

## Stats coursework solutions

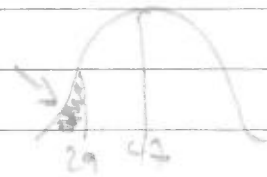
1. question gives variance, so need to square root:  $\sigma = 9$   
 a)  $P(T \geq 55)$

$$P\left(z > \frac{55 - 47}{9}\right) \\ = P(z > 0.89) = 0.1867 \quad A$$



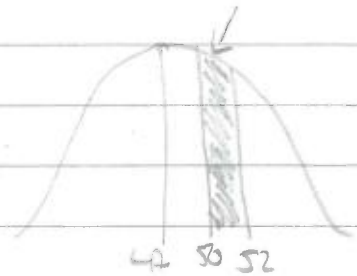
b)  $P(T < 29)$   
 $P\left(z < \frac{29 - 47}{9}\right)$

$$= P(z < -2) = 0.0228 \quad D$$



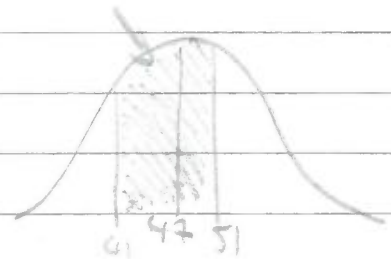
c)  $P(50 < T < 52)$

$$P\left(z > \frac{50 - 47}{9}\right) - P\left(z > \frac{52 - 47}{9}\right) \\ P(z < 0.33) - P(z > 0.56) \\ = 0.3707 - 0.2877 \\ = 0.083 \quad C$$



d)  $P(41 < T < 51)$

$$1 - P\left(z < \frac{41 - 47}{9}\right) - P\left(z > \frac{51 - 47}{9}\right) \\ 1 - P(z < -0.67) - P(z < 0.44) \\ = 1 - 0.2514 - 0.3300 \\ = 0.4186 \quad B$$



2.

$$\begin{aligned}
 a) P(X > 100) &= 1 - P(X < 100) \\
 &= 1 - \left( 1 - \exp \left( - \left( 10 / 3.5 \right)^{0.82} \right) \right) \\
 &= \exp \left( - \left( 10 / 3.5 \right)^{0.82} \right)
 \end{aligned}$$

$$= 0.093932388$$

$$\approx 0.0939$$

B

$$b) P(X < 10) = 1 - \exp \left( - \left( 1 / 3.5 \right)^{0.82} \right)$$

$$= 1 - 0.699083921$$

$$= 0.300916078$$

$$\approx 0.3009$$

A

$$\begin{aligned}
 c) P(X > 537) &= 1 - \left( 1 - \exp \left( - \left( 53.7 / 3.5 \right)^{0.82} \right) \right) \\
 &= \exp \left( - \left( 53.7 / 3.5 \right)^{0.82} \right)
 \end{aligned}$$

$$= 8.395823772 \times 10^{-5}$$

$$\approx 8.3958 \times 10^{-5}$$

D

$$d) P(X < 81) = 1 - \exp \left( - \left( 8.1 / 3.5 \right)^{0.82} \right)$$

$$= 1 - 0.136714941$$

$$= 0.863285058$$

$$\approx 0.8633$$

D

3.

a. per year  $\lambda = 2$ , so per month  $\lambda = \frac{2}{12} = \frac{1}{6}$ 

$$P(X=2) = \left(\frac{1}{6}\right)^2 \frac{e^{-\frac{1}{6}}}{2!} = \frac{e^{-\frac{1}{6}}}{72} = 0.01175669$$

$$\approx 0.0118 \quad D$$

b.  $P(X < 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$   
per year so  $\lambda = 2$ 

$$P(X < 4) = \frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!} + \frac{2^3 e^{-2}}{3!}$$

$$= 0.135335283 + 0.270670566 + 0.270670566 + 0.180447044$$

$$= 0.85712346$$

$$\approx 0.8571 \quad C$$

$$c. P(X=5) = \frac{2^5 e^{-2}}{5!} = 0.036089408$$

$$\approx 0.0361 \quad D$$

d. per week  $\lambda = \frac{2}{52} = \frac{1}{26}$ 

$$P(X=1) = \left(\frac{1}{26}\right)^1 \frac{e^{-\frac{1}{26}}}{1!} = 0.037010335$$

$$\approx 0.0370 \quad A$$

4.

