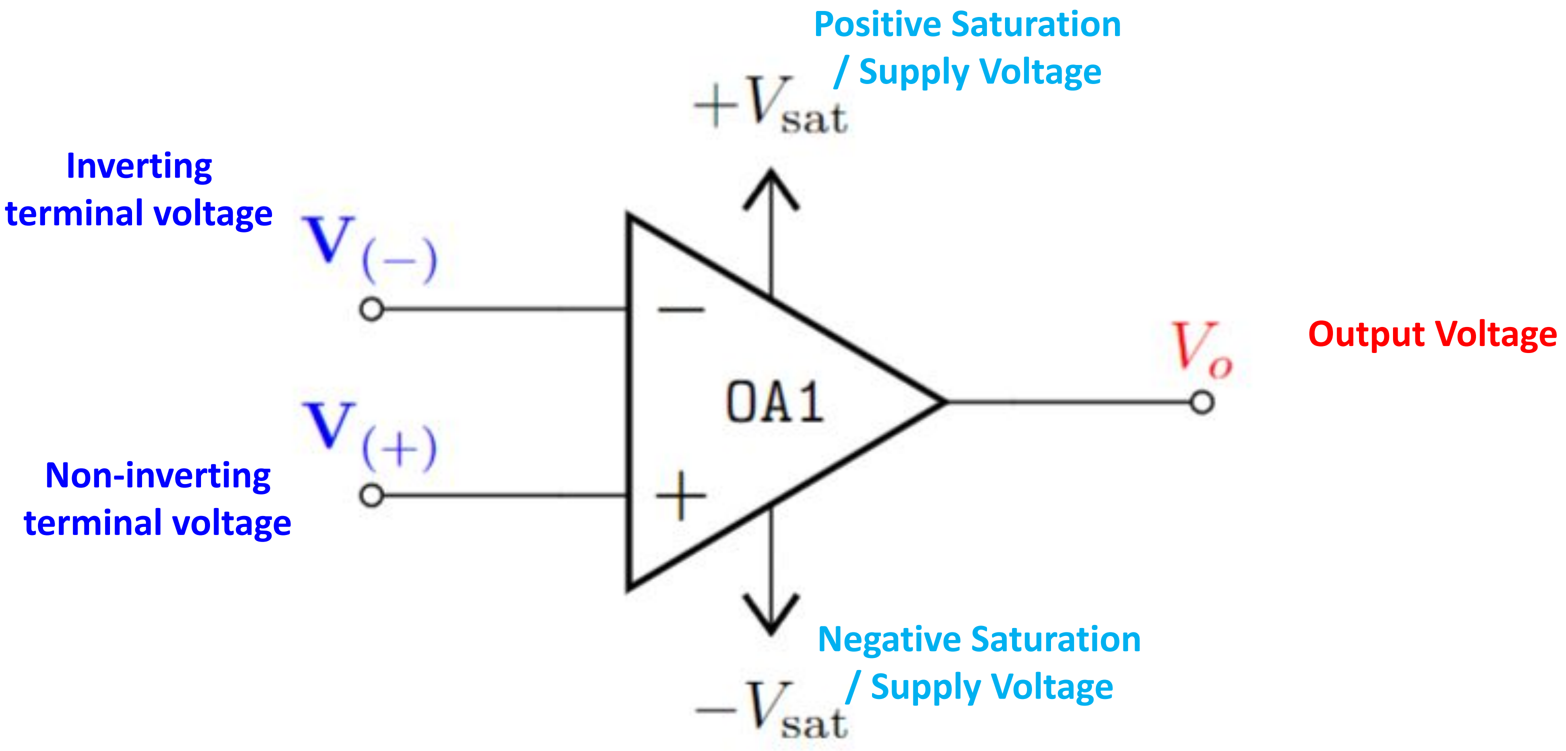
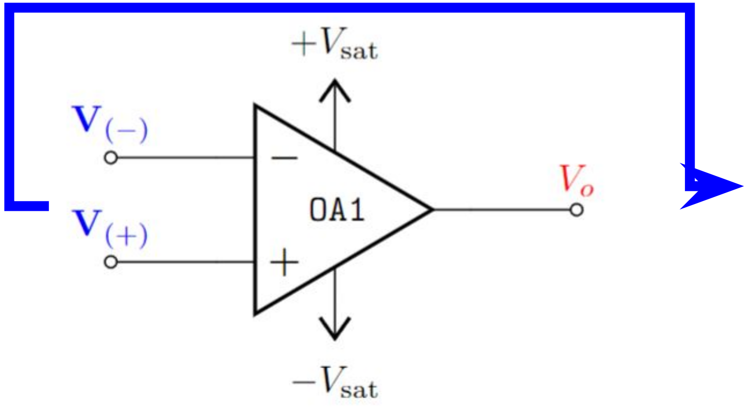
Lecture-2
Op-amps

Op-Amp: Circuit Symbols and terminal



Op-Amp: Circuit Symbols and terminal

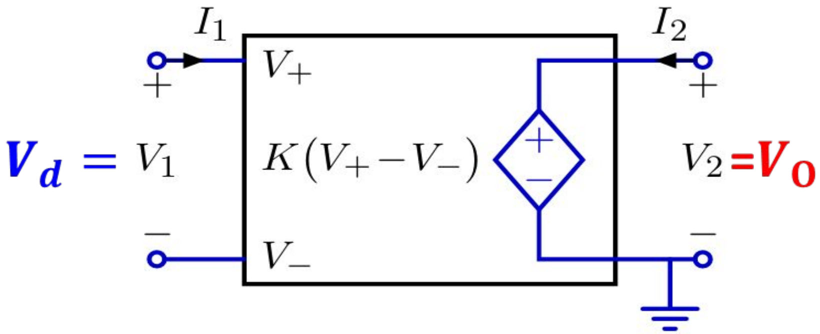


$$V_d = V_{(+)} - V_{(-)}$$

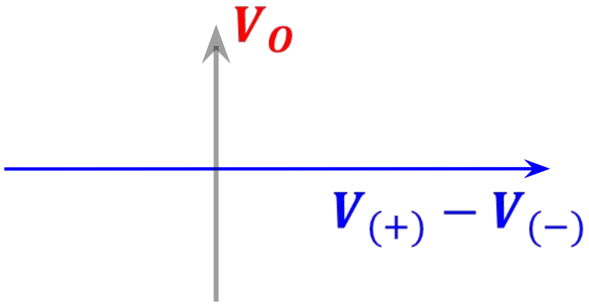
Difference Amplifier –

Amplifies the voltage difference between two terminals.

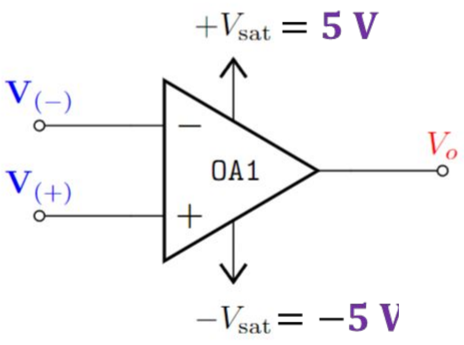
Differential input voltage



Voltage Transfer Characteristics (VTC)



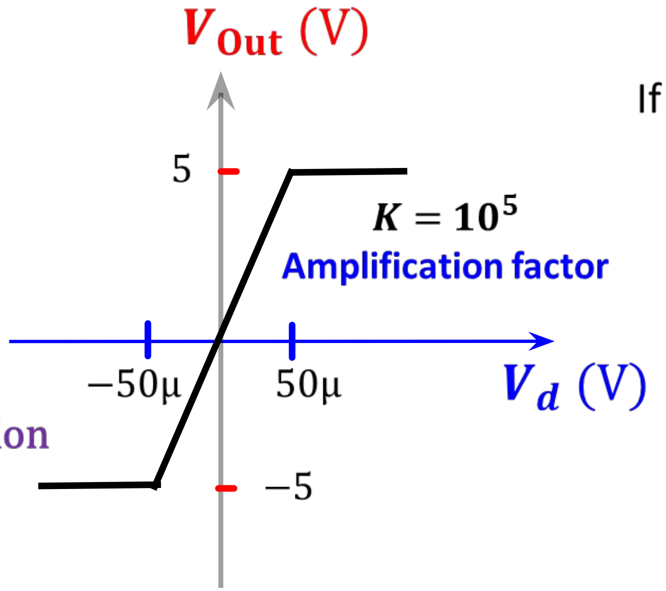
Op-Amp: VTC - Saturation



Non-Linear characteristics

$\boldsymbol{V_O = K \cdot V_{In}}$: **When** $\boldsymbol{-5\ \mathrm{V} < V_O < 5\ \mathrm{V}}$

$\boldsymbol{K \rightarrow 10^5}$**: Gain / Amplification**



If $\boldsymbol{V_d > 50\mu}$: Positive Saturation

$\Rightarrow \boldsymbol{V_O = 5}$

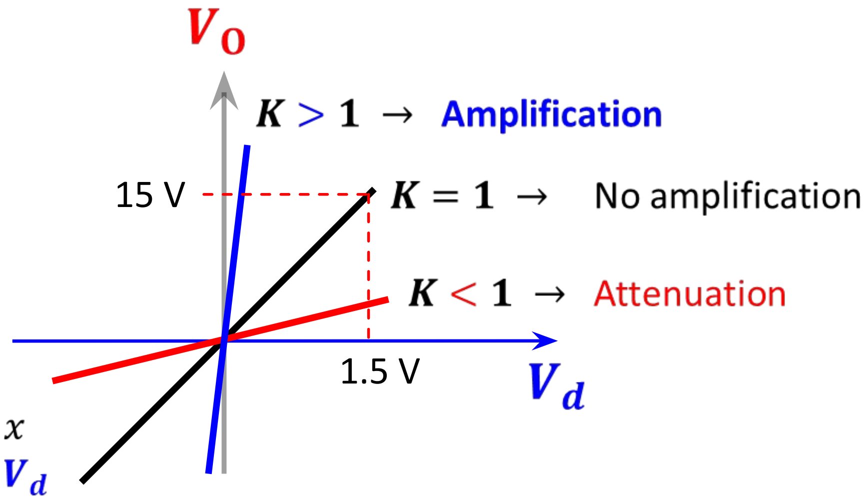
If $\boldsymbol{V_d < -50\mu}$: Negative Saturation

$\Rightarrow \boldsymbol{V_O = -5}$

Op-Amp: VTC – Linear Amplification

Voltage Transfer Characteristics (VTC)

- Shows how the output voltage varies with the input voltage $V_O(V_d)$
- y −axis → $\boldsymbol{V_O}$
- x −axis → $\boldsymbol{V_d}$
- Slope: $\boldsymbol{K} = \frac{\Delta \boldsymbol{V_O}}{\Delta \boldsymbol{V_d}} = \frac{\boldsymbol{V_O}}{\boldsymbol{V_d}}$



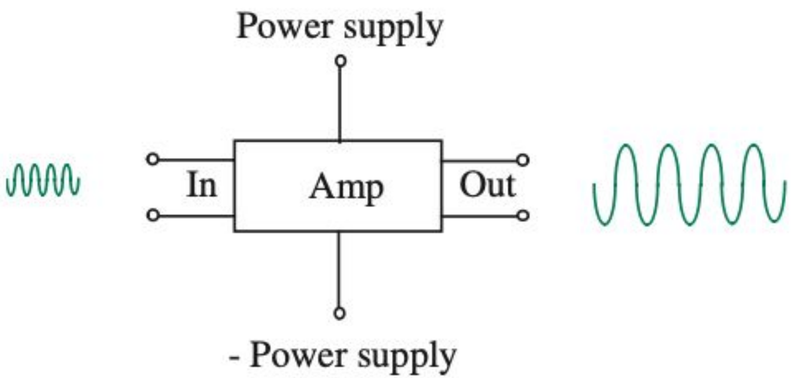
If the line passes through origin:

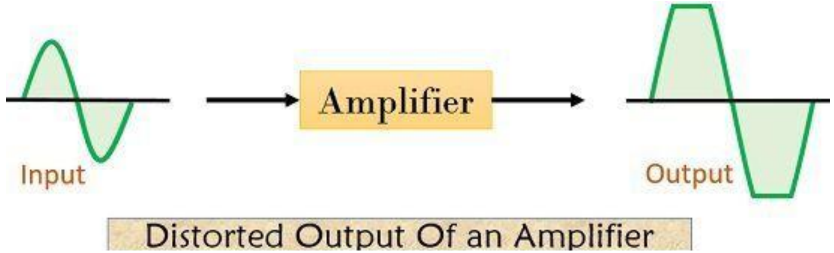
$$y = \boldsymbol{K} \cdot x$$
$$\boldsymbol{V_O} = \boldsymbol{K} \cdot \boldsymbol{V_d}$$

LINEAR AMPLIFICATION

Op-Amp: VTC – Linear Amplification

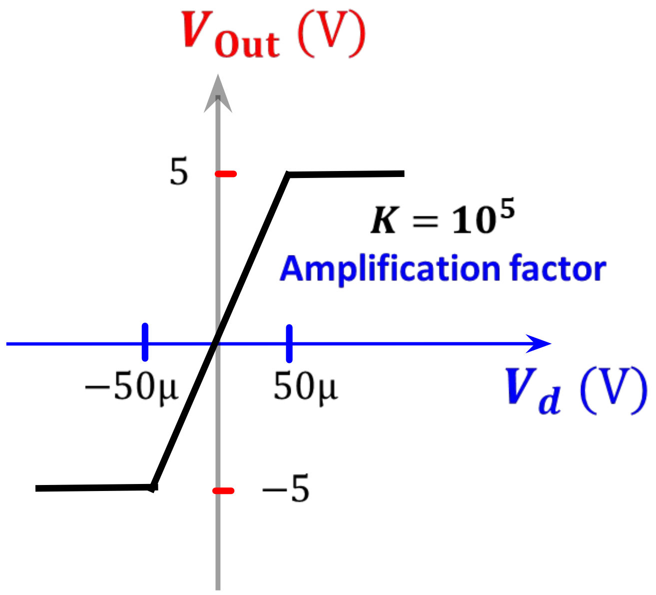
- **Linear Amplification** only takes place within a **valid input range.**
- Otherwise output will be distorted - - Saturation

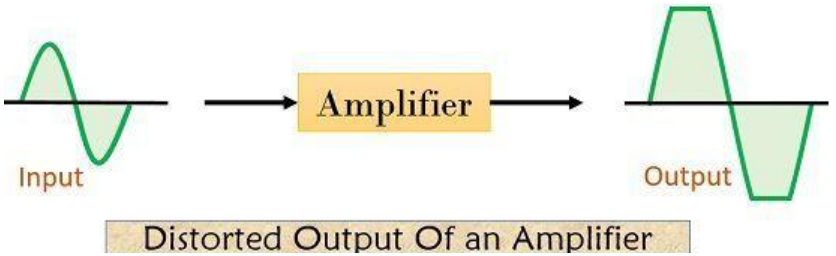


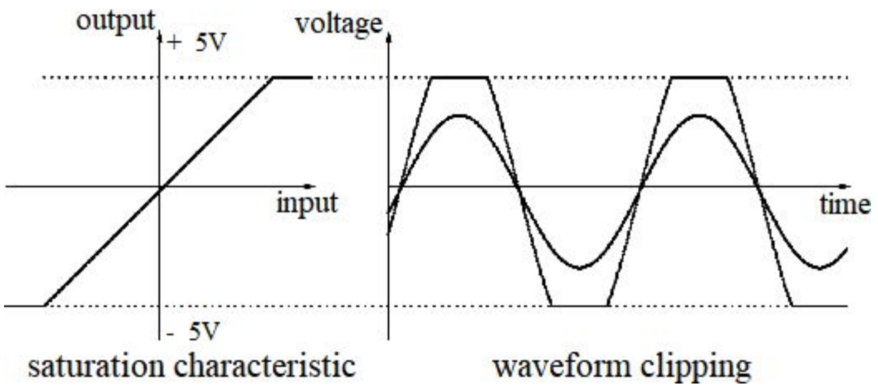


The limiting factor of **linear amplification** is determined by the **power supply** to the amplifier

Op-Amp: VTC - Saturation







Op-Amp: VTC - Summary

Voltage Transfer Characteristics (VTC)

Positive saturation:

If $V_d > \frac{+V_{sat}}{A}$: Positive

$\Rightarrow V_O = +V_{sat}$

Linear Region

$V_O = AV_d$: When V_d is very small

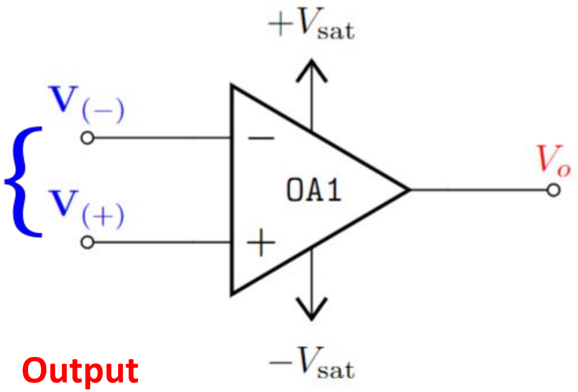
$-V_{sat} < V_o = AV_d < +V_{sat}$

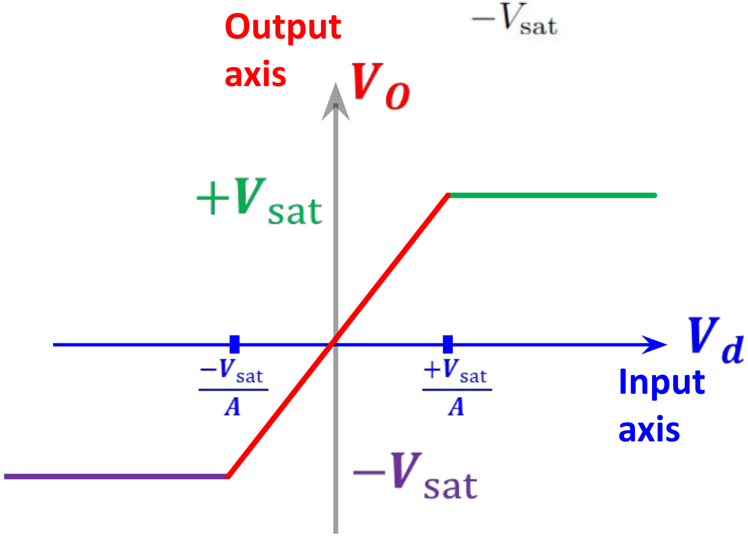
Negative saturation:

If $V_d < \frac{-V_{sat}}{A}$: Negative

$\Rightarrow V_O = -V_{sat}$

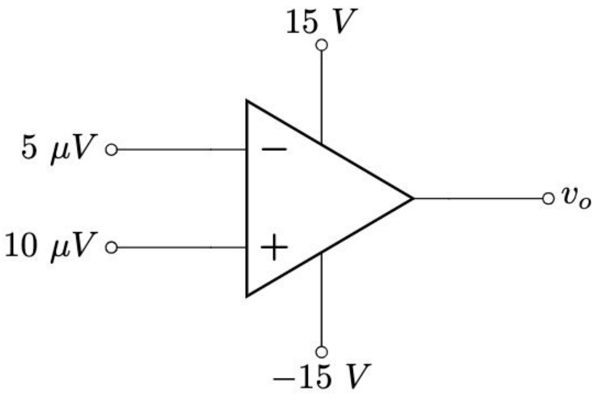
$V_d = V_{(+)} - V_{(-)}$



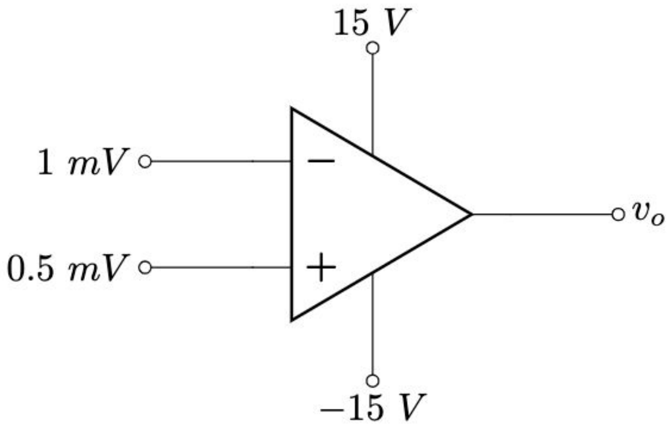


Op-Amp: Examples

Find v_0



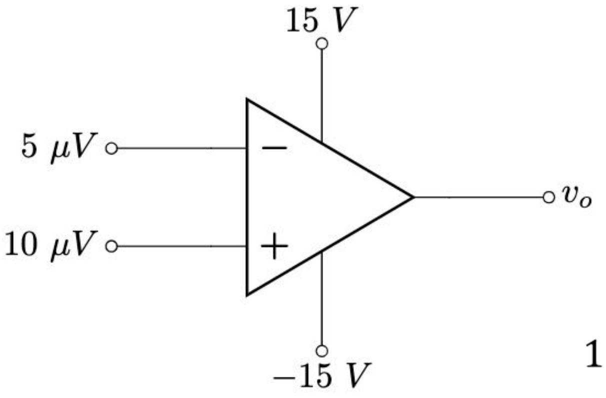
$A = 2 \times 10^5$



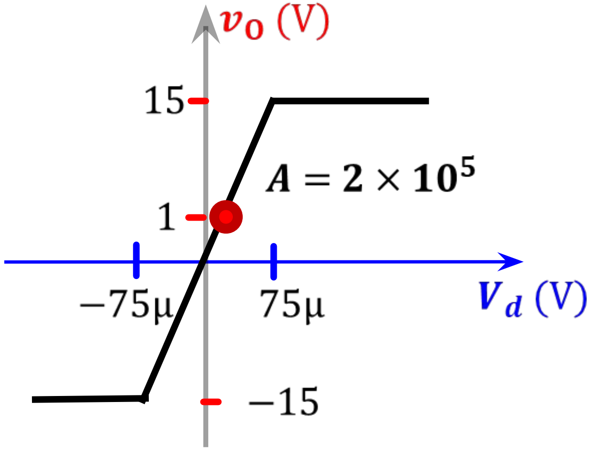
$A = 2 \times 10^5$

Example 1

- Find v_o



$A = 2 \times 10^5$



Solution:

$$V_d = V_{(+)} - V_{(-)} = (10 - 5)\ \mu\text{V} = 5\ \mu\text{V}$$

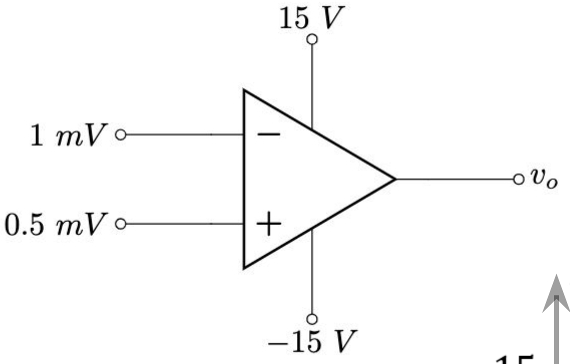
$$AV_d = (2 \times 10^5) \times (5 \times 10^{-6})\ \text{V} = 1\ \text{V}$$

