

DO NOT REMOVE FROM EXAM VENUE

Plymouth University

MODULE CODE: MATH237

TITLE OF PAPER: ENGINEERING MATHEMATICS AND STATISTICS

TIME ALLOWED THREE HOURS

DATE WEDNESDAY 24 JANUARY 2018

TIME 09:00 – 12:00

FACULTY SCIENCE AND ENGINEERING

SCHOOL COMPUTING, ELECTRONICS AND MATHEMATICS

ACADEMIC YEAR 2017 / 2018

STAGE TWO

INSTRUCTIONS TO CANDIDATES:

Candidates should attempt **ALL** questions. Questions do not carry equal marks. Marks for parts of questions are shown where appropriate.

Data Provided: Mathematical Formulae, Table of Standard Fourier, Laplace and Z Transforms, and Properties, Statistical Formulae and Tables are provided at the end of the examination paper on pages 6 – 17.

Candidates are not permitted to look at the examination paper until instructed to do so.

Semester 1 Exam

Q1. (a) The Laplace transform of a function is

$$e^{-3s}/s^2.$$

Find and sketch the function.

(5 Marks)

(b) Determine

$$\mathcal{L}^{-1} \left(\frac{s^2 - 2s + 3}{(s - 2)^3} \right).$$

(5 Marks)

Q2. For a particular circuit it can be shown that the transfer function $G(s)$ is given by

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{s + 2},$$

where $V_i(s)$ and $V_o(s)$ are the Laplace transforms of the input and output voltages $v_i(t)$ and $v_o(t)$, respectively.

(a) Find $v_o(t)$ when $v_i(t) = \delta(t)$.

(4 Marks)

(b) Use the convolution theorem to find $v_o(t)$ when

$$v_i(t) = \begin{cases} e^{-t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

(6 Marks)

(Over/...)

- Q3.** (a) The function $t \times H(t)$ is sampled at intervals $T = 1$ to give $k \times H(k)$, and subsequently shifted to the right by one sampling interval. Find the z-transform of the resulting function:

$$(k - 1)H(k - 1).$$

(3 Marks)

- (b) Solve the second-order difference equation

$$y[k + 2] - 5y[k + 1] + 6y[k] = 0,$$

given that $y[0] = 0$, and $y[1] = 2$.

(7 Marks)

- Q4.** (a) Find, if possible
(i) the Laplace transform
(ii) the Fourier transform
of

$$f(t) = H(t)e^{3t}.$$

(5 Marks)

- (b) Determine the amplitude and phase spectra of the signal

$$f(t) = e^{-at}H(t), \quad (a > 0).$$

(5 Marks)

- Q5.** Use Laplace transforms to solve the following ordinary differential equation:

$$x''(t) + 6x'(t) + 9x(t) = \sin(t), \quad (t \geq 0)$$

subject to the initial conditions $x(0) = 0$, and $x'(0) = 0$.

(10 Marks)

(Over/...)

- Q6.** A company which manufactures washing machines receives 60% of its parts from supplier A and 40% of its parts from supplier B. It is known that 5% of parts from supplier A are defective, while 9% of parts from supplier B are defective.
- (a) What is the probability that a randomly chosen part is defective? (3 Marks)
 - (b) What is the probability that a randomly chosen part is from supplier A and good? (3 Marks)
 - (c) Given that a randomly selected part is defective, what is the probability that it came from supplier B? (4 Marks)
- Q7.** A manufacturer designs a new component and claims that the mean lifetime of the components is 100 hours. A random sample of eight components are selected. The lifetime of these components is given below in hours:
- 89 100 150 95 60 76 80 66
- Assume that the lifetime of the components follows a normal distribution.
- (a) Using this sample, construct a 95% confidence interval for the true mean lifetime of these components. (6 Marks)
 - (b) Does the sample provide any evidence at the 95% confidence level that the mean lifetime is not 100 hours? Briefly justify your answer. (1 Mark)
- Q8.** Flaws occur on a cable randomly and independently with an average rate of two every 100 meters. Suppose a cable of 200 meters long.
- (a) What is the probability that there are no flaws in this cable? (3 Marks)
 - (b) What is the probability that there are no more than three flaws in this cable? (3 Marks)
 - (c) If there is no flaw in the first 100 meters, what is the probability that there is no more than one flaw in the next 100 meters? (5 Marks)

(Over/...)

