Lecture 8

CSE250 - Circuits and Electronics

SUPERPOSITION PRINCIPLE

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Inspiring Excellence

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Linearity

- *Linearity* is the property of an element describing a linear relationship between cause and effect, that is, the response in any point in a network is proportional to the stimulus which is causing it. This property is a combination of both the *homogeneity* (scaling) property and the *additivity* property.
- Homogeneity property
  A system is homogeneous if scaling the input by a constant factor scales the output by the same factor. For a resistor, for example, Ohm's law relates the input i to the output v as $v = iR$. If the current is increased by a constant k, then the voltage increases correspondingly by k; that is, $kv = (ki)R$.
- Additivity or Superposition property
  A system is additive if the response to the sum of inputs equals the sum of the responses to each input applied separately. For example, if applying v_1 & v_2 separately to a resistor gives rise to currents i_1 & i_2 respectively, then applying $(v_1 + v_2)$ should give rise to the current $(i_1 + i_2)$.

Superposition in Linear Circuits

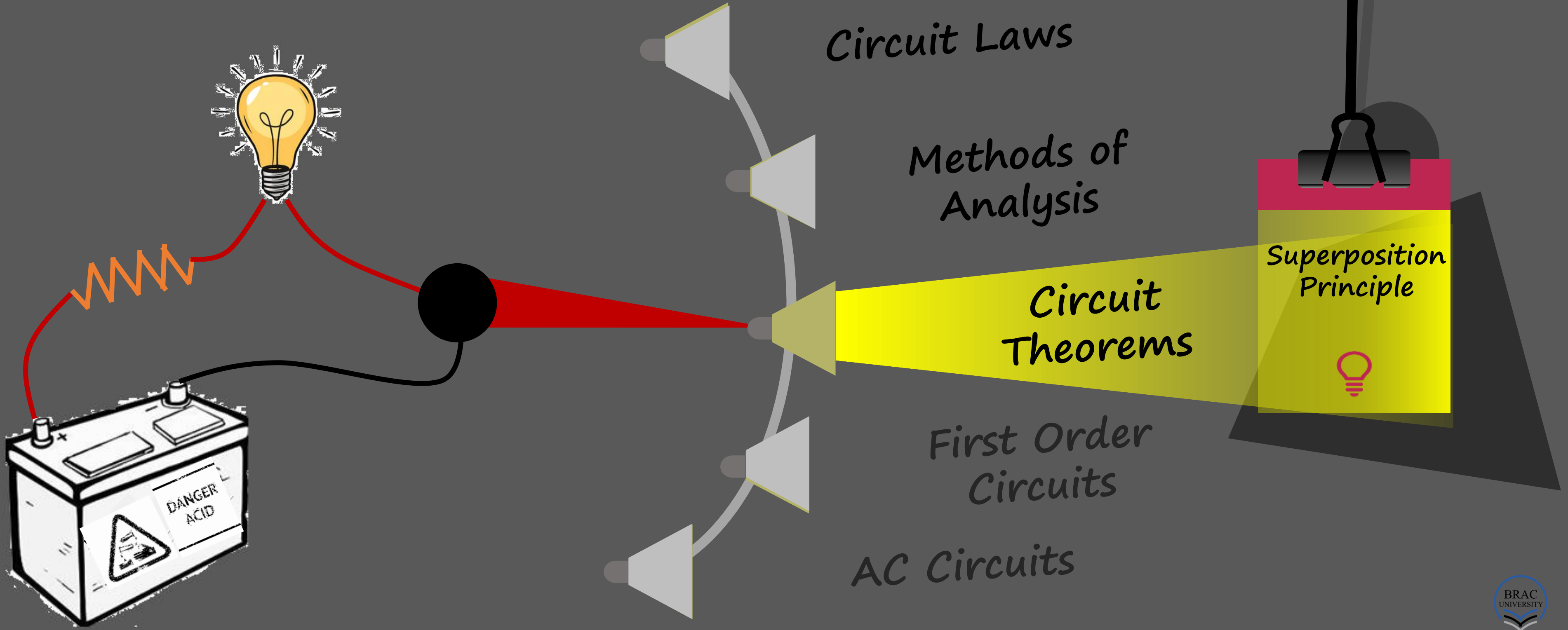
- For a linear function,

$$f(kx) = kf(x),$$

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

- A *linear circuit* is one whose input and output are related by a straight line. A linear circuit obeys the Superposition or Additivity property.