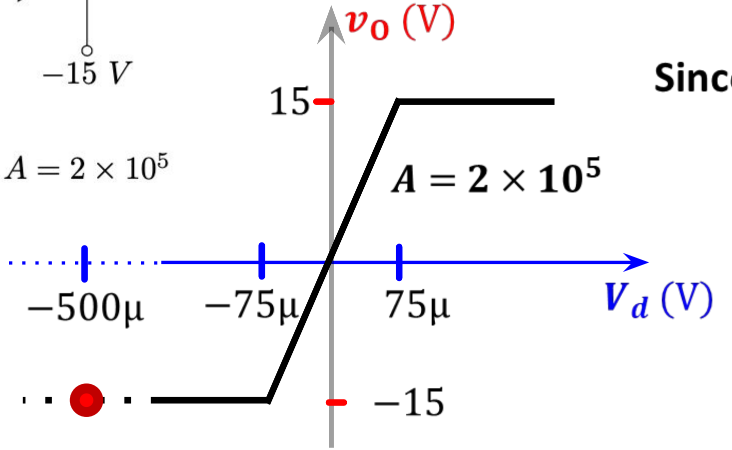
Since $-15\ \text{V} < AV_d < +15\ V$

$$V_O = AV_d = 1\ \text{V}$$

Example 2

- Find v_o





Solution:

$$V_d = V_{(+)} - V_{(-)} = (0.5 - 1)\ \mathrm{mV} = -0.5\ \mathrm{mV}$$

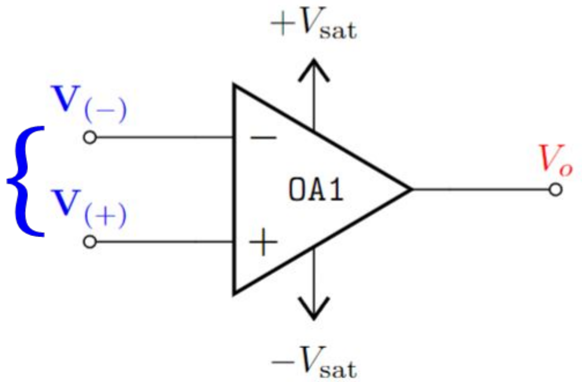
$$AV_d = \left(2 \times 10^5\right) \times \left(-0.5 \times 10^{-3}\right) \mathrm{V} = -100\ \mathrm{V}$$

Since $AV_d < -15\ \mathrm{V}$ **(Negative saturation)**

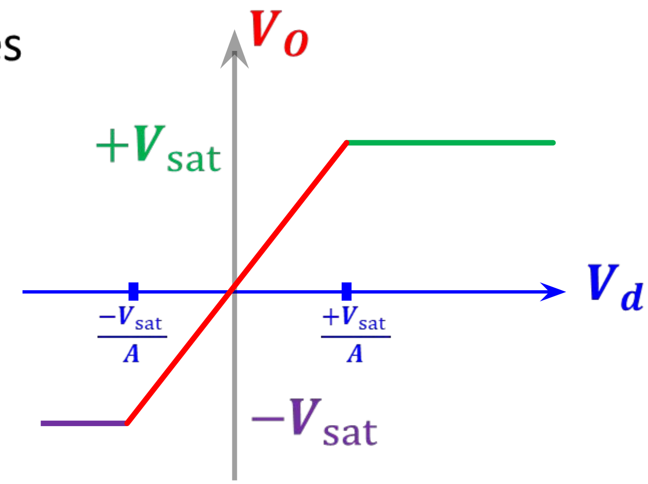
$$V_O = AV_d = -15\ \mathrm{V}$$

Op-Amp: Summary

Op-Amp **Amplifies** the difference between the voltages at it's two input terminals - $\boldsymbol{V_d}$



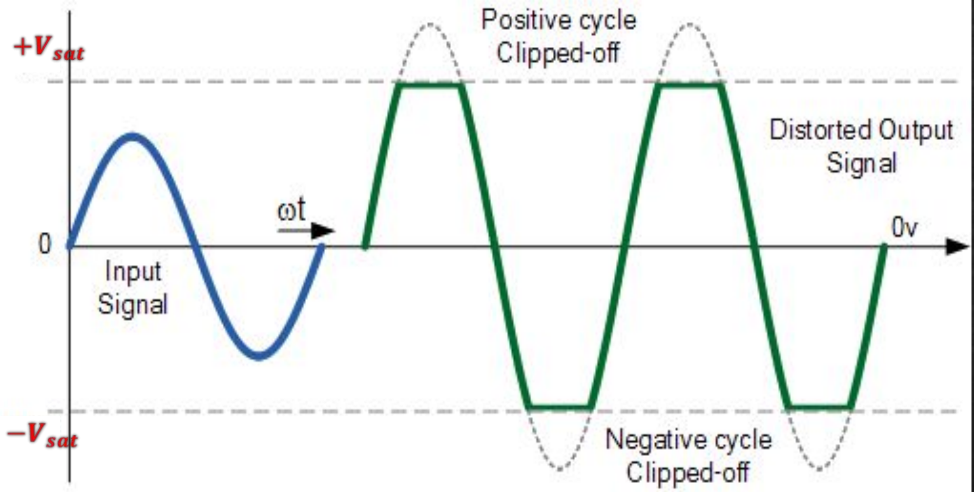
However, the **Amplification** is limited within voltage levels defined by the positive and negative saturation voltages $[-V_{sat}, +V_{sat}]$.

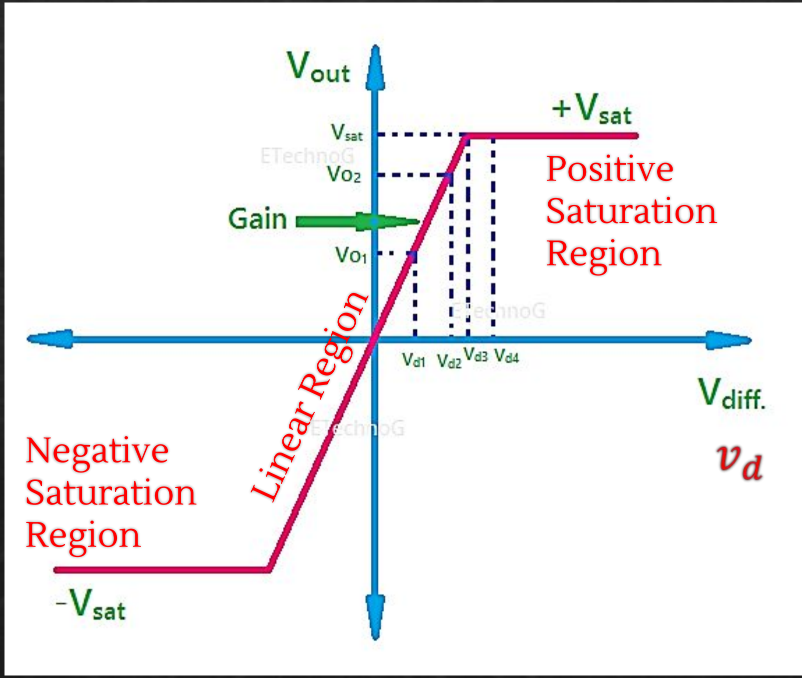


The "ideal" op-amp behaves like a **voltage dependent voltage source** within the linear region.

Operational Amplifier (Op-Amp)

- Saturation Voltages :





The dotted line shows what the output should have been

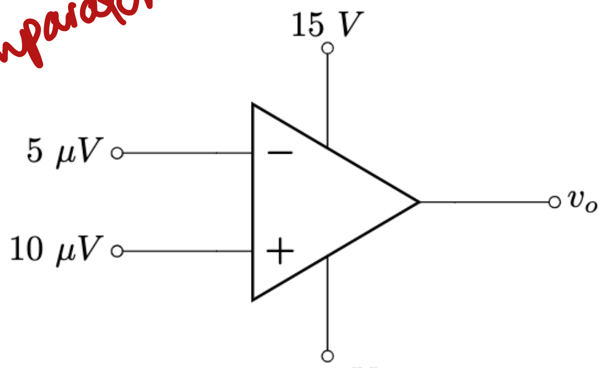
The green line shows what we actually get

Types of Op-Amp configuration

comparator

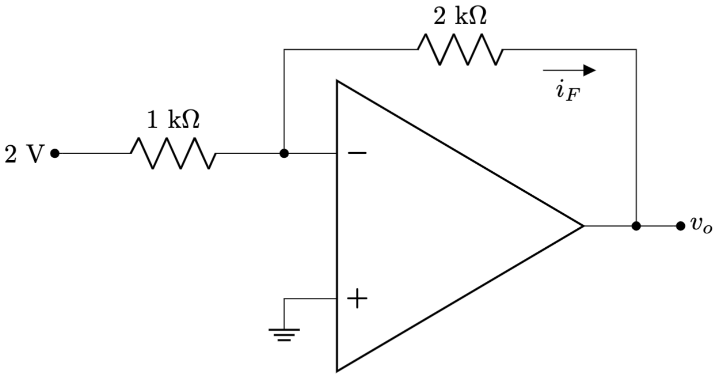
1. Open loop configuration:

   No physical connection between input and output



2. Closed loop configuration:

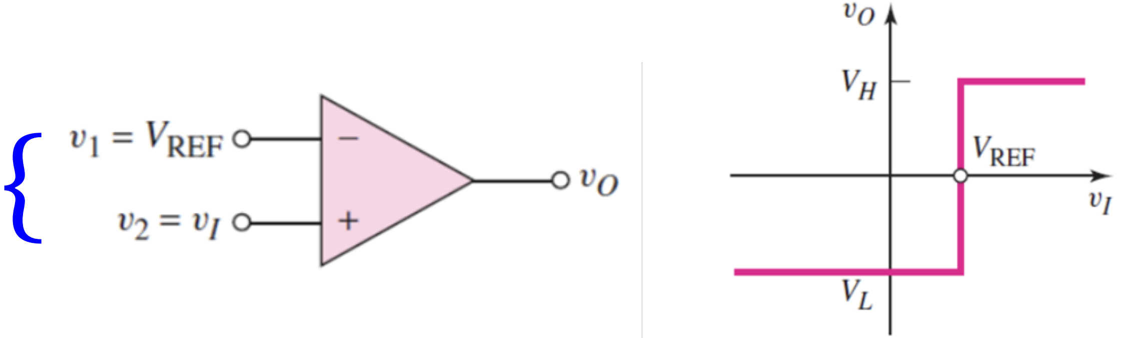
   Feedback from output terminal



Open Loop Configuration: Comparator

Level Crossing Detector / Comparator

$$V_d = v_I - V_{\text{REF}}$$



NON-INVERTING COMPARATOR

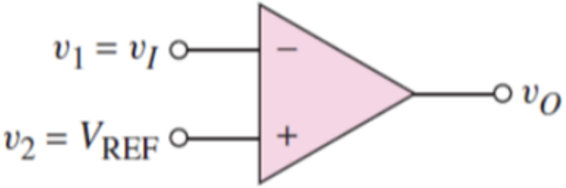
$$V_d = v_I - V_{\text{REF}} > 0 \quad \Rightarrow \quad v_O = V_H$$

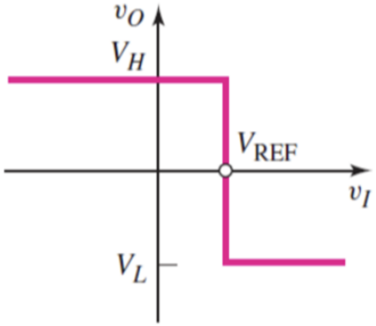
$$v_I > V_{\text{REF}} \quad \Rightarrow \quad v_O = V_H$$

Open Loop Configuration: Comparator

Level Crossing Detector / Comparator

$$V_d = V_{\mathrm{REF}} - v_I$$





INVERTING COMPARATOR

$$V_d = V_{\mathrm{REF}} - v_I > 0 \quad \Rightarrow \quad v_O = V_H$$

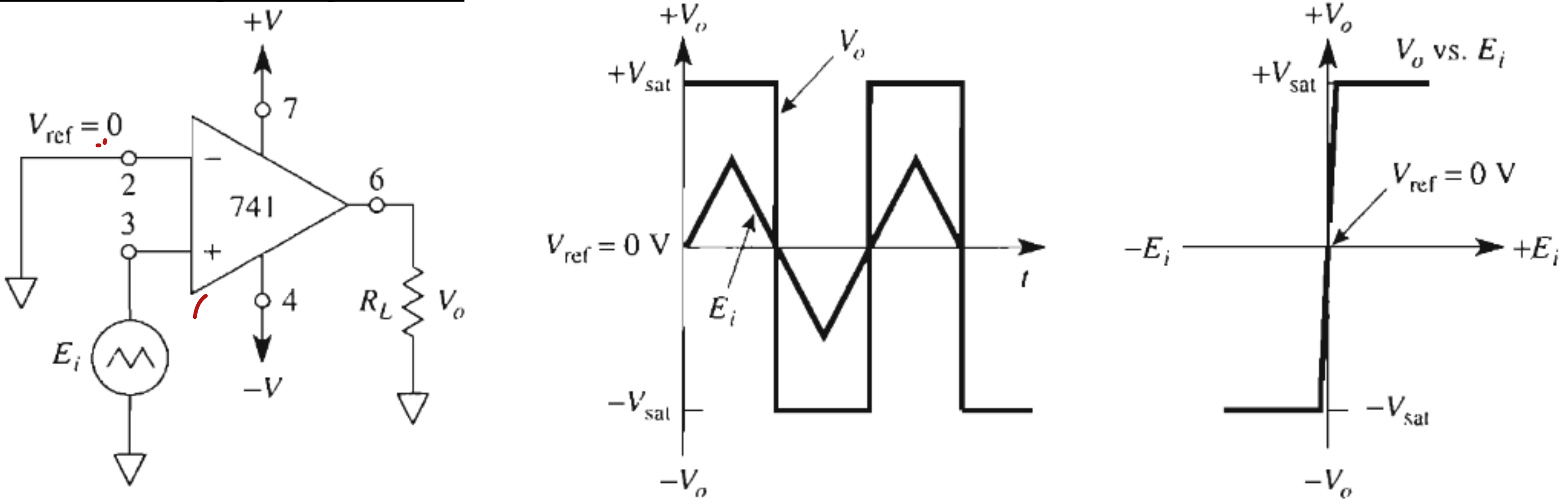
$$v_I < V_{\mathrm{REF}} \quad \Rightarrow \quad v_O = V_H$$

Open Loop Configuration: Comparator

Zero Crossing Detector

Compare values with a reference and pin value to $+\boldsymbol{V_{sat}}$ if voltage is above or to below that.

<u>Non-inverting configuration</u>

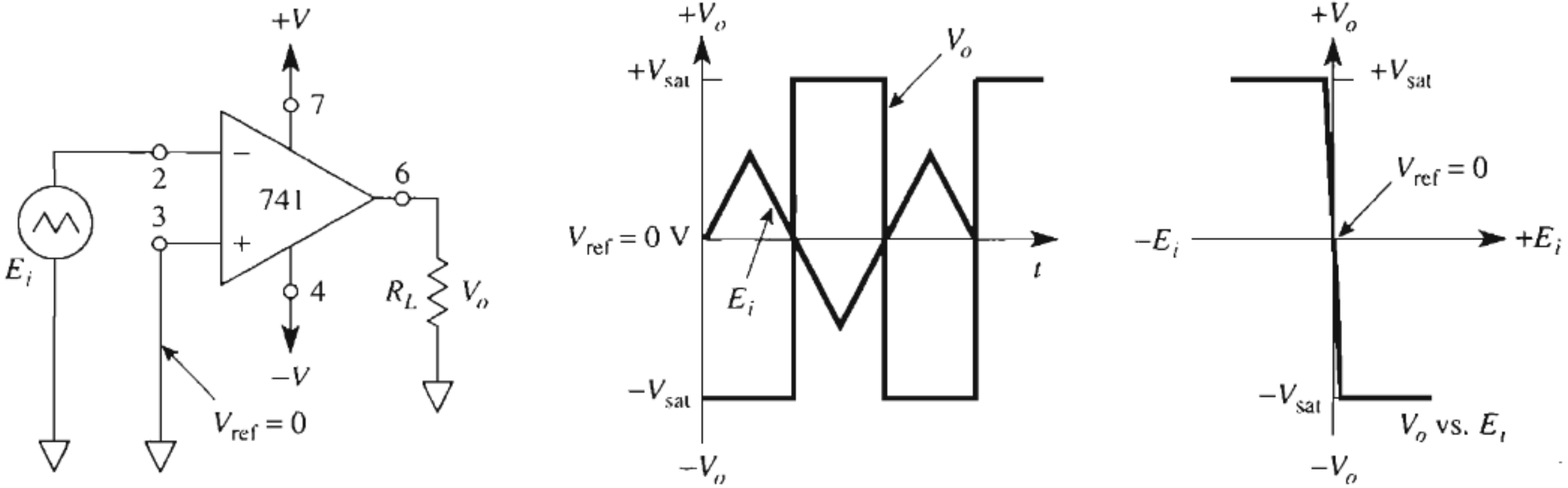


(a) Noninverting: When E_i is above V_{ref}, $V_o = +V_{sat}$.

Open Loop Configuration: Comparator

Zero Crossing Detector

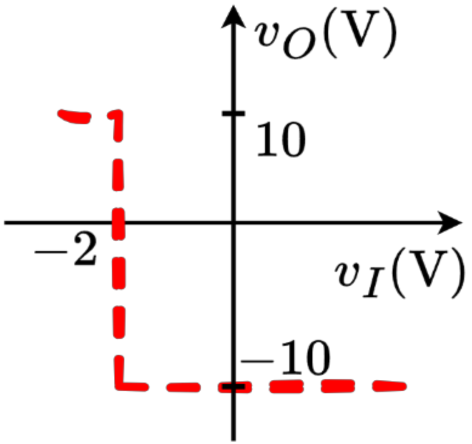
<u>Inverting configuration</u>



(b) Inverting: When E_i is above V_{ref}, $V_o = -V_{sat}$.

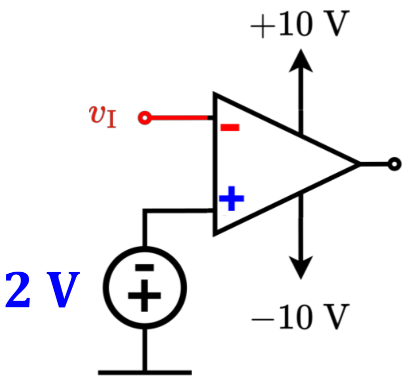
Open Loop Configuration: Example 1

- Design a circuit using **op-amp** that has the voltage transfer characteristics as shown in the figure below. $v_O(V)$ is the **output voltage** and $v_I(V)$ is the **input voltage**.



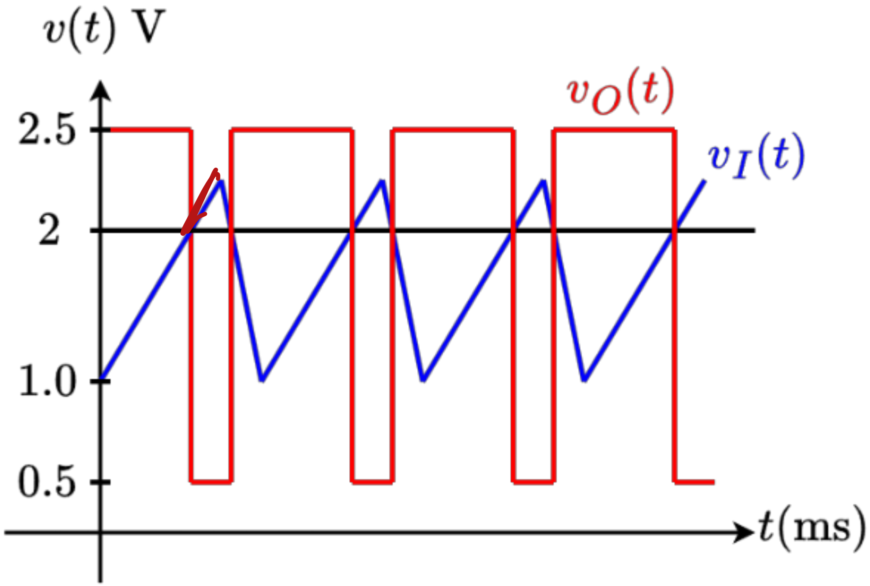
Solution:

Inverting comparator



Open Loop Configuration: Example 2

- Draw the voltage transfer characteristic (VTC) curve (v_O vs v_I) from the adjacent waveform graph. Also draw the **Op-Amp Circuit** that would give rise to such a VTC.

