Lecture 11

CSE250 - Circuits and Electronics

First Order Circuits

BRAC UNIVERSITY
Inspiring Excellence

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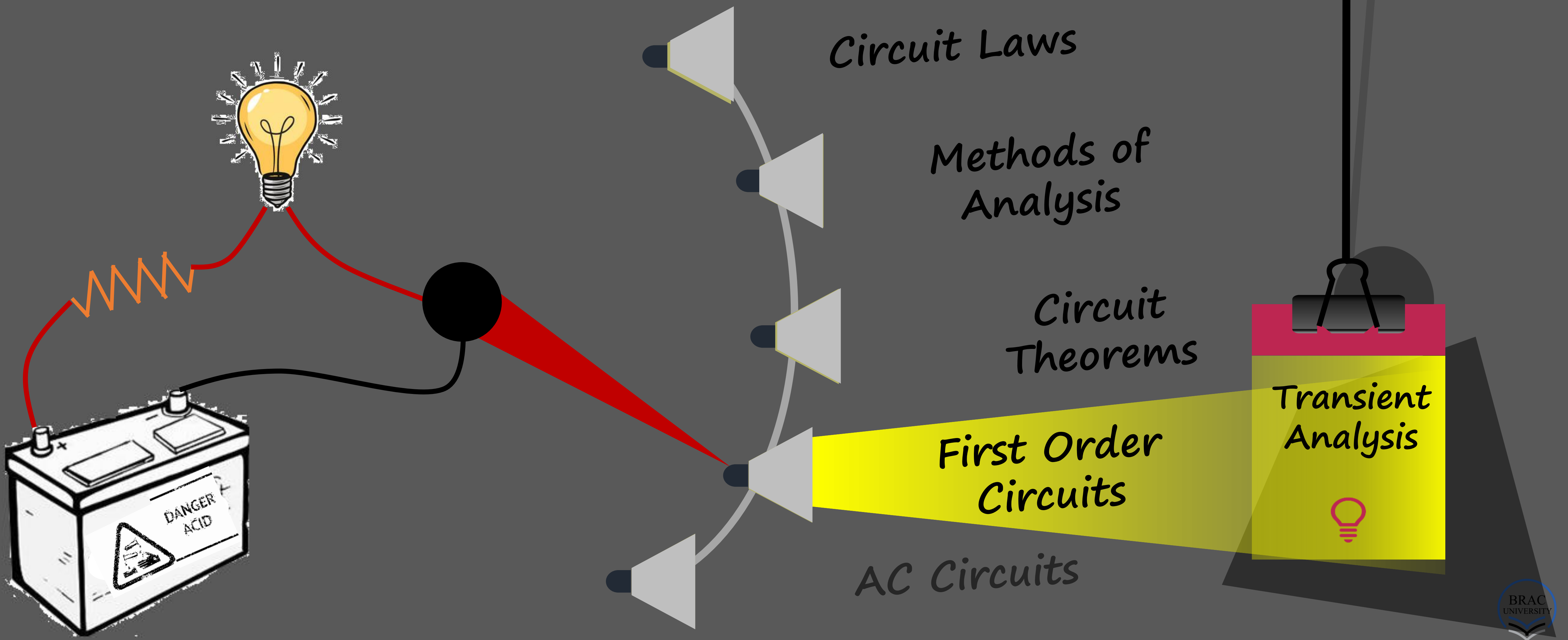
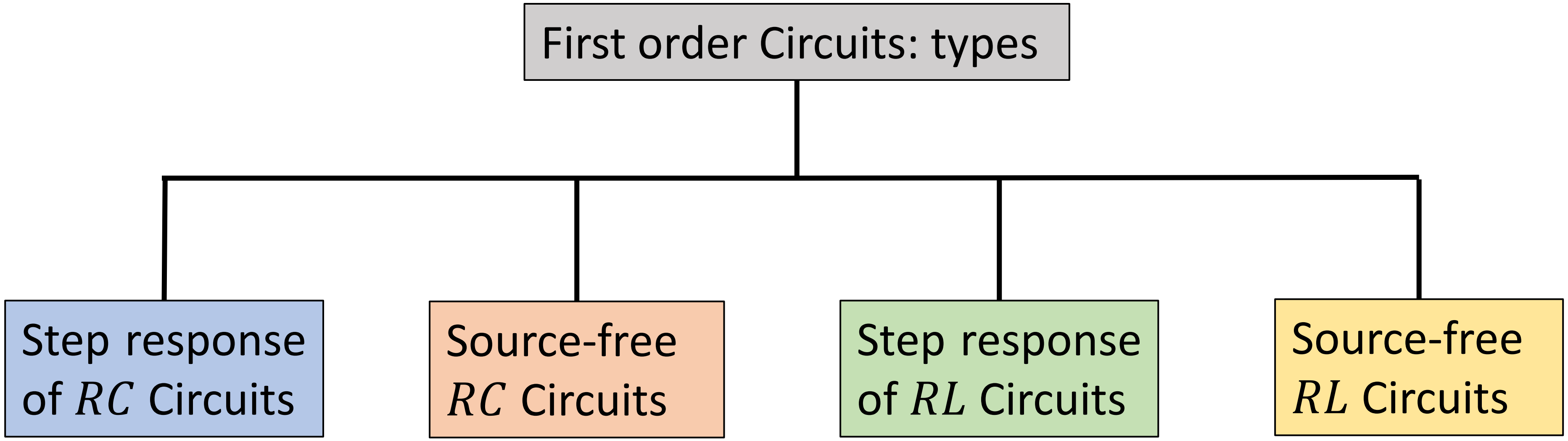
Course Outline: broad themes



- Circuit Laws
- Methods of Analysis
- Circuit Theorems
- First Order Circuits
- Transient Analysis
- AC Circuits

First Order Circuits

- A first-order circuit is characterized by a first-order differential equation.
- We shall examine two types of differential circuits: circuit comprising resistors and capacitors (RC circuit) and circuit comprising resistors and inductors (RL circuit).
- Two ways to excite the circuits: (i) by initial conditions of storage elements (source free circuits) and (ii) by independent sources (DC for this course).



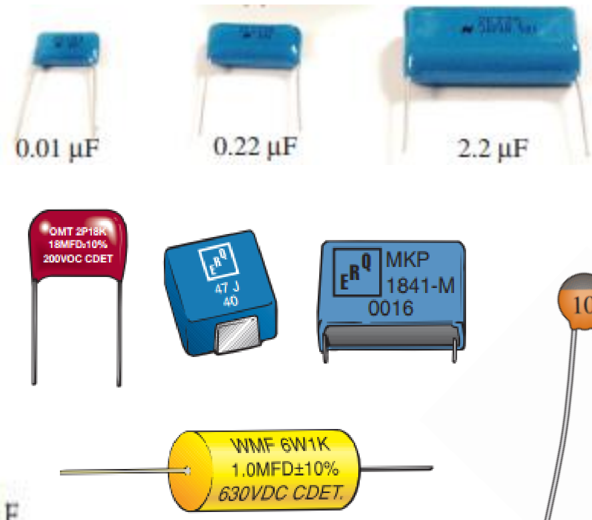
Circuit Elements

- **Active element**
  - An *active element* is capable of generating energy.
  - In other words, an element is said to be active if it can add some gain (in terms of voltage or current) to a circuit.
  - Active elements can absorb energy if they are forced to do so by other active elements.
  - Examples: *Voltage/current sources, generators, transistors, operational amplifiers.*
- **Passive element**
  - *Passive elements* cannot supply energy. They can only consume/dissipate/store energy.
  - Examples: *Resistors, capacitors, inductors, transformers.*
  - Transformers change the voltage or current levels, but the power is unchanged. This is why transformers are passive element.

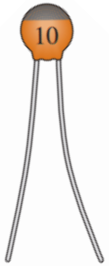
Capacitors

- A capacitor is a passive circuit element designed to store energy in its electric field.
- Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time. For this reason, capacitors and inductors are called *storage* elements.

Electrolytic

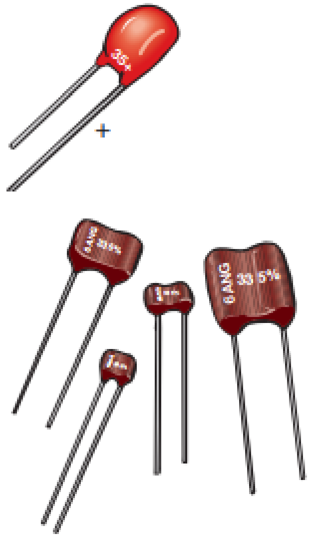


Polyester Film

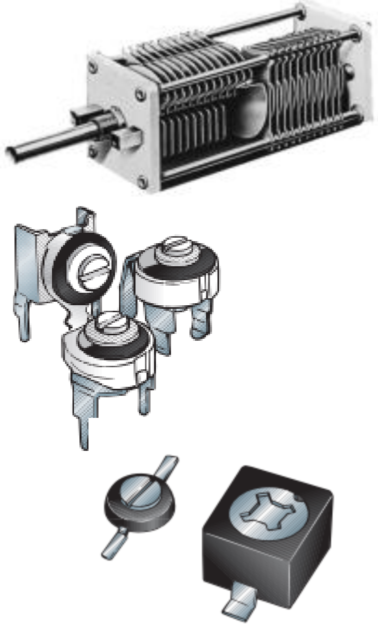


Ceramic

Mica Capacitors



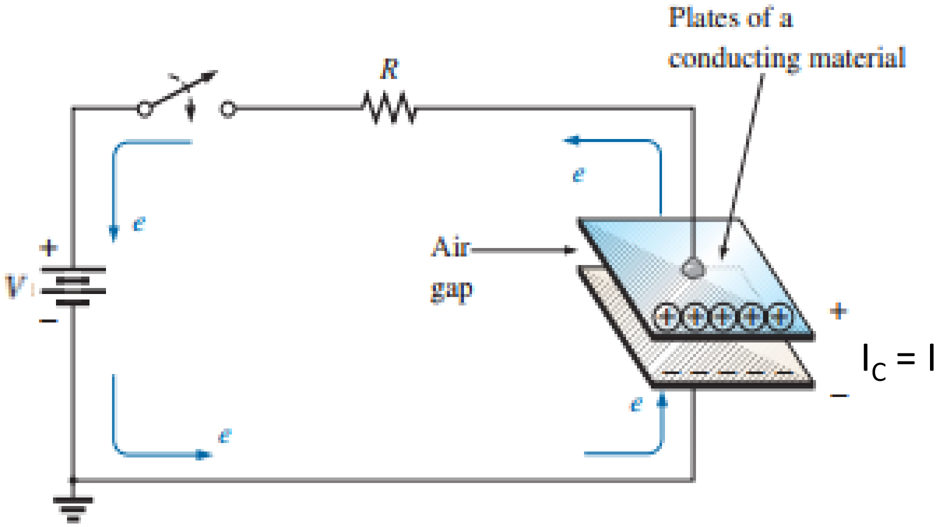
Dipped Capacitors

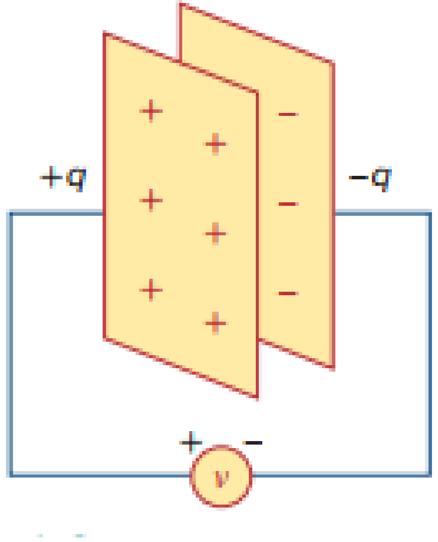


Variable Capacitors

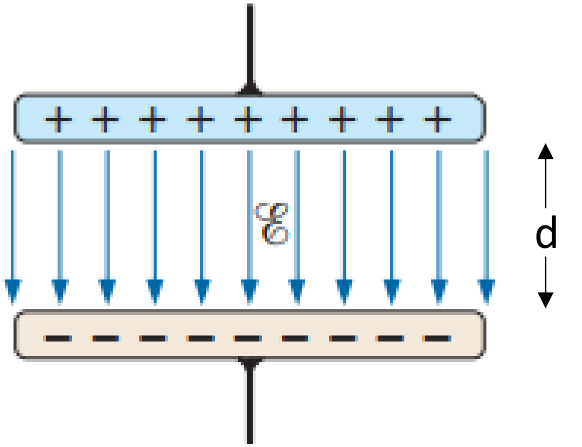
Parallel Plate Capacitor

- Most widely used configuration is the two conducting surfaces (aluminium mainly) separated by a dielectric (air, ceramic, paper, or mica).
- The switch is open initially (no net charge).
- Closing the switch causes electrons to flow from and to the upper and lower plates respectively as shown by the arrows.
- Electron flow continues until the potential difference between the plates equals the applied potential.
- The final result is a net positive charge on the top plate and a negative charge on the bottom plate.





Capacitance



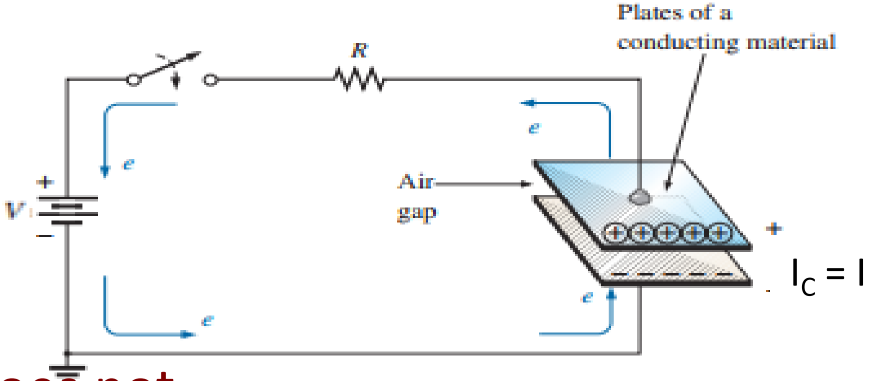
- *Capacitance* is a measure of a capacitor's ability to store charge.
- Increasing V increases E as $E \propto \frac{V}{d}$ as long as d is constant. An increase in E field causes increased charge separation i.e. increases q.
- So, $q \propto V$

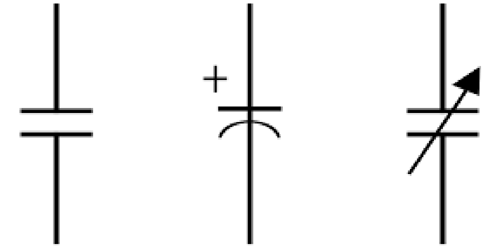
$\Rightarrow$ $q = CV$ [C is a proportionality constant ≡ Capacitance]

$\Rightarrow$ $C = \frac{q}{V}$ $[F\ (Farad),\ mF,\ \mu F]$

$\Rightarrow$ For a particular capacitor $\uparrow V, \uparrow q$ but $\frac{q}{V}$ = const. So, C does not depend on q or v. It depends on the physical dimension of the capacitor.

$\Rightarrow$ For the parallel plate capacitor, $C = \frac{\varepsilon A}{d}$





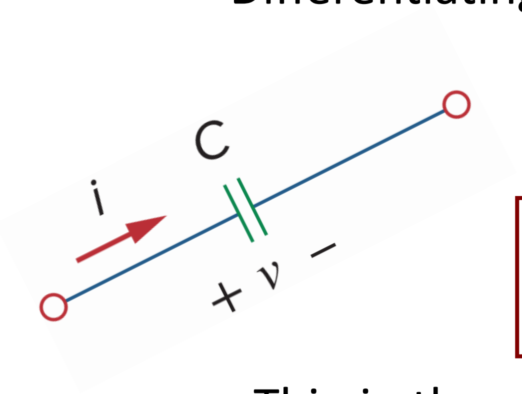
I-V relation of a Capacitor

- From the definition of the capacitance,

$$C = \frac{q}{v}$$

$$\Rightarrow \quad q = Cv$$

- Differentiating with respect to time,



$$\frac{dq}{dt} = C\frac{dv}{dt}$$

$$\boxed{i = C\frac{dv}{dt}}$$

- This is the characteristic equation of a capacitor.

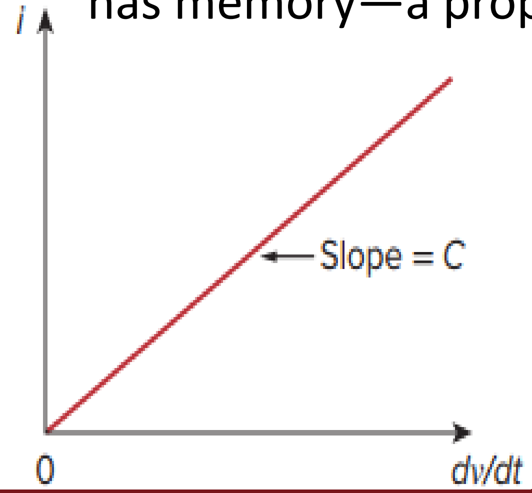
- Integrating with respect to time,

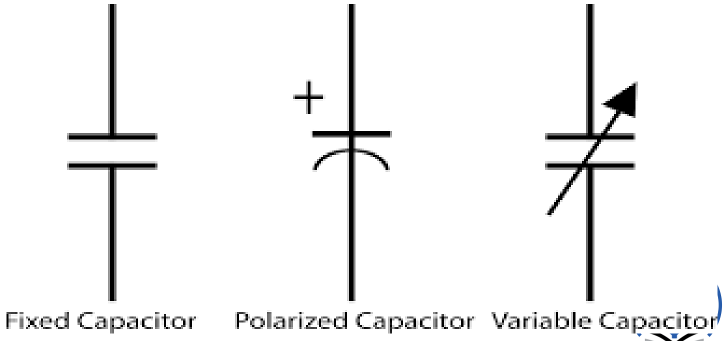
$$v(t) = \frac{1}{C}\int i(t)\,dt$$

- If the voltage of the capacitor at any time t_0 is $v(t_0) = q(t_0)/C$, then,

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt + v(i_0)$$

- It shows that capacitor voltage depends on the past history of the capacitor current. Hence, the capacitor has memory—a property that is often exploited.





Energy & Power of a Capacitor

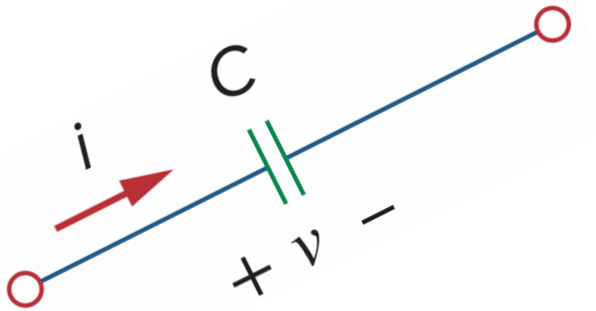
- The instantaneous power delivered to a capacitor according to the passive sign convention is,

$$p = v(t)i(t) = Cv(t)\frac{dv(t)}{dt}$$

- The energy stored in the capacitor is therefore

$$w(t) = \int_{-\infty}^{t} p(t)\,dt = \int_{-\infty}^{t} Cv(t)\frac{dv(t)}{dt}dt = \int_{v(-\infty)}^{v(t)} Cv(t)\,dv$$

$$\Rightarrow \quad w(t) = \frac{1}{2}Cv^2\Big|_{v(-\infty)=V_0}^{v(t)=V}$$



- If the voltage across the capacitor was initially (at $t = -\infty$) V_0, then,

$$\Rightarrow \quad w(t) = \frac{1}{2}CV^2 - \frac{1}{2}CV_0^2$$

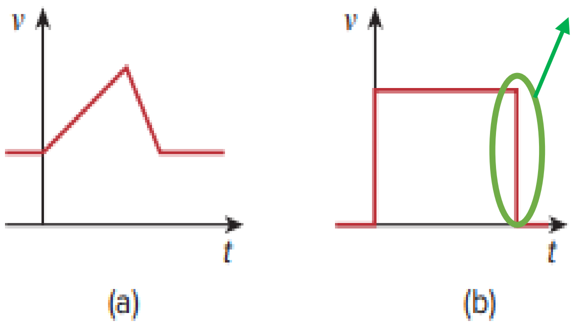
- In general, at any time t, if the voltage across a capacitor is V, then the stored energy can be found as,

$$w(t) = \frac{1}{2}Cv(t)^2 = \frac{1}{2}CV^2$$

Capacitor: important properties

1. A capacitor is an <u>open circuit</u> to dc. At dc, $i_C = C\frac{dV_{c\,dc}}{dt} = 0$ [Open circuit]
2. The <u>voltage</u> on a capacitor cannot change abruptly.

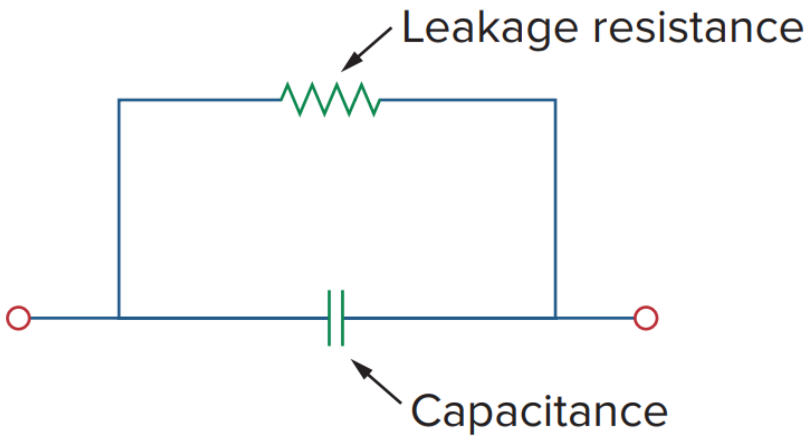
$$\frac{dV_c}{dt} = \infty$$



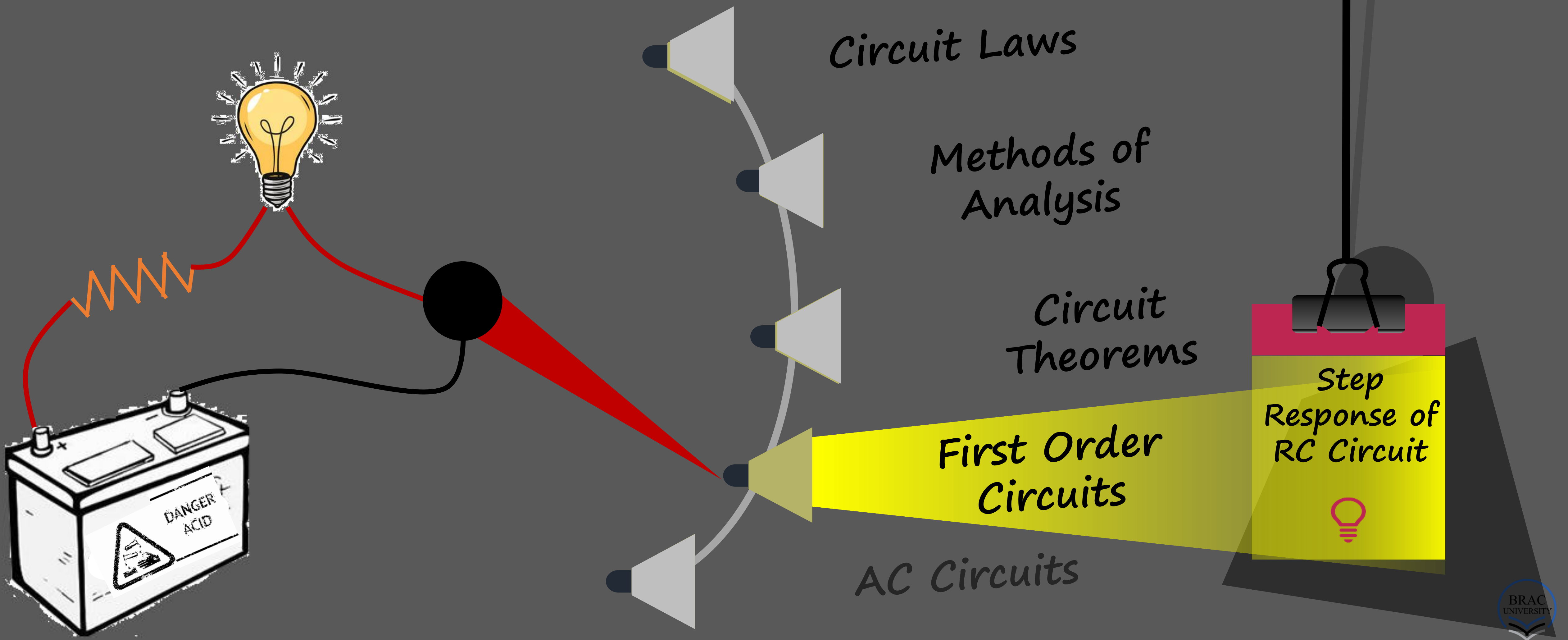
Voltage change across a capacitor (a) allowed and (b) not allowed

3. An ideal capacitor does not dissipate energy.

4. A real, nonideal capacitor has a parallel-model leakage resistance.



Course Outline: broad themes



Step Response of a RC circuit

- The *step response* of a circuit is its behaviour under the sudden application of dc voltage or current source. We assume the circuit response to be the capacitor voltage.

⇒ Since the voltage of a capacitor cannot change instantaneously,

⇒ $v(0^-) = v(0^+) = V_0$

⇒ Using KCL (for $t > 0$),

⇒ $C\frac{dv}{dt} + \frac{v - V_s}{R} = 0$

⇒ $\frac{dv}{dt} = -\frac{v - V_s}{RC}$

⇒ $\frac{dv}{v - V_s} = -\frac{1}{RC}dt$

Integrating both sides,

⇒ $[\ln(v - V_s)]_{V_0}^{v(t)} = -\left[\frac{t}{RC}\right]_0^t$

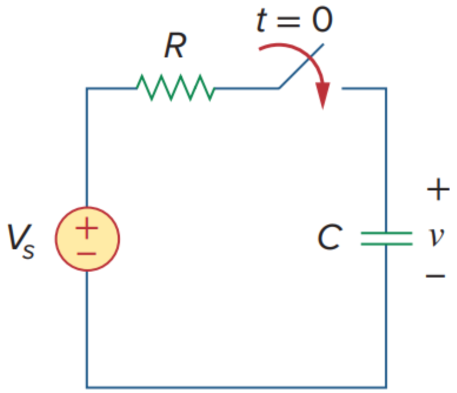
⇒ $\ln[v(t) - V_s] - \ln(V_0 - V_s) = -\frac{t}{RC} + 0$

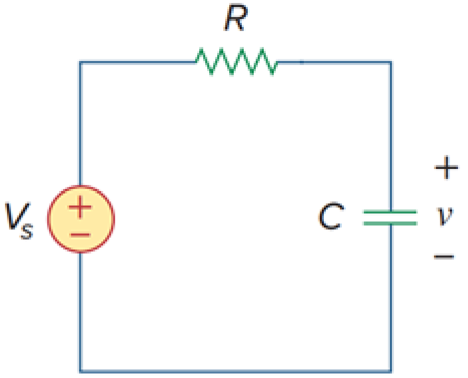
⇒ $ln\frac{v - V_s}{V_0 - V_s} = -\frac{t}{RC}$

⇒ $\frac{v - V_s}{V_0 - V_s} = e^{-t/RC}$

⇒ $v - V_s = (V_0 - V_s)e^{-t/RC}$

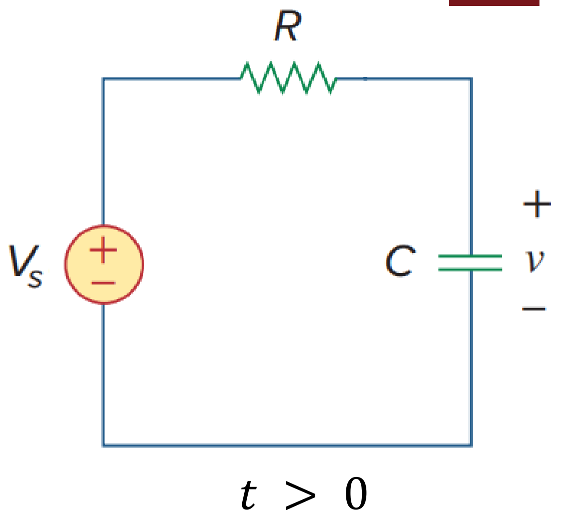
⇒ $v(t) = V_s + (V_0 - V_s)e^{-t/RC}$





Time Constant

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-\frac{t}{RC}}, & t > 0 \end{cases}$$



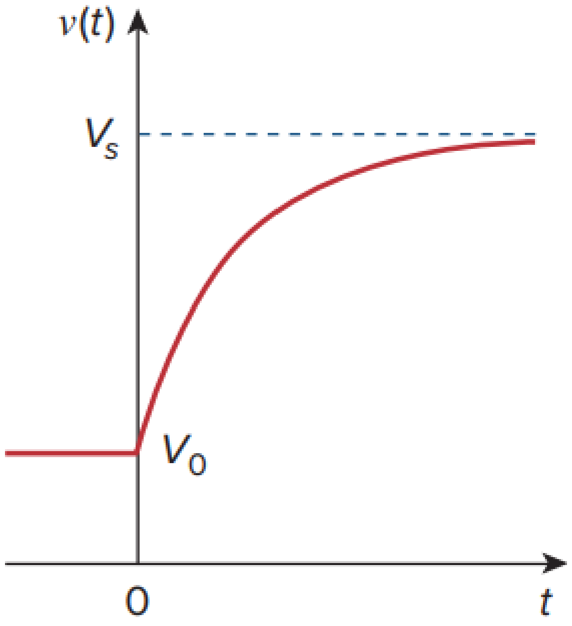
- This is known as the complete response (or total response) of the RC circuit to a sudden application of a dc voltage source. It is assumed that the capacitor was initially charged to V_0.

$$\Rightarrow \quad v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-\frac{t}{\tau}}, & t > 0 \end{cases}$$

$\Rightarrow$ where $\tau = RC$ is the *time constant* (unit in sec).

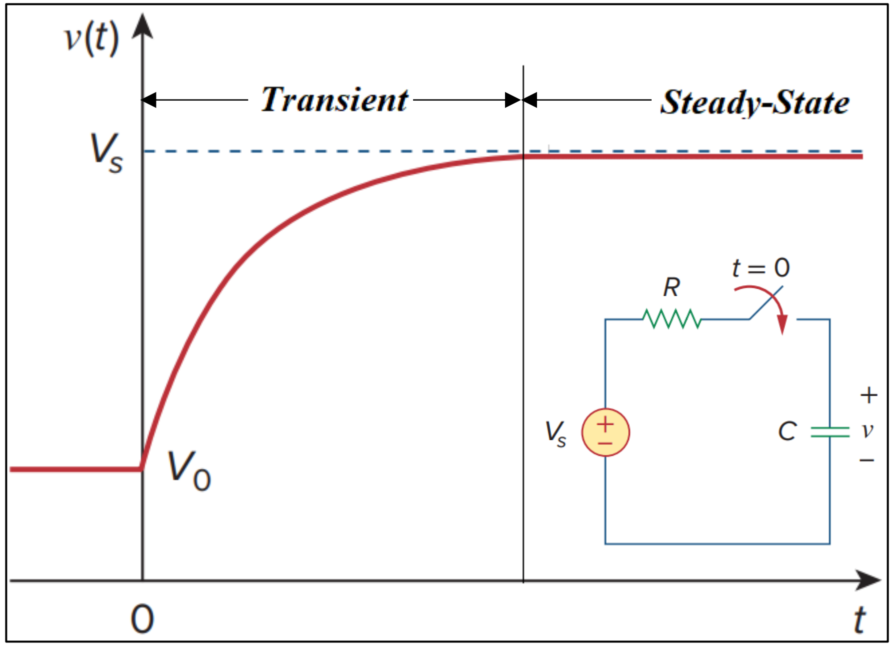
- Notice that, we write $\tau = RC$ for the circuit consisting of only a resistor R in series with the capacitor. As we know, all the linear two terminal circuits can be reduced to this form by Thevenin's Theorem, so the resistor R is actually the Thevenin Resistance R_{Th}. Therefore,

$$\boxed{\tau = R_{Th}C}$$



Transient and Steady-State Response

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-\frac{t}{\tau}}, & t > 0 \end{cases}$$



- The *complete response* can be broken into two parts—one temporary and the other permanent, that is,

$$v(t) = v_{ss} + v_t, \qquad \text{where,}$$

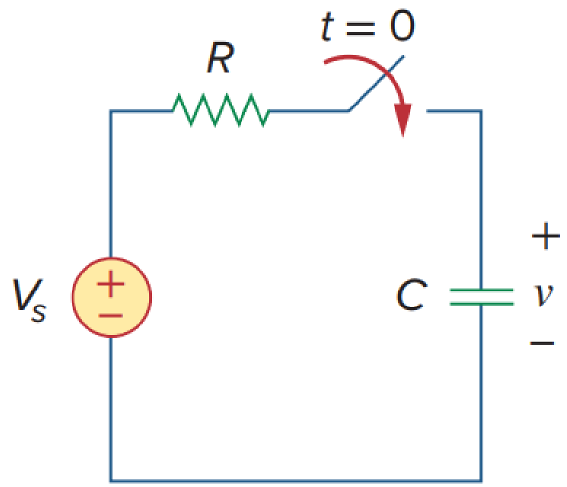
$$v_{ss} = V_s \qquad \& \qquad v_t = (V_0 - V_s)e^{-\frac{t}{\tau}}$$

- The *transient response* (v_t) is the circuit's temporary response that will die out with time.
- The *steady-state response* (v_{ss}) is the behaviour of the circuit a long time after an external excitation is applied.
- The complete response can be written as,

$$v(t) = V_{final} + [V_{initial} - V_{final}]e^{-\frac{t}{\tau}}$$

$$or, \qquad v(t) = V(\infty) + [V(0) - V(\infty)]e^{-\frac{t}{\tau}}$$

Definition of τ



$$v(t) = V_{final} + \left[V_{initial} - V_{final}\right]e^{-\frac{t}{\tau}}$$

At $t = \tau$,

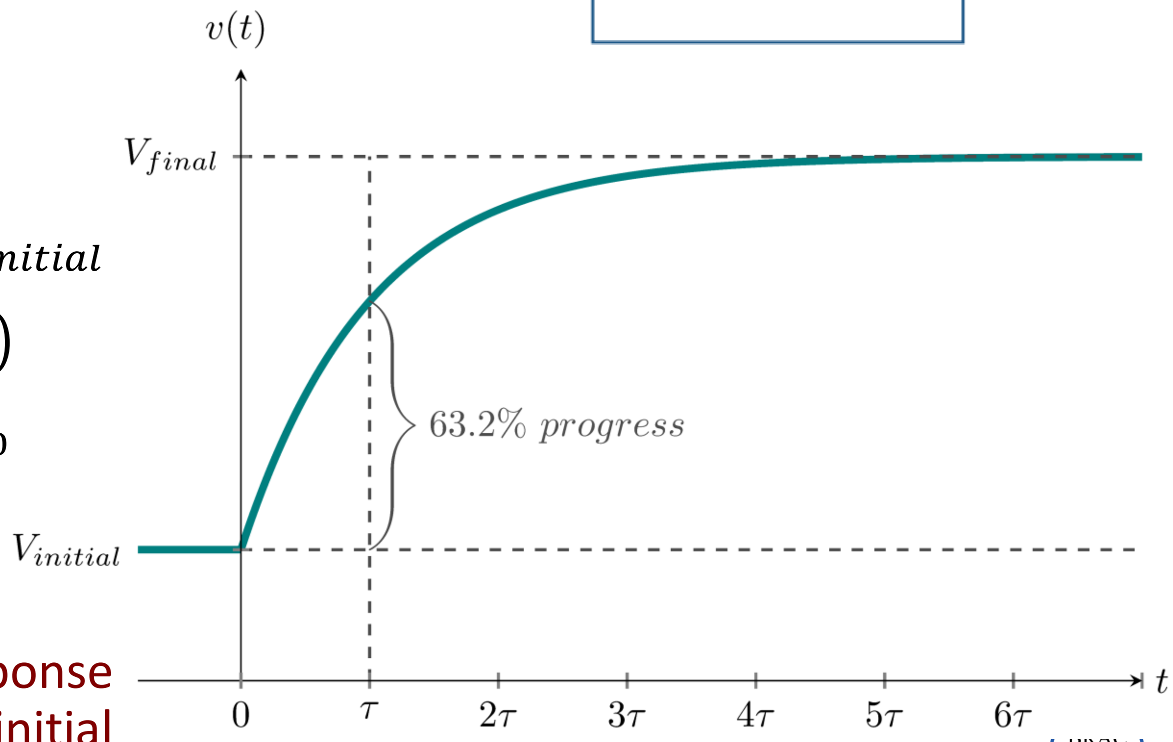
$$v(t) = V_{final} + \left[V_{initial} - V_{final}\right]e^{-1}$$

$$\Rightarrow \quad v(t) = V_{final}\left(1 - {}^{1}/_{e}\right) + V_{initial}\left({}^{1}/_{e}\right)$$

$$\Rightarrow \quad v(t) = V_{final}\left(1 - {}^{1}/_{e}\right) - V_{initial}\left(1 - {}^{1}/_{e}\right) + V_{initial}$$

$$\Rightarrow \quad v(t) = V_{initial} + \left[V_{final} - V_{initial}\right]\left(1 - {}^{1}/_{e}\right)$$

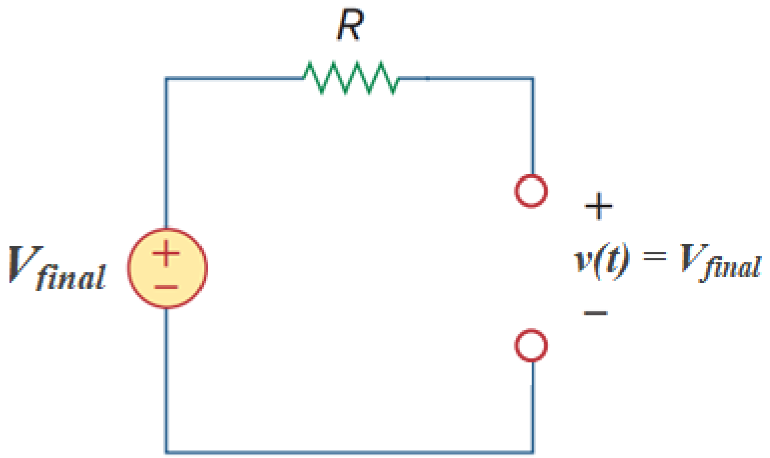
$$\Rightarrow \quad v(t) = V_{initial} + \left[V_{final} - V_{initial}\right] \times 63.2\%$$



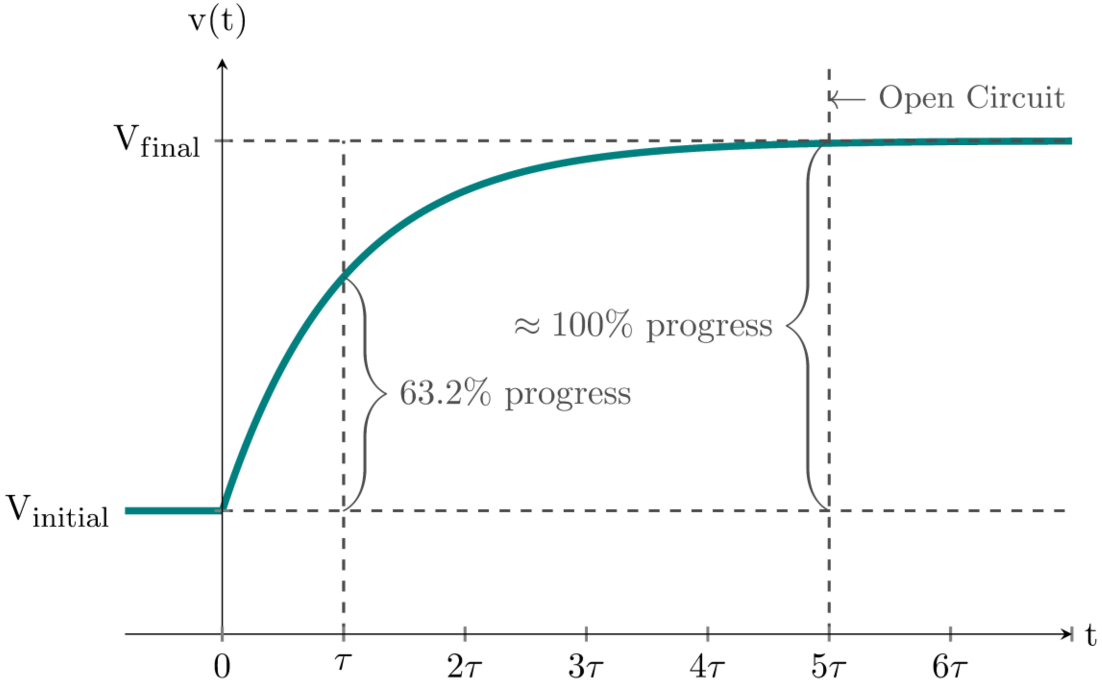
- We can define the time constant in this way,
- The *time constant* is the time required for the response to progress 63.2% towards V_{final} from an initial response $V_{initial}$.

