11 - Phasor

Sinusoids

- A sinusoid is a signal the form of the since or cosine function
- A sinusoidal signal is easy to generate and transmit
- Sinusoidal voltage generated through out the world and supplied to homes, factories, laboratories, and so on.

Consider the sinusoidal $v(t) = V_M \sin(\omega t)$
Where,
V_M = the amplitude of the sinusoid
ω = the angular frequency in radians/s

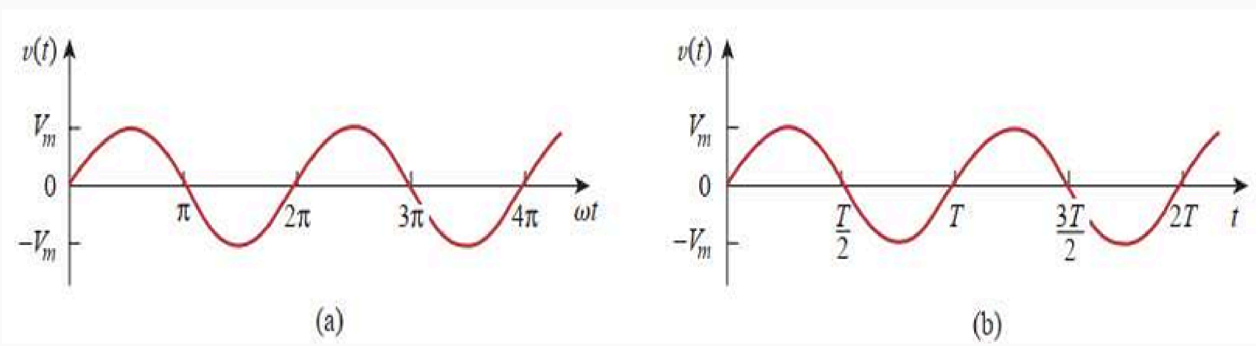


Fig. 1: A sketch of $V_m \sin \omega t$: (a) as a function of ωt (b) as a function of t.

T = Time period of the sinusoid

$$T = \frac{2\pi}{\omega}$$

$v(t)$ repeats itself every T seconds

$$v(t+T) = V_m \sin \omega(t+T)$$

$$= V_m \sin \omega \left(t + \frac{2\pi}{\omega}\right)$$

$$= V_m \sin(\omega t + 2\pi)$$

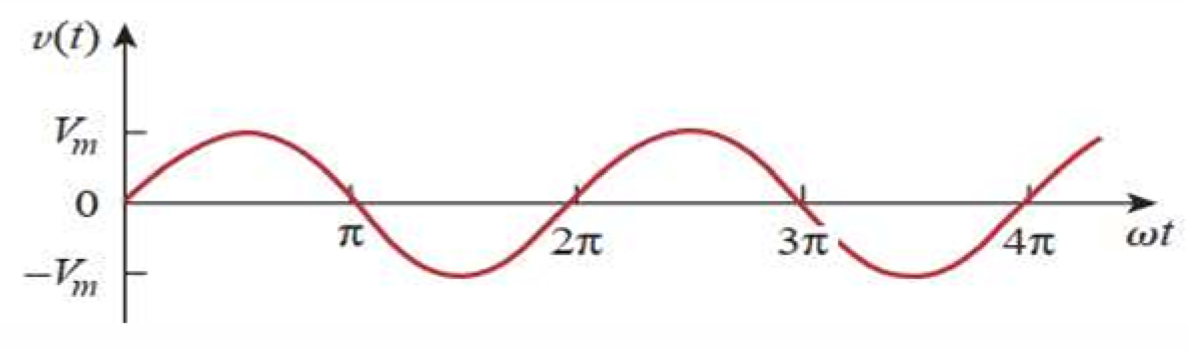
$$= V_m \sin \omega t$$

$$= v(t)$$

$$\therefore v(t+T) = v(t)$$

A periodic function is one that satisfied $f(t) = f(t + nT)$, for all t and all integers n.

Time period T of the periodic function is the time of one complete cycle



$$\omega T = 2\pi$$

$$T = \frac{2\pi}{\omega}$$

The number of cycles per second, known as the frequency f

$$f = \frac{1}{T}$$

Unit of f is hertz (Hz)

