#### **Boston Housing - Homework**

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#### **Importing Packages**

```
library(readr)
library(ggplot2)
library(corrplot)

## corrplot 0.84 loaded

library(mlbench)
library(reshape2)
library(caret)

## Loading required package: lattice

library(caTools)
library(sjPlot)
library(sjmisc)
library(car)

## Loading required package: carData
```

#### Read data from SCVDataset

```
#Reading CSV file
boston h=read.csv(file="C:/Users/azizi/Documents/CSVDatasets/Boston Housing.csv", sep = "
,")
#Attribute Information
str(boston h)
## 'data.frame':
                   506 obs. of 14 variables:
## $ CRIM : num 0.00632 0.02731 0.02729 0.03237 0.06905 ...
            : num 18 0 0 0 0 0 12.5 12.5 12.5 12.5 ...
## $ ZN
## $ INDUS : num 2.31 7.07 7.07 2.18 2.18 2.18 7.87 7.87 7.87 7.87 ...
## $ CHAS : int 0000000000...
## $ NOX : num 0.538 0.469 0.469 0.458 0.458 0.458 0.524 0.524 0.524 0.524 ...
           : num 6.58 6.42 7.18 7 7.15 ...
## $ RM
  $ AGE : num 65.2 78.9 61.1 45.8 54.2 58.7 66.6 96.1 100 85.9 ... $ DIS : num 4.09 4.97 4.97 6.06 6.06 ...
##
## $ DIS
         : int 1223335555...
##
  $ RAD
## $ TAX : int 296 242 242 222 222 311 311 311 311 ...
## $ PTRATIO: num 15.3 17.8 17.8 18.7 18.7 18.7 15.2 15.2 15.2 15.2 ...
## $ B
        : num 397 397 393 395 397 ...
## $ LSTAT : num 4.98 9.14 4.03 2.94 5.33 ...
## $ MEDV : num 24 21.6 34.7 33.4 36.2 28.7 22.9 27.1 16.5 18.9 ...
```

#### **Descriptive statistics**

```
#summarize dataframe
summary(boston_h)
##
         CRIM
                              ZN
                                              INDUS
                                                                CHAS
##
    Min.
           : 0.00632
                        Min.
                                  0.00
                                          Min.
                                                 : 0.46
                                                           Min.
                                                                   :0.00000
    1st Qu.: 0.08204
                        1st Ou.:
                                  0.00
                                          1st Qu.: 5.19
                                                           1st Ou.:0.00000
    Median : 0.25651
                        Median :
                                  0.00
                                          Median: 9.69
                                                           Median :0.00000
##
##
    Mean
           : 3.61352
                        Mean
                               : 11.36
                                          Mean
                                                 :11.14
                                                           Mean
                                                                  :0.06917
                                                           3rd Qu.:0.00000
    3rd Qu.: 3.67708
                        3rd Qu.: 12.50
                                          3rd Qu.:18.10
##
##
    Max.
           :88.97620
                        Max.
                               :100.00
                                          Max.
                                                  :27.74
                                                           Max.
                                                                  :1.00000
##
         NOX
                            RM
                                            AGE
                                                              DIS
    Min.
           :0.3850
                             :3.561
                                       Min.
                                              : 2.90
                                                                : 1.130
##
                      Min.
                                                         Min.
##
    1st Qu.:0.4490
                      1st Qu.:5.886
                                       1st Qu.: 45.02
                                                         1st Qu.: 2.100
    Median :0.5380
                                       Median : 77.50
##
                      Median :6.208
                                                         Median : 3.207
##
    Mean
                      Mean
                             :6.285
                                       Mean
                                             : 68.57
           :0.5547
                                                         Mean
                                                                : 3.795
    3rd Qu.:0.6240
                      3rd Qu.:6.623
                                       3rd Qu.: 94.08
                                                         3rd Qu.: 5.188
##
##
    Max.
           :0.8710
                      Max.
                             :8.780
                                       Max.
                                              :100.00
                                                         Max.
                                                                :12.127
##
         RAD
                           TAX
                                          PTRATIO
                                                              В
##
    Min.
           : 1.000
                      Min.
                             :187.0
                                              :12.60
                                                               : 0.32
                                       Min.
                                                        Min.
    1st Qu.: 4.000
                      1st Qu.:279.0
                                       1st Qu.:17.40
                                                        1st Qu.:375.38
##
    Median : 5.000
                      Median :330.0
                                       Median :19.05
                                                        Median :391.44
##
##
    Mean
           : 9.549
                      Mean
                             :408.2
                                       Mean
                                              :18.46
                                                        Mean
                                                               :356.67
    3rd Ou.:24.000
                      3rd Ou.:666.0
                                       3rd Ou.:20.20
                                                        3rd Ou.:396.23
##
##
    Max.
           :24.000
                      Max.
                             :711.0
                                       Max.
                                              :22.00
                                                        Max.
                                                               :396.90
        LSTAT
                          MEDV
##
           : 1.73
                            : 5.00
##
   Min.
                     Min.
##
    1st Qu.: 6.95
                     1st Qu.:17.02
    Median :11.36
##
                     Median :21.20
           :12.65
                          :22.53
##
   Mean
                     Mean
##
    3rd Qu.:16.95
                     3rd Qu.:25.00
    Max.
         :37.97
                     Max. :50.00
```

