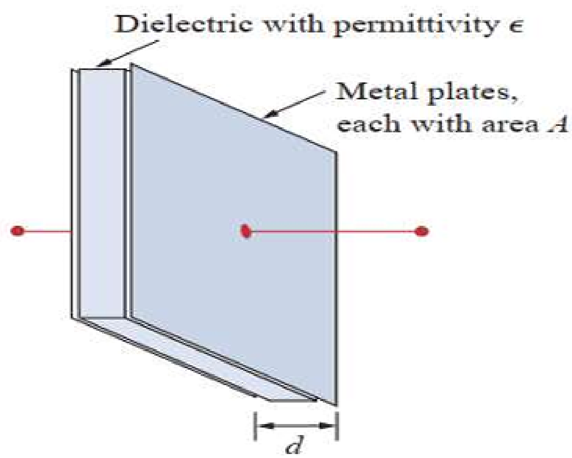
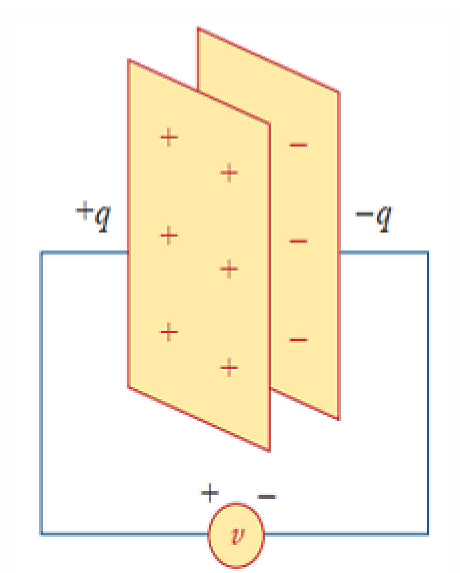
12 - Impedance

Capacitors

- A capacitor is a passive element
- Capacitor stores energy in its electric field
- A capacitor consists of two conducting plates separated by an insulator (or dielectric)
- Plates may be aluminum foil the dielectric may be air, ceramic, paper etc.



- When a voltage source is connected to the capacitor, the source deposits a positive charge q on one plate

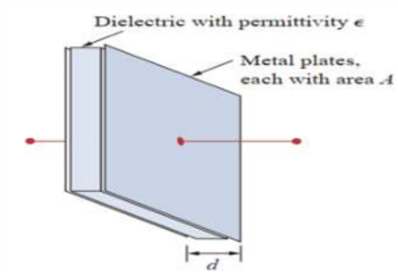


- A negative charge deposits on the other plate. The capacitor store the electric charge.
- The amount of charge stored, represented by q, is directly proportional to the applied voltage so that

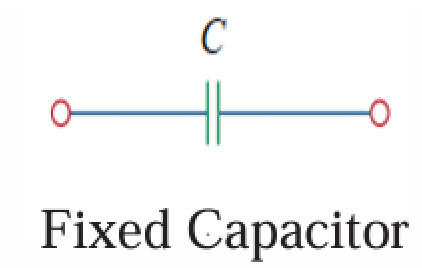
$$q = CV$$

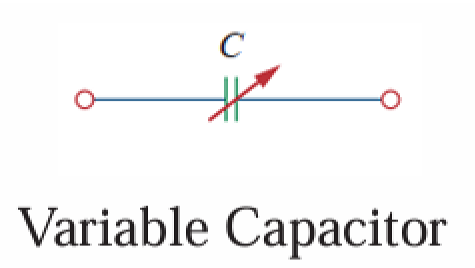
- Where, C, the constant of proportionality, is known as the capacitance of the capacitor. The unit of capacitance is the farad (F)
- Capacitance depends on the physical dimensions of the capacitor

$$C = \frac{\epsilon A}{d}$$



Where,
A = the surface area of each plate
d = distance between the plates
ϵ = the permittivity of the dielectric material between the plates





