Linear Programming Assignment – Optimization Techiniques

Author: Aziza Zhanabatyrova

OVERVIEW

The program consists in four functions written in MATLAB:

- SimplexSolver

  This is the only function the user will call by passing the problem in a certain format. SimplexSolver will then convert the problem into the standard form by calling the Standardization function, call the BigM function in case the Big M method has to be performed and finally call the Tableau function to iterate the tableau using the simplex algorithm in order to achieve optimality.

- Standardization

  The Standardization function is responsible for converting the problem passed in an easy and intuitive format by the user to the standard form.

- Tableau

  This function performs the iteration using the simplex algorithm. It also checks if the problem is bounded, unbounded, feasible or unfeasible.

- BigM

  The BigM function is responsible for applying the Big M method in the standard form of the problem.

HOW TO USE

Example I

max 2*x1 + x2
x1 + x2 <= 5
-x1 + x2 <= 0
6*x1 + 2*x2 <= 21

```
>> A = [1 1; -1 1; 6 2];      // Coefficients of constraints
>> b = [5; 0; 21];
>> c = [2 1];
>> type = 'max';
>> signs = {'<=','<=','<='};
>> simplexsolver(A,b,c,type,signs)
```

STANDARDIZED FORM

Type =
max

A =
```
   1    1    1    0    0
  -1    1    0    1    0
   6    2    0    0    1
```

b =
```
   5
   0
  21
```

C =
```
   2    1    0    0    0
```

x =
```
 x1
 x2
 s1
 s2
 s3
```

INITIALIZATION

There is an identity matrix in Slack variables

Basic variables = [s1, s2, s3]
Nonbasic variables = [x1, x2]

xb =
```
   5
   0
  21
```

xn =
```
   0
   0
```

B =
```
   1    0    0
   0    1    0
   0    0    1
```

N =
```
   1    1
  -1    1
   6    2
```

Coefficients of Basic var = 0 0 0

Coefficients of Non-basic var = -2 -1

TABLEAU

1 iteration

0	0	0	-2	-1	0
1	0	0	1	1	5
0	1	0	-1	1	0
0	0	1	6	2	21

2 iteration

0	0	0.3333	0	-0.3333	7.0000
1.0000	0	-0.1667	0	0.6667	1.5000
0	1.0000	0.1667	0	1.3333	3.5000
0	0	0.1667	1.0000	0.3333	3.5000

The cost has improved: 7.00 > 0.00

3 iteration

0.5000	0	0.2500	0	0	7.7500
1.5000	0	-0.2500	0	1.0000	2.2500
-2.0000	1.0000	0.5000	0	0	0.5000
-0.5000	0	0.2500	1.0000	0	2.7500

The cost has improved: 7.75 > 7.00

THE FINAL SOLUTION

x1 = 2.75
x2 = 2.25
x0 = 7.75

Elapsed time is 0.248772 seconds.

Example II

max x1 - x2
-2*x1 + x2 <= 2;
x2 <= 1;

```
>> A = [-2 1; 0 1];
>> b = [2; 1];
>> c = [1 -1];
>> type = 'max';
```

```
>> signs = {'<=','<='};
>> simplexsolver(A,b,c,type,signs)
```
STANDARDIZED FORM

```
Type =
max

A =
   -2     1     1     0
    0     1     0     1

b =
    2
    1

C =
    1    -1     0     0

x =
 x1
 x2
 s1
 s2
```

INITIALIZATION

There is an identity matrix in Slack variables

Basic variables = [s1, s2]

Nonbasic variables = [x1, x2]

```
xb =
    2
    1

xn =
    0
    0

B =
    1     0
    0     1

N =
   -2     1
    0     1
```

Coefficients of Basic var = 0 0

Coefficients of Non-basic var = -1 1

TABLEAU

1 iteration

```
   0    0   -1    1    0
   1    0   -2    1    2
   0    1    0    1    1
```

Warning: Problem is unbounded
Elapsed time is 0.137307 seconds.

Example III

$$\max x1 + 2*x2$$
$$x1 + x2 >= 2$$
$$x1 - x2 <= 0$$

```
>> A = [1 1; 1 -1];
>> b = [2; 0];
>> c = [1 2];
>> type = 'min';
>> signs = {'>=','<='};
>> simplexsolver(A,b,c,type,signs)
```

STANDARDIZED FORM

```
Type =
max

A =
    1    1   -1    0
    1   -1    0    1

b =
    2
    0

C =
   -1   -2    0    0

x =
 x1
 x2
 s1
 s2
```

INITIALIZATION

Using bigM

Basic variables = [s3, s4]

Nonbasic variables = [x1, x2, s1, s2]

xb =
2
0

xn =
0
0
0
0

B =
1 0
0 1

N =
1 1 -1 0
1 -1 0 1

Coefficients of Basic var = 0 0

Coefficients of Non-basic var = -7 2 4 -4

TABLEAU

1 iteration

0 0 -7 2 4 -4 -8
1 0 1 1 -1 0 2
0 1 1 -1 0 1 0

2 iteration

0 7 0 -5 4 3 -8
1 -1 0 2 -1 -1 2
0 1 1 -1 0 1 0

The cost has not improved

3 iteration

2.5000	4.5000	0	0	1.5000	0.5000	-3.0000
0.5000	-0.5000	0	1.0000	-0.5000	-0.5000	1.0000
0.5000	0.5000	1.0000	0	-0.5000	0.5000	1.0000

The cost has improved: -3.00 > -8.00

Big M has been applied to the problem.

THE FINAL SOLUTION

x1 = 1.00
x2 = 1.00
x0 = 3.00 (Minimum problem)
Elapsed time is 0.194190 seconds.

COMPARISON

Regarding the execution time taken, the SimplexSolver function is roughly twice faster than LINGO and the linear programming function in Matlab and about one and a half times faster than the Solver tool in Excel. The accuracy between all programs is very similar.