**DO NOT REMOVE FROM EXAM VENUE**

# Plymouth University

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|---|---|
| **MODULE CODE:** | **MATH237** |
| **TITLE OF PAPER:** | **ENGINEERING MATHEMATICS AND STATISTICS** |

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|---|---|
| **TIME ALLOWED** | **THREE HOURS** |
| **DATE** | **WEDNESDAY 25 JANUARY 2017** |
| **TIME** | **09:00 – 12:00** |
| **FACULTY** | **SCIENCE AND ENGINEERING** |
| **SCHOOL** | **COMPUTING, ELECTRONICS AND MATHEMATICS** |
| **ACADEMIC YEAR** | **2016 / 2017** |
| **STAGE** | **TWO** |

**INSTRUCTIONS TO CANDIDATES:**

Candidates should attempt **ALL** questions. Questions do not carry equal marks. Marks for parts of questions are shown where appropriate.

**Data Provided:** Mathematical Formulae, Table of Standard Fourier, Laplace and Z Transforms, and Properties, Statistical Formulae and a t-Distribution Table are provided at the end of the examination paper on pages 6 – 12.

**Candidates are not permitted to look at the examination paper until instructed to do so.**

**Semester 1 Exam**

**Q1.** Find the original signal from the following Laplace transforms:

(a) $X(s) = \dfrac{4}{s^3 + 6s^2 - 7s}$

(4 Marks)

(b) $X(s) = \dfrac{s}{s^2 + 8s + 25}$.

(4 Marks)

**Q2.** Find the Fourier transform of

$$x(t) = \begin{cases} t - 6, & 0 < t < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(8 Marks)

**Q3.** Solve the difference equation

$$x_{n+2} + 3x_{n+1} - 10x_n = 0,$$

where $x(0) = 0$ and $x(1) = 5$.

(7 Marks)

**Q4.** Find the original signals from the following transformed signals

(a) $X(f) = \dfrac{-je^{6j\pi f}}{\pi f}$

(3 Marks)

(b) $X(z) = \dfrac{z}{z - 10e^{-8T}}$.

(3 Marks)

**(Over/...)**

**Q5.** A discrete random variable $X$ has the probability distribution $f(x) = \dfrac{x+1}{21}$ where $x = 0, 1, 2, 3, 4, 5$.

(a) Find the probability that $X > 2$.

(3 Marks)

(b) Find the probability that $X \leq 1$.

(2 Marks)

(c) Find the probability of $X = 2$ or $X = 5$, that is $P(X = 2 \,\text{or}\, X + 5)$.

(2 Marks)

**Q6.** A manufacturer claims that a particular component has a mean resistance of 7.5 ohm which is normally distributed. You select a random sample of size 10 and find the following resistances, measured in ohms.

| | | | | |
|---|---|---|---|---|
| 9.2 | 5.5 | 3.9 | 3.8 | 6.6 |
| 9.8 | 6.5 | 5.4 | 7.8 | 5.9 |

Use this sample to construct a 95% confidence interval for the true mean resistance of all such components.

(8 Marks)

**Q7.** A new design for a light bulb is being tested for its lifetime measured in hundreds of hours. A random sample of size 15 is taken and the following results were obtained:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 41 | 31 | 67 | 49 | 54 | 41 | 62 | 59 |
| 43 | 32 | 61 | 50 | 53 | 34 | 60 | |

Find the 5 number summary for this sample and use it to construct a box plot for the sample.

(8 Marks)

**Q8.** The number of times a machine breaks down follows a Poisson distribution with a mean of 9 breakdowns per week. Find the probability that there are

(a) exactly 4 breakdowns in a week.

(1 Mark)

(b) no more than 2 breakdowns in a week.

(2 Marks)

(c) no breakdowns in a day.

(2 Marks)

(d) more than 1 breakdown but no more than 3 breakdowns in a day.

(3 Marks)

**(Over/...)**

**Q9.** A system is governed by the following ODE

$$y''(t) + 6y(t) = 2x'(t)$$

(a) Show that the transfer function for the system is

$$G(s) = \frac{2s}{s^2 + 6}$$

(5 Marks)

(b) Show that the system is critically stable.

(3 Marks)

(c) Find the impulse response for the system.