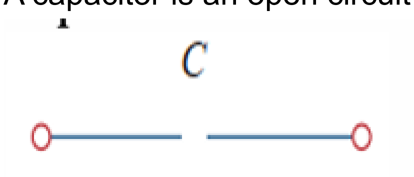
- Current-voltage relationship for a capacitor is

$$i = C\frac{dv}{dt}$$

- Energy stored in the capacitor is

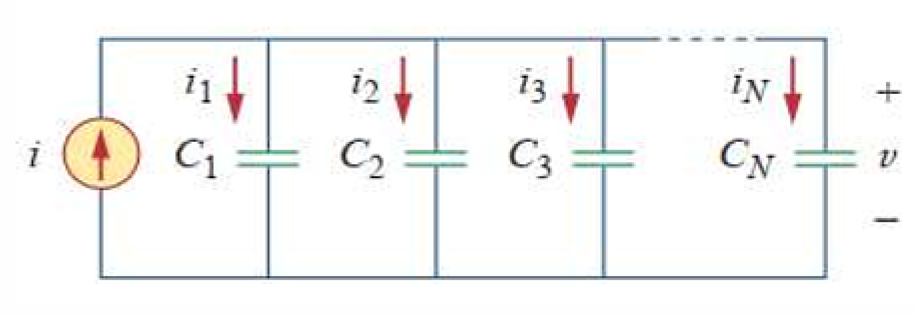
$$w = \frac{1}{2}Cv^2$$

- A capacitor is an open circuit to dc



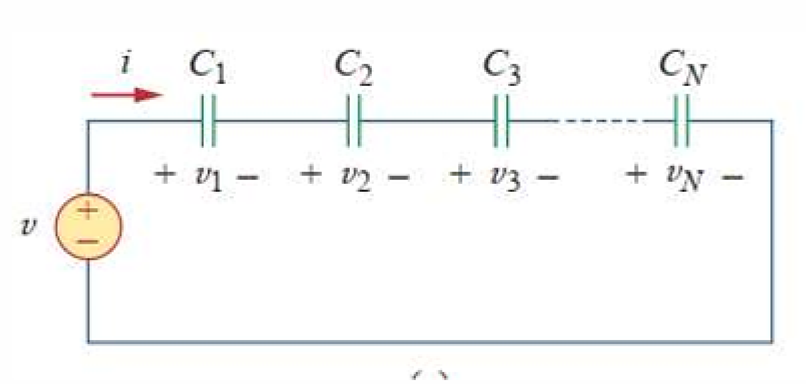
- The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and return previously stored energy when delivering power to the circuit

Equivalent Capacitance:



Equivalent capacitance C_{eq} of the parallel connected capacitor is

$$C_{eq} = C_1 + C_2 + C_3 + \ \dots \ + C_N$$

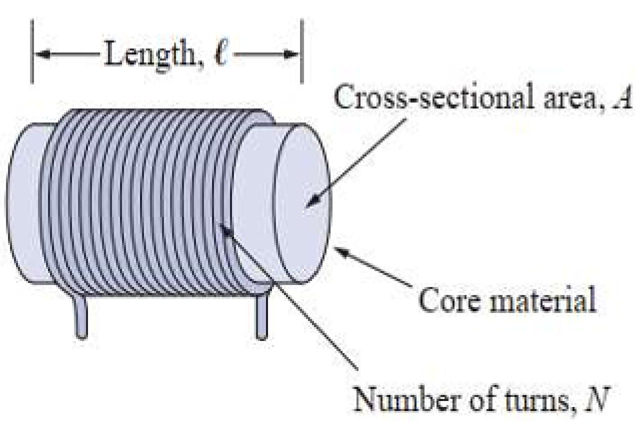


Equivalent capacitance C_{eq} of the series connected capacitor is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ \dots \ + \frac{1}{C_N}$$

Inductor

- Inductor is a passive element
- Inductor store energy in its magnetic field
- An inductor consists of a coil of conducting wire



