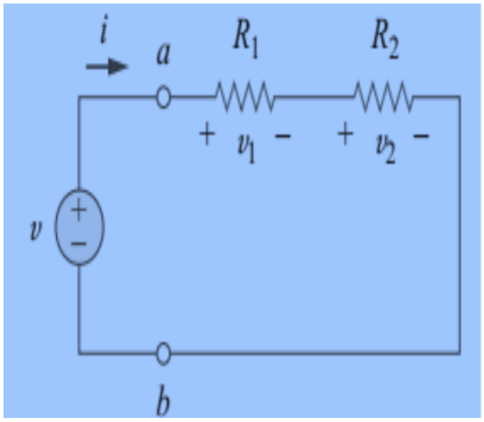
3 - Resistance

Series Resistors



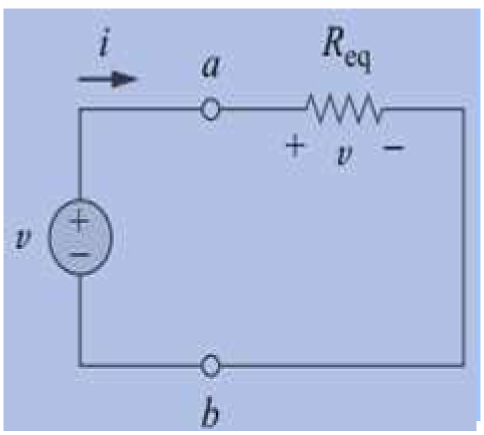
The two resistors are in series, since the same current i flows in both of them.

Voltage across in each resistors are $v_1 = iR_1$ and $v_2 = iR_2$

Applying KVL to loop, we obtain

$$-v + v_1 + v_2 = 0$$

$$\Rightarrow v = v_1 + v_2 = i(R_1 + R_2)$$



$$V = iR_{eq}$$

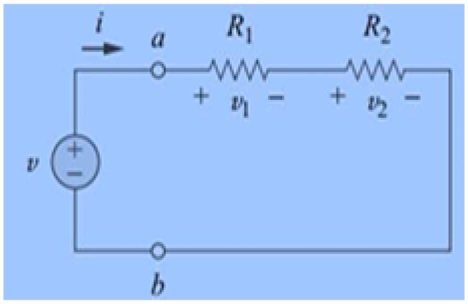
$$\therefore R_{eq} = R_1 + R_2$$

The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances

For N resistors in series then

$$R_{eq} = R_1 + R_2 + \ \dots\ \dots\ + R_N = \sum_{n=1}^{N} R_n$$

Voltage Divider Principle



Voltage across in each resistors are

$$v_1 = iR_1 \text{ and } v_2 = iR_2$$

$$i = \frac{v}{R_1 + R_2}$$

putting value of i in v_1 and v_2

$$v_1 = \frac{R_1}{R_1 + R_2} v$$

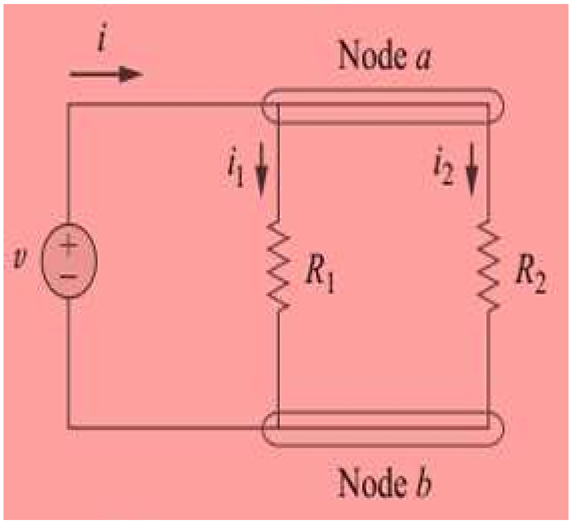
$$v_2 = \frac{R_2}{R_1 + R_2} v$$

If a voltage divider has N resistors in series with the source voltage v, the nth resistor (R_n) will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + R_3 + \ \dots\ \dots\ + R_N} v$$

Voltage divider rule is applied in the series connected circuit

Parallel Resistor



Two resistors are connected in parallel with voltage source and therefore have the same voltage across them

