Lecture-1

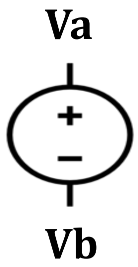
# KCL, KVL, Nodal Analysis & Line diagram

# Alternative Circuit Representation: Line diagrams

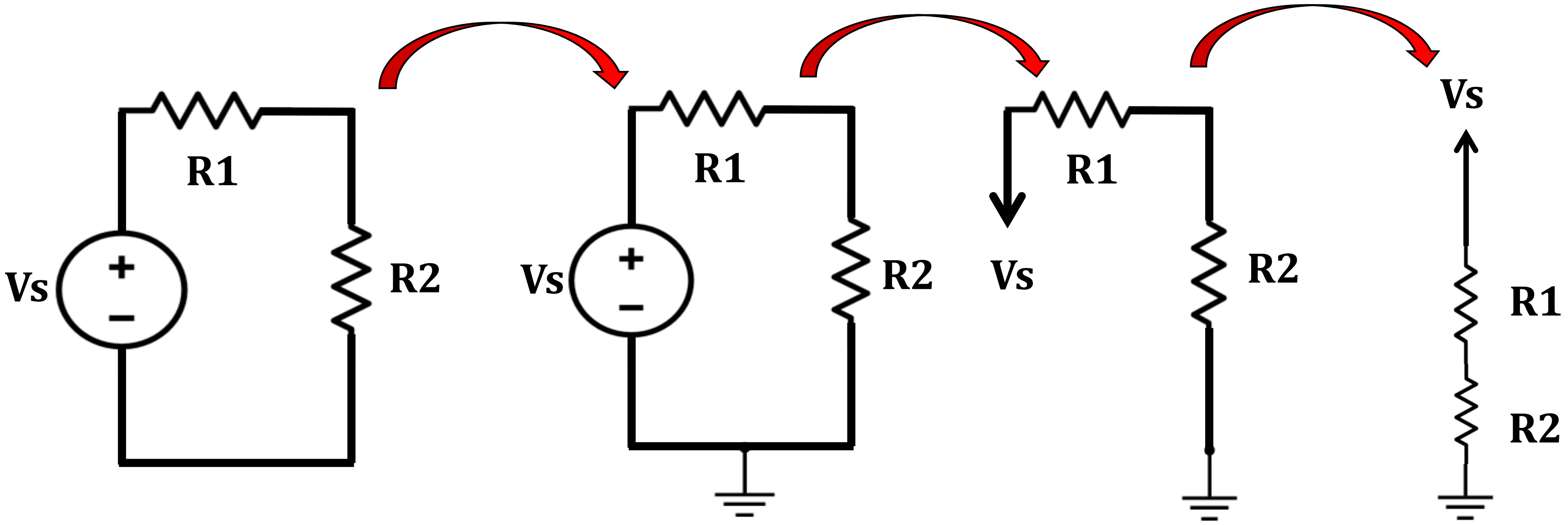
**Goal:** representing circuits using Short hand notations