#### **Data Exploration**

#### 1.a. Check for missing values in the dataset.

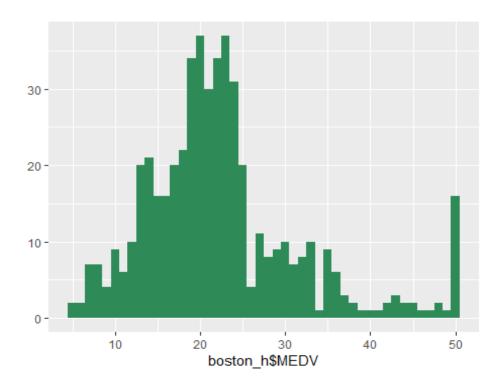
**Answer**: No missing values are found

```
#Searching NA in the dataframe
which(is.na(boston_h))
## integer(0)
```

#### 1.b. Plot the distribution of MEDV. What do you observe?

Answer: It can be seen that data is not normally distrubited around the mean of MEDV

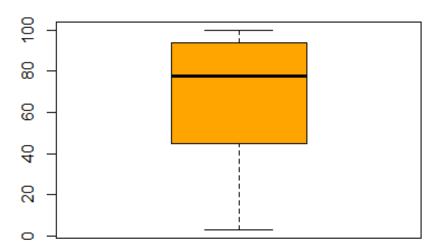
```
#Plot Histogram
qplot(boston_h$MEDV, geom="histogram", fill=I("seagreen"), binwidth=1)
```



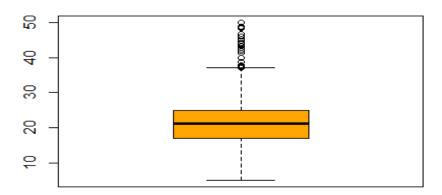
## 1.c. Generate box-plots of the AGE (proportion of owner-occupied units built prior to 1940) and MEDV (median home value) attributes and identify the cutoff values for outliers.

#Generate Boxplot from AGE data
boxplot(boston\_h\$AGE,col=c("orange"),main="Boxplot of owner-occupied units built prior to
1940")

#### Boxplot of owner-occupied units built prior to 194



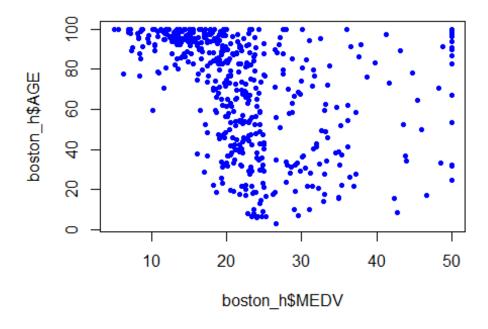
#### **Median Home Value**



#### 1.d.Generate a scatterplot of MEDV against AGE; comment on how inclusion of the outliers would affect a predictive model of median home value as a function of AGE.