- Superposition is an extension of the linearity principle to the case of multiple excitation. When the superposition principle applies, the output response from a number of sources acting simultaneously is simply the sum of responses which would be produced by each of the sources acting alone with all of the other sources dead.
- The superposition theorem includes the linearity theorem as a special case since increasing the value of a stimulus is equivalent to adding a second stimulus to it.

Course Outline: broad themes



Superposition Principle

- The *superposition principle* states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.
- Keep in mind that superposition is based on linearity. For this reason, it is not applicable to the effect on power due to each source.

$$P_{Total}^2 \neq P_1^2 + P_2^2 + \ldots + P_N^2$$

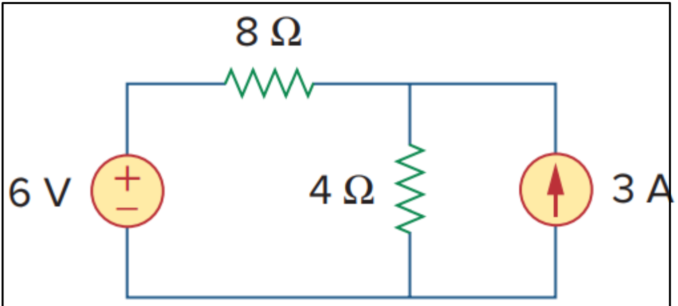
- If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

Steps to Apply Superposition Principle:

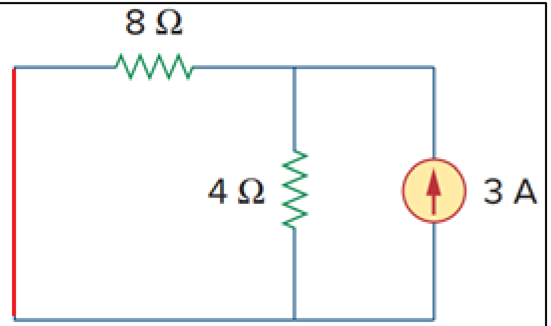
1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques covered in Chapters 2 and 3.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Killing independent sources

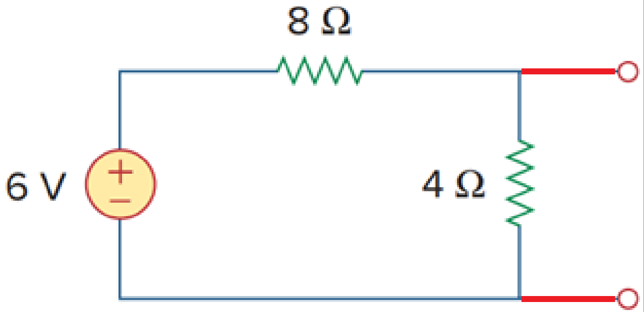
- In superposition principle, we consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by $0\ V$ (or a short circuit), and every current source by $0\ A$ (or an open circuit).
- Dependent sources are left intact because they are controlled by circuit variables.