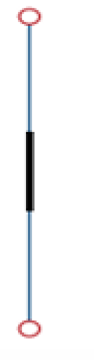
- Voltage across the inductor is directly proportion to the time rate of change of current

$$v = L\frac{di}{dt}$$

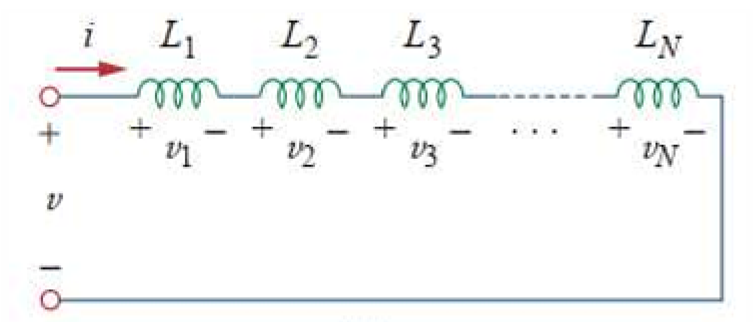
- Where L is the constant of proportionality called the inductance of the inductor. The unit of inductance is the henry (H)
- Energy stored in inductor is

$$w = \frac{1}{2}Li^2$$

- An inductor acts like a short circuit to dc



- Ideal inductor does not dissipate energy. The energy stored in it can be returned at a later time.



- The equivalent inductance of series-connected inductor is

$$L_{eq} = L_1 + L_2 + L_3 + \ \dots \ + L_N$$

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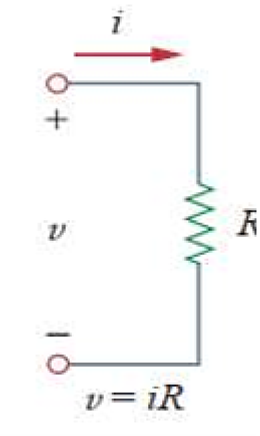
- The equivalent inductance of parallel-connected inductor is

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_2} + \dots + \frac{1}{L_N}$$

Phasor Relationships for Resistor

Assume, current through a resistor R is

$$i = I_m \cos(\omega t + \phi)$$



voltage across the resistor is

$$v = iR = RI_m \cos(\omega t + \phi)$$

Phasor form of this voltage is

$$V = RI_m \angle \phi$$

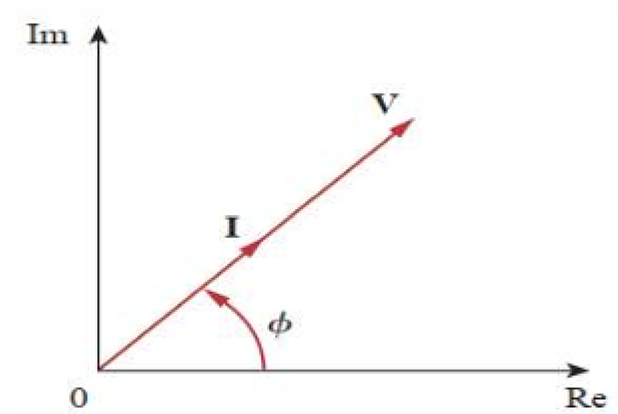
Phasor representation of the current is

$$I = I_m \angle \phi$$

Voltage-current relationship in phasor form is

$$V = RI$$

Voltage-current relation for the resistor in the phasor domain is shown in the following figure

