10 - Maximum Power

Maximum Power Transfer Theorem

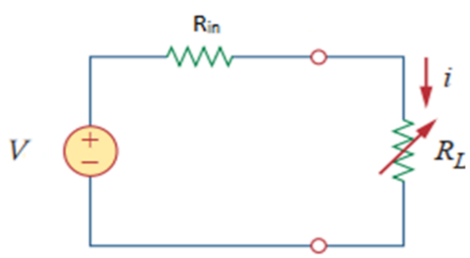
Practical source have internal resistance

Power loss occurred in the internal resistance

In any electric circuit, the electrical energy is delivered to the load where it is converted into a useful work.

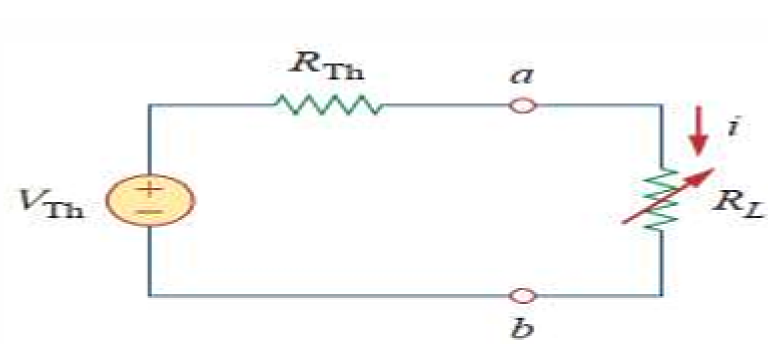
For give circuit with known internal resistance, the load size always affects the amount of power transferred from the supply source

Any change in the load resistance results to change in power transfer to the load.



Our main objective is to deliver maximum power to load. Maximum power transfer theorem determine the condition that transfer the maximum power to the load.

The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load

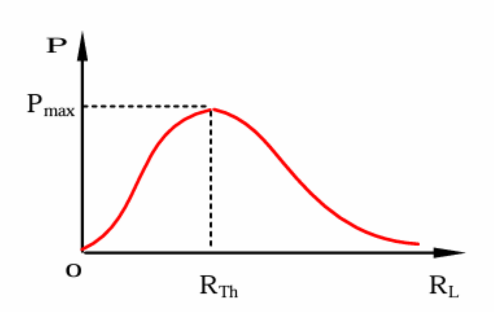


The power delivered to the load is

$$p = i_2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

For a given circuit, V_{Th} and R_{Th} are fixed

By varying the load resistance R_L, the power delivered to the load varies as sketched in the following figure.

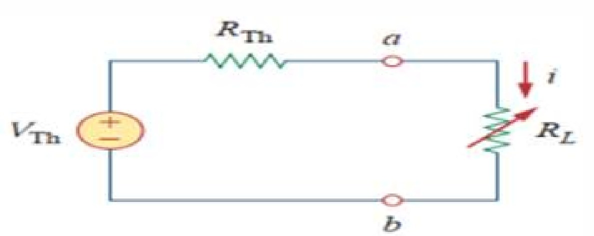


When, $R_L = R_{Th}$, P = P_{max}
When, $R_L < R_{Th}$, $P < P_{max}$
When, $R_L > R_{Th}$, $P < P_{max}$

Maximum power is transferred to load when $R_L = R_{Th}$

Maximum power transfer theorem states that maximum power is transferred to the load when the load resistance equals to the Thevenin Resistance. $(R_L = R_{Th})$

Derivation



Power delivered to the load R_L is

$$P = i^2 R_L$$

$$= \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R \;\; \ldots\ldots.(i)$$

We differentiate equation (i) with respect to R_L and set the result equal to zero

$$\frac{dP}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th+R_L})^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right]$$

$$\Rightarrow V_{Th}^2 \left[\frac{R_{Th} + R_L - 2R_L}{(R_{Th} + R_L)^3}\right] = 0$$

But $(R_{Th} + R_L)^3$ cannot be zero hence $R_{Th} + R_L - 2R_L = 0$ which further evolves to $R_{Th} = R_L$

The maximum power transferred to the load is

$$P = i_2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$