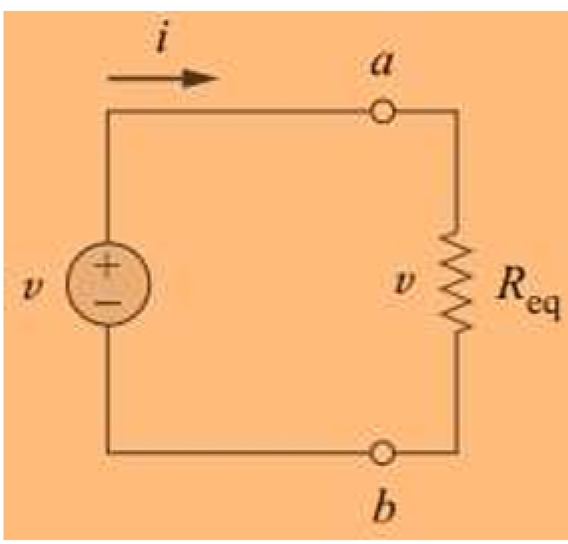
$$v = i_1 R_1 = i_2 R_2$$

$$i_1 = \frac{v}{R_1} \text{ and } i_2 = \frac{v}{R_2}$$

Applying KCL to node a, we obtain

$$i = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

$$\therefore \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

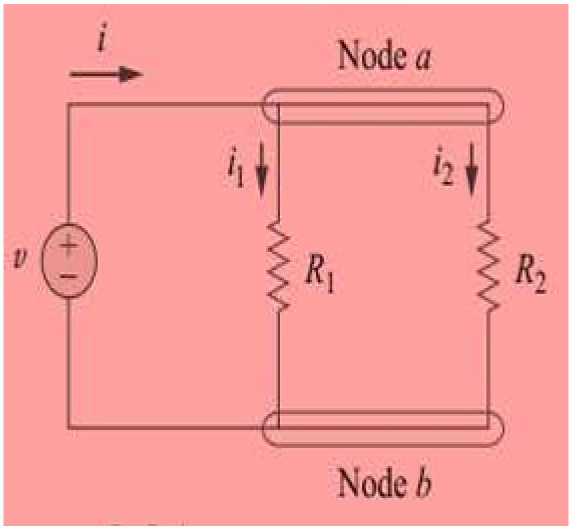
The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum

For N resistors in parallel then

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots\dots + \frac{1}{R_N}$$

R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination

Current Divider Principle



Voltage across the resistor is

$$v = R_{eq} i = \frac{R_1 R_2}{R_1 + R_2} i$$

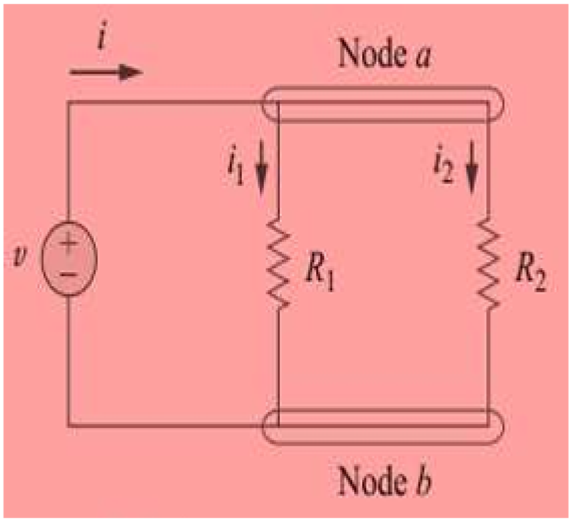
Current flow through in each resistors are