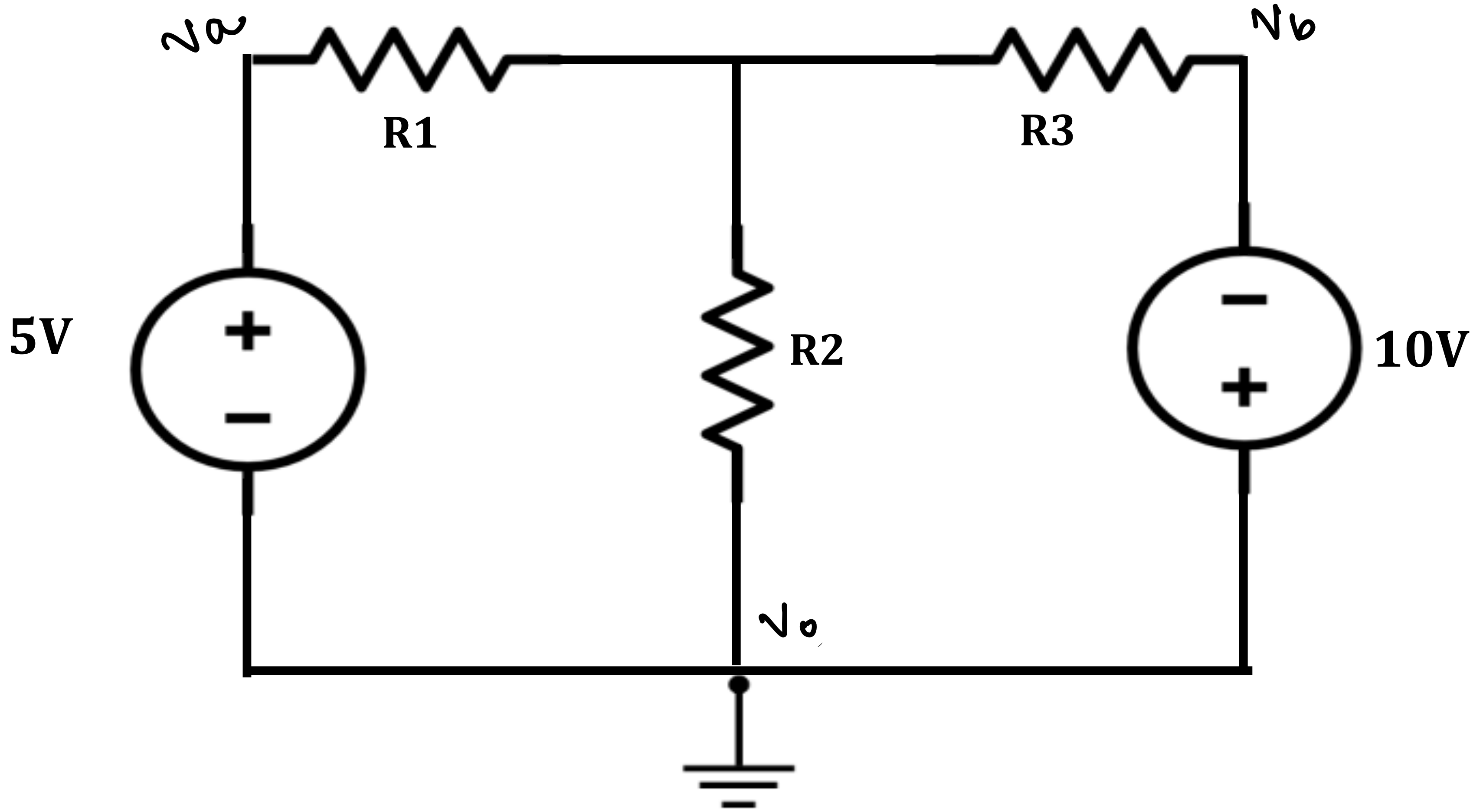
**Steps**

- Identify the 'Nodes'
- Select one node as 'Ground'
- Replace the other known voltage nodes with an arrow