- Q9.** The table below gives the batch size (number of components) and the cost (in dollars) for each batch. Suppose the variable **size** and **cost** are linearly related.

Size (number of components)	1	5	10	20	40
Cost (in dollars)	90	200	360	540	800

- (a) Find the linear equation which relates batch **size** and **cost**.
(7 Marks)
- (b) Calculate the residual for the batch of 20 components.
(3 Marks)
- (c) Predict the cost of producing a batch of 100 components.
(2 Marks)
- Q10.** A system is made up of four components (A, B, C and D), as shown in the figure below. The constant failure rates per year are 0.2, 0.3, 0.1 and 0.4 for components A, B, C and D respectively.

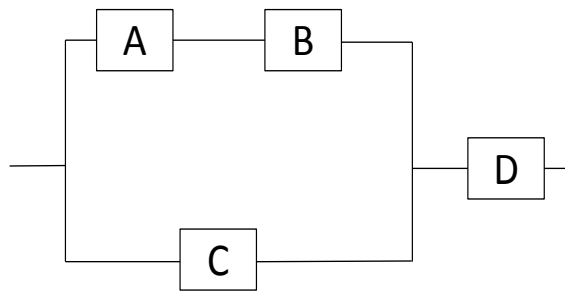


Figure: System for **Q10**.

- (a) Find the reliability of the system.
(6 Marks)
- (b) Find the probability that the system will still be operational after nine months.
(4 Marks)

END OF QUESTIONS

(Over/...)

Formulae and Tables of Transforms

Differentiation and Integration

$y(x)$	$\frac{dy}{dx}$	$\int y(x)dx$
k	0	$\int k dx = kx$
x^n	$nx^{n-1}, n \neq 0$	$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$
e^{kx}	ke^{kx}	$\int e^{kx} dx = \frac{e^{kx}}{k}$
$\ln(kx)$	$\frac{1}{x}$	$\int \frac{1}{x} dx = \ln(x) \text{ for } x > 0$
a^x	$a^x \ln(a)$	$\int a^x dx = \frac{a^x}{\ln(a)} \text{ for } a > 0$
$\sin(kx)$	$k \cos(kx)$	$\int \sin(kx) dx = -\frac{\cos(kx)}{k}$
$\cos(kx)$	$-k \sin(kx)$	$\int \cos(kx) dx = \frac{\sin(kx)}{k}$

Constants of integration have been omitted.

Rules of Differentiation

Sum Rule: if $y = u(x) + v(x)$, $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

Constant Rule: if $y = ku(x)$, $\frac{dy}{dx} = k\frac{du}{dx}$

Product Rule: if $y = u(x)v(x)$, $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Quotient Rule: if $y = \frac{u(x)}{v(x)} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Rules of Integration

Sum Rule: $\int [u(x) + v(x)] dx = \int u(x) dx + \int v(x) dx$

Constant Rule: $\int ky(x) dx = k \int y(x) dx$

Integration by Parts: $\int u\frac{dv}{dx} dx = uv - \int v\frac{du}{dx} dx$

Definite integral: $\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$

NOTE: a, b, k and n are constants.

(Over/...)

Table of Standard Fourier transforms

Description	Function	Transform
Definition	$v(t)$	$V(f) = \int_{-\infty}^{\infty} v(t)e^{-2j\pi ft} dt$
Scaling	$v\left(\frac{t}{T}\right)$	$ T \times V(fT)$
Time shift	$v(t - T)$	$V(f) \times e^{-2j\pi fT}$
Frequency shift	$v(t)e^{2j\pi f_0 t}$	$V(f - f_0)$
Reciprocity	$V(t)$	$v(-f)$
Addition	$A \times v(t) + B \times w(t)$	$A \times V(f) + B \times W(f)$
Multiplication	$v(t) \times w(t)$	$V(f) * W(f)$
Convolution	$v(t) * w(t)$	$V(f) \times W(f)$
Delta function	$\delta(t)$	1
Constant	1	$\delta(f)$
Rectangular function	$\text{rect}(t)$	$\text{sinc}(f) = \frac{\sin(\pi f)}{\pi f}$
Sinc function	$\text{sinc}(t)$	$\text{rect}(f)$
Heaviside function	$H(t)$	$\frac{1}{2}\delta(f) - \frac{j}{2\pi f}$
Signum function	$\text{sgn}(t)$	$-\frac{j}{\pi f}$
Decaying exponential, two-sided	$e^{- t }$	$\frac{2}{1 + (2\pi f)^2}$
Decaying exponential, one-sided	$e^{- t } \times H(t)$	$\frac{1 - 2j\pi f}{1 + (2\pi f)^2}$
Gaussian function	$e^{-\pi t^2}$	$e^{-\pi f^2}$

(Over/...)

