A crash course in binary trees

We'll revisit in MSDS689 but we need binary trees for projects now

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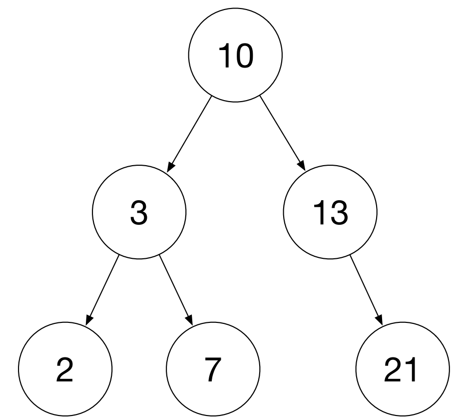
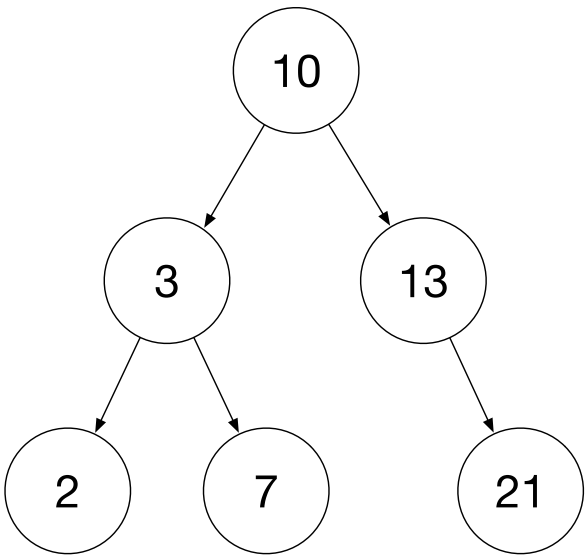
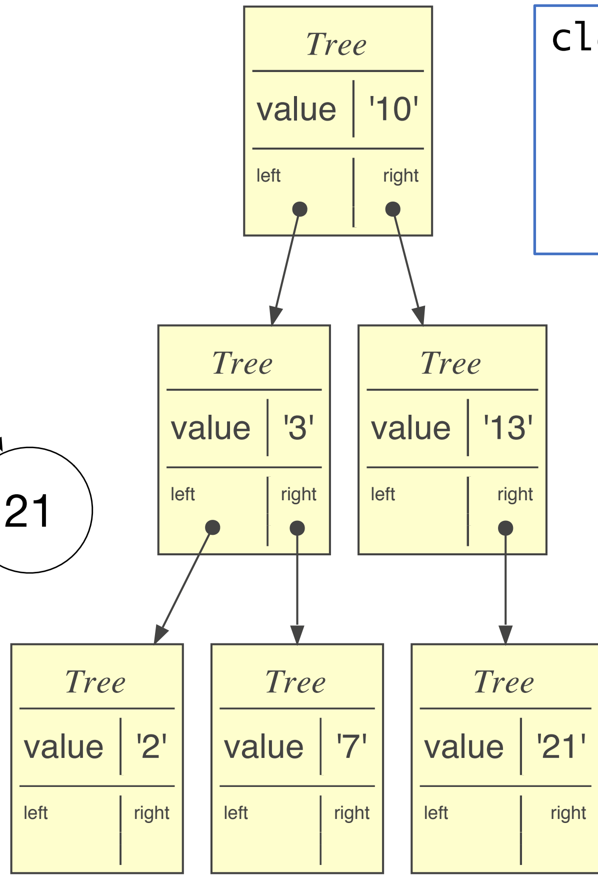
Binary tree abstract data structure



- A directed-graph with internal nodes and leaves
- No cycles and each node has at most one parent
- Each node has at most 2 child nodes
- For n nodes, there are n-1 edges
- A *full* binary tree: all internal nodes have 2 children
- Height of full tree with n internal nodes is about log2(n)
- Height defined as number of edges along path root->leaf
- Level 0 is root, level 1, …
- Note: binary tree doesn’t imply *binary search tree*

Concrete binary tree using pointers






```python
class TreeNode:
  def __init__(self, value, left, right):
    self.value = value
    self.left = left
    self.right = right
```