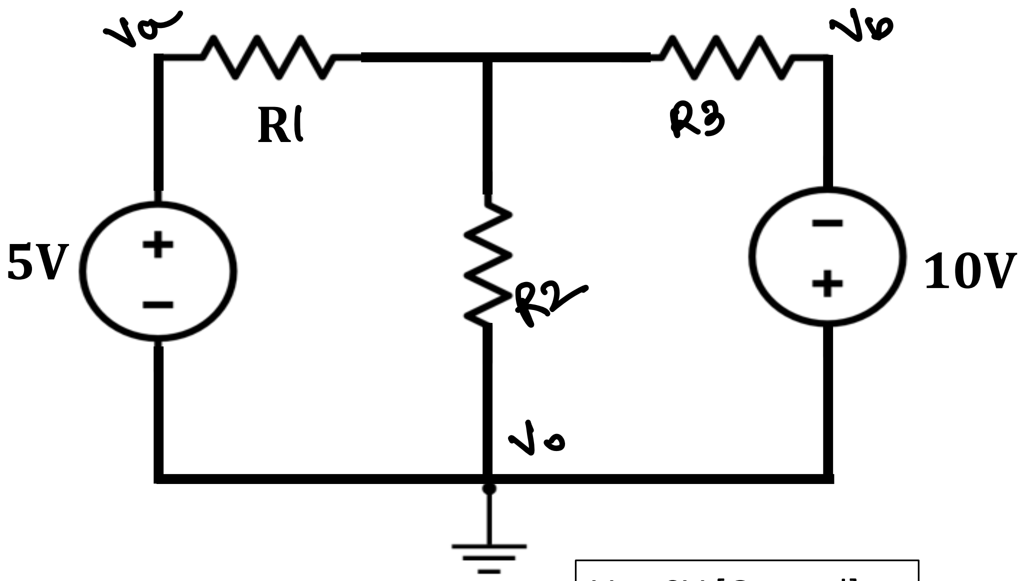



Vs= Va-Vb

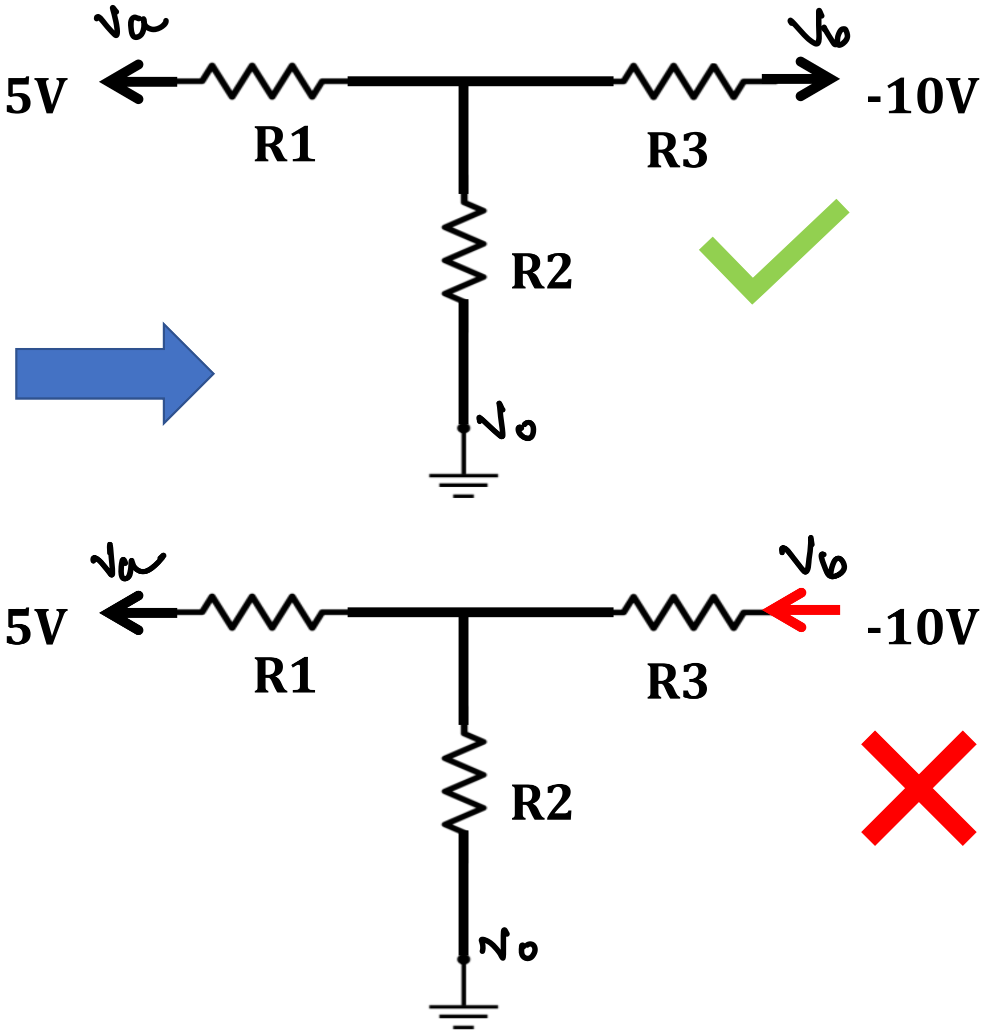



va
R1
R3
vb
5V
+
−
R2
−
+
10V
vo

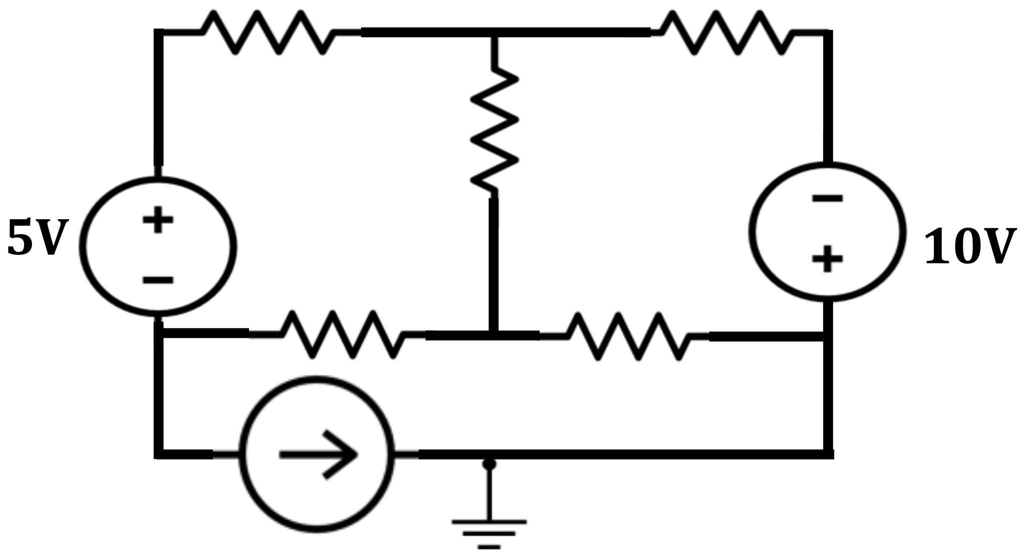
- Circuits with voltage sources of opposite polarities

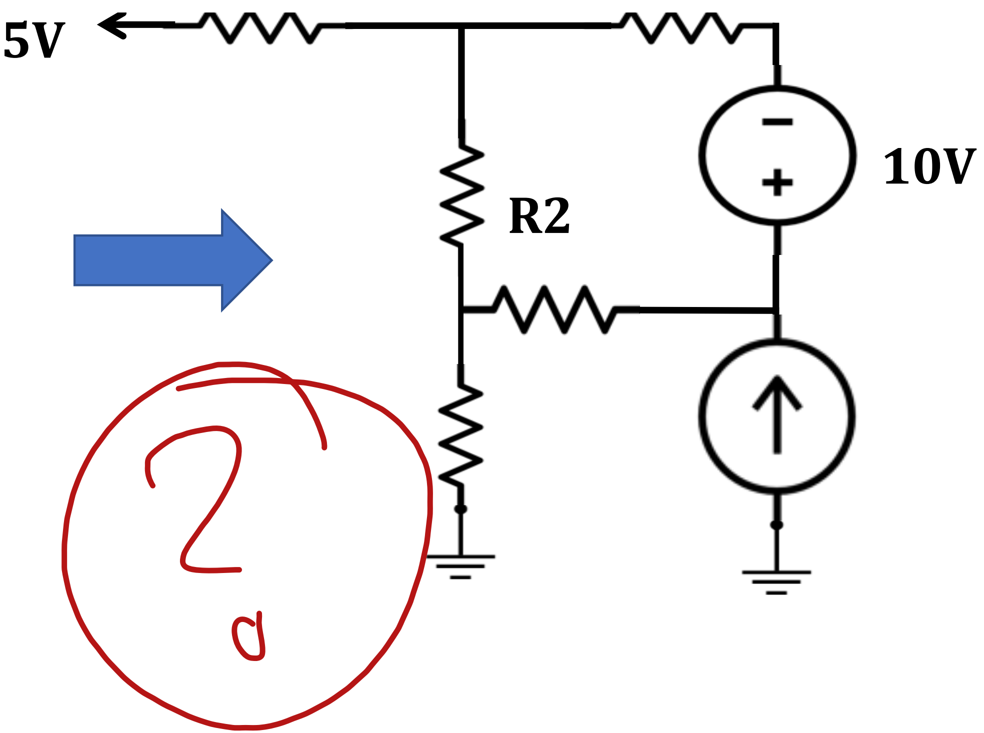



Vo= 0V [Ground]
**For 5V :**
Va-Vo= (5-0) V
=> Va = 5V
**For 10V :**
Vo-Vb= (0-10) V
=> Vb = -10 V