Table of Standard Laplace transforms

$v(t)$	$t > 0$	$V(s)$
$\delta(t)$	unit impulse	1
$\delta(t - T)$	delayed impulse	e^{-Ts}
e^{-at}		$\frac{1}{s + a}$
$H(t)$	Heaviside function	$\frac{1}{s}$
$H(t - T)$	delayed Heaviside function	$\frac{1}{s}e^{-Ts}$
$H(t) - H(t - T)$	rectangular pulse	$\frac{1}{s}(1 - e^{-Ts})$
t	unit ramp	$\frac{1}{s^2}$
$\sin(\omega t)$		$\frac{\omega}{s^2 + \omega^2}$
$\cos(\omega t)$		$\frac{s}{s^2 + \omega^2}$
t^n (n a positive integer)		$\frac{n!}{s^{n+1}}$
$\sinh(at)$		$\frac{a}{s^2 - a^2}$
$\cosh(at)$		$\frac{s}{s^2 - a^2}$
$L \left[\frac{d^{(n)}v}{dt^{(n)}} \right]$		$s^n V(s) - s^{n-1}v(0) \dots - v^{(n-1)}(0)$
$e^{-at}v(t)$	s-shift	$V(s + a)$
$v(t - T)H(t - T)$	time-shift	$e^{-Ts}V(s)$
$v(at)$	time-scaling	$\frac{1}{a}V\left(\frac{s}{a}\right)$
$v(t) * g(t)$	convolution	$V(s) \times G(s)$

(Over/...)

Table of Standard Z transforms

$x(nT)$ -sampled	Z transform
a^n	$\frac{z}{z - a}$
$\delta(n)$	1
$H(n)$	$\frac{z}{z - 1}$
nT	$\frac{Tz}{(z - 1)^2}$
$(nT)^2$ i.e. when $x(t) = t^2$	$\frac{T^2 z(z + 1)}{(z - 1)^3}$
e^{-anT}	$\frac{z}{z - e^{-aT}}$
$1 - e^{-anT}$	$\frac{z(1 - e^{-aT})}{(z - 1)(z - e^{-aT})}$
nTe^{-anT}	$\frac{Tze^{-aT}}{(z - e^{-aT})^2}$
$(1 - anT)e^{-anT}$	$\frac{z[z - e^{-aT}(1 + aT)]}{(z - e^{-aT})^2}$
$\sin(n\omega T)$	$\frac{z \sin(\omega T)}{z^2 - 2z \cos(\omega T) + 1}$
$\cos(n\omega T)$	$\frac{z(z - \cos(\omega T))}{z^2 - 2z \cos(\omega T) + 1}$
$\sinh(at)$	$\frac{z \sinh(\omega T)}{z^2 - 2z \cosh(\omega T) + 1}$
$\cosh(at)$	$\frac{z(z - \cosh(\omega T))}{z^2 - 2z \cosh(\omega T) + 1}$
$e^{-anT}x(nT)$	$X(ze^{aT})$
$x(nT - n_0T)$	$\frac{X(z)}{z^{n_0}}$
$nx(n)$	$-z \frac{dX(z)}{dz}$
$x(nT + T)$	$zX(z) - zx(0)$
$x(nT + 2T)$	$z^2X(z) - zx(T) - z^2x(0)$

(Over/...)

Statistics formulae and tables

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A|B) \times P(B) = P(B|A) \times P(A)$$

Probability Distributions

The binomial distribution $P(k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$ for $k = 0, 1, 2, \dots, n$.

The Poisson distribution $P(k) = \frac{\lambda^k \exp(-\lambda)}{k!}$ for $\lambda > 0$ for $k = 0, 1, 2, \dots, n$.

The exponential distribution (pdf) $f(t) = \lambda \exp(-\lambda t)$ for $\lambda > 0$; MTBF = $\frac{1}{\lambda}$ for $t > 0$.