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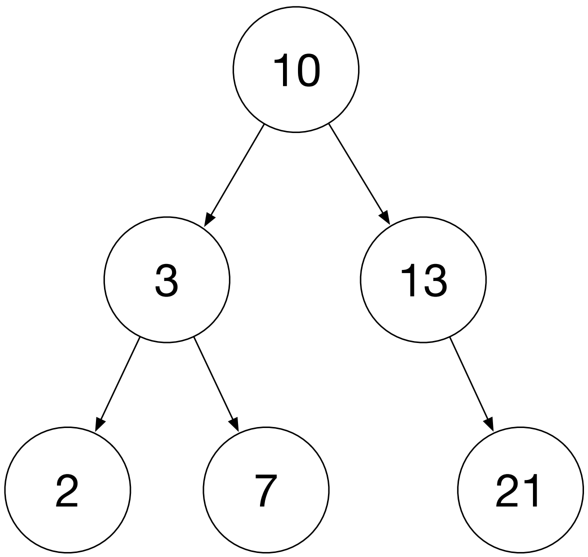
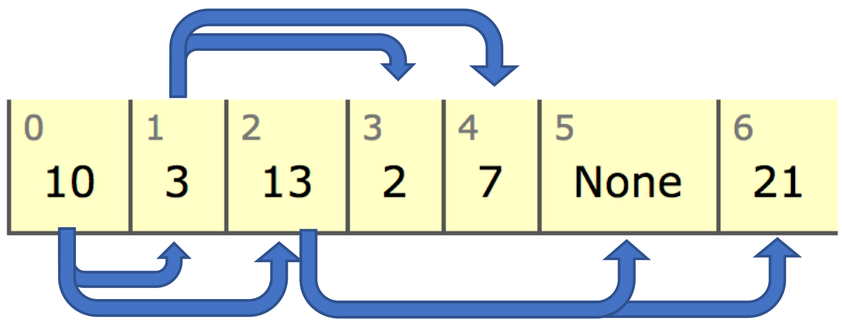
Concrete binary tree using contiguous array



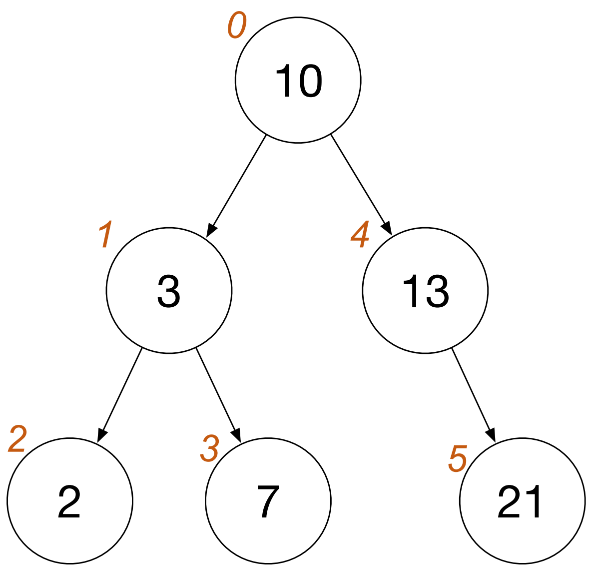
We don't need child pointers!

left child is 2i+1
right child is 2i+2



Indexes as pointers

- sklearn doesn't use nodes with pointers
- Uses node ID's and parallel arrays like *left*, *right*, *value*



left[0] = 1	right[0] = 4	value[0] = 10
left[1] = 2	right[1] = 3	value[1] = 3
left[2] = -1	right[2] = -1	value[2] = 2
left[3] = -1	right[3] = -1	value[3] = 7
left[4] = -1	right[4] = 5	value[4] = 13
left[5] = -1	right[5] = -1	value[5] = 21

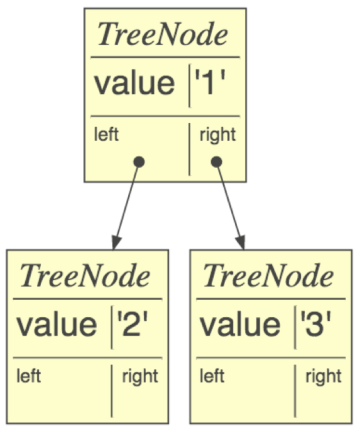
https://scikit-learn.org/stable/auto_examples/tree/plot_unveil_tree_structure.html