**Answer**: IT is affecting definetely not postively as there can be observed more noise in the data.

```
#Generate Scatter for MEDV vs AGE
plot(boston_h$MEDV, boston_h$AGE, type="p", pch=20, col="blue", lty=3)
```



2. Try to fit an MLR to this dataset, with MEDV as the dependent variable. MEDV has a somewhat long tail and is not so Gaussian-like, so we will take a log transform, (use LMEDV = log(MEDV)), and then predict LMDEV instead. (You should convince yourself that this is a better idea by looking at the histograms and quantile plots to assess normality; however, no need to submit such plots). Keep the first 356 records as a training set (call it Bostrain) which you will use to fit the model; the remaining 150 will be used as a test set (Bostest). Use only LSTAT, RM, TAX, AGE and ZN as independent (predictor) variables and LMEDV as dependent (target) variable as follows when constructing a linear regression model:

```
#Convert MEDV data to log and split data into two parts
boston_h$LMEDV = log(boston_h$MEDV)
bos train<-boston h[1:356,]
bos_test<-boston_h[357:506,]</pre>
#Run regression with the given model
fit1<-lm(LMEDV~LSTAT+RM+TAX+AGE+ZN, data=bos_train)</pre>
summary(fit1)
##
## Call:
## lm(formula = LMEDV ~ LSTAT + RM + TAX + AGE + ZN, data = bos_train)
##
## Residuals:
        Min
##
                  1Q
                      Median
                                    3Q
                                            Max
## -0.37400 -0.08262 -0.01016 0.07685 0.45421
##
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.396e+00 1.119e-01 12.480 < 2e-16 ***
              -1.003e-02 1.990e-03 -5.041 7.46e-07 ***
## LSTAT
              3.251e-01 1.514e-02 21.478 < 2e-16 ***
## RM
## TAX
              -5.077e-04 1.064e-04 -4.772 2.69e-06 ***
## AGE
              -7.358e-04 3.522e-04 -2.089
                                              0.0374 *
## ZN
              -3.036e-05 3.187e-04 -0.095
                                              0.9242
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1291 on 350 degrees of freedom
## Multiple R-squared: 0.8255, Adjusted R-squared: 0.823
## F-statistic: 331.1 on 5 and 350 DF, p-value: < 2.2e-16
anova(fit1)
## Analysis of Variance Table
##
## Response: LMEDV
##
              Df Sum Sq Mean Sq
                                   F value
                                              Pr(>F)
              1 18.1100 18.1100 1086.6838 < 2.2e-16 ***
## LSTAT
## RM
              1 8.9216 8.9216 535.3371 < 2.2e-16 ***
## TAX
              1 0.4717 0.4717
                                  28.3030 1.855e-07 ***
## AGE
              1 0.0887 0.0887
                                   5.3198
                                            0.02167 *
                                   0.0091
                                            0.92416
## ZN
              1 0.0002 0.0002
## Residuals 350 5.8329 0.0167
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

## 3. Do any variables have to be dropped because of multicollinearity?(Use VIF criteria to check for multicollinearity)

**Answer**: Favorably, all the values of VIF are way below critical point which implies that variables are independent and thus are not multicollinear.