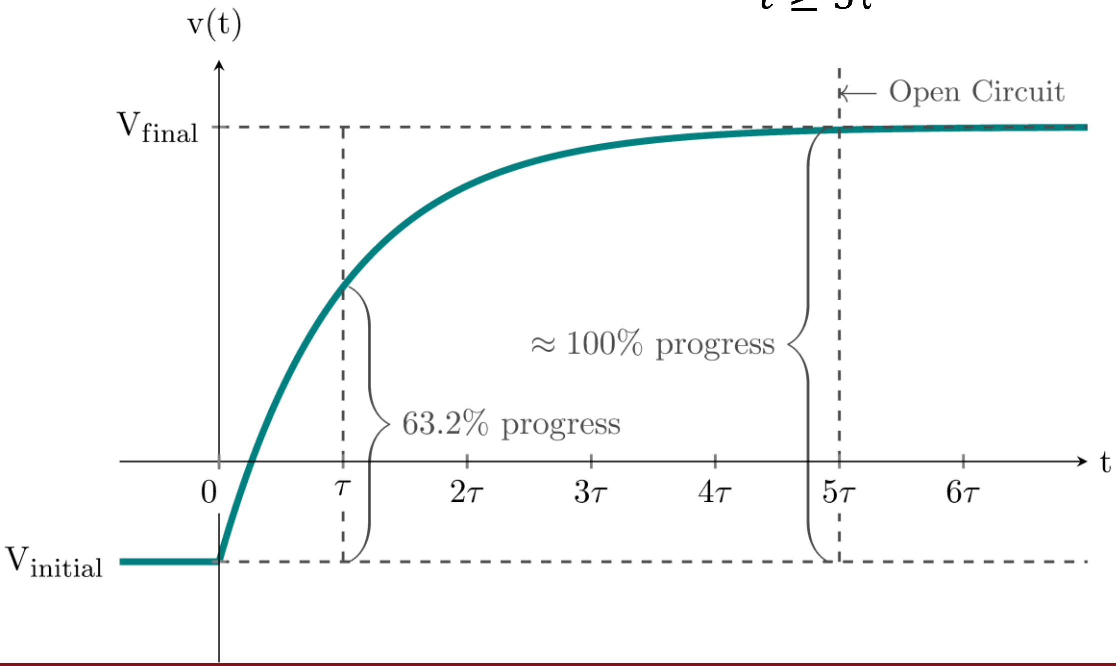
Significance of τ

- The significance of τ is that it determines how fast or slow a capacitor will be charged or discharged. Mathematically, a capacitor voltage reaches the final voltage approximately after 5 times the Time Constant (τ). **The capacitor is fully charged and acts as open circuit from 5τ time onward.** So, when designing circuits, the charging time of a capacitor under the application of a certain dc supply can be set by choosing R_{Th}.



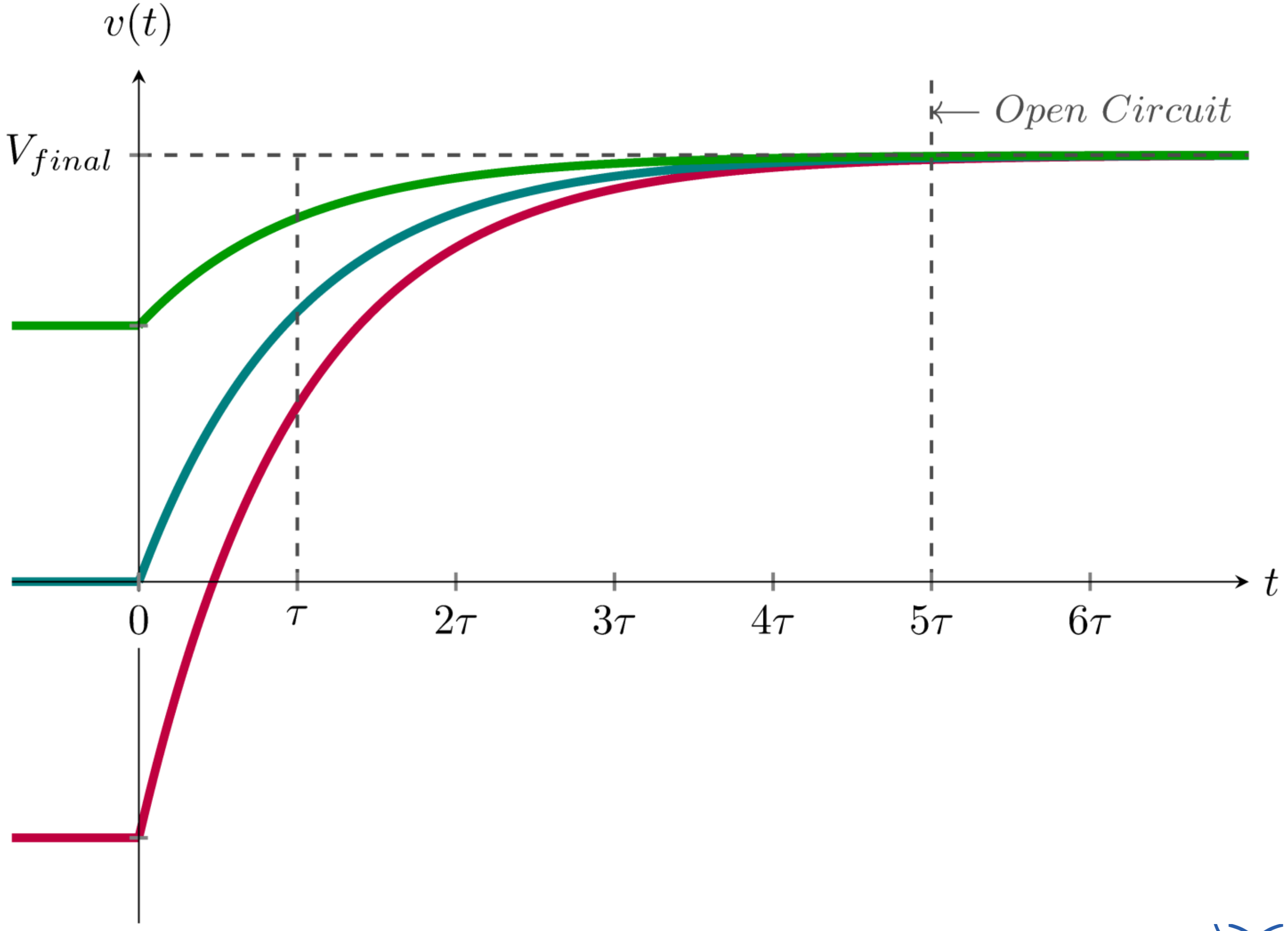
$t \geq 5\tau$





τ's independency from initial condition

- The time constant does not depend on the initial voltage or initial charge of the capacitor. For a given circuit, that is, for a fixed R_{Th} and C, the time needed for the capacitor voltage to rise to its final value is the same whether or not the capacitor is initially charged.



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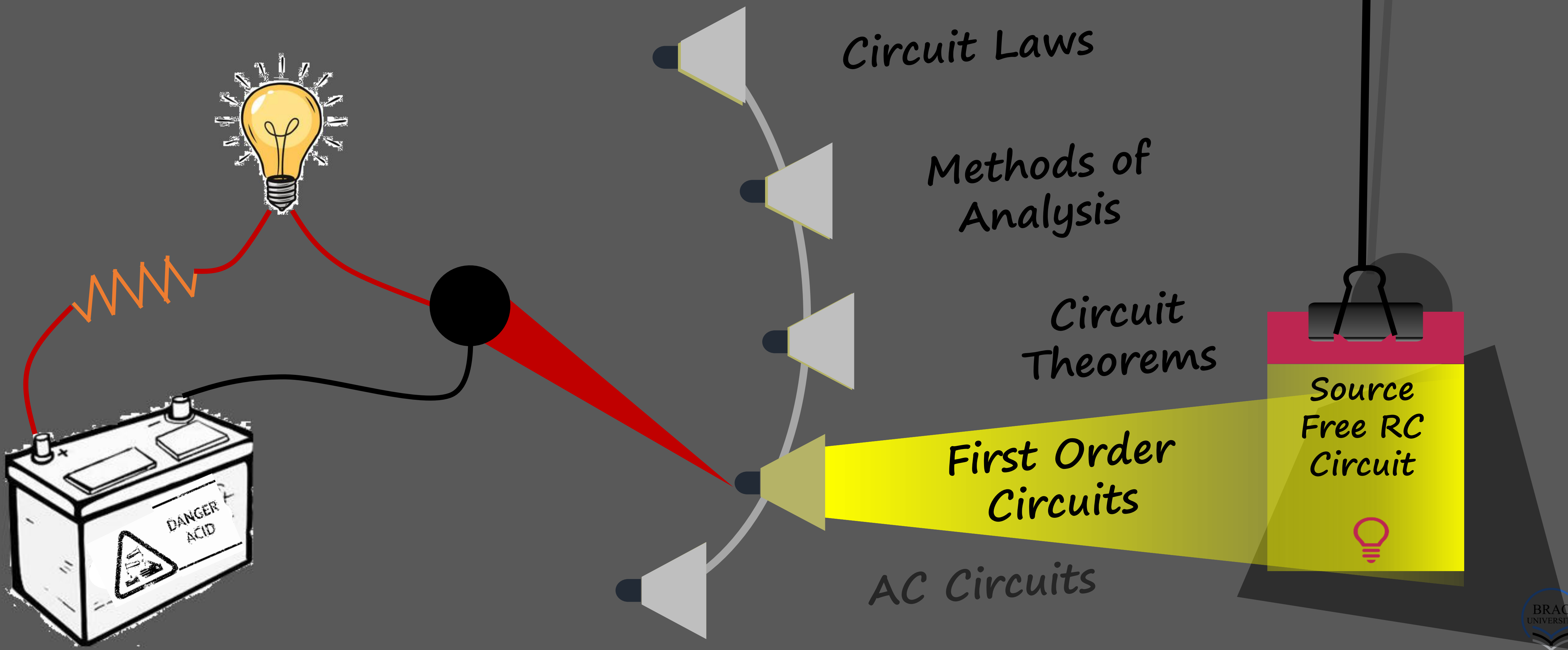


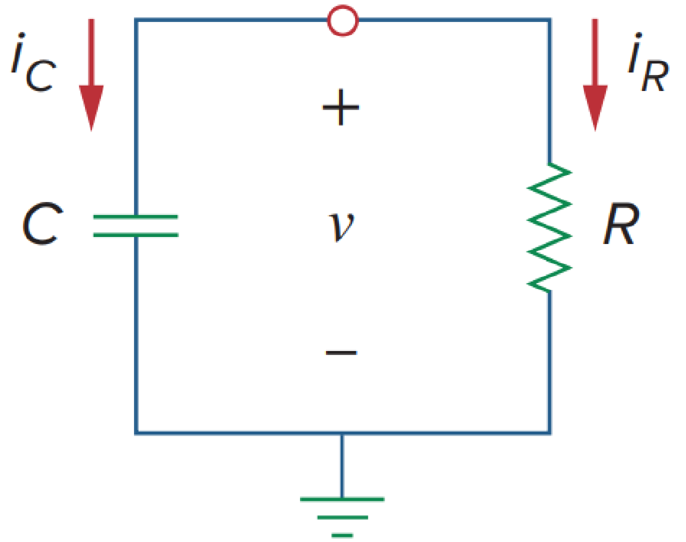
Source-Free RC circuit

- A *source-free RC circuit* occurs when its dc source is suddenly disconnected. The energy already stored in the capacitor is released to the resistors.

⇒ Assume that a capacitor is charged to V_0 and then it is connected to a resistor as shown. The capacitor starts to discharge the stored energy to the resistor.

⇒ Initially stored charge, $w(0) = \frac{1}{2}CV_0^2$

⇒ From the figure using KCL, $i_C + i_R = 0$



$\Rightarrow C\frac{dv}{dt} + \frac{v}{R} = 0$

$\Rightarrow \frac{dv}{dt} + \frac{v}{RC} = 0$

$\Rightarrow \frac{dv}{v} = -\frac{1}{RC}dt$

Integrating both sides,

$\Rightarrow lnv = -\frac{t}{RC} + lnA$

$\Rightarrow ln\frac{v}{A} = -\frac{t}{RC}$

$\Rightarrow v = Ae^{-\frac{t}{RC}}$

At $t = 0, v(0) = A = V_0$. So, $\boxed{v(t) = V_0e^{-\frac{t}{RC}}}$

Time Constant

$$v(t) = V_0 e^{-\frac{t}{RC}}$$

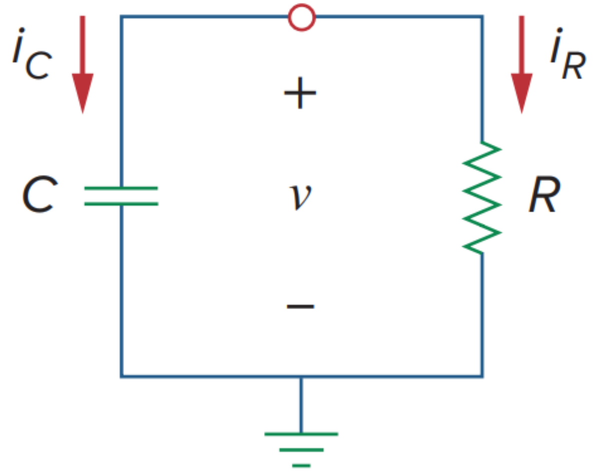
- This shows that the voltage response of the RC circuit is an exponential decay of the initial voltage. It is called the *natural response* of the circuit.

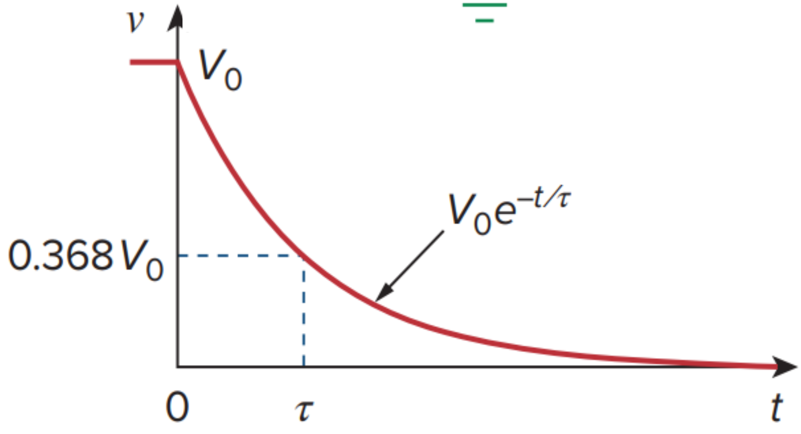
$$\Rightarrow \quad v(t) = V_0 e^{-\frac{t}{\tau}}$$

- where $\tau \ = \ RC$ is the time constant (unit in sec).

- Notice that, we write τ = RC for the circuit consisting of only a resistor R in series with the capacitor. As we know, all the linear two terminal circuits can be reduced to this form by Thevenin's Theorem, so the resistor R is actually the Thevenin Resistance R_{Th}. Therefore,

$$\boxed{\tau = R_{Th}C}$$





Definition of τ

$$v(t) = V_0 e^{-\frac{t}{\tau}}$$

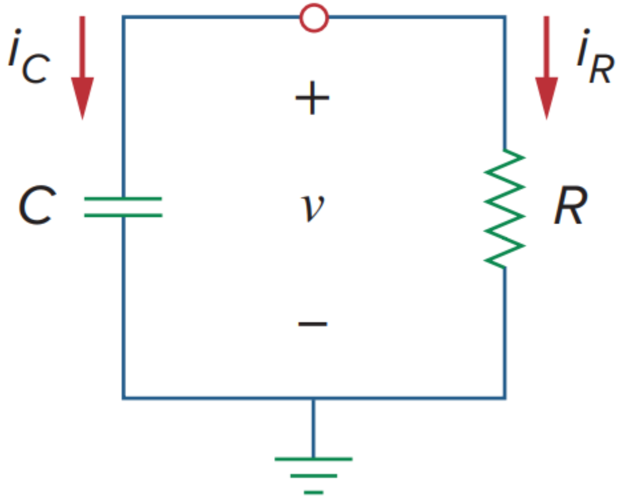
At $t = \tau$,

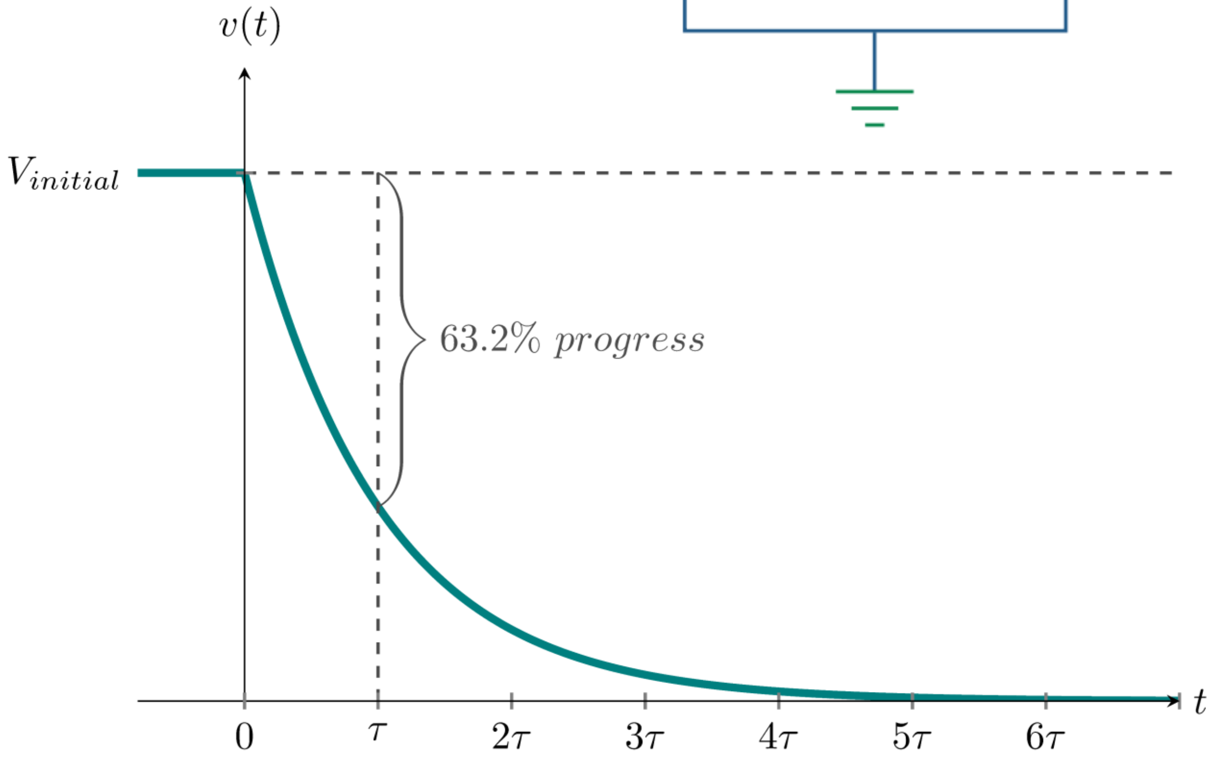
$$v(t) = V_0 e^{-1}$$

$$\Rightarrow \quad v(t) = 0.368 \times V_0$$

$$\Rightarrow \quad v(t) = V_{initial} - 0.632 \times [V_{initial} - 0]$$

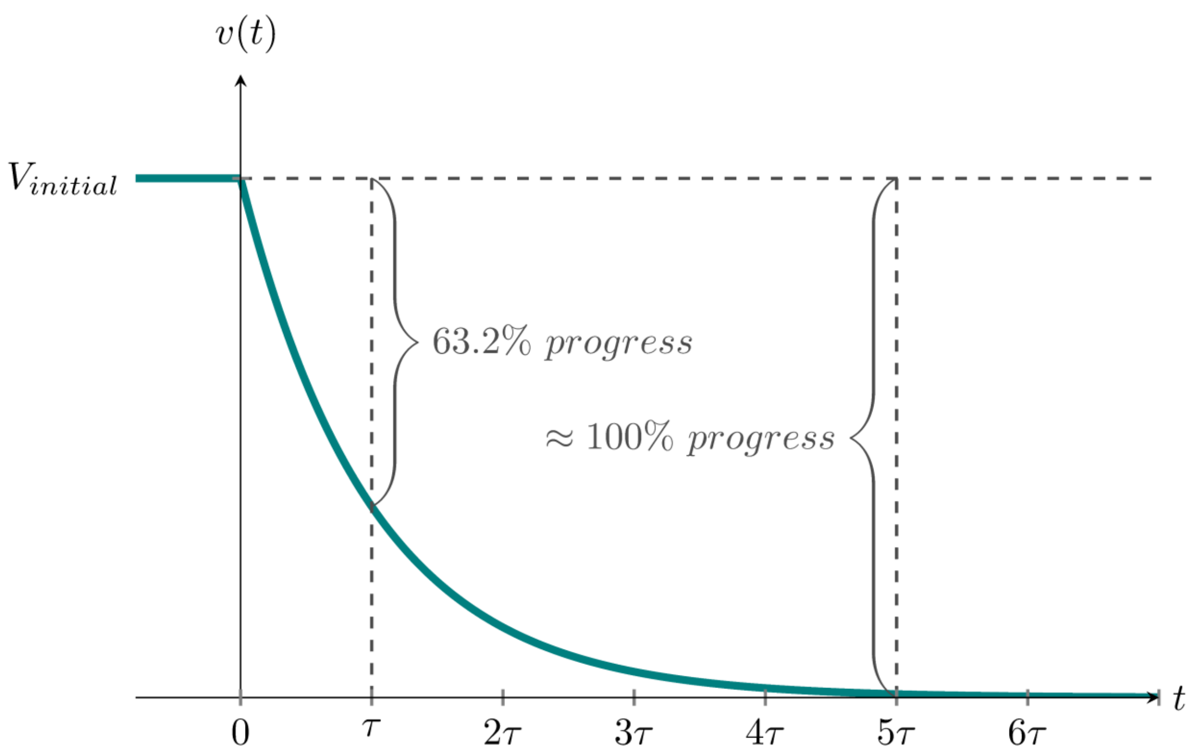
- We can define the time constant in the same way as we did in the case of the step response:
- The *time constant* is the time required for the response to progress 63.2% towards a final voltage from an initial response $V_{initial}$ or V_0.

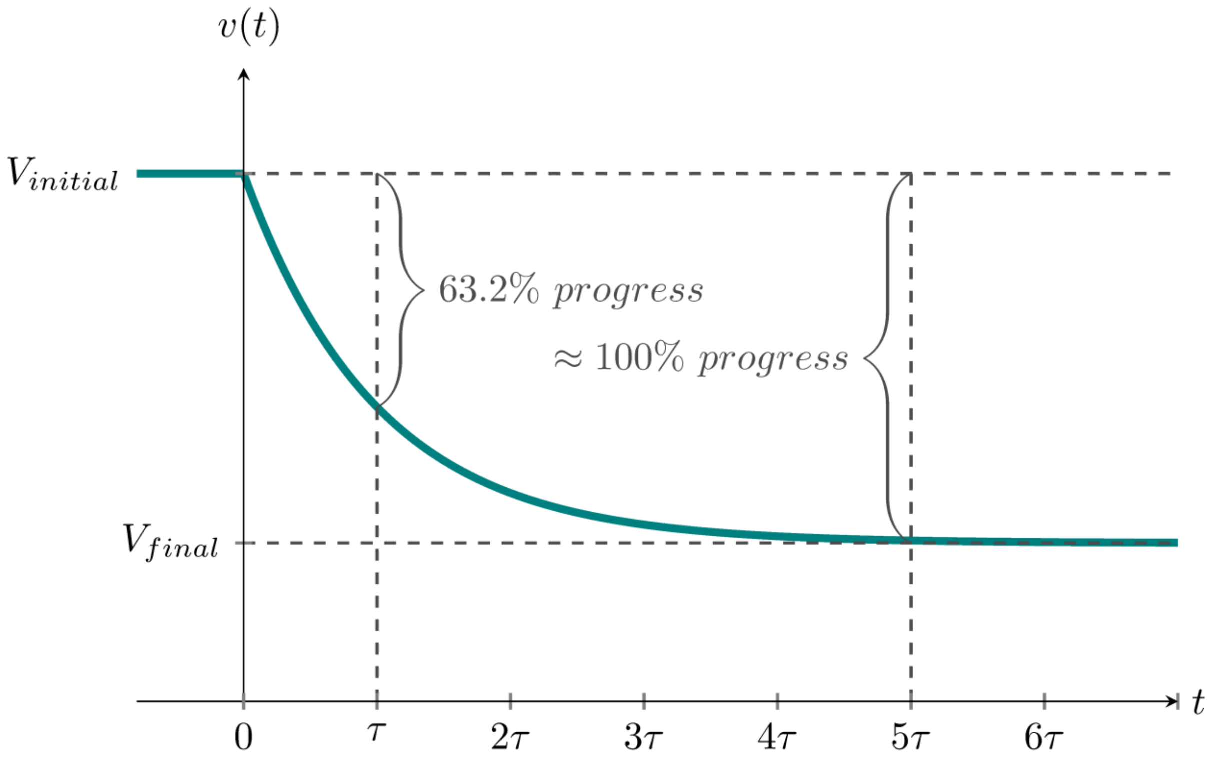




Significance of τ

- When a capacitor's initial voltage ($V_{initial}$) falls to zero ($V_{final} = 0$) **(if no independent sources are present in the circuit to charge it back)**, it can be shown mathematically that it takes approximately 5τ time to reach the final voltage. In fact, irrespective of the values of $V_{initial}$ and V_{final}, a capacitor will always take $\approx 5\tau$ time to reach the final voltage.





τ's independency from initial condition

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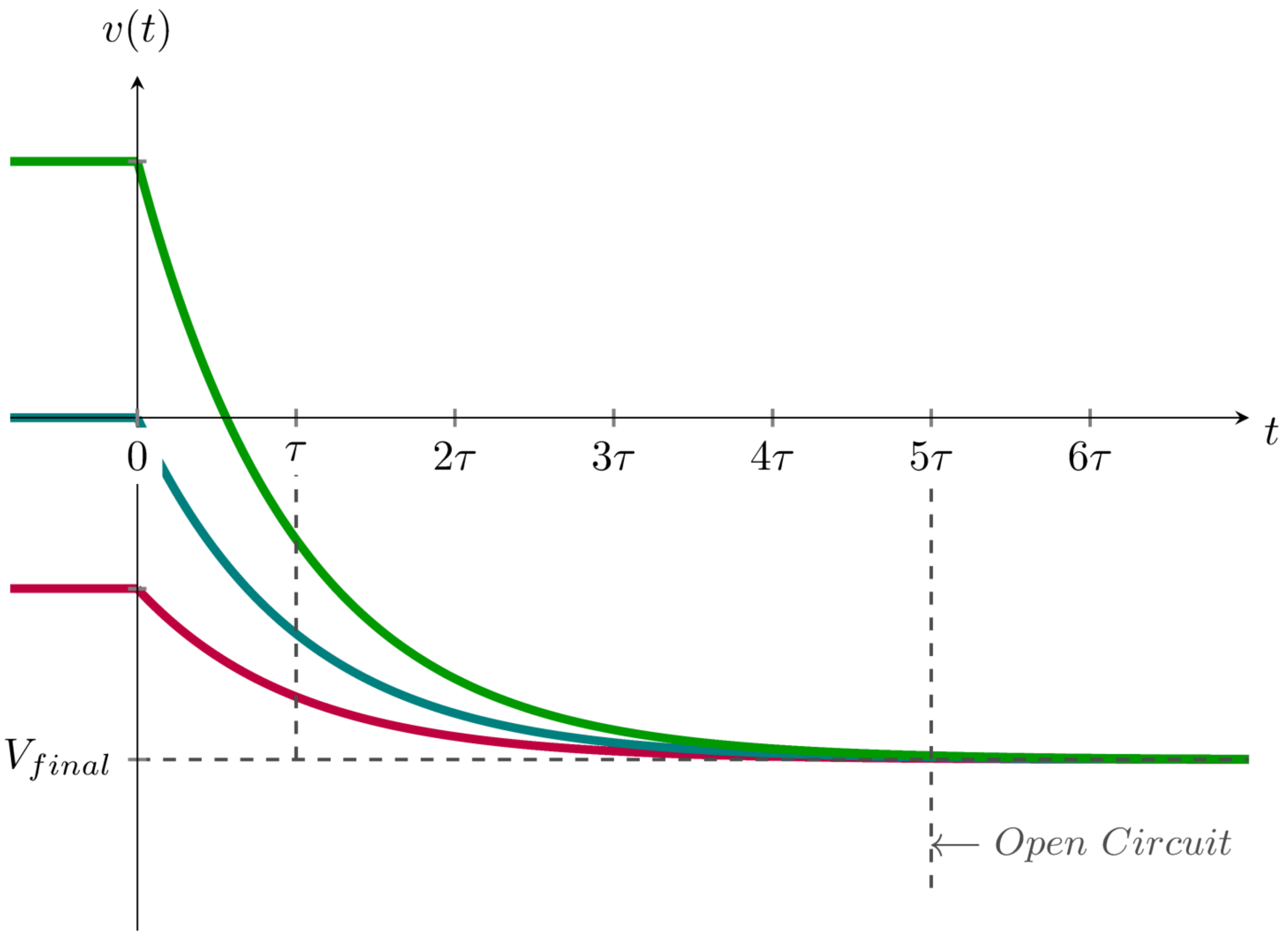
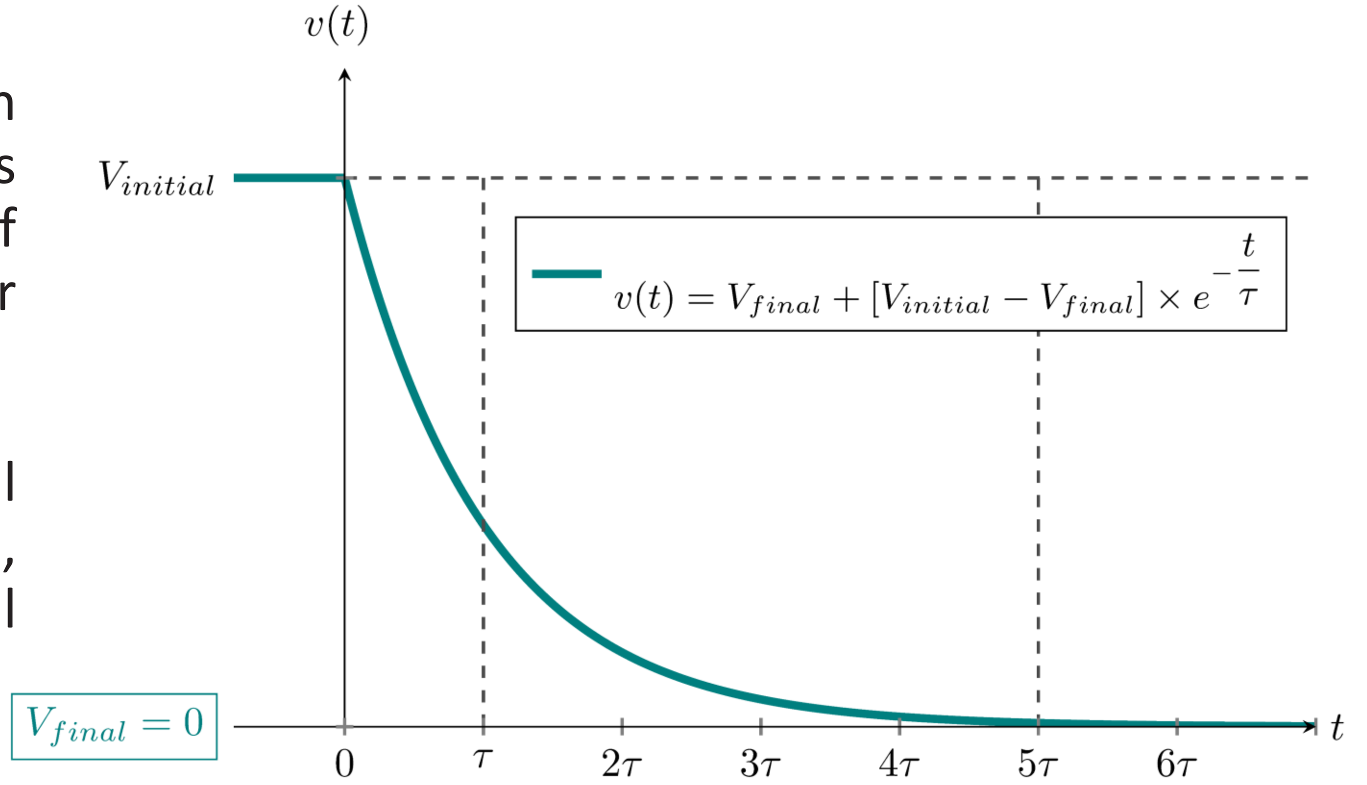


General equation

- It is interesting to interpret this way that the source-free RC circuit is a condition where $V_{final} = V(\infty) = 0$ in the step response $\left[v(t) = V(\infty) + [V(0) - V(\infty)]e^{-\frac{t}{\tau}}\right]$ or $\left[v(t) = V_{final} + [V_{initial} - V_{final}]e^{-\frac{t}{\tau}}\right]$ equation we derived earlier.

- We can in fact use this equation in general to find a capacitor's voltage response irrespective of the capacitor getting charged or discharged or both.

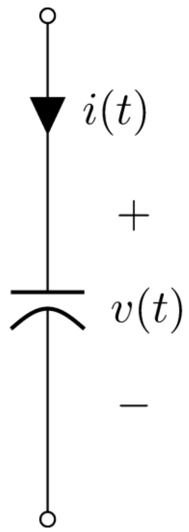
- A capacitor's energy will increase if $|V(\infty)| > |V(0)|$, otherwise the energy will decrease.