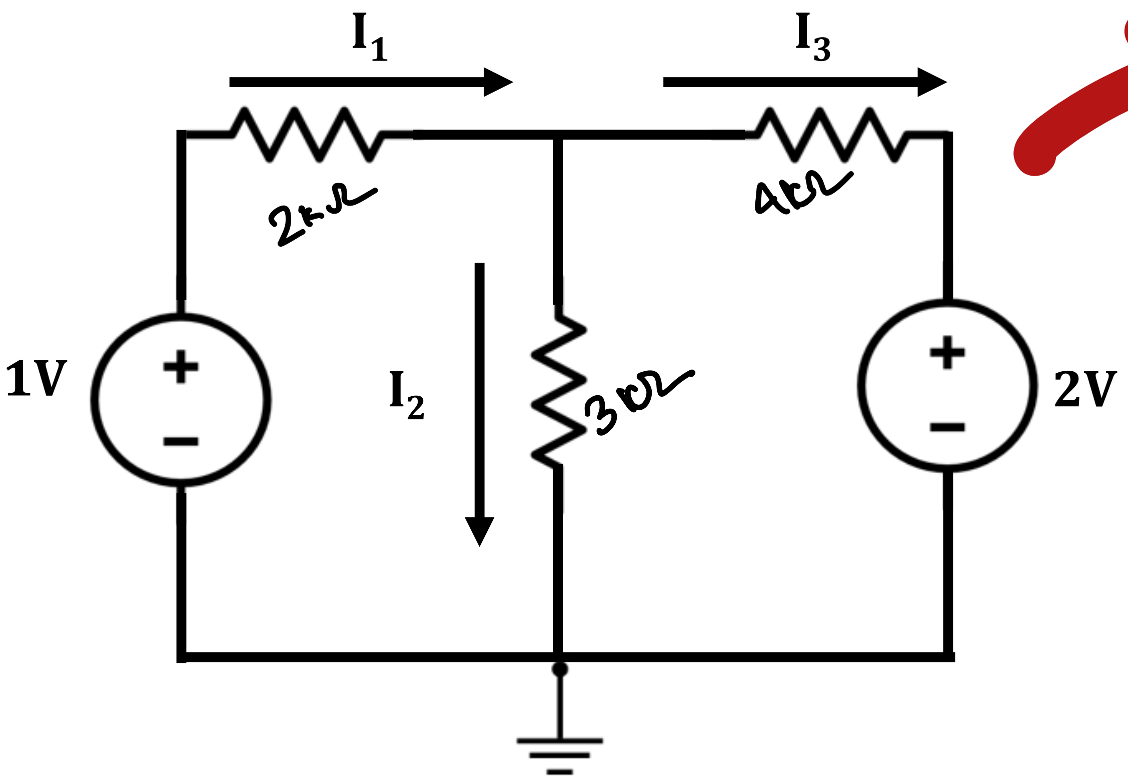
- Circuit with a current source/ floating voltage source: Keep them as they are!
- **Floating voltage sources:** None of the terminals of the voltage source is connected to the reference i.e. ground node

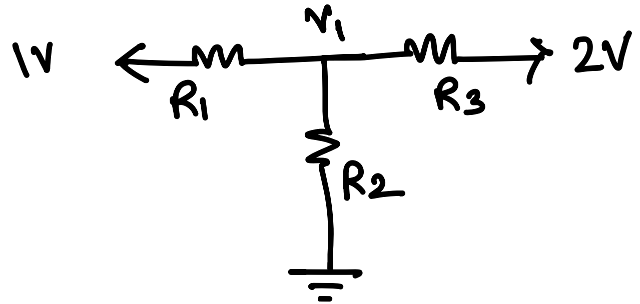


**Kirchhoff's Current Law (KCL):**

- “The algebraic sum of all currents entering and exiting a node must equal zero.”
- “Currents flowing into a node (or a junction) must be equal to the currents flowing out of it.”



