<u>DO NOT REMOVE FROM EXAM VENUE</u>

Plymouth University

MODULE CODE:	**MATH237**
TITLE OF PAPER:	**ENGINEERING MATHEMATICS AND STATISTICS**

TIME ALLOWED	**THREE HOURS**
DATE	**TUESDAY 19 JANUARY 2016**
TIME	**09:00 – 12:00**
FACULTY	**SCIENCE AND ENGINEERING**
SCHOOL	**COMPUTING ELECTRONICS AND MATHEMATICS**
ACADEMIC YEAR	**2015 / 2016**
STAGE	**TWO**

INSTRUCTIONS TO CANDIDATES:

Candidates should attempt **ALL** questions. Questions do not carry equal marks. Marks for parts of questions are shown where appropriate. Question 8 requires use of prepared charts at the end of this paper - **remember to enter your student number on this page and add it to the answer booklet.**

Data Provided: Mathematical Formulae, Table of Standard Fourier, Laplace and Z Transforms, and Properties, Statistical Formulae and a Normal Distribution Table are provided at the end of the examination paper on pages 7 – 13.

Candidates are not permitted to look at the examination paper until instructed to do so.

Release to library? Yes

Semester 1 Exam

Q1. (a) For each of the following signals,find its Fourier Transform using the table of standard transforms:

(i) $2sgn(t-3)$

(3 Marks)

(ii) $5H(t+1)$

(3 Marks)

(iii) $\frac{1}{2}\,\text{rect}\left(\frac{t}{4}\right)$

(3 Marks)

(b) Find the amplitude and phase of the following

(i) $Y(f) = e^{-3j\pi f^2}$

(4 Marks)

(ii) $Y(f) = \dfrac{j(\cos(8\pi f) - 2\cos(4\pi f)}{2(\pi f)^2}$

(4 Marks)

Q2. Use the Laplace transform to find the current, i, which flows in the circuit governed by the differential equation,

$$i + 5\frac{di}{dt} = \sin(t),$$

and where no current flows for $t < 0$.Investigate the stability of the system.

(10 Marks)

(Over/...)

Q3. (a) Using the Table of Standard z Transforms, find the following:

(i) $Z[e^{-4nT}6^n]$ where T=1

(2 Marks)

(ii) $Z[3(nT-4T)]$ where T=1

(2 Marks)

(iii) $Z[\cos(3nT)-e^{-7nT}]$ where $T=\pi$

(3 Marks)

(b) Find the overall pulse transfer function for a system consisting of two systems in series with transfer functions $\dfrac{1}{s+2}$ and $\dfrac{1}{s-3}$.

(9 Marks)

Q4. (a) A component has a probability of failure being 0.01 independently of the other components. The components are grouped in batches of size 50.

(i) What is the probability that there are no failures in a batch?

(2 Marks)

(ii) What is the probability that there are at least two failures in a batch?

(3 Marks)

(iii) What is the probability of having exactly 5 failures in a batch?

(2 Marks)

(b) A component has a lifetime measured in 1000s of hours which follows a Weibull distribution with α =3 and β=11.

(i) Given that Γ(1.33)=0.8934, find the Mean Time Between Failure.

(3 Marks)

(ii) What is the probability that a component survives longer than 3000 hours?

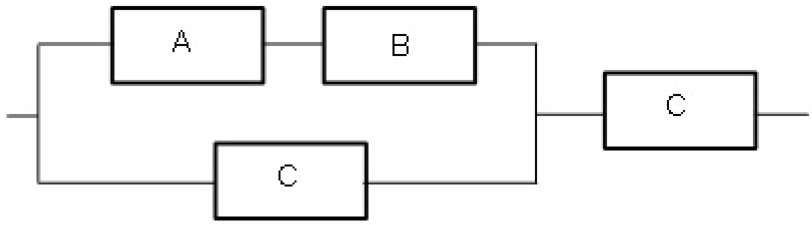
(2 Marks)

(iii) What is the probability that a component survives for less than 1500 hours?

(2 Marks)

(Over/...)

Q5. A device is constructed from components of types A, B and C. They are connected as shown in the diagram below. Components A and C have a constant failure rate of 0.05 per year and component B has a constant failure rate of 0.01 per year.



(a) Find the overall reliability of the system.

