Basic Probability and Reliability of systems

For Questions 1-5, select **one** answer only. If you select the other option, you need to also provide the correct numerical answer.

1. A probability plot suggests that the lifetime, T, in years of a machine has a normal distribution with mean and variance 47 and 81 respectively.
   (a) What is the probability that the machine will survive for more then 55 years?
   A. 0.1867 B. 0.2199 C. 0.1894 D. Other ☐
   (b) What is the probability that the machine will fail before 29 years?
   A. 0.0315 B. 0.244 C. 0.075 D. Other ☐
   (c) What is the probability that the machine will survive longer than 50 years but fail before 52 years?
   A. 0.0864 B. 0.0795 C. 0.083 D. Other ☐
   (d) What is the probability that the machine will survive longer than 41 years but fail before 51 years?
   A. 0.0786 B. 0.4186 C. 0.4154 D. Other ☐

   (4 Marks)

2. The lifetime of a component, in tens of hours, follows a Weibull distribution with $\alpha = 0.82$ and $\beta = 3.5$.
   (a) What is the probability that the component survives longer than 100 hours?
   A. 0.906 B. 0.0939 C. 0.849 D. Other ☐
   (b) What is the probability that the component fails before 10 hours?
   A. 0.3009 B. 0.1349 C. 0.699 D. Other ☐
   (c) What is the probability that the component survives longer then 537 hours?
   A. 0.544 B. 8.8396 x10^{-5} C. 0.9999 D. Other ☐
   (d) What is the probability that the component fails before 81 hours?
   A. 0.1367 B. 0.22 C. 0.076 D. Other ☐

   (4 Marks)

3. The rate at which a particular component fails follows a Poisson distribution with an average of 2 per year. What is the probability that there are
   (a) 2 breakdowns in a month?
   A. 0.2707 B. 0.5413 C. 0.752 D. Other ☐
   (b) less than 4 breakdowns in a year?
   A. 0.0902 B. 0.7218 C. 0.8571 D. Other ☐
   (c) exactly 5 breakdowns in a year?
   A. 0.0282 B. 0.0842 C. 0.6767 D. Other ☐
   (d) one breakdown in a week?
   A. 0.037 B. 0.1353 C. 0.8669 D. Other ☐

   (4 Marks)

4. Let X be the number of faults on a manufactured item at random. The probability distribution for X is $f(x) = \dfrac{3x^2 + 4x + 2}{k}$ for $x = 0, 1, 2, 3, 4, 5$.

   (a) Find the value of k.

   A. 9 B. 41 C. 500 D. Other ☐

   (b) What is the probability of an item having three faults?

   A. 0.1729 B. 0.1730 C. 0.227 D. Other ☐

   (c) What is the expected value?

   A. 3.436 B. 4 C. 3.975 D. Other ☐

   (d) What is the probability of an item having either 1 or 4 faults?

   A. 0.901 B. 0.3165 C. 0.731 D. Other ☐

(4 marks)

5. The frequency at which a wire has a fault, per meter, follows an exponential distribution with $\lambda = 0.25$.

   (a) What is the expected distance between faults?

   A. 4 B. 0.25 C. 2 D. Other ☐

   (b) What is the probability of having a fault in less than 2 meters?

   A. 0.3935 B. 0.4759 C. 0.2078 D. Other ☐

   (c) What is the probability of having a fault in more than 5 meters?

   A. 0.0337 B. 0.0573 C. 0.253 D. Other ☐

   (d) What is the variance?

   A. 0.0625 B. 4 C. 16 D. Other ☐

(4 Marks)

6. (a) The manufacture of a particular component is controlled by its thickness in mm. Construct the Means and Range charts for the component using the data below.

Sample number					
1	7	8	6	5	5
2	9	1	8	4	5
3	1	4	3	8	5
4	6	4	1	1	1
5	6	7	1	4	4
6	9	3	8	9	2
7	4	5	8	7	7
8	1	1	5	8	8
9	1	7	5	3	2
10	7	6	3	3	8
11	4	2	3	3	1
12	6	8	8	6	8
13	7	2	7	8	1
14	6	1	2	8	4
15	5	9	7	6	1
16	9	3	3	5	3
17	1	9	7	5	8
18	6	4	3	4	4
19	6	1	1	8	2
20	5	6	8	4	4
21	4	3	4	6	9
22	2	3	1	2	3
23	7	5	1	3	5
24	7	8	1	5	4
25	6	2	6	9	9
26	3	8	2	3	5
27	4	7	7	8	7
28	2	4	6	6	2
29	7	1	4	3	2
30	4	3	5	5	1

(b) Is the process in control?

(c) Below are the next 5 samples, add these to your charts and assess the state of control of the process.

Sample number					
1	3	7	7	8	5
2	9	7	9	4	4
3	6	2	4	9	2
4	8	2	7	6	7
5	2	8	6	7	1

(30 Marks)

Coursework Deadline: **Thursday 13th December 3pm** to be submitted via the Faculty Office.