Building binary trees

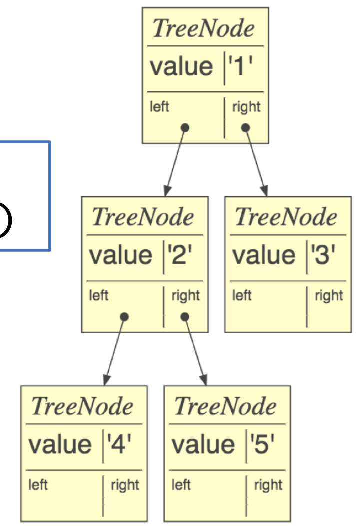
```
class TreeNode:
  def __init__(self, value, left, right):
    self.value = value
    self.left = left
    self.right = right
```

- Manual construction is a simple matter of creating nodes and setting left/right child pointers
- **Exercise**: work through notebook given below

```
root = TreeNode(1)
root.left = TreeNode(2)
root.right = TreeNode(3)
```




```
root.left.left = TreeNode(4)
root.left.right = TreeNode(5)
```



https://github.com/parrt/msds621/blob/master/notebooks/trees/basics.ipynb

Recursion detour

Math recurrence relations => recursion

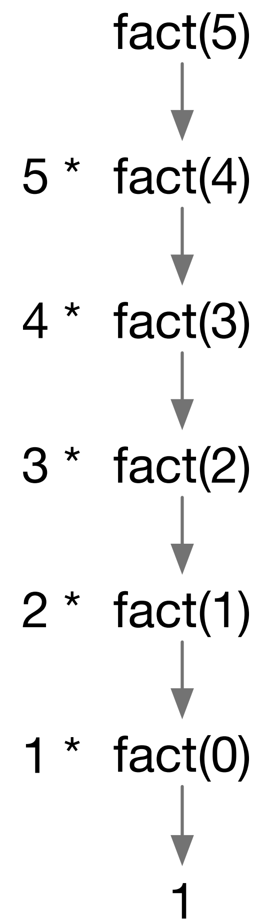
- Factorial definition:
  - Let 0! = 1
  - Define n! = n * (n-1)! for n>=1
- Recurrent math function calls become recursive function call in Python
- Non-recursive version is harder to understand and less natural

```
def fact(n):
    if n==0: return 1
    return n * fact(n-1)
```

```
def factloop(n):
    r = 1
    for i in range(1,n+1):
        r *= i
    return r
```

Recursion traces out a call graph

- Think of each call to function as node in chain or graph of calls
- Result of function call is a piece of the result and each call combines subresult(s) to create more complete answer and passes it back



Formula for writing recursive functions

def **f**(input):
1. check termination condition
2. process the active input region / current node, etc…
3. invoke f on subregion(s)
4. combine and return results

Steps 2 and 4 are optional

```
def fact(n):
    if n==0: return 1
    return n * fact(n-1)
```

Terminology: *currently-active region* or *element* is what function is currently trying to process. Here, that is argument n (the “region” is the numbers 0..n)

Don't let the recursion scare you

- Just pretend that you are calling a different function
- Or, pretend that you are calling the same function except that it is known to be correct
- We call this the *recursive leap of faith*
- Follow the “Formula for recursive functions” and all will be well!

Recursive tree procedures

An analogy for recursive tree walking

- Imagine searching for an item in a maze of rooms connected by doors (no cycles)
- Each room has at most 2 doors, some have none
- Search procedure that works in ANY room:

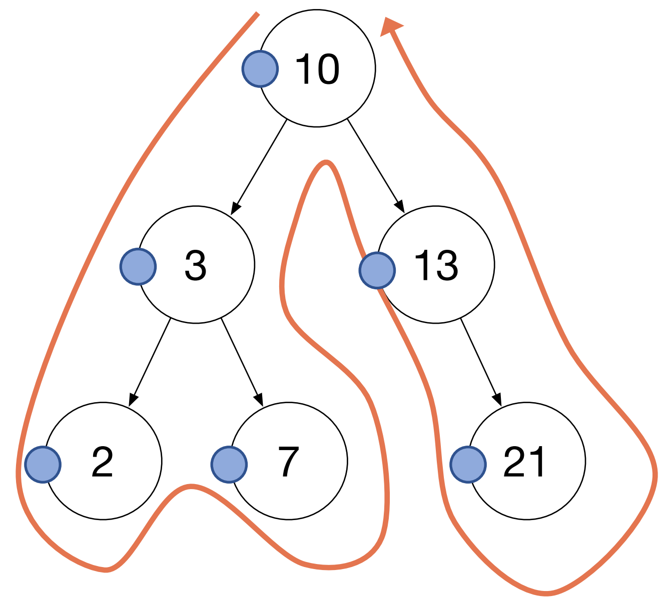
```
def visit(room):
    if item in room: print("rejoice!")
    if room.left exists: visit(room.left)
    if room.right exists: visit(room.right)
```

