PYC

16 Series

Use separate answer script for each section.

SECTION-A

Q1. (a) Define limit of a function. A function $f(x)$ is given that: **6**

$$f(x) = x \sin\frac{1}{x} \quad \text{for } x \neq 0$$
$$= 0 \quad \text{for } x = 0.$$

Show that $f(x)$ is continuous at $x=0$ but its derivative does not exist.

(b) If **6**

$$f(x) = \begin{cases} -\dfrac{x^2}{2} & \text{for } x \leq 0 \\ x^n \operatorname{Sin}\left(\dfrac{1}{x}\right) & \text{for } x > 0 \end{cases}$$

Find whether $f'(x)$ exists for $n=1$ and 2 or not.

Q2. (a) If $y=(x^2-1)^n$, then prove that **6**

$$(x^2-1)y_{n+2}+2xy_{n+1}-n(n+1)y_n=0$$

(b) State and prove Rolle's theorem. **6**

Q3. (a) Examine whether $x^{1/x}$ possesses a maximum or a minimum and determine the same. **6**

(b) Evaluate $\underset{x \to 0}{\mathrm{lt}}\left(\dfrac{\tan x}{x}\right)^{1/x}$ **6**

Q4. (a) If V be a function of x and y, prove that **6**

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=\frac{\partial^2 V}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}+\frac{1}{r^2}\frac{\partial^2 V}{\partial r^2}$$

(b) Define radius of curvature. Find the radius of curvature at the origin of $4x^4+3y^3-8x^2y+2x^2-3xy-6y^2-8y=0$. **6**

SECTION-B

Q5. Integrate the following integrals (any three): **12**

i) $\int \frac{x^2+1}{\sqrt{x^2+4x+20}}\,dx$, ii) $\int \frac{dx}{a+b\cos x}$, iii) $\int \frac{dx}{(1+x)\sqrt{1+x-x^2}}$

iv) $\int (2x+1)\sqrt{2x^2-8x+5}\,dx$

Q6. (a) Evaluate: $\lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)------\left(1+\frac{n}{n}\right)\right\}^{1/n}$ **4**

(b) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2} = \frac{\pi}{8}\log 2$ **3**

(c) Find the reduction formula for $\int \tan^n\theta\, d\theta$ and then also evaluate $\int_0^{\frac{\pi}{4}} \tan^6\theta\, d\theta$. **5**

Q7. (a) What are the Beta and Gamma functions? Show that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$. **4**

(b) Show that $\int_0^1 \frac{x^2\,dx}{(1-x^4)^{1/2}} \times \int_0^1 \frac{dx}{(1+x^4)^{1/2}} = \frac{\pi}{4\sqrt{2}}$ **5**

Evaluate: $\int_0^a \frac{x^4}{(1+x^2)^{\cdots}}\,dx$ **3**

. area of the segment of the parabola **6**
Y=(x-1)(4-x) cut by the x-axis.

Find the volume and surface area of the solid generated by revoluting the cycloid, $x=a(\theta+\sin\theta)$ **6**
and $y=a(1+\cos\theta)$ is about its base.

17 Series

SECTION : A

Q.1. (a) A function $f(x)$ is given by $f(x)=\begin{cases} 1 & \text{for } x<0 \\ 1+\sin x & \text{for } \cancel{0 \le x \le \frac{\pi}{2}} \quad 0 \le x < \frac{\pi}{2} \\ 2+(x-\frac{\pi}{2})^2 & \text{for } \frac{\pi}{2} \le x \end{cases}$ 7

Does $f'(x)$ exists for $x=\frac{\pi}{2}$ and $x=0$? Justify your answer. Plot $f(x)$ for $0-1<x<2$

(b) Find $\frac{dy}{dx}$ for (i) $y=x^{\cot x}+(\sin x)^{\tan x}$ (ii) $y=x^{x^{x}}$ 5

Q.2. (a) State and prove Taylor's series in finite form. Hence drive Maclaurin's series. 6

(b) If $y=\sin(m\sin^{-1}x)$ then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$. Find y_n at $x=0$. 6

Q.3. (a) State and prove Euler's theorem for three variables. 6

(b) Define maxima and minima of a function at $x=1$. Examine whether $x^{\frac{1}{x}}$ possesses a maximum or minimum and determine the same. 6

Q.4. (a) Find the condition that the conics $ax^2+by^2=1$ and $a_1x^2+b_1y^2=1$ cut each other orthogonally. 6