(6 Marks)

(b) Find the probability that the system survives for 5 years.

(2 Marks)

Q6. A pair of components, A and B, are to be connected in series to form a circuit. The probabilities that A and B are faulty are 0.05 and 0.02, respectively and are independent of each other.

(a) What is the probability that both of a randomly selected pair of components is faulty?

(1 Marks)

(b) What is the probability that both of a randomly selected pair of components is faultless?

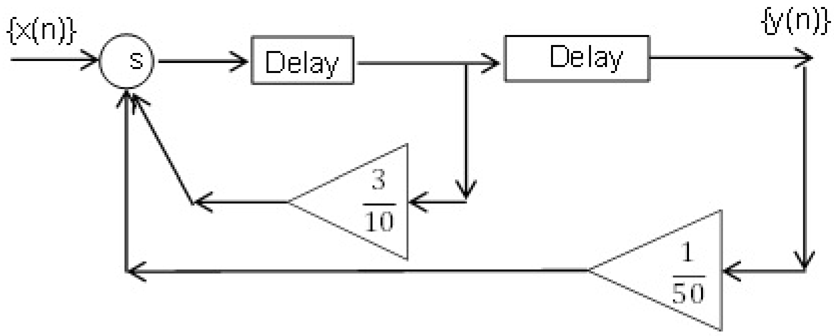
(2 Marks)

(c) What is the probability that exactly one component of a randomly selected pair of components is faulty?

(2 Marks)

(Over/...)

Q7. Consider the system represented by the block diagram below:



(a) Show that the transfer function for the system is

$$G(z) = \frac{1}{z^2 + 0.3z + 0.02}.$$

(5 Marks)

(b) Show the system is stable.

(5 Marks)

(c) Find the impulse response for the system.

(10 Marks)

(Over/...)

Q8. The resistance of a component is used to control a production process. Samples of size 4 have been taken and the means and ranges of these samples are shown below:

$\bar{x}$	208	208	214	216	210	209	213	217	209	211
R	12	16	15	7	11	12	14	8	16	15
$\bar{x}$	203	207	213	210	210	211	214	215	213	209
R	4	13	18	20	5	18	13	15	12	11

(a) Find the Grand means and the control limits and add these to the prepared charts in **Appendix A** at the end of this question paper.

(10 Marks)

(b) Is the process in statistical control?

(2 Marks)

(c) The next two samples have been taken and the results are shown below:

sample 21	209	215	214	218
sample22	204	214	212	218

Find the mean and range for these samples and add them to your chart. What do you conclude?

(8 Marks)

END OF QUESTIONS

(Over/...)

Formulae and Tables of Transforms

Differentiation and Integration

y(x)	$\frac{dy}{dx}$	$\int y(x)dx$
k	0	$\int kdx = kx$
x^n	$nx^{n-1}, n \neq 0$	$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$
e^{kx}	ke^{kx}	$\int e^{kx} dx = \frac{e^{kx}}{k}$
$\ln(kx)$	$\frac{1}{x}$	$\int \frac{1}{x} dx = \ln(x)$
a^x	$a^x \ln(a)$	$\int a^x dx = \frac{a^x}{\ln(a)}$
$\sin(kx)$	$k\cos(kx)$	$\int \sin(kx)dx = -\frac{\cos(kx)}{k}$
$\cos(kx)$	$-k\sin(kx)$	$\int \cos(kx)dx = \frac{\sin(kx)}{k}$

Constants of integration have been omited.

Rules of Differentiation

Sum Rule: if $y = u(x) + v(x)$, $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

Constant Rule: if $y = ku(x)$, $\frac{dy}{dx} = k\frac{du}{dx}$

Product Rule: if $y = u(x)v(x)$, $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Quotient Rule: if $y = \frac{u(x)}{v(x)} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Rules of Integration

Sum Rule: = $\int [u(x) + v(x)]dx = \int u(x)dx + \int v(x)dx$

Constant Rule: $\int ky(x)dx = k\int y(x)dx$

Integration by Parts: $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$

Definite integral: $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

NOTE: a**,** b**,** k **and** n **are constants.**

(Over/...)