- This approach is called *backtracking*

Recursive tree walk is natural

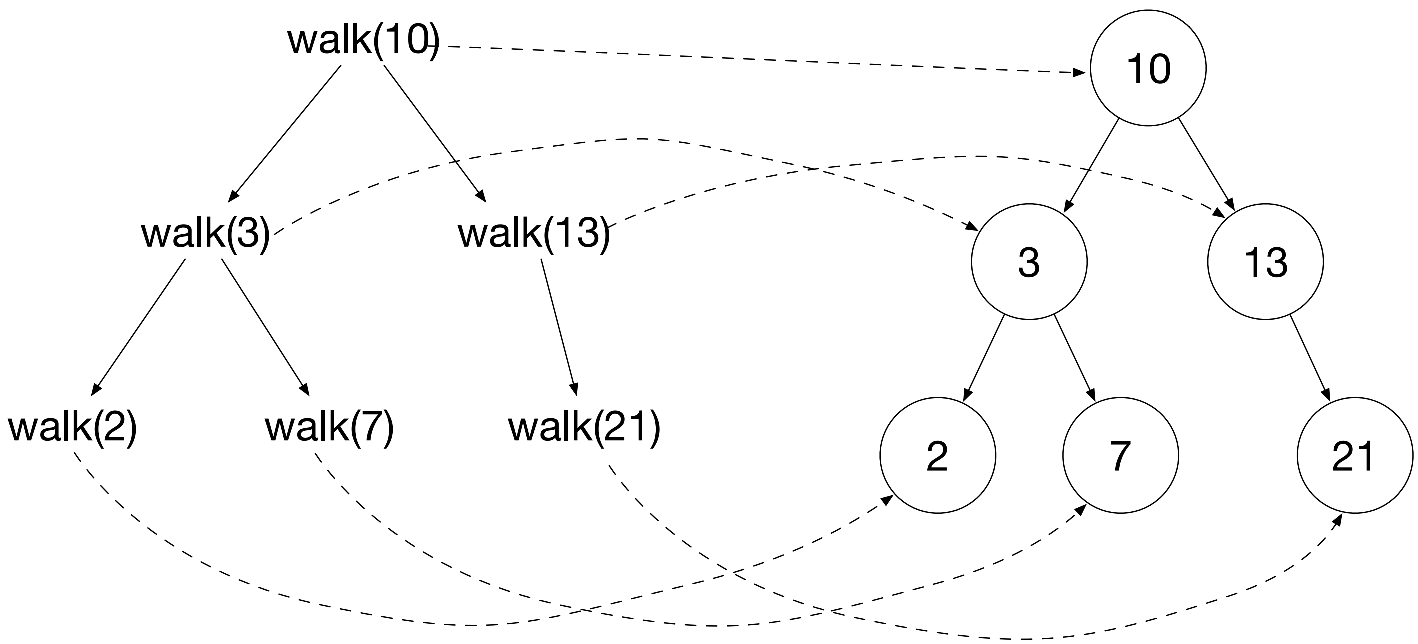
- *Depth-first search* is how we walk (visit) through nodes
- *Pre-order traversal*: executing an action at discovery time, before visiting kids

```
def walk(p:TreeNode):
    if p is None: return
    print(p.value) # preorder
    walk(p.left)
    walk(p.right)
```



Recursion call tree vs tree

```
def walk(p:TreeNode):
    if p is None: return
    walk(p.left)
    walk(p.right)
```



Exhaustive search of all nodes

Searching in binary tree

- Let's modify the tree walker to search for an element and compare to unrestricted depth-first tree walk