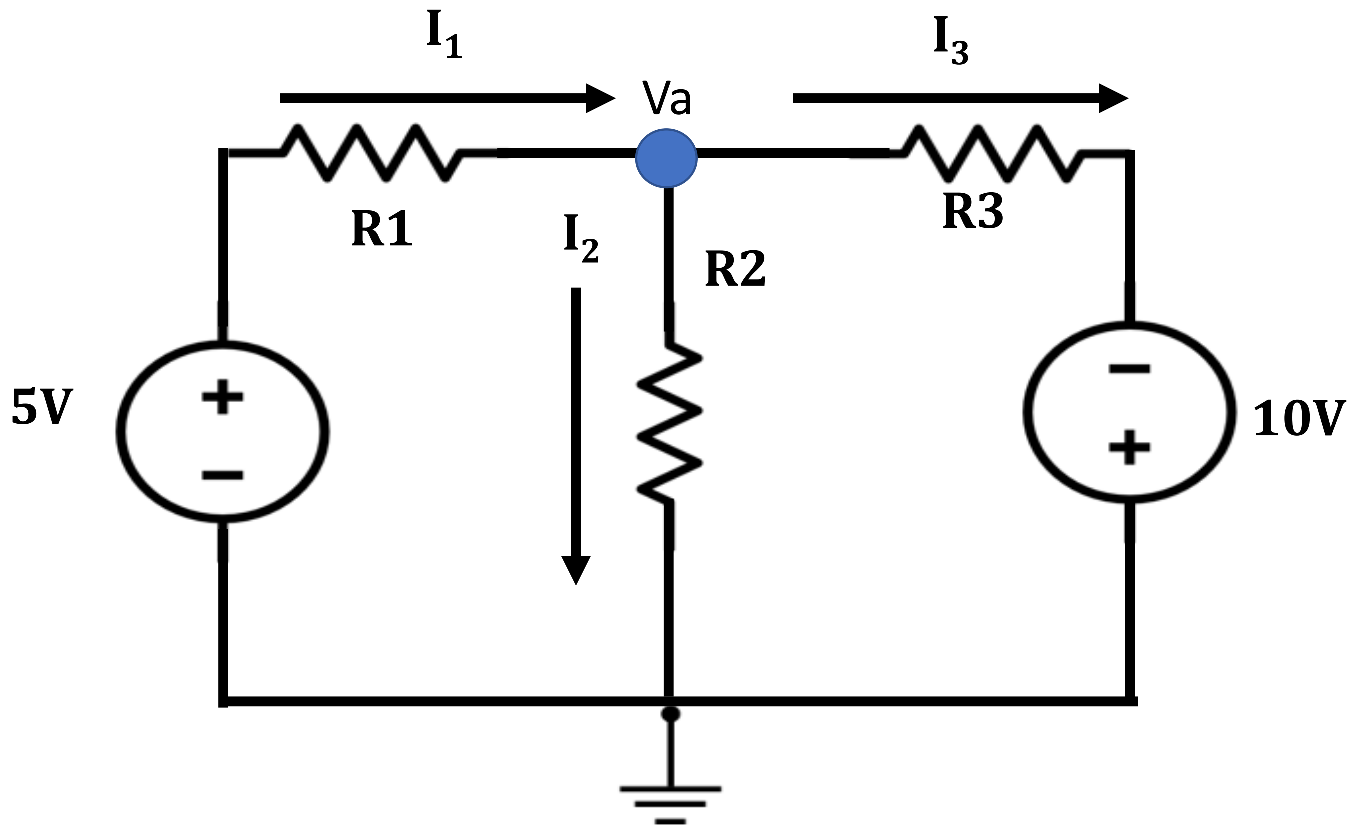




$$I_1 = I_2 + I_3$$

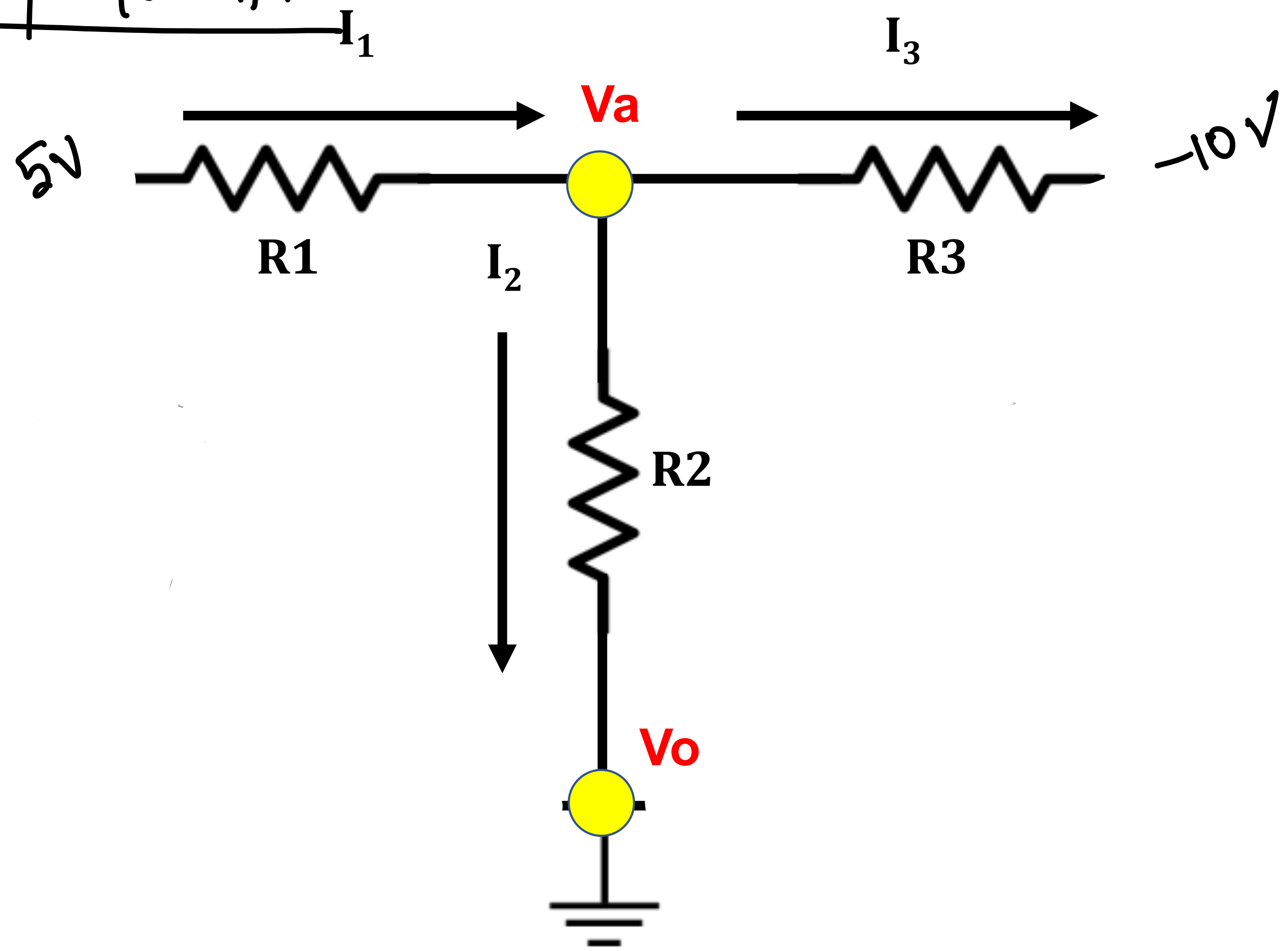
$$\frac{1 - V_1}{R_1} = \frac{V_1 - 2}{R_3} + \frac{V_1 - 0}{R_2}$$

*This is also applicable for supernodes!