$$i_1 = \frac{v}{R_1} \Rightarrow i_1 = \frac{R_2}{R_1 + R_2} i$$

$$i_2 = \frac{v}{R_2} \Rightarrow i_2 = \frac{R_1}{R_1 + R_2} i$$

Current divider rule using conductance

$$G = \frac{1}{R}$$



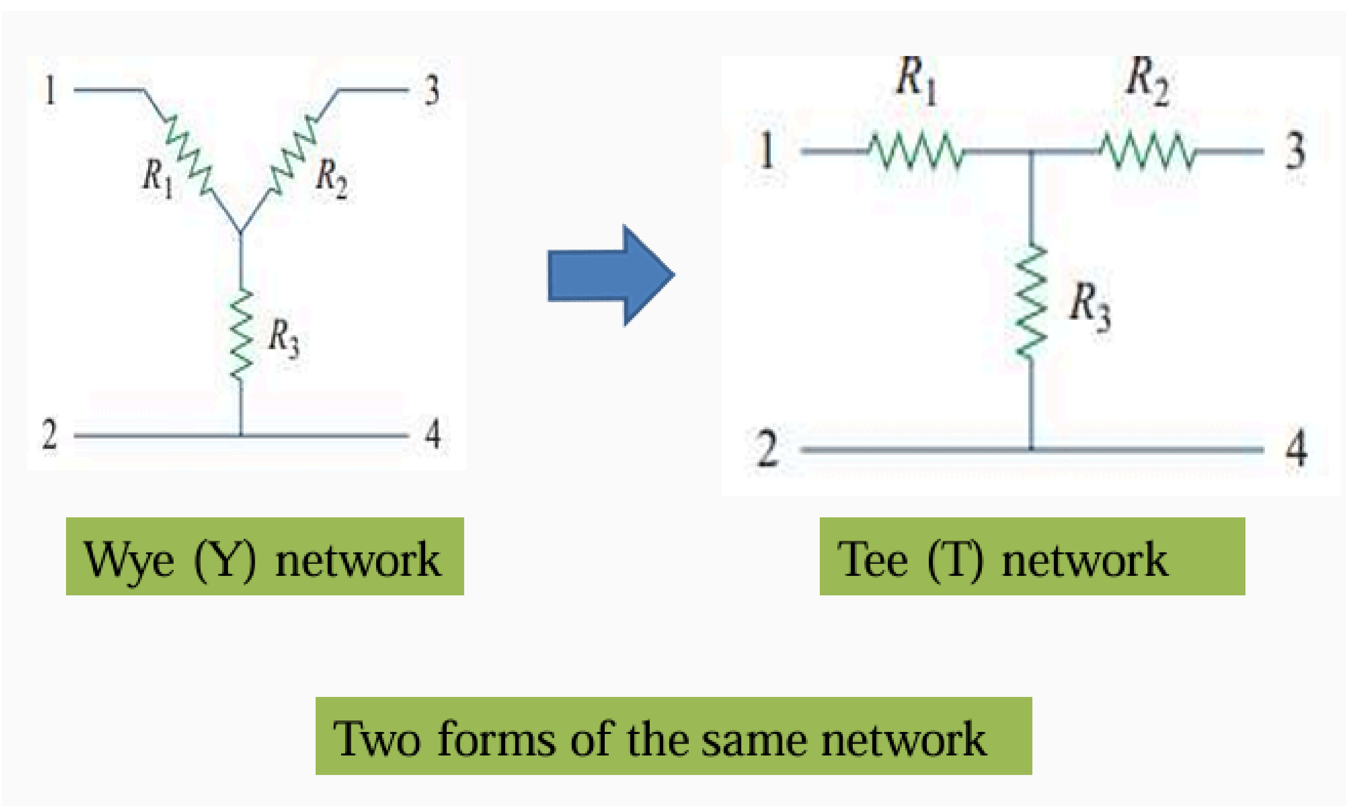
$$i_1 = \frac{G_1}{G_1 + G_2} i$$

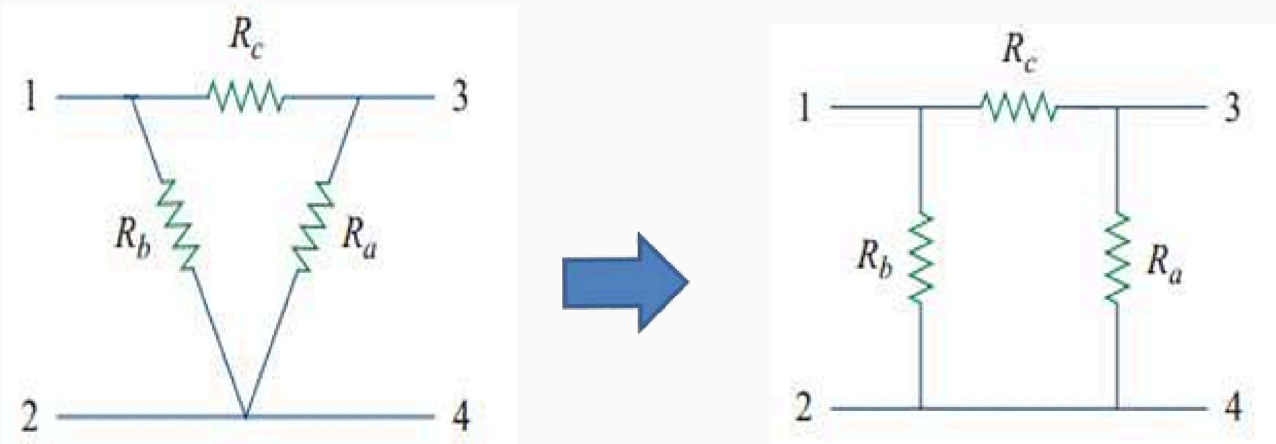
$$i_2 = \frac{G_2}{G_1 + G_2} i$$

If a current divider has N resistors in parallel with the source current i, the n-th resistors R_n will have current