Table of Standard Fourier transforms

Description	Function	Transform
Definition	$v(t)$	$V(f) = \int_{-\infty}^{\infty} v(t)e^{-2j\pi ft}dt$
Scaling	$v\left(\frac{t}{T}\right)$	$\lvert T\rvert \times V(fT)$
Time shift	$v(t-T)$	$V(f) \times e^{-2j\pi fT}$
Reciprocity	$V(t)$	$v(-f)$
Addition	$A.v(t) + B.w(t)$	$A.V(f) + B.W(f)$
Multiplication	$v(t).w(t)$	$V(f) * W(f)$
Convolution	$v(t) * w(t)$	$V(f).W(f)$
Delta function	$\delta(t)$	1
Constant	1	$\delta(f)$
Rectangular function	$rect(t)$	$sinc(f) = \frac{\sin(\pi f)}{\pi f}$
Sinc function	$sinc(t)$	$rect(f)$
Heaviside function	$H(t)$	$\frac{1}{2}\delta(f) - \frac{j}{2\pi f}$
Signum function	$sgn(t)$	$-\frac{j}{\pi f}$
Decaying exponential, two-sided	$e^{-\lvert t\rvert}$	$\frac{2}{1+(2\pi f)^2}$
Decaying exponential, one-sided	$e^{-\lvert t\rvert}.H(t)$	$\frac{1-2j\pi f}{1+(2\pi f)^2}$
Gaussian function	$e^{-\pi t^2}$	$e^{-\pi f^2}$

(Over/...)

Table of Standard Laplace transforms

$v(t)$	$t > 0$	$V(s)$
$\delta(t)$	unit impulse	1
$\delta(t-T)$	delayed impulse	e^{-Ts}
e^{-at}		$\dfrac{1}{s+a}$
$H(t)$	Heaviside function	$\dfrac{1}{s}$
$H(t-T)$	delayed Heaviside function	$\dfrac{1}{s}e^{-Ts}$
$H(t)-H(t-T)$	rectangular pulse	$\dfrac{1}{s}(1-e^{-Ts})$
t	unit ramp	$\dfrac{1}{s^2}$
$\sin(\omega t)$		$\dfrac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$		$\dfrac{s}{s^2+\omega^2}$
t^n (n a positive integer)		$\dfrac{n!}{s^{n+1}}$
$\sinh(at)$		$\dfrac{a}{s^2-a^2}$
$\cosh(at)$		$\dfrac{s}{s^2-a^2}$
$L\left[\dfrac{d^{(n)}v}{dt^{(n)}}\right]$		$s^nV(s)-s^{n-1}v(0)...-v^{n-1}(0)$
$e^{-at}v(t)$	s-shift	$V(s+a)$
$v(t-T)H(t-T)$	time-shift	$e^{-Ts}V(s)$
$v(at)$	time-scaling	$\dfrac{1}{a}V(\dfrac{s}{a})$
$v(t)*g(t)$	convolution	$V(s)\times G(s)$

(Over/...)

Table of Standard Z transforms

$x(nT)$-**sampled**	**Z transform**
a^n	$\frac{z}{z-a}$
$\delta(n)$	1
$H(n)$	$\frac{z}{z-1}$
nT	$\frac{Tz}{(z-1)^2}$
$(nT)^2$ ie when $x(t)=t^2$	$\frac{T^2z(z+1)}{(z-1)^3}$
e^{-anT}	$\frac{z}{z-e^{-aT}}$
$1-e^{-anT}$	$\frac{z(1-e^{-aT}}{(z-1)(z-e^{-aT})}$
nTe^{-anT}	$\frac{Tze^{-aT}}{(z-e^{-at})^2}$
$(1-anT)e^{-anT}$	$\frac{z[z-e^{-aT}(1+aT)]}{(z-e^{-aT})^2}$
$\sin(n\omega T)$	$\frac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$
$\cos(n\omega T)$	$\frac{z(z-\cos(\omega T))}{z^2-2z\cos(\omega T)+1}$
$\sinh(at)$	$\frac{z\sinh(\omega T)}{z^2-2z\cosh(\omega T)+1}$
$\cosh(at)$	$\frac{z(z-\cosh(\omega T))}{z^2-2z\cosh(\omega T)+1}$
$e^{-anT}x(nT)$	$X(ze^{at})$
$x(nT-n_0T)$	$\frac{X(z)}{z^{n_0}}$
$nx(n)$	$-z\frac{dX(z)}{dz}$
$x(nT+T)$	$zX(z)-zx(0)$
$x(nT+2T)$	$z^2X(z)-zx(T)-z^2x(0)$

(Over/...)