(b) Determine the radii of curvature of $y^2=\frac{ax^2+x^3}{a-x}$ at (0,0) and (-a,0) respectively. 6

SECTION : B

Q.5. (a) Integrate the following integrals: 12

(i) $\int \frac{dx}{3+\cos\alpha\cos 3x}$ (ii) $\int \frac{dx}{(x+1)\sqrt{1-x+x^2}}$ (iii) $\int \frac{3\cos x+5\sin x}{4\cos x+7\sin x}dx$

$\cos 3x$

Q.6. Evaluate:

(i) $\lim_{n\to\infty}\left\{(1+\frac{1^2}{n^2})(1+\frac{2^2}{n^2})\text{.........}\text{.....}(1+\frac{n^2}{n^2})\right\}^{\frac{1}{n}}$ 4

(ii) $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$ 3

(iii) $\int_0^a \frac{a(x-\sqrt{a^2-x^2})}{(2x^2-a^2)^2}dx$ 5

Q.7. (a) What are the Beta and Gamma functions? Show that $B(m,n)=\int_0^\infty \frac{P^{n-1}}{(1+P)^{m+n}}dP$ 6

(b) Find a reduction formula to evaluate $I_n=\int_0^{\frac{\pi}{2}}\sin^n\theta d\theta$. Hence find I_n in terms of n. 6

Q.8. (a) Find the area bounded by the curve $y^2=x^3$ and the line $y=x$. 6

(b) Find volume and surface area of the solid generated by revolving the cardioids $r=a(1-\cos\theta)$ about the initial line. 6

$f(x) = hf(x) + \frac{h^2}{2!}f'(x) + \frac{h^3}{3!}f''(x)$ $\frac{h^{n}}{(n)!}f^n(x)$

$f(x)+ hf'(x) + \frac{h^2}{2}f''(x)$

18 Series

Marks

SECTION : A

Q.1. (a) Find from first principles the derivative of logcosx. 4

(b) A function is given as: 5

$$f(x)=\begin{cases} 3+2x & for \quad -\frac{3}{2}\le x<0 \\ 3-2x & for \quad 0\le x<\frac{3}{2} \\ -3-2x & for \quad x\ge\frac{3}{2} \end{cases}$$

Is it continuous at x=0 and x=3/2? Justify your answer.

(c) Differentiate $x^{Sin^{-1}x}$ with respect to $Sin^{-1}x$. 3

Q.2. (a) State and prove Rolle's theorem. 6

(b) If $y=x^{n-1}\log x$ then prove that $y_n=\dfrac{(n-1)!}{x}$. 6

Q.3. (a) Find the extremum values of (i) xy and (ii) x^2+y^2 under the condition $\dfrac{x}{a}+\dfrac{y}{b}=1$, a>0, b>0. 6

(b) State Euler's theorem. If $u=\tan^{-1}\dfrac{x^3+y^3}{x-y}$ then show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=Sin2u$. 6

Q.4. (a) Find the condition that the conics $ax^2+by^2=1$ and $a_1x^2+b_1y^2=1$ shall cut orthogonally. 6

(b) What is the radius of curvature? 6
Find the radius of curvature of $3x^4-2y^4+5x^2y+2xy-2y^2+4x=0$ at x=0

SECTION : B

Q.5. Integrate any three 12

(i) $\displaystyle\int\frac{2x^2+1}{\sqrt{3x^2+12x+27}}dx$ (ii) $\displaystyle\int Sin^{-1}\sqrt{\frac{x}{a+x}}dx$

(iii) $\displaystyle\int\frac{dx}{p+q\cos x}$ (iv) $\displaystyle\int\frac{dx}{(1+x)\sqrt{1-x+x^2}}$

Q.6. Evaluate

(i) $\displaystyle\lim_{n\to\infty}\left[\left(1+\frac{1^2}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\cdots\cdots\cdots\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}}$ 6

(ii) $\displaystyle\int_0^1\frac{\log(1+x)}{1+x^2}dx$ 6

Q.7. (a) What are the Beta and Gamma functions? Obtain the relation between Beta and Gamma functions. 6

(b) Show that

(i) $\Gamma(n+1)=n!$ 3

(ii) $\displaystyle\int_0^\infty e^{-\frac{x^2}{2}}dx=\sqrt{\frac{\pi}{2}}$ 3

Q.8. (a) Find the area above the x-axis, included between the curves $y^2=ax$ and $x^2+y^2=2ax$. 6

(b) Find the volume of the solid generated by revolving the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ about the major axis. 6

****

19 Series

SECTION : A

Marks

Q.1. (a) Define differentiability of a function at any point. Show that every differentiable function is continuous but converse is not always true. 4

(b) A function is defined as $f(x)=\begin{cases} x^2 Sin\left(\frac{1}{x}\right) & for\ x\neq 0 \\ 0 & for\ x=0 \end{cases}$ 4

Does $f'(0)$ exist?