Original circuit



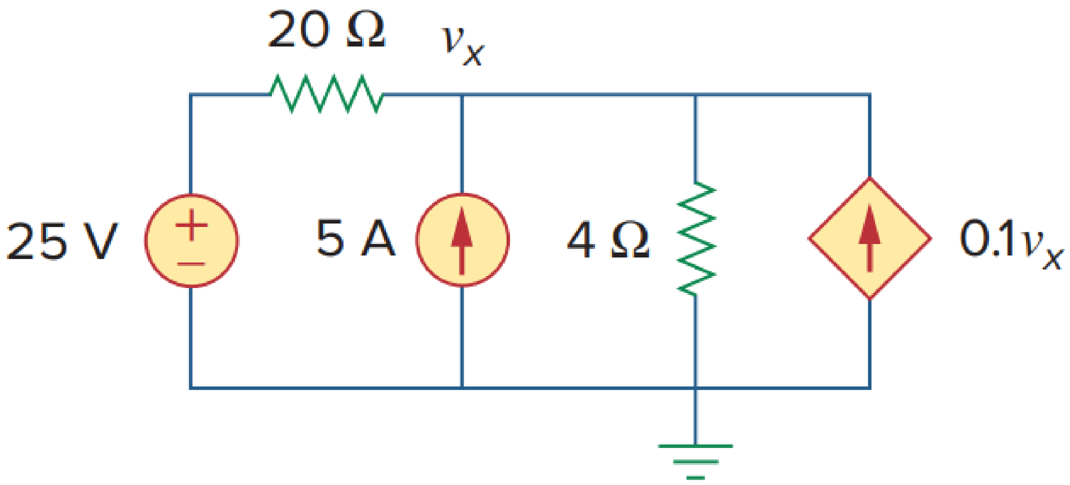
The voltage source has been replaced with a short circuit



The current source has been replaced with an open circuit

Example 1

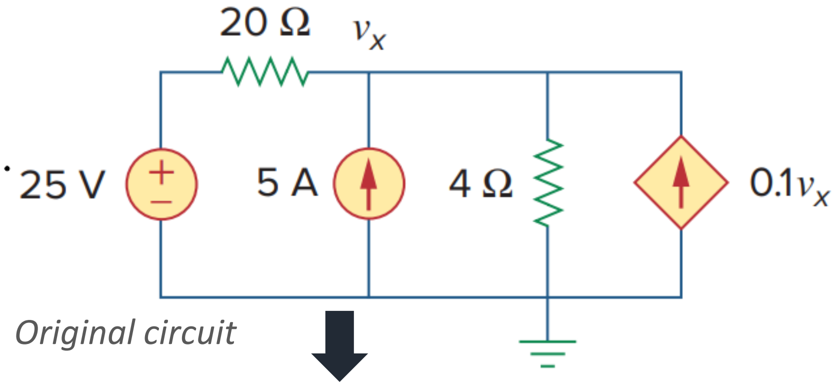
- Use Superposition Principle to find v_x.



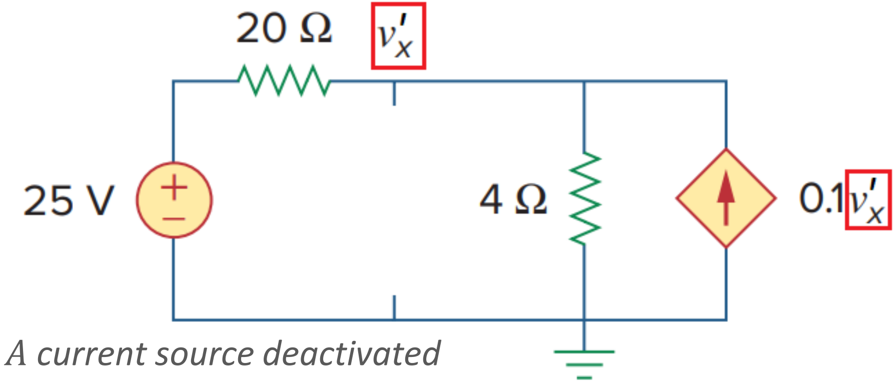
There are two independent and one dependent sources. The principle requires us to determine the individual contributions of the two independent sources to the node voltage v_x. If v_x' and v_x'' are the contributions from the $25\ V$ voltage source and $5\ A$ current source respectively, then

