The Heston model is a popular stochastic volatility model used in financial mathematics to describe the evolution of the volatility of an asset over time. Unlike the Black-Scholes model, which assumes constant volatility, the Heston model allows for volatility to change dynamically and is particularly useful for capturing market phenomena such as volatility clustering and the leverage effect.

Heston Model Overview

1. Model Formulation

The Heston model consists of two stochastic differential equations (SDEs):

1. **Stock Price Dynamics**:

$$dS_t = \mu S_t \, dt + \sqrt{V_t} S_t \, dW_t^S$$

   - S_t: Stock price at time t
   - μ: Drift rate of the stock price (expected return)
   - V_t: Volatility of the stock price at time t
   - W_t^S: Brownian motion driving the stock price

2. **Volatility Dynamics**:

$$dV_t = \kappa(\theta - V_t) \, dt + \sigma_V \sqrt{V_t} \, dW_t^V$$

   - κ: Rate at which the volatility reverts to its long-term mean
   - θ: Long-term mean of the volatility
   - σ_V: Volatility of the volatility (vol-of-vol)
   - W_t^V: Brownian motion driving the volatility
   - **Correlation**: dW_t^S and dW_t^V are correlated with correlation coefficient ρ, i.e., $\text{Cov}(dW_t^S, dW_t^V) = \rho \, dt$

2. Key Parameters

- **Drift** (μ): Expected return of the stock price.
- **Long-term Mean Volatility** (θ): The mean level that the volatility reverts to.
- **Volatility of Volatility** (σ_V): The standard deviation of the volatility process.
- **Rate of Mean Reversion** (κ): The speed at which volatility reverts to its mean.
- **Correlation** (ρ): Correlation between the Brownian motions driving the stock price and volatility.