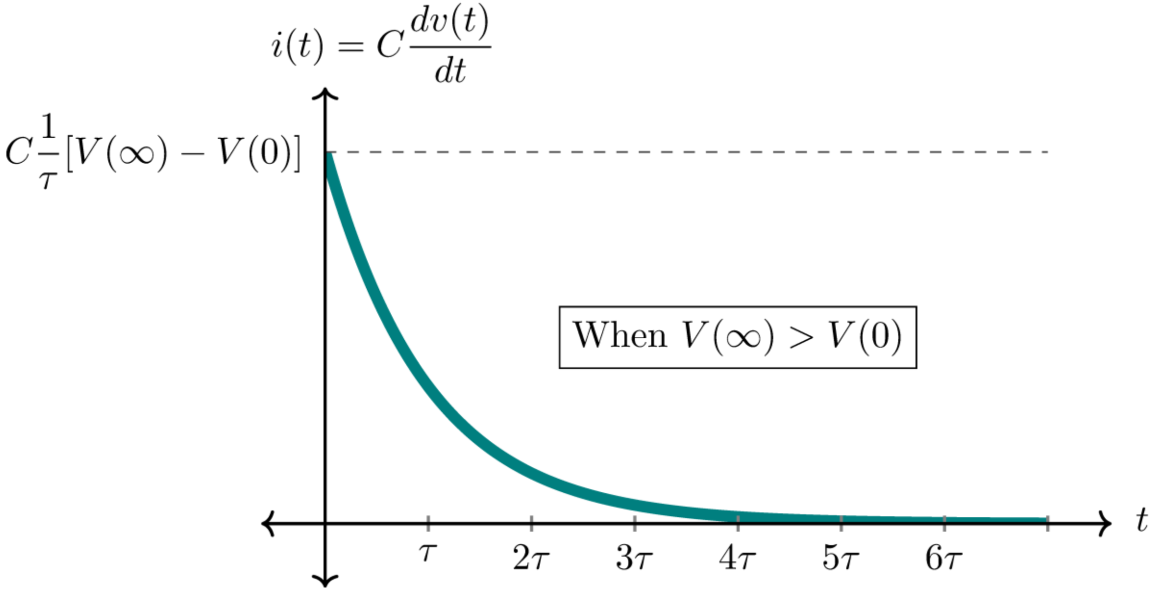
Capacitor Current

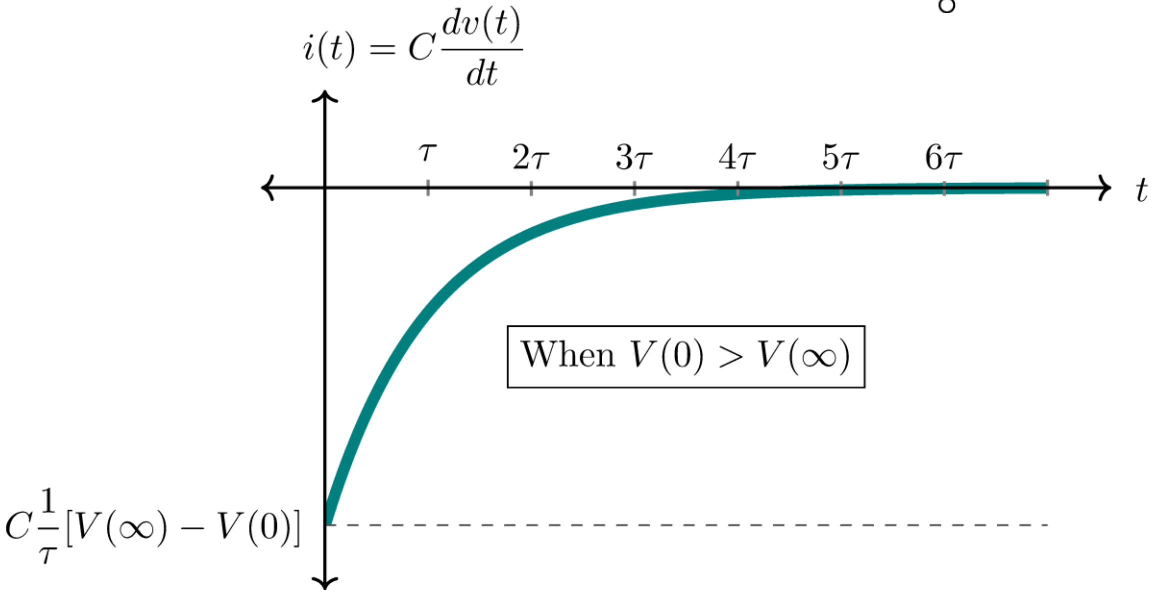
- The capacitor current can be found from the $I - V$ characteristic equation as

$$I = C\frac{dv(t)}{dt} = C\frac{d}{dt}\left\{v(t) = V(\infty) + [V(0) - V(\infty)]e^{-\frac{t}{\tau}}\right\} = C\frac{1}{\tau}[V(\infty) - V(0)]e^{-\frac{t}{\tau}}$$



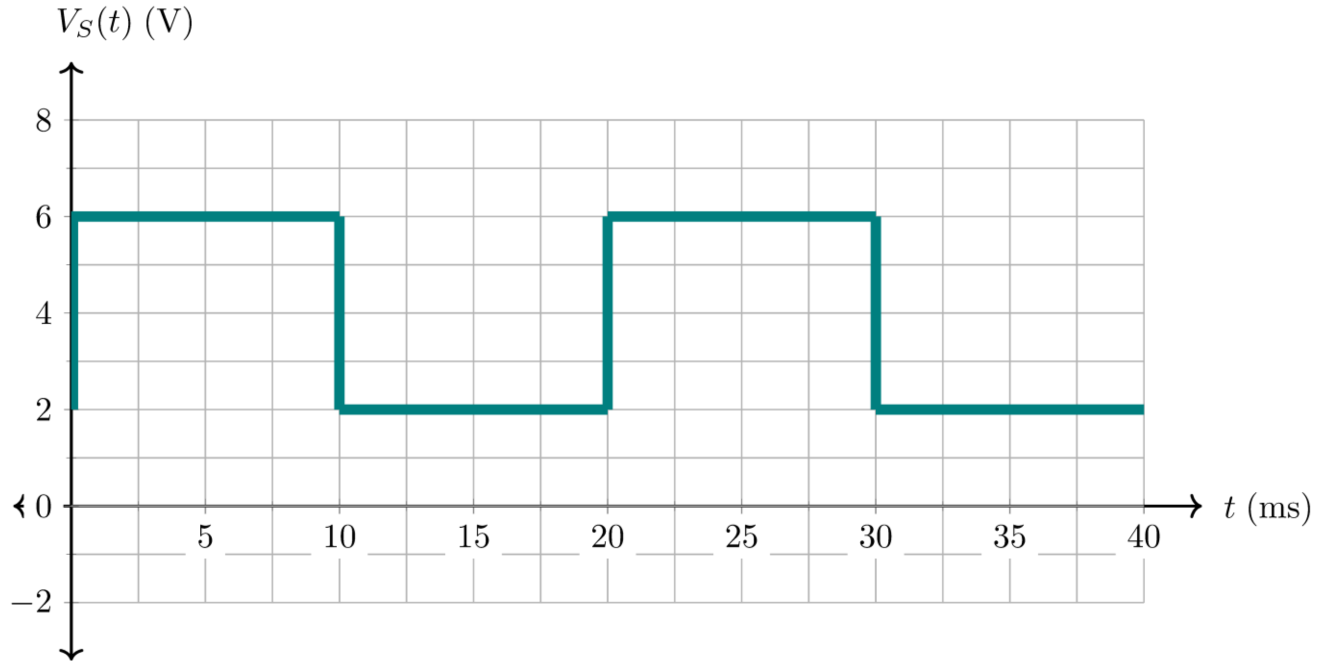
- Note that regardless of whether the capacitor voltage is increasing or decreasing, the current—whether flowing into or out of the capacitor always decreases exponentially and approaches zero at $t = 5\tau$.

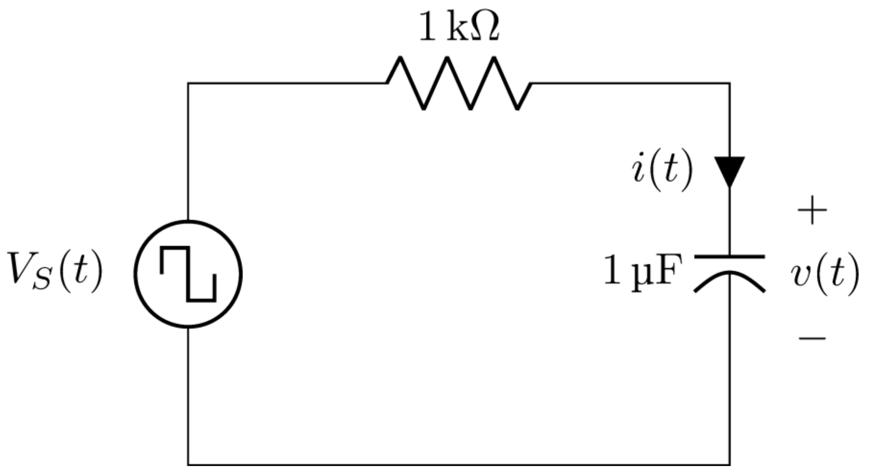




RC Circuit Under Periodic Square Voltage

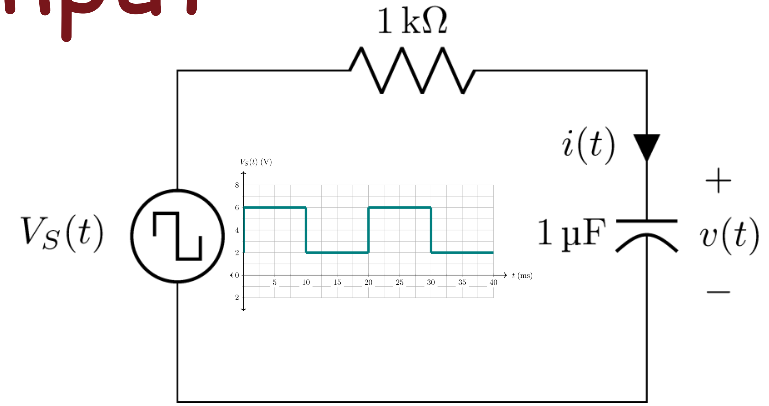
- To observe the voltage-current response of a capacitor in the laboratory, a square wave voltage—periodically switching between two levels—can be applied across it, as shown.
- Here, the input voltage stays at each level for $10\,ms$, while the time constant is $\tau = 1\,k\Omega \times 1\,\mu F = 1\,ms$. This allows the capacitor sufficient time to reach its steady-state voltage after $5\tau = 5\,ms$ each time the input switches.

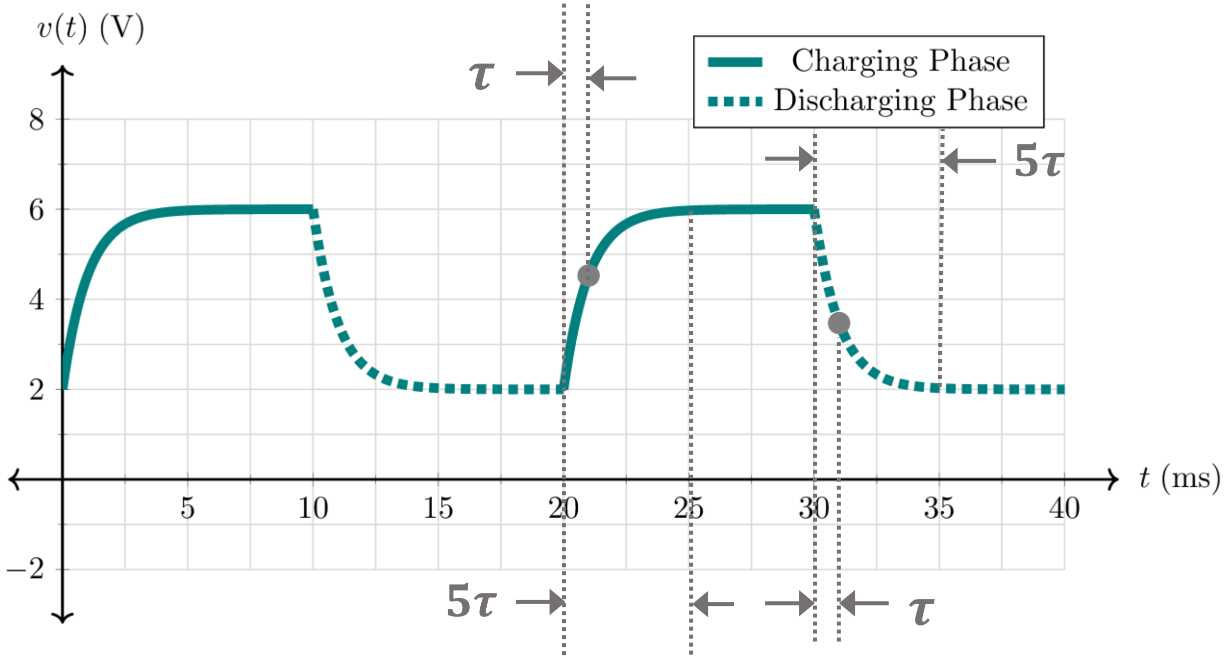


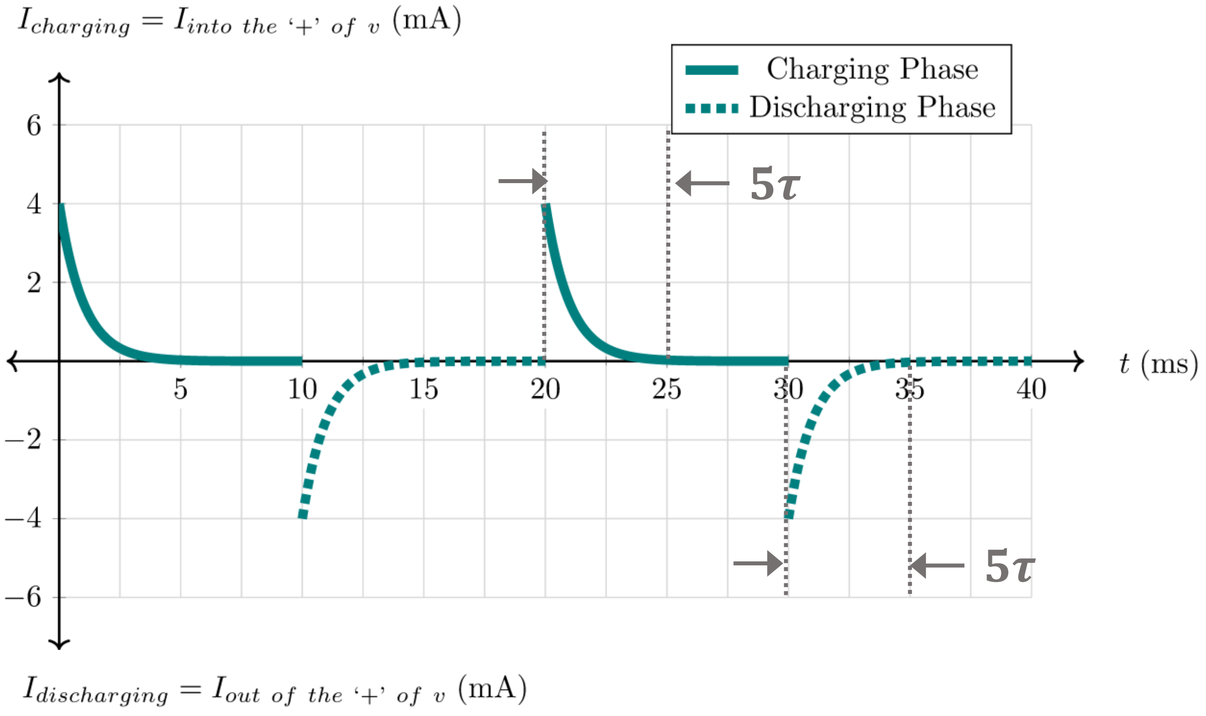


V-I Response to Square Wave Input

- When a periodic square wave voltage is applied across a capacitor, the voltage across the capacitor changes exponentially during each high and low phase. Depending on the polarity and direction of change, the capacitor charges or discharges, and the current flows accordingly







Procedure

$$v(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

Determine the initial voltage of the capacitor $V_{initial}$ or $V(0)$

Consider only the active‡ portion of the circuit before switching. For example, if switching occurs at $t = 0$, consider the circuit for $t < 0$.

If the circuit includes any dc source (current or voltage), open the capacitor and determine the voltage at the open terminal. This is the $V(0)$. $V(0) = 0$ if there is no independent source in the circuit.

Determine the final voltage of the capacitor V_{final} or $V(\infty)$

Now consider the active‡ portion of the circuit after switching. For example, for $t > 0$.

Repeat the step. This time, the voltage across the capacitor is $V(\infty)$. Circuits with $V(\infty) = 0$ are called source free.

Determine the time constant (τ)

Again, only consider the active‡ portion after switching. For example, for $t > 0$.

Determine the Thevenin resistance (R_{Th}) as seen from the capacitor terminals

$\tau = R_{Th}C$

Determine $v(t)$

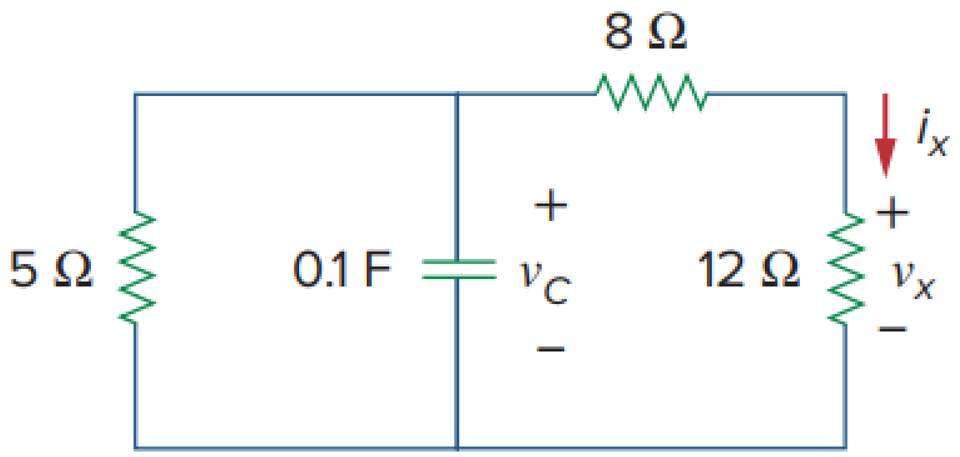
Plug in $V(0)$, $V(\infty)$, and τ into the equation for $v(t)$

Determine any other voltages or currents in the circuit using $v(t)$ and the circuit laws.

‡ active portion of the circuit excludes everything that has no influence on the capacitor

Example 1

- Let $V_C(0) = 15\ V$, Determine v_C, v_x, and i_x for $t > 0$.



<u>Solution</u>

The equivalent resistance as seen from the capacitor terminal is,

$$R_{eq} = (8+12)\,||\,5 = 4\,\Omega$$

Time constant, $\tau = R_{eq}C = 4 \times 0.1 = 0.4\ s$

Thus, for a source-free RC circuit, $V(\infty) = 0$. So,

$$v_C(t) = V(0)e^{-\frac{t}{\tau}} = 15e^{-2.5t}\ (V)$$

The voltage v_x can be found by simple voltage division.

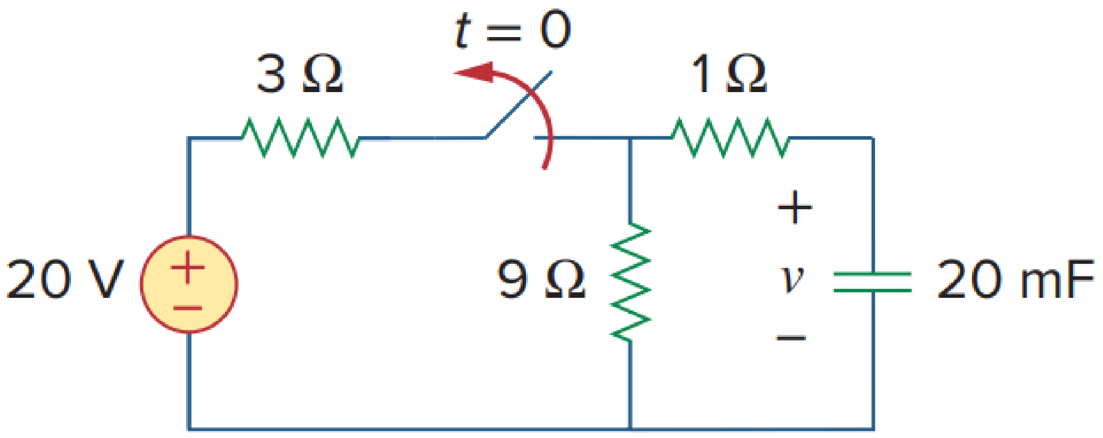
$$v_x(t) = \frac{12}{12+8} \times v_C(t) = 9e^{-2.5t}\ (V)$$

According to the Ohm's law,

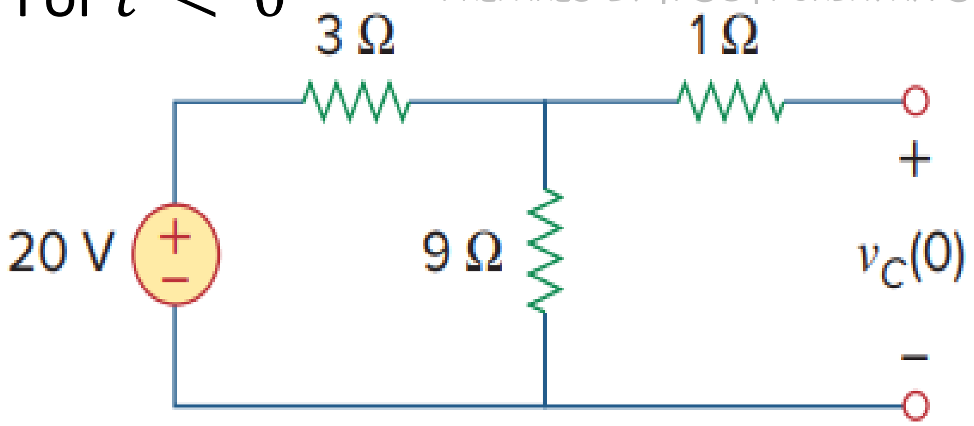
$$i_x = \frac{v_x}{12} = \frac{9e^{-2.5t}}{12} = 0.75e^{-2.5t}\ (\mathrm{A})$$

Example 2: $t < 0$

- The switch in the circuit has been closed for a long time, and it is opened at $t = 0$. Find $v(t)$ for $t > 0$. Calculate the initial energy stored in the capacitor.



For $t < 0$



For $t < 0$, the switch is closed. With the capacitor open at dc, the circuit transforms into the one shown above.

No current flows through the $1\,\Omega$. So, the voltage across the $9\,\Omega$ is the $v_C(t)$ for $t < 0$,

$$v_C(t) = \frac{9}{9+3} \times 20 = 15\,V, \qquad t < 0$$

Since the voltage across the capacitor cannot change instantaneously,

$$v_C(0) = v_C(0^-) = 15\,V$$

Example 2: $t > 0$

For $t > 0$, the switch is open. The circuit transforms into the one shown above. As there is no independent source in the circuit, $V(\infty) = 0$.

The Thevenin resistance as seen from the capacitor terminal,

$$R_{Th} = 1 + 9 = 10\,\Omega$$

The time constant is,

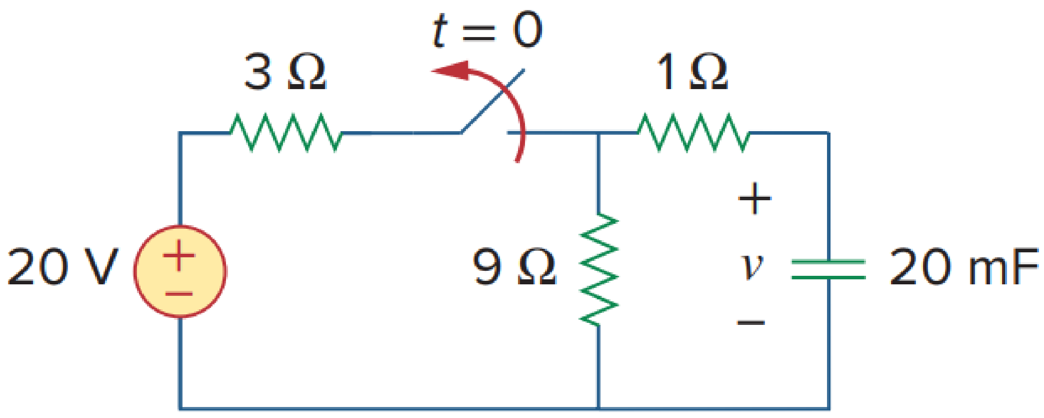
$$\tau = R_{Th}C = 10 \times 20 \times 10^{-3} = 0.2\ s$$

So, the voltage across the capacitor for t > 0 is,

$$v_C(t) = V(0)e^{-\frac{t}{\tau}}$$
$$= 15e^{-5t}\ (V)$$

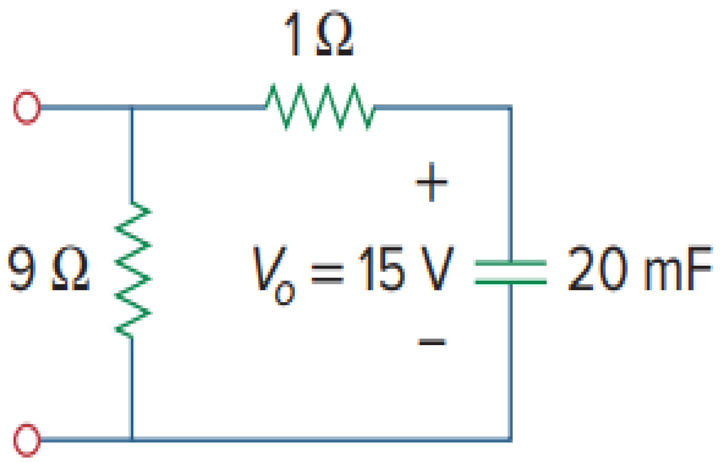
The initial energy stored in the capacitor is,

$$w_C(t) = \frac{1}{2}CV(0)^2$$
$$= \frac{1}{2} \times 20 \times 10^{-3} \times 15^2 = 2.25\ J$$



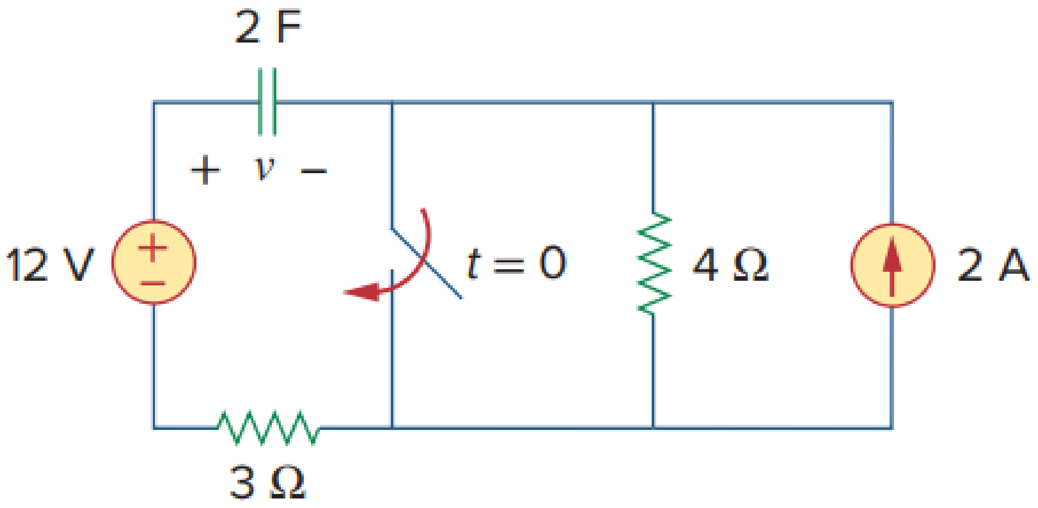
Original circuit

For $t > 0$

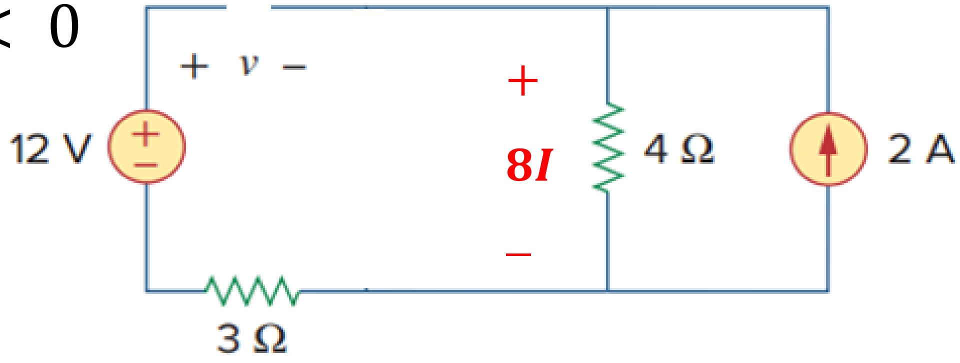


Example 3

- Calculate the capacitor voltage $v(t)$ for $t < 0$ and for $t > 0$.



For $t < 0$



For $t < 0$, the switch is open. With the capacitor open at dc, the circuit transforms into the one shown above.

The $2\ A$ current from the current source will flow only through the $4\ \Omega$ resistance. The voltage drop across the $4\ \Omega$ resistance is, $4 \times 2 = 8\ V$.

There is no voltage drop across the $3\ \Omega$ ($i = 0$ at open circuit). So,

$$v(t) = 12 - 8 = 4\ V, \qquad \mathrm{t} < 0$$

Since the voltage across the capacitor cannot change instantaneously,

$$v(0) = v(0^-) = 4\ V$$

Example 3: $t > 0$

For $t > 0$, the switch is closed. With the capacitor again open at dc, the circuit transforms into the one shown above.

Again, there is no voltage drop across the 3 Ω ($i = 0$ at open circuit). So,

$$v(t) = 12\,V, \qquad t > 0$$

This is the steady-state voltage across the capacitor for $t > 0$.

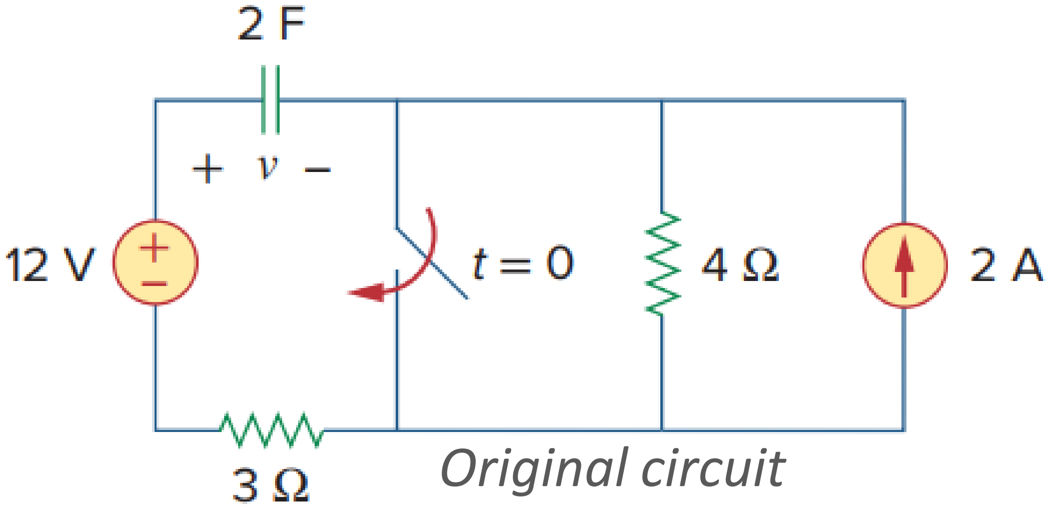
$$v(\infty) = 12\,V$$

The time constant is, $\tau = R_{Th}C = 3 \times 2 = 6\,s$

So,

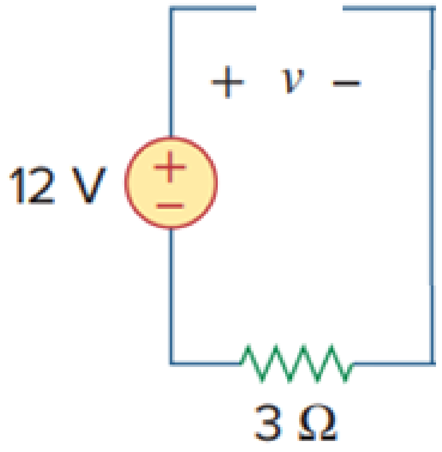
$$v(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

$$= 12 + [4 - 12]e^{-\frac{t}{6}} = 12 - 8e^{-\frac{t}{6}}$$

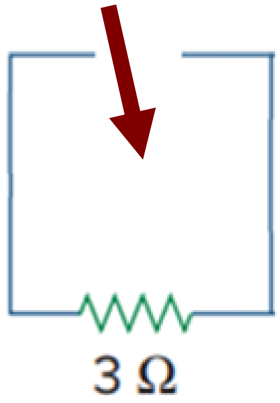


Original circuit

For $t > 0$

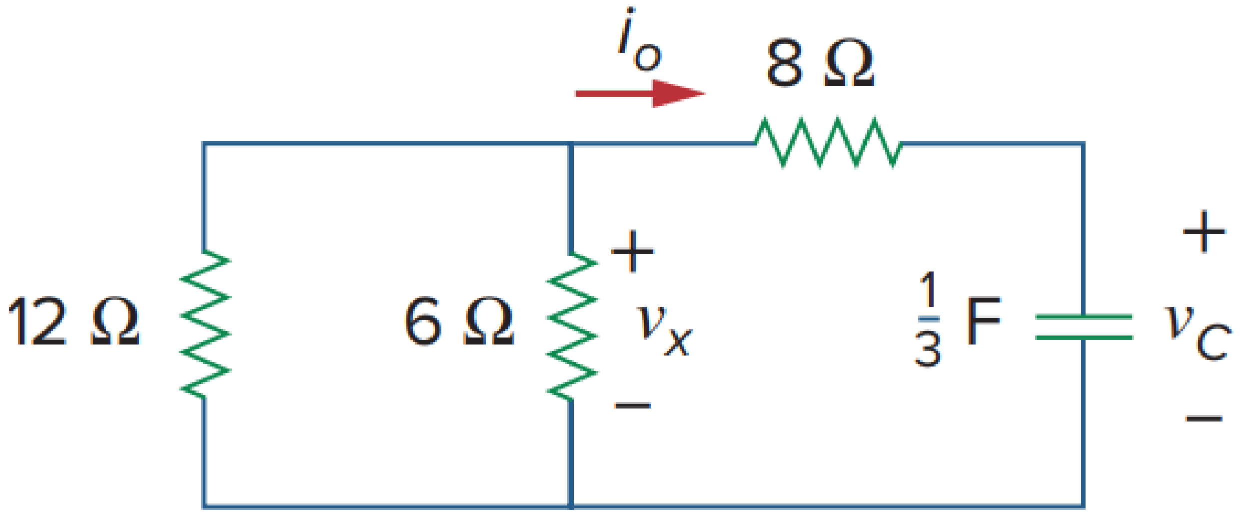


$R_{Th} = 3\,\Omega$



Problem 1

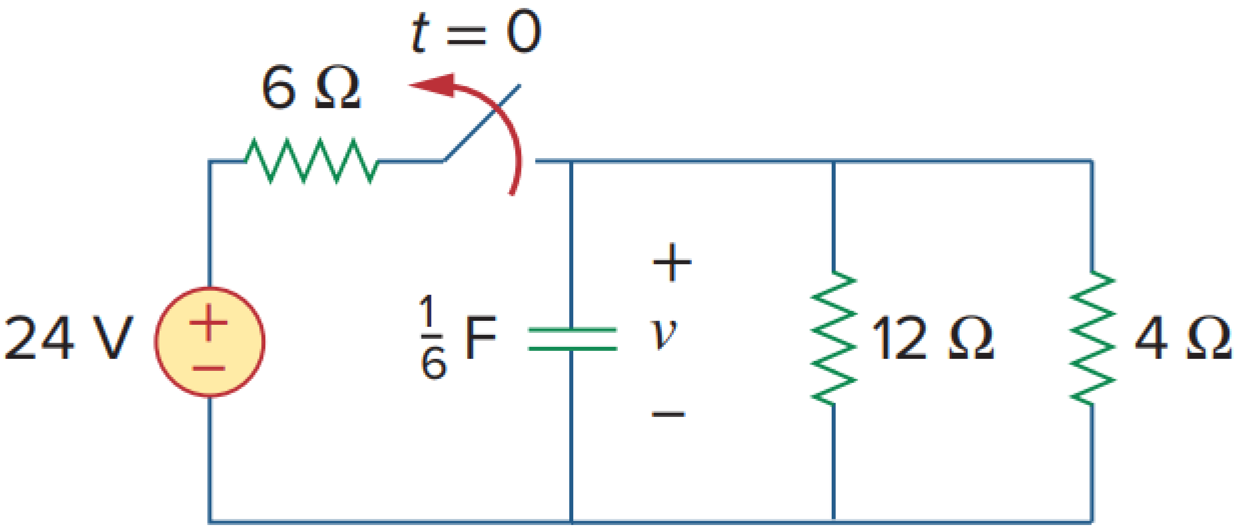
- Let $V_C\ (0)\ =\ 60\,V$, Find v_C, v_x, and i_x for $t\ >\ 0$.



Ans: $\boldsymbol{v_C\ =\ 60e^{-0.25t}\ V;\ v_x\ =\ 20e^{-0.25t}\ V;\ i_x\ =\ -5e^{-0.25t}\ A}$

Problem 2

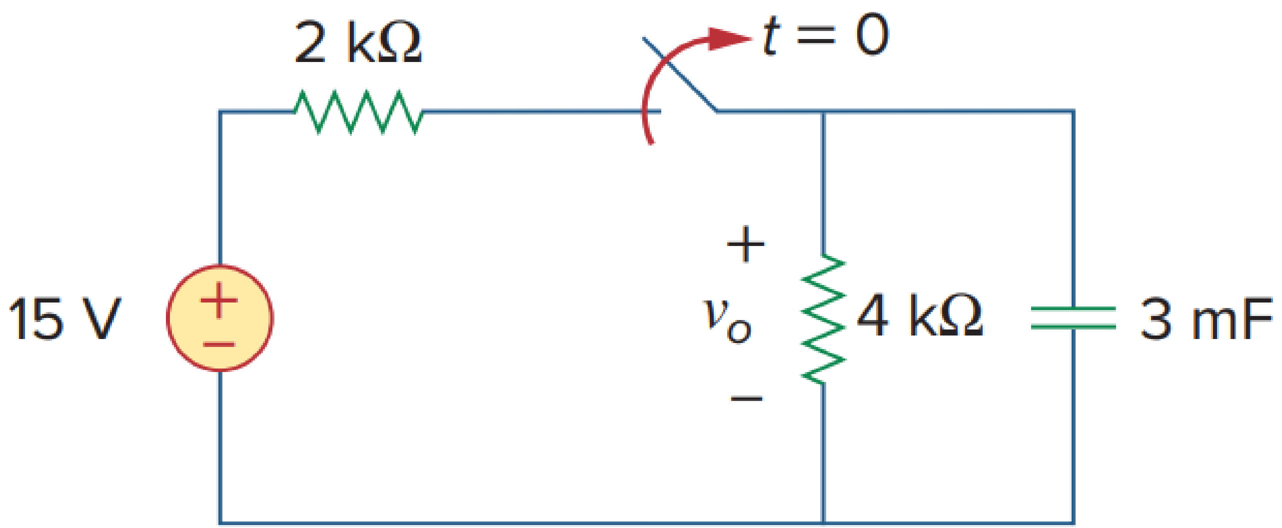
- The switch in the circuit has been closed for a long time, and it is opened at $t = 0$. Find $v(t)$ for $t > 0$. Calculate the initial energy stored in the capacitor.



$\underline{\text{Ans}}$: $\boldsymbol{v(t) = 8e^{-2t}\ V;\ w_C(0) = 5.333\ J}$

Problem 3

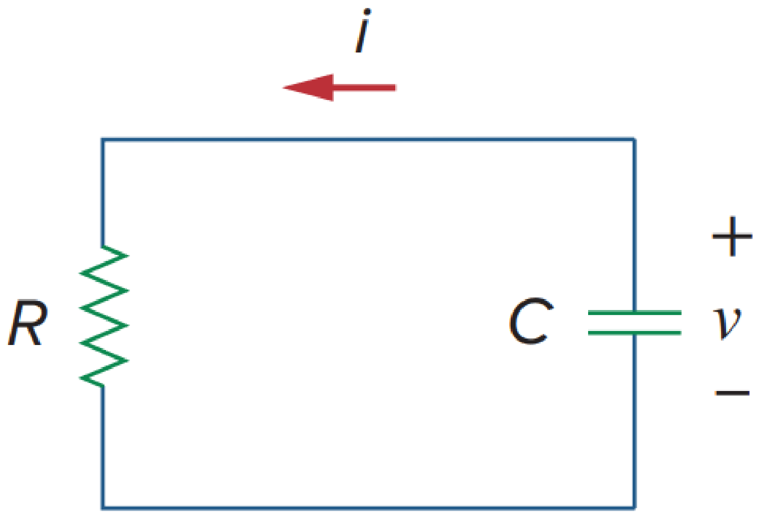
- The switch opens at $t = 0$. Find $v_0(t)$ for $t > 0$.