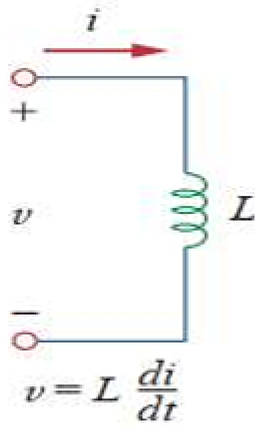


Voltage and current are in phase

Phasor Relationships for Inductor

Assume the current through the inductor is

$$i = I_m \cos(\omega t + \phi)$$



Voltage across the inductor is

$$v = L\frac{di}{dt}$$

$$= -\omega L I_m \cos(\omega t + \phi)$$

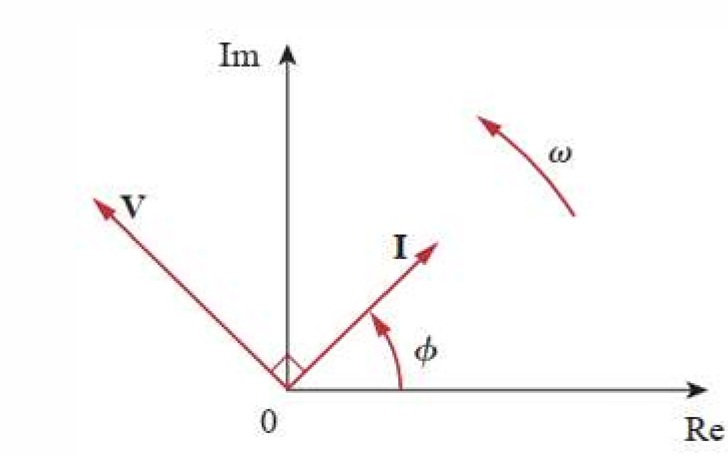
$$= \omega L I_m \sin(\omega t + \phi + 90^\circ)$$

$$= \omega L I_m \angle(\phi + 90^\circ)$$

$$\omega L I_m \angle\phi \cdot 1\angle 90^\circ$$

$$\therefore V = j\omega L I$$

Phasor diagram for inductor is



Phase angle between voltage and current = $(\phi + 90^\circ) - \phi$ = 90°
Voltage leads current by 90°
Current lags voltage by 90°

Phasor Relationships for Capacitor

Assume the voltage across capacitor is

$$v(t) = V_m \cos(\omega t + \phi)$$

Current through the capacitor is

$$i = C\frac{dv}{dt}$$

$$= -\omega C V_m \sin(\omega t + \phi)$$

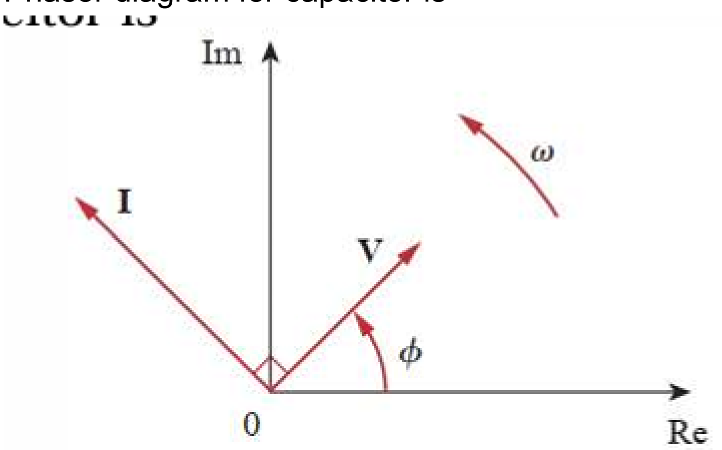
$$= \omega C V_m \cos(\omega t + \phi + 90^\circ)$$

$$\omega C V_m \angle(\phi + 90^\circ)$$

$$I = j\omega C V$$

phase angle between voltage and current = $\phi - (\phi + 90^\circ)$ = -90°
Voltage lags current by 90°

Phasor diagram for capacitor is



Element	Time Domain	Frequency Domain
R	$v = Ri$	$V = RI$
L	$v = L\frac{di}{dt}$	$V = j\omega LI$
C	$i = C\frac{dv}{dt}$	$V = \frac{I}{j\omega C}$

Impedance

Voltage-current relations for the three passive elements are given in the above table, those equations may be written in terms of the ration of the phasor voltage to the phasor current as

$$\frac{V}{I} = R$$

$$\frac{V}{I} = j\omega L$$

$$\frac{V}{I} = \frac{1}{j\omega C}$$

Ohm's law in phasor form for any type of element as

$$Z = \frac{V}{I} \; or, \; V = ZI$$

Where Z is a frequency-dependent quantity known as impedance, measure in ohms

The impedance Z of a circuit is the ration of the phasor voltage V to the phasor current I