```
# viffunction to check wether multicollinearity exists amoung variables
vif(fit1)

## LSTAT RM TAX AGE ZN
## 2.907403 2.218182 1.110602 2.146126 1.506499
```

## 4.Report the coefficients obtained by your model. Would you drop any of the variables used in your model (based on the t-scores or p-values)?

**Answer**: From estimates it is clear that, for every unit increase of LSTAT, TAX, AGE, ZN, the predicted value of MEDV would be around -0.0100329, -0.0005077, -0.0007358, -0.0000304 units lower respectively. However, for every unit increase in RM, the MEDV is predicted to be 3.25 unit higher. Let's test null hypothesis that the parameter is equal to zero. In our case coefficients having p values less than 0.05 or less LSTAT, TAX, AGE, ZN, RM are significant. This means that the slope and the t value are in the region of rejection for the null hypothesis. At the 0.05 level of signi???cance, there is evidence that the relationship between prior mentioned independent variables and MEDV indeed exists. While ZN is statistically insignificant as its p-values is higher than 0.05 therefore it is guilty one. Therefore I would drop ZN.

# 5.Rerun your regression model after removing variables (if any) based on your analysis in the previous question. What is the value of R2 ? What does it signify? What is the overall F-statistic and the corresponding p-value of this final model? What does it signify?

**Answer**: if we look at R square for goodness of fit, it shows around 83%. So, overall measure of strength association is relatively strong. This shows that the fit of the regression line to the points is fairly good. An R square of 0.83 means that 0.83 or 83% of the variation in the values of Y can be explained on the basis of the regression line. The explanation is a statistical one, meaning that 83% of the differences in MEDV's rate in the different provinces are explained statistically by differences in selected predictors

## 6. What is the overall F-statistic and the corresponding p-value of this final model? What does it signify?

**Answer**: The important step in a multiple regression analysis is to compute F-statistics and investigate associated p-value. The p-value for F-test is very small, that is smaller than 0.05 and therefore significant. It implies that our group of independent variables reliably predict the dependent variable. In other words, at least one of the predictors is related to the response, then it is natural to wonder which the guilty ones are (for that we looked at T-test above).R

```
#Regression analysis after removing variables
fit2= lm(LMEDV~LSTAT+RM+TAX+AGE, data=bos train)
summary(fit2)
##
## Call:
## lm(formula = LMEDV ~ LSTAT + RM + TAX + AGE, data = bos train)
##
## Residuals:
      Min
               10 Median
                               30
                                      Max
## -0.37337 -0.08208 -0.01022 0.07642 0.45535
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.3971629 0.1110078 12.586 < 2e-16 ***
## LSTAT -0.0100453 0.0019833 -5.065 6.61e-07 ***
## RM
            ## TAX
## AGE
           -0.0007203 0.0003119 -2.310 0.0215 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1289 on 351 degrees of freedom
## Multiple R-squared: 0.8255, Adjusted R-squared: 0.8235
## F-statistic: 415.1 on 4 and 351 DF, p-value: < 2.2e-16
```

### 7. Report the MSE obtained on Bostrain. How much does this increase when you score your model (i.e., predict) on Bostest?

**Answer**: As it is indicated below, There is a slight increase in MSE for test data(bos\_test) compared to train data(bos\_train)

```
#Find MSE for the bos train
MSE_bostrain=mean(fit1$residuals^2)
MSE_bostrain
## [1] 0.01638446

# model to predict MSE for bostest
test_predict = predict(fit1, bos_test)
MSE_bostest= mean((bos_test$LMEDV-test_predict)^2)
MSE_bostest
## [1] 0.1781114
```

(Bonus 1 point). Use the stepwise regression considering to reach your final model (LMEDV as dependent variable and all but MEDV as independent variables). Try different model section criteria (i.e., AIC, Cp, BIC, adj R^2) and see if you can come up with the same model even with the different criteria. Determine the best model if you get different models with different criteria? We will consider a model that gives the highest accuracy (in terms of MSE) in the test set as the best model.

**Answer**: The best model is as follows:  $fit4=lm(formula = LMEDV \sim ZN + INDUS + CHAS + NOX + RM + AGE + DIS + RAD + TAX + PTRATIO + B + LSTAT, data = bos_train), however based on the MSE statistics the first model(fit3) is preferred.$ 