The Weibull distribution (cdf) $F(x) = 1 - \exp[-(\frac{x}{\beta})^\alpha]$; MTBF = $\beta \Gamma(1 + \frac{1}{\alpha})$ for $x > 0$.

The reliability function for the exponential distribution:

$$R(t) = 1 - \int_0^t f(s) ds = \exp(-\lambda t).$$

Confidence Intervals

$\mu \in (\bar{X} - z \frac{\sigma}{\sqrt{n}}, \bar{X} + z \frac{\sigma}{\sqrt{n}})$ with the probability $1 - 2\alpha$, where α is the confidence level;

$$P(X > z) = \frac{1}{2\pi} \int_z^\infty \exp(-\frac{x^2}{2}) dx = \alpha.$$

Sample standard deviation

$$s = \sqrt{\frac{1}{n-1} \sum_{k=1}^n (x_k - \bar{x})^2} = \sqrt{\frac{1}{n-1} \left(\sum_{k=1}^n (x_k)^2 - n\bar{x}^2 \right)}.$$

(Over/...)

Mean charts

When μ and σ are known, the warning limits are given by:

$$\mu \pm 1.96 \times \frac{\sigma}{\sqrt{n}}.$$

When μ and σ are known, the action limits are given by:

$$\mu \pm 3.09 \times \frac{\sigma}{\sqrt{n}}.$$

When μ and σ are not known, the warning limits are given by:

$$\bar{\bar{x}} \pm 1.96 \times \frac{\bar{R}}{d_n \sqrt{n}}.$$

When μ and σ are not known, the action limits are given by:

$$\bar{\bar{x}} \pm 3.09 \times \frac{\bar{R}}{d_n \sqrt{n}}.$$

Linear equation

A simple linear regression equation is

$$y = a + bx,$$

where $a = \bar{y} - b\bar{x}$, $b = \frac{s_{xy}}{s_{xx}}$ with $s_{xx} = \sum_{i=1}^n x_i^2 - n(\bar{x})^2$ and $s_{xy} = \sum_{i=1}^n x_i y_i - n\bar{x}\bar{y}$.

(Over/...)

Standard normal distribution table

Table entries are $P(X \leq z) = 1 - \alpha$, where α is the confidence level.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441

(Standard normal distribution table continued/.....)

(Standard normal distribution table continued.....)

<i>z</i>	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

(Over/...)

Hartley's Constants

Sample size	d_n	$D_{0.999}$	$D_{0.975}$	$D_{0.025}$	$D_{0.001}$
2	1.128	0.00	0.04	2.81	4.12
3	1.693	0.04	0.18	2.17	2.98
4	2.059	0.10	0.29	1.93	2.57
5	2.326	0.16	0.37	1.81	2.34
6	2.534	0.21	0.42	1.72	2.21
7	2.704	0.26	0.46	1.66	2.11
8	2.847	0.29	0.50	1.62	2.04
9	2.970	0.32	0.52	1.58	1.99
10	3.078	0.35	0.54	1.56	1.93
11	3.173	0.38	0.56	1.53	1.91
12	3.258	0.40	0.58	1.51	1.87

(Over/...)

t-distribution table

Table entries are $t_{p,v}$ where $P(X > t_{p,v}) = p$.

v	p=0.1	p=0.05	p=0.025	p=0.01	p=0.005	p=0.001	p=0.0005
1	3.0777	6.3138	12.7062	31.8205	63.6567	318.3088	636.6192
2	1.8856	2.92	4.3027	6.9646	9.9248	22.3271	31.5991
3	1.6377	2.3534	3.1824	4.5407	5.8409	10.2145	12.924
4	1.5332	2.1318	2.7764	3.7469	4.6041	7.1732	8.6103
5	1.4759	2.015	2.5706	3.3649	4.0321	5.8934	6.8688
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.2076	5.9588
7	1.4149	1.8946	2.3646	2.998	3.4995	4.7853	5.4079
8	1.3968	1.8595	2.306	2.8965	3.3554	4.5008	5.0413
9	1.383	1.8331	2.2622	2.8214	3.2498	4.2968	4.7809
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.1437	4.5869
11	1.3634	1.7959	2.201	2.7181	3.1058	4.0247	4.437
12	1.3562	1.7823	2.1788	2.681	3.0545	3.9296	4.3178
13	1.3502	1.7709	2.1604	2.6503	3.0123	3.852	4.2208
14	1.345	1.7613	2.1448	2.6245	2.9768	3.7874	4.1405
15	1.3406	1.7531	2.1314	2.6025	2.9467	3.7328	4.0728
16	1.3368	1.7459	2.1199	2.5835	2.9208	3.6862	4.015
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.6458	3.9651
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.6105	3.9216
19	1.3277	1.7291	2.093	2.5395	2.8609	3.5794	3.8834
20	1.3253	1.7247	2.086	2.528	2.8453	3.5518	3.8495

