Signals and Systems

Experiment No. # 11

Objective:

The objective of this lab is to create a practical understanding of the Discrete Time Fourier Transform (Chapter 5 of textbook) and to prove some properties of the DTFT.

Theoretical Background:

The Discrete Time Fourier Transform is used for representation of discrete-time a-periodic signals: DTFT representation of a finite discrete signal (Analysis equation):

�(� ��) = ∑ �[�]� −��� �−1 �=0

Calculation of a signal from its DTFT (Synthesis equation):

�[�] = 1/2� ∫ �(� ��)� ��� �� 2� 0

Properties of the Discrete Time Fourier Transform:

There are many properties associated with the DTFT, in this lab the students will prove the following two properties, where:

- x[n] and y[n] are two discrete time a-periodic signals
- �(� ��) and �(� ��) are the DTFT representation of x[n] and y[n] respectively

1. Convolution Property:

This property states that the DTFT of convolution of two discrete time sequences is equal to multiplication of the DTFTs of the sequences.

�[�] * �[�] = ����� (�(� ��)�(� ��))

2. Multiplication Property:

The DTFT of multiplication of two discrete time a-periodic signals is equal to the periodic convolution of the DTFT of the individual signals.

�[�]�[�] = ����� (1/2� ∫ �(� ��)�(� �(�−�)) �� 2� 0)

Tasks:

The following tasks are to be performed by each student.

Task 1:

Create separate functions in MATLAB for Discrete Time Fourier Transform (DTFT), i.e. analysis equation, and Inverse Discrete Time Fourier Transform (IDTFT), i.e. synthesis equation.

Task 2:

Consider the rectangular pulse: �[�] = { 1, |�| ≤ �1 0, |�| > �1 which is illustrated below for �1 = 2. Find the DTFT of x[n] using the DTFT function created in Task 1. Then, find the IDFT of this result using the IDTFT function, also created in Task 1. Using subplot, display the input signal x[n], and the output of the IDTFT function. The result in both subplots should be same. This task is an implementation of Example 5.3 of your textbook.

Task 3:

Using the functions created in task 1, prove the convolution and multiplication properties of the DTFT in separate codes. Display the time domain (n- domain) results in each case using the subplot command. The specifications of the two signals are given below for both properties separately.

For Convolution Property:

• x[n]=[1 0 1 0 1]

• y[n]=[1 1 0 1 0]

• N is the length of the signal.

For Multiplication Property:

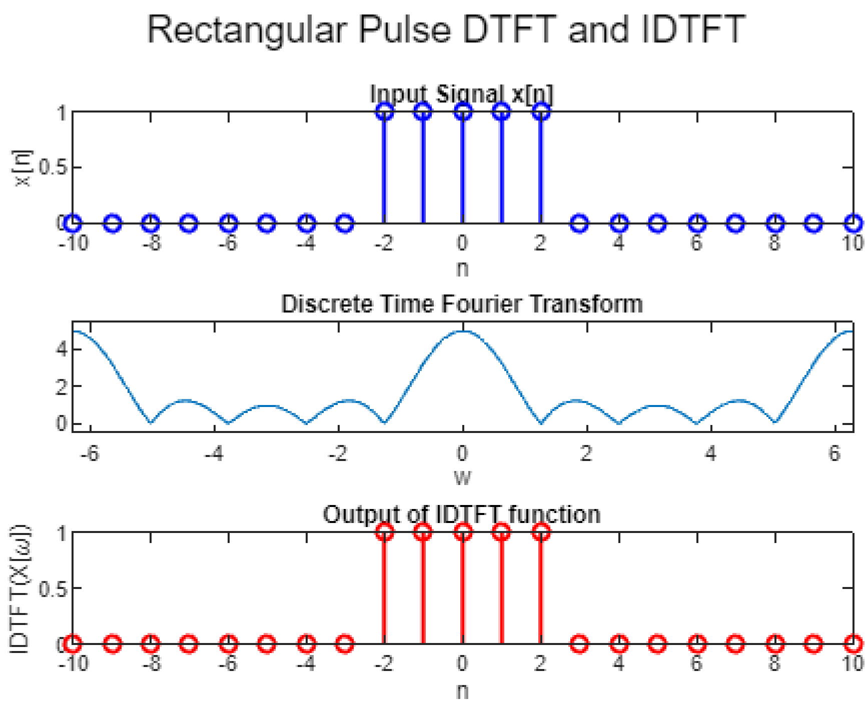
• x[n]=[1 2 3 1 3]

• y[n]=[3 4 3 3 2]

• N is the length of the signal.