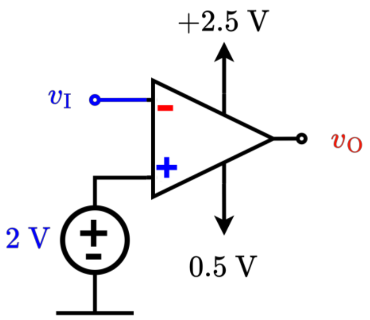


Solution:

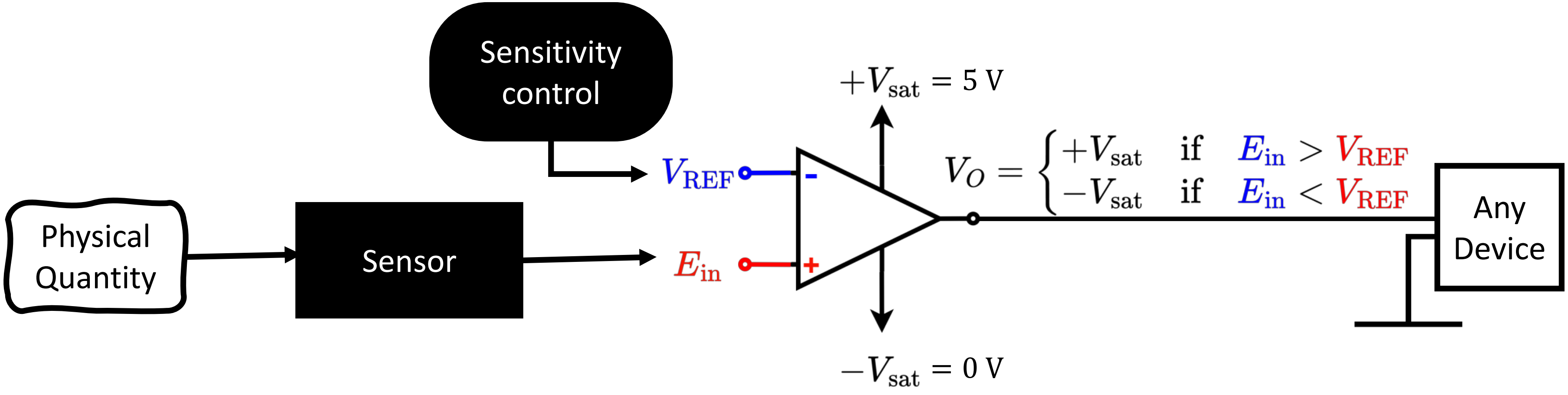
v_I **smaller** than $2\text{ V} \Rightarrow v_O = 2.5\text{ V}$ → **Positive Saturation**

v_I **larger** than $2\text{ V} \Rightarrow v_O = -2.5\text{ V}$ → **Negative Saturation**

INVERTING CONFIGURATION



Summary



- Comparators can be used to switch on **any device** based on a **physical quantity**.
- The sensitivity of **switching** is determined by V_{REF}

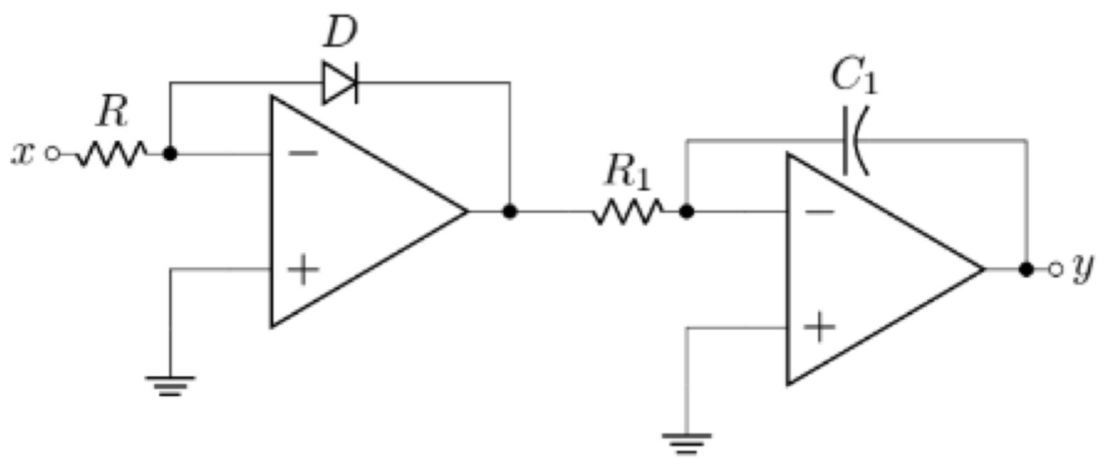
(a) **Design** a circuit using **Op-Amp comparator** to automatically turn ON (or OFF) the street lights. For this, you have a lux sensor installed on top of the street lights (facing above) that outputs a voltage proportional to amount of natural light, as listed below:

$v_{\text{night, 0 lux}} = 1$ V	$v_{\text{dusk, 20 lux}} = 2$ V	$v_{\text{dawn, 80 lux}} = 3$ V

The lights require 20 V and should be ON if the amount of light goes ***below*** 20 lux (at dusk). [3]

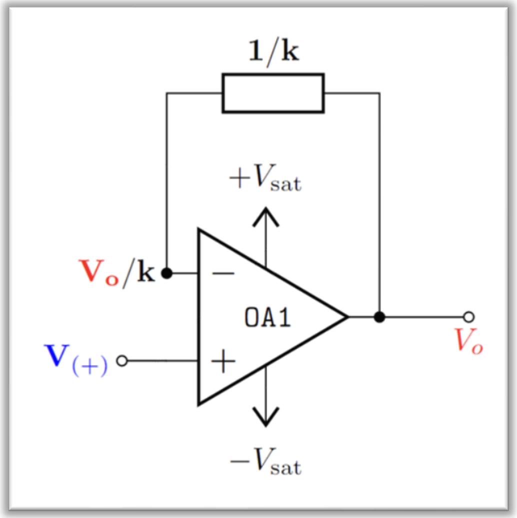
(b) **Design** a circuit using Op-Amp to implement the expression: $f = -3\frac{dx}{dt} + 2\exp y + 4z$ [4]

(c) **Analyze** the circuit below to find y as a function of x. For the diode, $I_S R = 1$ and $V_T = 1$. [3]



Closed Loop Configuration

Feedback



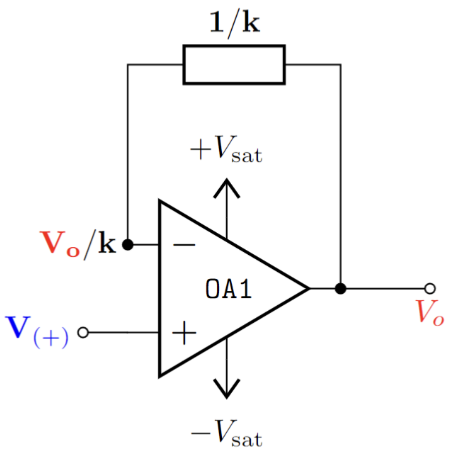
Feedback in Op-Amp circuit

Two types of feedback

1. Negative Feedback:

   Output voltage is fed to the inputs **negatively**

   The output voltage is connected to the **<u>inverting</u>** terminal

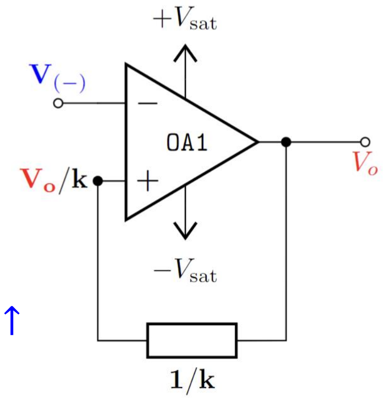


$$V_O \uparrow \quad \Rightarrow \frac{V_O}{k}\uparrow \quad \Rightarrow V_{(-)}\uparrow \quad \boxed{\Rightarrow V_d \downarrow = V_{(+)} - V_{(-)}\uparrow} \quad \Rightarrow V_O \propto V_d \downarrow$$

2. Positive Feedback:

   Output voltage is fed to the inputs **positively**

   The output voltage is connected to the **<u>non-inverting</u>** terminal



$$V_O \uparrow \quad \Rightarrow \frac{V_O}{k}\uparrow \quad \Rightarrow V_{(+)}\uparrow \quad \boxed{\Rightarrow V_d \uparrow = V_{(+)}\uparrow - V_{(-)}} \quad \Rightarrow V_O \propto V_d \uparrow$$

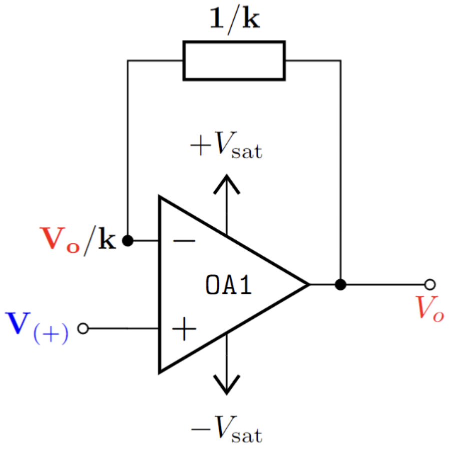
Feedback in Op-Amp circuit

Two types of feedback

1. Negative Feedback:

   Output voltage is fed to the inputs **negatively**

   The output voltage is connected to the **<u>inverting</u>** terminal

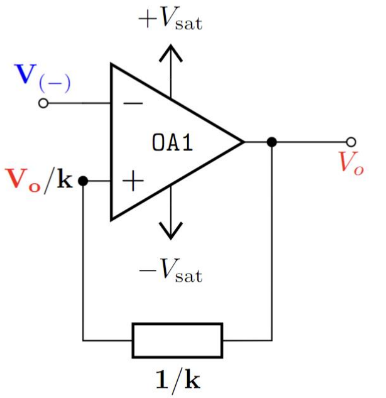


$$V_o \uparrow \;\Rightarrow V_o \propto V_d \downarrow$$

2. Positive Feedback:

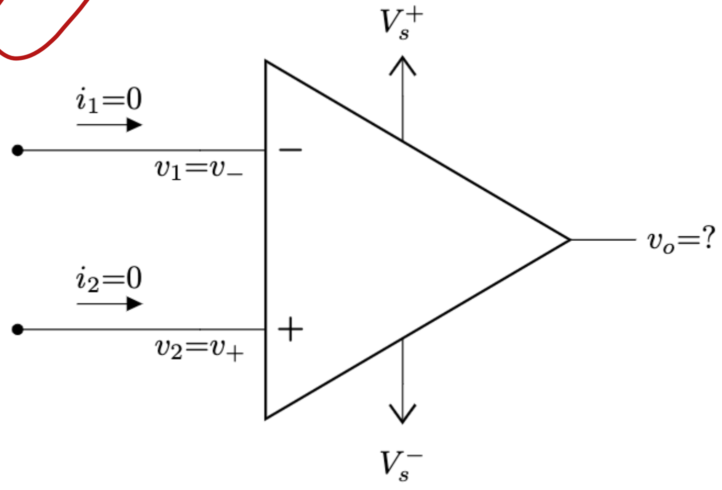
   Output voltage is fed to the inputs **positively**

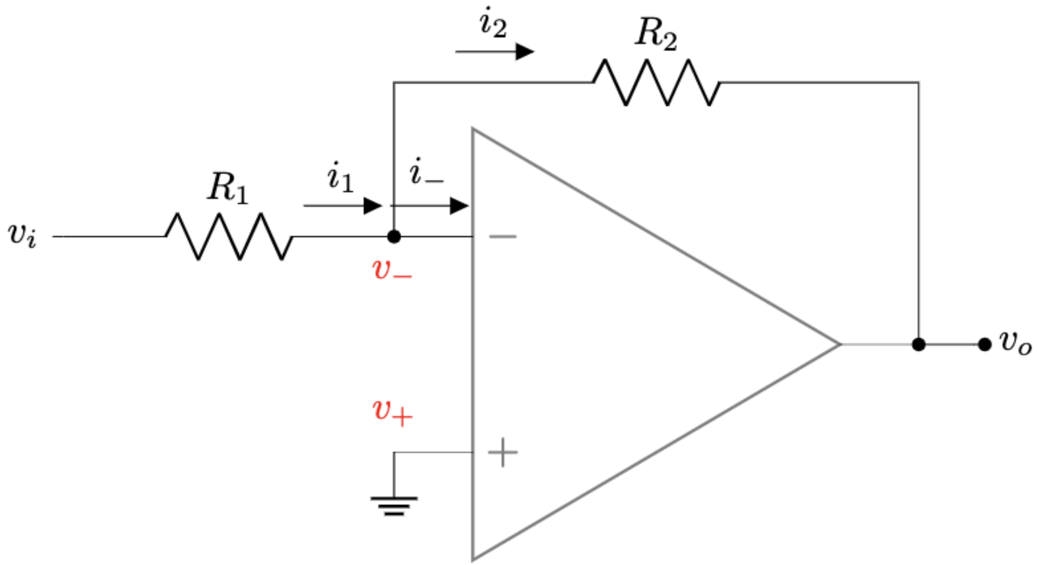
   The output voltage is connected to the **<u>non-inverting</u>** terminal



$$V_o \uparrow \;\Rightarrow V_o \propto V_d \uparrow$$

Solving Closed Loop Op-Amp Circuit





Two Rules:

1. Virtual Shorting:
2. Zero input bias current:

$$\boxed{\begin{aligned} v_+ &= v_- \\ i_- = i_+ &= 0 \end{aligned}}$$

Feedback in Op-Amp circuit

Negative Feedback:

Output voltage is fed to the inputs **negatively**

The output voltage is connected to the **inverting** terminal

Here, $\boldsymbol{V}_{(-)} = \frac{\boldsymbol{V_O}}{\mathbf{k}}$

We know, $\boldsymbol{V_o} = A\boldsymbol{V_d}$

$$V_o = A\left(\boldsymbol{V}_{(+)} - \boldsymbol{V}_{(-)}\right)$$

$$= A\left(\boldsymbol{V}_{(+)} - \frac{\boldsymbol{V_O}}{\mathbf{k}}\right)$$

$$= A\boldsymbol{V}_{(+)} - \frac{A}{k}V_O$$

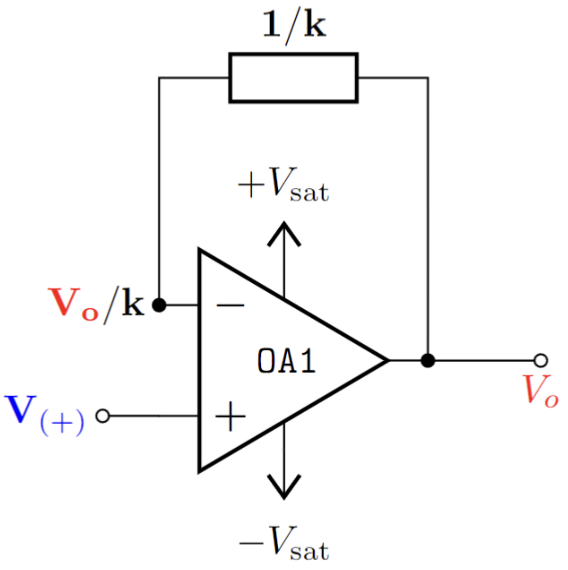
$$\Rightarrow \boldsymbol{V_o}\left(1 + \frac{A}{k}\right) = A\boldsymbol{V}_{(+)}$$

$$\frac{\boldsymbol{V_O}}{\boldsymbol{V}_{(+)}} = \frac{A}{1 + \frac{A}{k}} = \frac{1}{\frac{1}{A} + \frac{1}{k}}$$

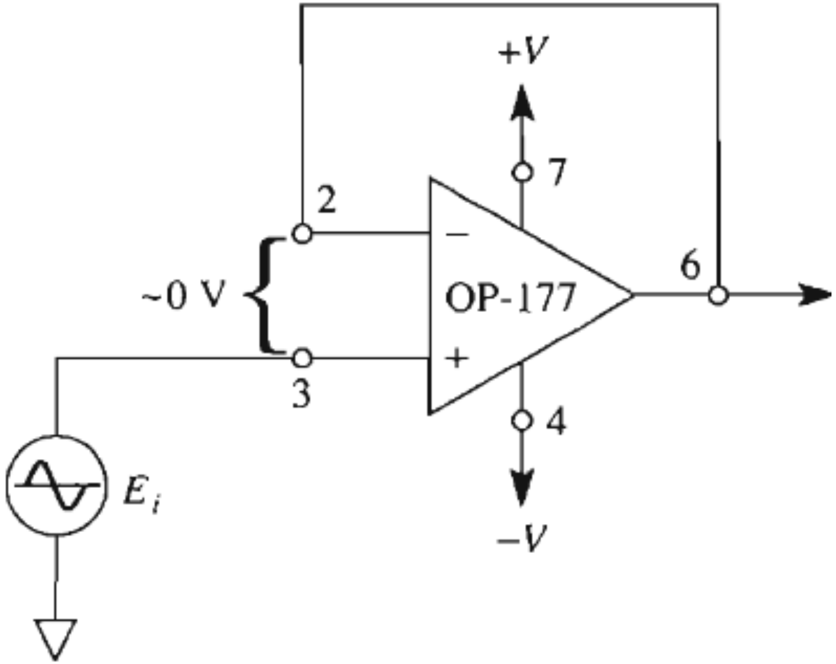
If $A \to \infty$ then $\frac{1}{A} \to 0$.

$$\therefore \frac{\boldsymbol{V_O}}{\boldsymbol{V}_{(+)}} = k$$

This is the new amplification factor / Gain

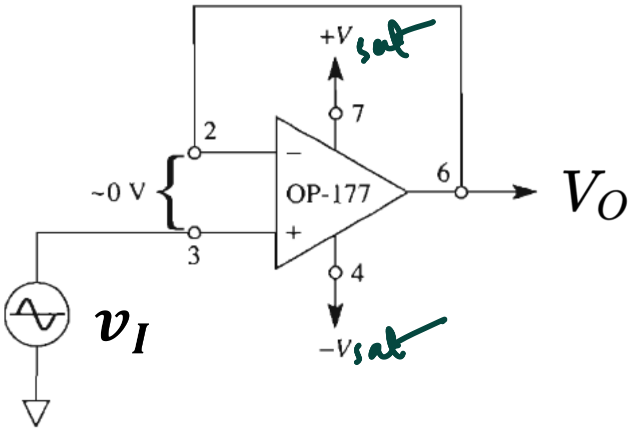


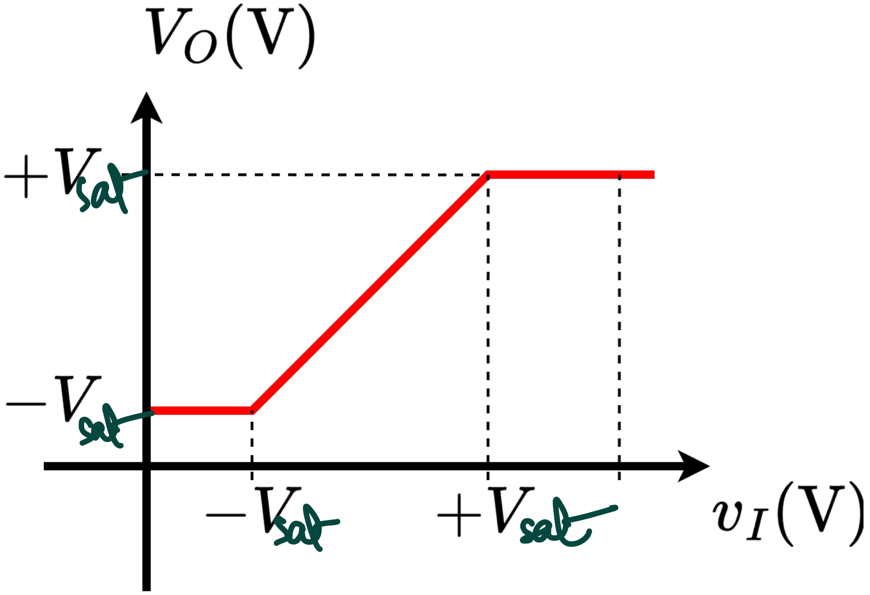
Voltage Follower / Buffer:



$V_O = E_i$

Voltage Follower – VTC

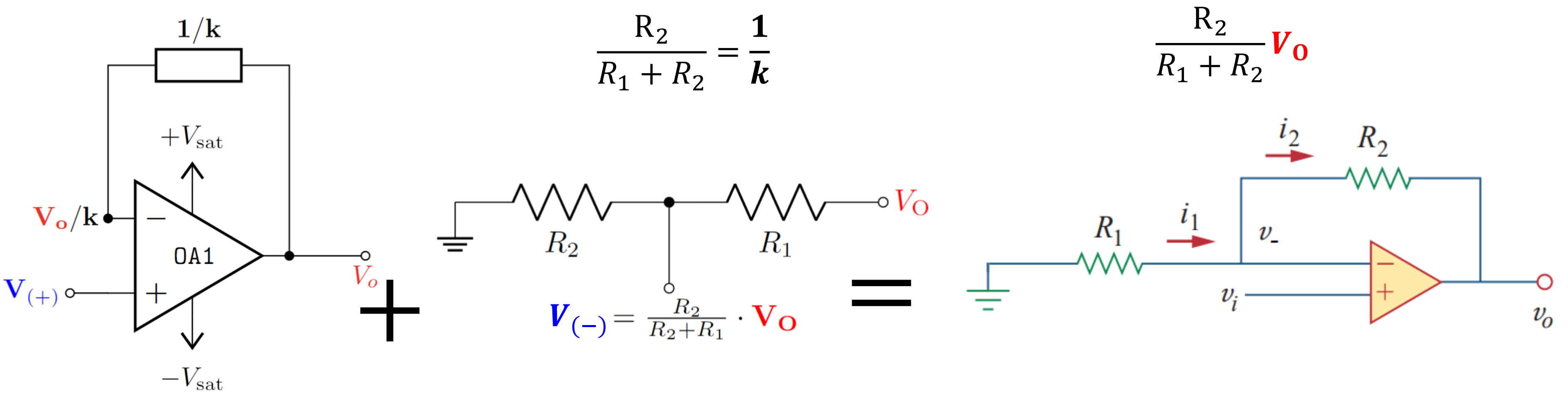




$$V_O = \begin{cases} +V_{sat} & \text{if } v_I \geq +V_{sat} \\ v_I, & \text{if } -V_{sat} \leq v_I \leq +V_{sat} \\ -V_{sat} & \text{if } v_I \leq -V_{sat} \end{cases}$$

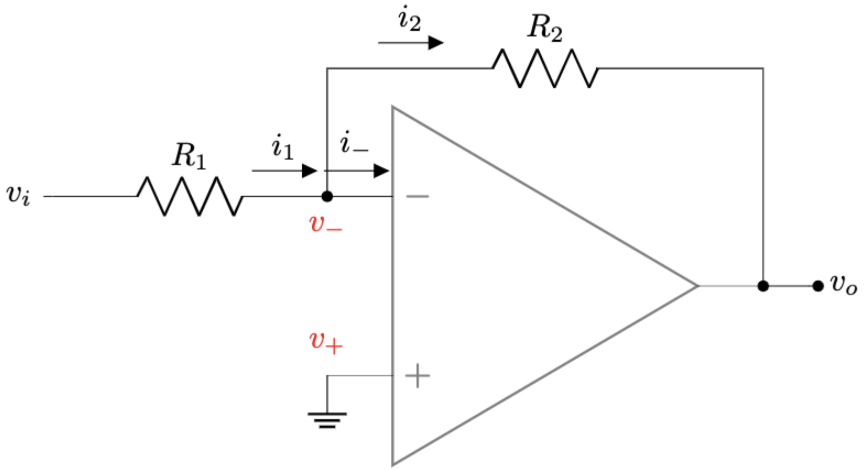
Negative Feedback in Op-Amp circuit

A voltage divider can act as a **multiplier/factor** in the **feedback** branch



If $\boldsymbol{k}$ = 10 (meaning we feed back one tenth of the output to negative input), we will get v_o=10*v_i.
that is 10-fold gain.