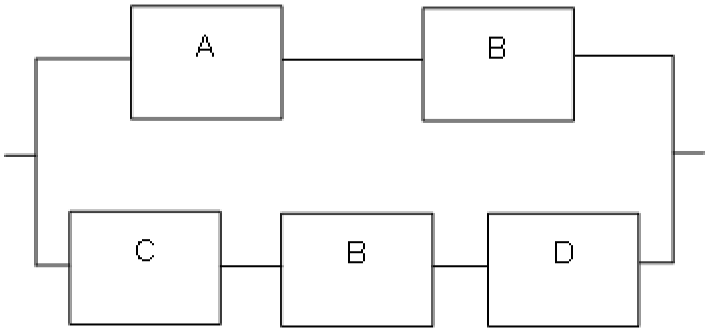
(2 Marks)

(d) Find the system response to an input of $x(t) = 3t$.

(10 Marks)

**(Over/...)**

**Q10.** (a) The system below is constructed of four components A, B, C and D. The failure rate, per hour, of each component is 0.05, 0.15, 0.02 and 0.13, respectively.




(i) Show that the reliability function for this system is

$$R(t) = e^{-0.3t} + e^{-0.2t} - e^{-0.5t}.$$

(9 Marks)

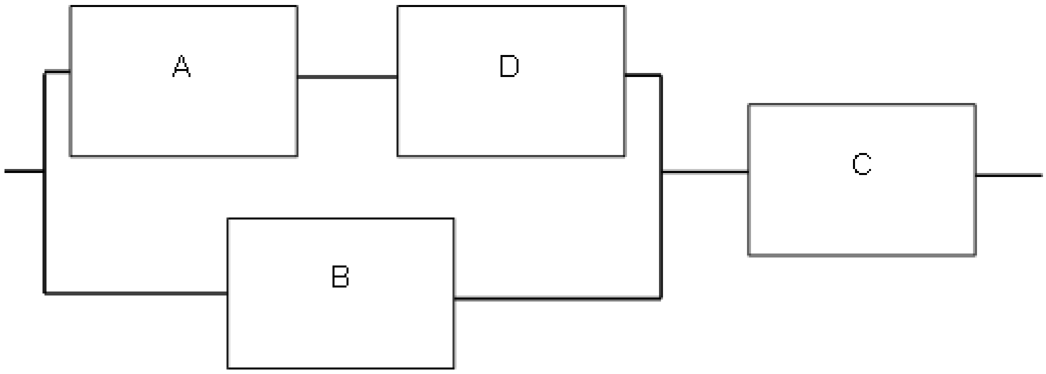
(ii) What is the reliability of the system at 10 hours?

(2 Marks)

(iii) What is the reliability of the system at 2 hours?

(2 Marks)

(b) The same components are set up to create a new system, which is believed to be more reliable and is shown below




(i) Find the reliability function for this system.

(5 Marks)

(ii) What is the reliability for this system at 10 hours?

(2 Marks)

**END OF QUESTIONS**

**(Over/...)**

# Formulae and Tables of Transforms

### Differentiation and Integration

| y(x) | $\frac{dy}{dx}$ | $\int y(x)dx$ |
|---|---|---|
| k | 0 | $\int kdx = kx$ |
| $x^n$ | $nx^{n-1}, n \neq 0$ | $\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$ |
| $e^{kx}$ | $ke^{kx}$ | $\int e^{kx}dx = \frac{e^{kx}}{k}$ |
| $\ln(kx)$ | $\frac{1}{x}$ | $\int \frac{1}{x}dx = \ln(x)$ for $x > 0$ |
| $a^x$ | $a^x \ln(a)$ | $\int a^x dx = \frac{a^x}{\ln(a)}$ for $a > 0$ |
| $\sin(kx)$ | $k\cos(kx)$ | $\int \sin(kx)dx = -\frac{\cos(kx)}{k}$ |
| $\cos(kx)$ | $-k\sin(kx)$ | $\int \cos(kx)dx = \frac{\sin(kx)}{k}$ |

Constants of integration have been omitted.