$$i_n = \frac{G_n}{G_1 + G_2 + \ \ldots \ \ldots \ + G_n} i$$

Wye-Delta Transformations



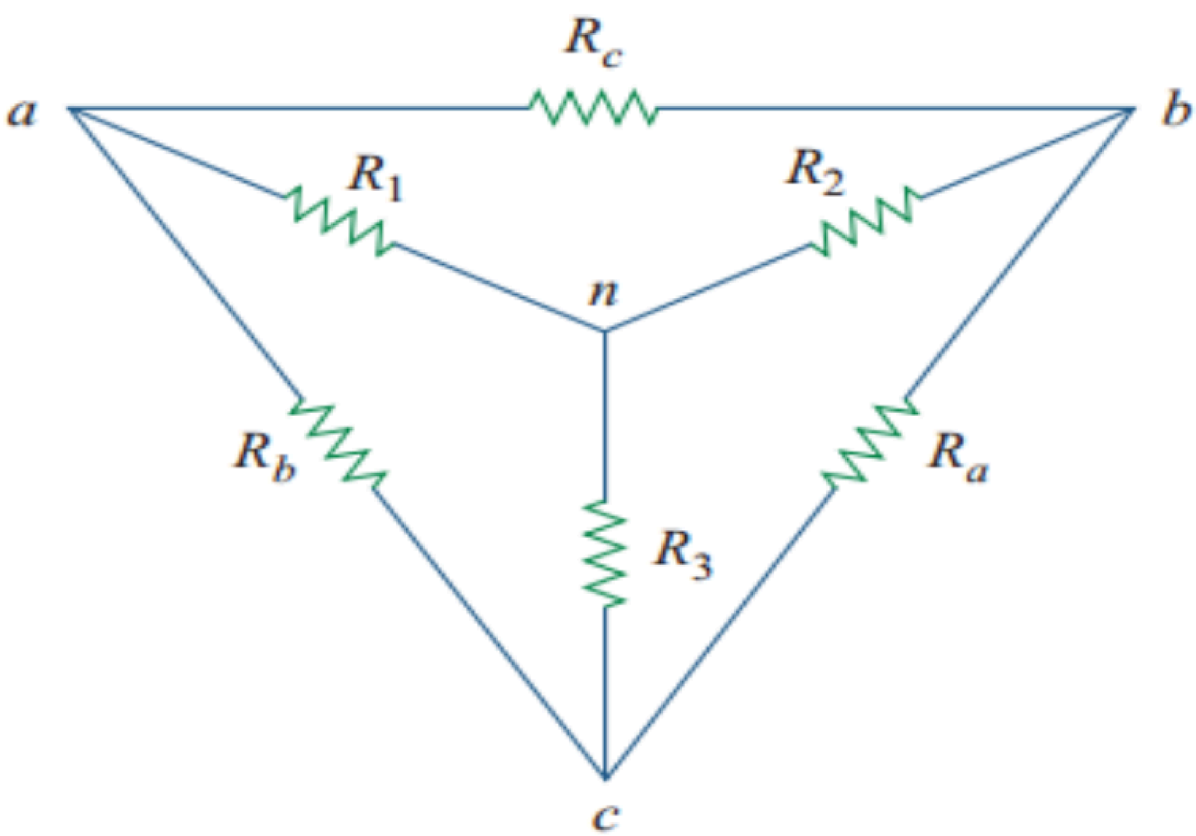


Delta (Δ) network

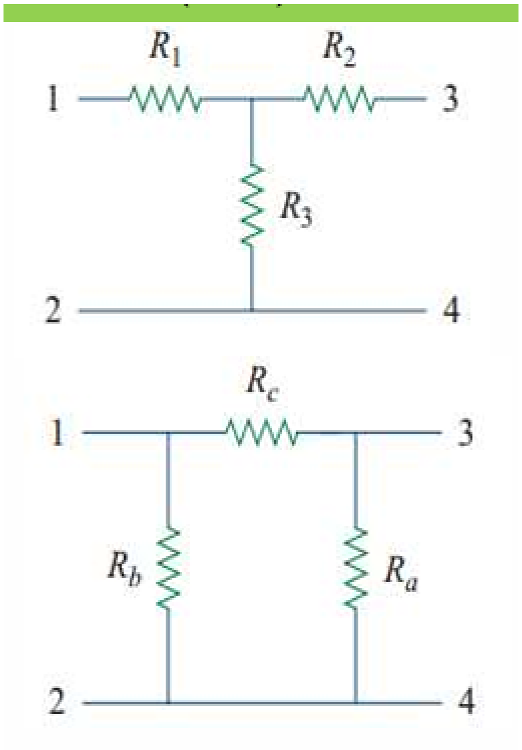
Pi (π) network

Two forms of the same network

Delta to Wye Conversion



To obtain the equivalent resistances in the wye (Y) network, we compare the two networks and make sure that the resistance between each pair of nodes in the Δ (or π) network is the same as the resistance between the same pair of nodes in the Y (or T) network.



$$R_{12}(Y) = R_1 + R_3$$

$$R_{12}(\Delta) = R_b||(R_a + R_c) = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$