$$v_x = v_x' + v_x''$$

Example 1: 25 V source is active



Original circuit



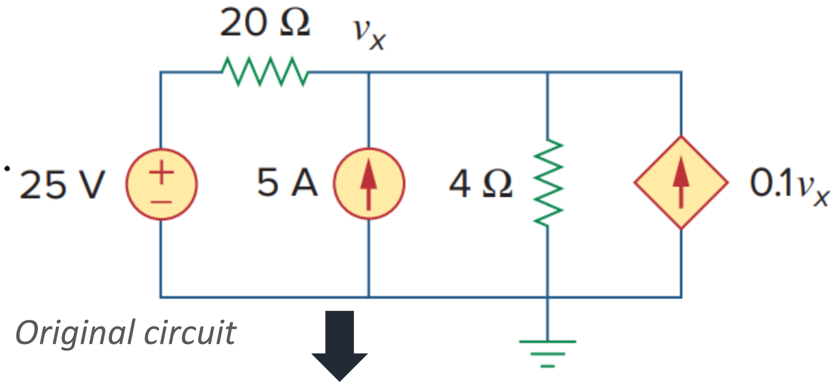
5 *A current source deactivated*

- The $5\ A$ current source has been replaced by an open circuit. The notation v_x is replaced by v_x'.
- Different circuit solving techniques (nodal analysis or mesh analysis or source transformation or voltage division) can be applied to solve for v_x'. Nodal analysis may be the easiest one.
- KCL at the node v_x',

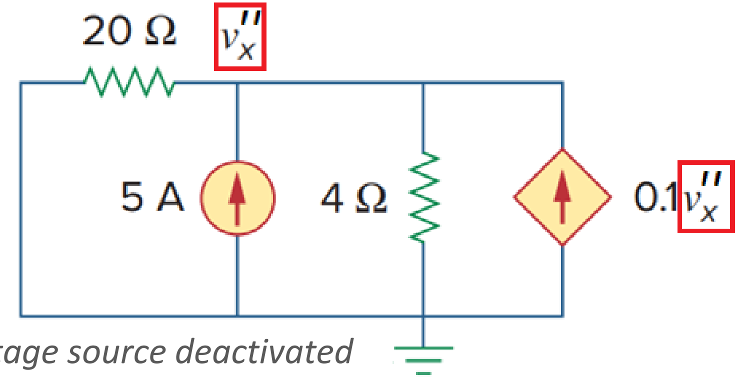
$$\frac{v_x' - 25}{20} + \frac{v_x'}{4} = 0.1v_x'$$

Simplification yields, $v_x' = 6.25\ V$

Example 1: 5 A source is active



Original circuit



25 V voltage source deactivated

- The 25 V voltage source has been replaced by a short circuit. The notation v_x is replaced by v_x''.
- KCL at the node v_x'',

$$\frac{v_x''}{20} + \frac{v_x''}{4} = 5 + 0.1v_x''$$

Simplification yields, $v_x'' = 25\ V$

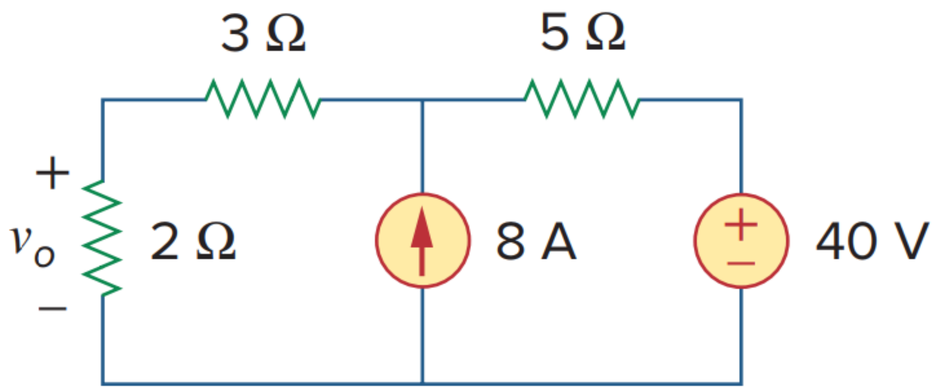
So, according to the Superposition Principle,

