

A simulation problem

Consider a graph showing a stereotype connection system (e.g. circuit) with adjacency matrix X with a source and a sink. We want to use simulation to calculate the probability (average time) that the system works, i.e. a unit can traverse from source to sink. Also i^{th} edge works with probability p_i and does not work with probability q_i . We use stratification and post stratification conditioning on number of edges that works. Let $Y = \sum_k X_k$ in which X_k is 1 if k^{th} edge works and otherwise 0. Then let $P_j(i) = P(X_1 + X_2 + \dots X_i = j)$ then

$$P_j(i) = P_{j-1}(i-1)p_i + P_j(i-1)q_i$$

so $P_0(1) = q_1$ and $P_1(1) = p_1$.

How to calculate the post stratified simulation variance???

We simulate the adjacency matrix for a graph with n edges given $Y = k$ as follows.

$$P(X_n = 1|Y = k) = \frac{p_n P_{k-1}(n-1)}{P_k(n)}$$

$$P(X_{n-1} = 1|Y = k, X_n = i_n) = \frac{p_{n-1} P_{k-i_n-1}(n-2) P(X_n = i_n)}{P(X_n = i_n) P_{k-i_n}(n-1)} ???$$

$$P(X_{n-2} = 1|Y = k, X_n = i_n, X_{n-1} = i_{n-1}) = \frac{p_{n-2} P_{k-i_n-i_{n-1}-1}(n-3)}{P_{k-i_n-i_{n-1}}(n-2)}$$

So

$$P(X_{n-j} = 1|Y = k, X_n = i_n, \dots, X_{n-j+1} = i_{n-j+1}) = \frac{p_{n-j} P_{k-i_n-\dots-i_{n-j+1}-1}(n-j-1)}{P_{k-i_n-\dots-i_{n-j+1}}(n-j)}$$

what if Y after simulation is not k ? It can happen because of the computational error, since the probability can be so small so the round-off gives zero.