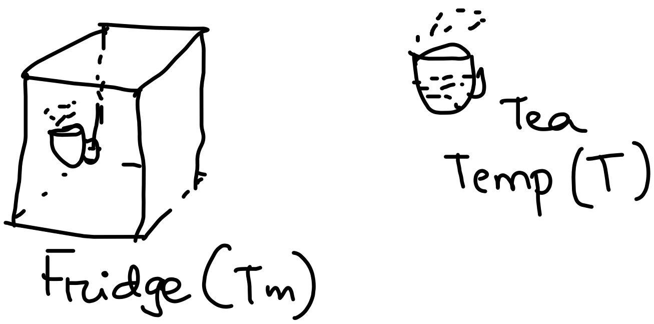
Modelling : Newton's Law of cooling:

28.09.2025




$T > T_m$

$t$: time

Newton's Law

$$\frac{dT}{dt} \propto T - T_m$$

$$\frac{dT}{dt} = k(T - T_m)$$

($k$ is constant)

$$\frac{1}{T - T_m}\, dT = k \, dt$$

variable separable:

**Integrating:**

$$\int \frac{1}{T - T_m}\, dT = \int k \, dt$$

$$\ln|T - T_m| = kt + c$$

$$T - T_m = e^{kt + c} = e^{kt} \cdot e^{c} = c_0 e^{kt}$$

where $c_0 = e^{c}$

$$T - T_m = c_0 e^{kt}$$

$$\boxed{T = T_m + c_0 e^{kt}}$$ ✓

## Problem:

Suppose that in winter the daytime temperature in a certain office building is maintained at 70°F. The heating is shut off at 10 P.M. and turned on again at 6 A.M.

On a certain day the temperature inside the building at 2 A.M. was found to be 65°F. Consider that the outside temperature varies between 50°F to 40°F by this time period. What was the temperature inside the building when the heat was turned on at 6 A.M.?

## Solution:

Room Temp: $T$

Outside Temp: $T_m$

time $t$

$$T = T_m + C_0 e^{kt} \qquad \text{--- (1)}$$

Outside Temp is $T_m = \text{average}(40^\circ, 50^\circ)\,F$

$= 45^\circ F$

* When $t=0$, $T = 70$

Then (1): $\underbrace{70}_{T} = \underbrace{45}_{T_m} + C_0 e^{k \cdot 0}$ ← $t=0$

$70 - 45 = C_0$

$$\boxed{C_0 = 25}$$

When $t = 4$ hours, $T = 65^\circ F$

Eqn (1) becomes: $65 = 45 + (25)e^{k \cdot 4}$

$\boxed{k = -0.056}$

We need to know $T$ when $t = 8$ hours

(6 A.M – 10 P.M last night)

$= 8$ hours

$$T = T_m + c_0 e^{kt}$$

$$T = 45 + (25)e^{(-0.056)8}$$

$$= 45 + 25e^{-0.448}$$

$$= 60.9^\circ F$$

$$\approx 61^\circ F$$

Ans

# Exact Differential equations

## (Exact ODE)

Def$^n$:

$$M(x,y)\,dx + N(x,y)\,dy = 0 \quad \text{--- (1)}$$

(1) is Exact ODE iff

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{--- (2)}$$

$\boxed{\text{Solution of (1)}}$

$\Downarrow$

Let us consider the general solution of (1) is $f(x,y,c) = 0$

Then $f$ must satisfy:

$$\frac{\partial f}{\partial x} = M(x,y) \quad \text{--- (3.1)}$$

$$\frac{\partial f}{\partial y} = N(x,y) \quad \text{--- (3.2)}$$

Integrating (3.1) w.r.to $x$

$$\int \frac{\partial f}{\partial x} \cancel{dx} = \int M(x,y)\,dx \checkmark$$

$$\Rightarrow f = \int M(x,y)\,dx + c \text{ (constant)}$$

$$\Rightarrow f(x,y,c) = \int M(x,y)\,dx + g(y) \quad \text{—④}$$

$$\left[c = \underset{\text{const.}}{g}(y)\right]$$

$\frac{d}{dx}(xy)$

$= y \cdot \frac{d}{dx}(x)$

$= y \cdot 1 = y$

$\int xy\,dx$

$= y \cdot \int x\,dx$

$= y \cdot \frac{x^2}{2} + c$

$= \frac{x^2 y}{2} + c$

We get $g'(y)$ from:

$$g'(y) = N(x,y) - \frac{\partial}{\partial y} \int M(x,y)\,dx \quad \text{—⑤}$$

Then, $g(y) = \int g'(y)\,dy$

Then ④ gives the

$f(x,y,c) = 0$ Ans

How I get ⑤

$$f(x,y,c) = \int M(x,y)\,dx + g(y) \rightarrow ④$$

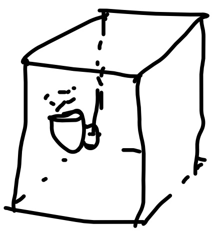
Then
$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\int M(x,y)\,dx + \frac{\partial}{\partial y} g(y)$$

$$N(x,y) = \frac{\partial}{\partial y}\int M(x,y)\,dx + g'(y)$$

Hence: $g'(y) = N(x,y) - \frac{\partial}{\partial y} \int M(x,y)\,dx \longrightarrow (5)$

# Modelling : Newton's Law of cooling:

28.09.2025



Fridge ($T_m$)

Tea
Temp ($T$)

$T > T_m$

$t$: time

**Newton's Law**

$$\frac{dT}{dt} \propto T - T_m$$

$$\frac{dT}{dt} = k(T - T_m)$$

($k$ is constant)

$$\frac{1}{T - T_m}\, dT = k\, dt$$

variable separable:

**Integrating:**

$$\int \frac{1}{T - T_m}\, dT = \int k\, dt$$

$$\ln|T - T_m| = kt + c$$

$$T - T_m = e^{kt + c} = e^{kt} \cdot e^{c} = c_0 e^{kt}$$

where $c_0 = e^{c}$

$$T - T_m = c_0 e^{kt}$$

$$\boxed{T = T_m + c_0 e^{kt}} \checkmark$$