$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

① $y(n) = x(n) * h(n) = \sum_{k=0}^{\infty} h(n)x(n-k)$

② $r_{yy}(m) = r_{xx}(m) * h(m) * h(-m) = r_{xy}(m) * h(-m)$

③ $P_{yy}(w) = P_{xx}(w) \cdot H(w) \cdot H^*(w) = P_{xx}(w)|H(w)|^2$

④ $r_{xx}(m) \longleftrightarrow P_{xx}(w)$ （傅里叶） $r_{xx}(m) \xleftrightarrow{z} \Phi_{xx}(z)$

⑤ $P_{xx}(w) = \Phi_{xx}(z)\big|_{z=e^{jw}}$

$r_{xx}(m)$

① 对称 ② 斜对角线元素相等 ③ R为非负定

④ $r_{xx}(0)$ 最大为正数 （$r_{xx}(0) = m_x^2 + \sigma_x^2$）

⑤ $r_{xx}(m)$ 为实数

$P_{xx}(w)$

① 非负 ② 实偶函数 ③ 2π 为周期

相关卷积定理

$$\begin{cases} e = a * b \\ f = c * d \\ r_{ef}(m) = r_{ac}(m) * r_{bd}(m) \end{cases}$$

Vina

1. 维纳-霍夫方程

$$\phi_{xs}(k)=\sum_{m=0}^{\infty} h_{opt}(m)\,\phi_{xx}(k-m) \qquad k\geq 0$$

2. 维纳滤波(时)

$$J(w)=E[e_n^2]=\sigma_d^2-2P^TW+W^TRW$$

(都是列向量)　　(大是负)

$$J_{min}=\sigma_d^2-P^TW_{opt}=\sigma_d^2-W_{opt}^T\cdot P \qquad W_{opt}=R^{-1}P \Longrightarrow P=R\cdot W_{opt}$$

$$R=E[X(n)X^T(n)], \quad P=E[X(n)d(n)]$$

3. 维纳滤波(z)(习题)　($x=s+v$, s与v不相关)

① $\Phi_{xx}(z)=\sigma_w^2B(z)B(z^{-1})$

② $H_{opt}(z)=\dfrac{1}{\sigma_w^2B(z)}\left[\dfrac{\Phi_{xs}(z)}{B(z^{-1})}\right]_+$ 因果　　($H_{opt}=\dfrac{\Phi_{xs}(z)}{\Phi_{xx}(z)}$ 非因果)

③ $E[e^2]_{min}=\dfrac{1}{2\pi j}\oint\left[\Phi_{ss}(z)-H_{opt}(z)\cdot\Phi_{xs}(z^{-1})\right]z^{-1}dz$

4. 因果维纳预测值.(N步)($v=0$)　　$d(n)=s(n+N)$

① $\Phi_{xx}(z)=\Phi_{xs}(z)=\Phi_{ss}(z)=\sigma_w^2B(z)B(z^{-1})$

② $H_{opt}(z)=\dfrac{1}{\sigma_w^2B(z)}\left[\dfrac{\Phi_{xs}(z)}{B(z^{-1})}\cdot z^N\right]=\dfrac{1}{B(z)}\left[\underset{\updownarrow\; b(n+N)u(n)}{z^NB(z)}\right]_+$

③ $J_{min}=\sigma_w^2\sum_{k=0}^{N-1}b^2(k)$

④ $a_{pk}=-h(k-1)$ (P步预测)(Yule-walker方程)

LMS

① $y(n) = \hat{W}^T(n) X(n)$

② $e(n) = d(n) - y(n)$

③ $\hat{W}(n+1) = \hat{W}(n) + 2\mu X(n) e(n)$

④ $\hat{\nabla} J(w) = -2 X(n) e(n)$

最速下降法

$0 < \mu < \frac{1}{\lambda_{max}}$，λ为R的特征值

$W(n+1) = W(n) + \mu[-\nabla J(W(n))]$

$= W(n) + 2\mu[P - RW(n)]$

$\nabla J(w) = -2P + 2RW(n)$

谱估计

1. 经典：BT法

$\hat{\Phi}'_{xx}(m) = \frac{1}{N-|m|} \sum_{n=0}^{N-1-|m|} x(n) x(n+m)$，无偏，方差大

$\hat{\Phi}_{xx}(m) = \frac{1}{N} \sum_{n=0}^{N-1-|m|} x(n) x(n+m)$，有偏，方差小，均方误差小

2. 周期图法

$X_N(e^{jw}) = \sum_{n=0}^{N-1} x(n) e^{-jwn}$

$I_N(w) = \frac{1}{N} |X_N(e^{jw})|^2$，有偏，不是一致估计

$\hat{P}_{xx}(w) = I_N(w)$

↓ 改进

Bartlette 平均周期图法，方差降为原来的 $\frac{1}{L}$.

Yule-walker

$$\begin{bmatrix} \Phi_{xx}(0) & \Phi_{xx}(-1) & \Phi_{xx}(-2) \\ \Phi_{xx}(1) & \Phi_{xx}(0) & \Phi_{xx}(-1) \\ \Phi_{xx}(2) & \Phi_{xx}(1) & \Phi_{xx}(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_{p_1} \\ a_{p_2} \end{bmatrix} = \begin{bmatrix} E[e^2]_{min} \\ 0 \\ 0 \end{bmatrix}$$

二阶预测

$$w(n) \longrightarrow \boxed{h(n)} \longrightarrow x(n)$$

3. 参数化模型

$$\begin{cases} ARMA(p,q) \\ MA(q) \\ AR(p) \longrightarrow H(z) = \dfrac{1}{1+\sum_{k=1}^{p} a_k z^{-k}} \end{cases}$$

$$x(n) = -\sum_{k=1}^{p} a_k x(n-k) + w(n)$$

$$\hat{P}_{xx}(w) = \sigma_w^2 \, |H(w)|^2$$