Solving Closed Loop Op-Amp Circuit



$$\text{Gain} = \frac{v_o}{v_i} = -\frac{R_2}{R_1}$$

Since $\boldsymbol{v}_+$ is connected to ground, $\boldsymbol{v}_+ = \mathbf{0\ V}$

Since there is negative feedback, from virtual short, $\boldsymbol{v}_- = \boldsymbol{v}_+ = \mathbf{0\ V}$

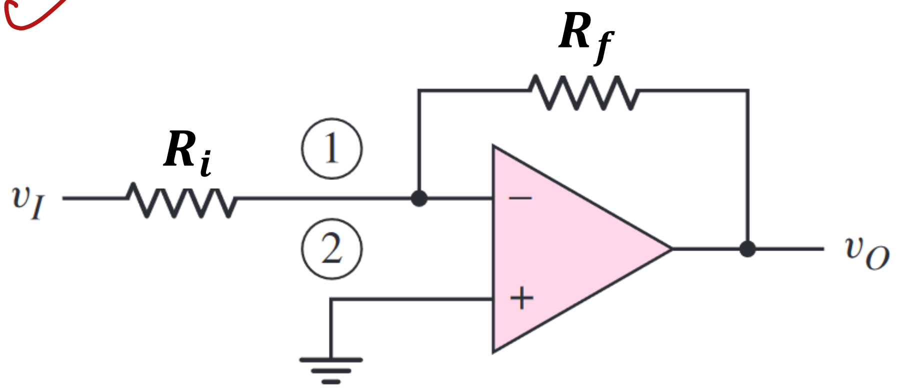
From Ohm's law for $\mathrm{R}_1 \Rightarrow i_1 = \frac{v_i - 0}{R_1} = \frac{v_i}{R_1}$

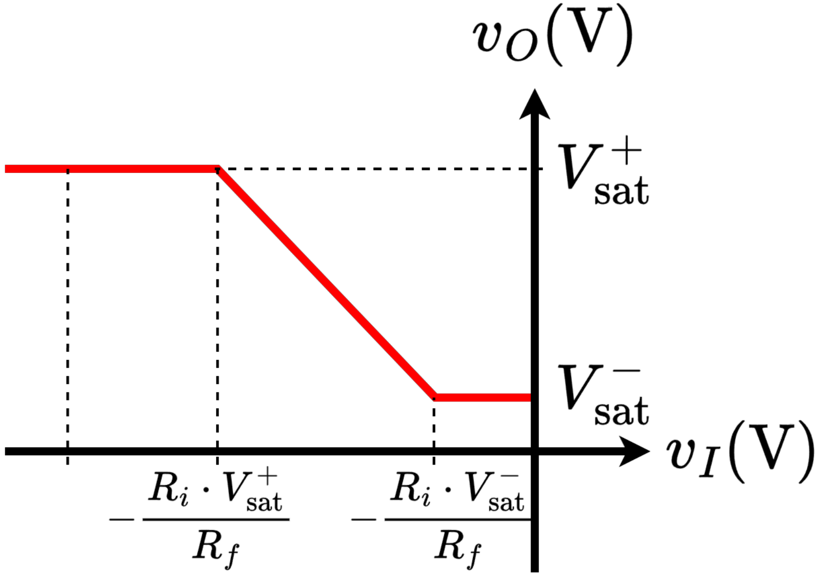
Since ideal op-amp, $i_- = i_+ = 0$

From KCL at v_-, $i_1 = i_- + i_2 \Rightarrow i_1 = i_2 = v_i/R_1$

From Ohm's law for $R_2 \Rightarrow i_2 = \frac{v_- - v_0}{R_2} = \frac{v_i}{R_1} \Rightarrow v_o = -i_2 \times R_2 \Rightarrow v_o = -\frac{R_2}{R_1} v_i$ [ANS]

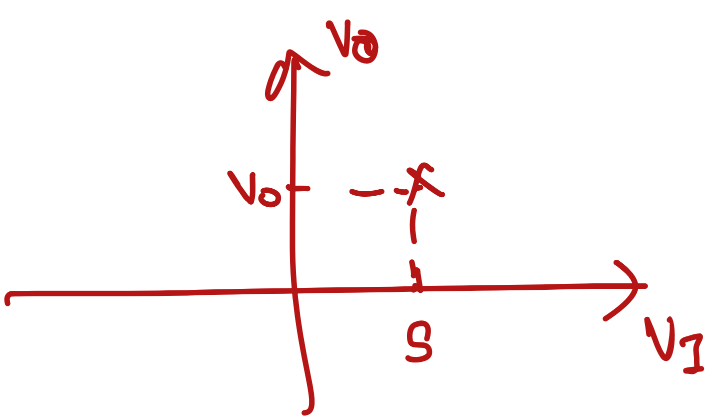
Inverting Amplifier – VTC

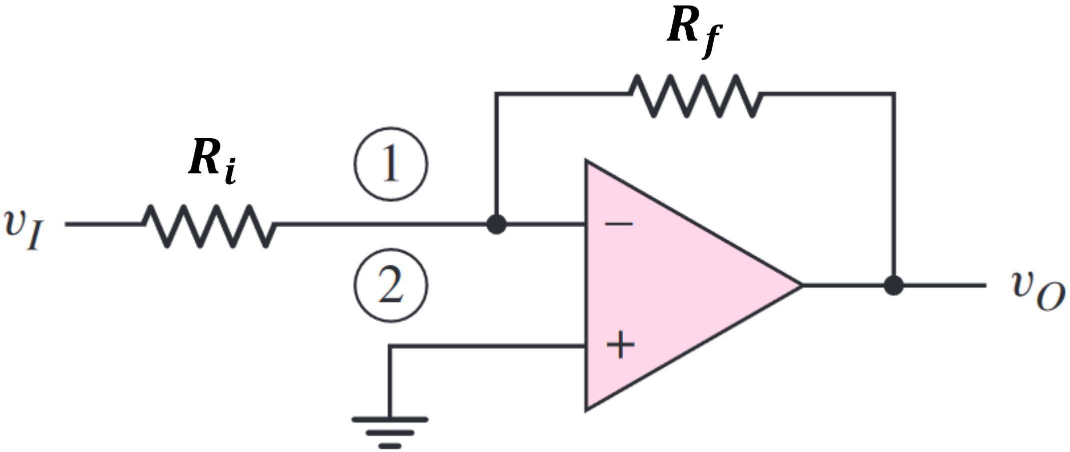




$$v_O = \begin{cases} V_{\mathrm{sat}}^{+}, & \text{if } v_O \geq V_{\mathrm{sat}}^{+} \\ -v_I \cdot \frac{R_f}{R_i}, & \text{if } V_{\mathrm{sat}}^{-} \leq v_O \leq V_{\mathrm{sat}}^{+} \\ V_{\mathrm{sat}}^{-}, & \text{if } v_O \leq V_{\mathrm{sat}}^{-} \end{cases}$$

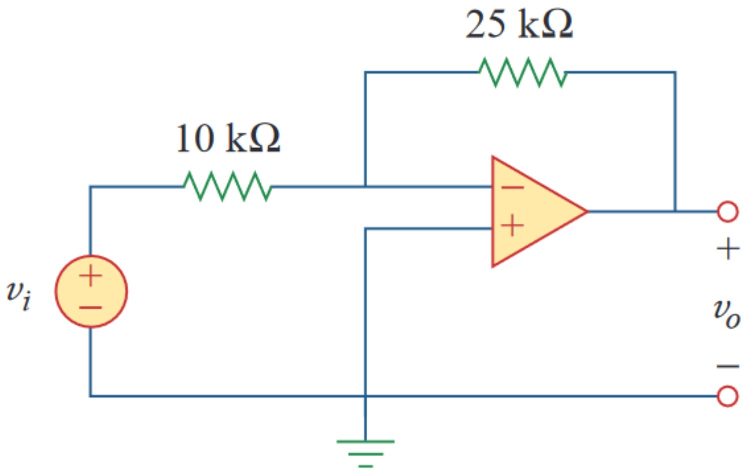
Inverting Amplifier





$$v_O = -\frac{R_f}{R_i} \cdot v_I$$

Example - 1



If $\boldsymbol{v_i} = \mathbf{0.5}$ V, calculate:

(a) Output voltage $\boldsymbol{v_o}$.

(b) Current in the $\mathbf{10\ k\Omega}$ resistor.

(a)

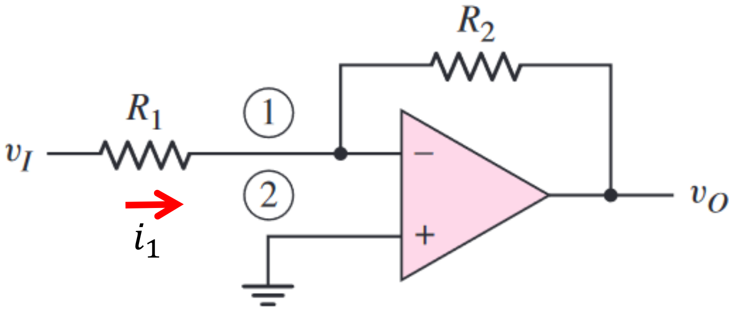
$$\boldsymbol{v_o = -\frac{R_f}{R_i} \cdot v_i = -2.5 v_i = -1.25\ \mathrm{V}}$$

(b) Current through the $\mathbf{10\ k\Omega}$ resistor is

$$i = \frac{v_i}{R_i} = \frac{0.5}{10}\ \mathrm{mA} = \mathbf{50\ \mu A}$$

Example - 2

Design the circuit such that the closed loop voltage gain is $\boldsymbol{A_{\mathrm{CL}} = -5}$. Assume the op-amp is driven by an ideal sinusoidal source, $\boldsymbol{v_I = 0.1 \sin(\omega t)\ (V)}$, that can supply a maximum current of $\mathbf{5\ \mu A}$.



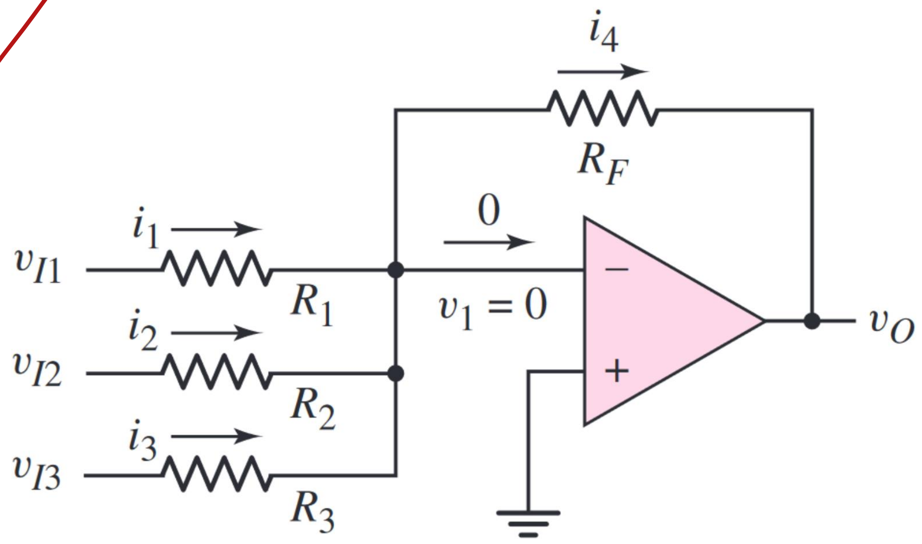
$$\boldsymbol{i_1 = \frac{v_I}{R_1}}$$

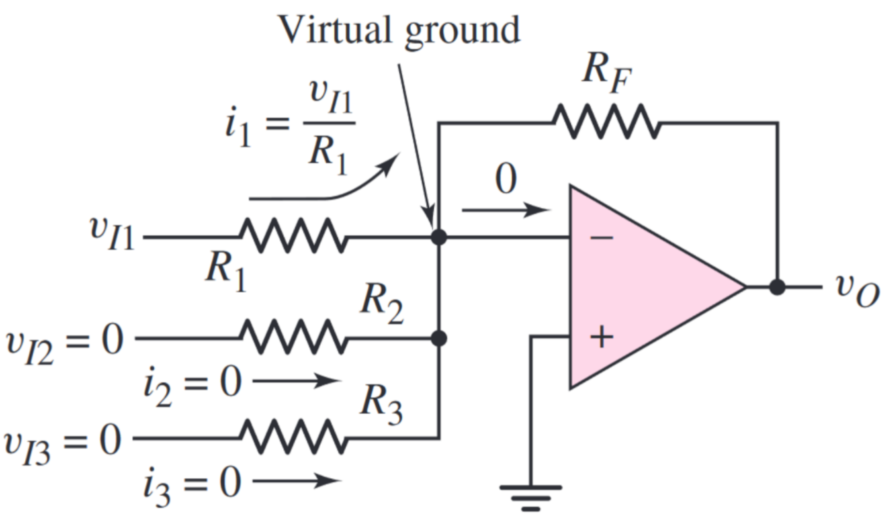
$$\boldsymbol{R_1} = \frac{v_I(\max)}{i_1(\max)} = \frac{0.1}{5 \times 10^{-3}} = \mathbf{20\ k\Omega}$$

$$\boldsymbol{R_2} = |-A_{\mathrm{CL}}| \cdot R_1 = 5 \times 20 = \mathbf{100\ k\Omega}$$

Inverting Summer

- Multichannel Amplifier

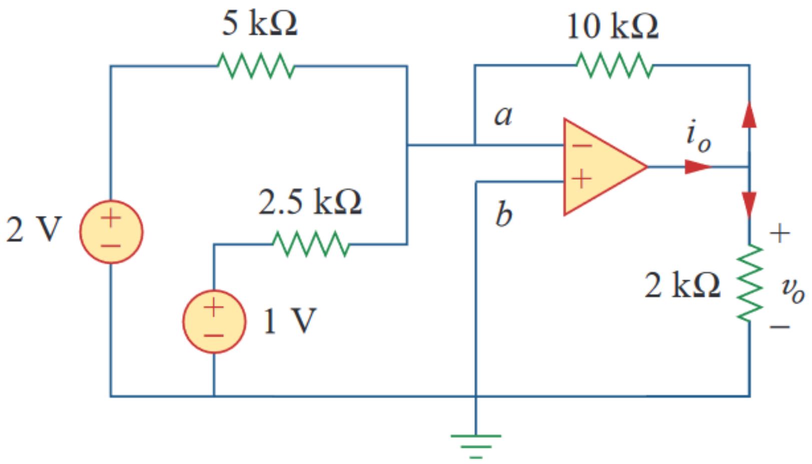




$$v_O(v_{I1}) = -i_1 R_F = -\left(\frac{R_F}{R_1}\right) v_{I1}$$

$$v_O = -\left(\frac{R_F}{R_1}v_{I1} + \frac{R_F}{R_2}v_{I2} + \frac{R_F}{R_3}v_{I3}\right)$$

Example - 3



Calculate:

(a) Output voltage $\boldsymbol{v_o}$.

(b) Output current $\boldsymbol{i_o}$.

(a)

$$\boldsymbol{v_o} = -\left(\frac{\mathbf{10}}{\mathbf{5}} \cdot \mathbf{2} + \frac{\mathbf{10}}{\mathbf{2.5}} \cdot \mathbf{1}\right) = \mathbf{-8\ V}$$

(b)

$$i = \frac{v_o}{10} + \frac{v_o}{2} = (-0.8 - 4\ )\ \text{mA} = \mathbf{-4.8\ mA}$$

Example 4

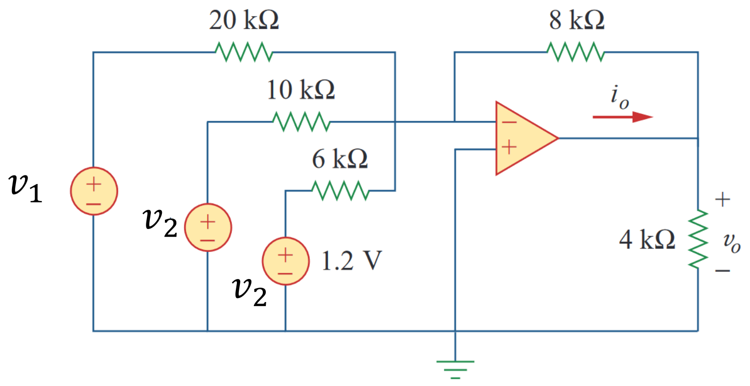
Design an op-amp circuit with inputs $\boldsymbol{v_1}$, $\boldsymbol{v_2}$ and $\boldsymbol{v_3}$ such that, output voltage $\boldsymbol{v_o}$:

$$\boldsymbol{v_o = -\frac{2}{5}v_1 - \frac{4}{5}v_2 - \frac{4}{3}v_3}$$

Example 4

Design an op-amp circuit with inputs $\boldsymbol{v_1}$, $\boldsymbol{v_2}$ and $\boldsymbol{v_3}$ such that, output voltage $\boldsymbol{v_o}$:

$$\boldsymbol{v_o = -\frac{2}{5}v_1 - \frac{4}{5}v_2 - \frac{4}{3}v_3}$$



Solution:

The given function can be achieved by an **inverting summing amplifier**. Having the voltage transfer formula as:

$$v_o = \left(-\frac{R_f}{R_1}v_1 - \frac{R_f}{R_2}v_2 - \cdots \frac{R_f}{R_n}v_n\right)$$

Here, the numerators of all the coefficients of input voltages are same ($\boldsymbol{R_f}$). As per the given problem, this can be achieved by setting the numerator to the LCM of 2 and 4 (i.e., to **8**).

$$\boldsymbol{v_o = -\frac{2}{5}v_1 - \frac{4}{5}v_2 - \frac{4}{3}v_3}$$

$$\boldsymbol{= -\frac{8}{20}v_1 - \frac{8}{10}v_2 - \frac{8}{6}v_3}$$

Example 4

Design an op-amp circuit with inputs v_1, v_2 and v_3 such that, output voltage v_o:

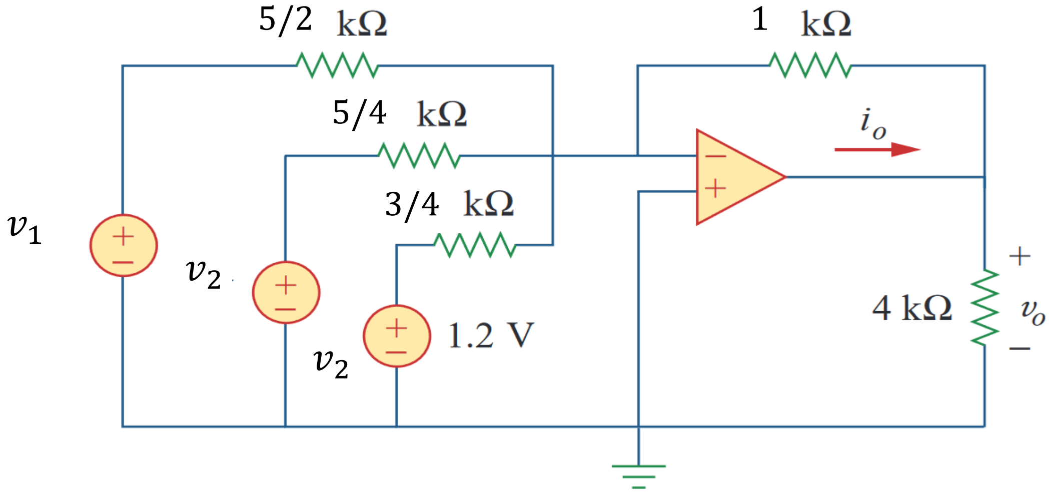
$$v_o = -\frac{2}{5}v_1 - \frac{4}{5}v_2 - \frac{4}{3}v_3$$