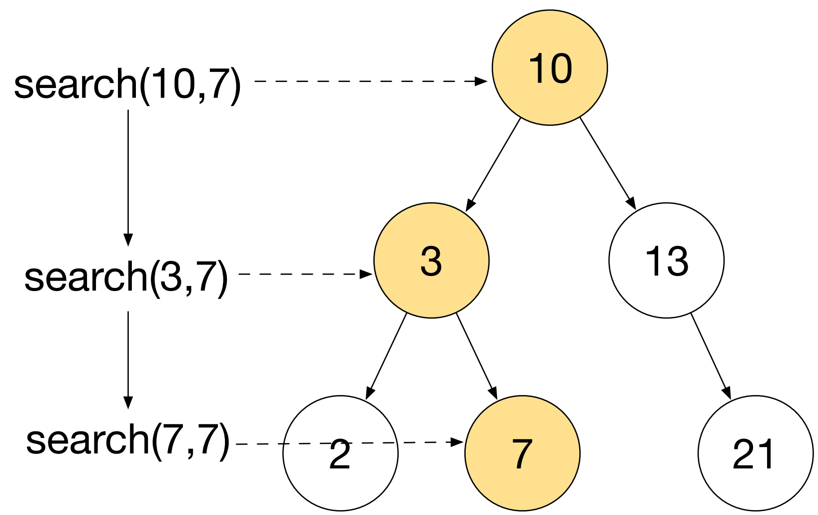
```
def walk(p:TreeNode):
    if p is None: return
    walk(p.left)
    walk(p.right)
```

```
def search(p:TreeNode, x:object):
    if p is None: return None
    if x==p.value: return p
    q = search(p.left, x)
    if q is not None: return q
    q = search(p.right, x)
    return q
```

Now restrict to *binary search tree* structure

- *BST* does a restricted walk using node values
- *binary search tree*: all elements < current node live in left subtree & all elements > current node live in right subtree

```
def search(p:TreeNode, x:object):
    if p is None: return None
    if x<p.value:
        return search(p.left, x)
    if x>p.value:
        return search(p.right, x)
    return p
```



Compare BST search to tree walk

- Conditional recursion; we only recurse to ONE child not both

```
def walk(p:TreeNode):
    if p is None: return
    walk(p.left)
    walk(p.right)
```

O(n) complexity

```
def search(p:TreeNode, x:object):
    if p is None: return None
    if x<p.value:
        return search(p.left, x)
    if x>p.value:
        return search(p.right, x)
    return p
```

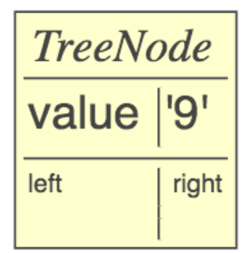
O(log(n)) complexity

Constructing binary search trees

- Result of add() function is the modified tree

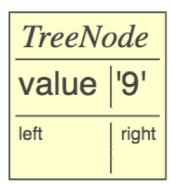
```
def add(p:TreeNode, value) -> TreeNode :
    if p is None:          return TreeNode(value)
    if value < p.value:    p.left = add(p.left, value)
    elif value > p.value: p.right = add(p.right, value)
    return p # do nothing if equal (already there)
```

- Initial condition: add to None: `root = add(None, 9)`



- If node.value==value, return that node:

`root = add(root, 9)`



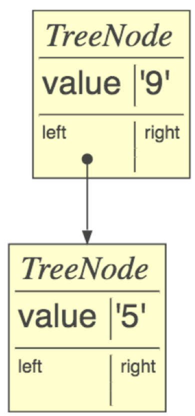
Constructing binary search trees cont'd

- If value < current node, add to the left

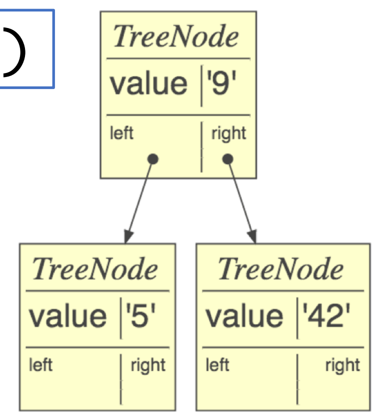
```
if value < p.value: p.left = add(p.left, value)
```

- If value > current node, add to the right

```
root = add(root, 5)
```




```
root = add(root, 42)
```



Consider similarity of search / build

```
def search(p:TreeNode, x:object):
    if p is None: return None
    if x < p.value:
        return search(p.left, x)
    if x > p.value:
        return search(p.right, x)
    return p
```

```
def add(p:TreeNode, v):
    if p is None: return TreeNode(v)
    if v < p.value:
        p.left = add(p.left, v)
    elif v > p.value:
        p.right = add(p.right, v)
    return p
```

Key takeaways

- Binary tree: acyclic tree structure with at most two children, constructed by hooking nodes together (`root.left = TreeNode(2)`)
- Self-similar data structures walked and built with recursion
- Each recursive call does a piece of the work and returns its piece combined with results obtained from recursive calls
- Recursion traces out a tree that looks like the data structure
- Remember the recursive function template
- Depth-first-search visits each node through backtracking
- Binary search tree constrains node values