**Nodal analysis:**



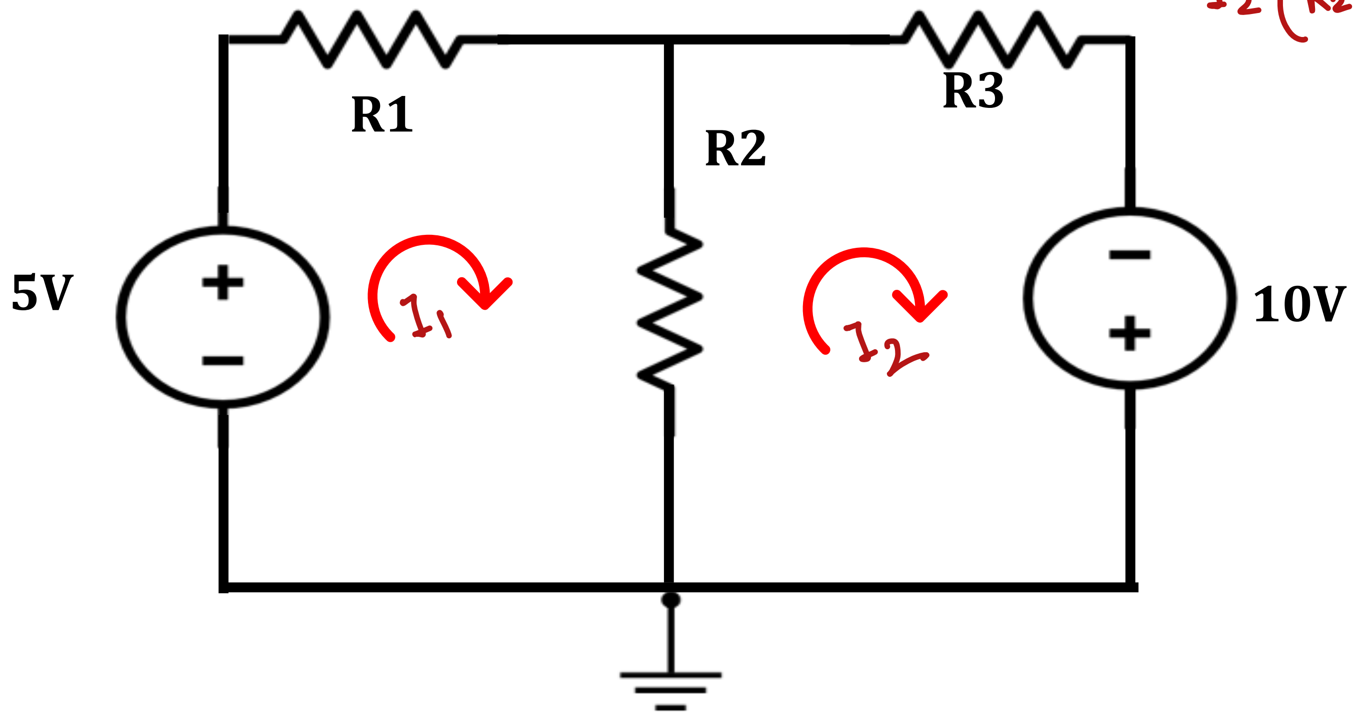
Do it by yourself:



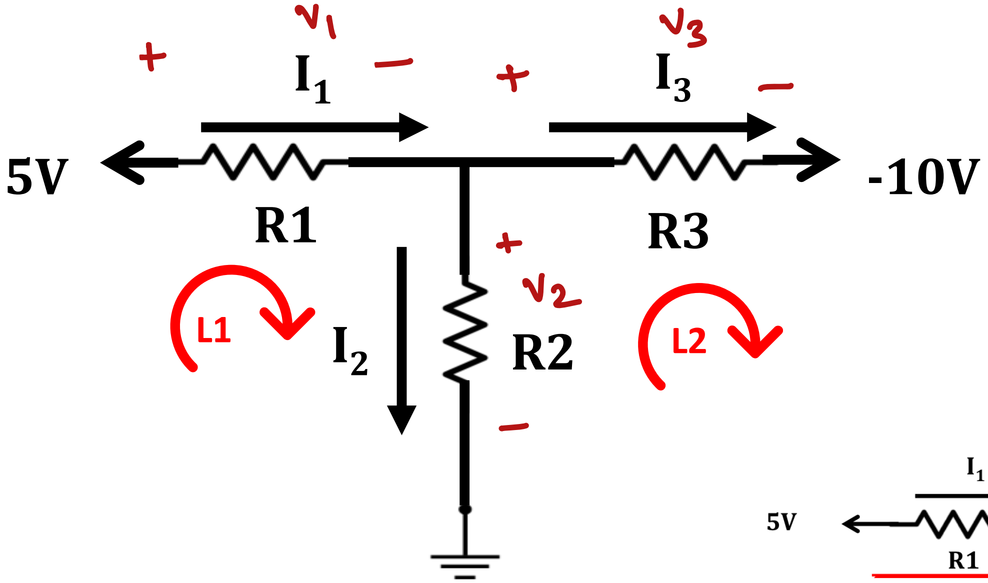
**Kirchhoff's Voltage Law (KVL):** The algebraic sum of all voltages in a loop must equal zero

$$-5 + I_1R_1 + I_1R_2 - I_2R_2 = 0$$

$$I_2(R_2 + R_3) - 10 - I_1R_2 = 0$$



$$\sum V\ [along\ line] = Voltage\ at\ the\ starting\ of\ the\ node - Voltage\ at\ the\ ending\ of\ the\ node$$