Statistics formulae and tables

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A|B) \times P(B) = P(B|A) \times P(A)$$

Probability Distributions

The binomail distribution $P(k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$

The Poisson distribution $P(k) = \frac{\lambda^k \exp(-\lambda)}{k!}$ for $\lambda > 0$

The exponential distribution (pdf) $f(t) = \lambda \exp(-\lambda t)$ for $\lambda > 0$; MTBF $= \frac{1}{\lambda}$

The Weibull distribution (cdf) $F(x) = 1 - \exp[-(\frac{x}{\beta})^\alpha]$; MTBF=$\beta\Gamma(1 + \frac{1}{\alpha})$

The reliability function for the exponential distribution $R(t) = 1 - \int_0^t f(s)ds = \exp(-\lambda t)$

Confidence Intervals

$\mu \in (\bar{X} - z\frac{\sigma}{\sqrt{n}}, \bar{X} + z\frac{\sigma}{\sqrt{n}})$ with the probability $1 - 2\alpha$, where α is the confidence level; $P(X > z) = \frac{1}{2\pi}\int_z^\infty \exp(-\frac{x^2}{2})dx = \alpha$

Standard deviation of sample

$$s = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}(x_k - \bar{x})^2} = \sqrt{\frac{1}{n-1}\left(\sum_{k=1}^{n}(x_k)^2 - n\bar{x}^2\right)}$$

Correlation and Regression

The least-squares straight line is $\hat{y} = a + bx$, where: $a = \bar{y} - b\bar{x}$, $b = \frac{S_{xy}}{S_{xx}}$, $S_{xx} = \sum_{i=1}^{n} x_i^2 - n(\bar{x})^2$, $S_{xy} = \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}$.
The correlation coefficient $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$.

(Over/...)

Table of Hartleys Constants

sample size n	d_n	$D_{0.999}$	$D_{0.975}$	$D_{0.025}$	$D_{0.001}$
2	1.128	0	0.04	2.81	4.12
3	1.693	0.04	0.18	2.17	2.98
4	2.059	0.1	0.29	1.93	2.57
5	2.326	0.16	0.37	1.81	2.34
6	2.534	0.21	0.42	1.72	2.21
7	2.704	0.26	0.46	1.66	2.11
8	2.847	0.29	0.5	1.62	2.04
9	2.97	0.32	0.52	1.58	1.99
10	3.078	0.35	0.54	1.56	1.93
11	3.173	0.38	0.56	1.53	1.91
12	3.258	0.4	0.58	1.51	1.87