$$v_x = v_x' + v_x''$$

$$\Rightarrow \quad v_x = 6.25 + 25 = 31.25\ V$$

Problem 1

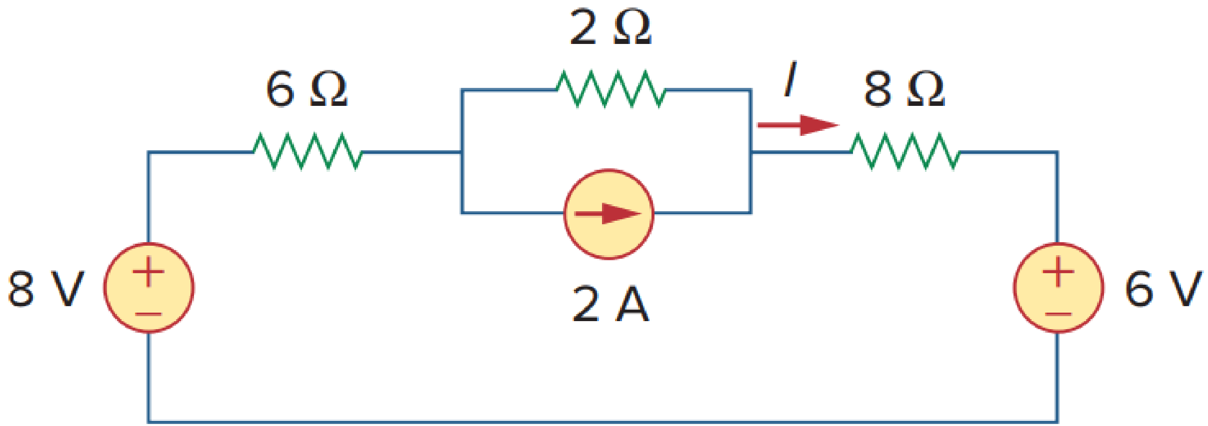
- Using the Superposition Theorem, find v_o.



Ans: $\boldsymbol{v_0 = 16\ V}$

Problem 2

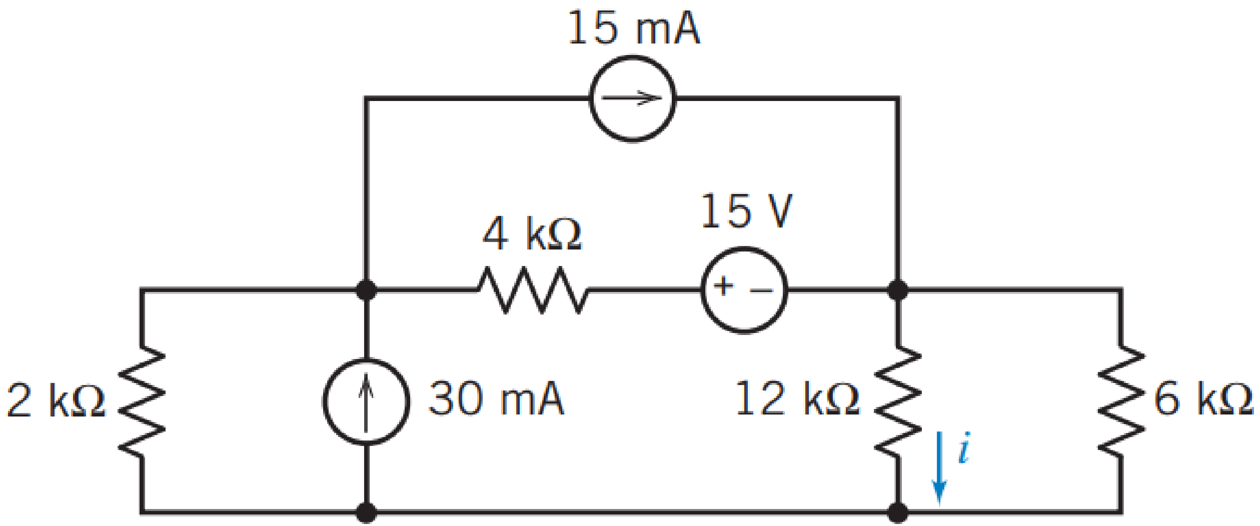
- Find I in the circuit below using the Superposition Principle.