The impedance of resistor is

$$Z = R$$

The impedance of inductor is

$$Z = j\omega L$$

The impedance of capacitor is

$$Z = \frac{1}{j\omega C}$$

Impedances may be expressed in the rectangular form as

$$Z = R + jX$$

where,
R = $Re(Z)$, is called the resistance
X = $Im(Z)$, is called the reactance

Impedance is inductive when X is positive or capacitive when X is negative

Impedance $Z = R + jX$ is said to be inductive or lagging since current lags voltage

Impedance $Z = R - jX$ is said to be capacitive or leading since current leads voltage

The impedance may also be expressed in polar form as

$$Z = |Z| \angle \theta = R + jX$$

where

$$|Z| = \sqrt{R^2 + X^2}$$

$$\theta = \tan^{-1} \frac{X}{R}$$

and

$$R = |Z| \cos \theta$$

$$X = |Z| \sin \theta$$

The admittance Y is the reciprocal of impedance

The admittance Y of an element (or a circuit) is the ration of the phasor current through to it to the phasor voltage across it

$$Y = \frac{1}{Z} = \frac{I}{V}$$

$$Y = G + jB$$

Where $G = Re(Y)$ is called the conductance and $B = Im(Y)$ is called the susceptance

Unit of admittance, conductance, susceptance is siemens (or mhos)

Element	Impedance	Admittance
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

Circuit Theorem

Kirchhoff's voltage law and current law can also be applied in phasor form resulting in following two equations
In a mesh,

$$V_1 + V_2 + \ \dots \ + V_n = 0$$

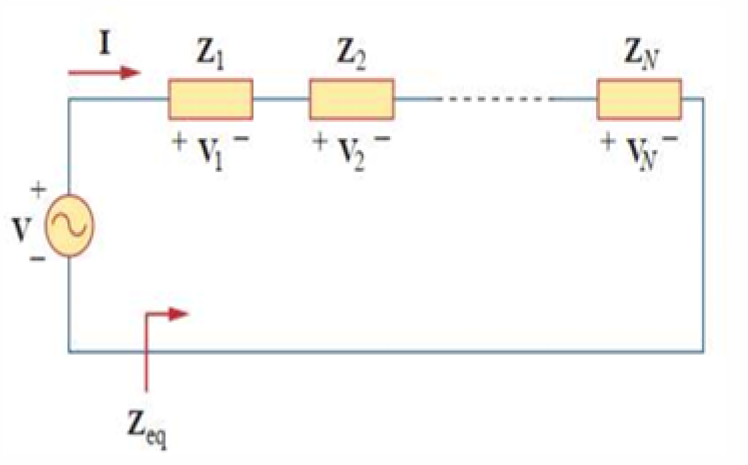
In a node,

$$I_1 + I_2 + \ \dots \ I_n = 0$$

Ohm's law in phasor form

$$V = ZI$$

Impedance Combinations



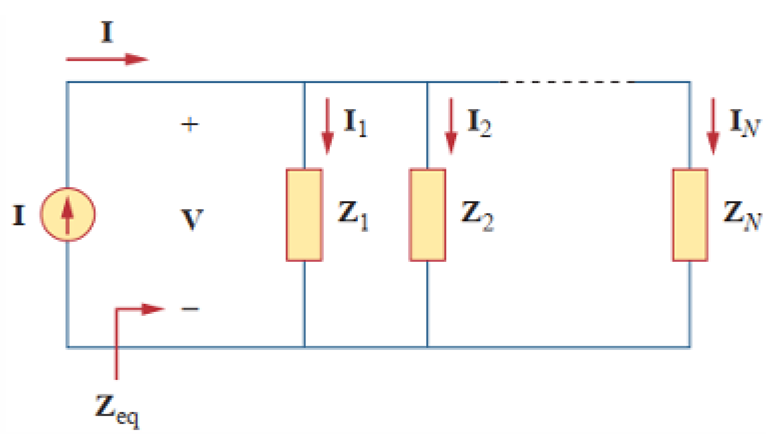
The same current I flows through the impedances