Mean and Range Control Charts

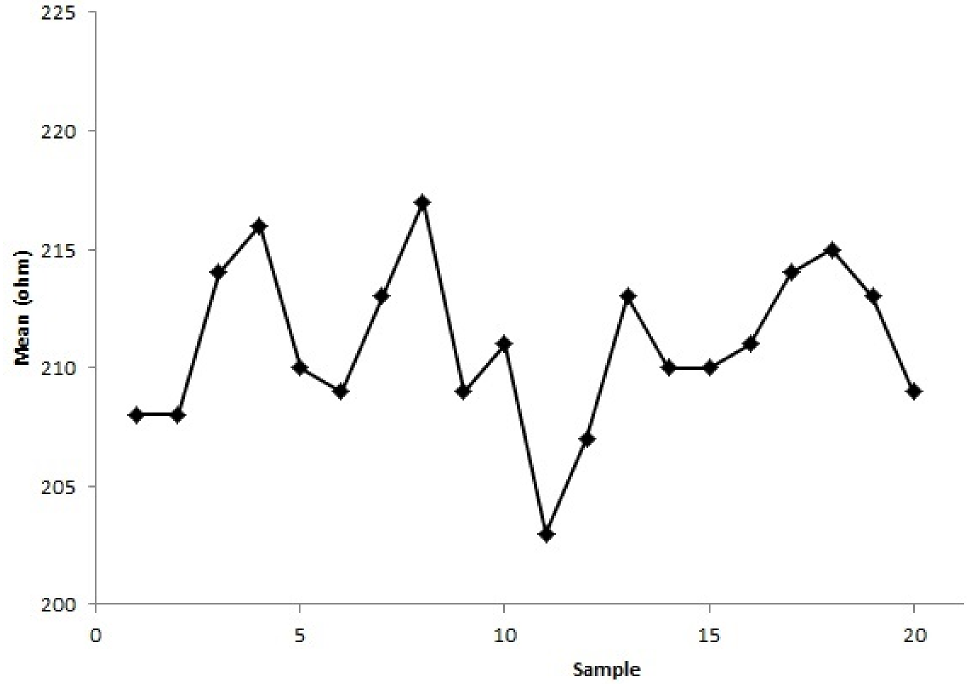
Limit	Ranges	Means	
		μ and σ known	μ and σ unknown
UAL	$D_{0.001}\overline{R}$	$\mu + 3.09\frac{\sigma}{\sqrt{n}}$	$\overline{\overline{x}} + 3.09\frac{\overline{R}}{d_n\sqrt{n}}$
UWL	$D_{0.025}\overline{R}$	$\mu + 1.96\frac{\sigma}{\sqrt{n}}$	$\overline{\overline{x}} + 1.96\frac{\overline{R}}{d_n\sqrt{n}}$
Grand mean	$\overline{R}$	μ	$\overline{\overline{x}}$
LWL	$D_{0.975}\overline{R}$	$\mu - 1.96\frac{\sigma}{\sqrt{n}}$	$\overline{\overline{x}} - 1.96\frac{\overline{R}}{d_n\sqrt{n}}$
LAL	$D_{0.999}\overline{R}$	$\mu - 3.09\frac{\sigma}{\sqrt{n}}$	$\overline{\overline{x}} - 3.09\frac{\overline{R}}{d_n\sqrt{n}}$

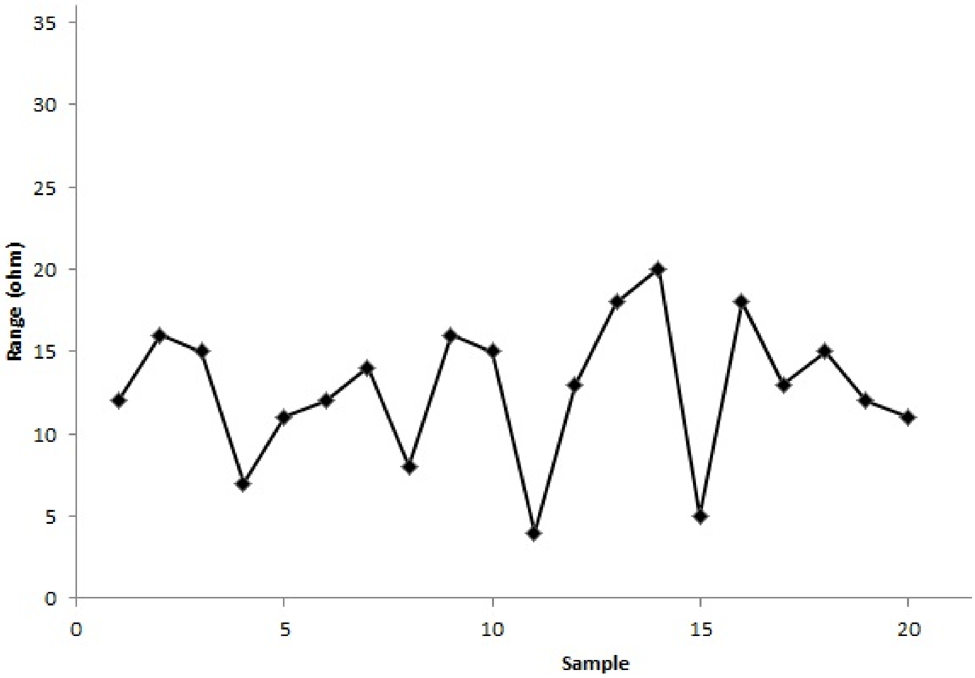
(Over/...)

MATH237 Appendix A

Student Registration No. . . . .

For questions **Q8.** (a) and (c)





Attach this page to your answer sheet

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