### Rules of Differentiation

Sum Rule: if $y = u(x) + v(x)$, $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

Constant Rule: if $y = ku(x)$, $\frac{dy}{dx} = k\frac{du}{dx}$

Product Rule: if $y = u(x)v(x)$, $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Quotient Rule: if $y = \frac{u(x)}{v(x)} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

### Rules of Integration

Sum Rule: $\int [u(x) + v(x)]dx = \int u(x)dx + \int v(x)dx$

Constant Rule: $\int ky(x)dx = k\int y(x)dx$

Integration by Parts: $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$

Definite integral: $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

**NOTE:** $a$, $b$, $k$ **and** $n$ **are constants.**

**(Over/...)**

## Table of Standard Fourier transforms

| Description | Function | Transform |
|---|---|---|
| Definition | $v(t)$ | $V(f) = \int_{-\infty}^{\infty} v(t)e^{-2j\pi f t} dt$ |
| Scaling | $v\left(\frac{t}{T}\right)$ | $\lvert T\rvert \times V(fT)$ |
| Time shift | $v(t-T)$ | $V(f) \times e^{-2j\pi f T}$ |
| Frequency shift | $v(t)e^{2\pi j f_0 t}$ | $V(f - f_0)$ |
| Reciprocity | $V(t)$ | $v(-f)$ |
| Addition | $A.v(t) + B.w(t)$ | $A.V(f) + B.W(f)$ |
| Multiplication | $v(t).w(t)$ | $V(f) * W(f)$ |
| Convolution | $v(t) * w(t)$ | $V(f).W(f)$ |
| Delta function | $\delta(t)$ | $1$ |
| Constant | $1$ | $\delta(f)$ |
| Rectangular function | $\mathrm{rect}(t)$ | $\mathrm{sinc}(f) = \frac{\sin(\pi f)}{\pi f}$ |
| Sinc function | $\mathrm{sinc}(t)$ | $\mathrm{rect}(f)$ |
| Heaviside function | $H(t)$ | $\frac{1}{2}\delta(f) - \frac{j}{2\pi f}$ |
| Signum function | $\mathrm{sgn}(t)$ | $-\frac{j}{\pi f}$ |
| Decaying exponential, two-sided | $e^{-\lvert t\rvert}$ | $\frac{2}{1+(2\pi f)^2}$ |
| Decaying exponential, one-sided | $e^{-\lvert t\rvert}.H(t)$ | $\frac{1-2j\pi f}{1+(2\pi f)^2}$ |
| Gaussian function | $e^{-\pi t^2}$ | $e^{-\pi f^2}$ |

**(Over/...)**

## Table of Standard Laplace transforms

| $v(t)$ | $t > 0$ | $V(s)$ |
|---|---|---|
| $\delta(t)$ | unit impulse | $1$ |
| $\delta(t-T)$ | delayed impulse | $e^{-Ts}$ |
| $e^{-at}$ | | $\dfrac{1}{s+a}$ |
| $H(t)$ | Heaviside function | $\dfrac{1}{s}$ |
| $H(t-T)$ | delayed Heaviside function | $\dfrac{1}{s}e^{-Ts}$ |
| $H(t)-H(t-T)$ | rectangular pulse | $\dfrac{1}{s}(1-e^{-Ts})$ |
| $t$ | unit ramp | $\dfrac{1}{s^2}$ |
| $\sin(\omega t)$ | | $\dfrac{\omega}{s^2+\omega^2}$ |
| $\cos(\omega t)$ | | $\dfrac{s}{s^2+\omega^2}$ |
| $t^n$ (n a positive integer) | | $\dfrac{n!}{s^{n+1}}$ |
| $\sinh(at)$ | | $\dfrac{a}{s^2-a^2}$ |
| $\cosh(at)$ | | $\dfrac{s}{s^2-a^2}$ |
| $L\left[\dfrac{d^{(n)}v}{dt^{(n)}}\right]$ | | $s^nV(s)-s^{n-1}v(0)...-v^{n-1}(0)$ |
| $e^{-at}v(t)$ | s-shift | $V(s+a)$ |
| $v(t-T)H(t-T)$ | time-shift | $e^{-Ts}V(s)$ |
| $v(at)$ | time-scaling | $\dfrac{1}{a}V(\dfrac{s}{a})$ |
| $v(t)*g(t)$ | convolution | $V(s)\times G(s)$ |