Applying KVL around the loop

$$V = V_1 + V_2 + \ \ldots \ V_N = I(Z_1 + Z_2 + \ \ldots \ Z_N)$$

The equivalent impedance at the input terminals is

$$Z_{eq} = \frac{V}{I} = Z_1 + Z_2 + \ \ldots \ + Z_N$$



The voltage across each impedance is the same

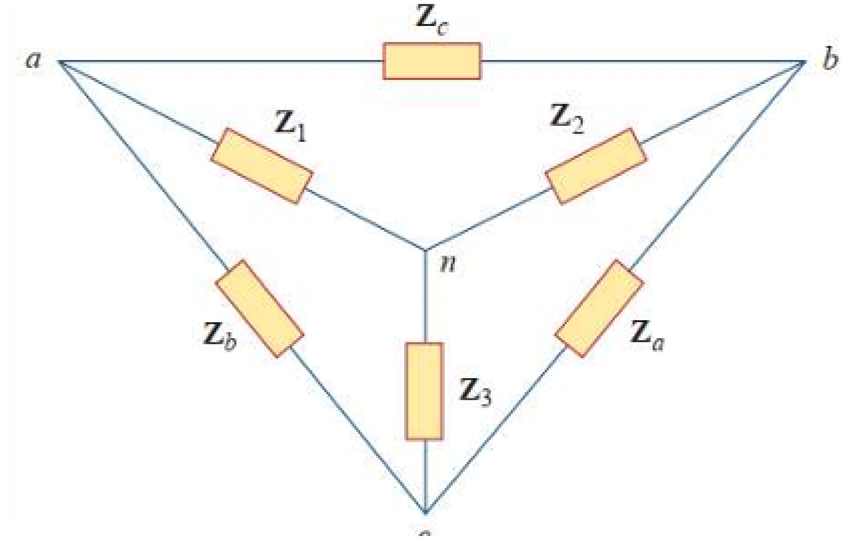
Applying KCL at the top node,

$$I = I_1 + I_2 + \ \ldots \ I_N = V(\frac{1}{Z_1} + \frac{1}{Z_2} + \ \ldots \ + \frac{1}{Z_N})$$

The equivalent impedance is,

$$\frac{1}{Z_{eq}} = \frac{I}{V} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ \ldots \ + \frac{1}{Z_N}$$

Delta-to-wye and wye-to-delta transformations for impedance is same as resistive circuit

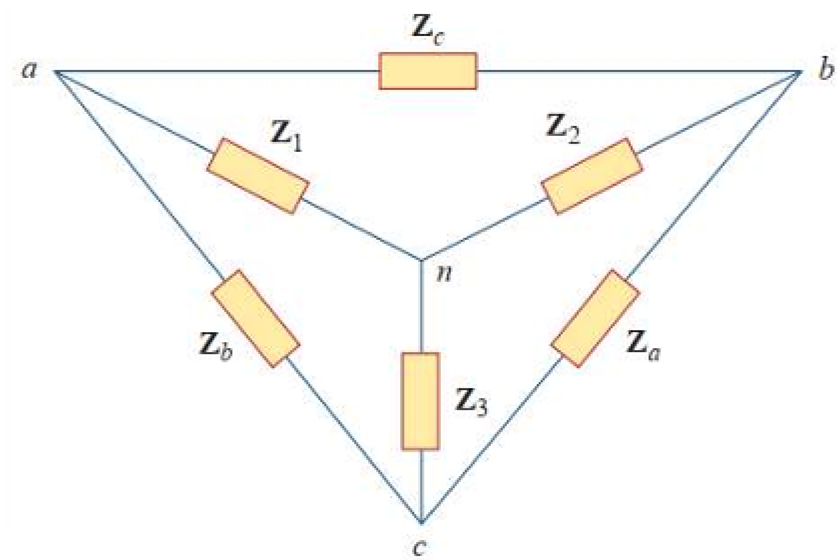


$Y-\Delta$ conversion:

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_1}$$

$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_2}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_3}$$



$\Delta - Y$ conversion:

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

A delta or wye circuit is said to be balanced if it has equal impedances in all three branches.

When a $\Delta - Y$ circuit is balanced, the equation may write

$$Z_\Delta = 3Z_Y$$