Training decision trees

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Training overview

- Training partitions feature space into rectangular hypervolumes chasing reduced y impurity in subregions
- Hypervolumes are specified by sequence of *splits* that test a single feature and value at a time
- Each split becomes a decision node in decision tree
- Records in an "atomic" hypervolume form a single leaf
- Hypervolume described by conditionals on path from root to leaf

How to create a decision node

- Each split chosen greedily to minimize impurity in subregion y's
  - Regressor: variance or MSE
  - Classifier: gini criterion or entropy
- To choose split, exhaustively try each (variable,value) pair and pick the pair with min weighted average impurity for subregions created by that split

Fitting decision trees

subsets

MSE or gini

Algorithm: $dtreefit(X, y, min\_samples\_leaf, loss)$

if $|X| < min\_samples\_leaf$ **then** return Leaf(y)
$col, split = bestsplit(X, y, loss)$
if $col =$ *-1* **then** return Leaf(y) $\quad$ $(No\ better\ split?)$
$lchild = dtreefit(X[X \leq split], y[X \leq split])$
$rchild = dtreefit(X[X > split], y[X > split])$
return $DecisionNode(col, split, lchild, rchild)$

Overall fit: pass in full X, y to dtreefit() and get back the decision tree

Best split var/value

Algorithm: *bestsplit(X, y, loss)*

$best = (col = -1, split = -1, loss = loss(y))$
for $j = 1..p$ **do**
  foreach $split \in X_{\_,j}$ **do**
    $yl = y[X \leq split]$
    $yr = y[X > split]$
    if $|yl| = 0$ *or* $|yr| = 0$ **then continue**
    $l = \frac{|yl| \times loss(yl) + |yr| \times loss(yr)}{|y|}$ $(weighted\ average\ of\ subregion\ losses)$
    if $l = 0$ **then return** *col, split*
    if $l < best[loss]$ **then** $best = (col, split, l)$
  end
end
return *col, split*

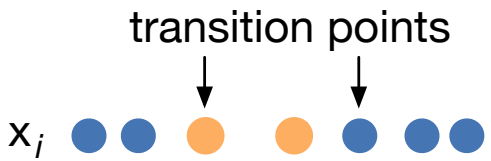
Should pick midpoint between split value and next smallest x

Decision tree prediction

```
1 Algorithm: predict(node,x)
2    if node is leaf then
3       if classifier then return mode(node.y)
4       return mean(node.y)
5    end
6    if x[node.col] ≤ node.split then return predict(node.lchild, x)
7    return predict(node.rchild, x)
```

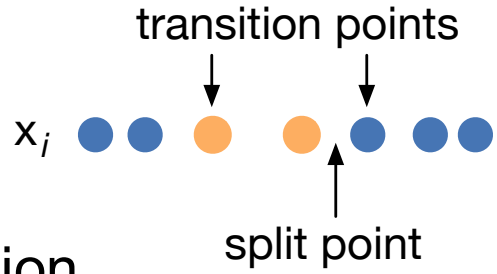
The usual bestsplit() is inefficient

- It has a nested loop; tries all combinations of p variables and worst-case n unique values in each variable at root: O(n*p)
- Cost of computing loss on all values in subregion each iteration is also expensive
- For classification, can mitigate by sorting by ith var then we know at a specific x value, everything to left is less and right is greater; keep track of class counts to left/right
- Reduce computation by focusing on transitions points in x, effectively focusing on unique(x)



Improving generality and efficiency

- Select a subset of values as candidates, k; then we reduce O(n*p) to O(k*p) for k << n (n is often huge) (our project k=11)
- We should really pick split point between two x values: $(x^{(i)}+x^{(i-1)})/2$ (if sorted)
- More likely split point is between, not on, x values, so midpoint is good guess as to underlying distribution
- And, of course, we can reduce tree height with min_samples_leaf to restrict complexity



Decision tree prediction via x subset

Algorithm: *bestsplit(X, y, loss)*

$best = (col = -1, split = -1, loss = loss(y))$
for $j = 1..p$ **do**
  $candidates =$ randomly pick $k \ll n$ values from $X_{-,j}$
  foreach $split \in candidates$ **do**
    $yl = y[X \leq split]$
    $yr = y[X > split]$
    if $|yl| = 0$ *or* $|yr| = 0$ **then continue**
    $l = \frac{|yl| \times loss(yl) + |yr| \times loss(yr)}{|y|}$ $(weighted\ average\ of\ subregion\ losses)$
    if $l = 0$ **then return** *col, split*
    if $l < best[loss]$ **then** $best = (col, split, l)$
  end
end
return *col, split*

Can even pick just 1 split randomly or in min..max range (see "Extremely random trees"); any small k value works.

Prediction

- Start at the root node and descend through the decision nodes to the appropriate leaf
- At each decision node, test a specific variable's value against the split value stored in the decision node

```
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