Clique Cover

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1 General Explanation:

Take the graph $G(V, E)$ as the following:

1. Each vertex in the graph is assigned a unique color from the available set of colors. (number of colors $= n$)

2. The subset of vertices corresponding to each color, are denoted as $W_1, W_2, ..., W_n$. These subsets partition the vertex set v based on the assigned colors.

3. The edge set is then restricted to edges that connect vertices within the same color subset. The restricted edge set is denoted as $EW_1, EW_2, ..., EW_n$.

2 The Main Objective:

To determine whether each subset W_i, along with its corresponding edge set EW_i, forms a clique[1].

3 Hamiltonian:

$$H = A \sum_v (1 - \sum_{i=1}^n x_{v,i})^2 + B \sum_{i=1}^n \left[\frac{1}{2}(-1 + \sum_v x_{v,i})(\sum x_{v,i}) - \sum_{uv} x_{u,i} x_{v,i} \right] \tag{1}$$

$\sum_v (1 - \sum_{i=1}^n x_{v,i})^2$: enforcing the constraint that each vertex has exactly one color, if not the Hamiltonian will give an energy penalty.

$\frac{1}{2}(-1 + \sum_v x_{v,i})(\sum x_{v,i})$: highest possible number of edges that could exist with color i.

$\sum_{uv} x_{u,i} x_{v,i}$: checks if this number of edges in fact exist.

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[1]Every per of vertices within the subset is connected by an edge.