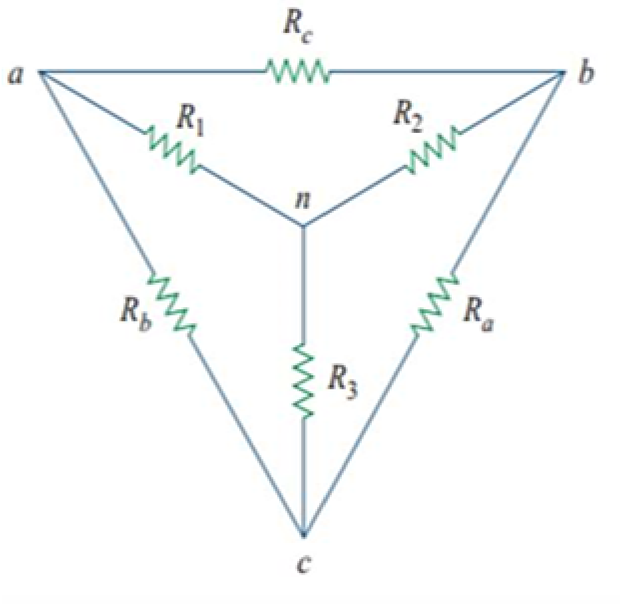
$$\therefore R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \ldots\ldots(i)$$

Similarly,

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \ldots\ldots(ii)$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \ldots\ldots(iii)$$

Solving equation (i), (ii) and (iii) we get delta to wye conversion as follows

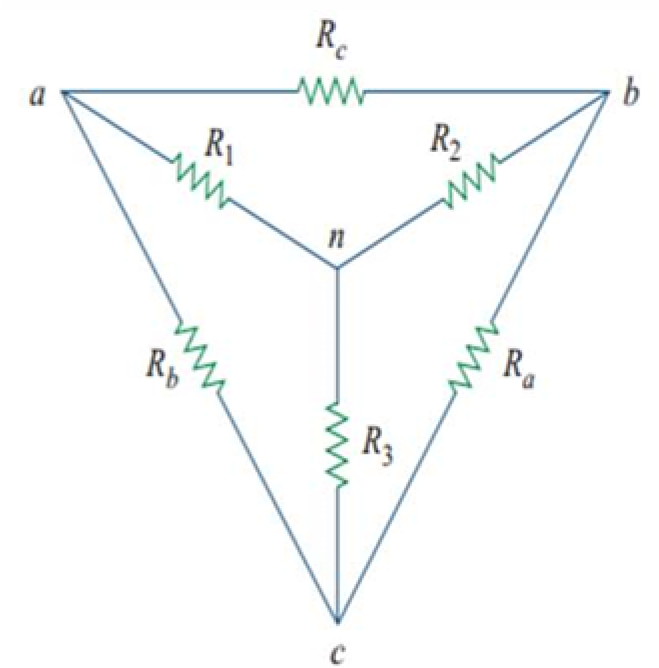


$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

and similarly we also get the equation for wye to delta conversion



$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$