

The Black-Scholes model is a mathematical model used for pricing European-style options. It provides a theoretical estimate of the price of options based on several key variables. The model assumes that the stock price follows a geometric Brownian motion with constant volatility and that the option can only be exercised at expiration.

Black-Scholes Formula

The Black-Scholes formula for a European call and put option is:

- Call Option Price:

$$C = S_0 \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2)$$

- Put Option Price:

$$P = K \cdot e^{-rT} \cdot N(-d_2) - S_0 \cdot N(-d_1)$$

where:

- S_0 = Current stock price
- K = Strike price
- T = Time to expiration (in years)
- r = Risk-free interest rate (annualized)
- σ = Volatility of the underlying asset (annualized)
- $N(\cdot)$ = Cumulative distribution function of the standard normal distribution

- d_1 and d_2 are calculated as:

$$d_1 = \frac{\log(S_0/K) + (r + \sigma^2/2) \cdot T}{\sigma \cdot \sqrt{T}}$$

$$d_2 = d_1 - \sigma \cdot \sqrt{T}$$

Assumptions of the Black-Scholes Model

1. **Constant Volatility:** The volatility of the underlying asset is constant over the life of the option.
2. **Constant Risk-Free Rate:** The risk-free interest rate is constant.
3. **European Option:** The option can only be exercised at expiration, not before.
4. **Geometric Brownian Motion:** The stock price follows a log-normal distribution, and returns are normally distributed.
5. **No Dividends:** The model assumes that the underlying asset does not pay dividends during the option's life.
6. **Efficient Markets:** No transaction costs or taxes, and markets are frictionless.