Easier Solution:

$$v_o = -\frac{2}{5}v_1 - \frac{4}{5}v_2 - \frac{4}{3}v_3$$

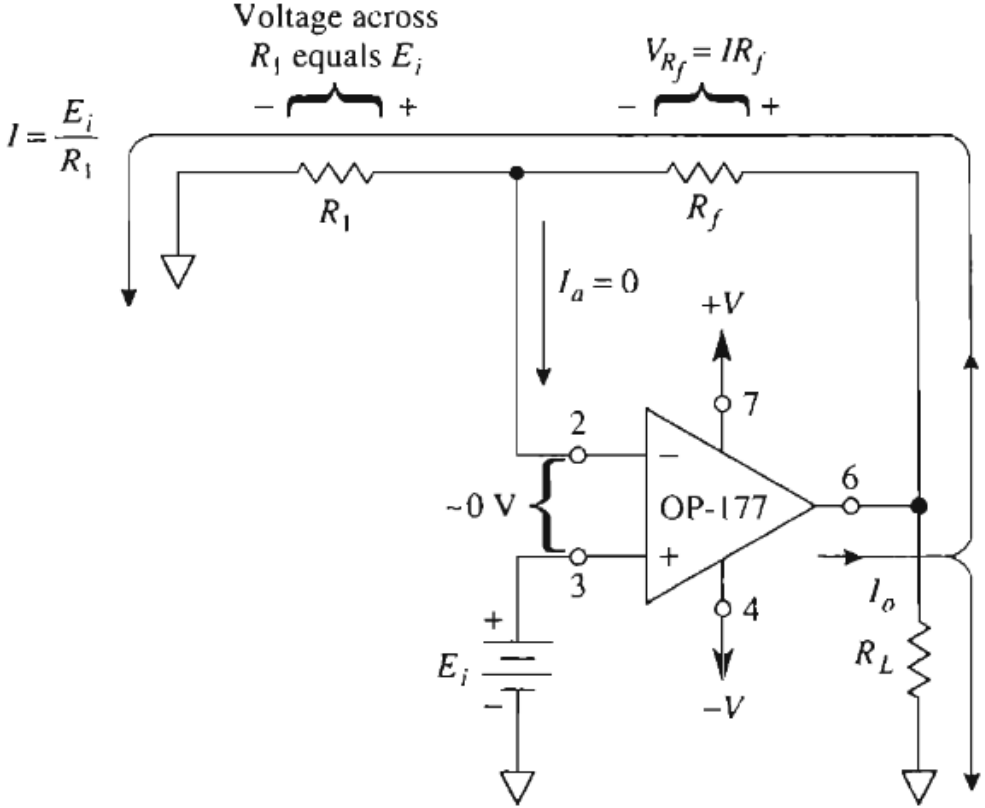
$$\cancel{= -\frac{8}{20}v_1 - \frac{8}{10}v_2 - \frac{8}{6}v_3}$$

$$= -\frac{1}{5/2}v_1 - \frac{1}{5/4}v_2 - \frac{1}{3/4}v_3$$



Non-Inverting Amplifier

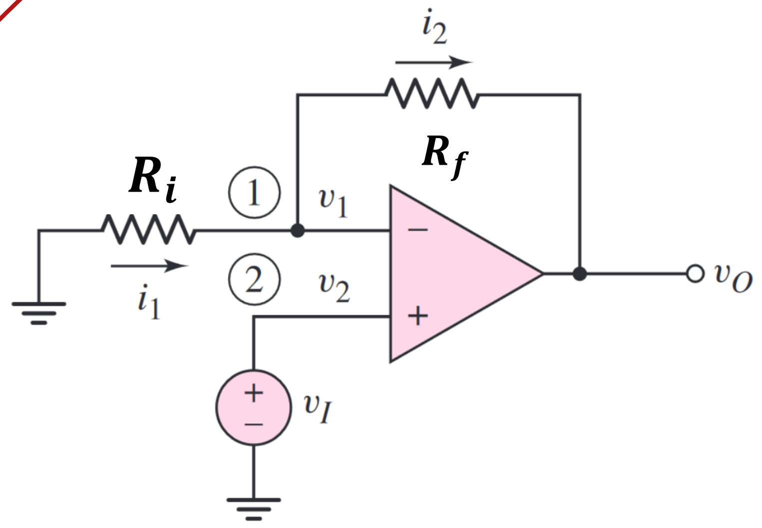
$$\frac{0 - E_i}{R_1} = \frac{E_i - V_o}{R_f}$$

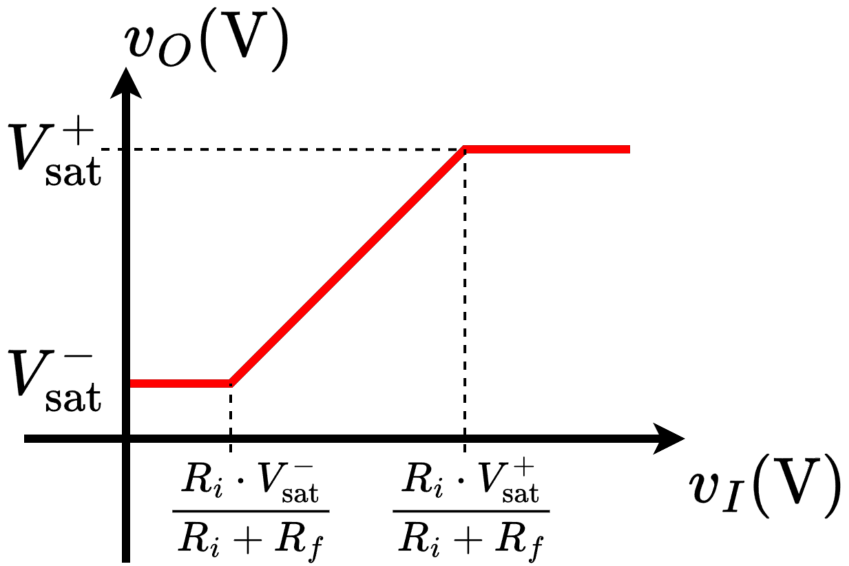


$$V_O > E_i$$

$$V_o = (1 + \frac{R_f}{R_1})E_i$$

Non-Inverting Amplifier – VTC

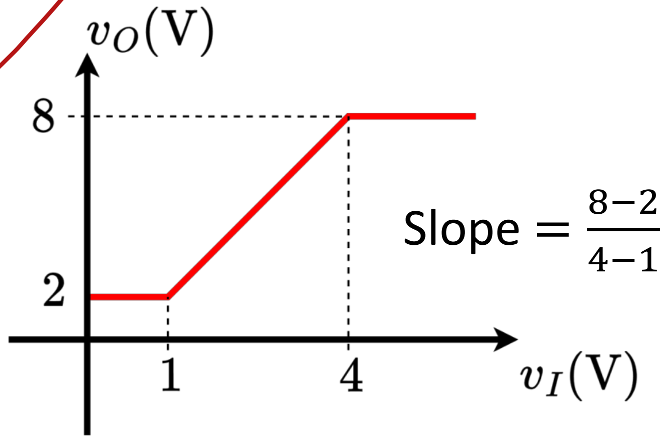




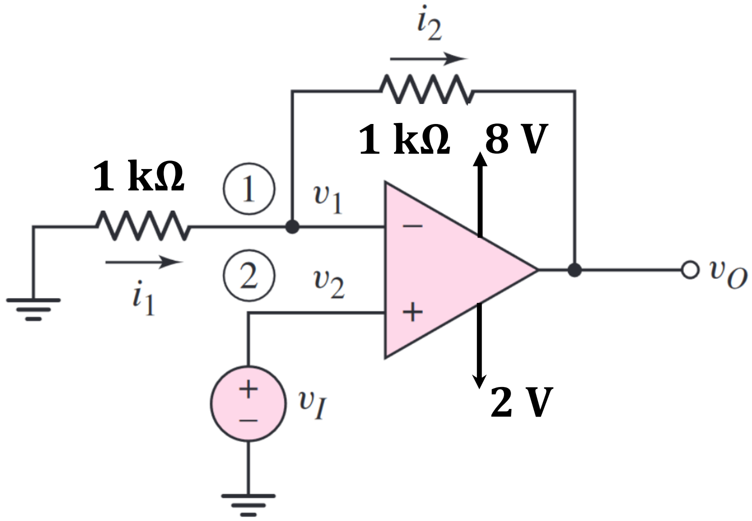
$$v_O = \begin{cases} V_{\mathrm{sat}}^{+}, & \text{if } v_O \geq V_{\mathrm{sat}}^{+} \\ v_I \cdot (1 + \frac{R_f}{R_i}), & \text{if } V_{\mathrm{sat}}^{-} \leq v_O \leq V_{\mathrm{sat}}^{+} \\ V_{\mathrm{sat}}^{-}, & \text{if } v_O \leq V_{\mathrm{sat}}^{-} \end{cases}$$

Non-Inverting Amplifier – VTC

Draw an Op-Amp Circuit with the following VTC



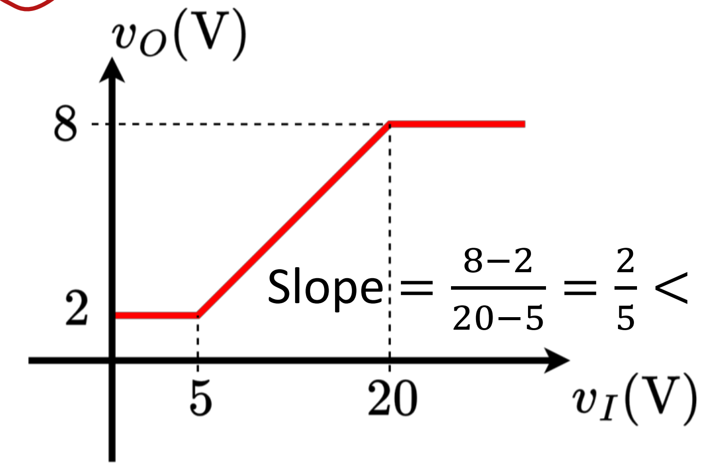
$$\text{Slope} = \frac{8-2}{4-1} = 2 = \left(1 + \frac{R_f}{R_i}\right)$$



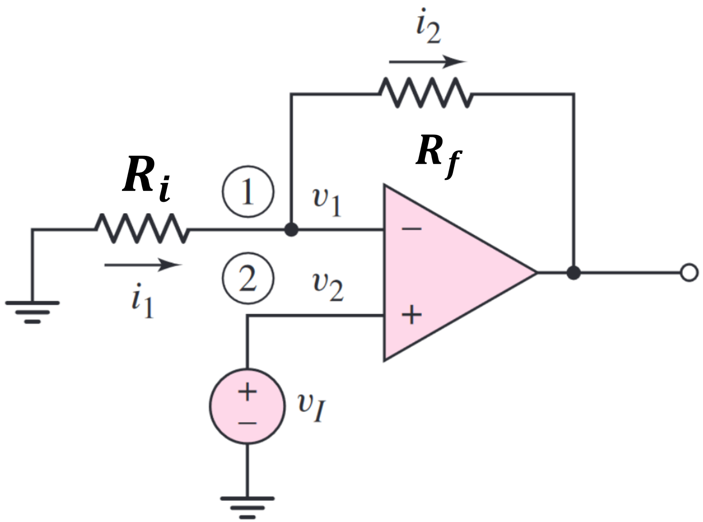
$$\left(1 + \frac{R_f}{R_i}\right) = 2 \qquad \Rightarrow \frac{R_f}{R_i} = 1$$

Non-Inverting Amplifier – VTC

Draw an Op-Amp Circuit with the following VTC. (What if the slope is less than **1?**)



$$\text{Slope} = \frac{8-2}{20-5} = \frac{2}{5} < 1$$

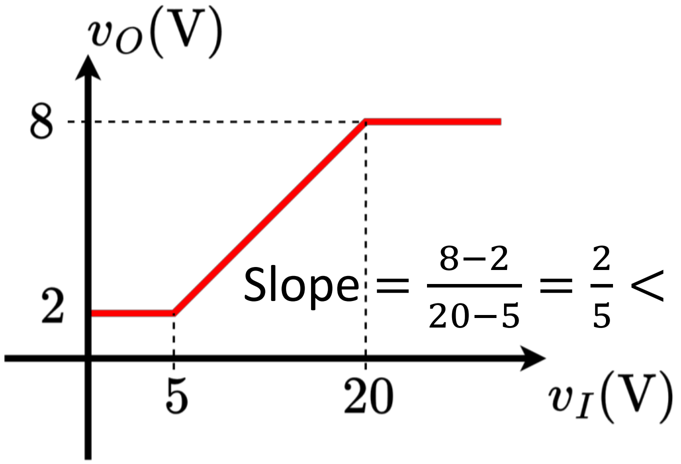


$$v_O = v_I \cdot \left(1 + \frac{R_f}{R_i}\right)$$

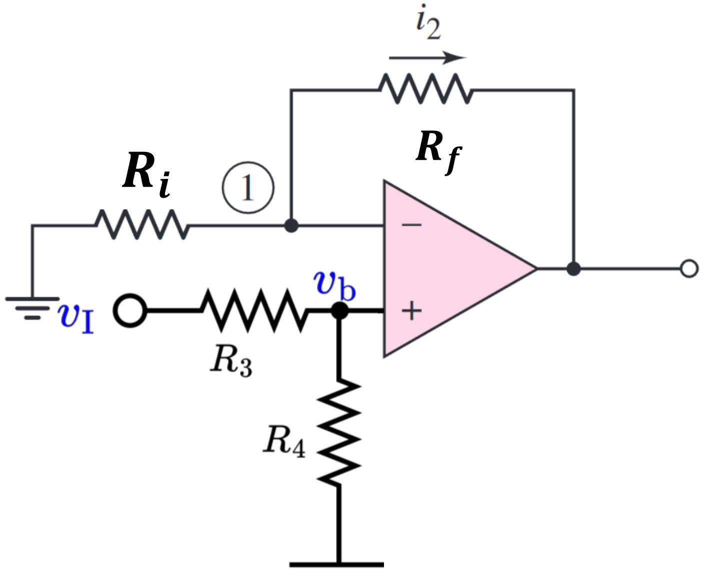
A non-inverting amplifier closed loop gain $\left(1 + \frac{R_f}{R_i}\right) > 1$. So, it is not possible to use this configuration for less than unity gain.

Non-Inverting Amplifier – VTC

Draw an Op-Amp Circuit with the following VTC. (What if the slope is less than **1?**)



$\text{Slope} = \frac{8-2}{20-5} = \frac{2}{5} < 1$

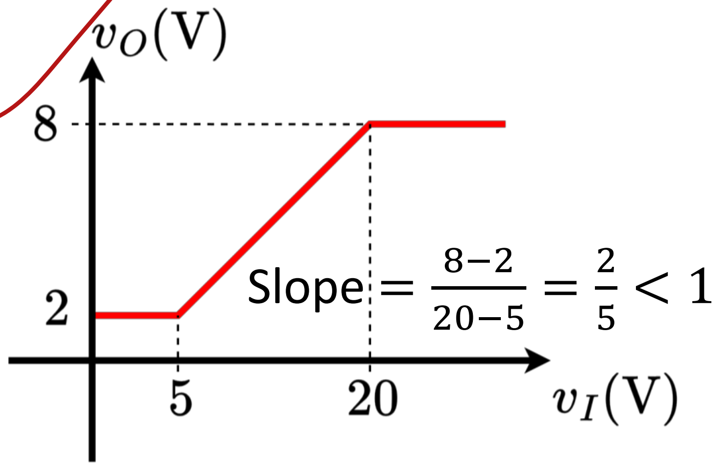


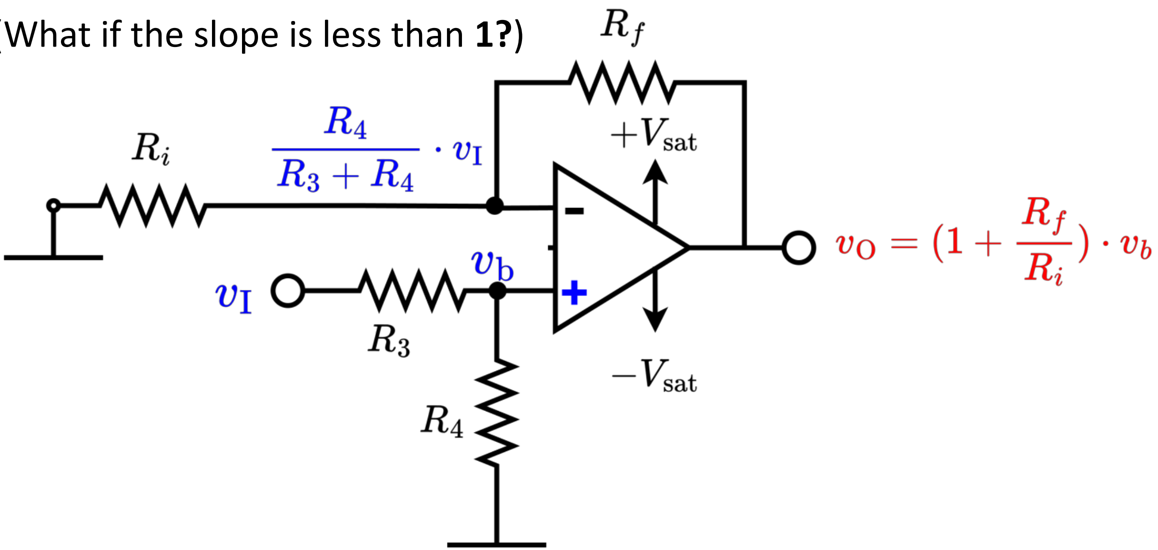
$v_O = v_I \cdot \left(1 + \frac{R_f}{R_i}\right)$

If Slope < 1, then, an additional voltage divider network should be added to the non-inverting terminal.

Non-Inverting Amplifier – VTC

Draw an Op-Amp Circuit with the following VTC. (What if the slope is less than **1?**)



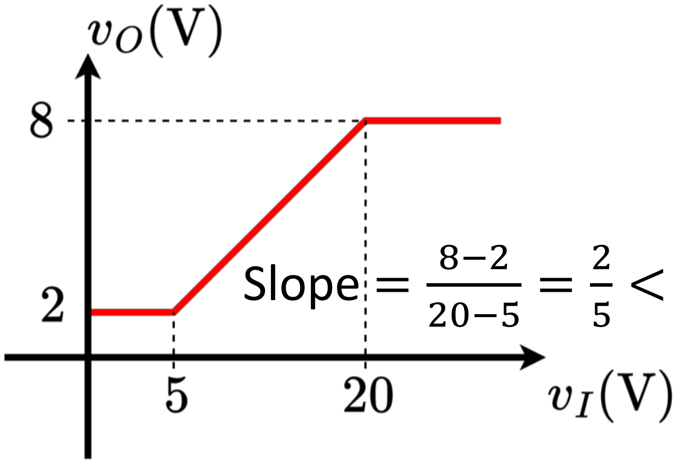


Voltage at the non-inverting terminal is converted from v_I to $v_I \cdot \left(\frac{R_4}{R_3+R_4}\right)$. So, the overall gain becomes: $\left(1 + \frac{R_f}{R_i}\right)\left(\frac{R_4}{R_3+R_4}\right)$ which can be less than **1.**

Non-Inverting Amplifier – VTC

Draw an Op-Amp Circuit with the following VTC. (What if the slope is less than **1?**)

So, $\left(1+\frac{R_f}{R_i}\right)\left(\frac{R_4}{R_3+R_4}\right)=\frac{2}{5}$ can be true if:



$$\text{Slope} = \frac{8-2}{20-5} = \frac{2}{5} < 1$$

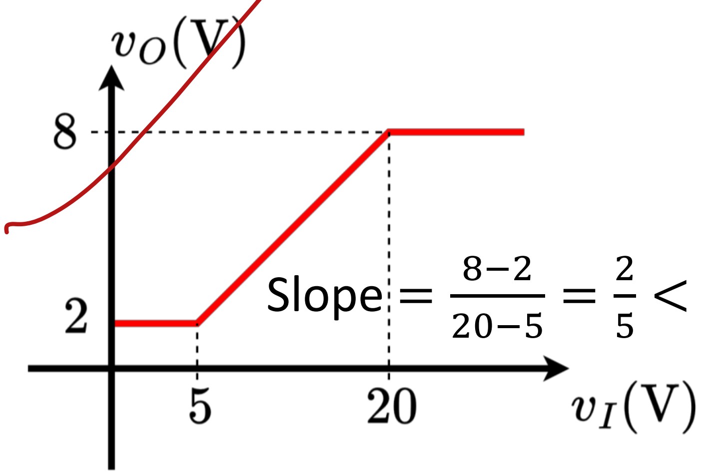
$$\left(1+\frac{R_f}{R_i}\right) = 2$$

and

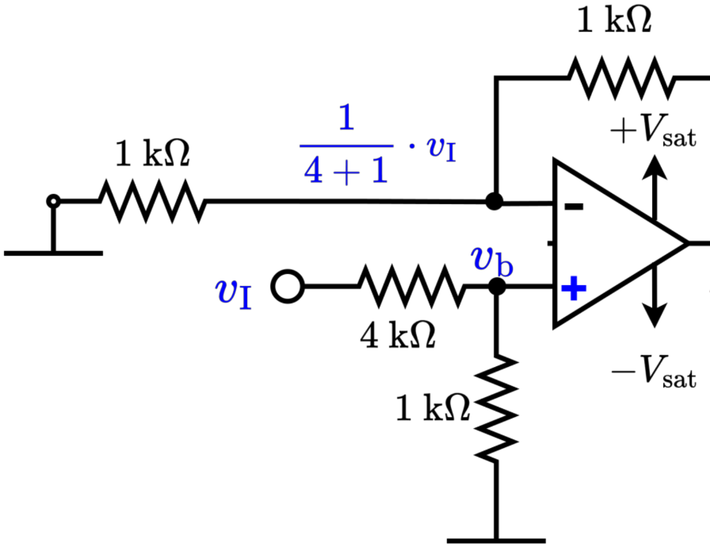
$$\left(\frac{R_4}{R_3+R_4}\right) = \frac{1}{5}$$