Ans: $\boldsymbol{i_0} = \mathbf{0.375}\ \boldsymbol{A}$

Problem 3

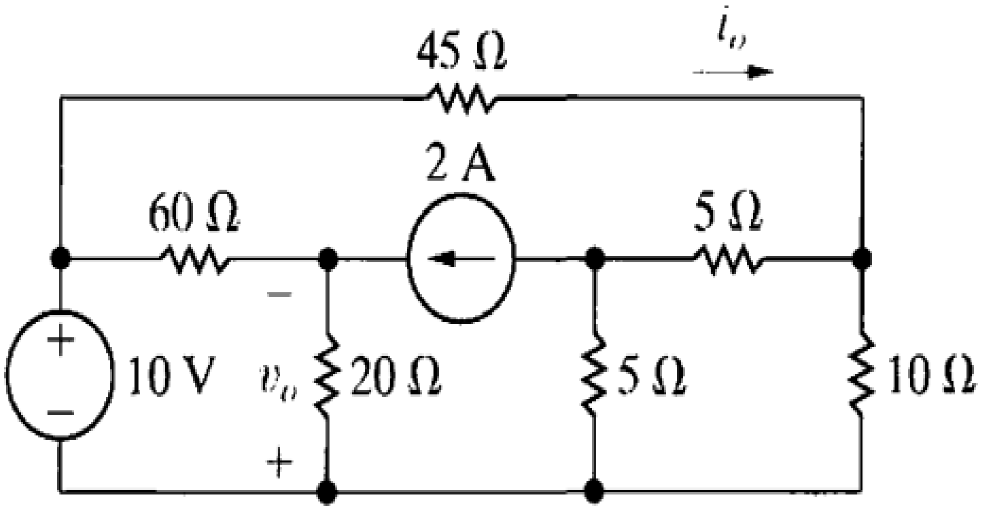
- Use Superposition Principle to find i in the circuit shown below.



Ans: $\boldsymbol{i} = \mathbf{2 + 2 - 0.5 = 3.5}\ \boldsymbol{mA}$

Problem 4

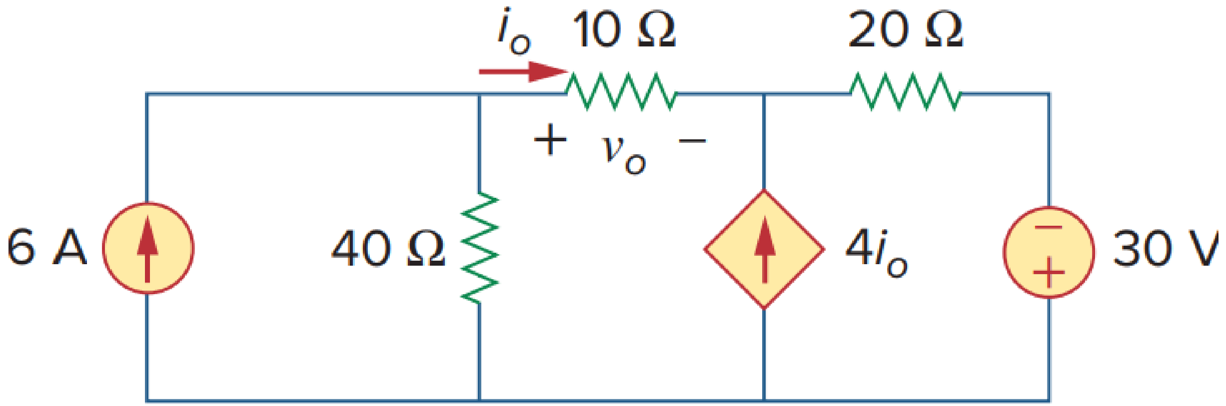
- Use Superposition Principle to solve for i_0 and v_0.