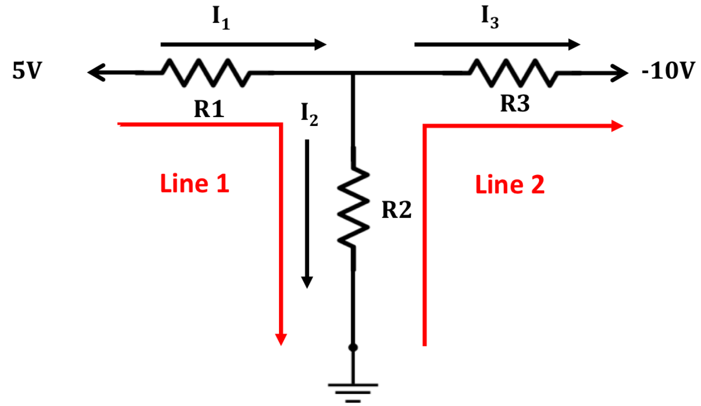

Line 1: $5-0 = I_1R_1 + I_2R_2 = 0$

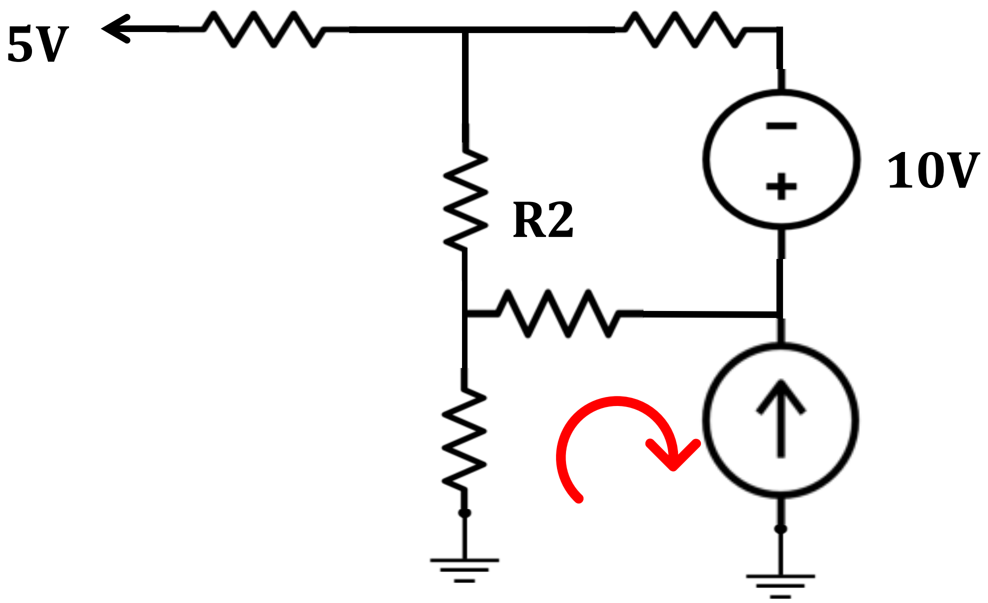
Line 2: $0+10 = -I_2R_2 + I_3R_3$

$V_1 = I_1R_1$

$V_2 = I_2R_2$

$V_3 = I_3R_3$

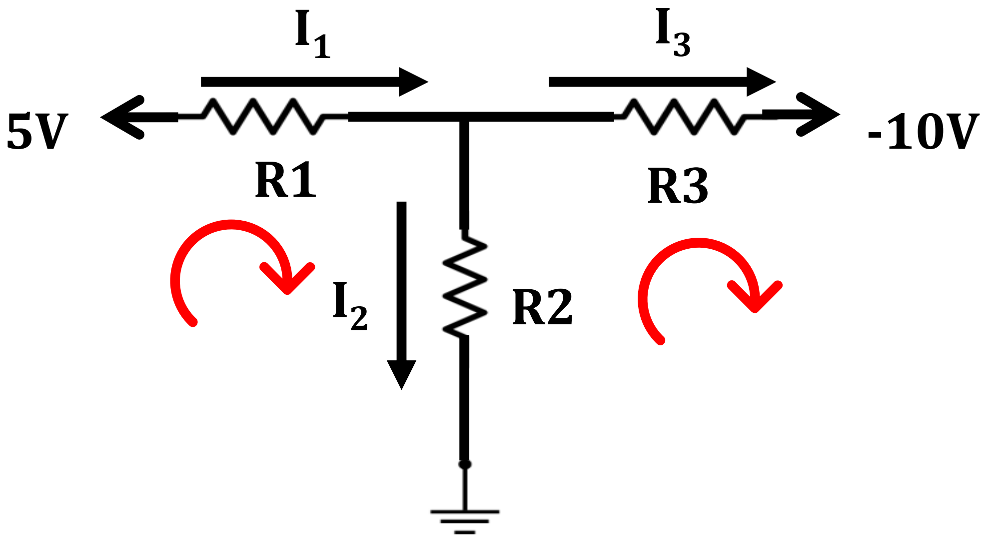




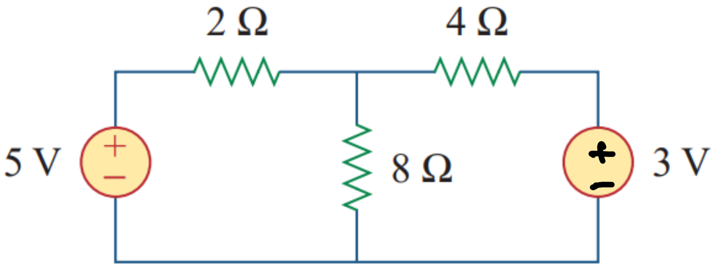

Can you write a KVL equation along this line?

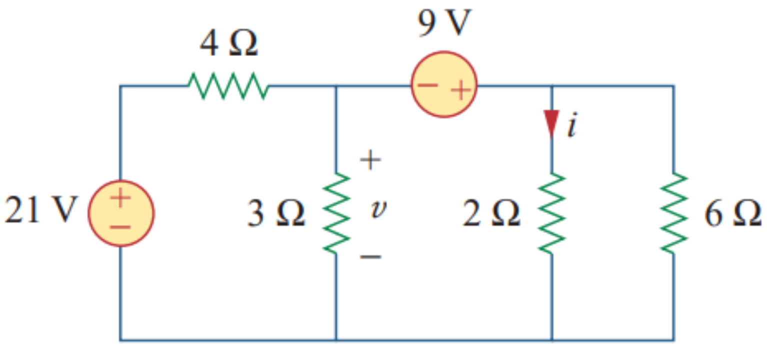
**Nodal analysis:**

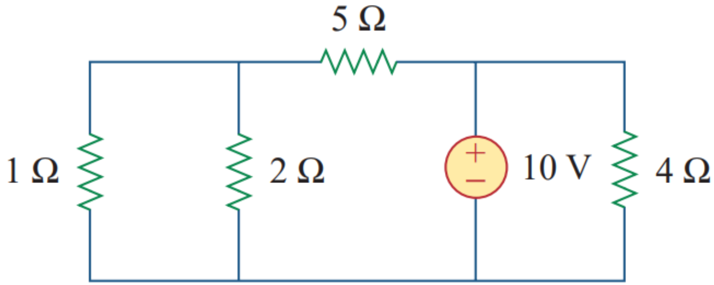
$$V_a\left(\frac{1}{R1}+\frac{1}{R2}+\frac{1}{R3}\right)-\frac{5}{R1}-\frac{0}{R2}-\frac{-10}{R3}=0$$