Non-Inverting Amplifier – VTC

Draw an Op-Amp Circuit with the following VTC. (What if the slope is less than **1?**)



Slope $= \frac{8-2}{20-5} = \frac{2}{5} < 1$

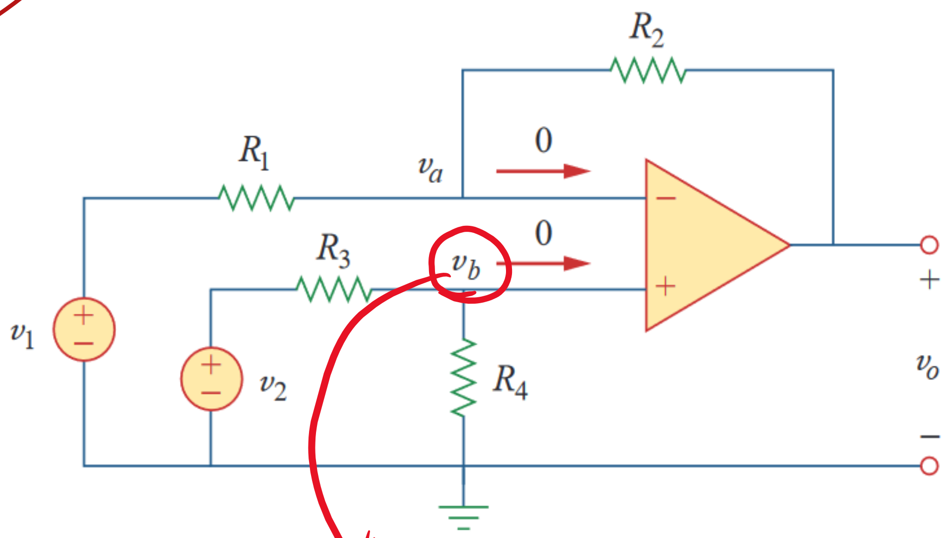


$v_{\mathrm{O}} = (1+1) \cdot (\frac{1}{5}) \cdot v_I$

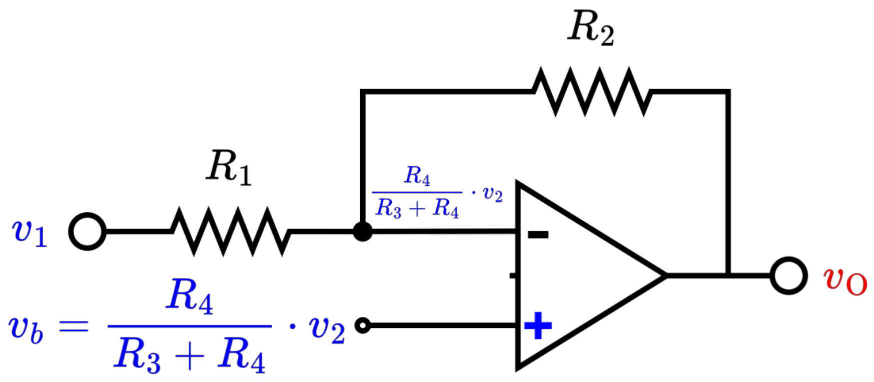
$v_{\mathrm{O}} = (\frac{2}{5}) \cdot v_I$

$\left(1 + \frac{R_f}{R_i}\right) = 2$ and $\left(\frac{R_4}{R_3+R_4}\right) = \frac{1}{5}$

Difference Amplifier / find V_o in terms of V_1, V_2, R_1, R_2, R_3 & R_4



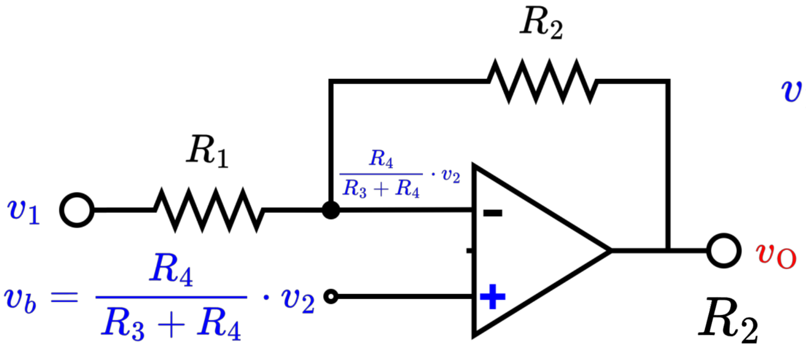
$$v_b = v_a = \frac{R_4}{R_3 + R_4} v_2$$

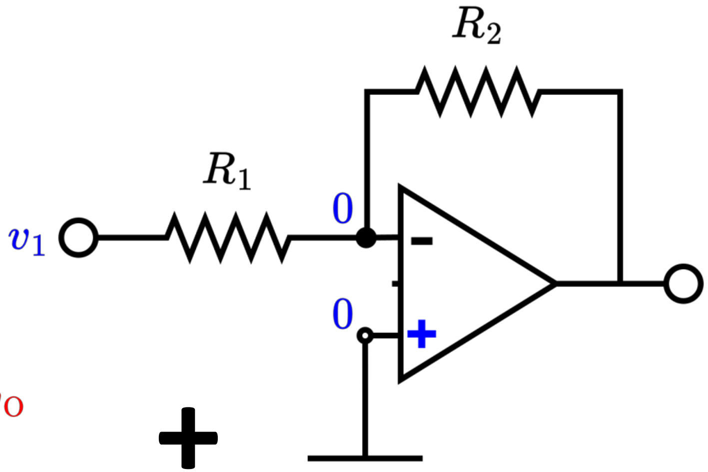


Simplified Circuit

Difference Amplifier

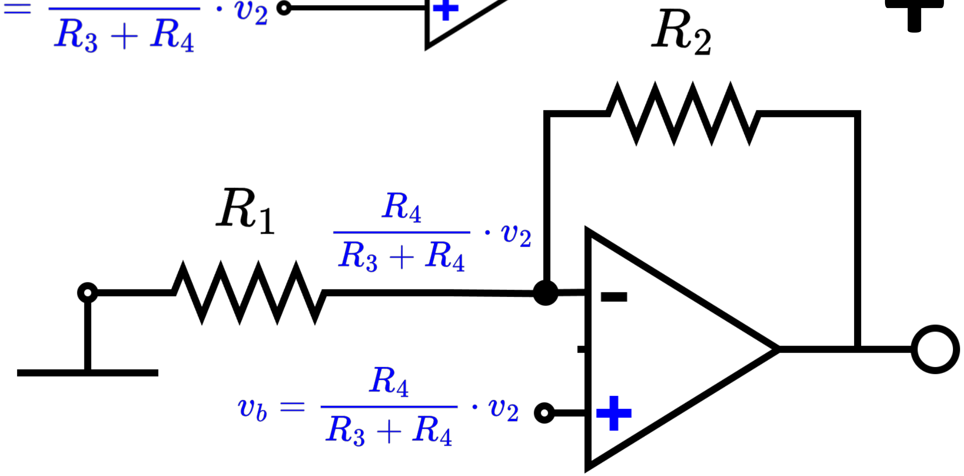
Apply superposition principle:





$$v_O' = -\frac{R_2}{R_1}\cdot v_1$$

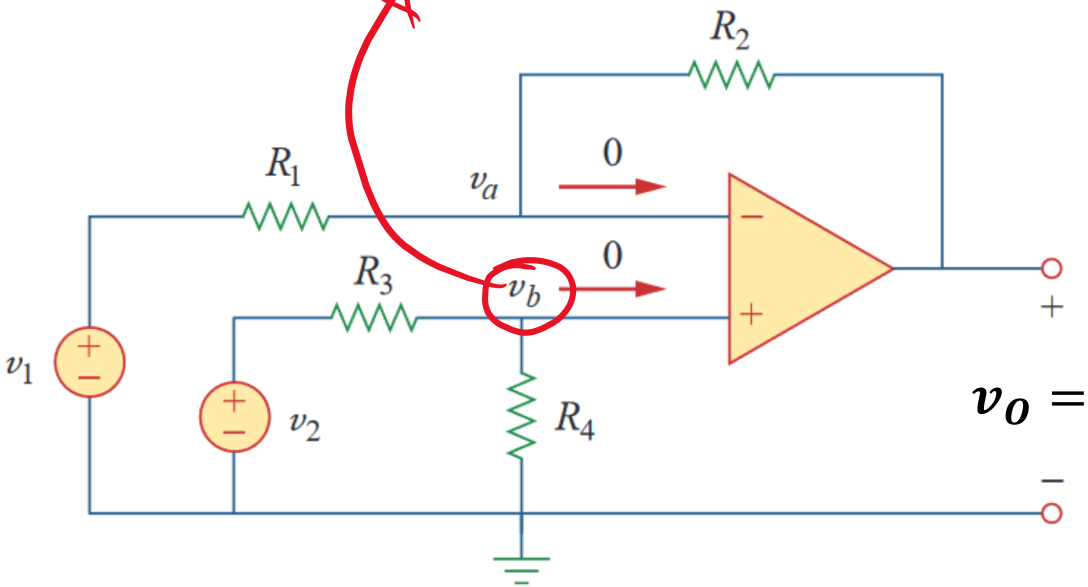
+



$$v_O'' = \left(1+\frac{R_2}{R_1}\right)\cdot\frac{R_4}{R_3+R_4}v_2$$

Difference Amplifier

$$v_b = v_a = \frac{R_4}{R_3 + R_4} v_2$$



$$v_O = v_O'' + v_O'$$

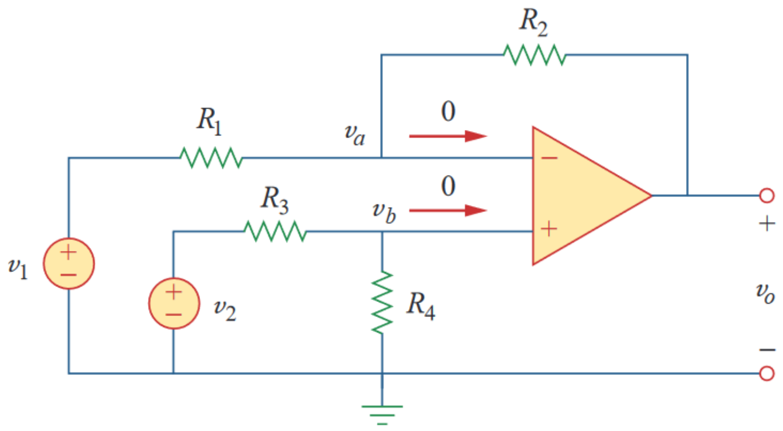
$$v_O = \left(1 + \frac{R_2}{R_1}\right) v_b - \frac{R_2}{R_1} \cdot v_1$$

$$v_O = \left(1 + \frac{R_2}{R_1}\right) \cdot \frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1} \cdot v_1$$

Difference Amplifier – Example 5

Design an op amp circuit with inputs v_1 and v_2 such that

$$v_o = -5v_1 + 3v_2.$$



Solution: Method 1

$$v_O = -\frac{R_2}{R_1} \cdot v_1 + \left(1 + \frac{R_2}{R_1}\right) \cdot \frac{R_4}{R_3 + R_4} v_2$$

$$\therefore \frac{R_2}{R_1} = 5$$

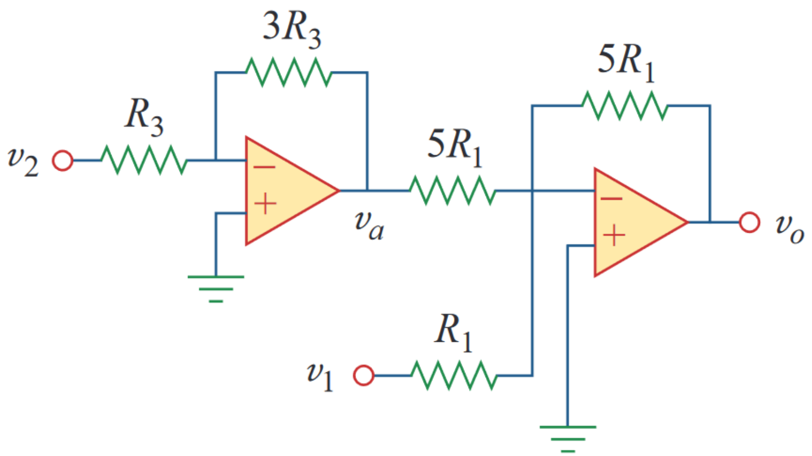
$$\therefore (1 + 5) \cdot \frac{R_4}{R_3 + R_4} = 3$$

$$\Rightarrow R_3 = R_4$$

Difference Amplifier – Example 6

Design an op amp circuit with inputs $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ such that

$$\boldsymbol{v_o = -5v_1 + 3v_2.}$$

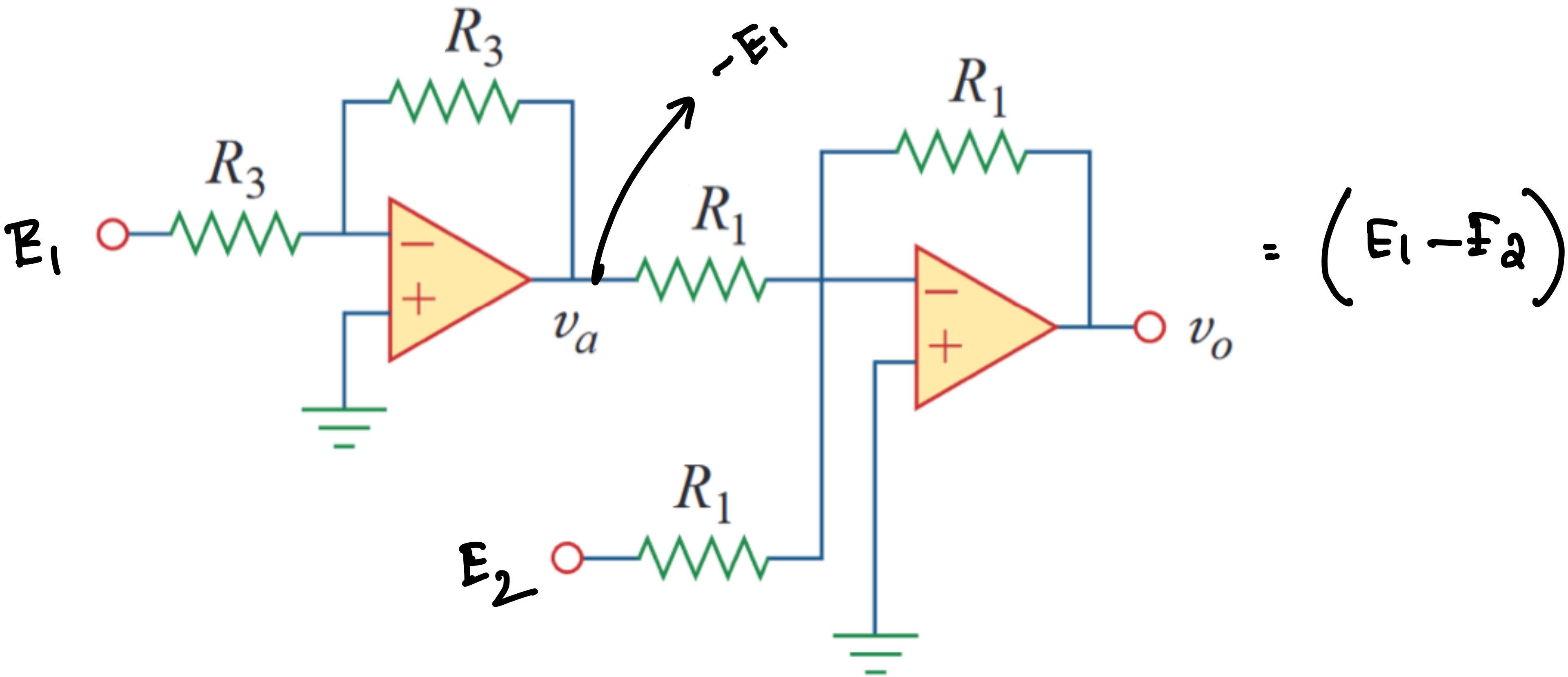


Solution: Method 2
Using two stages, we can implement this function.

$\boldsymbol{-5v_1}$: Can be achieved with one stage inverting amplifier

$\boldsymbol{+3v_2}$: Can be achieved by cascading two inverting amplifiers → $(-\times - = +)$

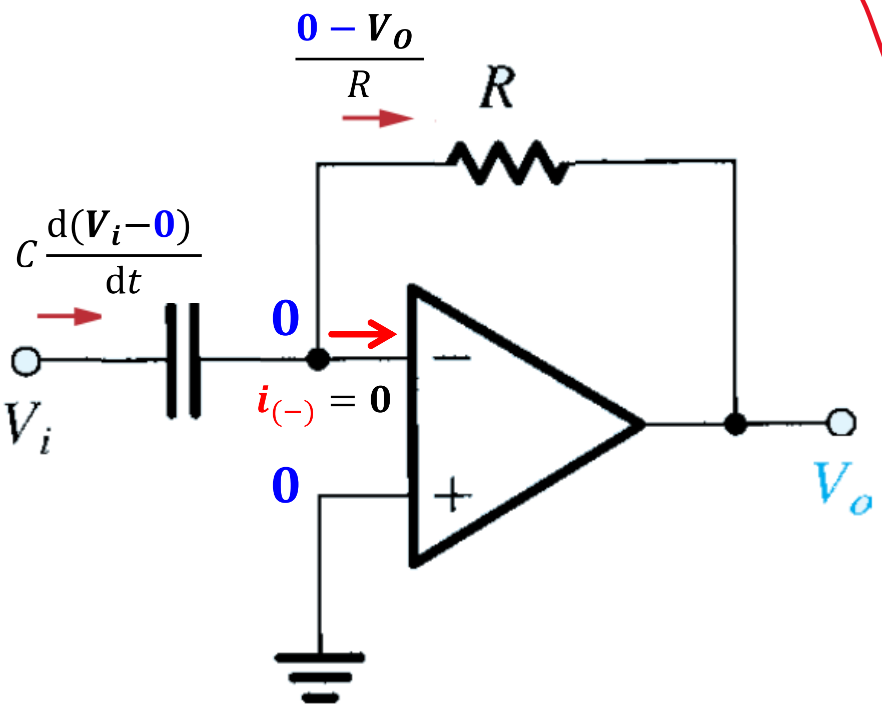
Subtractor $(E_1 - E_2)$



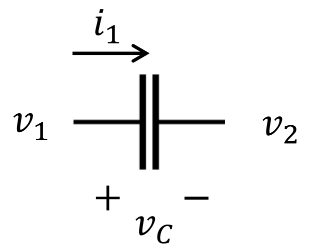
R3
R3
E1
-E1
va
R1
R1
R1
E2
vo
= (E1 − E2)

Op Amp as Differentiator

Since ideal op-amp, $\boldsymbol{i_- = i_+ = 0}$, so $\boldsymbol{i_1 = i_2}$



Review – Capacitor



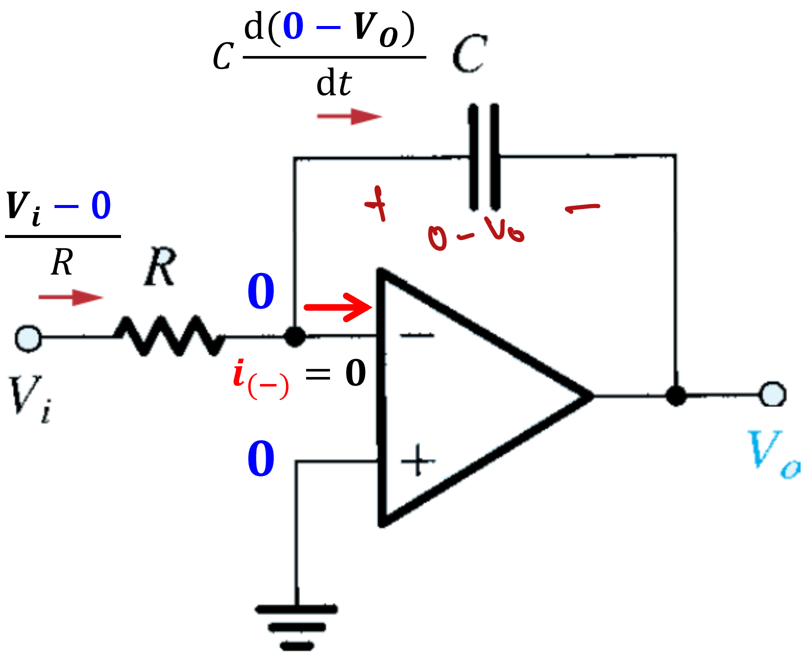
$$i_1 = C\frac{dv_C}{dt} = C\frac{d(v_1 - v_2)}{dt}$$

$$\Rightarrow -\frac{V_o}{R} = C\frac{dV_i}{dt}$$

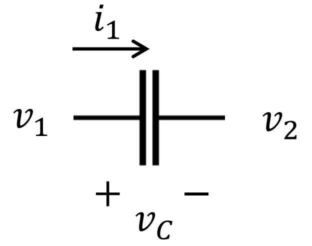
$$\boxed{\Rightarrow V_O = -RC\frac{dv_i}{dt}}$$