Ans: $\boldsymbol{i_0 = 0.2 + 0.1 = 0.3\ A;\ v_0 = -2.5 - 30\ V = -32.5\ V}$

Problem 5

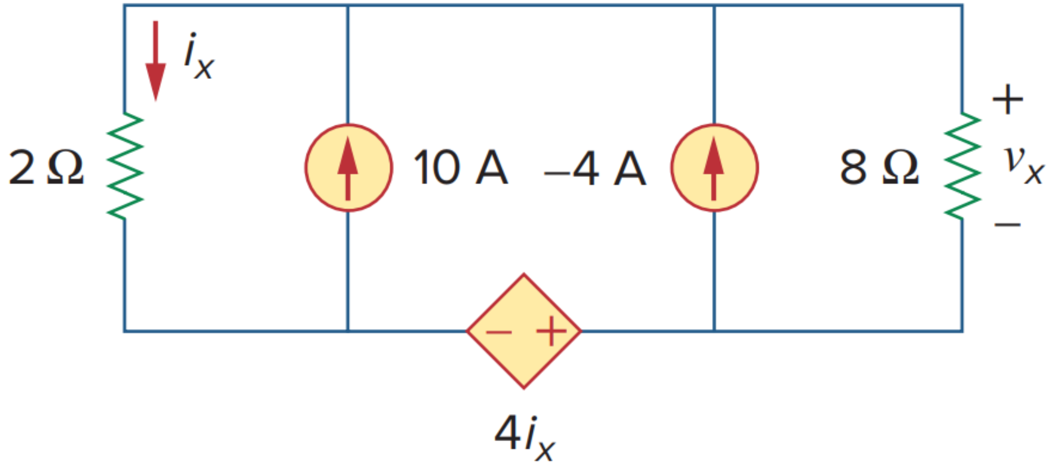
- Use the Superposition Principle to find i_o and v_o.



Ans: $\boldsymbol{i_0 = 1.8\,A;\ v_0 = 18\,V}$

Problem 6

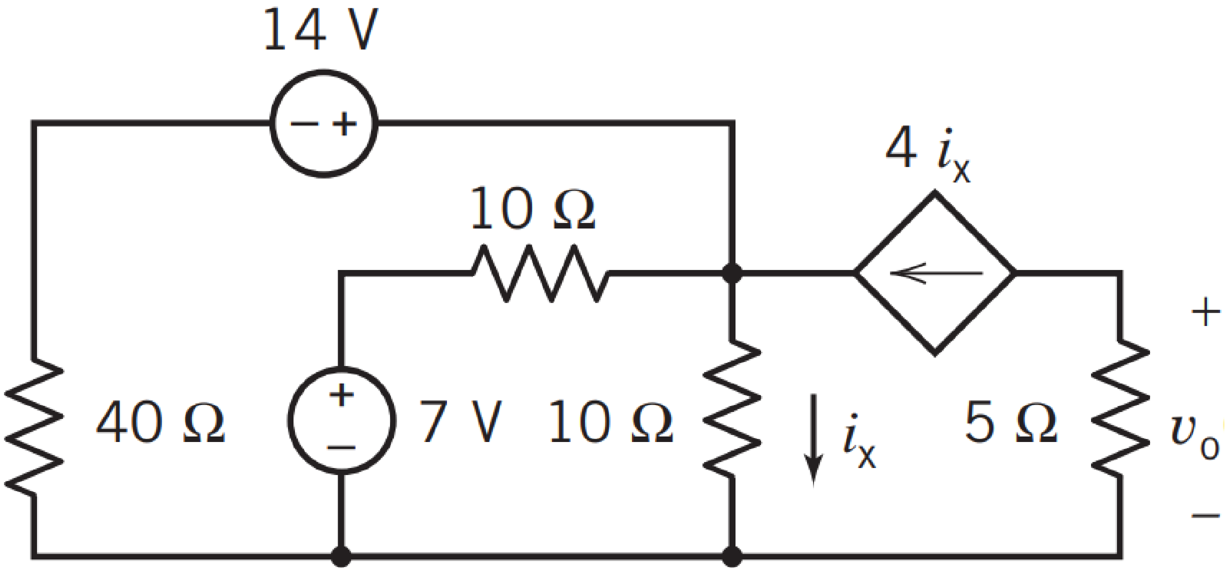
- Use Superposition Principle to solve for v_x.



