Clique Cover

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1 General Explanation:

Take the graph G(V, E) as the following:

- 1. Each vertex in the graph is assigned a unique color from the available set of colors. (number of colors = n)
- 2. The subset of vertices corresponding to each color, are denoted as $W_1, W_2, ..., W_n$. These subsets partition the vertex set v based on the assigned colors.
- 3. The edge set is then restricted to edges that connect vertices within the same color subset. The restricted edge set is denoted as $EW_1, EW_2, ..., EW_n$.

2 The Main Objective:

To determine whether each subset W_i , along with its corresponding edge set EW_i , forms a clique¹.

3 Hamiltonian:

$$H = A \sum_{v} (1 - \sum_{i=1}^{n} x_{v,i})^{2} + B \sum_{i=1}^{n} \left[\frac{1}{2} (-1 + \sum_{v} x_{v,i}) (\sum_{i=1}^{n} x_{v,i}) - \sum_{uv} x_{u,i} x_{v,i} \right]$$
(1)

 $\sum_{v} (1 - \sum_{i=1}^{n} x_{v,i})^2$: enforcing the constraint that each vertex has exactly one color, if not the Hamiltonian will give an energy penalty. $\frac{1}{2}(-1 + \sum_{v} x_{v,i})(\sum_{i} x_{v,i})$: highest possible number of edges that could exist with color i.

 $\sum_{uv} x_{u,i} x_{v,i}$: checks if this number of edges in fact exist.

¹Every per of vertices within the subset is connected by an edge.