Op Amp as Integrator

Since ideal op-amp, $i_{-} = i_{+} = 0$, so $i_1 = i_2$



Review – Capacitor



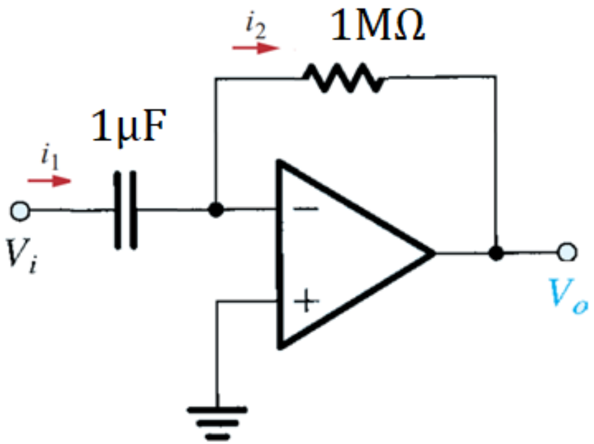
$$i_1 = C\frac{dv_C}{dt} = C\frac{d(v_1 - v_2)}{dt}$$

$$\Rightarrow \frac{V_i}{R} = -C\frac{dV_O}{dt}$$

$$\boxed{\Rightarrow V_O = -\frac{1}{RC}\int V_i(t)dt}$$

Example 8

Observe the following Figure. If V_i = $5 \cdot \sin(6t)$, Find the value of $\boldsymbol{V_0}$



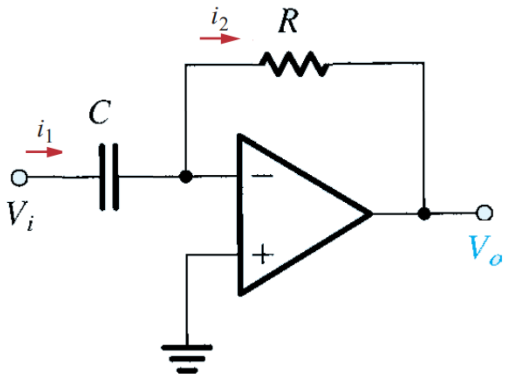
Solution:

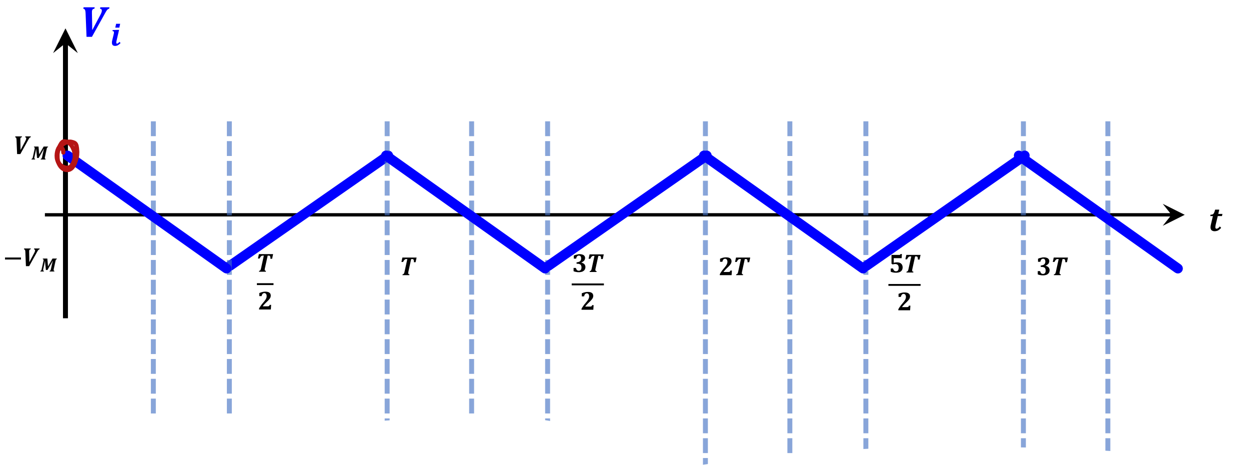
This is a **differentiator**.

So, $v_o = -RC\frac{\mathbf{d}V_i}{\mathbf{d}t} = -1 \times 10^6 \times 1 \times 10^{-6} \times \frac{\mathbf{d}(5 \cdot \sin(6t))}{\mathbf{d}t}$

$\Rightarrow v_o = -1 \times (5 \times 6\cos(6t)) = \mathbf{-30\,cos(6}\boldsymbol{t}\mathbf{)}$ [Ans.]

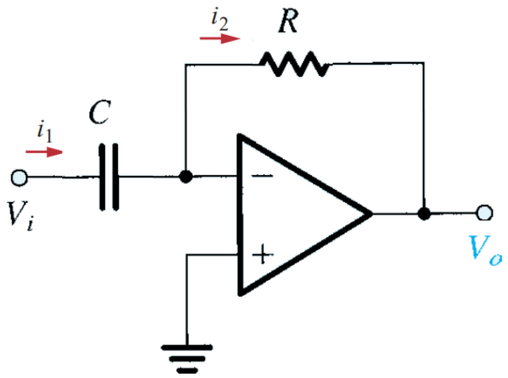
Example 9





Find v_o .

Example 9



Slope: $\left|\frac{\mathrm{d}v}{\mathrm{d}t}\right| = \frac{V_M-(-V_M)}{T/2} = \frac{4V_M}{T}$

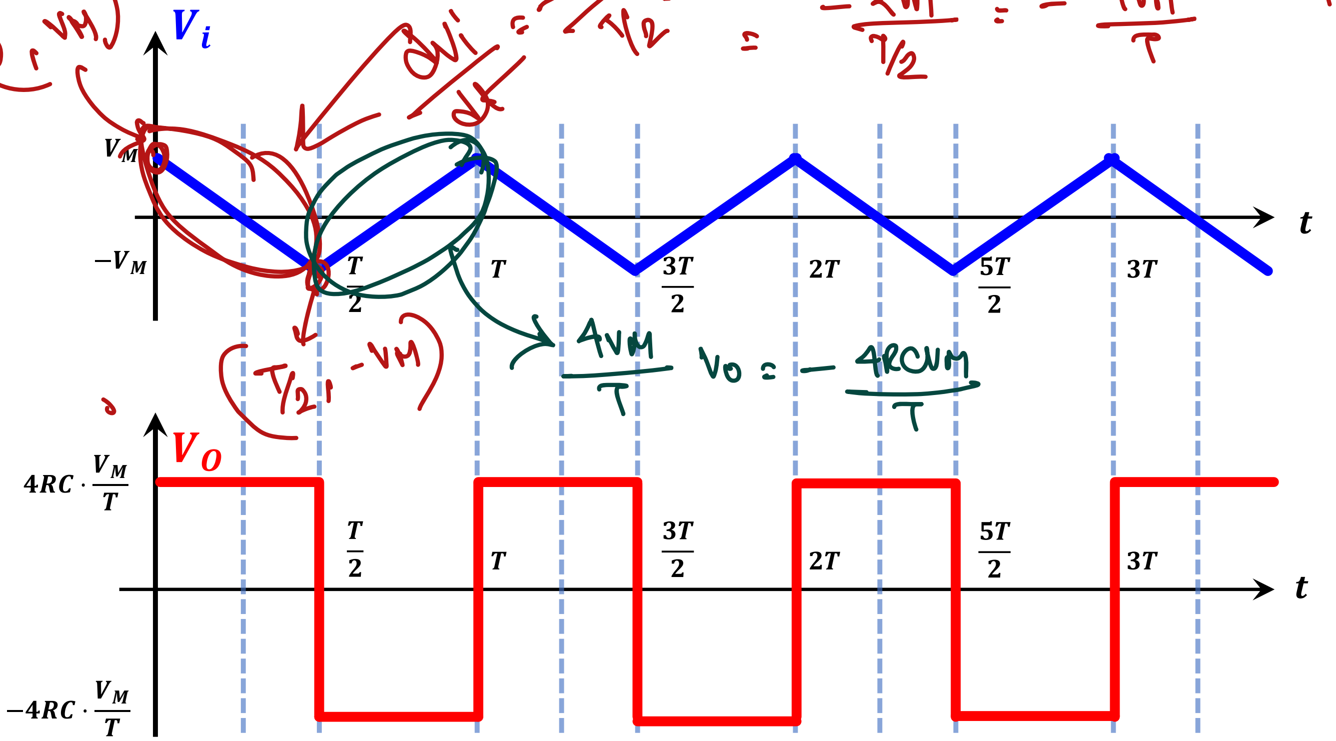
$$V_o = -RC\frac{dv_i}{dt} = \cancel{-}RC \times \frac{\cancel{-}4V_M}{T} = \frac{4RCV_M}{T}$$

$$\frac{dv_i}{dt} = \frac{-V_M - V_M}{T/2 - 0} = \frac{-2V_M}{T/2} = -\frac{4V_M}{T}$$

$(0, V_M)$

$(T/2, -V_M)$

$\frac{4V_M}{T}$ $\quad V_o = -\frac{4RCV_M}{T}$



APPLICATIONS:

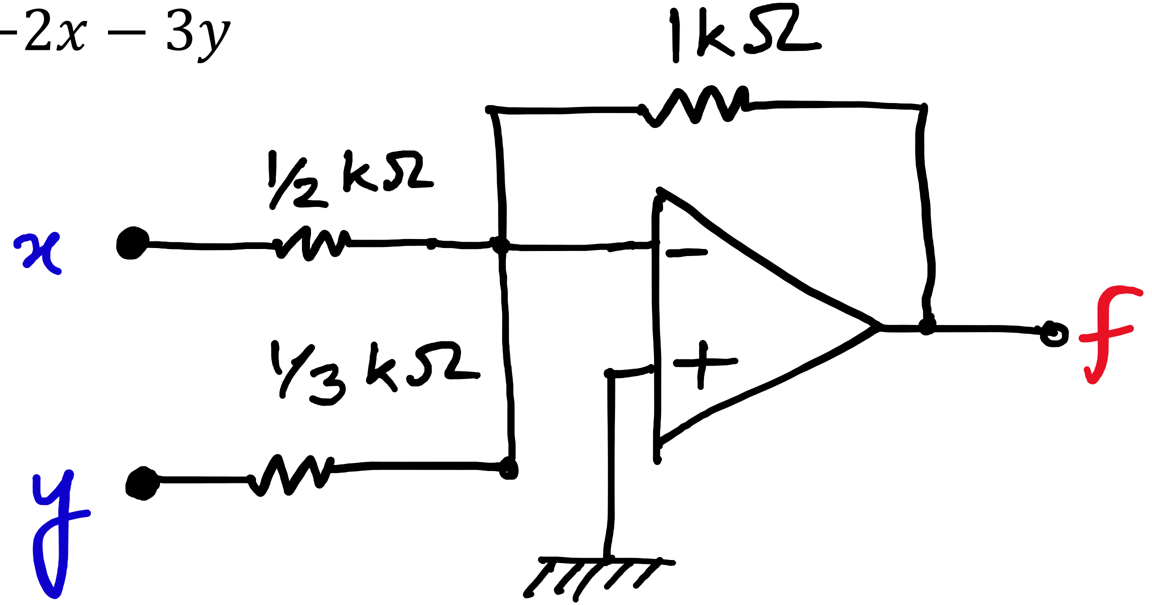
Implementing operational functions

- $f = -2x - 3y$
- $f = -4x + 5y$
- $f = -7x + \frac{d}{dt}y$
- $f = -\frac{1}{3}\int x \cdot dt + 2\ln y + 4z$
- $f = -3\frac{dx}{dt} + 2\exp(y) + 4z$
- $\cancel{f = xy/z}$

APPLICATIONS:

Implementing operational functions

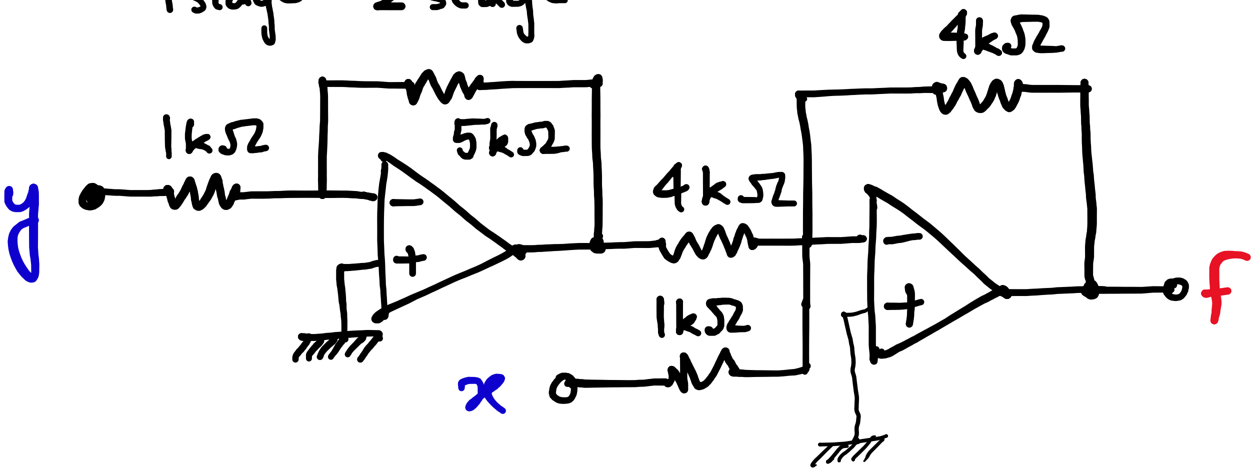
- $f = -2x - 3y$

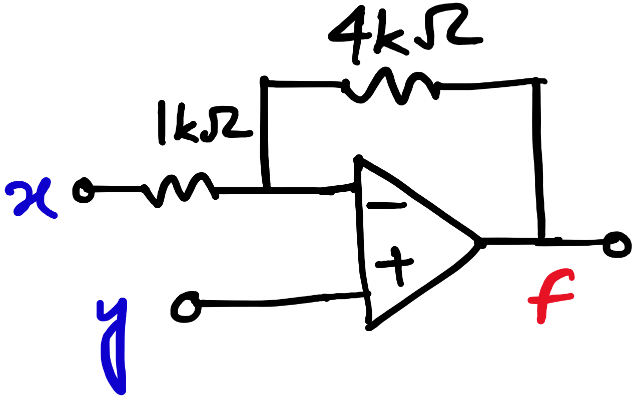


APPLICATIONS:

Implementing operational functions

- $f = \underbrace{-4x}_{\text{1 stage}} + \underbrace{5y}_{\text{2 stage}}$

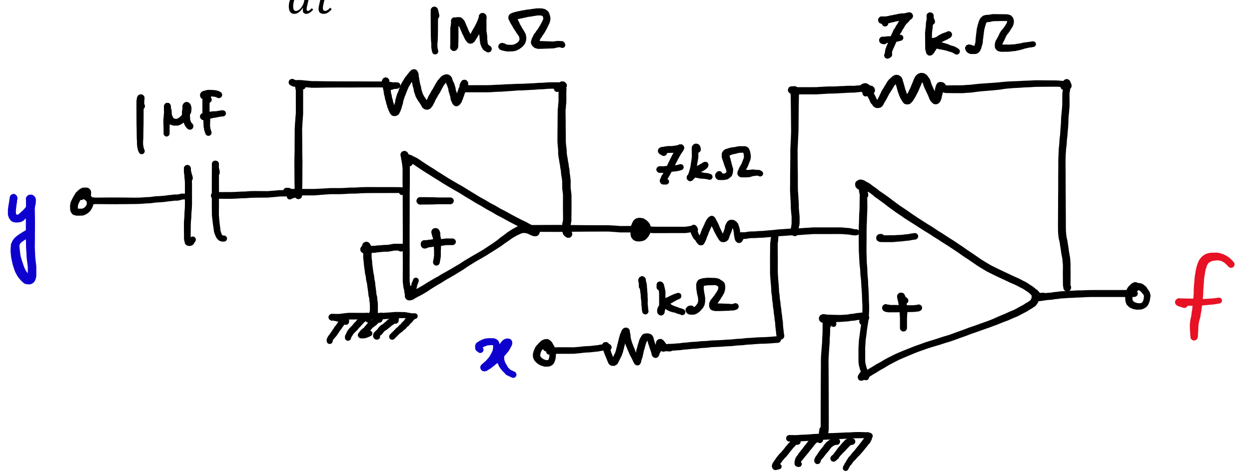




APPLICATIONS:

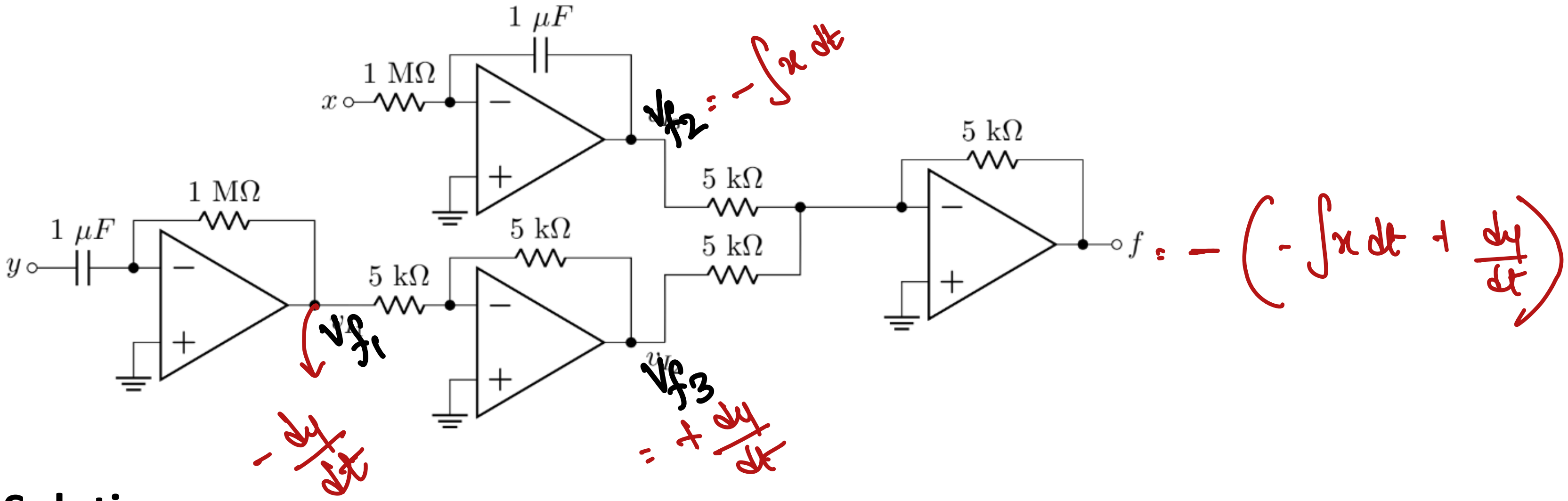
Implementing operational functions

- $f = -7x + \frac{d}{dt}y$



Example

Analyze the circuit below to **find** an expression of f in terms of inputs x and y.