(c) Evaluate $lt_{x\to 0}\dfrac{(1+x)^{1/x}-e}{x}$ 4

Q.2. (a) If $y=A\left(x+\sqrt{x^2+a^2}\right)^n+B\left(x+\sqrt{x^2+a^2}\right)^{-n}$ then prove that $(x^2+a^2)y_{m+2}+(2m+1)xy_{m+1}+(m^2-n^2)y_m=0$ 6

(b) Expand the following function in a finite series in power of x with the remainder in Lagrange's form: $e^x Cosx$. 6

Q.3. (a) Determine maximum and minimum values of $1+2Sinx+3Cos^2x$ where $0\leq x\leq \Pi/2$ 6

(b) Define homogeneous function. State and prove the Euler's theorem on homogeneous function. 6

Q.4. (a) If $xCos\alpha+ySin\alpha=p$ touch the curve $\dfrac{x^m}{a^m}+\dfrac{y^m}{b^m}=1$ then show that 6

$$(aCos\alpha)^{\frac{m}{m-1}}+(bSin\alpha)^{\frac{m}{m-1}}=p^{\frac{m}{m-1}}$$

(b) Find the radius of curvature at any point of the curves 6

(i) $e^{y/a}=Sec(x/a)$ s(ii) $y=e^{-x^2}$ at (0, 1)

SECTION : B

Q.5. (a) Integrate the following integrals 6

(i) $\int Sin^2 x Cos 2x dx$ (ii) $\int \frac{dx}{x^4-1}$

(b) Write down first principle formula for definite integral. Evaluate from first principle $\int_0^1 x^2 dx$ 6

Q.6. (a) Evaluate $\lim_{n\to\infty}\left[\frac{1^2}{n^3+1^3}+\frac{2^2}{n^3+2^3}+\ldots\ldots\ldots\ldots+\frac{n^2}{n^3+n^3}\right]$ 4

(b) Integrate $\int_0^{\pi/2}\frac{\sqrt{Sinx}}{\sqrt{Sinx}+\sqrt{Cosx}}dx$ 4

(c) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2}dx=\frac{\pi}{8}\log 2$ 4

Q.7. (a) Obtain Reduction formula for $\int Sec^n x dx$ hence deduce $\int Sec^6 x dx$ and $\int Sec^7 x dx$ 6

(b) Show that $\int_0^{\pi/2} Sin^p\theta Cos^q\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$ 6

Q.8. (a) Find the area between the curves $y^2=\frac{(a-x)^3}{a+x}$ and the asymptote. 3

(b) Find the volume and surface area of the solid generated by revolving the cardioids *r=a(1-Cosθ)* about the initial line. 4

****

20 Series

			COs	Marks

SECTION : A

			COs	Marks
Q.1.	(a)	Define differentiability of a function at any point. Prove that if $f'(a)$ is finite, then $f(x)$ must be continuous at $x = a$.	CO_1	3
	(b)	A function is $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & for\ x \neq 0 \\ 0 & for\ x = 0 \end{cases}$ Does $f'(0)$ exist?	CO_1	3
	(c)	i) Find $\frac{dy}{dx}$ for $(x)^y = (y)^x$.	CO_2	2
		ii) Differentiate x^x with respect to $sin^{-1}x$.	CO_2	2
Q.2.	(a)	If $y = e^{asin^{-1}x}$, using Leibnitz's theorem show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$, Also find y_n for $x = 0$.	CO_1	5
	(b)	Explain the geometrical interpretation of mean value theorem.	CO_1	5
Q.3.	(a)	Is mean value theorem applicable to the function $f(x) = 4 - (6-x)^{2/3}$ in the intervals (-1,1) and (5,7). Justify your answer.	CO_1	5
	(b)	Examine whether $x^{\frac{1}{x}}$ possesses a maximum or minimum and determine the same.	CO_2	5
Q.4.	(a)	Find the tangent and normal to the curve $xy^2 = 4(4-x)$ at the point where it is cuts by the line $y = x$.	CO_2	5
	(b)	Discuss and sketch the curve $ay^2 = x^2(a-x)$, where a is constant; choose $a = 2$.	CO_2	5

