7 - Superposition

Homogeneity Property

The homogeneity property states that if the input is multiplied by a constant, then the output(also called the response) is multiplied by the same constant.

For a resistor, Ohm's law relates the input i to the output v.

$$v = iR$$

If the current is increased by a constant k, then the voltage increases correspondingly by k; that is,

$$kv = kiR$$

Additivity Property

The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately

If i_1 ampere current separately applied to the resistor then output voltage is $v_1 = i_1 R$

If i_2 ampere current separately applied to the resistor then output voltage is $v_2 = i_2 R$

If $(i_1 + i_2)$ ampere current applied to the resistor then output voltage is

$$v = (i_1 + i_2)R = i_1 R + i_2 R = v_1 + v_2$$

Linearity Property

The linearity property is a combination of both the homogeneity property and the additivity property

A circuit is linear if it has both additive and homogenous property

A linear circuit consists of only linear elements, linear dependent sources, and independent sources.

A linear circuit is one whose output is linearly related (or directly proportional) to its input.

A resistor is a linear element because the voltage-current relationship satisfied both the homogeneity and the additivity property

Relationship between power and voltage (or current) is nonlinear

$p_1 = i_1^2 R$, $p_2 = i_2^2 R$

$$p = (i_1 + i_2)^2 R = i_1^2 R + i_2^2 R + 2 i_1 i_2 R$$

$$\therefore p \neq p_1 + p_2$$

Superposition Theorem

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately

We must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by $0V$ (or a short circuit), and every current source by $0A$ (or an open circuit). This way we obtain a simpler and more manageable circuit.
2. Dependent sources are left intact because they are controlled by circuit variables

Steps to apply superposition principle:

1. Turn of all independent sources except one source. Find the output (voltage or current) due to that active source using mesh analysis of nodal analysis
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to each the independent sources.