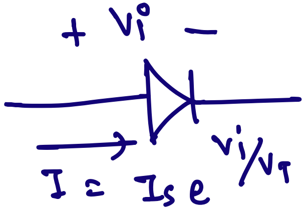


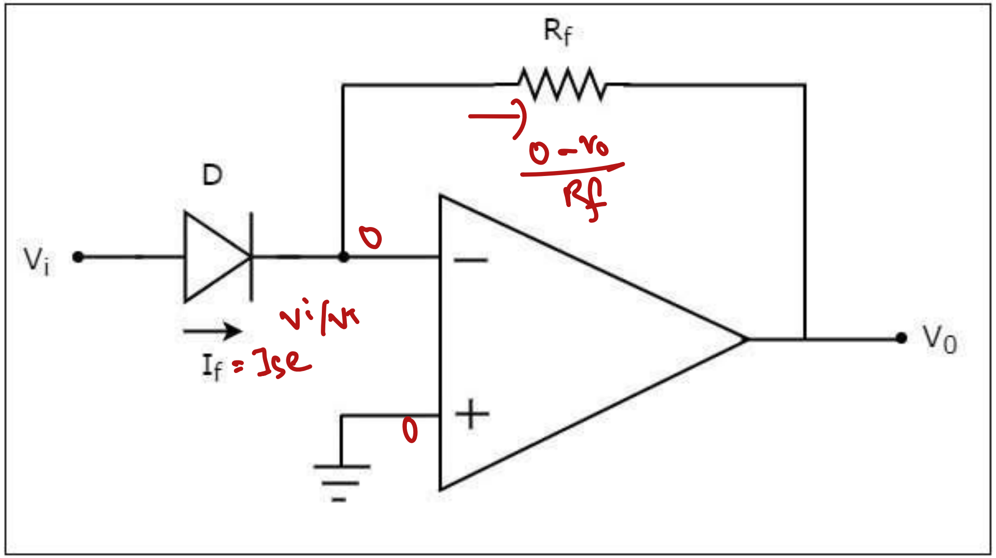
Solution:

$$v_{f1} = -\frac{dy}{dt}\,;\, v_{f2} = -\frac{1}{RC}\int x\,dt\,;\, v_{f3} = -v_{f1} = \frac{dy}{dt}\,;\; v_o = -(\cancel{v_{f1}} + v_{f2} + v_{f3})$$

$$= -\int x\,dt \qquad\qquad = \int x\,dt - \frac{dy}{dt}$$

Exponential (Anti-log) Converter





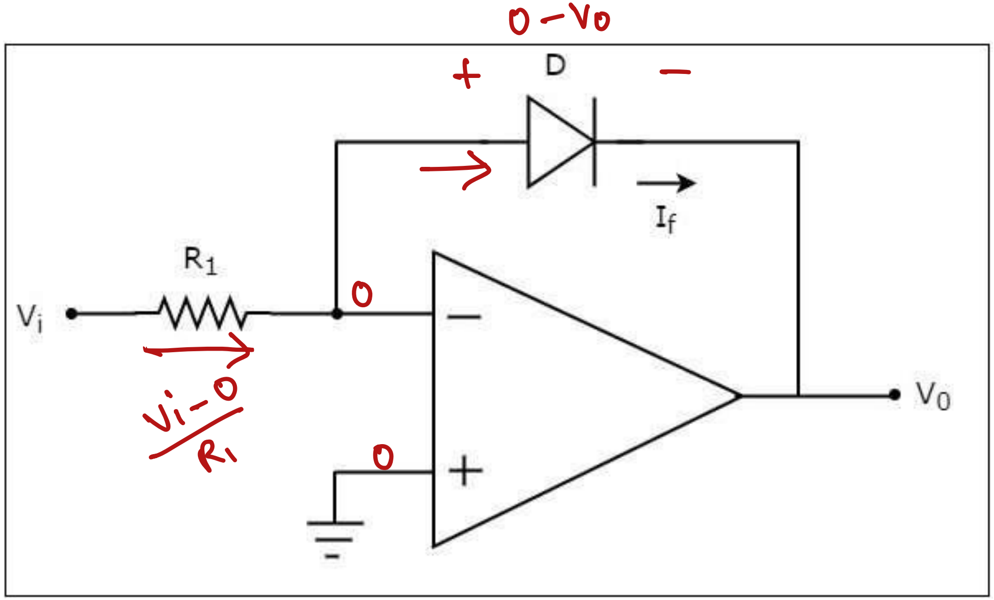
constants

$$I_f = I_S \exp\left(\frac{V_i - 0}{V_T}\right)$$

$$\frac{0 - V_O}{R_f} = I_S \exp\left(\frac{V_i}{V_T}\right)$$

$$\boxed{\boldsymbol{V_O = -I_s R_f \cdot \exp\left(\frac{V_i}{V_T}\right)}}$$

Logarithmic Amplifier



$$I_f = I_S \exp\left(-\frac{V_O}{V_T}\right)$$

$$\frac{V_i}{R_1} = I_S \exp\left(-\frac{V_O}{V_T}\right)$$

$$\frac{V_i}{I_S R_1} = \exp\left(-\frac{V_O}{V_T}\right)$$

$$\boxed{\boldsymbol{V_O = -V_T \cdot \ln\left(\frac{V_i}{I_S R_1}\right)}}$$

$V_T = 0.025\ V$

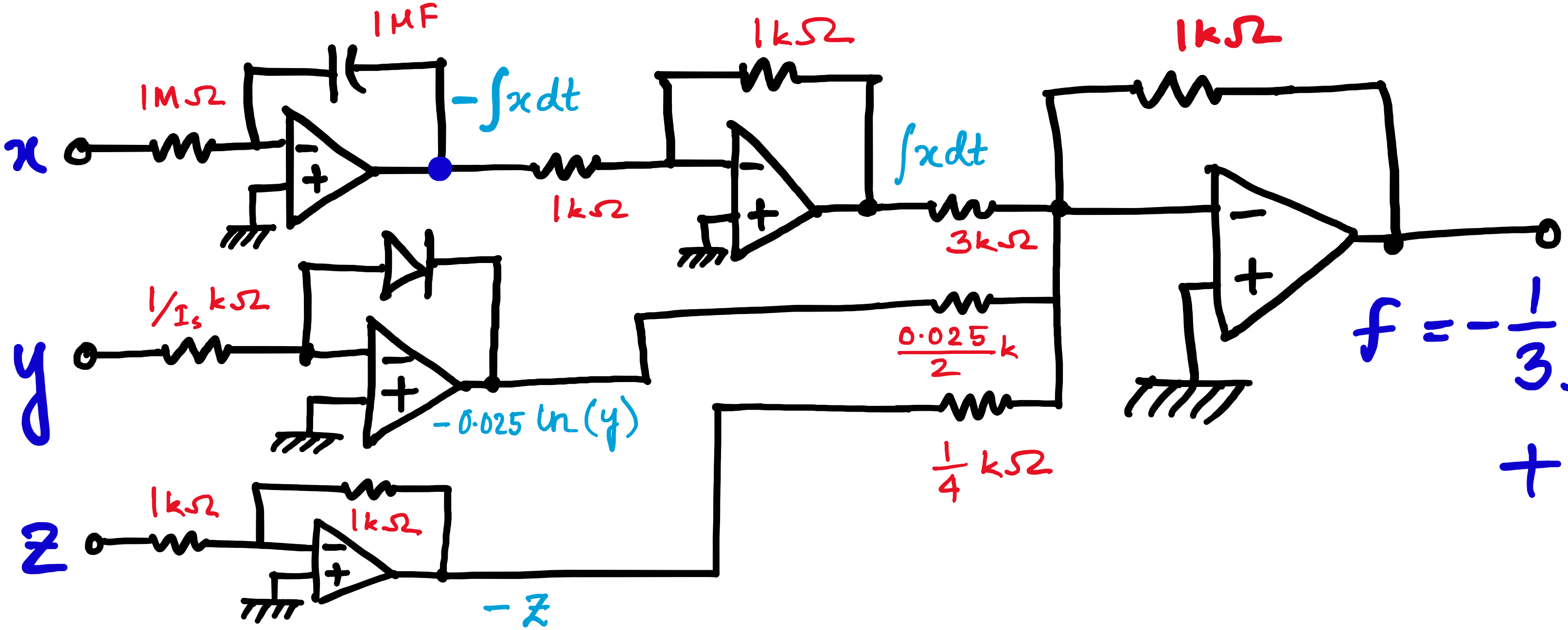
$I_s R_f = 1\ V$

$\therefore R_f = \frac{1}{I_s} k$

APPLICATIONS:

Implementing operational functions

- $f = -\frac{1}{3}\int x \cdot dt + 2\ln y + 4z$ $= -\left(\frac{1}{3}\int x dt - 2\ln(y) - 4z\right)$



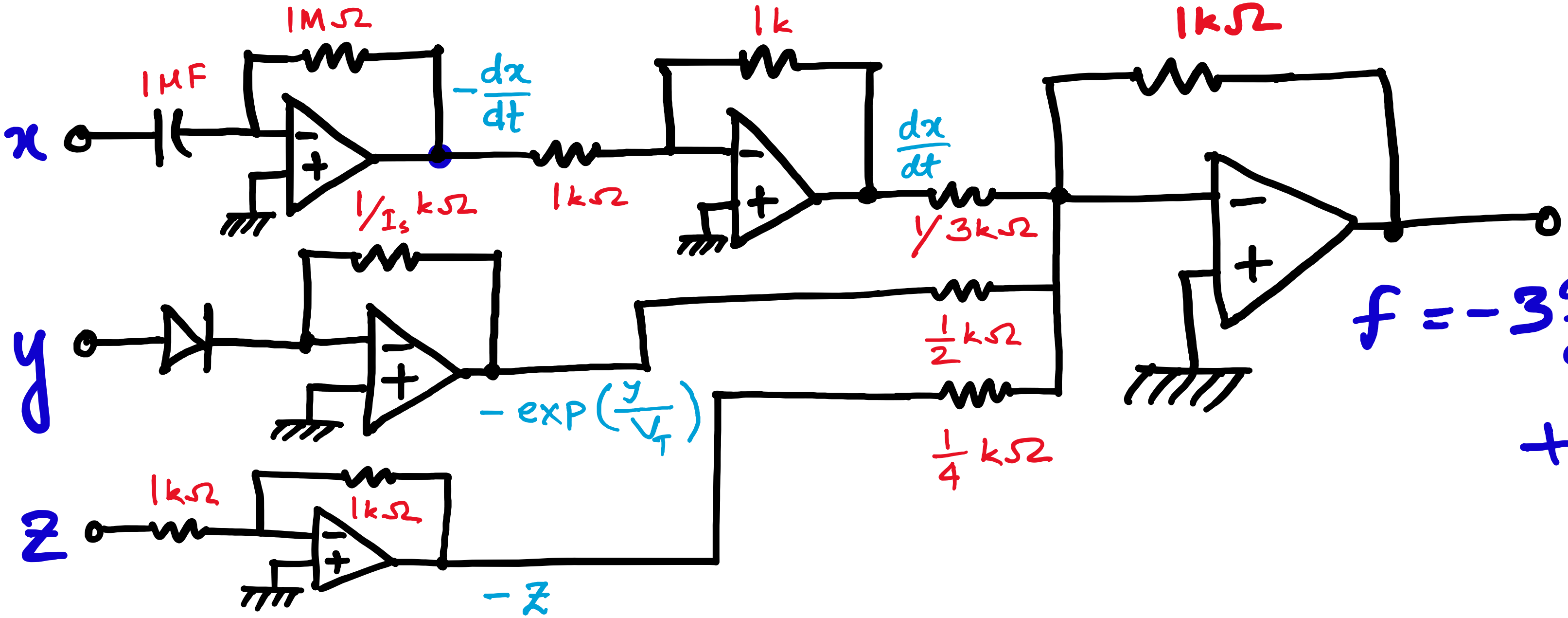
$f = -\frac{1}{3}\int x dt + 2\ln y + 4z$

APPLICATIONS:

Implementing operational functions

- $f = -3\dfrac{dx}{dt} + 2\exp(y) + 4z$

$V_T = 1\text{ V}$
$I_s R_f = 1\text{ V}.$



$$f = -3\frac{dx}{dt} + 2\exp(y) + 4z.$$

APPLICATIONS:

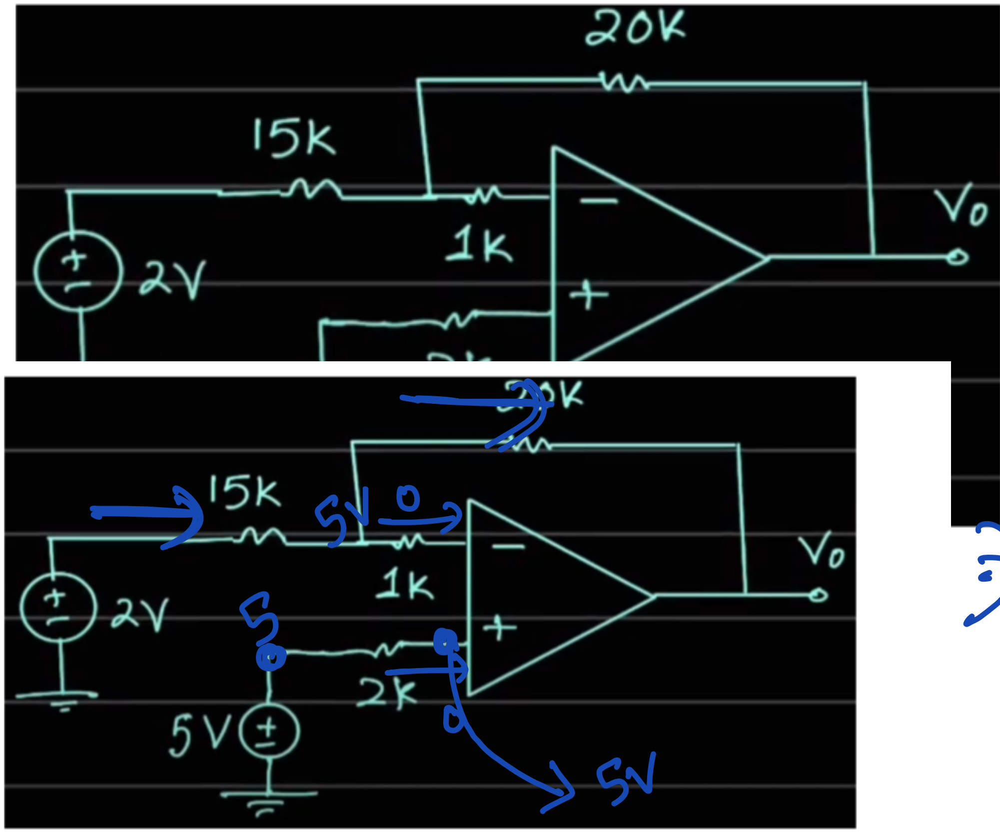
Implementing operational functions

- $f = -\frac{1}{3}\int x \cdot dt + 2\ln y + 4z$

$v_+ = v_-$

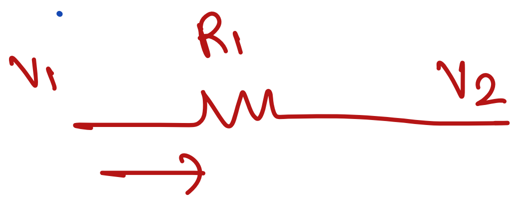
$i_+ = i_- = 0$

Determine the output voltage, v_0



$$\frac{2-5}{15} = \frac{5-V_o}{20}$$

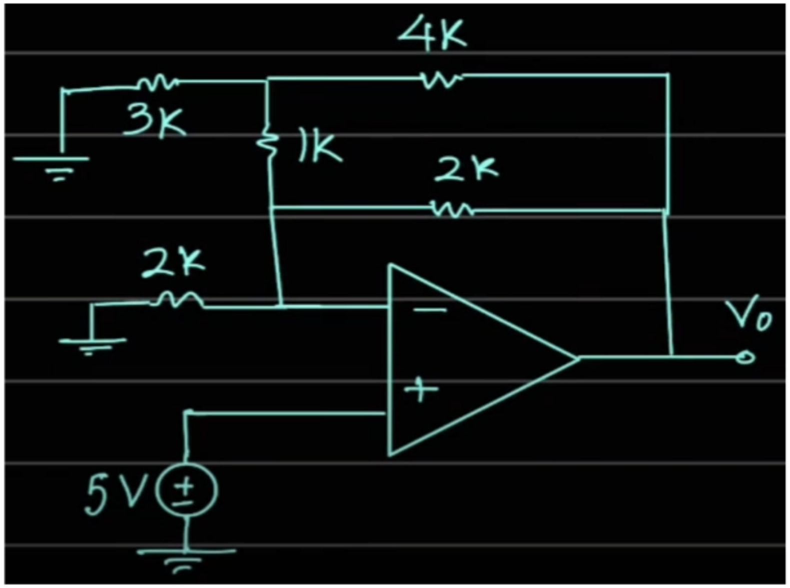
$\Rightarrow V_o = ?$



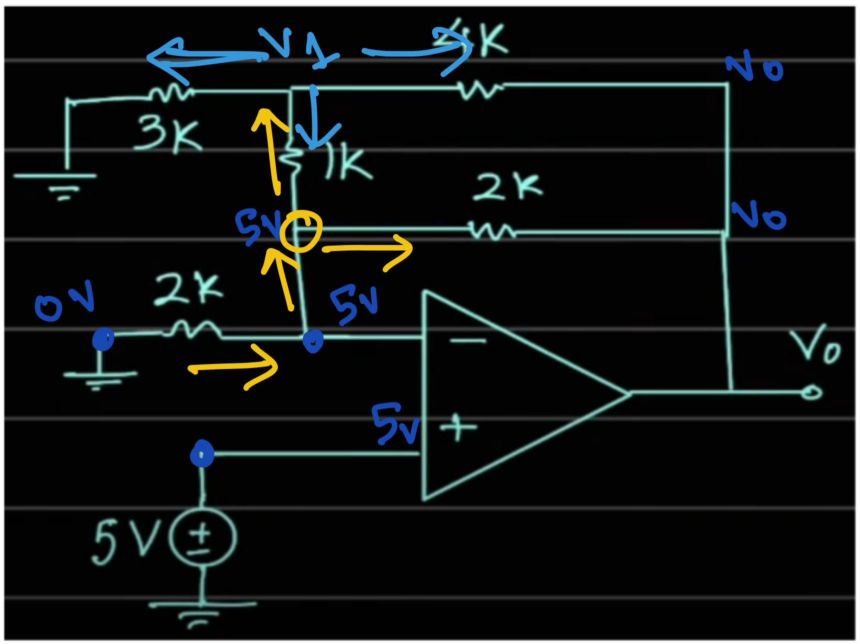
$$I = \frac{V_1 - V_2}{R_1}$$

$$I = \frac{V_1 - V_2 - V_s}{R}$$

Determine the output voltage, v_0



Determine the output voltage, v_0

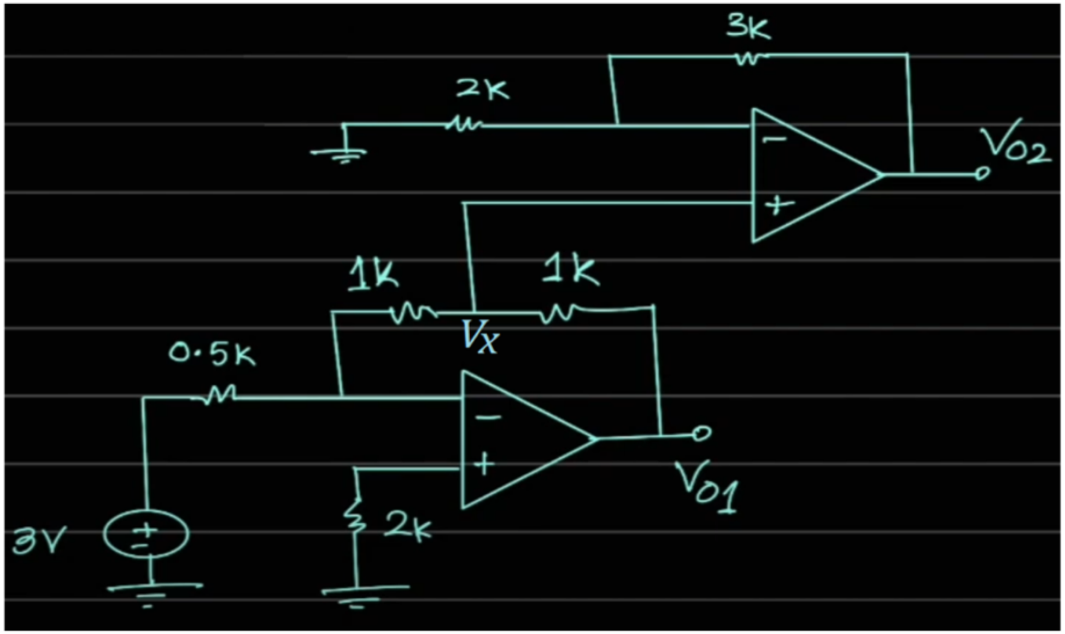


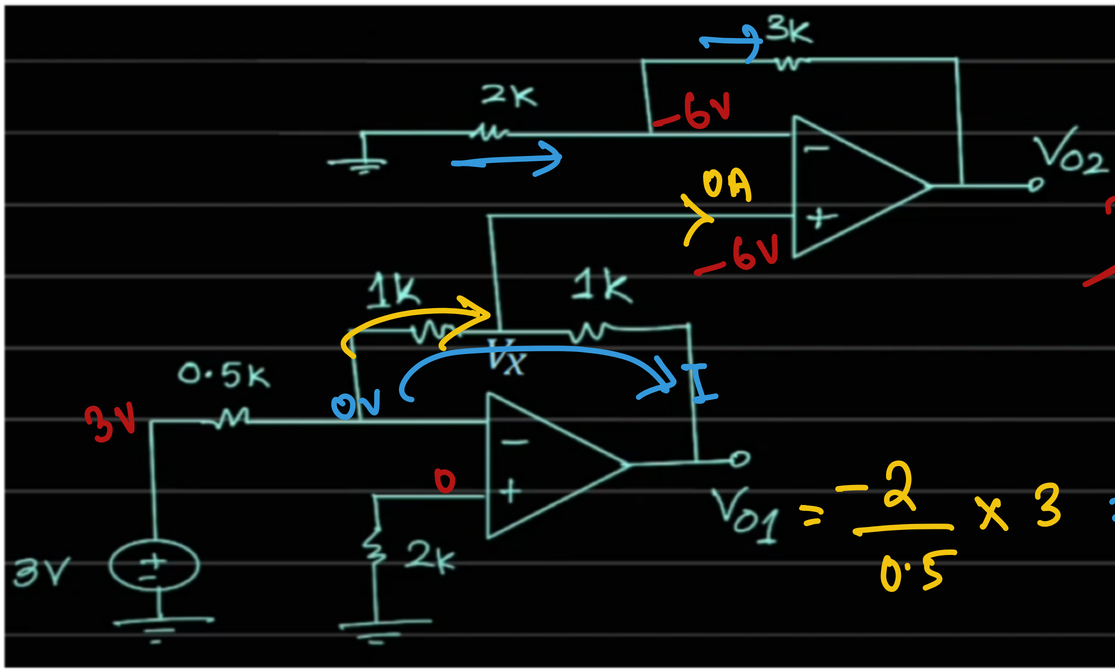
$$\frac{0-5}{2} = \frac{5-V_0}{2} + \frac{5-V_1}{1} \quad \text{— (I)}$$

$$\frac{V_1-0}{3} + \frac{V_1-V_0}{4} + \frac{V_1-5}{1} = 0 \quad \text{— (II)}$$

Solve $\rightarrow$ v_0, v_1

Determine the voltages: V_{01}, V_X, V_{02}





3rd

$$\frac{0-(-6)}{2} = \frac{-6-V_{o2}}{3}$$

$$\Rightarrow V_{o2} = ✓$$

2nd

$$\frac{0-V_x}{1k} = 6m$$

$$\Rightarrow V_x = -6V$$

1st

$$V_{o1} = \frac{-2}{0.5} \times 3 = -4 \times 3 = -12$$

$$I = \frac{0-(-12)}{2} = 6mA$$