Ans: $\boldsymbol{v(t) = 10e^{-t/12}\ V}$

Problem 4

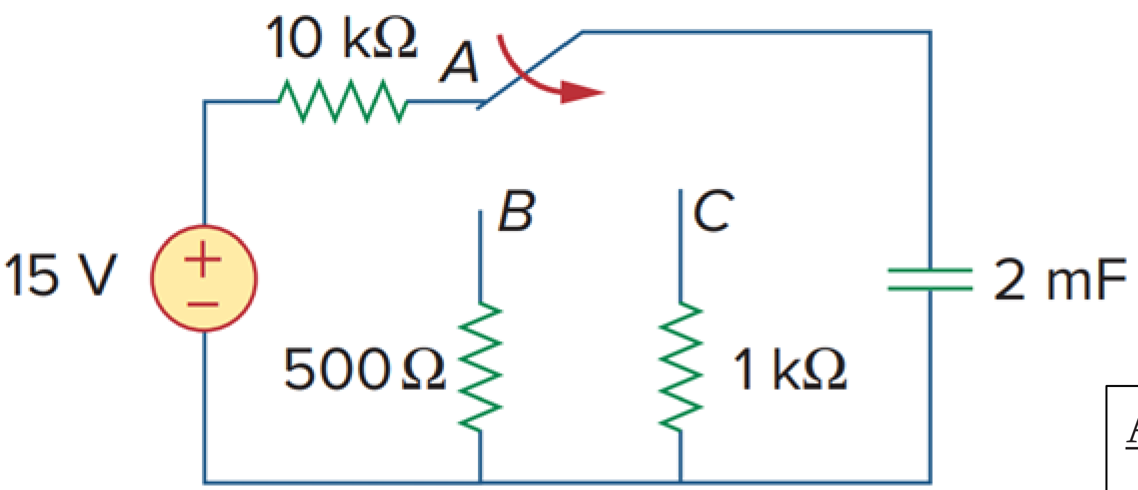
- For the circuit below, $v = 10e^{-4t}\ V$ and $i = 0.2e^{-4t} A$
  - (a) Find R and C.
  - (b) Determine the time constant.
  - (c) Calculate the initial energy in the capacitor.
  - (d) Obtain the time it takes to dissipate 50% of the initial energy.



Ans: $\boldsymbol{R = 50\ \Omega;\ C = 5\ mF;\ \tau = 0.25\ s;\ w_{C(0)} = 0.25\ J;\ t = 86\ ms}$

Problem 5

- Assume that the switch has been in position A for a long time and is moved to position B at $t = 0$. Then at $t = 1s$, the switch moves from B to C. Find $I_C(t)$ for $t > 0$.

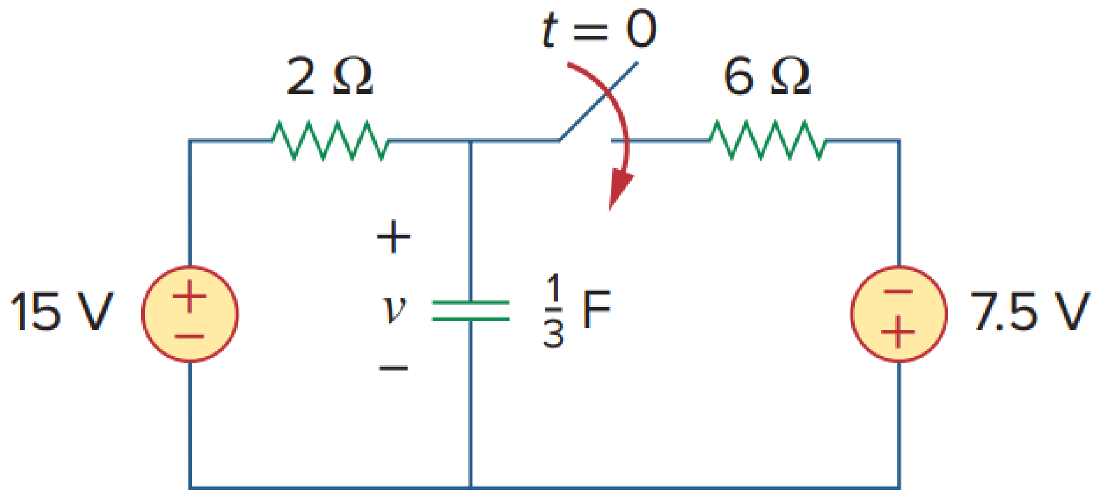


Ans: $\boldsymbol{v(t) = 15e^{-t}\,V\ for\ 0 < t < 1\ sec;}$

$\boldsymbol{v(t) = 5.518e^{-(t-1)/2}\,V\ for\ 1 < t < \infty\ sec;}$

Problem 6

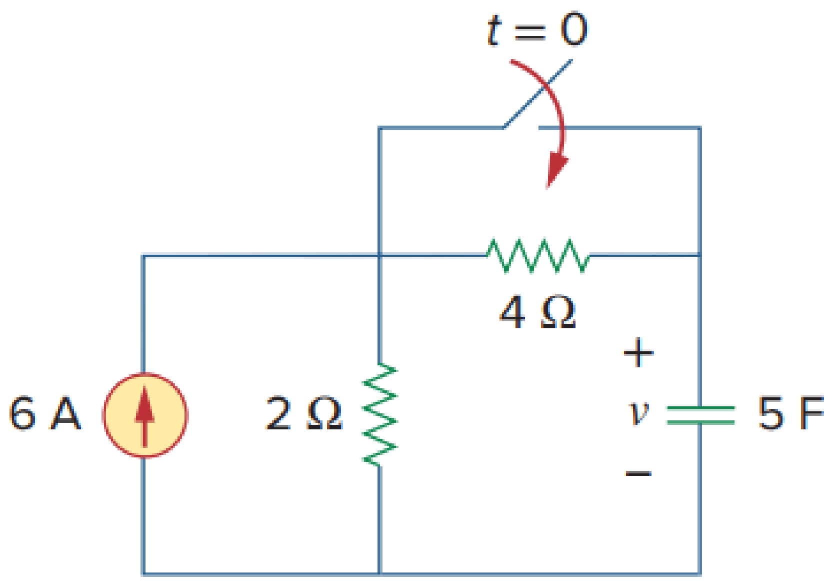
- Find $v(t)$ for $t > 0$ in the circuit shown below. Assume the switch has been open for a long time and is closed at $t = 0$. Calculate $v(t)$ at $t = 0.5s$.



Ans: $\boldsymbol{v_c(t) = 9.375 + 5.625e^{-2t}\,V\ for\ t > 0;\ v_c(0.5) = 11.444\,V}$

Problem 7

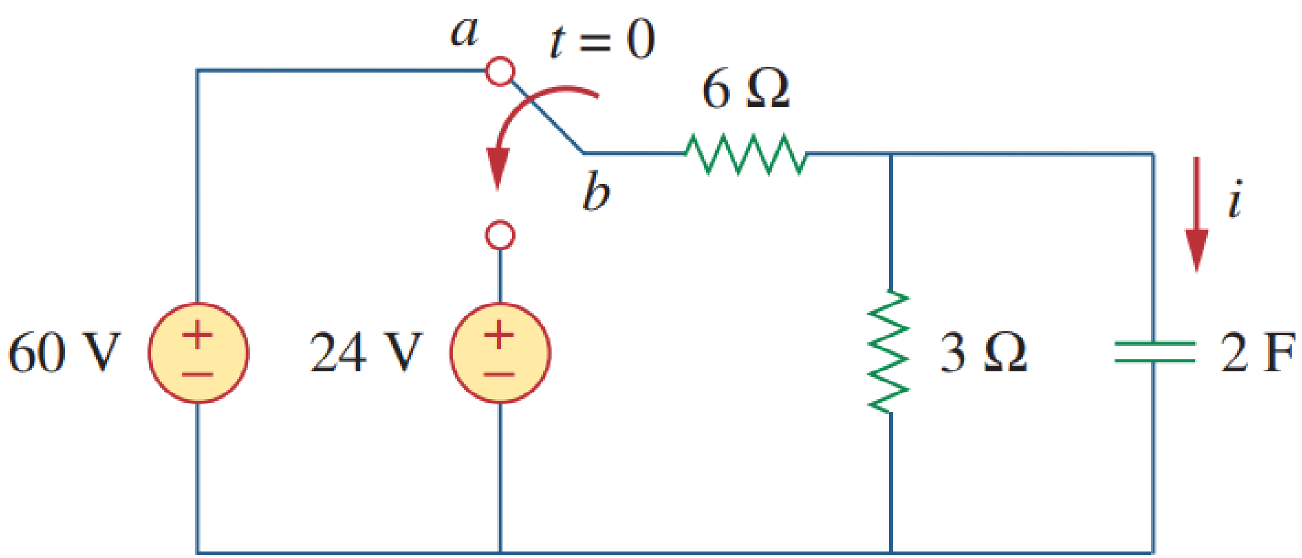
- Calculate the capacitor voltage for $t < 0$ and for $t > 0$.



Ans: $\boldsymbol{v(t) = 12\,V\ for\ t < 0;\ v(t) = 12\,V\ for\ t > 0}$

Problem 8

- The switch has been in position a for a long time. At $t = 0$ it moves to position b. Calculate $i(t)$ for all $t > 0$.



Ans: $\boldsymbol{i(t) = -6e^{-0.25t}\ A\ for\ t > 0}$

Problem 9

The figure below shows the voltage response of an RC circuit to a sudden DC voltage applied through an equivalent resistance of $4\ k\Omega$.

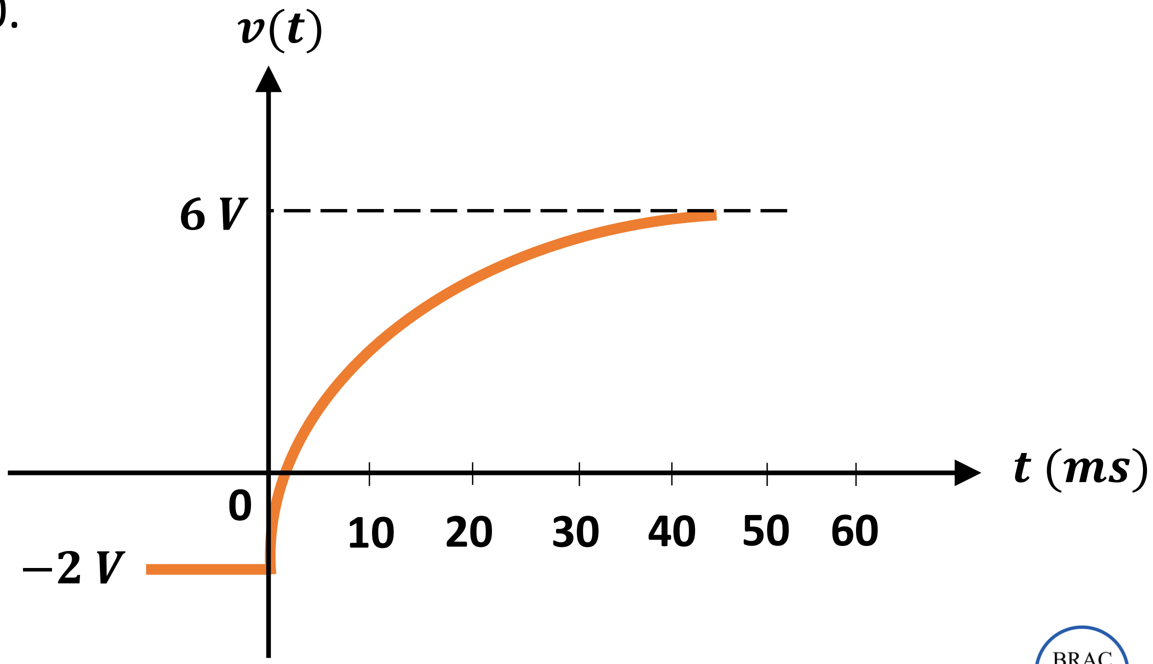
I. Define time constant.

II. Determine the approximate time constant from the figure.

III. Find the mathematical expression of $v(t)$ for t > 0.

IV. What is the initial energy stored in the capacitor?

V. Draw the circuit diagram.

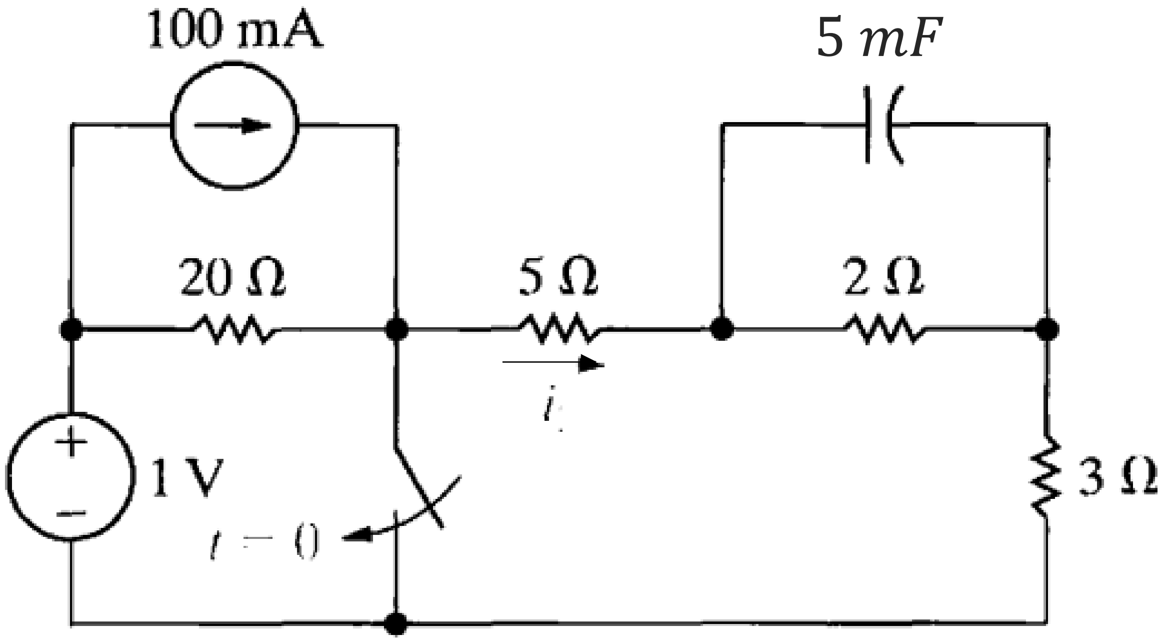


Ans: $(ii)\ \tau = 9\ ms$; $(iii)\ v(t) = 6 - 8e^{-1000t/9}\ V\ for\ t > 0$; $(iv)\ w = 4.5 \times 10^{-6}\ J$

Problem 10

- Determine the current $i(t)$ for $t > 0$.

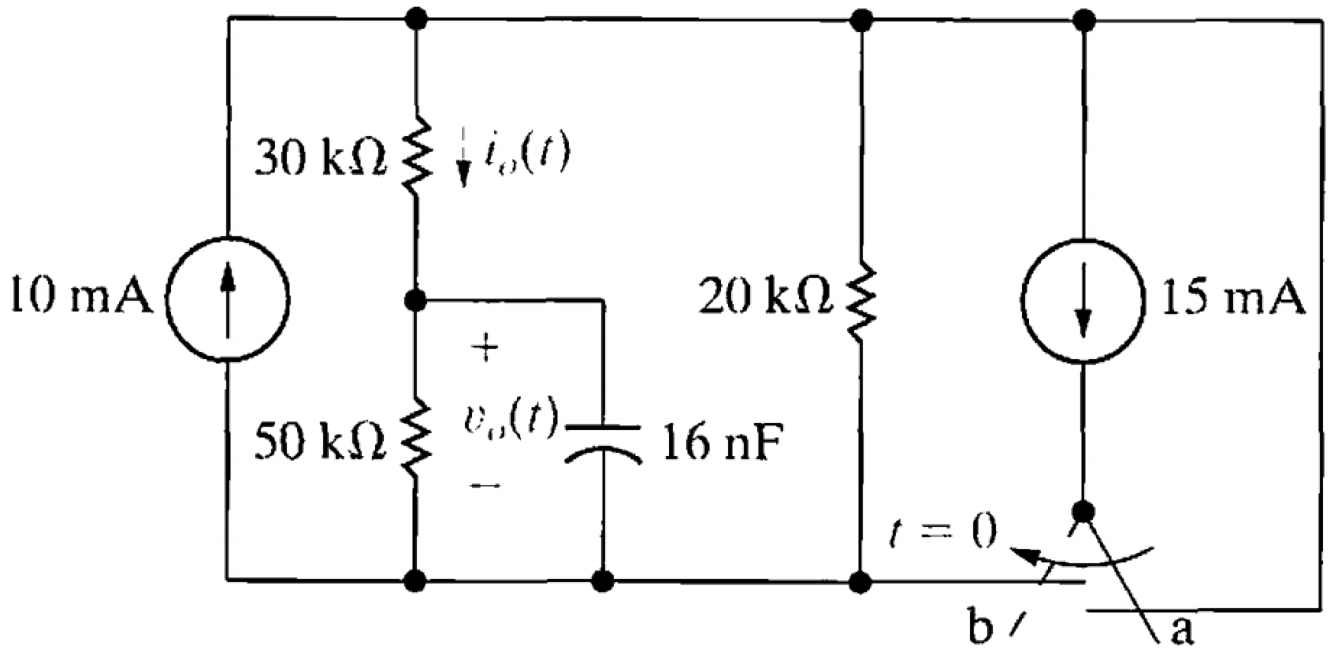
[Hint: always calculate the capacitor voltage for $t > 0$ first, then determine anything else.



Ans: $\boldsymbol{i(t) = 25\, e^{-1/8\times10^{-3}}\ (mA), t > 0}$

Problem 11

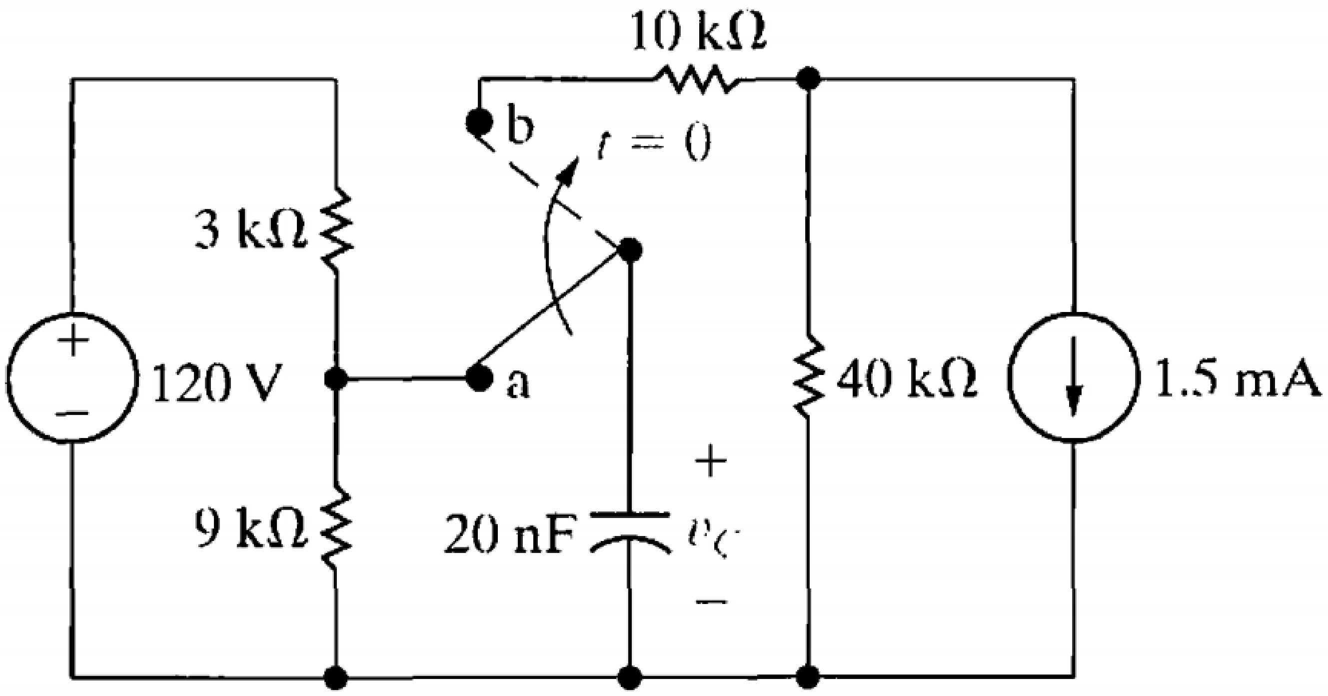
- The switch in the following circuit moves from position a to b at $t = 0$. Determine $v_o(t)$ and $i_o(t)$ for $t > 0$.



Ans: $\boldsymbol{v_o(t) = -50 + 150\, e^{-2500t}\ (V), t > 0; i_o(t) = -1 - 3\, e^{-2500t}\ (mA), t > 0}$

Problem 12

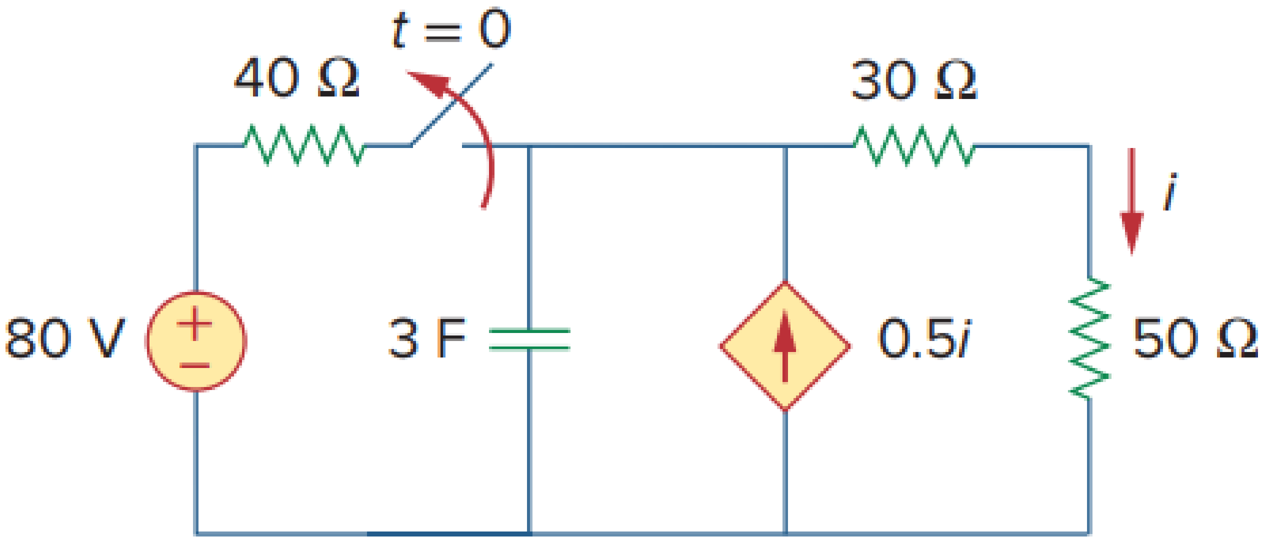
- The switch in the following circuit moves from position a to b at $t = 0$. Determine the voltage $v_C(t)$ for $t > 0$.



Ans: $v_C(t) = -60 + 150\, e^{-1000t}\ (V), t > 0$

Problem 13

- Consider the circuit shown below. Find $i(t)$ for $t < 0$ and $t > 0$.

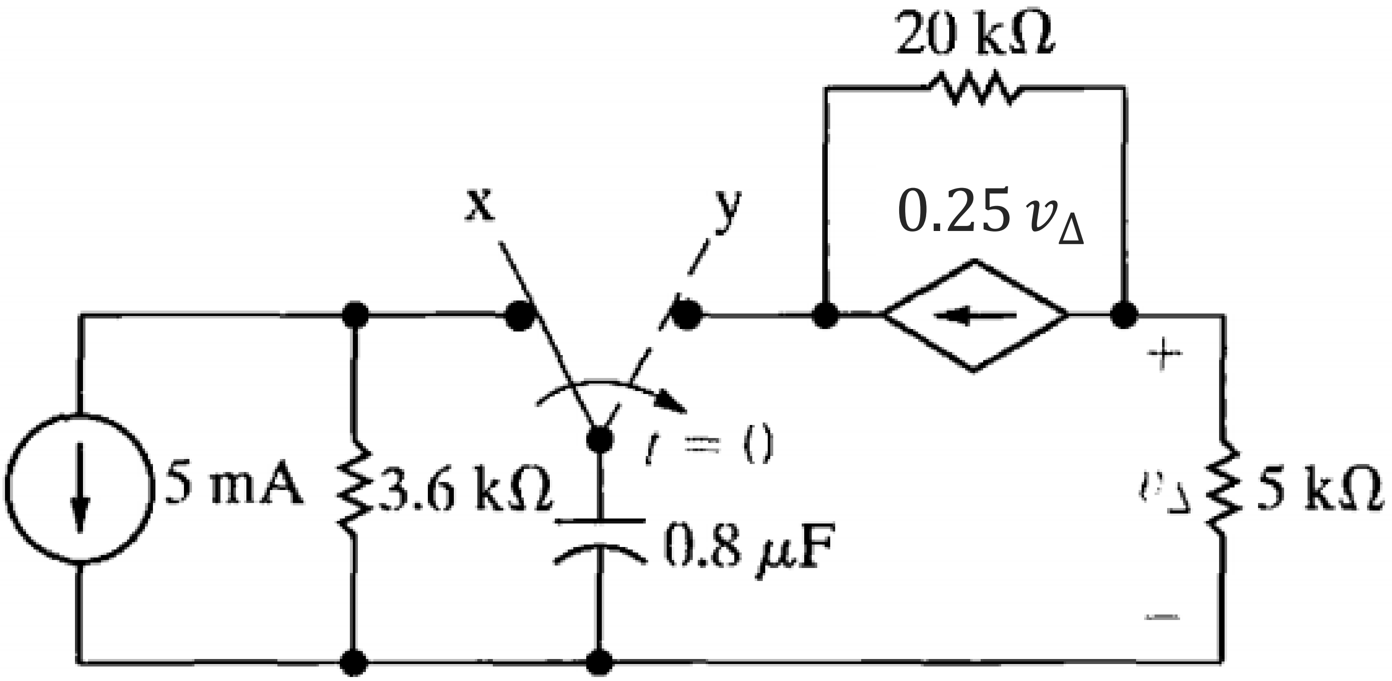


<u>Ans</u>: $\boldsymbol{i(t) = 0.8\ A\ for\ t < 0;\ i(t) = 0.8e^{-t/480}\ A\ for\ t > 0}$

Problem 14

- Determine the voltage $v_{\Delta}(t)$ for $t > 0$ if the switch moves from x to y at $t = 0$.

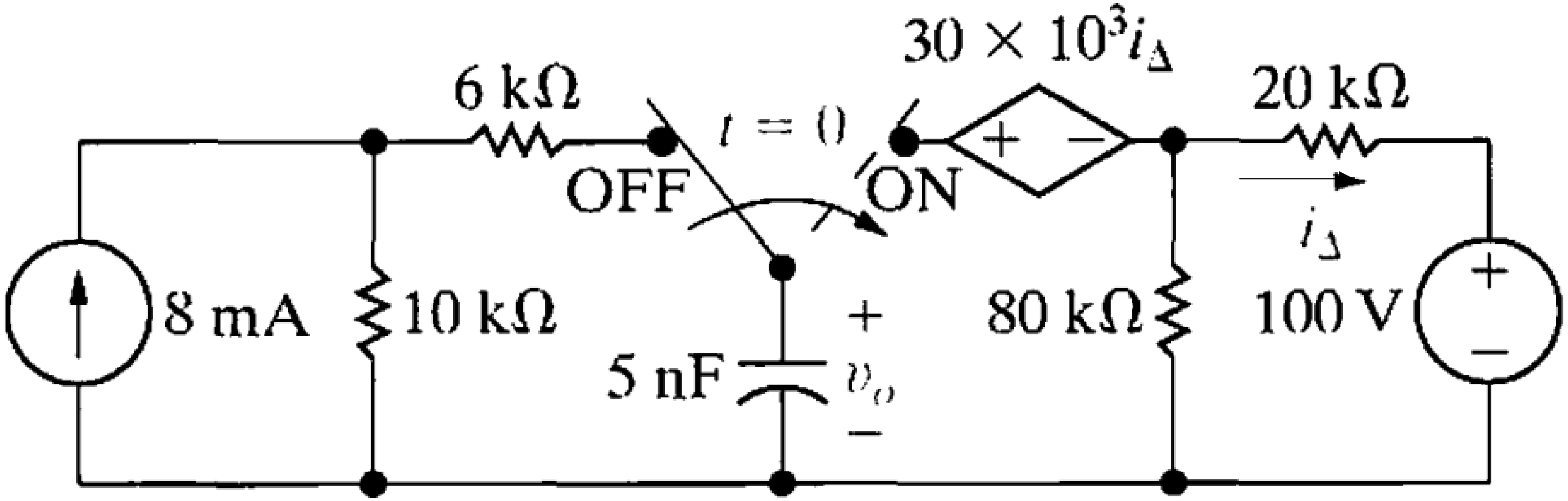
[Hint: always calculate the capacitor voltage for $t > 0$ first, then determine anything else.



Ans: $v_{\Delta}(t) = -1.8\, e^{-25t}\ (V), t > 0$

Problem 15

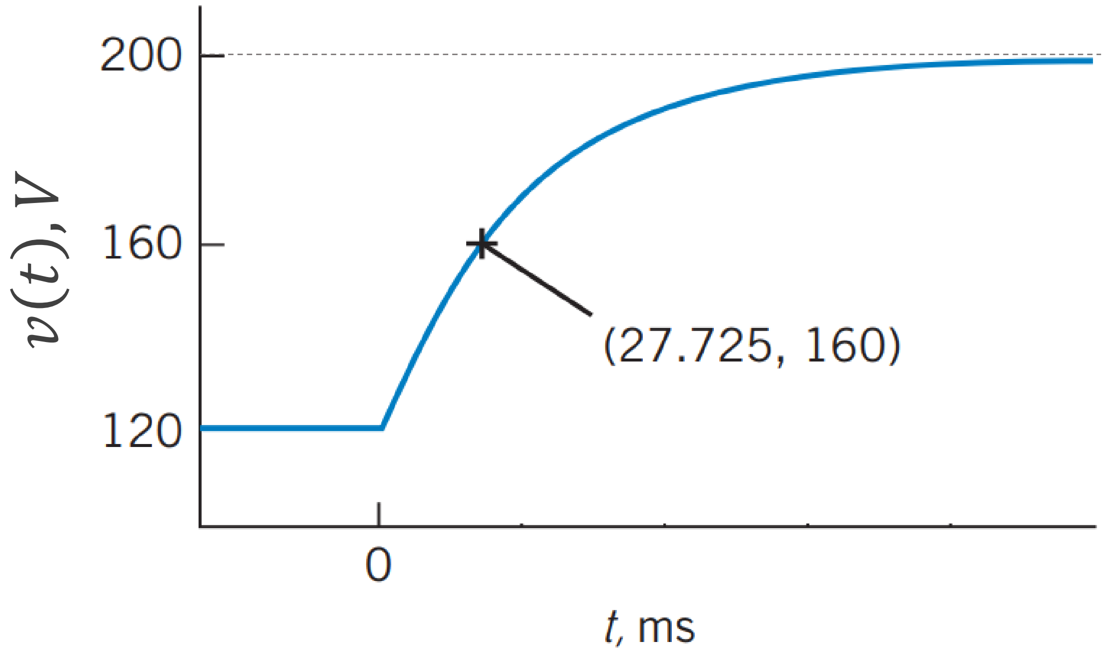
- The switch in the following circuit has been in OFF position for a long time. At $t = 0$, the switch moves to position ON. Determine $v_o(t)$ for $t > 0$.

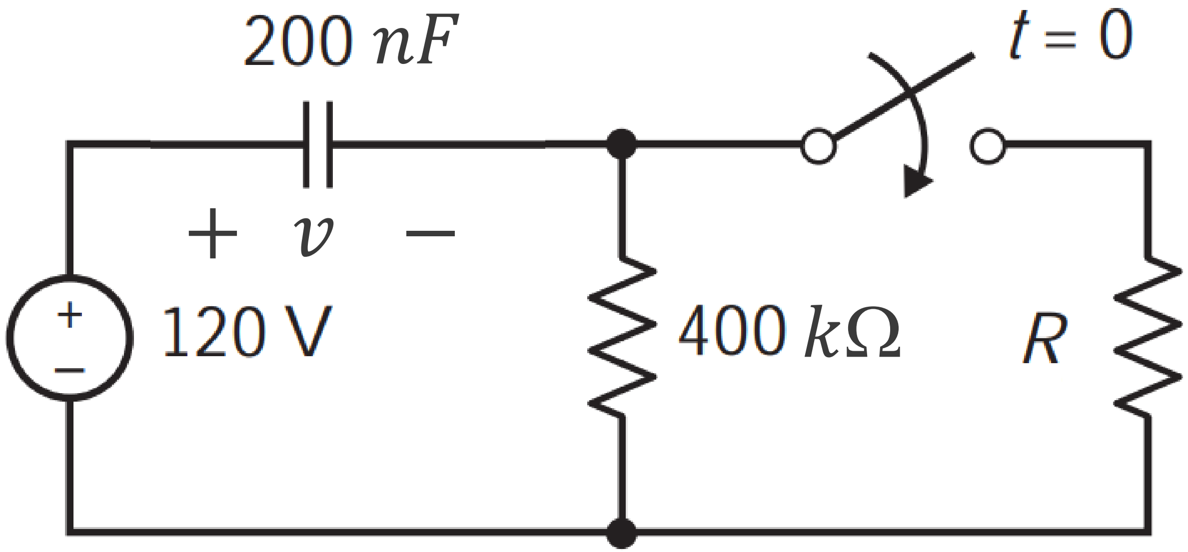


Ans: $v_o(t) = 50 + 30\, e^{-5000t}\ (V), t > 0$

Problem 16

- The voltage v across the capacitor has the following plot. Determine the value of R.



