimetric channels. Where do you see the biggest differences between ground and forest? Select an area on the forest and plot the interferometric coherences of different polarimetric channels on the complex plane (i.e. the polar plot).

5.2 Forest height retrieval

The interferometric phase difference between polarimetric channels keeps information regarding the extent of the volume (i.e. forest). We want to use this information to retrieve the height of the forest.

Firstly, try to convert the phase difference in meters with the k_z and obtain a first estimate of the height. Which are the best polarimetric channels to use in this situation and why?

Then apply the 3 stage inversion of Cloude and Papathanassiou [1]. Plot coherences on complex plane, fit a line (use IDL LINFIT routine). TIP: to avoid infinite slopes one can also try shifting the points 90 degs (when needed). Find intersection(s) between best-fit line and the unit circle and project the points onto the line. TIP: use the equation of a line and a circle to find intersections. Determine the ground phase (e.g. the one further from XX coherence). Plot the ground phase of all pixels in a histogram. Create and plot lookup table (LUT) for coherence of vegetation: assume XX coherence (projected to best-fit line) has no ground contribution. Perform integral from eqn. 8 (Cloude & Papathanassiou 2003). Compute 2D LUT (look-up table) with different values of extinction (eg. vary from 0 to 0.2 m^{-1}) and height (eg. vary from 0 to 30 m or up to interferometry sensitivity height h_π). Estimate the vegetation height and extinction from the closest point on the Look-up-table. Plot vegetation height histogram and 3D-plot (shade surface or contour).
The input parameters are:
topographic phase = 0.0
extinction = 0.1 dB/m
height = 10 m

H	= 3e3	(m)	sensor height
lambda	= 0.24	(m)	wavelength, L-band
B_{12}	= -10	(m)	horizontal baseline between image 1 & 2
B_{13}	= -20	(m)	horizontal baseline between Image 1 & 3
W	= 100e6]	(Hz)	bandwidth in range
grng_res	= 0.5	(m)	ground range pixel spacing
θ_0	= 45	$(^\circ)$	angle of incidence to image centre, assume constant for small area
c	= 3e8	(m/s)	speed of light
R_m	$= H/\cos\theta_0$	(m)	broadside range
α	= 0	$(^\circ)$	local slope

Table 1: Parameters for the calculation and of sim_data_rvog

References

[1] Cloude & Papathanassiou ”Three-stage inversion process for polarimetric SAR interferometry”, 2003.

[2] Cloude & Papathanassiou ”Polarimetric SAR Interferometry”,1998.

[3] Papathanassiou & Cloude ”Single-Baseline Polarimetric SAR Interferometry”, 2001.