Expression for the sinusoid

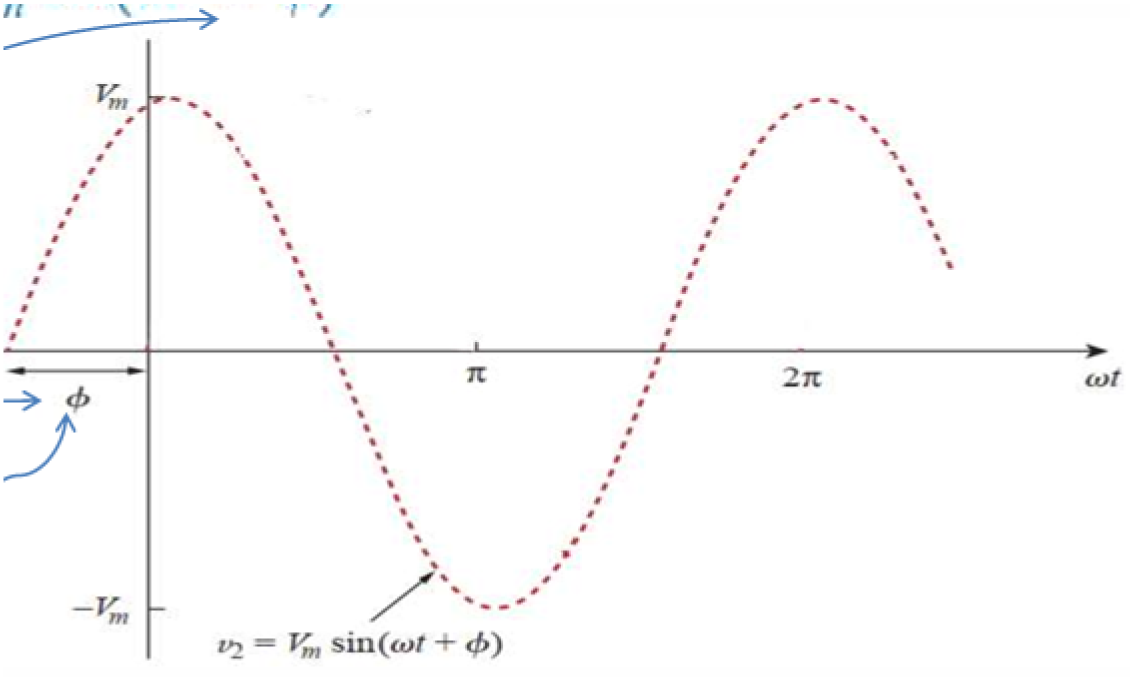
$$v(t) = V_m \sin(\omega t + \phi)$$

Where,
ϕ is the phase

$$\omega t + \phi = 0$$

$$\omega t = -\phi$$

"-" sign indicate signal start from the left side of origin



Likewise, "+" sign indicate signal start from the right side of origin and if $\omega t = 0$ then signal start from origin.

A sinusoid can be expressed in either sine or cosine form. For example,

$$v_2 = 12 \sin(\omega t - 10^\circ) = 12 \cos(\omega t - 100^\circ)$$

When comparing two sinusoids, both sinusoids is expressed as either sine or cosine with positive amplitudes.

Sine to cosine conversion or vice versa are achieved by using the following trigonometric formula