$x$	0	1	2	3	4	5
$p$	$\frac{2}{10}$	$\frac{9}{10}$	$\frac{22}{10}$	$\frac{41}{10}$	$\frac{66}{10}$	$\frac{97}{10}$

$$a) \sum p_i = 1 \quad \sum p_i = \frac{2 + 9 + 22 + 41 + 66 + 97}{10} = \frac{237}{10}$$

$$10 = 237 \quad \text{D}$$

$$b) P(x=3) = f(3) = \frac{3 \times 3^2 + 4 \times 3 + 2}{237} = \frac{41}{237}$$

$$\approx 0.17299578$$

$$\approx 0.1730$$

B

c) Expected value:  $\sum x_i p_i$ 

$$E(x) = \left(0 \times \frac{2}{237}\right) + \left(1 \times \frac{9}{237}\right) + \left(2 \times \frac{22}{237}\right) + \left(3 \times \frac{41}{237}\right) + \left(4 \times \frac{66}{237}\right) + \left(5 \times \frac{97}{237}\right)$$

$$= \frac{9}{237} + \frac{44}{237} + \frac{123}{237} + \frac{264}{237} + \frac{485}{237}$$

$$= \frac{925}{237} = 3.902953587$$

$$\approx 3.9030$$

D

$$d) P(x=104) = P(x=1) + P(x=4) \\ = \frac{9}{237} + \frac{66}{237} = \frac{75}{237}$$

$$= 0.316455696$$

$$\approx 0.3165$$

B

5

a) Expected distance =  $\frac{1}{\lambda}$ ,  $\lambda = 0.25$

$$\frac{1}{0.25} = 4 \text{ m} \quad A$$

b)  $P(x < 2) = 1 - \exp(-2 \times \frac{1}{4})$

$$= 1 - 0.606530659$$

$$= 0.39346934$$

$$\approx 0.3935 \quad A$$

c)  $P(x > 5) = 1 - (P(x < 5)) = 1 - (1 - \exp(-5 \times \frac{1}{4}))$

$$= \exp(-\frac{5}{4})$$

$$= 0.286504796$$

$$\approx 0.2865 \quad D$$

d)  $\text{Var}(x) = \frac{1}{\lambda^2}$

$$\frac{1}{(0.25)^2} = 16 \text{ m}^2 \quad C$$

6.

a)	sample	mean	range	sample	mean	range
	1	6.2	3	16	4.6	6
	2	5.4	8	17	6	8
	3	4.2	7	18	4.2	3
	4	2.6	5	19	3.6	7
	5	4.4	6	20	5.4	4
	6	6.2	7	21	5.2	6
	7	6.2	4	22	2.2	2
	8	4.6	7	23	4.2	6
	9	3.6	6	24	5	7
	10	5.4	5	25	6.4	7
	11	2.6	3	26	4.2	6
	12	7.2	2	27	6.6	4
	13	5	7	28	4	4
	14	4.2	7	29	3.4	6
	15	5.6	8	30	3.6	4
	$\Sigma$	73.4	85	$\Sigma$	68.6	80

$$\text{Grand mean: } \frac{73.4 + 68.6}{30} = \frac{142}{30} \approx 4.7333$$

$$\text{mean range: } \frac{85 + 80}{30} = \frac{165}{30} \approx 5.5$$

$$\text{Samples of size 5: } d_n = 2.326, D_{0.999} = 0.16, D_{0.975} = 0.37 \\ D_{0.025} = 1.81, D_{0.001} = 2.34$$

UCL	$\frac{142}{30} + 3.09 \times \frac{5.5}{2.326\sqrt{5}} \approx 8.0009$	$2.34 \times 5.5 = 12.87$
UWL	$\frac{142}{30} + 1.96 \times \frac{5.5}{2.326\sqrt{5}} \approx 6.8060$	$1.81 \times 5.5 = 9.955$
LWL	$\frac{142}{30} - 1.96 \times \frac{5.5}{2.326\sqrt{5}} \approx 2.6607$	$0.37 \times 5.5 = 2.035$
LAL	$\frac{142}{30} - 3.09 \times \frac{5.5}{2.326\sqrt{5}} \approx 1.4658$	$0.16 \times 5.5 = 0.88$



see plots at the end.

- b. Range Chart: no samples that are between the action and warning lines.  
 no samples above the action lines  
 no runs of 5 or more that cross a warning or action limit  
 no runs of 6 or more lying entirely above or below the grand mean  
 $\Rightarrow$  in control with respect to the range.

means Chart: no samples above the action limit  
 no two consecutive values outside the same warning limit  
 no runs of 5 or more that cross a warning or action limit  
 no runs of 6 or more lying entirely above or below the grand mean  
 $\Rightarrow$  in control with respect to the mean

$\Rightarrow$  process is in control

c. next 5 samples

sample	mean	range
31	6	5
32	6.6	5
33	4.6	7
34	6	6
35	4.8	7

see plots at the end.