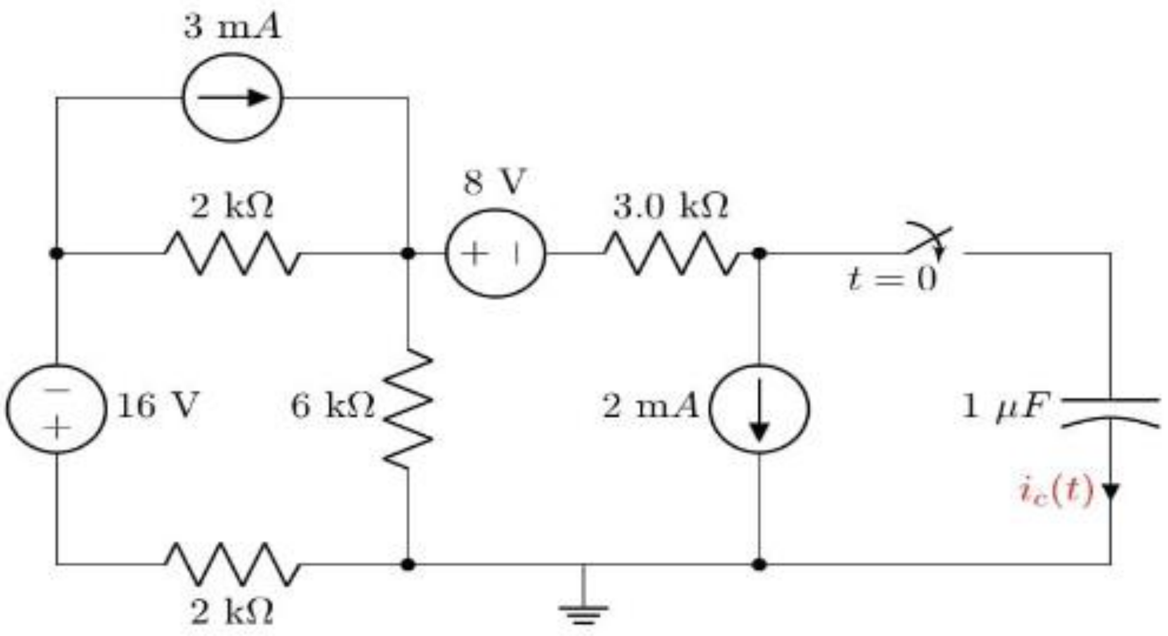


Ans: $\boldsymbol{R = 400\ k\Omega}$

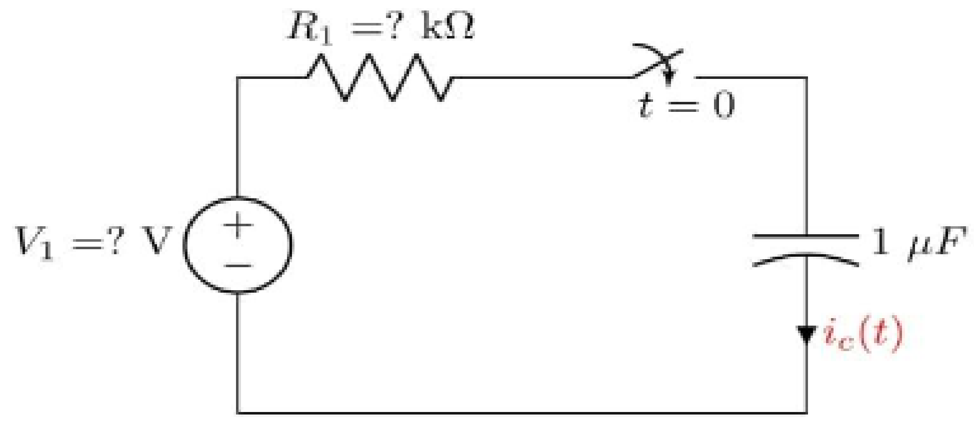
Problem 17

I. Simplify the circuit 1 below so that it takes the form of the circuit 2. Determine the values of V_1 and R_1.

II. Perform transient analysis to determine $i_c(t)$ through the capacitor for $t > 0$.



Circuit 1

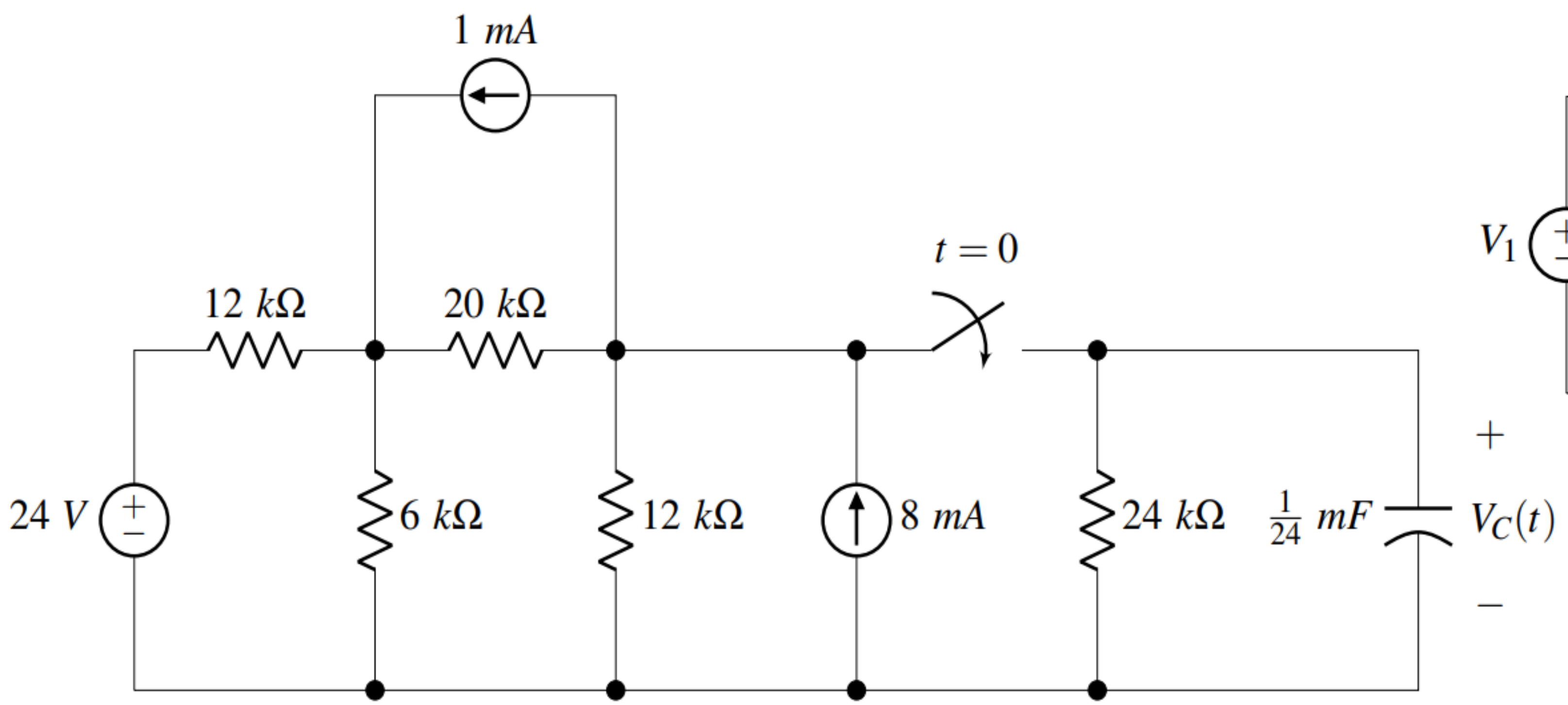


Circuit 2

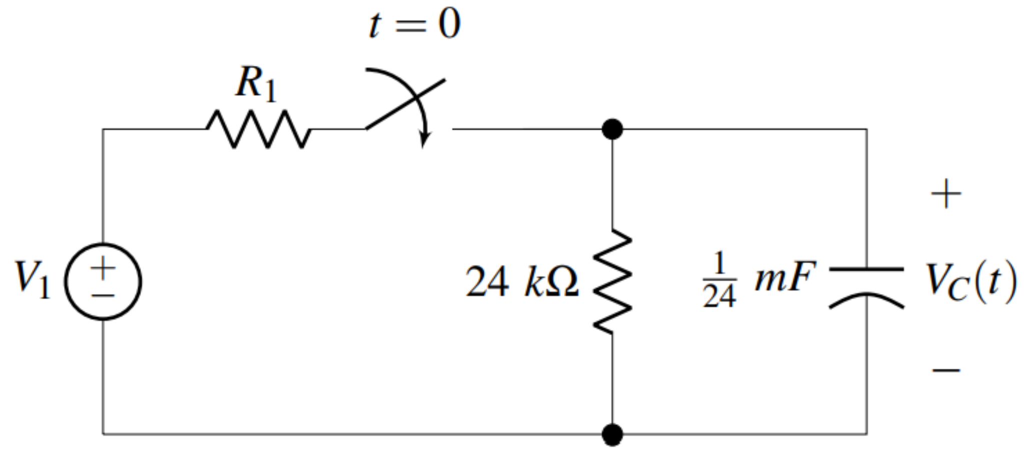
$\underline{Ans}$: $\mathbf{V_1 = -24.8\ V}$; $\boldsymbol{R_1} = \mathbf{5.4\ k\Omega}$; $\boldsymbol{i_c(t) = -\,4.6e^{-1000t/5.4}\ A}$

Problem 18

- Simplify the Circuit 1 below so that it takes the form of the Circuit 2. Determine the values of V_1 and R_1. Perform transient analysis to determine $V_C(t)$ across the capacitor for $t > 0$.



Circuit 1



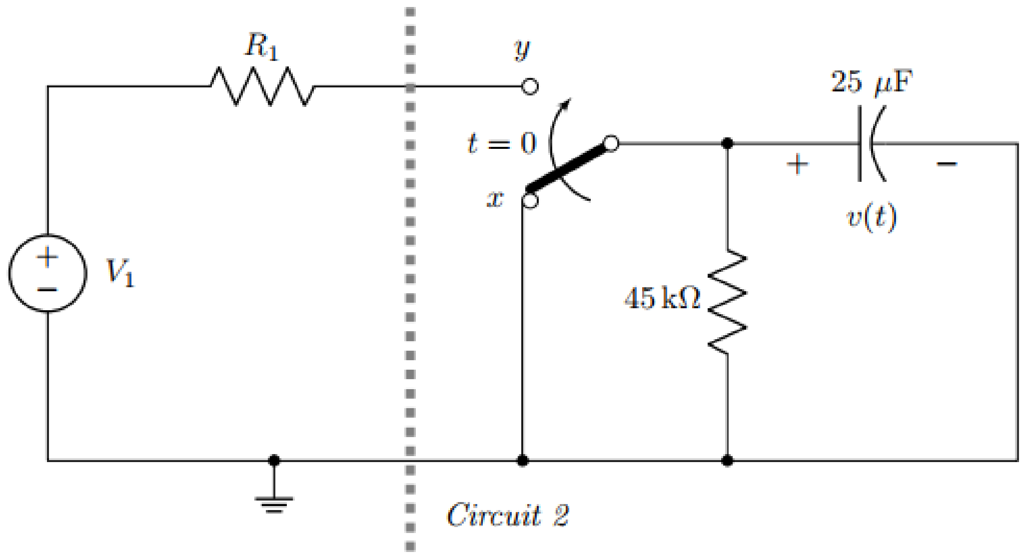
Circuit 2

Fall 2022

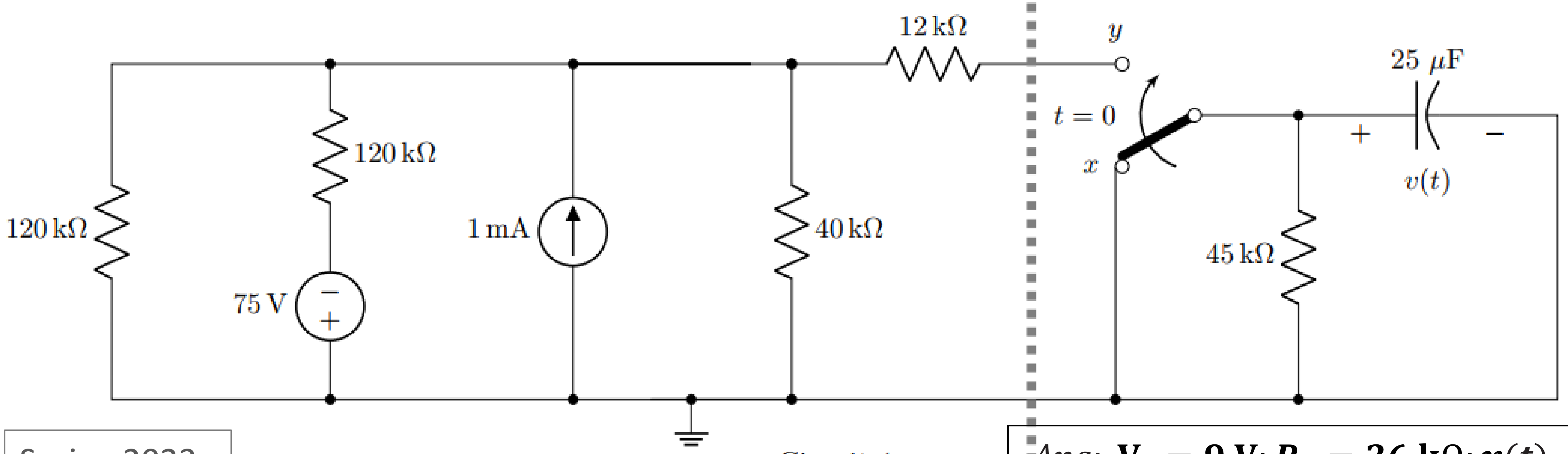
Ans: $\mathbf{V_1 = 60\ V; R_1 = 8\ k\Omega; V_C(t) = 45(1 - e^{-4t})\ V}$

Problem 19

- Reduce the left portion with respect to the dashed grey line of Circuit 1 so that it takes the form of Circuit 2 as shown. Write down the values of V_1 and R_1.
- Now, analyse the Transient Behaviour of the circuit assuming that the switch moves from position x to position y at $t = 0$. Determine $v(t)$ for $t > 0$.



Circuit 2



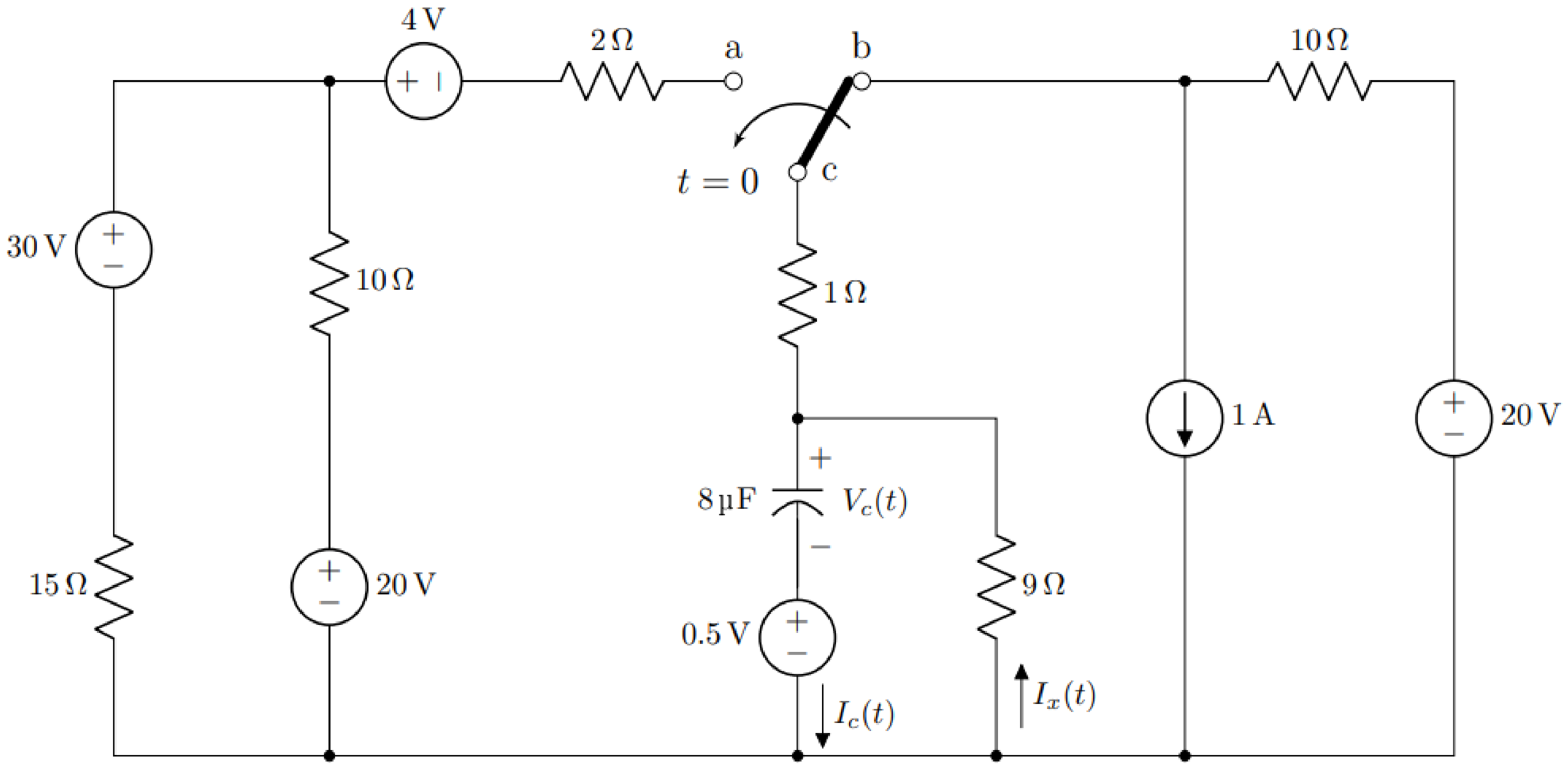
Circuit 1

Spring 2023

Ans: $\mathbf{V_1 = 9\ V; R_1 = 36\ k\Omega; v(t) = 5(1 - e^{-2t})\ V}$

Problem 20

- The switch in the following circuit moves from position a to b at $t = 0$ where position c remains unchanged. Determine the $V_C(t)$, $I_C(t)$, and $I_x(t)$ for $t > 0$.



$\underline{Ans}$: $\boldsymbol{V_C(t) = 9.5 - 5.5e^{-1000t/36}\ V}$

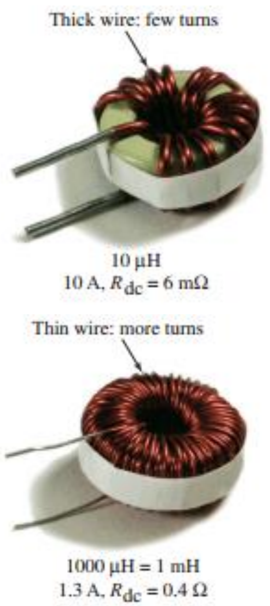
$\boldsymbol{I_c(t) = 1.22e^{-1000t/36}\ mA}$

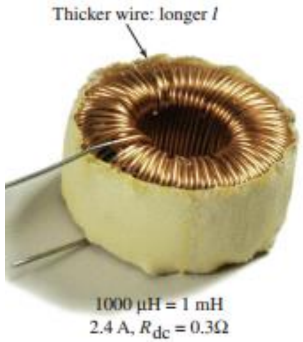
$\boldsymbol{I_x(t) = -1.11 + 0.61e^{-1000t/36}\ A}$

Fall 2023

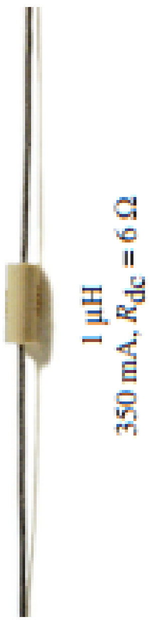
Inductors

- An inductor is a passive circuit element designed to store energy in its magnetic field.
- Inductors are designed to set up a strong magnetic field linking the unit, whereas capacitors are designed to set up a strong electric field between the plates.



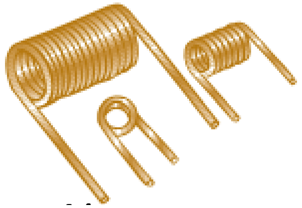


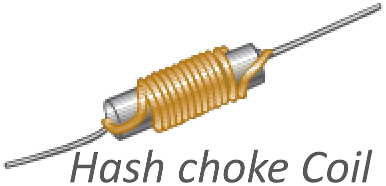
Toroid



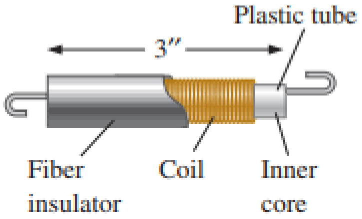
Plastic core

Ferrite core

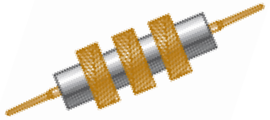




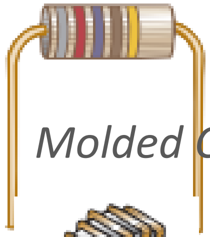
Hash choke Coil



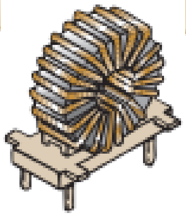
Delay line Coil



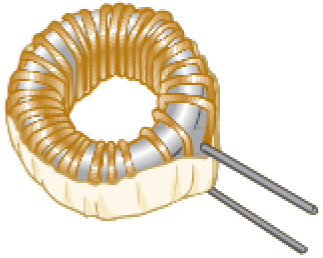
RF chokes



Molded Coil

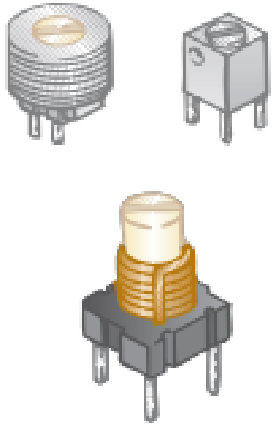


Common-mode choke Coil



Toroid Coil

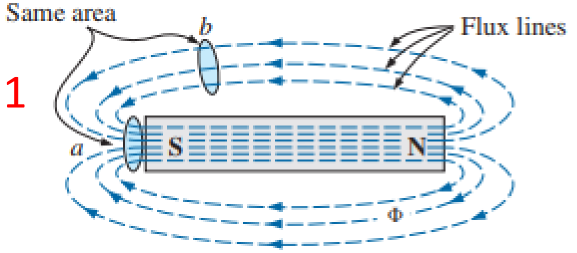
Surface mount



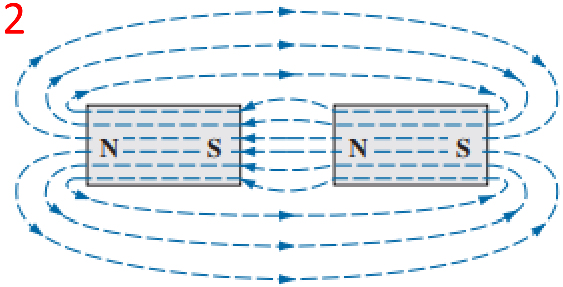
Variable inductors

Magnetic Field

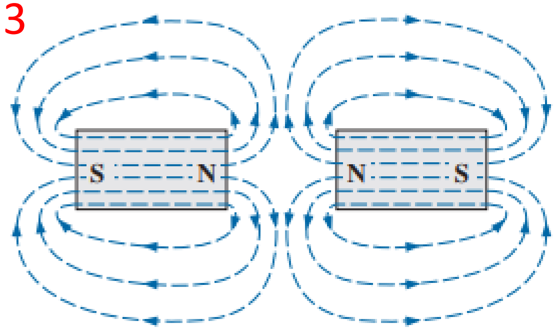
- A magnetic field exists in the region surrounding a permanent magnet, which can be represented by magnetic flux lines (imaginary) similar to electric flux lines. Magnetic flux lines, however, do not have origins or terminating points as do electric flux lines but exist in continuous loops
- In 1820, the Danish physicist Hans Christian Oersted discovered that the needle of a compass deflects if brought near a current-carrying conductor. This was the first demonstration that electricity and magnetism were related.

1


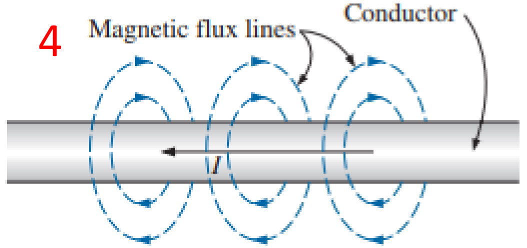
Flux distribution for a permanent magnet.

2


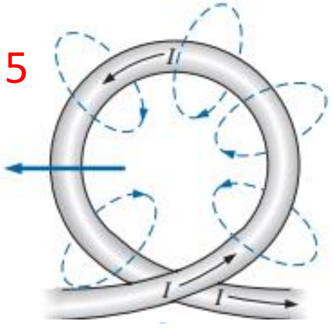
Flux distribution for two adjacent, opposite poles.

3


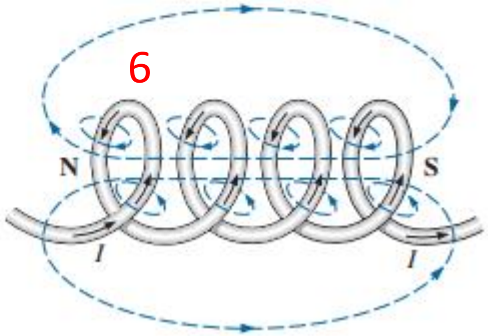
Flux distribution for two adjacent, like poles.

4


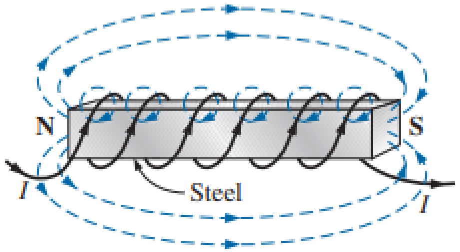
Magnetic flux lines around a current-carrying conductor.

5


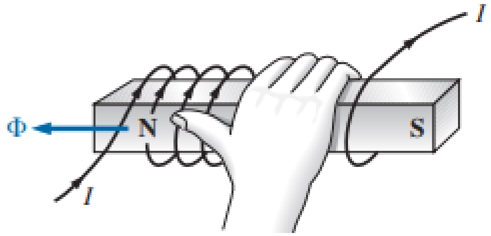
Flux distribution of a single-turn coil.

6


Flux distribution of a current-

7


Electromagnet.

8


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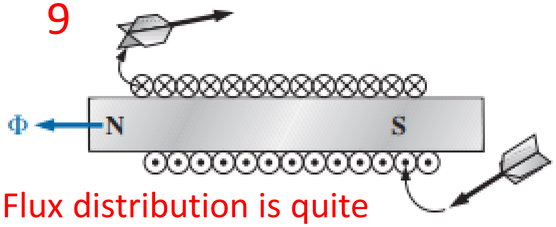
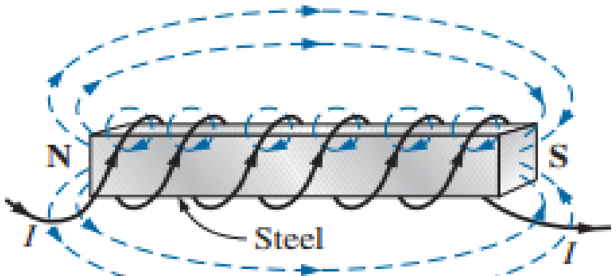


Flux distribution is quite similar to that of permanent magnet

Magnetic Flux Density

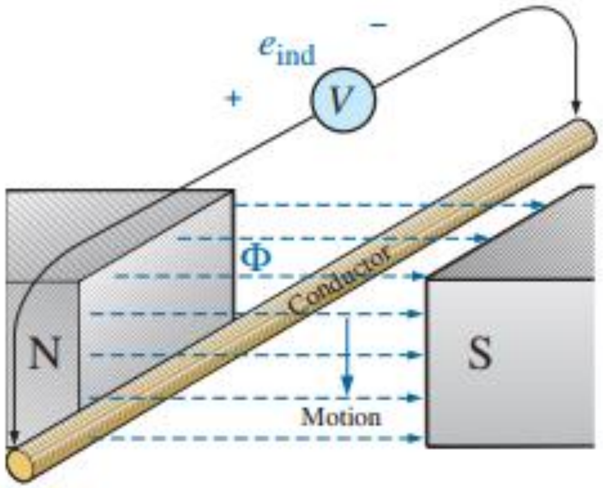


Electromagnet.

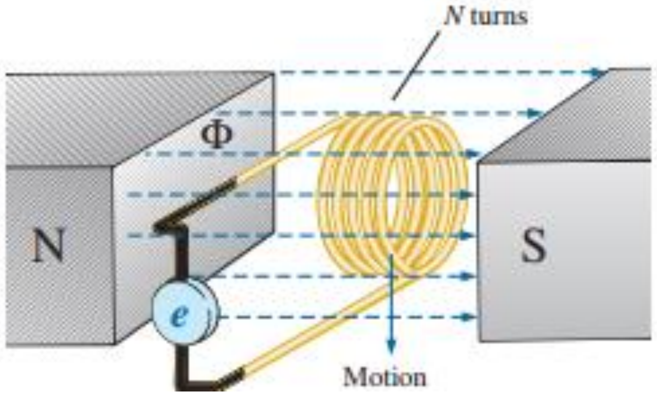
- *Magnetic flux* (ϕ) is measured in *webers* (Wb) as derived from the surname of Wilhelm Eduard Weber.
- The number of flux lines per unit area is called *flux density* (B). Measured in *tesla* (T).
- In equation form,
- $B = \dfrac{\phi}{A} \quad (Wb/m^2 \equiv T \equiv 10^4\ Gauss)$
- B of an electromagnet is directly proportional to the number of turns of, and current through, coil. Increasing either one (or both) results in increasing magnetic field.
- Another factor that affects the magnetic field strength is the type of core used. Ferromagnetic materials such as iron, cobalt, nickel, steel, alloys provide higher magnetic flux, high permeability (μ).

Faraday's Law of Electromagnetic Induction

- *Faraday's law* states that, a conductor exposed to a **changing magnetic flux** will develop an induced voltage across it.
- It doesn't matter whether the changing flux is due to moving the magnetic field or moving the coil in the vicinity of a magnetic field: The only requirement is that the flux linking (passing through) the coil changes with time.
- In the form of equation,
- $e = N\frac{d\phi}{dt} \quad (volts, V)$
- This important phenomenon can now be applied to an inductor



Generating an induced voltage by moving a conductor through a magnetic field.

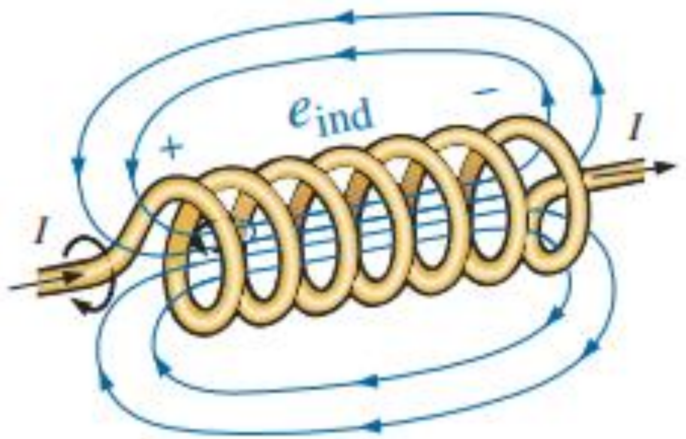


Demonstrating Faraday's law.

Inductance

- We found that the magnetic flux linking the coil of N turns with a current I has the distribution shown in the figure.
- If the current (I) through the coil increases in magnitude, the flux (ϕ) linking the coil also increases.
- So, $\phi \propto I$

$\Rightarrow$ $\phi = LI$ [L is a proportionality constant ≡ Self-inductance]



Demonstrating the effect of Lenz's law.

- If the loop has N number of turns,

$\Rightarrow$ $N\phi = LI$

$\Rightarrow$ $L = \frac{N\phi}{I}$ $(Henry,\ H,\ mH,\ \mu H)$; $\left[L = \frac{N^2\mu A}{l}\right.$ for a solenoid$\left.\right]$

- For a particular inductor, $\uparrow I, \uparrow \phi$ but $\frac{\phi}{I} = const.$ So, L does not depend on ϕ or I. It depends on the physical dimension (length, # of turns, area, material) of the inductor.

Lenz's Law of Electromagnetic Induction

- It is very important to note in the figure that the polarity of the induced voltage across the coil is such that it opposes the increasing level of current in the coil.
- The quicker the change in current through the coil, the greater is the opposing induced voltage to squelch the attempt of the current to increase in magnitude
- Lenz's law says that an induced effect is always such as to oppose the cause that produced it.

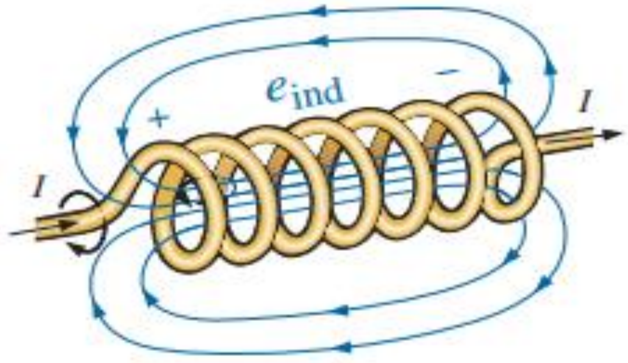
$$L = \frac{N^2 \mu A}{l}$$

$\Rightarrow \quad L = \frac{N\phi}{I}$

$\Rightarrow$ Differentiating with respect to t, $\frac{d\phi}{dt} = \frac{L}{N}\frac{dI}{dt}$

$\Rightarrow$ Substituting into Faraday's law, $e = N\frac{d\phi}{dt} = N\frac{L}{N}\frac{dI}{dt}$

$\Rightarrow \quad e = L\frac{dI}{dt}$



Demonstrating the effect of Lenz's law.

I-V relation of an Inductor

- In network analysis, $e = L\frac{dI}{dt}$ is expressed as,

$$\boxed{v = L\frac{di}{dt}}$$

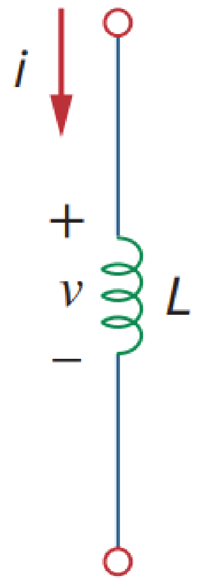
- This is the characteristic equation of an inductor.
- The current can be found as,

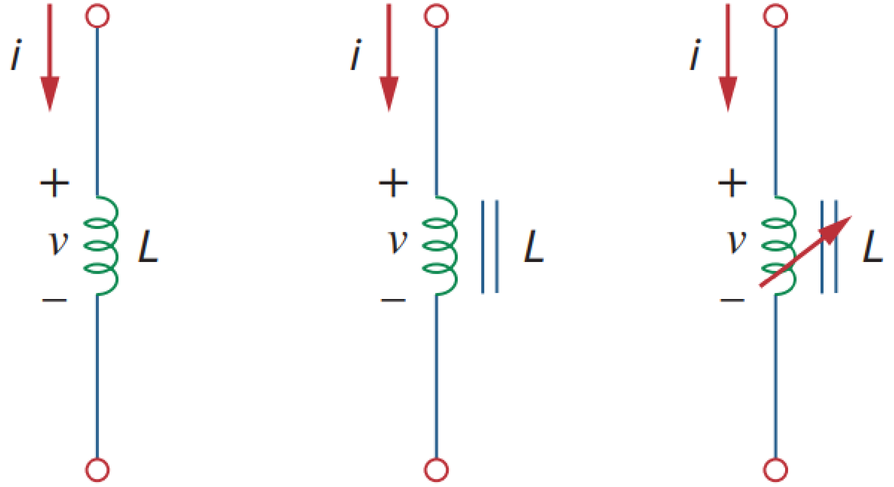
$$\Rightarrow \quad di = \frac{1}{L} v\, dt$$

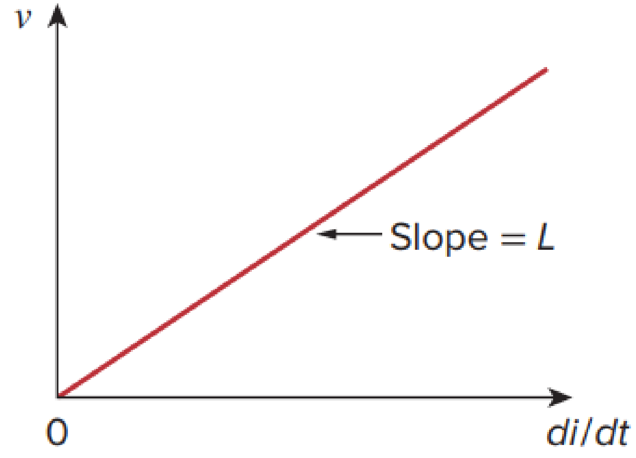
- Integrating both sides,

$$i(t) = \frac{1}{L}\int_{\infty}^{t} v(t)\, dt$$

$$\Rightarrow \quad i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\, dt + i(t_0)$$



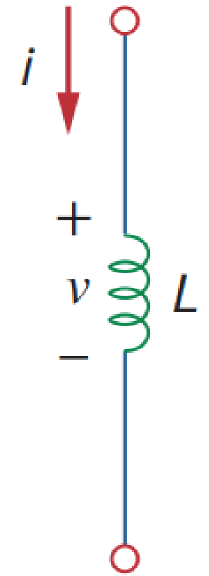




Energy stored & Power of an Inductor

- The instantaneous power delivered to an inductor according to the passive sign convention is,

$$p = v(t)i(t) = Li(t)\frac{di(t)}{dt}$$



- The energy stored in the inductor is therefore

$$w(t) = \int_{-\infty}^{t} p(t)\,dt = \int_{-\infty}^{t} Li(t)\frac{di(t)}{dt}dt = \int_{v(-\infty)}^{v(t)} Li(t)\ di$$

$$\Rightarrow \quad w(t) = \frac{1}{2}Li^2\Big|_{i(-\infty)=I_0}^{i(t)=I}$$

- If the current through the inductor was initially (at $t = -\infty$) I_0, then,

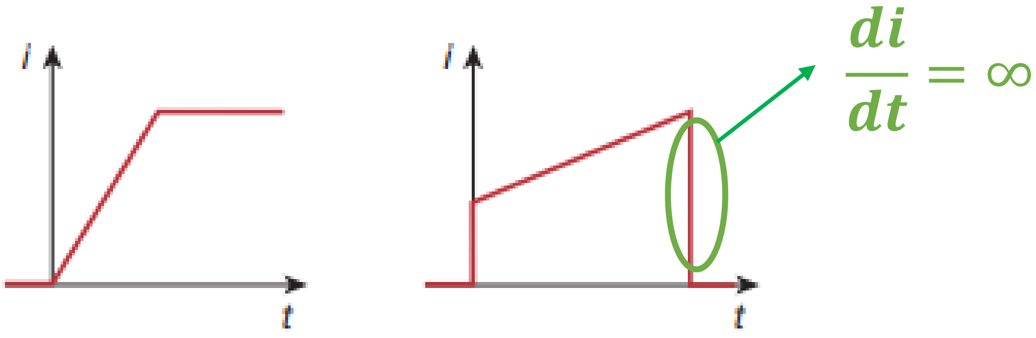
$$\Rightarrow \quad w(t) = \frac{1}{2}LI^2 - \frac{1}{2}LI_0^2$$

- In general, at any time t, if the current through an inductor is I, then the stored energy can be found as,

$$w(t) = \frac{1}{2}Li(t)^2 = \frac{1}{2}LI^2$$

Inductor: important properties

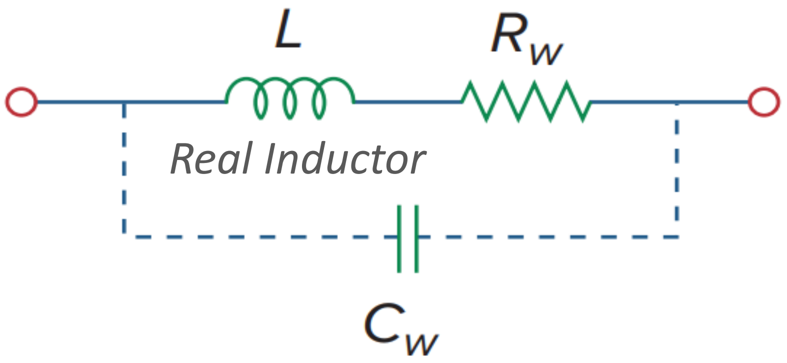
1. An inductor is a short circuit to dc. At dc, $v_L = L\frac{di_{L\_dc}}{dt} = 0$ [Short circuit]
2. The current through an inductor cannot change abruptly.



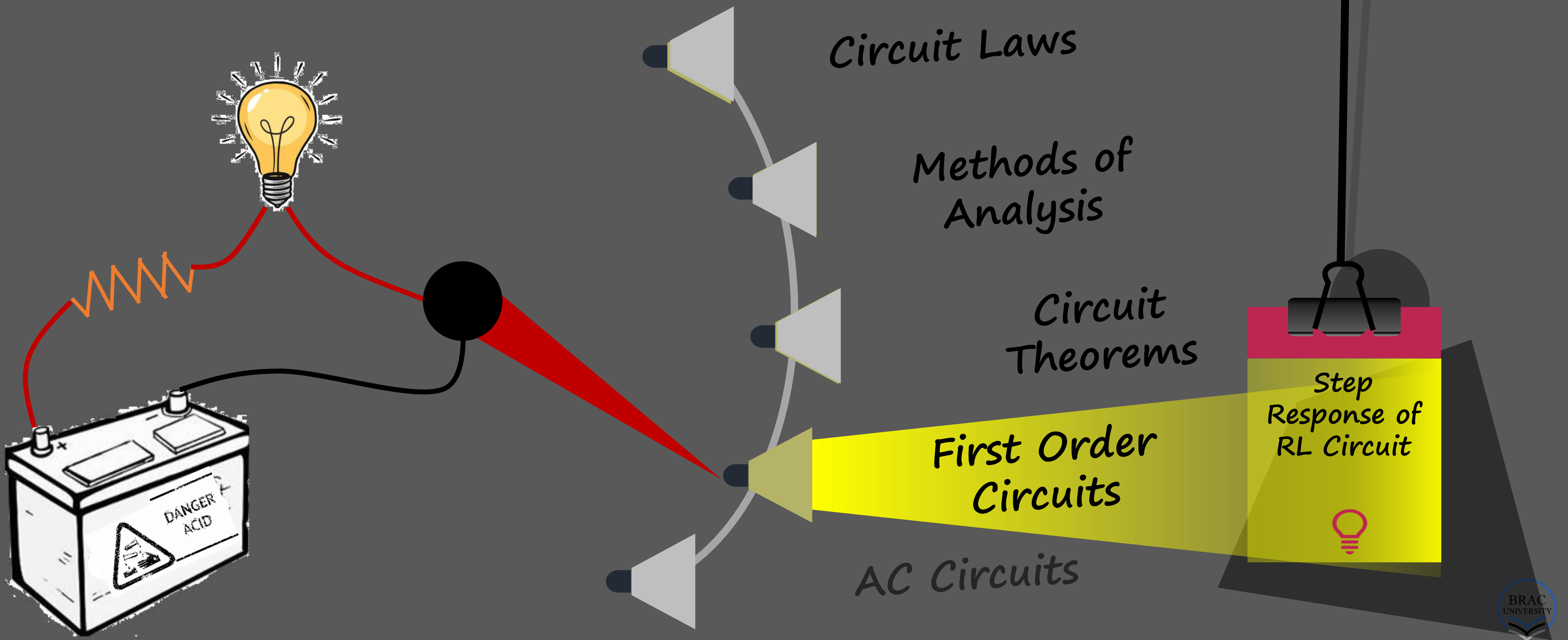
Current through an inductor (a) allowed and (b) not allowed

3. An ideal inductor does not dissipate energy.

4. A real, nonideal inductor has a series winding resistance and a capacitive coupling between the conducting coils.



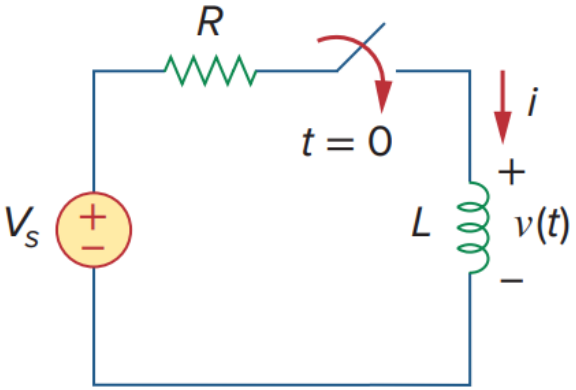
Course Outline: broad themes



Step Response of a RL circuit

- The *step response* of a circuit is its behaviour under the sudden application of dc voltage or current source. We assume the circuit response to be the inductor current.