SECTION : B

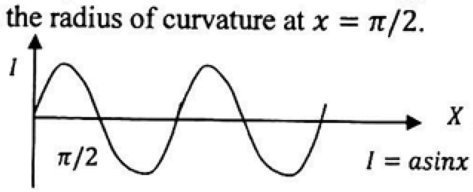
			COs	Marks
Q.5.	(a)	Integrate: i) $\int \frac{dx}{x^4+1}$ ii) $\int \frac{dx}{p+q\cos x}$	CO_4	5
	(b)	Show that $\int_0^{\pi/4} \log(1+\tan\theta)d\theta = \frac{\pi}{8}log2$.	CO_4	5
Q.6.	(a)	Show that $\lim_{n\to\infty}\left\{\frac{1^2}{n^3+1^3} + \frac{2^2}{n^3+2^3} + \dots\dots + \frac{n^2}{2n^3}\right\} = \frac{1}{3}ln\,2$	CO_3	4
	(b)	Show that $\int_0^1 \frac{\log(1+x)}{1+x^2}dx = \frac{\pi}{8}\log 2$.	CO_3	4
	(c)	Using walli's formula integrate, i) $\int_0^{\pi/2} sin^4x\, cos^5x\, dx$ ii) $\int_{-3}^{3} x\sqrt{a^2 - x^2}\,dx$	CO_3	2
Q.7.	(a)	Show that $\beta(m,n) = \int_0^\infty \frac{y^{n+1}}{(1+y)^{m+n}}dy$	CO_3	4
	(b)	Using 7(a), Evaluate, i) $\int_0^\infty \frac{y^7}{(1+y)^{15}}dy$ ii) $\int_0^\infty \frac{y^6}{(1+y)^{15}}dy$	CO_3	2
	(c)	Determine the area of the cardioid $r = a(1-\cos\theta)$.	CO_4	4
Q.8.	(a)	Determine the area above the x-axis included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$.	CO_4	5
	(b)	Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the major axis.	CO_4	5

*** END ***

21 Series

SECTION : A

			COs	Marks
Q.1.	(a)	Describe Limit and Continuity of a function at $x = a$.	CO_1	2
	(b)	A function is described as $$f(x) = \begin{cases} 1 + x & for\ x < 0 \\ 1 & for\ 0 \le x \le 1 \\ 2x^2 + 4x + 5 & for\ x > 1 \end{cases}$$ Evaluate $f'(x)$ for all values of x for which it exists.	CO_2	5
	(c)	Differentiate x^x with respect to $sin^{-1}x$.	CO_2	3
Q.2.	(a)	If $y = \sin\,(m\,sin^{-1}x)$ then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$	CO_1	5
	(b)	State and prove Tailor's Series in finite form, also formulated it to Maclaurin's series.	CO_1	5
Q.3.	(a)	Find a necessary condition for a maxima or minima of a function $f(x)$. Why for the maxima the value of $f''(x)$ is less than 0 and for the minima $f''(x)$ is greater than 0.	CO_2	5
	(b)	The total cost function of a firm is $c(x) = \frac{1}{3}x^3 - 5x^2 + 28x + 10$ where c is the total cost and x is output unit. A tax at the rate of 2 tk. per unit of output is imposed and the producer adds it to his cost. If the market demands function is given by $p = 2530 - 5x$ where p is the price per unit of the output, find the profit maximizing output and price.	CO_2	5
Q.4.	(a)	Write the statement of Euler's theorem.	CO_2	2
	(b)	If $u = f\left(\frac{x}{y}\right) + g\left(\frac{y}{x}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.	CO_2	4
	(c)	Find the radius of curvature at $x = \pi/2$.	CO_2	4