Does the process remain in control?

Range chart: no sample infringes the action limits

no run of 5 or more infringing a warning limit

no run of 6 or more entirely above or below the mean

→ remains in control

mean chart: no sample infringes the action limits

no run of 5 or more crossing a warning limit

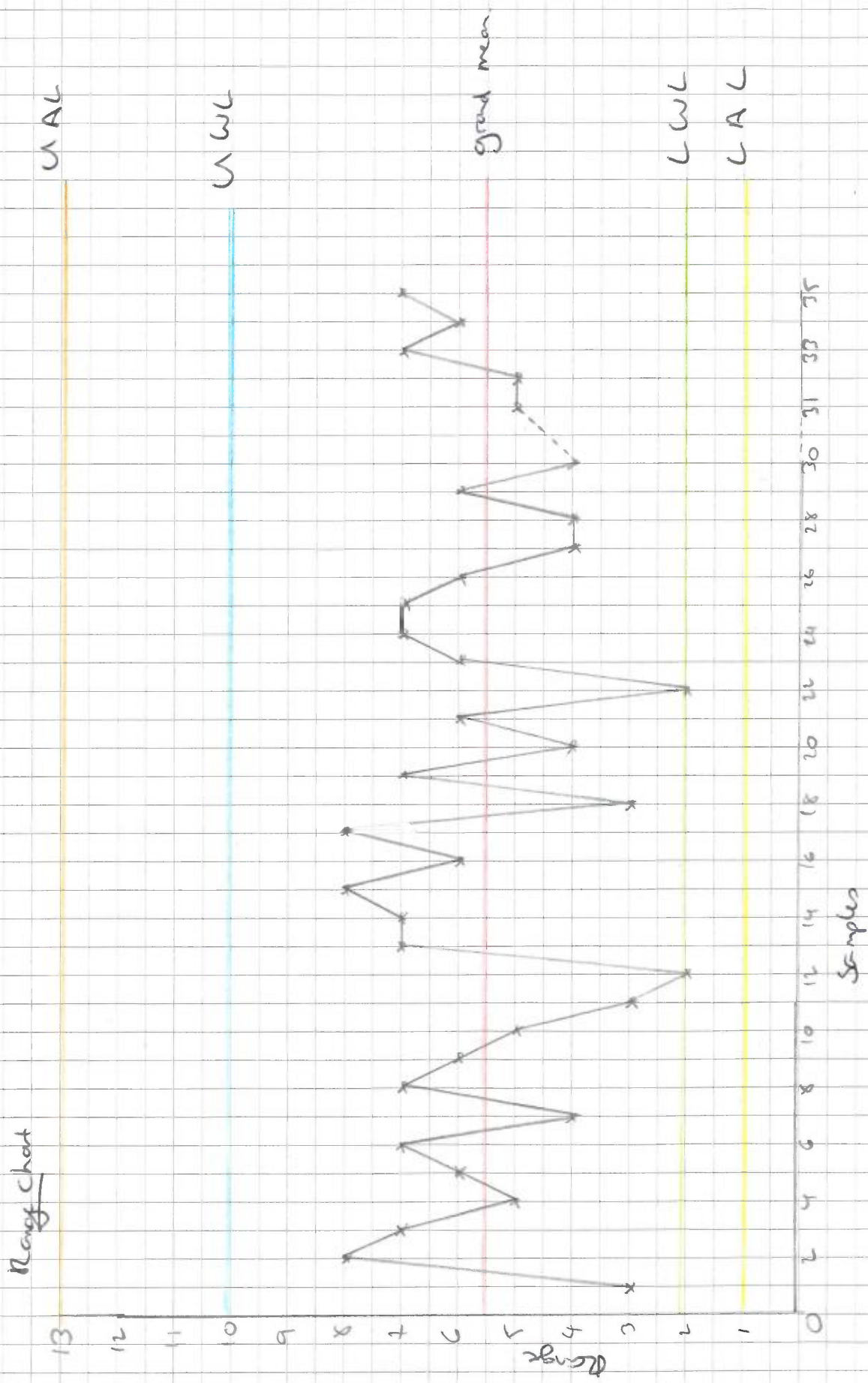
no run of 6 or more entirely above or below the mean

→ remains in control

⇒ The process remains in control



5



# Means Chart