⇒ Since the current through an inductor cannot change instantaneously,



$$\Rightarrow \quad i(0^-) = i(0^+) = I_0$$

⇒ Using KVL (for $t > 0$),

$$\Rightarrow \quad L\frac{di}{dt} + iR = V_s$$

$$\Rightarrow \quad \frac{di}{dt} + \frac{R}{L}i = \frac{V_s}{L}$$

Multiplying both sides by $e^{\frac{R}{L}t}$,

$$\Rightarrow \quad e^{\frac{R}{L}t}\frac{di}{dt} + e^{\frac{R}{L}t}\frac{R}{L}i = \frac{V_s}{L}e^{\frac{R}{L}t}$$

$$\Rightarrow \quad \frac{d}{dt}\left[e^{\frac{R}{L}t}.i\right] = \frac{V_s}{L}e^{\frac{R}{L}t}$$

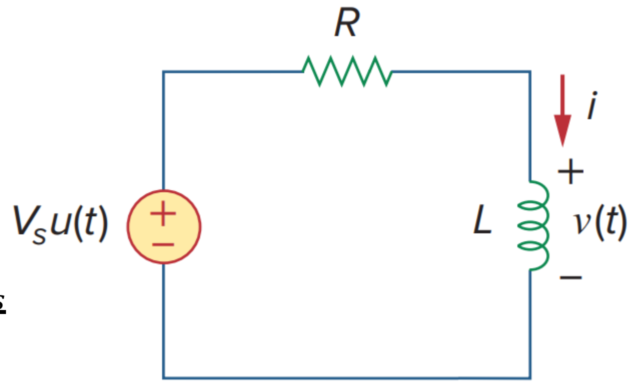
Integrating both sides with respect to t,

$$\Rightarrow \quad e^{\frac{R}{L}t}.i = \frac{V_s}{R}e^{\frac{R}{L}t} + C$$

$$\Rightarrow \quad i = \frac{V_s}{R} + Ce^{-\frac{R}{L}t}$$

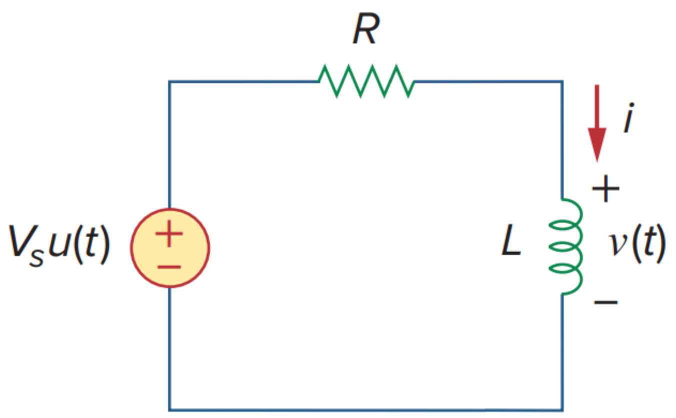
At $t = 0,\ i = I_0$. $\quad So,\ C = i - \frac{V_s}{R}$

Substituting, $i = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-\frac{R}{L}t}$



Time Constant

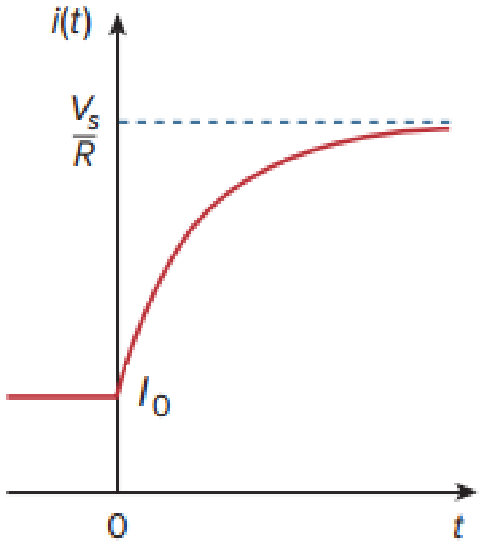
$$i(t) = \begin{cases} I_0, & t < 0 \\ \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}t}, & t > 0 \end{cases}$$



- This is known as the complete response (or total response) of the RL circuit to a sudden application of a dc voltage source. It is assumed that the inductor was initially charged to I_0.

$$\Rightarrow \quad i(t) = \begin{cases} I_0, & t < 0 \\ \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{t}{\tau}}, & t > 0 \end{cases}$$

$\Rightarrow$ where $\tau = {}^{L}/_{R}$ is the *time constant* (unit in sec).

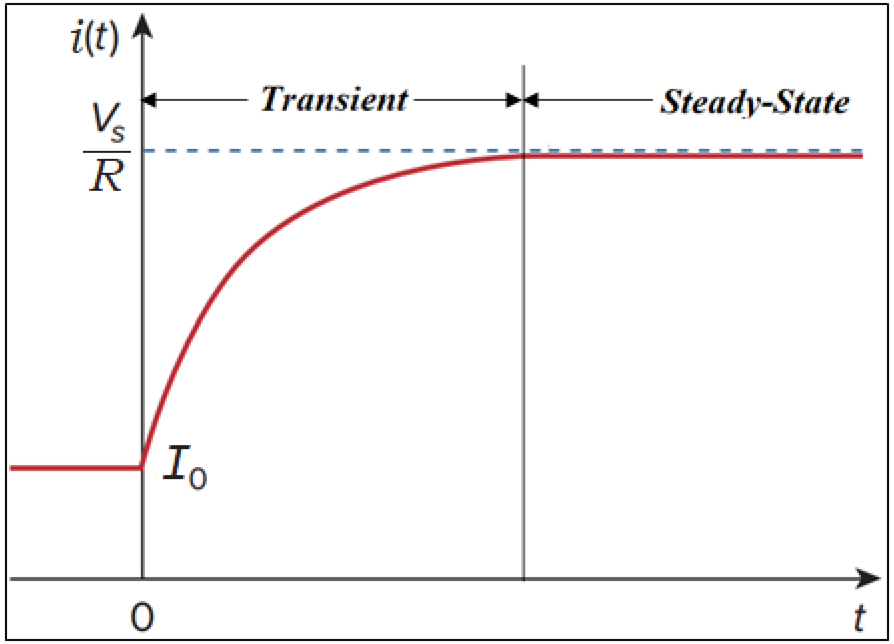


- Notice that, we write $\tau = {}^{L}/_{R}$ for the circuit consisting of only a resistor R in series with the inductor. As we know, all the linear two terminal circuits can be reduced to this form by Thevenin's Theorem, so the resistor R is actually the Thevenin Resistance R_{Th}. Therefore,

$$\boxed{\tau = \frac{L}{R_{Th}}}$$

Transient and Steady-State Response

$$i(t) = \begin{cases} I_0, & t < 0 \\ \dfrac{V_s}{R} + \left(I_0 - \dfrac{V_s}{R}\right) e^{-\frac{R}{L}t}, & t > 0 \end{cases}$$



- The *complete response* can be broken into two parts—one temporary and the other permanent, that is,

$$i(t) = i_{ss} + i_t, \quad \text{where,}$$

$$i_{ss} = \frac{V_s}{R} \quad \& \quad i_t = \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{t}{\tau}}$$

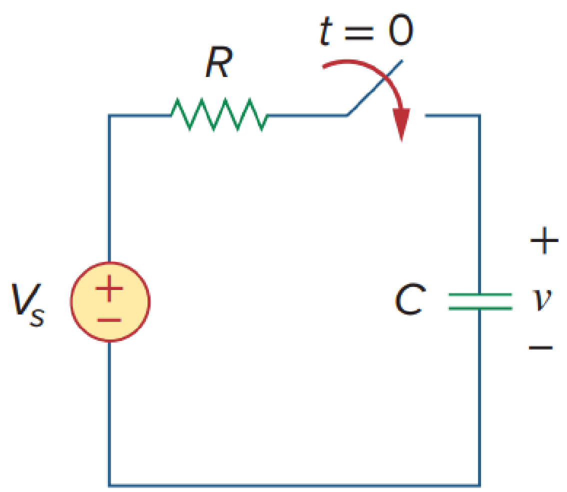
- The *transient response* (i_t) is the circuit's temporary response that will die out with time.
- The *steady-state response* (i_{ss}) is the behaviour of the circuit a long time after an external excitation is applied.

- The complete response can be written as,

$$i(t) = I_{final} + [I_{initial} - I_{final}]e^{-\frac{t}{\tau}}$$

$$or, \quad i(t) = I(\infty) + [I(0) - I(\infty)]e^{-\frac{t}{\tau}}$$

Definition of τ



$$i(t) = I_{final} + [I_{initial} - I_{final}]e^{-\frac{t}{\tau}}$$

At $t = \tau$,

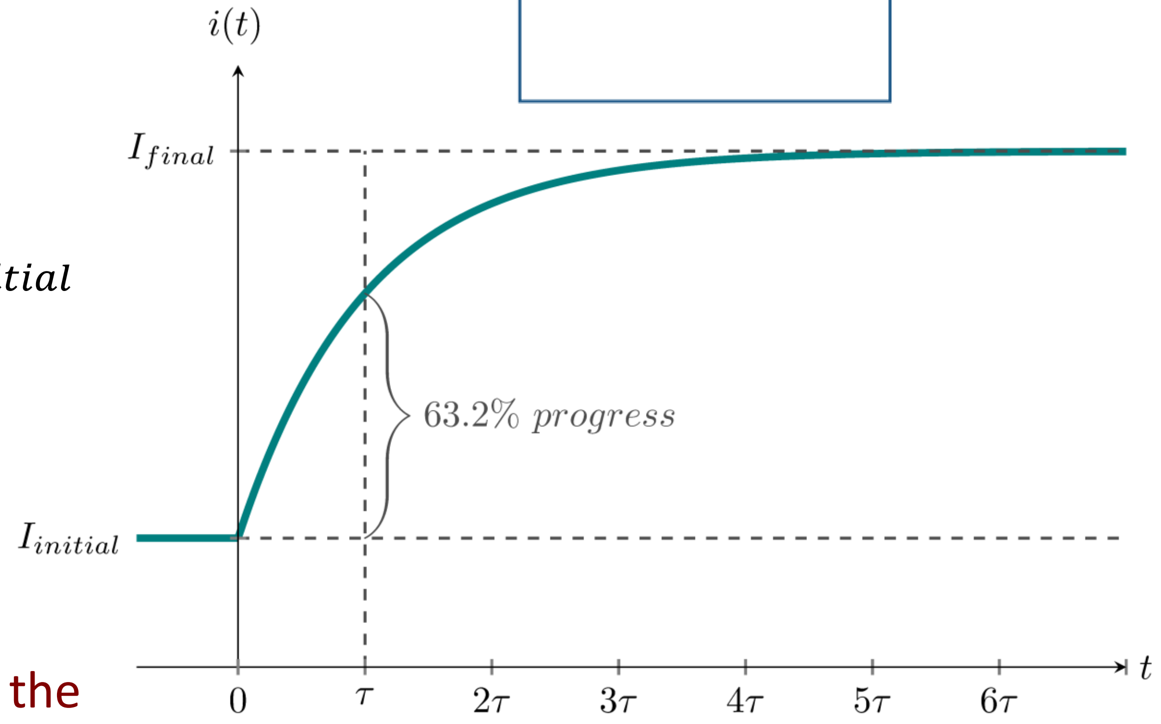
$$i(t) = I_{final} + [I_{initial} - I_{final}]e^{-1}$$

$$\Rightarrow \quad i(t) = I_{final}(1 - {}^1\!/_e) + I_{initial}({}^1\!/_e)$$

$$\Rightarrow \quad i(t) = I_{final}(1 - {}^1\!/_e) - I_{initial}(1 - {}^1\!/_e) + I_{initial}$$

$$\Rightarrow \quad i(t) = I_{initial} + [I_{final} - I_{initial}](1 - {}^1\!/_e)$$

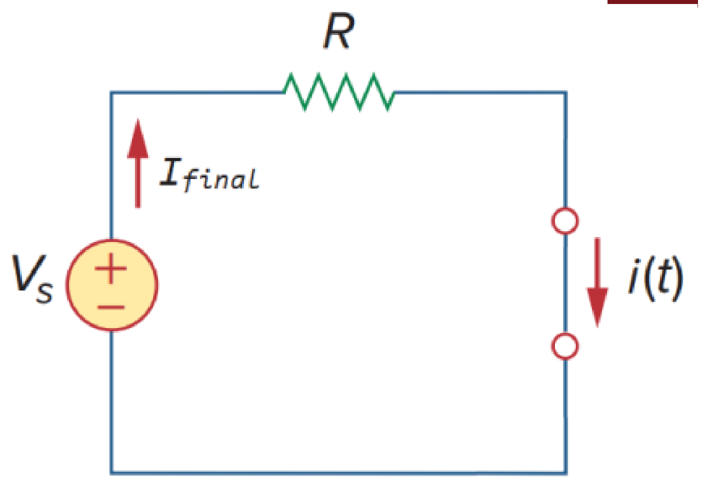
$$\Rightarrow \quad i(t) = I_{initial} + [I_{final} - I_{initial}] \times 63.2\%$$



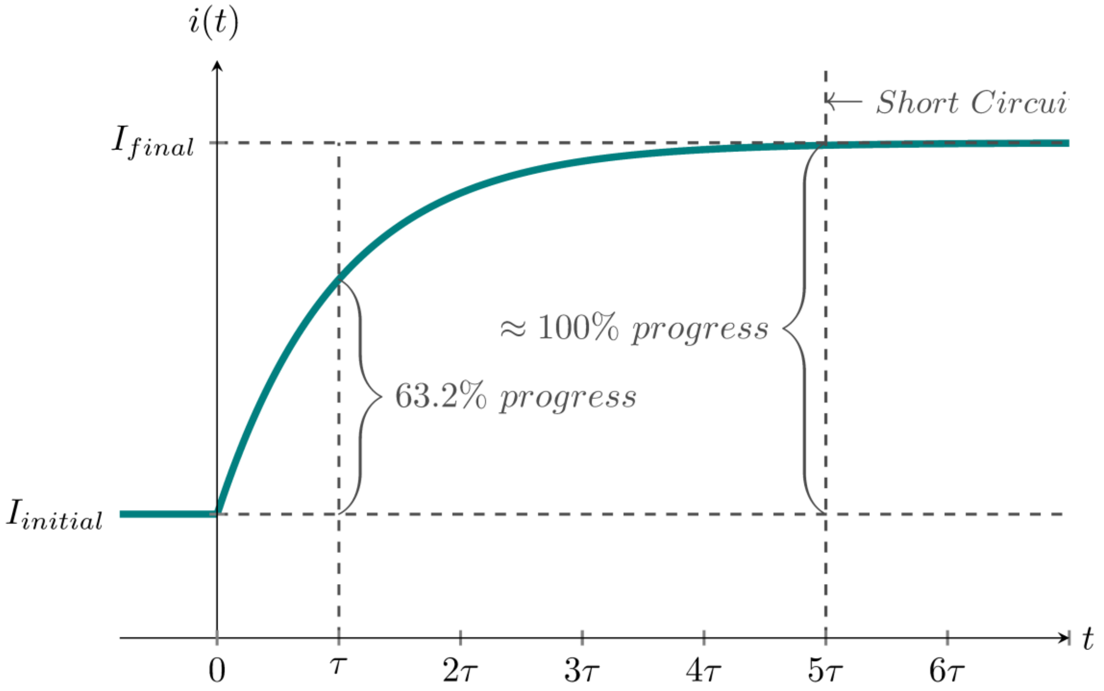
- We can define the time constant in this way,
- The *time constant* is the time required for the response to progress 63.2% towards I_{final} from an initial response $I_{initial}$.

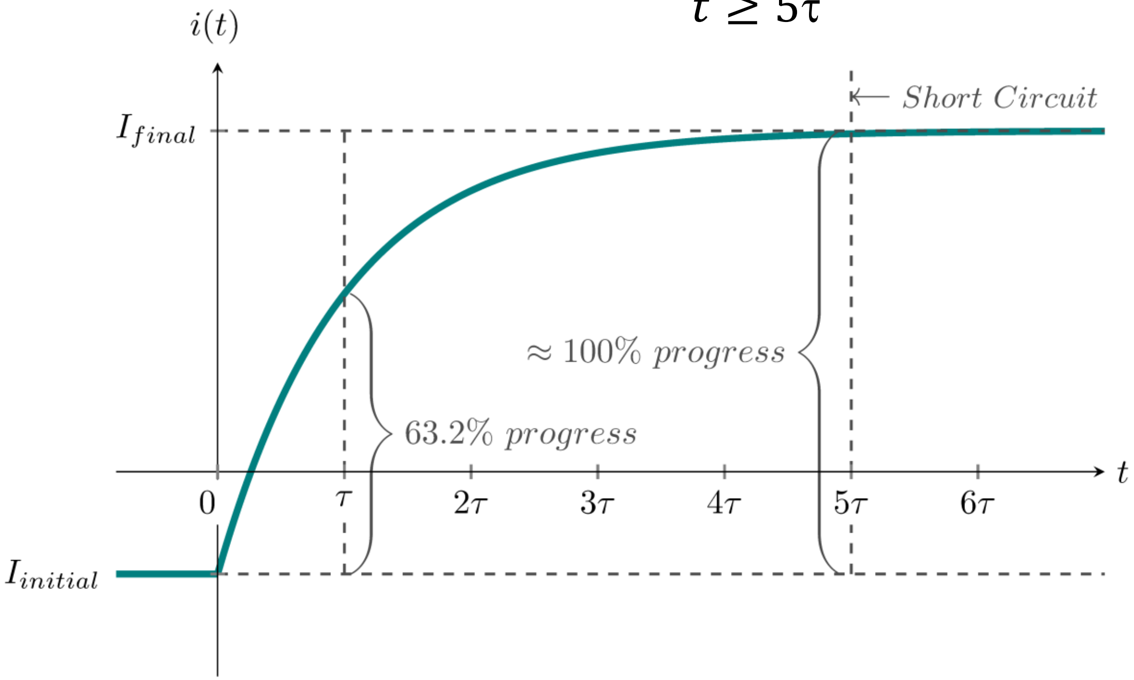
Significance of τ

- The significance of τ is that it determines how fast or slow an inductor will be charged or discharged. Mathematically, an inductor current reaches the steady-state current approximately after 5 times the Time Constant (τ). **The inductor is fully charged and acts as short circuit from 5τ time onward**. So, when designing circuits, the charging time of an inductor under the application of a certain dc supply can be set by choosing R_{Th}.



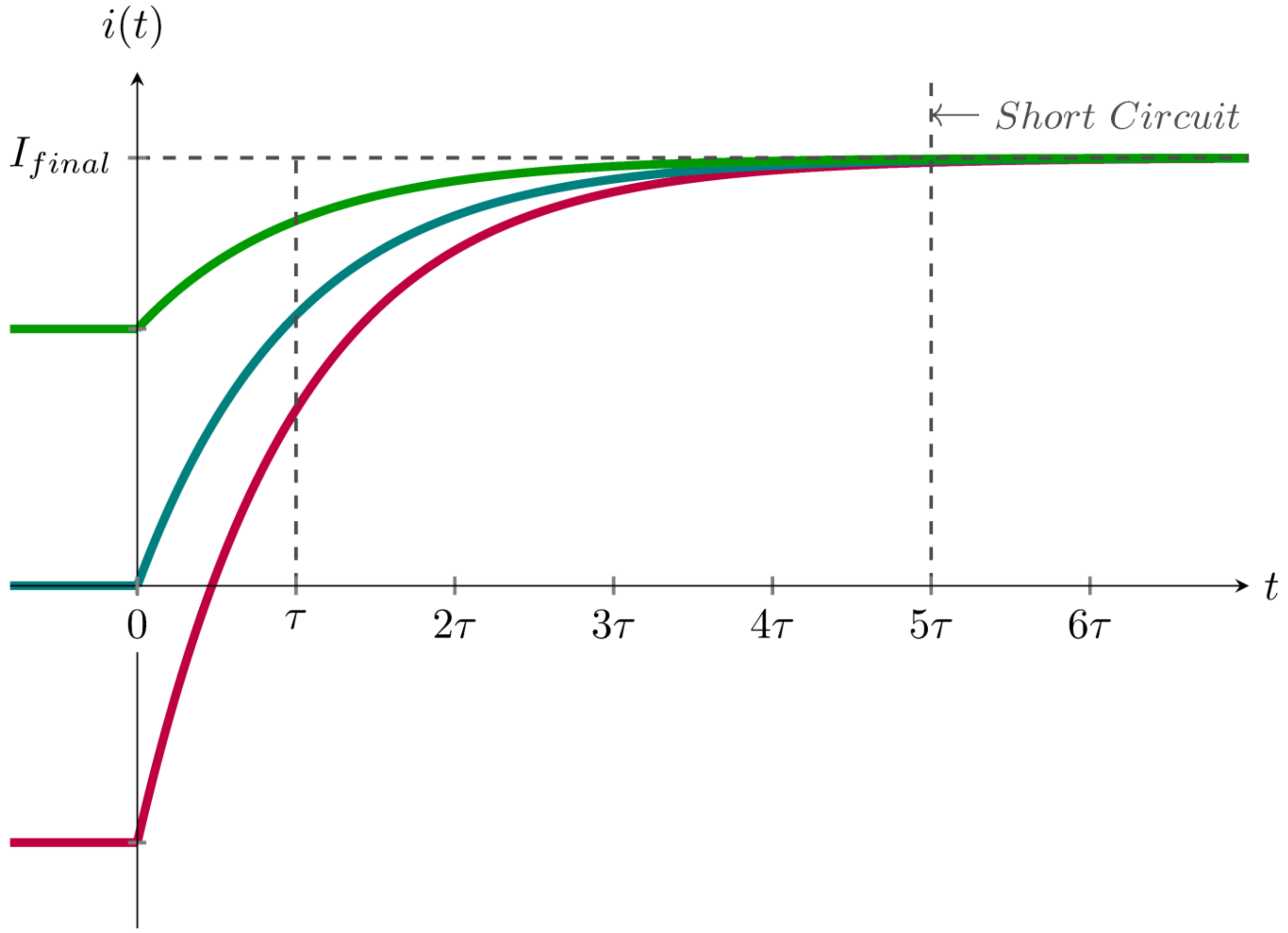
$t \geq 5\tau$



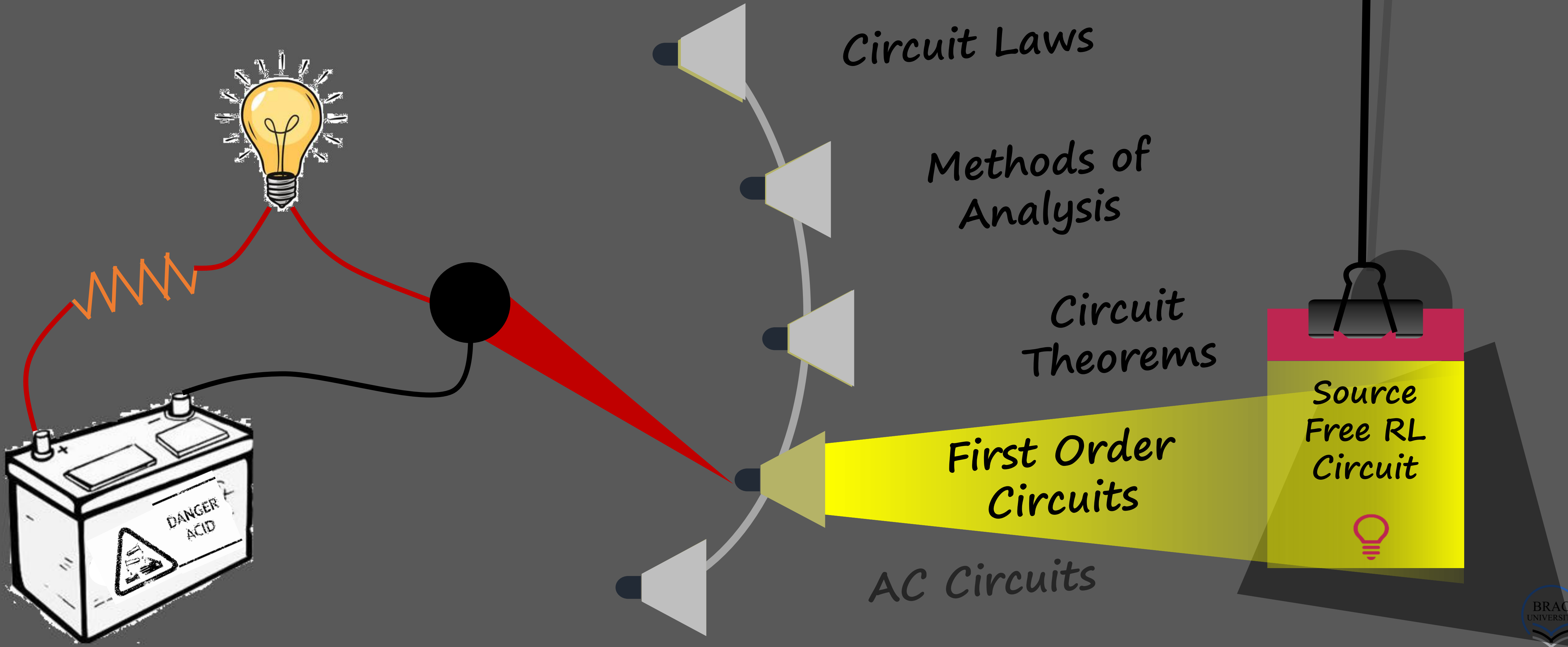


τ's independency from initial condition

- The time constant does not depend on the initial current or initial field of the inductor. For a given circuit, that is, for a fixed R_{Th} and L, the time needed for the inductor current to rise to its final value is the same whether or not the it is initially charged.



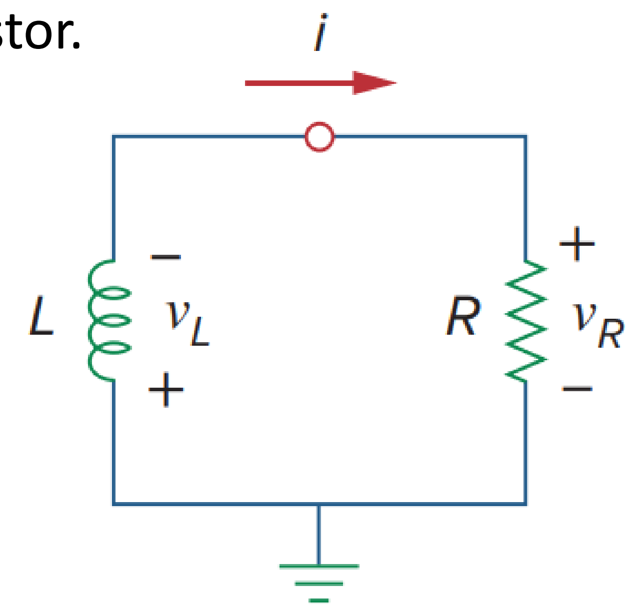
Course Outline: broad themes



Source-Free RL circuit

- A *source-free RL circuit* occurs when its dc source is suddenly disconnected. The energy already stored in the capacitor is released to the resistors.

⇒ Assume that an inductor is charged to I_0 and then it is connected to a resistor as shown. The inductor starts to discharge the stored energy to the resistor.



⇒ Initially stored charge, $w(0) = \frac{1}{2}LI_0^2$

⇒ From the figure using KVL, $v_L + v_R = 0$

$$\Rightarrow \mathrm{L}\frac{di}{dt} + Ri = 0$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = 0$$

$$\Rightarrow \frac{di}{i} = -\frac{R}{L}dt$$

Integrating both sides,

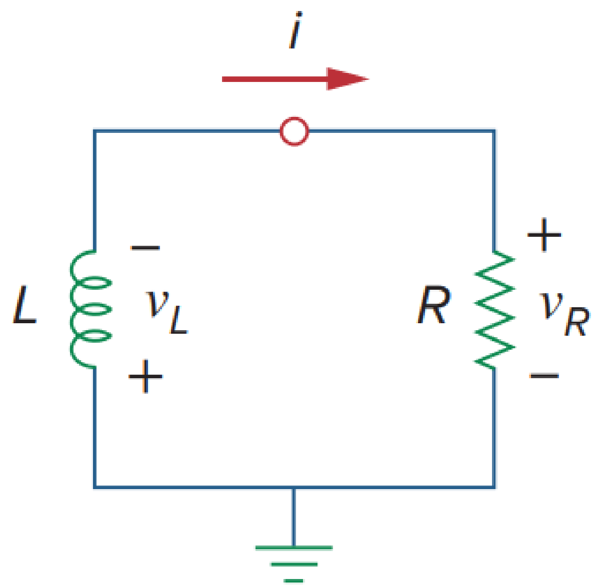
$$\Rightarrow lni = -\frac{R}{L}t + lnA$$

$$\Rightarrow ln\frac{i}{A} = -\frac{R}{L}t$$

$$\Rightarrow i = Ae^{-\frac{R}{L}t}$$

At t = 0, $i(0) = \mathrm{A} = I_0$ So, $\boxed{i(t) = I_0e^{-\frac{R}{L}t}}$

Time Constant

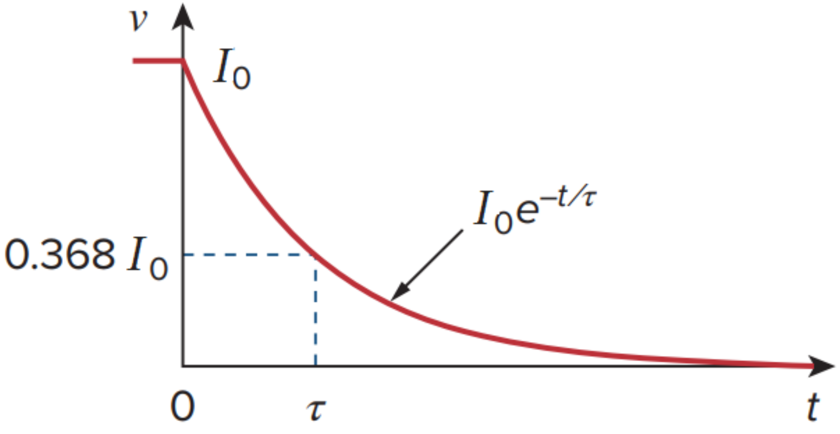


$$i(t) = I_0 e^{-\frac{R}{L}t}$$

- This shows that the voltage response of the RL circuit is an exponential decay of the initial voltage. It is called the *natural response* of the circuit.

$$\Rightarrow \quad i(t) = I_0 e^{-\frac{t}{\tau}}$$

- where $\tau = {}^{L}/_{R}$ is the time constant (unit in sec).



- Notice that, we write $\tau = {}^{L}/_{R}$ for the circuit consisting of only a resistor R in series with the capacitor. As we know, all the linear two terminal circuits can be reduced to this form by Thevenin's Theorem, so the resistor R is actually the Thevenin Resistance R_{Th}. Therefore,

$$\boxed{\tau = {}^{L}/_{R_{Th}}}$$

Definition of τ

$$i(t) = I_0 e^{-\frac{t}{\tau}}$$

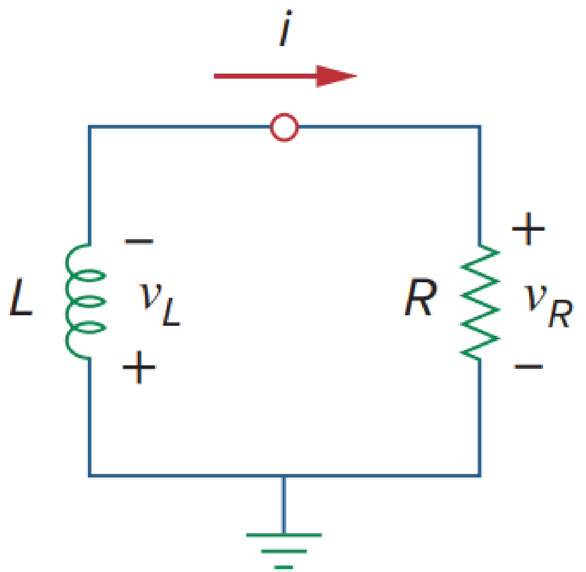
At $t = \tau$,

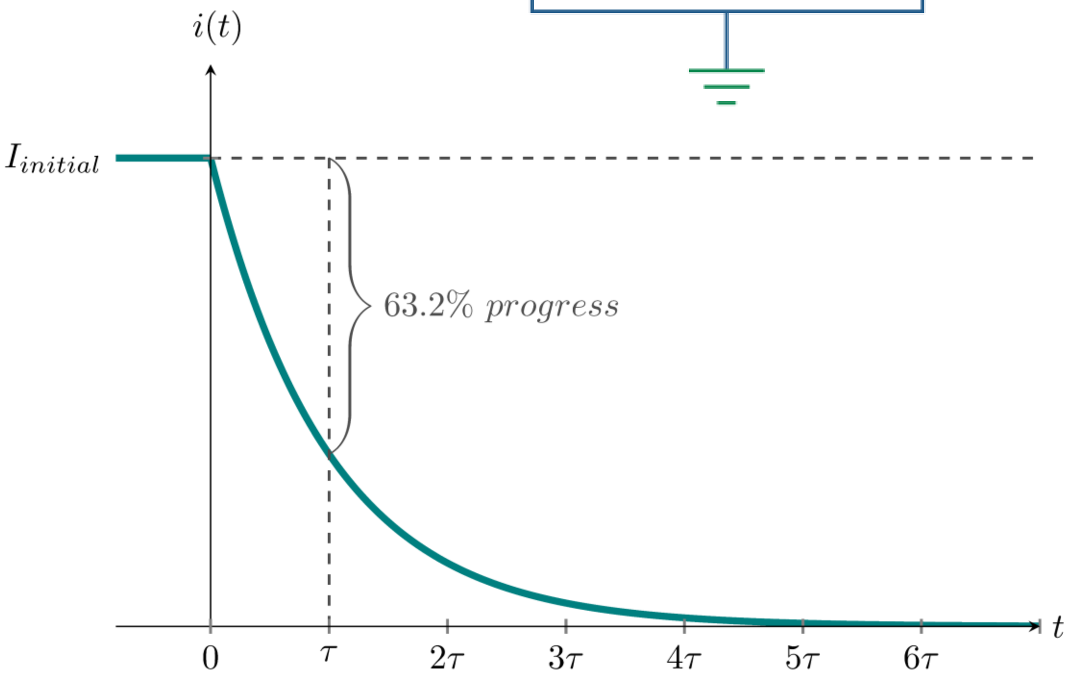
$$i(t) = I_0 e^{-1}$$

$$\Rightarrow \quad i(t) = 0.368 \times I_0$$

$$\Rightarrow \quad i(t) = I_{initial} - 0.632 \times [I_{initial} - 0]$$

- We can define the time constant in the same way as we did in the case of the step response:
- The *time constant* is the time required for the response to progress 63.2% towards a final current from an initial response $I_{initial}$ or I_0.



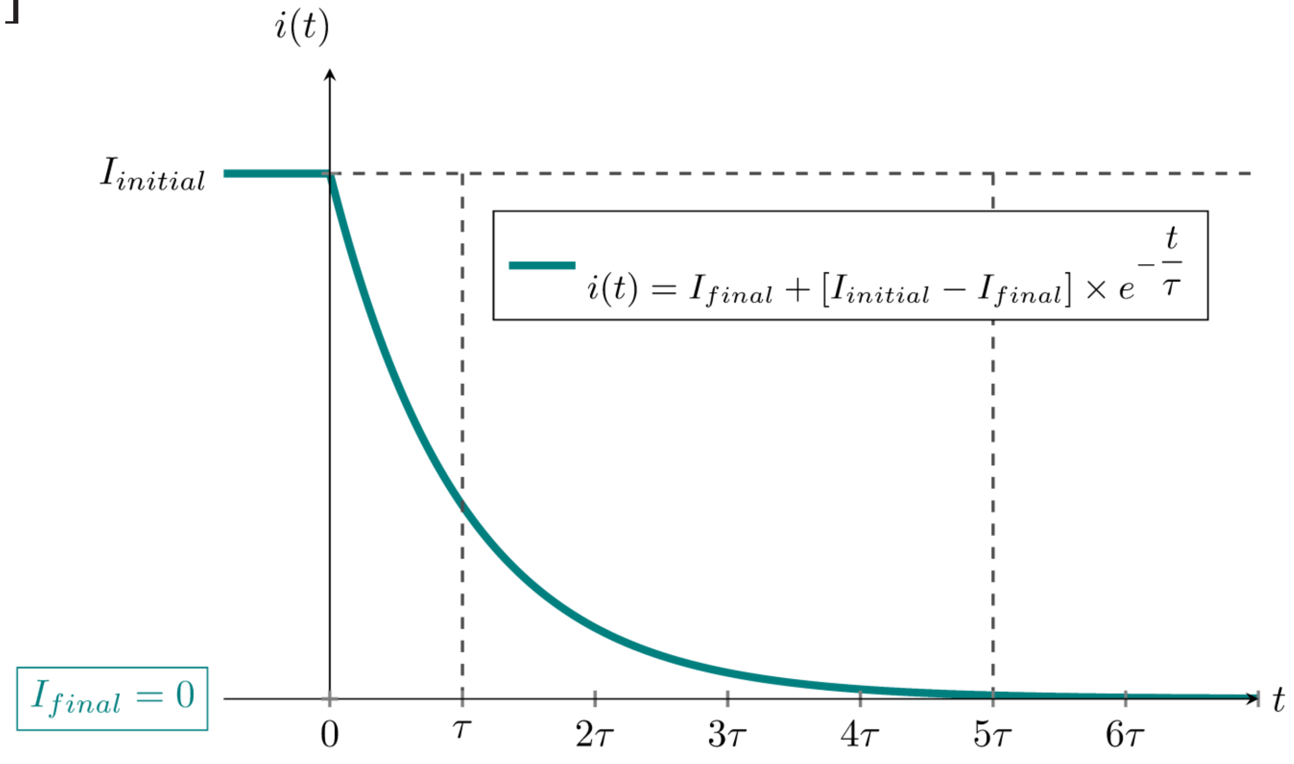


General equation

- It is interesting to interpret this way that the source-free RL circuit is a condition where $I_{final} = I(\infty) = 0$ in the step response $\left[i(t) = I(\infty) + [I(0) - I(\infty)]e^{-\frac{t}{\tau}}\right]$ or $\left[i(t) = I_{final} + [I_{initial} - I_{final}]e^{-\frac{t}{\tau}}\right]$ equation we derived earlier.