**(Over/...)**

## Table of Standard Z transforms

| $x(nT)$-sampled | Z transform |
|---|---|
| $a^n$ | $\frac{z}{z-a}$ |
| $\delta(n)$ | $1$ |
| $H(n)$ | $\frac{z}{z-1}$ |
| $nT$ | $\frac{Tz}{(z-1)^2}$ |
| $(nT)^2$ ie when $x(t) = t^2$ | $\frac{T^2 z(z+1)}{(z-1)^3}$ |
| $e^{-anT}$ | $\frac{z}{z-e^{-aT}}$ |
| $1-e^{-anT}$ | $\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$ |
| $nTe^{-anT}$ | $\frac{Tze^{-aT}}{(z-e^{-aT})^2}$ |
| $(1-anT)e^{-anT}$ | $\frac{z[z-e^{-aT}(1+aT)]}{(z-e^{-aT})^2}$ |
| $\sin(n\omega T)$ | $\frac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$ |
| $\cos(n\omega T)$ | $\frac{z(z-\cos(\omega T))}{z^2-2z\cos(\omega T)+1}$ |
| $\sinh(at)$ | $\frac{z\sinh(\omega T)}{z^2-2z\cosh(\omega T)+1}$ |
| $\cosh(at)$ | $\frac{z(z-\cosh(\omega T))}{z^2-2z\cosh(\omega T)+1}$ |
| $e^{-anT}x(nT)$ | $X(ze^{aT})$ |
| $x(nT-n_0T)$ | $\frac{X(z)}{z^{n_0}}$ |
| $nx(n)$ | $-z\frac{dX(z)}{dz}$ |
| $x(nT+T)$ | $zX(z)-zx(0)$ |
| $x(nT+2T)$ | $z^2X(z)-zx(T)-z^2x(0)$ |

**(Over/...)**

# Statistics formulae and tables

## Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A|B) \times P(B) = P(B|A) \times P(A)$$

## Probability Distributions

The binomial distribution $P(k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$ for $k = 0, 1, 2, ..n$

The Poisson distribution $P(k) = \frac{\lambda^k \exp(-\lambda)}{k!}$ for $\lambda > 0$ for $k = 0, 1, 2, .., n$

The exponential distribution (pdf) $f(t) = \lambda \exp(-\lambda t)$ for $\lambda > 0$; MTBF $= \frac{1}{\lambda}$ for $t > 0$

The Weibull distribution (cdf) $F(x) = 1 - \exp[-(\frac{x}{\beta})^\alpha]$; MTBF=$\beta\Gamma(1 + \frac{1}{\alpha})$ for $x > 0$

The reliability function for the exponential distribution $R(t) = 1 - \int_0^t f(s)ds = \exp(-\lambda t)$

## Confidence Intervals

$\mu \in (\bar{X} - z\frac{\sigma}{\sqrt{n}}, \bar{X} + z\frac{\sigma}{\sqrt{n}})$ with the probability $1 - 2\alpha$, where $\alpha$ is the confidence level; $P(X > z) = \frac{1}{2\pi} \int_z^\infty \exp(-\frac{x^2}{2})dx = \alpha$

Standard deviation of sample

$$s = \sqrt{\frac{1}{n-1} \sum_{k=1}^{n} (x_k - \bar{x})^2} = \sqrt{\frac{1}{n-1} \left( \sum_{k=1}^{n} (x_k)^2 - n\bar{x}^2 \right)}$$

**(Over/...)**

## t-distribution table

Critical values of the t-distribution are given for areas $\alpha$ in one tail.

| $\alpha$ | 0.100 | 0.050 | 0.025 | 0.010 | 0.005 | 0.001 | 0.0005 |
|---|---|---|---|---|---|---|---|
| DF | | | | | | | |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.30 | 636.619 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 | 31.599 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| 7 | 1.4150 | 1.8950 | 2.3650 | 2.998 | 3.499 | 4.7850 | 5.408 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| 13 | 1.3500 | 1.771 | 2.1600 | 2.6500 | 3.012 | 3.852 | 4.221 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 |
| 25 | 1.316 | 1.708 | 2.0600 | 2.4850 | 2.787 | 3.4500 | 3.72500 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | 3.659 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| 50 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 3.261 | 3.496 |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| 80 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 | 3.416 |
| 120 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 3.160 | 3.373 |
| infinity | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.091 | 3.291 |

**END OF PAPER**