Ans: $\boldsymbol{v_x = -16\ V}$

Problem 7

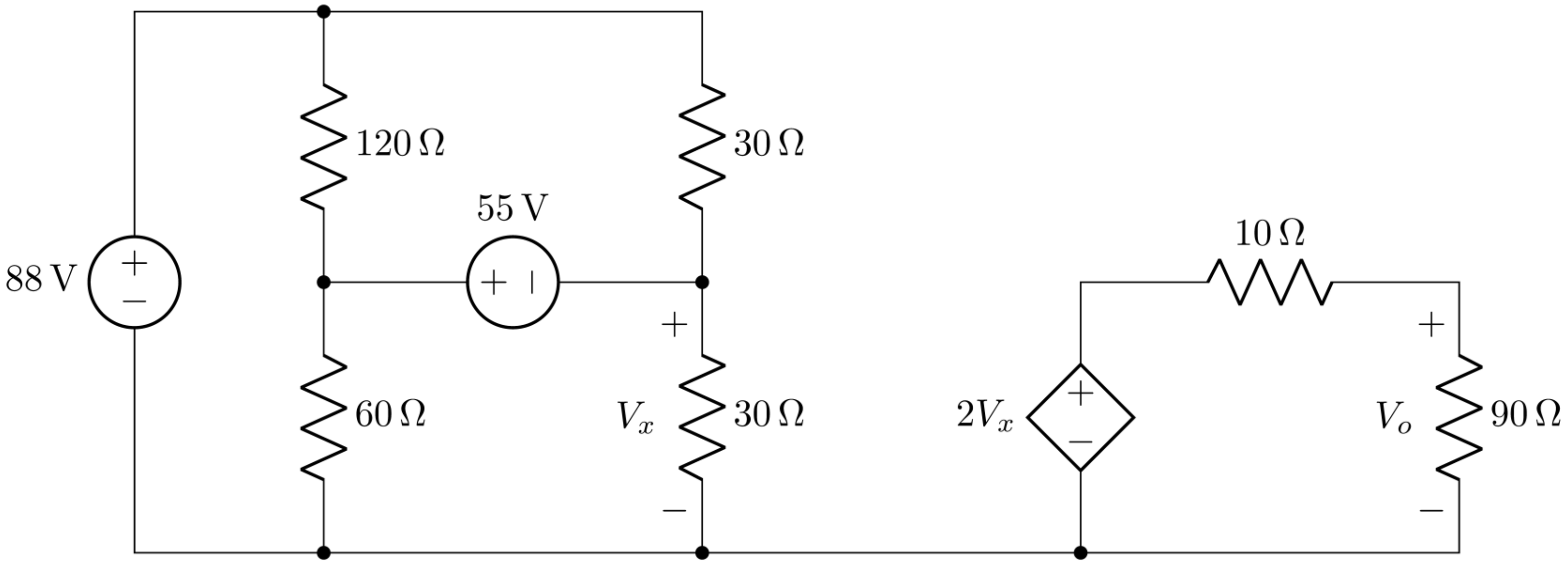
- Use Superposition Principle to solve for v_o.



Ans: $v_o = 8 + 4 = 12\ V$

Problem 8

- Use Superposition Principle to solve for V_o.



<u>Ans</u>: $\boldsymbol{V_o = -27V + 72V = 45\ V}$

Practice Problems

- Additional recommended practice problems: here
- Other suggested problems from the text book: here

Thank you for your attention