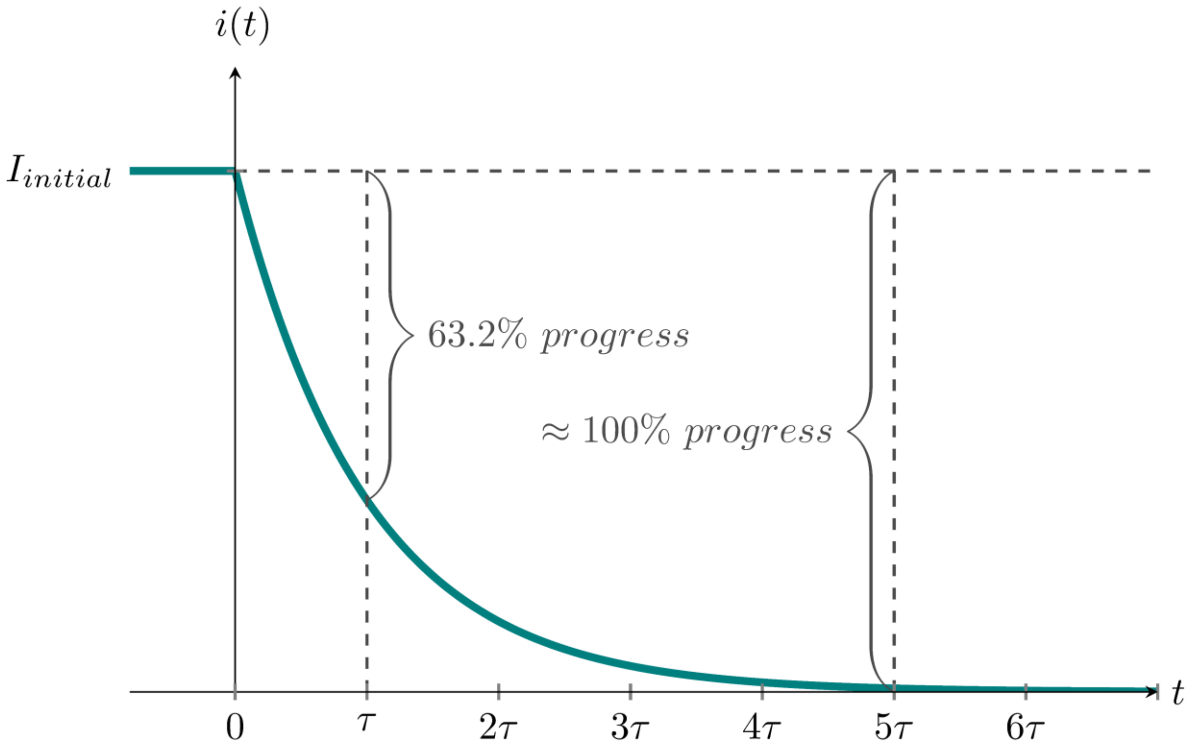
- We can in fact use this equation in general to find an inductor's current response irrespective of the inductor getting charged or discharged or both.

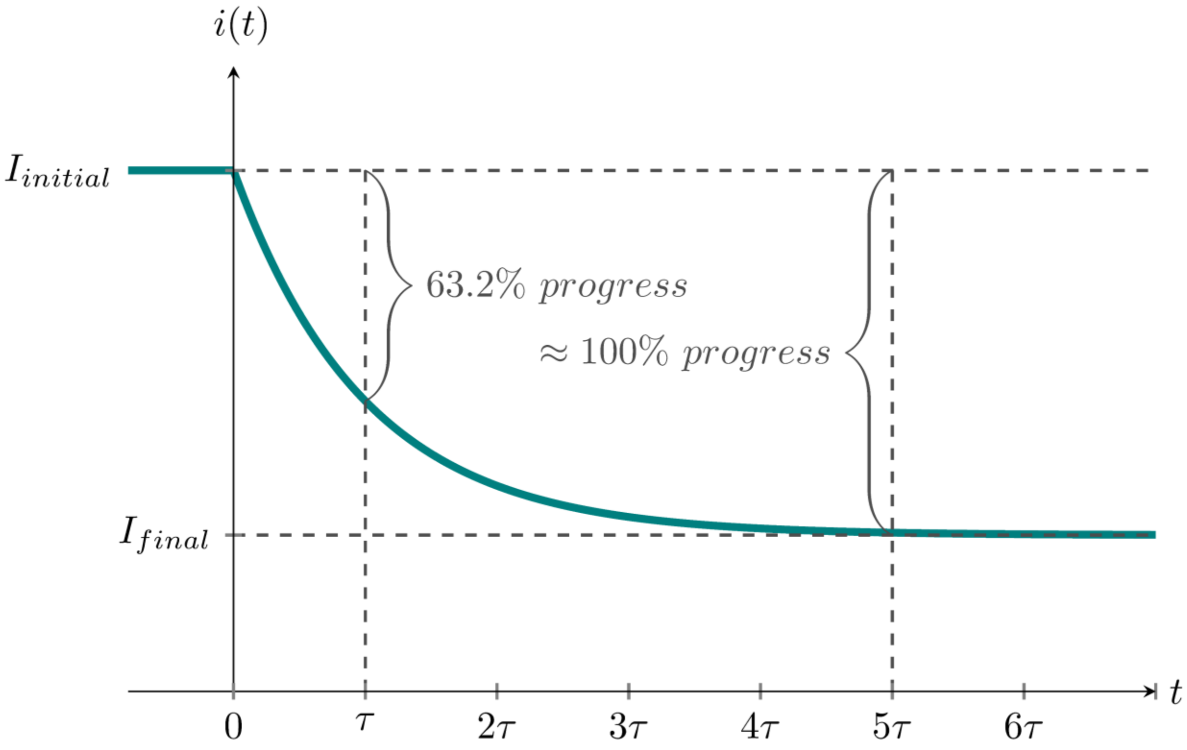
- An inductor's energy will increase if $|I(\infty)| > |I(0)|$, otherwise the energy will decrease.



Significance of τ

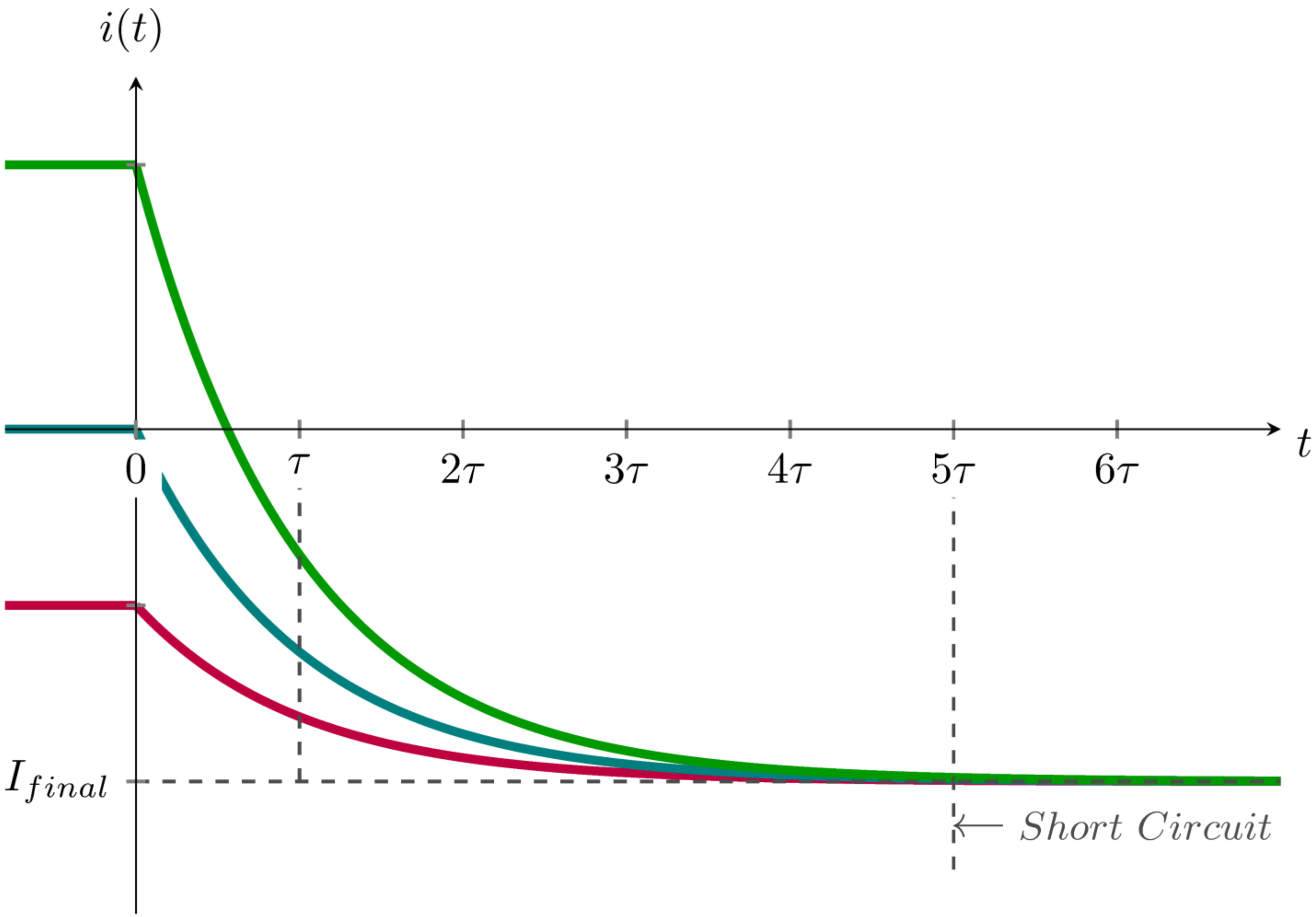
- When an inductor's initial voltage $(I_{initial})$ falls to zero $(I_{final} = 0)$ **(if no independent sources are present in the circuit to charge it back)**, it can be shown mathematically that it takes approximately 5τ time to reach the final current. In fact, irrespective of the values of $I_{initial}$ and I_{final}, an inductor will always take $\approx 5\tau$ time to reach the final current.





τ's independency from initial condition

- The time constant does not depend on the initial current or initial field of the inductor. For a given circuit, that is, for a fixed R_{Th} and L, the time needed for the inductor current to fall to its final current is the same whether or not the inductor is initially charged.



Summary: Capacitor vs. Inductor

	Capacitor	Inductor
Capacitance/ Inductance	$C = \frac{q}{v}$	$L = \frac{N\varphi}{I}$
$I - V$ Characteristics	$I = C\frac{dV}{dt}$	$V = L\frac{dI}{dt}$
Behavior	$open\ circuit\ at\ dc$	$short\ circuit\ at\ dc$
Energy	$W = \frac{1}{2}CV^2$	$W = \frac{1}{2}LI^2$
Step response	$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-\frac{t}{\tau}}$	$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}}$
Time constant	$\tau = R_{Th}C_{eq}$	$\tau = \frac{L_{eq}}{R_{Th}}$

Procedure

$$i(t) = I(\infty) + [I(0) - I(\infty)]e^{-t/\tau}$$

Determine the initial current of the inductor $I_{initial}$ or $I(0)$

Consider only the active‡ portion of the circuit before switching. For example, if switching occurs at $t = 0$, consider the circuit for $t < 0$.

If the circuit includes any dc source (current or voltage), short the capacitor and determine the voltage at the open terminal. This is the $I(0)$. $I(0) = 0$ if there is no independent source in the circuit.

Determine the final voltage of the inductor I_{final} or $I(\infty)$

Now consider the active‡ portion of the circuit after switching. For example, for $t > 0$.

Repeat the step. This time, the voltage across the capacitor is $I(\infty)$.

Determine the time constant (τ)

Again, only consider the active‡ portion after switching. For example, for $t > 0$.

Determine the Thevenin resistance (R_{Th}) as seen from the capacitor terminals

$\tau = L/R_{Th}$

Determine $i(t)$

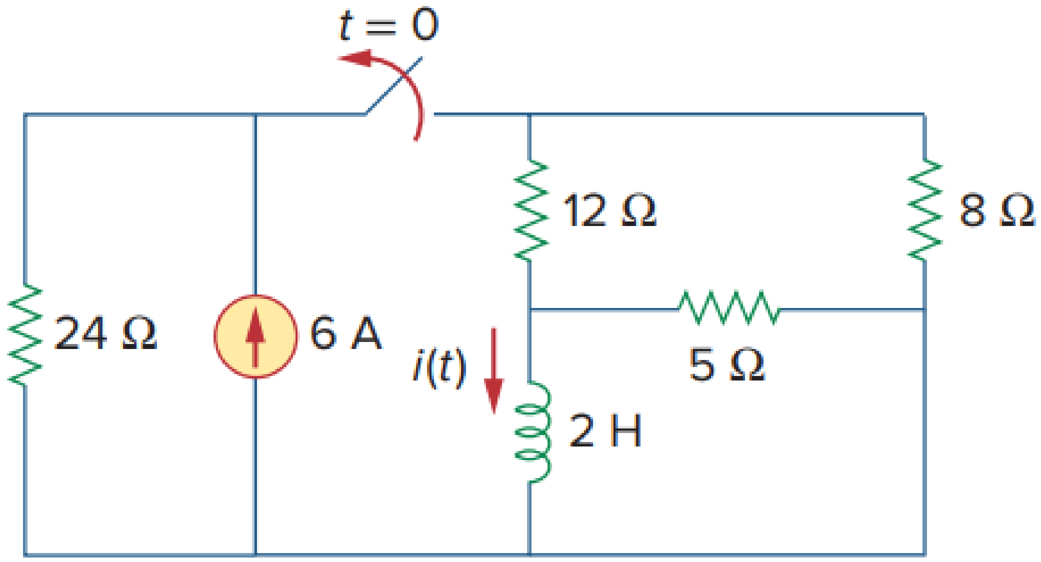
Plug in $I(0)$, $I(\infty)$, and τ into the equation for $I(t)$

Determine any other voltages or currents in the circuit using $i(t)$ and the circuit laws.

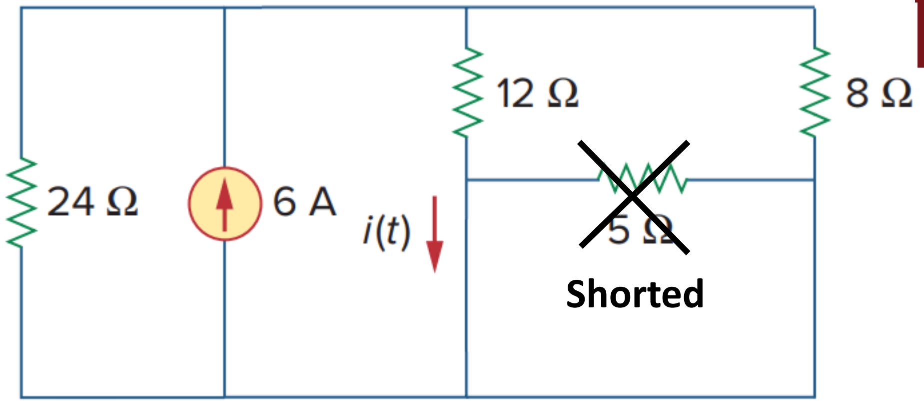
‡ active portion of the circuit excludes everything that has no influence on the inductor

Example 4 - 1/2

- Find $i(t)$ for $t > 0$. Find the initial energy stored in the inductor.



For $t < 0$



For $t < 0$, the switch is closed. With the inductor shorted at dc, the circuit transforms into the one shown above.

The $5\,\Omega$ resistance has zero potential difference, hence, shorted out. The current through the $12\,\Omega$ can be found using current division rule.

$$i(t) = \frac{12^{-1}}{12^{-1} + 24^{-1} + 8^{-1}} \times 6 = 2\,A, \qquad t < 0$$

Since the current though the inductor cannot change instantaneously,

$$i(0) = i(0^{-}) = 2\,A$$

Example 4 - 2/2

For $t > 0$, the switch is open. transforms into the one shown above. As there is no independent source in the circuit, $I(\infty) = 0$. The Thevenin resistance as seen from the inductor terminal,

$$R_{Th} = (12 + 8) \,||\, 5 = 4\,\Omega$$

The time constant is,

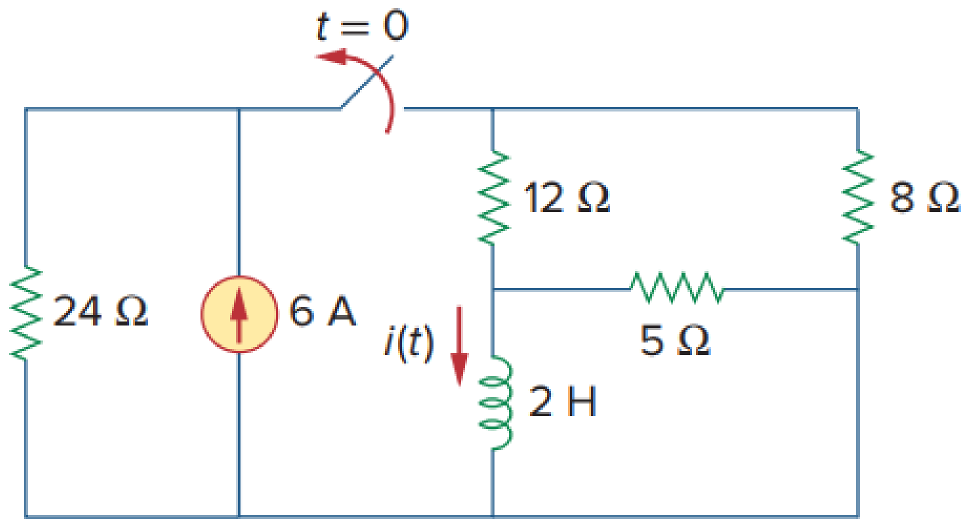
$$\tau = \frac{L}{R_{Th}} = \frac{2}{4} = 0.5\ s$$

So, the current through the inductor for t > 0 is,

$$i(t) = i(0)e^{-\frac{t}{\tau}}$$
$$= 2e^{-2t}\ (A)$$

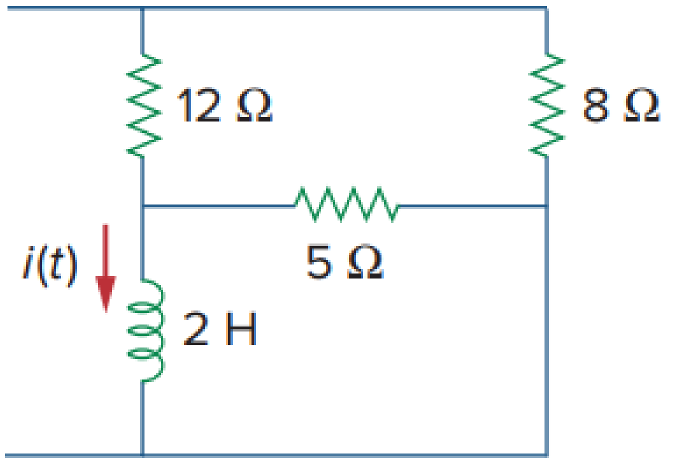
The initial energy stored in the inductor is, $w_L(0) = \frac{1}{2}LI(0)^2$

$$\Rightarrow \quad \frac{1}{2} \times 2 \times 2^2 = 4\ J$$



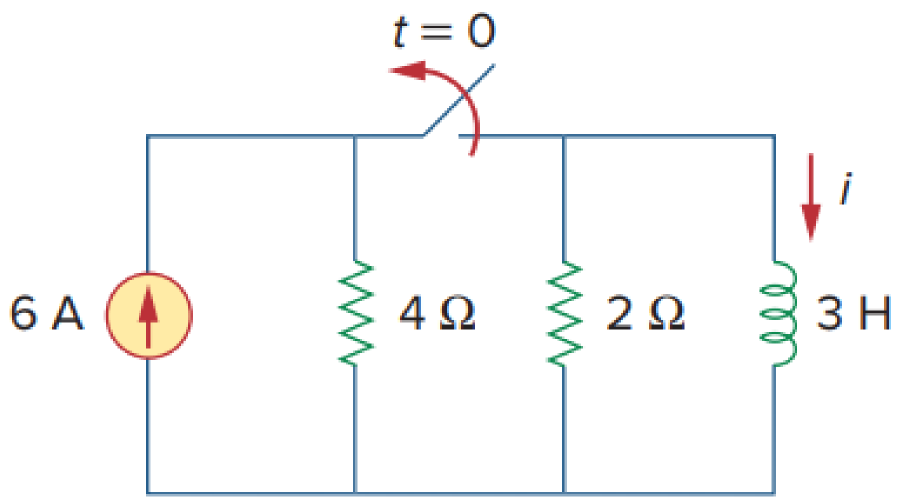
Original circuit

For $t > 0$

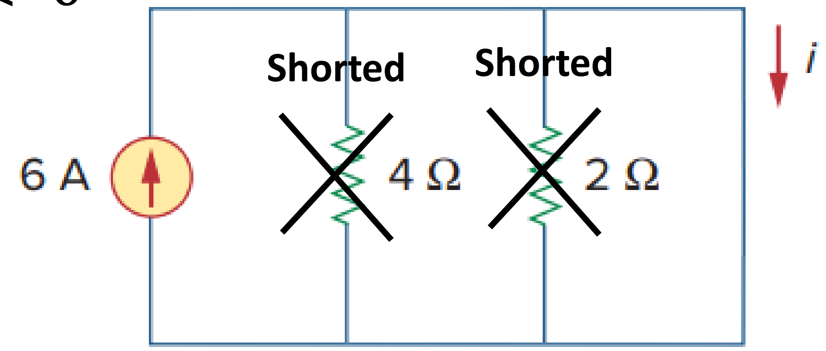


Example 5 : $t < 0$

- Determine the inductor current $i(t)$ for both $t < 0$ and $t > 0$.



For $t < 0$



For $t < 0$, the switch is closed. With the inductor shorted at dc, the circuit transforms into the one shown above.

As the potentials across each of the $4\,\Omega$ and $2\,\Omega$ resistances are equal due to the short circuit at the inductor, no current will flow through them.

The current $6\ A$ will flow only through the short circuit at the inductor.

$$i(t) = 6\ A, \qquad t < 0$$

Since the current through the inductor cannot change instantaneously,

$$i(0) = i(0^-) = 6\ A$$

Example 5: $t > 0$

For $t > 0$, the switch is open. The circuit transforms into the one shown above. As there is no dc source, inductor is not short circuited and $I(\infty) = 0$.

The time constant is,

$$\tau = \frac{L}{R_{Th}} = \frac{3}{2} s$$

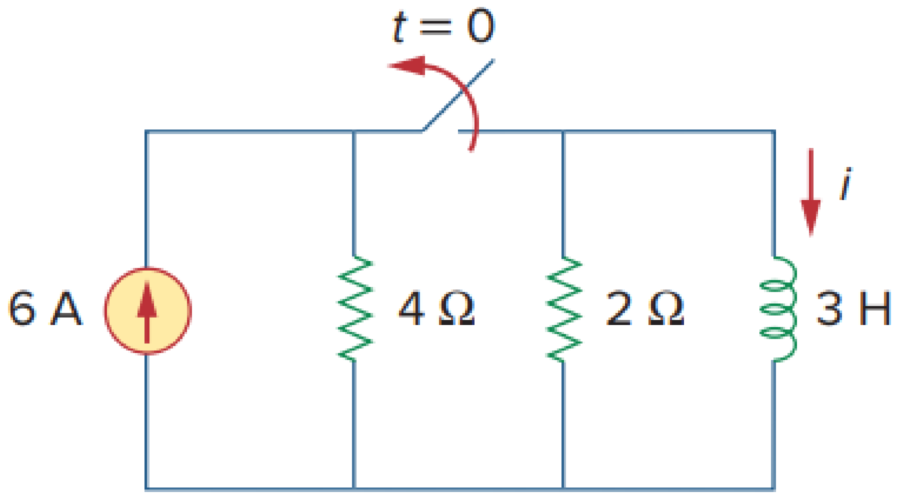
The current through the inductor for t > 0 is thus,

$$i(t) = i(0)e^{-\frac{t}{\tau}} = 6e^{-\frac{2t}{3}}\,(A), \qquad t > 0$$

The voltage across the inductor for $t > 0$ is,

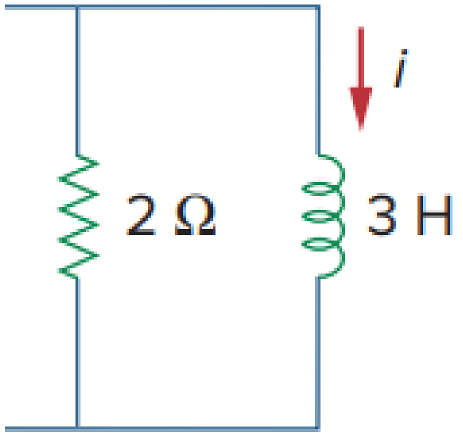
$$v_L = L\frac{di}{dt} = 3\frac{d}{dt}\left(6e^{-\frac{2t}{3}}\right) = 3 \times 6 \times \left(-\frac{2}{3}\right)e^{-\frac{2t}{3}}$$

$$= -12e^{-\frac{2t}{3}}, \qquad t > 0$$



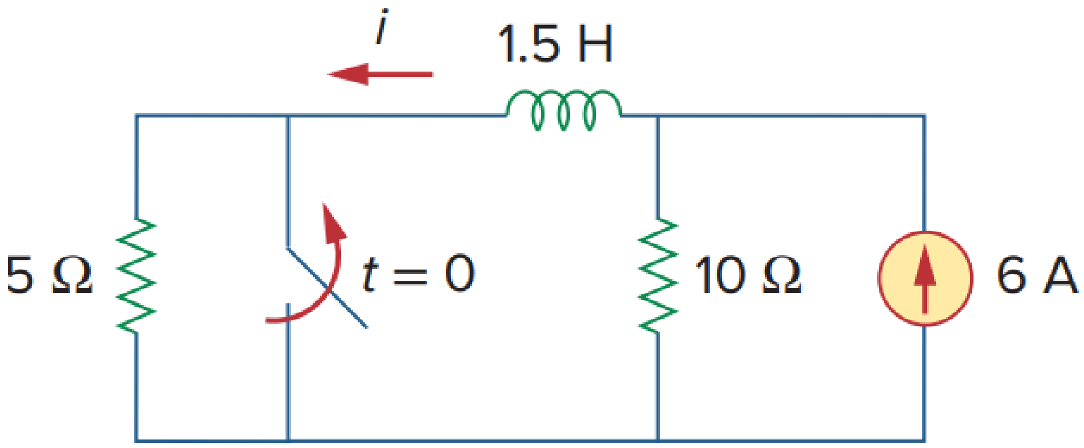
Original circuit

For $t > 0$

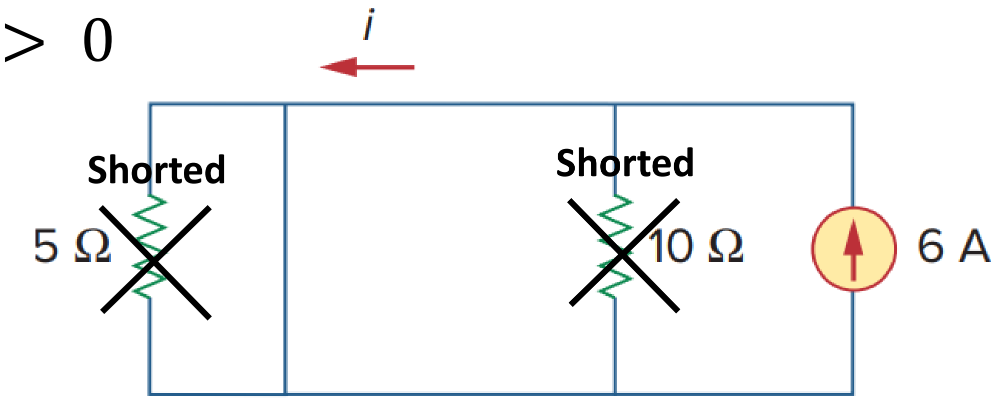


Example 6 : $t < 0$

- The switch has been closed for a long time. It opens at $t = 0$. Find $i(t)$ for $t > 0$.



For $t > 0$



For $t < 0$, the switch is closed. With the inductor shorted at dc, the circuit transforms into the one shown above.

As the potentials across $5\ \Omega$ resistor are equal due to the short circuit at the switch, no current will flow through it.

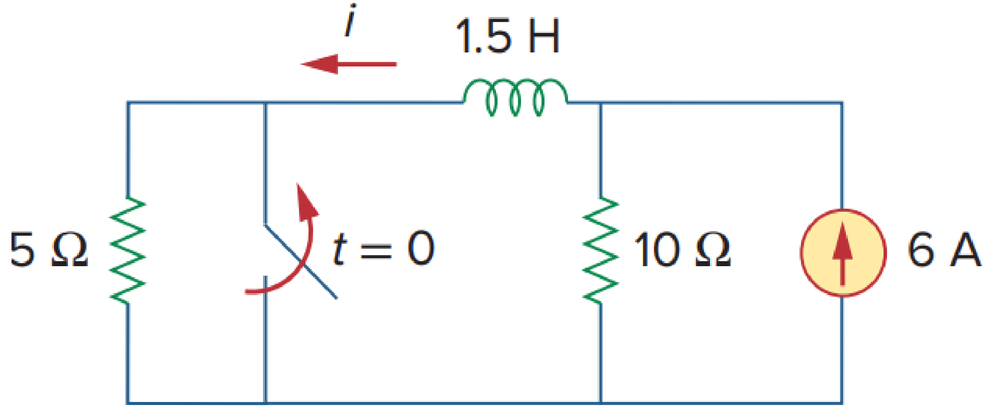
The $10\ \Omega$ resistor is also shorted out due the short circuits at the inductor and switch. Consequently the $6\ A$ current will flow through the short circuit. So,

$$i(t) = 6\ A, \qquad t < 0$$

Since the current through the inductor cannot change instantaneously,

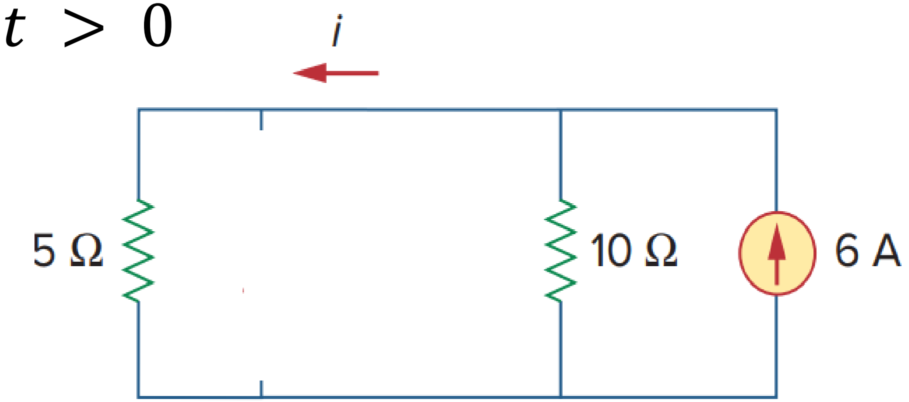
$$i(0) = i(0^-) = 6\ A$$

Example 6 : $t > 0$

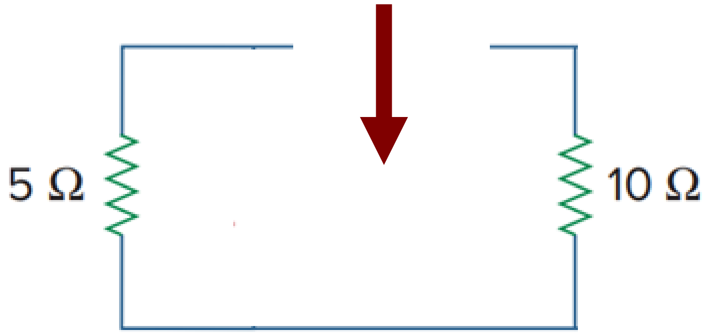


Original circuit

For $t > 0$



$$R_{Th} = 5 + 10 = 15\,\Omega$$



For $t > 0$, the switch is open. With the inductor shorted at dc, the circuit transforms into the one shown above.

The current, $i(t)$ through the $5\,\Omega$ resistor can be found using voltage division as,

$$i(t) = \frac{10}{10+5} \times 6 = 4\,A, \qquad t > 0$$

This is the inductor's steady-state current until the circuit is modified. So,

$$i(\infty) = 4\,A$$

The time constant is, $\tau = \frac{L}{R_{Th}} = \frac{1.5}{5+10} = 0.1\,s$

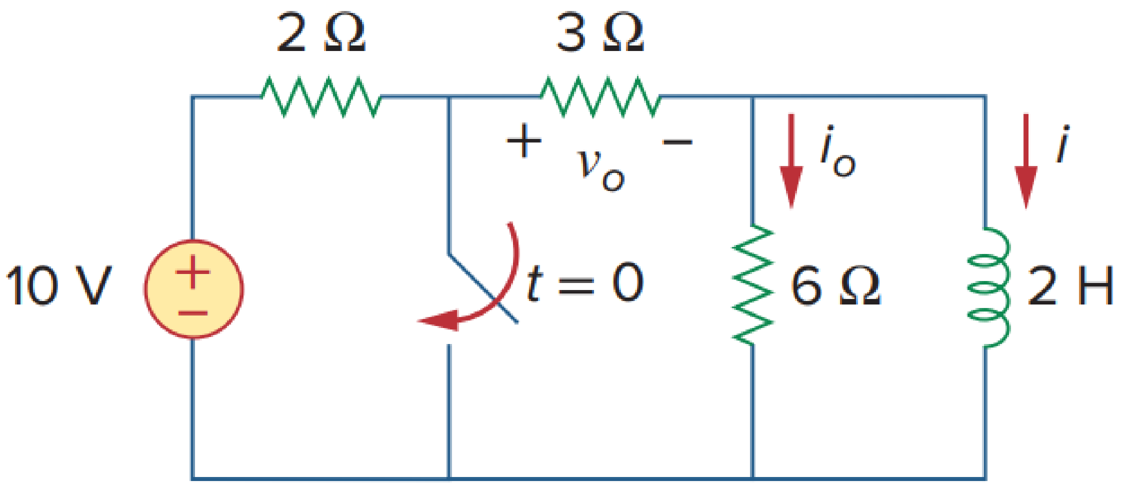
So,

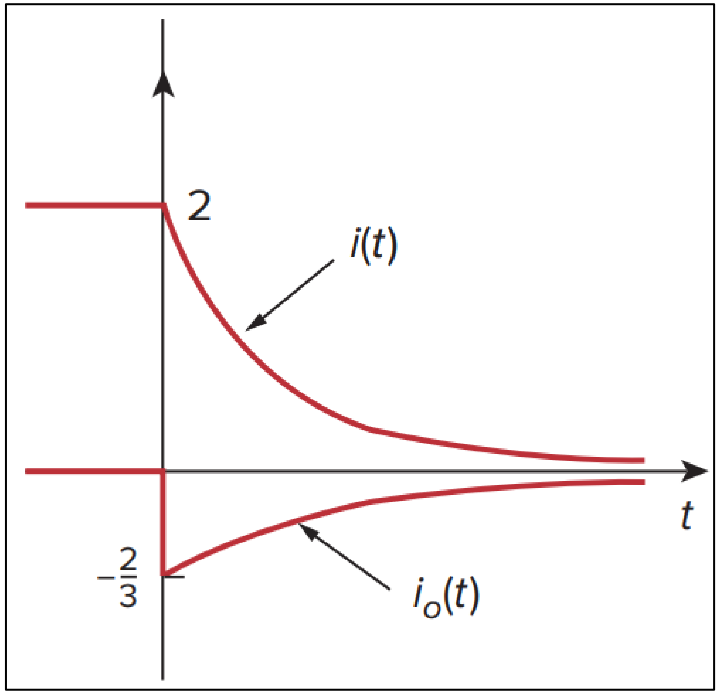
$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

$$= 4 + [6-4]e^{-t/0.1} = 4 + 2e^{-10t}\,(A), \quad t > 0$$

Problem 21

- Find i_0, v_0, and i for all time, assuming that the switch was open for a long time.





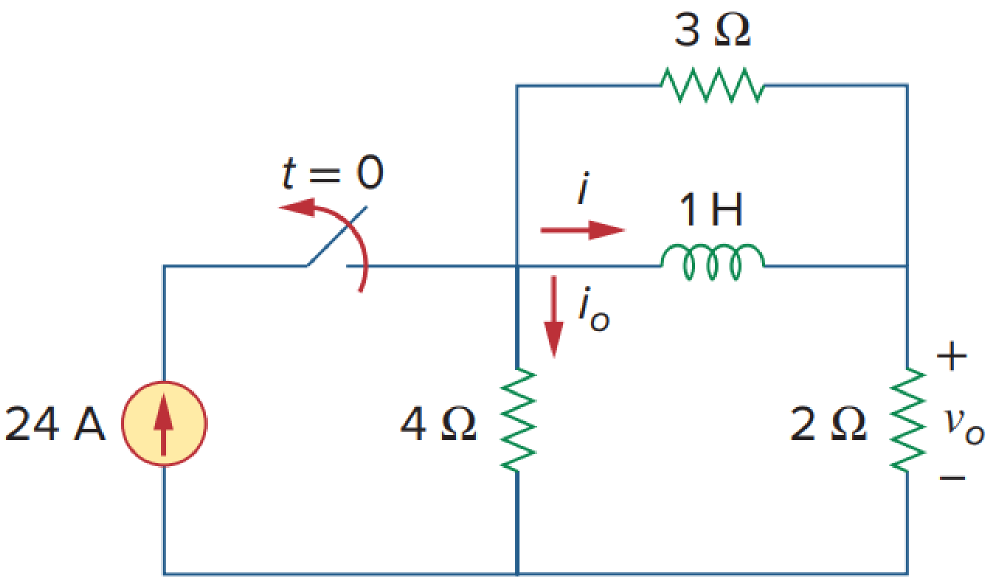
Ans:

$$i_o(t) = \begin{cases} 0 \text{ A}, & t < 0 \\ -\frac{2}{3}e^{-t} \text{ A}, & t > 0 \end{cases}, \quad v_o(t) = \begin{cases} 6 \text{ V}, & t < 0 \\ 4e^{-t} \text{ V}, & t > 0 \end{cases}$$

$$i(t) = \begin{cases} 2 \text{ A}, & t < 0 \\ 2e^{-t} \text{ A}, & t \geq 0 \end{cases}$$

Problem 22

- Determine i, i_0, and v_0 for all t in the circuit shown below. Assume that the switch was closed for a long time.