(t-distribution table continued/.....)

(t-distribution table continued.....)

v	p=0.1	p=0.05	p=0.025	p=0.01	p=0.005	p=0.001	p=0.0005
21	1.3232	1.7207	2.0796	2.5176	2.8314	3.5272	3.8193
22	1.3212	1.7171	2.0739	2.5083	2.8188	3.505	3.7921
23	1.3195	1.7139	2.0687	2.4999	2.8073	3.485	3.7676
24	1.3178	1.7109	2.0639	2.4922	2.7969	3.4668	3.7454
25	1.3163	1.7081	2.0595	2.4851	2.7874	3.4502	3.7251
26	1.315	1.7056	2.0555	2.4786	2.7787	3.435	3.7066
27	1.3137	1.7033	2.0518	2.4727	2.7707	3.421	3.6896
28	1.3125	1.7011	2.0484	2.4671	2.7633	3.4082	3.6739
29	1.3114	1.6991	2.0452	2.462	2.7564	3.3962	3.6594
30	1.3104	1.6973	2.0423	2.4573	2.75	3.3852	3.646
40	1.3031	1.6839	2.0211	2.4233	2.7045	3.3069	3.551
60	1.2958	1.6706	2.0003	2.3901	2.6603	3.2317	3.4602
120	1.2886	1.6577	1.9799	2.3578	2.6174	3.1595	3.3735
300	1.2844	1.6499	1.9679	2.3388	2.5923	3.1176	3.3233

Gamma function $\Gamma(x)$

x	$\Gamma(x)$	x	$\Gamma(x)$	x	$\Gamma(x)$	x	$\Gamma(x)$	x	$\Gamma(x)$
1.01	0.994	1.21	0.916	1.41	0.887	1.61	0.895	1.81	0.934
1.02	0.989	1.22	0.913	1.42	0.886	1.62	0.896	1.82	0.937
1.03	0.984	1.23	0.911	1.43	0.886	1.63	0.897	1.83	0.940
1.04	0.978	1.24	0.909	1.44	0.886	1.64	0.899	1.84	0.943
1.05	0.974	1.25	0.906	1.45	0.886	1.65	0.900	1.85	0.946
1.06	0.969	1.26	0.904	1.46	0.886	1.66	0.902	1.86	0.949
1.07	0.964	1.27	0.903	1.47	0.886	1.67	0.903	1.87	0.952
1.08	0.960	1.28	0.901	1.48	0.886	1.68	0.905	1.88	0.955
1.09	0.955	1.29	0.899	1.49	0.886	1.69	0.907	1.89	0.958
1.10	0.951	1.30	0.897	1.50	0.886	1.70	0.909	1.90	0.962
1.11	0.947	1.31	0.896	1.51	0.887	1.71	0.911	1.91	0.965
1.12	0.944	1.32	0.895	1.52	0.887	1.72	0.913	1.92	0.969
1.13	0.940	1.33	0.893	1.53	0.888	1.73	0.915	1.93	0.972
1.14	0.936	1.34	0.892	1.54	0.888	1.74	0.917	1.94	0.976
1.15	0.933	1.35	0.891	1.55	0.889	1.75	0.919	1.95	0.980
1.16	0.930	1.36	0.890	1.56	0.890	1.76	0.921	1.96	0.984
1.17	0.927	1.37	0.889	1.57	0.890	1.77	0.924	1.97	0.988
1.18	0.924	1.38	0.889	1.58	0.891	1.78	0.926	1.98	0.992
1.19	0.921	1.39	0.888	1.59	0.892	1.79	0.929	1.99	0.996
1.20	0.918	1.40	0.887	1.60	0.894	1.80	0.931	2.00	1.000

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