$$\sin(\omega t \pm 180^\circ) = -\sin(\omega t)$$

$$\cos(\omega t \pm 180^\circ) = -\cos(\omega t)$$

$$\sin(\omega t \pm 90^\circ) = \pm \cos(\omega t)$$

$$\cos(\omega t \pm 90^\circ) = \mp \sin(\omega t)$$

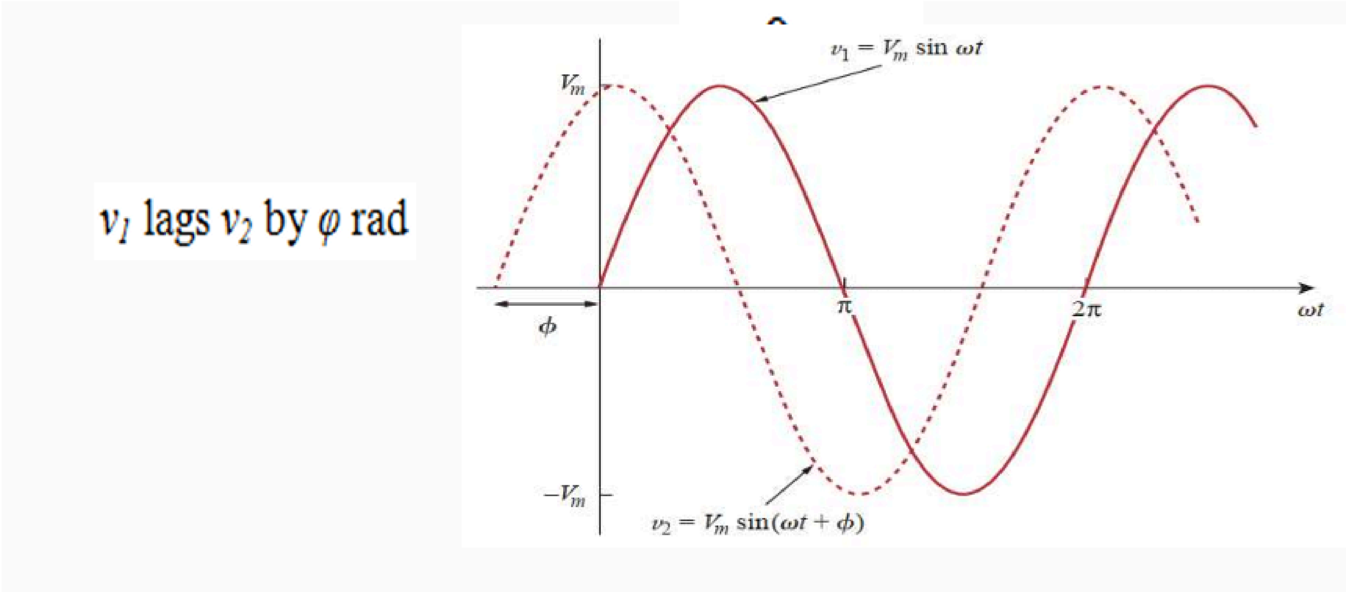
Consider the following two equations

$$v_1 = 10 \sin(\omega t - 40^\circ)$$

$$v_2 = 12 \sin(\omega t - 10^\circ)$$

Phase angle between v_1 and v_2 = phase angle of v_1 - phase angle of v_2 = $(-40^\circ) - (-10^\circ)$ = -30°

As the sign is negative, it means that v_1 lags v_2 by 30° or vice versa which is v_2 leads v_1 by 30°



Phasors

- Sinusoids are easily expressed in terms of phasors
- Phasors are more convenient to work with than sine and cosine functions
- A phasor is a complex number that represents the amplitude and phase of a sinusoid
- Phasors provide a simple means of analyzing linear circuits excited by sinusoidal sources
- A complex number z can be written in rectangular form as

$$z = x + jy$$

  Where $j = \sqrt{-1}$; x is the real part of z; y is the imaginary part of z
  z can be represented in three ways

1. Rectangular form

$$z = x + jy$$

2. Polar form

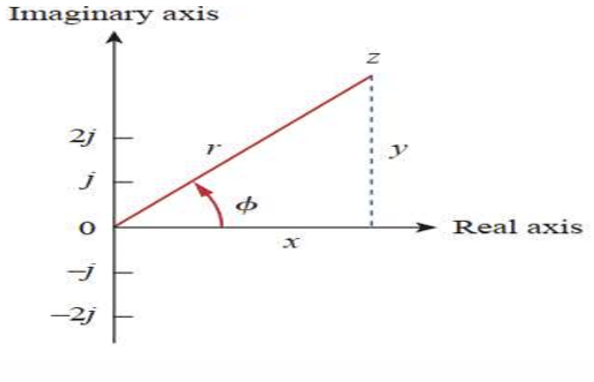
$$z = r\angle(\phi)$$

3. Exponential form

$$z = re^{j\phi}$$

Where, r is the magnitude of z, and ϕ is the phase of z

The relationship between the rectangular form and the polar form is shown in following figure



We can get r and ϕ as

$$r = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}(\frac{y}{x})$$

We can obtain x and y as

$$x = r\cos\phi$$

$$y = r\sin\phi$$

Complex number z may be written as

$$z = x + jy = r\angle(\phi) = r(\cos\phi + j\sin\phi)$$

Addition and subtraction of complex numbers are better performed in rectangular form

$$z_1 = x_1 + jy_1$$

$$z_2 = x_2 + jy_2$$

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

Multiplication and division of complex numbers are better done in polar form

$$z_1 = r_1\angle(\phi_1)$$

$$z_2 = r_2\angle(\phi_2)$$

Multiplication:

$$z_1 z_2 = r_1 r_2\angle(\phi_1 + \phi_2)$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle(\phi_1 - \phi_2)$$

Reciprocal:

$$\frac{1}{z} = \frac{1}{r}\angle(-\phi)$$

Square Root:

$$\sqrt{z} = \sqrt{r}\angle(\phi/2)$$

Complex Conjugate:

$$z^* = x - jy = r\angle(-\phi)$$

$$\frac{1}{j} = -j$$

Time Domain vs phasor domain:

$$\frac{dv}{dt} \Leftrightarrow j\omega V$$

$$\int v dt \Leftrightarrow \frac{V}{j\omega}$$

$$v(t) = V_m \cos(\omega t + \phi) \Leftrightarrow V = V_m\angle\phi$$

Before representing in phasor domain we make our sinusoidal function to cosine.

$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

Draw phasor diagram of

$$\mathbf{V} = V_m \angle \phi \text{ and } \mathbf{I} = I_m \angle -\theta.$$