SECTION : B

Q.	Part	Question	CO	Marks
Q.5.		Evaluate: i) $\int e^x(\log x + \frac{1}{x})dx$ ii) $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$ iii) $\int_0^{\pi/2} \frac{dx}{5+3\cos x}$	CO_4	10
Q.6	(a)	Show that $\int_0^{\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$	CO_3	3
	(b)	Show that $\int_0^{\pi/2} \sin^p\theta \cos^q\theta d\theta = \frac{\Gamma\frac{p+1}{2}\,\Gamma\frac{q+1}{2}}{2\,\Gamma\frac{p+q+2}{2}}$ Hence evaluate i) $\int_0^{\frac{\pi}{2}} \sin^5\theta \cos^6\theta\ d\theta$ ii) $\int_0^{\frac{\pi}{2}} \sin^6\theta \cos^4\theta\ d\theta$	CO_3	7
Q.7.	(a)	State and prove Walli's formula.	CO_3	4
	(b)	Evaluate from first principle $\int_a^b e^x dx$	CO_3	3
	(c)	Prove that $\Gamma(n+1) = (n)!$	CO_3	3
Q.8.	(a)	Find the perimeter of the circle $x^2 + y^2 = a^2$.	CO_4	4
	(b)	Find the volume and area of the curved surface of a paraboloid of revolution form by revolving the parabola $y^2 = 4ax$ about the x axis bounded by x=4.	CO_4	6

***** END *****

22 Series

SECTION : A

			Marks	CO	PO
Q.1.	(a)	What are limits and continuity of a function?	2	CO1	PO1
	(b)	Given the function $$f(x) = \begin{cases} \pi - x; & x \le \pi \\ c \sin x; & x > \pi \end{cases}$$ i) For the value $x = \pi$ verify whether the 3 conditions for continuity are satisfied. ii) Draw a graph of $f(x)$ from $x = -\pi$ to $x = 3\pi$.	4	CO1	PO1
	(c)	Sketch a graph of the function $$f(x) = \begin{cases} x^2; & 0 < x < 1 \\ 2; & x = 1 \\ x^2 + 2; & 1 < x < 2 \\ -x + 8; & 2 \le x \le 4 \end{cases}$$ Answer the following questions by yes or no: i) Is $f(x)$ continuous at 1) $x = 1$ 2) $x = 2$? ii) Is $f(x)$ differentiable at 3) $x = 3/2$ 4) $x = 2$?	4	CO1	PO1
Q.2.	(a)	State and prove Leibnitz's theorem.	5	CO1	PO1
	(b)	If $y = \sin(m \sin^{-1} x)$ then show that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$.	5	CO1	PO1
Q.3.	(a)	If $u = e^x(x \cos y - y \sin y)$ then show that $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0$.	5	CO2	PO2
	(b)	Prove that the curves $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{a'} + \frac{y^2}{b'} = 1$ cut each of them orthogonally if $a - b = a' - b'$.	5	CO2	PO2
Q.4.	(a)	Find the radius of curvature of the curve $y^2 = \frac{x^2(a+x)}{a-x}$ at $(0,0)$ and $(-a, 0)$.	5	CO2	PO2
	(b)	Trace the curve $ay^2 = x^2(a - x)$. Choose a value of $a > 0$.	5	CO2	PO2

SECTION : B

			Marks	CO	PO
Q.5.	(a)	Evaluate: (i) $\int \sqrt{x^2 + a^2}\, dx$ (ii) $\int \frac{dx}{a + b\cos x} dx$.	5+5	CO3	PO2
Q.6.	(a)	Show that $\int_0^{\pi/2} \log \sin x\, dx = \int_0^{\pi/2} \log \cos x\, dx = \pi/2 \log \frac{1}{2}$.	5	CO3	PO1
	(b)	Show that i) $\int_0^{\propto} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$. ii) $\Gamma(n) = \int_0^1 \left[\log\left(\frac{1}{x}\right)\right]^{n-1} dx$ and hence evaluate $\int_0^1 (\log(\frac{1}{x}))^7 dx$.	5	CO3	PO1
Q.7.	(a)	Find a relation between Beta and Gamma functions.	5	CO3	PO1
	(b)	Obtain the reduction formula for $\int \tan^n x dx$ and hence evaluate $\int \tan^6 x dx$.	5	CO3	PO1
Q.8.	(a)	Find the length of the curve $y = \frac{1}{2}x^2 - \frac{1}{4}\log x$ from $x = 1$ to $x = 2$.	5	CO4	PO1
	(b)	Find the volume and area of the curved surface of a paraboloid of revolution by revolving the parabola $y^2 = 4ax$ about the x-axis bounded by $x = 4$.	5	CO4	PO1

****