```
#Determine the best model
step(lm(LMEDV~.-MEDV,data=bos_train))
## Start: AIC=-1578.21
## LMEDV ~ (CRIM + ZN + INDUS + CHAS + NOX + RM + AGE + DIS + RAD +
      TAX + PTRATIO + B + LSTAT + MEDV) - MEDV
##
##
##
            Df Sum of Sq
                            RSS
## - CRIM
             1
                  0.0129 3.9212 -1579.0
## - INDUS
                  0.0191 3.9274 -1578.5
## <none>
                         3.9083 -1578.2
## - CHAS
             1
                  0.0284 3.9367 -1577.6
## - ZN
                  0.0302 3.9385 -1577.5
             1
## - RAD
             1
                  0.0618 3.9701 -1574.6
## - NOX
            1
                  0.1036 4.0119 -1570.9
             1
## - B
                  0.1719 4.0802 -1564.9
## - AGE
             1 0.2807 4.1890 -1555.5
## - TAX
             1
                  0.3537 4.2620 -1549.4
## - LSTAT
             1 0.3994 4.3077 -1545.6
             1 0.5407 4.4490 -1534.1
## - DIS
## - PTRATIO
             1
                  0.7464 4.6547 -1518.0
## - RM
             1
                  4.9822 8.8906 -1287.6
##
## Step: AIC=-1579.04
## LMEDV ~ ZN + INDUS + CHAS + NOX + RM + AGE + DIS + RAD + TAX +
      PTRATIO + B + LSTAT
##
##
            Df Sum of Sq
##
                            RSS
                                    AIC
                         3.9212 -1579.0
## <none>
## - INDUS
                  0.0224 3.9436 -1579.0
                  0.0298 3.9510 -1578.3
## - ZN
             1
## - CHAS
             1
                  0.0300 3.9512 -1578.3
## - RAD
             1
                  0.0647 3.9859 -1575.2
## - NOX
             1
                  0.0929 4.0141 -1572.7
## - B
             1
                  0.1592 4.0804 -1566.9
## - AGE
             1
                  0.2814 4.2026 -1556.4
## - TAX
                  0.3435 4.2647 -1551.1
             1
## - LSTAT
             1
                  0.3929 4.3141 -1547.0
## - DIS
             1
                  0.5296 4.4508 -1535.9
## - PTRATIO 1
                  0.7902 4.7114 -1515.7
## - RM
             1 4.9839 8.9051 -1289.0
```

```
##
## Call:
## lm(formula = LMEDV ~ ZN + INDUS + CHAS + NOX + RM + AGE + DIS +
       RAD + TAX + PTRATIO + B + LSTAT, data = bos_train)
##
## Coefficients:
## (Intercept)
                         ΖN
                                   INDUS
                                                 CHAS
                                                               NOX
##
     2.2524769
                  0.0005252
                               0.0020575
                                            0.0359232
                                                        -0.3095658
##
            RM
                        AGE
                                     DIS
                                                  RAD
                                                               TAX
##
     0.2775077
               -0.0016655
                              -0.0333158
                                            0.0091700
                                                        -0.0005494
##
       PTRATIO
                          В
                                   LSTAT
   -0.0266376
               0.0005924
##
                              -0.0097952
#Using the 1st model calculate MSE
fit3=lm(LMEDV ~ (CRIM + ZN + INDUS + CHAS + NOX + RM + AGE + DIS + RAD +TAX + PTRATIO + B
+ LSTAT + MEDV) - MEDV, data=bos_train)
fit3_predict = predict(fit3, bos_train)
mse_fit3 = mean(bos_train$LMEDV-fit3_predict)^2
#Using the 2st model calculate MSE
fit4=lm(LMEDV ~ ZN + INDUS + CHAS + NOX + RM + AGE + DIS + RAD + TAX + PTRATIO + B + LSTA
T, data= bos_train)
fit4_predict = predict(fit4, bos_train)
mse fit4 = mean(bos train$LMEDV-fit4 predict)^2
#Print out all models MSE
options(scipen = 100, digits=4)
mse fit3
## [1] 0.00000000000000000000000000000003758
mse fit4
## [1] 0.00000000000000000000000000000007779
```