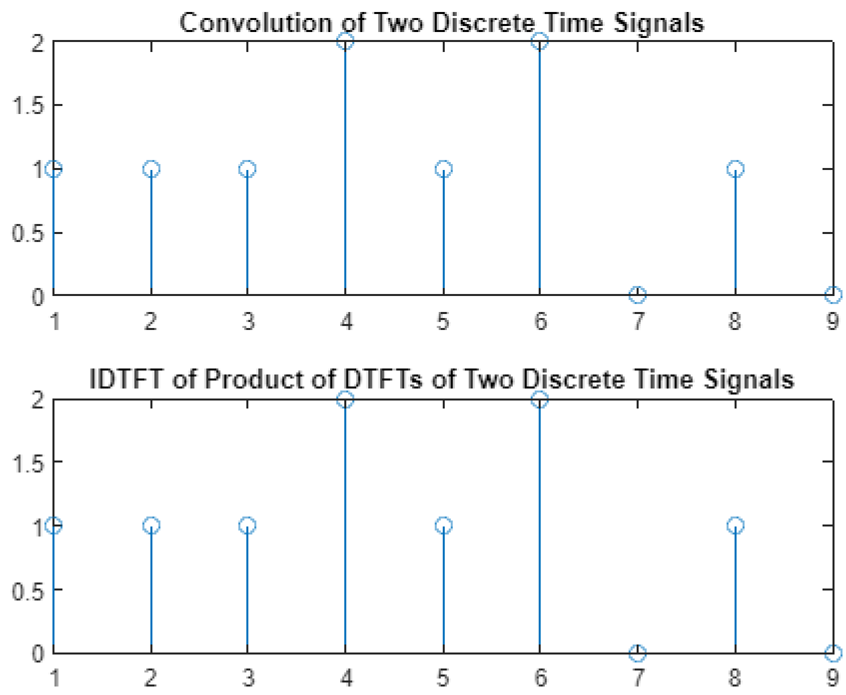
```
N1 = 2;
n = -10:10;
x_n = (abs(n) <= N1);
N = length(n);

% Find DTFT of x[n]
X_w = DTFT(x_n, N);

% Find IDFT of X(w)
x_reconstructed = IDTFT(X_w, N);

% Plotting
figure;

% Plot input signal x[n]
subplot(3,1,1);
stem(n, x_n, 'b', 'LineWidth', 1.5);
title('Input Signal x[n]');
xlabel('n');
ylabel('x[n]');

%Plot of DTFT function output
subplot(3,1,2)
ezplot(abs(X_w));
title('Discrete Time Fourier Transform');

% Plot output of IDTFT function
subplot(3,1,3);
stem(n, abs(x_reconstructed), 'r', 'LineWidth', 1.5);
```

```
title('Output of IDTFT function');
xlabel('n');
ylabel('IDTFT(X[\omega])');

% Adjust subplot layout
sgtitle('Rectangular Pulse DTFT and IDTFT');

% Ensure the plots are on the same scale
axis tight;
```




```
x = [1 0 1 0 1];
N1 = length(x);
y = [1 1 0 1 0];
N2 = length(y);

figure;
% LHS
LHS = conv(x,y);
subplot(2,1,1);
stem(abs(LHS));
title('Convolution of Two Discrete Time Signals');

% RHS
A = DTFT(x,N1);
B = DTFT(y,N2);
C = A .* B;
RHS = IDTFT(C, length(LHS));
```

```
subplot(2,1,2);
stem(abs(RHS));
title('IDTFT of Product of DTFTs of Two Discrete Time Signals');
```




```
x = [1 2 3 1 3];
N1 = length(x);
y = [3 4 3 3 2];
N2 = length(y);

figure;
% LHS
LHS = x .* y;
subplot(2,1,1);
stem(abs(LHS));
title('Product of Two Discrete Time Signals');

% RHS
syms w theta
A = DTFT(x, N1);
C = DTFT(y, N2);
B = subs(A, w, theta);
D = subs(C, w, w-theta);
E = B * D;
F = 1/(2.*pi) .* int(E, theta, 0, 2*pi);
RHS = IDTFT(F,length(LHS));

subplot(2,1,2);
stem(abs(RHS));
```

```
title('Periodic Convolution of DTFT of Two Discrete Time Signals');
```