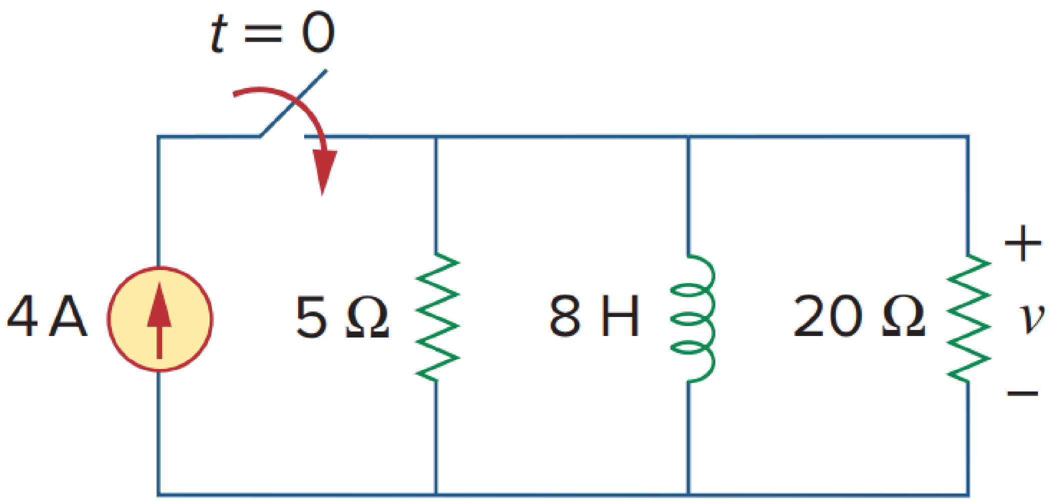
Ans:

$$i = \begin{cases} 16 \text{ A}, & t < 0 \\ 16e^{-2t} \text{ A}, & t \geq 0 \end{cases}, \qquad i_o = \begin{cases} 8 \text{ A}, & t < 0 \\ -5.333e^{-2t} \text{ A}, & t > 0 \end{cases},$$

$$v_o = \begin{cases} 32 \text{ V}, & t < 0 \\ 10.667e^{-2t} \text{ V}, & t > 0 \end{cases}$$

Problem 23

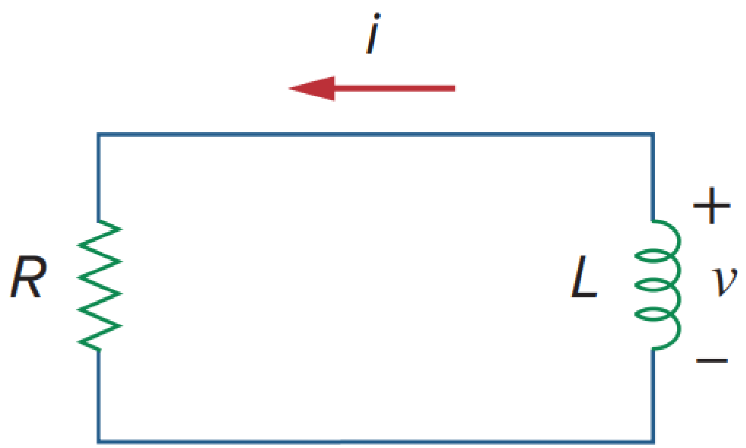
- Find $v(t)$ for $t > 0$.



Ans: $\boldsymbol{v(t) = 4(1 - e^{-t/2})\ V\ for\ t > 0}$

Problem 24

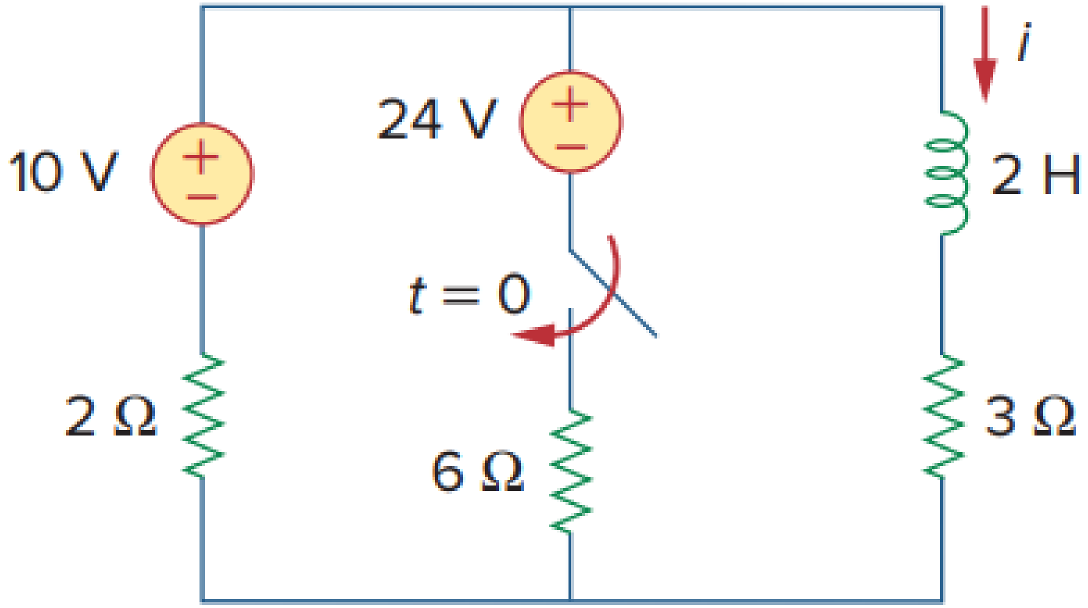
- For the circuit below, $v = 90e^{-50t}\ V$ and $i = 30e^{-50t}$ A for $t > 0$
  a) Find L and R.
  b) Determine the time constant.
  c) Calculate the initial energy in the inductor.
  d) What fraction of the initial energy is dissipated in 10 ms.



Ans: $\boldsymbol{R = 3\ \Omega;\ L = 60\ mH;\ \tau = 0.02\ s;\ w_L(0) = 27\ J;\ \% = 94.7}$

Problem 25

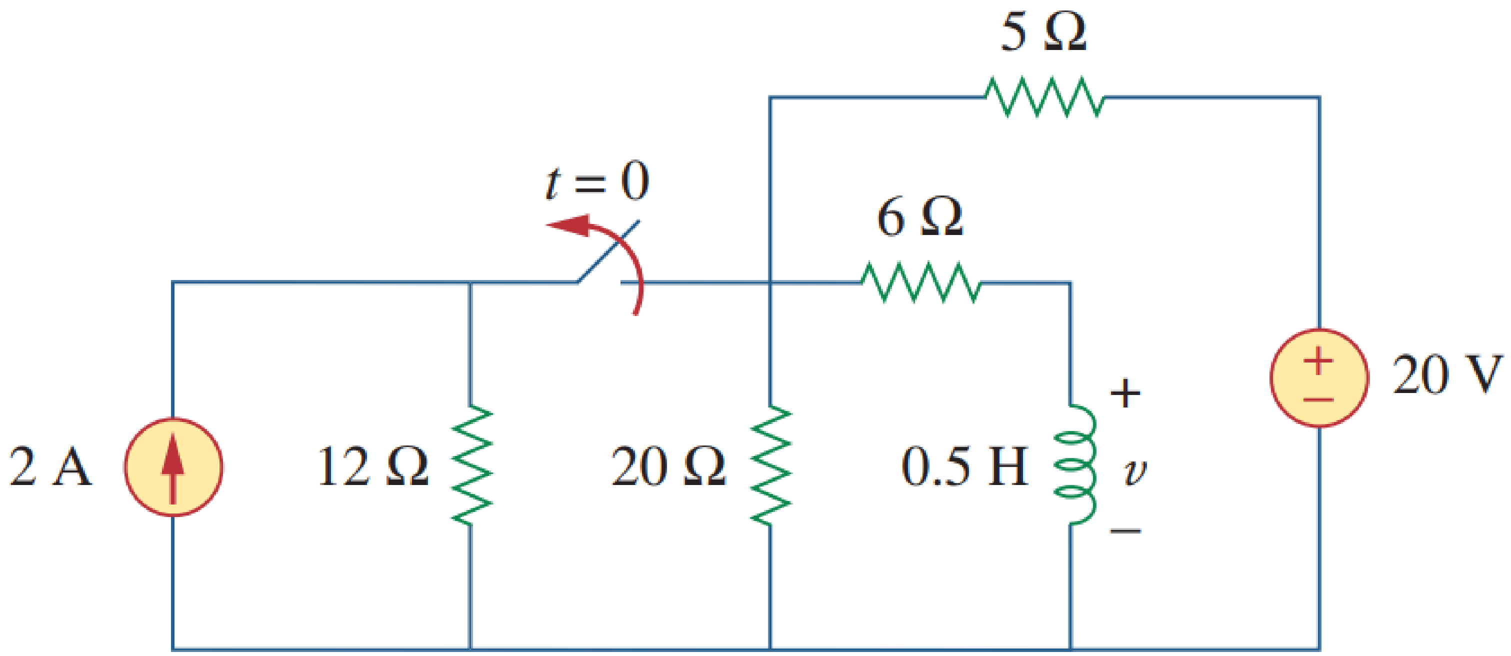
- Obtain the inductor current $i(t)$ for $t > 0$.



Ans: $i(t) = 3 - e^{-9t/4}\ A\ for\ t > 0$

Problem 26

- Find $v(t)$ for $t > 0$.



Ans: $\boldsymbol{v(t) = -4e^{-20t}\ V\ for\ t > 0}$

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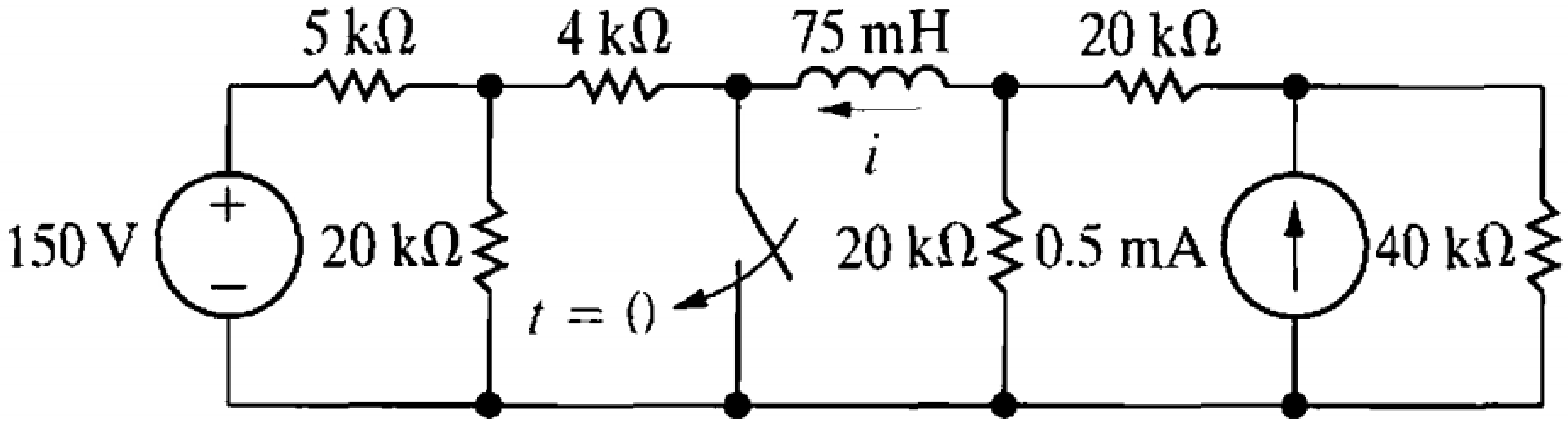
Problem 27

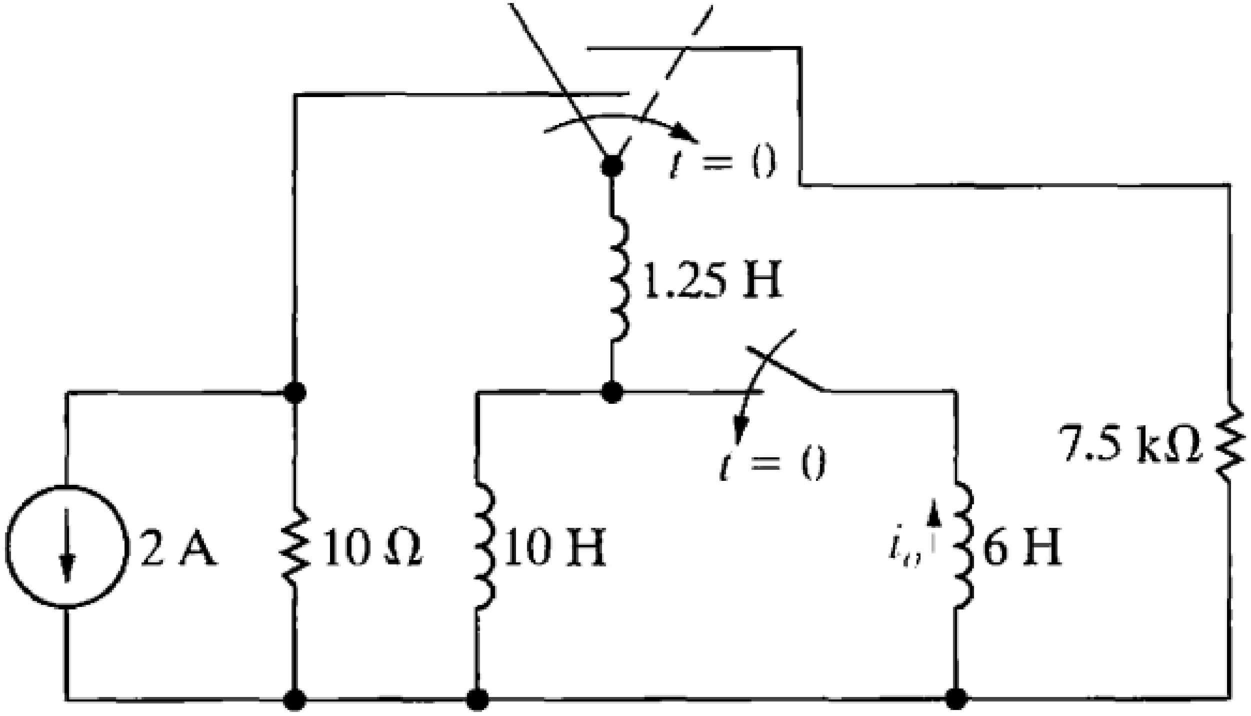
- The switch in the following circuit has been open for a long time, and it is closed at $t = 0$. Find $i(t)$ for $t > 0$.



Ans: $\boldsymbol{i(t) = 0.333 - 5.333e^{-200{,}000t}\ mA\ for\ t > 0}$

Problem 28

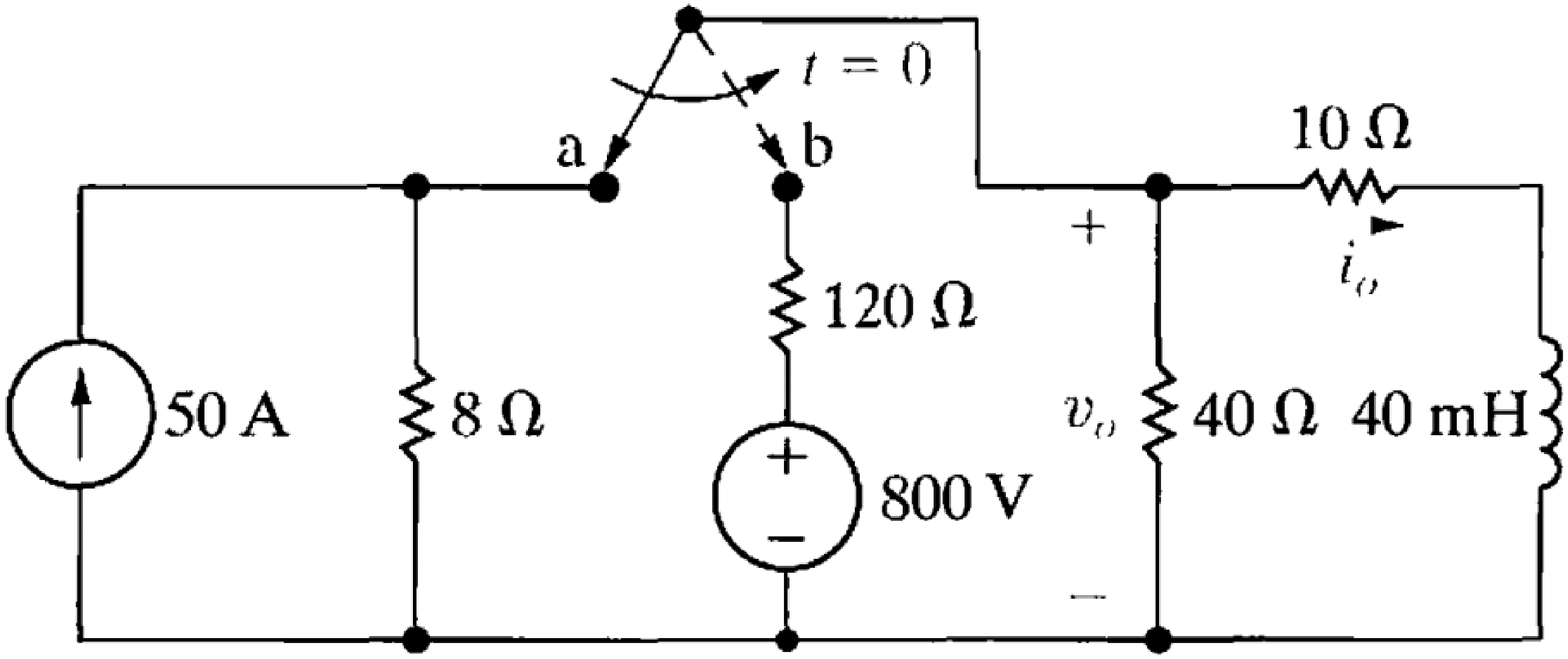
- The two switches in the following circuit operate simultaneously. Prior to $t = 0$ each switch has been in its indicated position for a long time. At t — 0 the two switches move instantaneously to their new positions. Determine $i_o(t)$ for $t > 0$.



Ans: $\boldsymbol{i_o(t) = 2e^{-1500t}\ A\ for\ t > 0}$

Problem 29

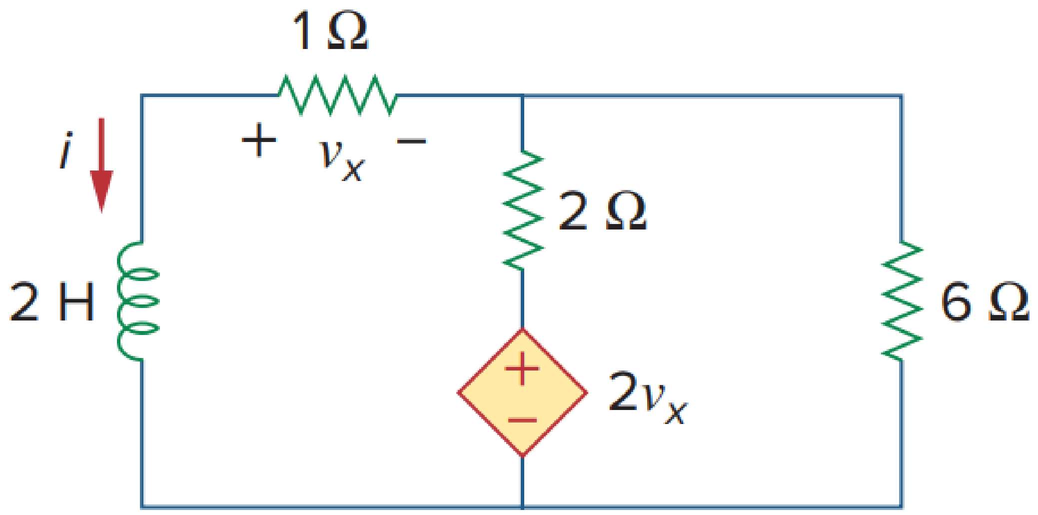
- The switch in the following circuit has been in position a for a long time, and it moves to position b at $t = 0$. Determine $i_o(t)$ and $v_o(t)$ for $t > 0$.



Ans: $\boldsymbol{i_o(t) = 5 + 15e^{-2000t}\ A, t > 0; v_o(t) = 50 - 450e^{-2000t}\ V, t > 0}$

Problem 30

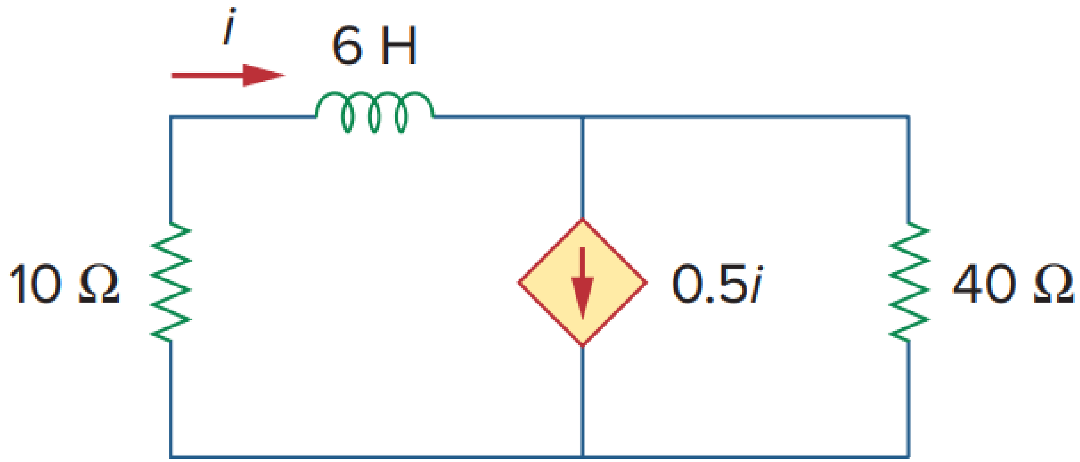
- Find i and v_x if $i(0) = 7\,A$.



Ans: $\boldsymbol{i(t) = 7e^{-2t}A;\ v_x(t) = -7e^{-50t}\ V}$

Problem 31

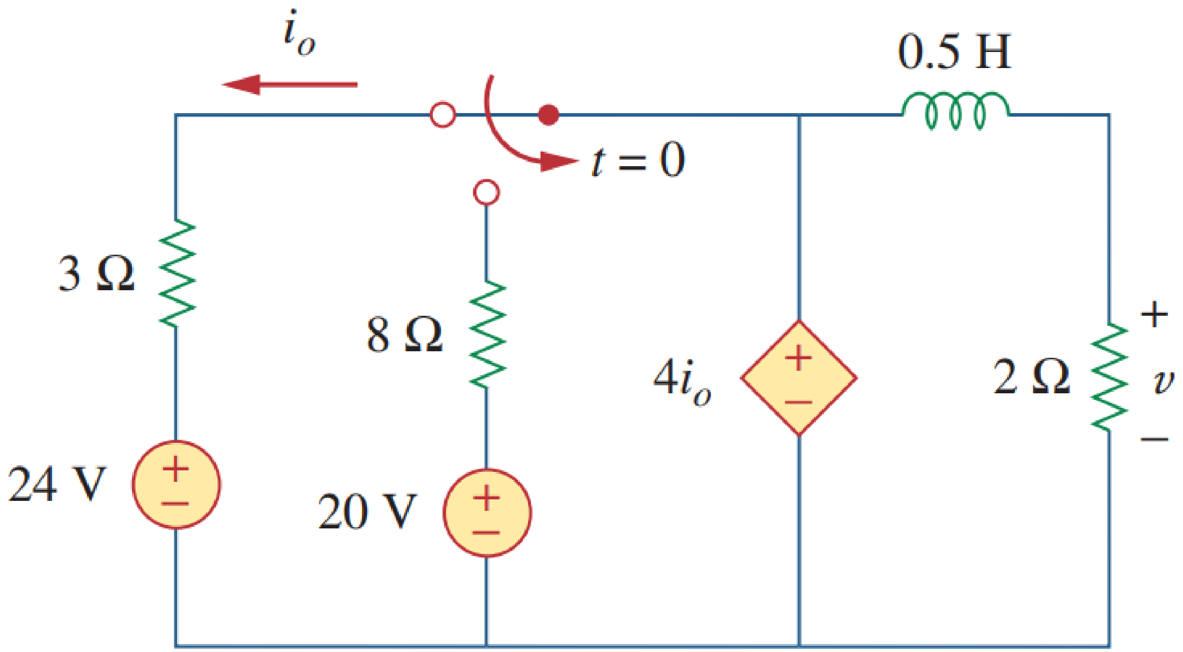
- Find $i(\mathrm{t})$ if $i(0) \ = 5\ A.$



$\underline{\text{Ans}}$: $\boldsymbol{i(t) \ = \ 5e^{-5t}\ A}$

Problem 32

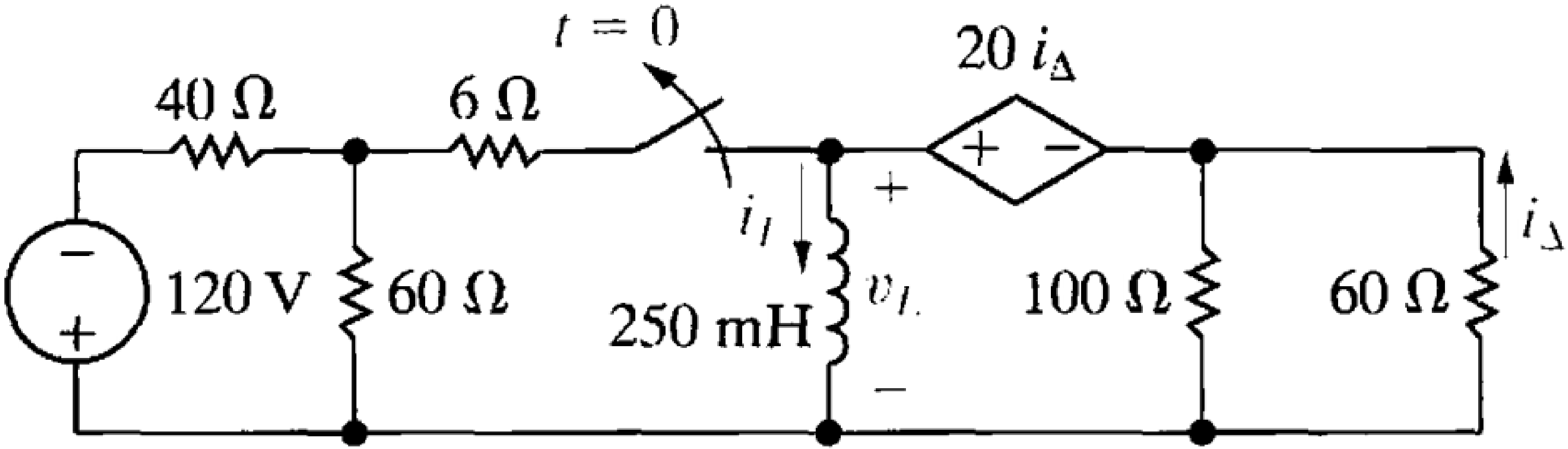
- Find $v(t)$ for both $t < 0$ and $t > 0$.



Ans: $v(t) = 96\,V\ for\ t < 0;\ v(t) = 96e^{-4t}\,V\ for\ t > 0$

Problem 33

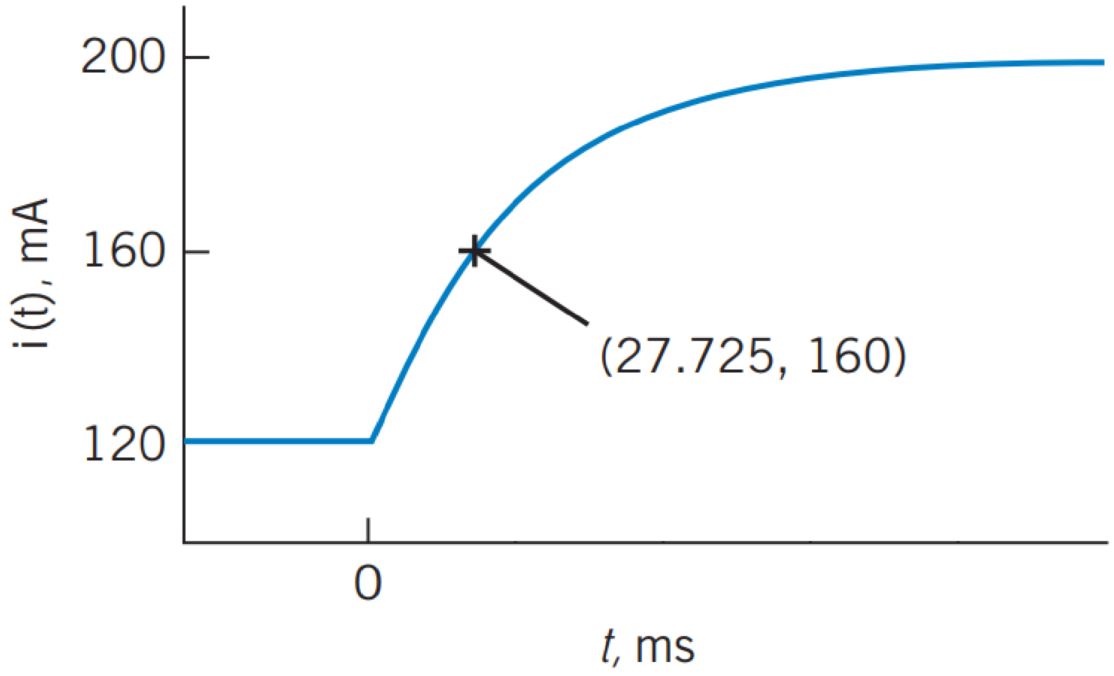
- Find $i_L(t)$, $v_L(t)$, and $i_\Delta(t)$ for $t > 0$.

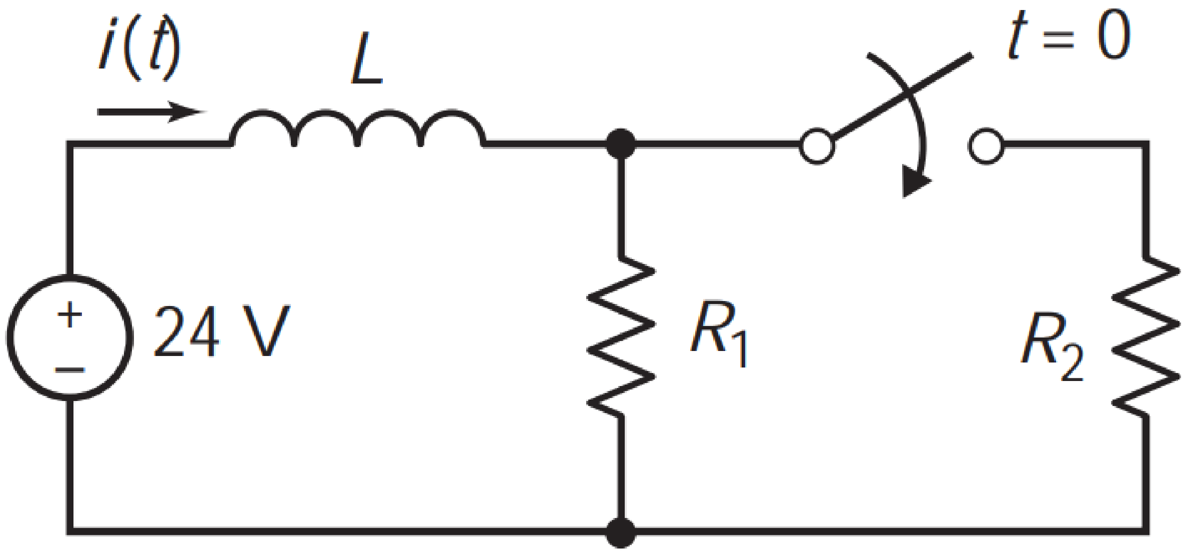


Ans: $i_L(t) = -2.4e^{-100t}\ A, t > 0$; $v_L(t) = -60e^{-100t}\ V, t > 0$; $i_\Delta(t) = -21.5e^{-100t}\ A, t > 0$

Problem 34

- The inductor current $i(t)$ has the following plot. Determine the values of L, R_1, and R_2.

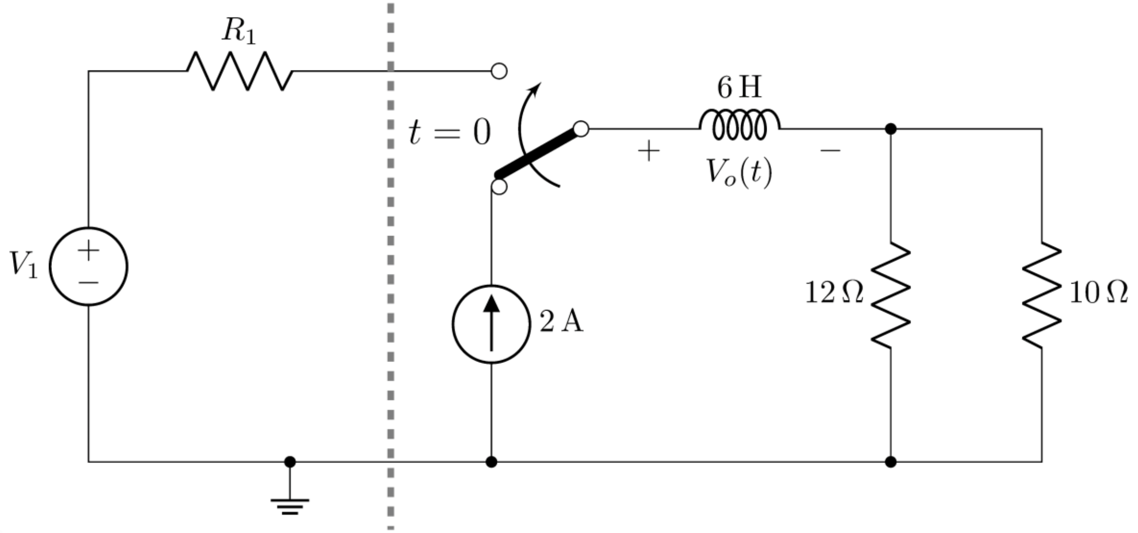




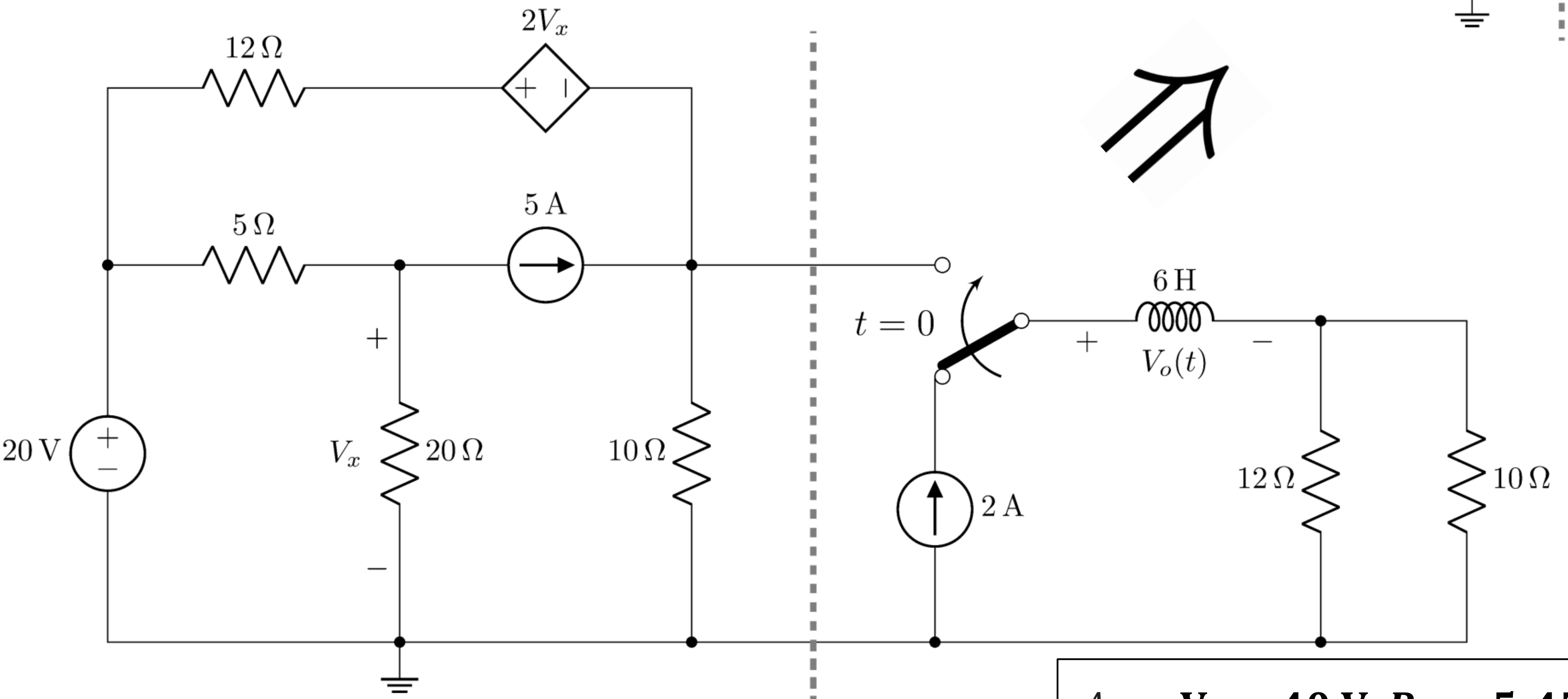
Ans: $\boldsymbol{L = 4.8\ H, R_1 = 200\ \Omega, R_2 = 300\ \Omega}$

Problem 35

- Reduce Circuit 1 so that it takes the form of Circuit 2. Determine V_1, and R_1.
- Find $V_o(t)$ for $t > 0$.



Circuit 2



Ans: $\mathbf{V_1 = 40\ V;\ R_1 = 5.45\ \Omega;\ V_o(t) = 18.22e^{-t/0.55}\ V}$

Thank you for your attention