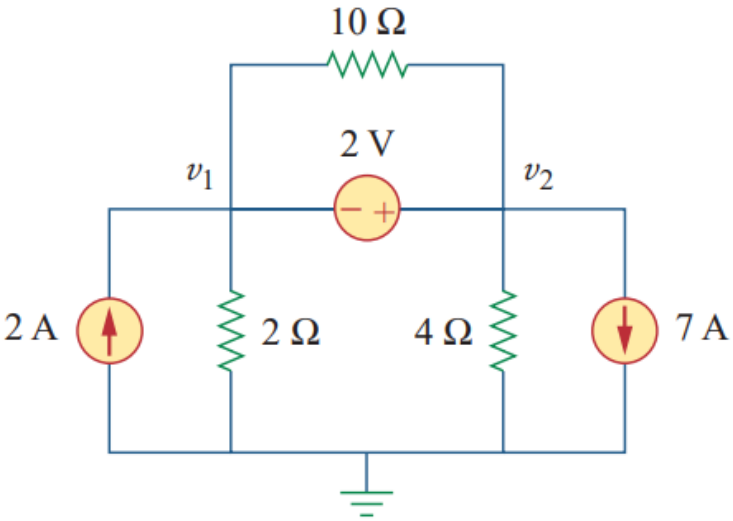


**Practice Problems:** i) Draw Alternative Circuit Diagrams , ii) Write down KCL equations, iii) Write down KVL equations and iv) Nodal equation



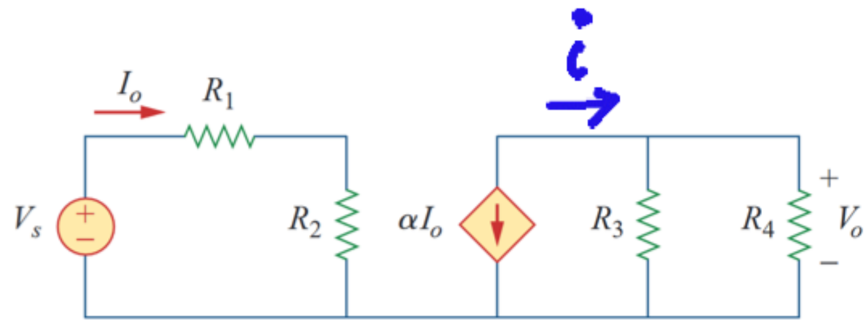



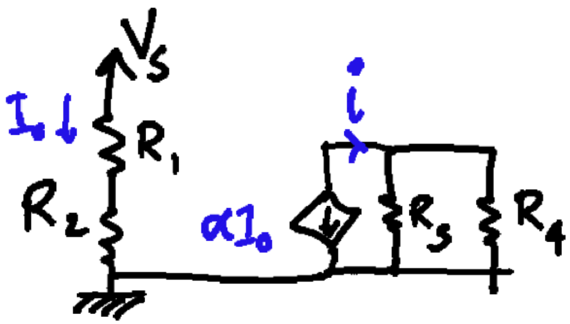



# Practice Problems

① Question:



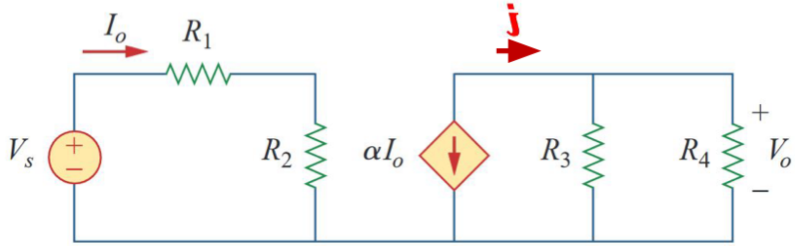
For the circuit, find $\left|\frac{V_o}{V_s}\right|$ in terms of $\alpha, R_1, R_2, R_3$ and $R_4$.

If $R_1 = R_2 = R_3 = R_4$ what value of $\alpha$ will produce $\left|\frac{V_o}{V_s}\right| = 10$?

**Solution:**

Ohm's Law across $R_1 + R_2$.

$$I_O = \frac{V_S}{R_1 + R_2}$$




$$i = -\alpha I_O$$

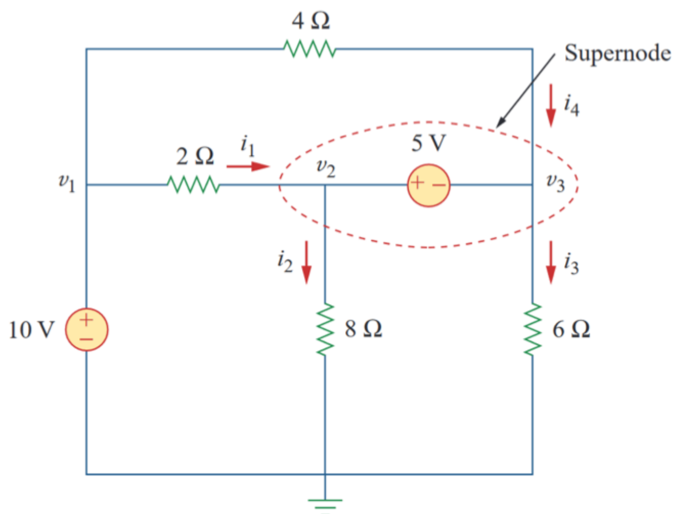
Voltage across **Parallel Resistors** $R_3, R_4$

$$V_O = i(R_3||R_4) = -\frac{\alpha V_S}{R_1 + R_2} \cdot \frac{R_3 R_4}{R_3 + R_4}$$

$$\left|\frac{V_O}{V_S}\right| = \frac{\alpha}{R_1 + R_2} \cdot \frac{R_3 R_4}{R_3 + R_4}$$

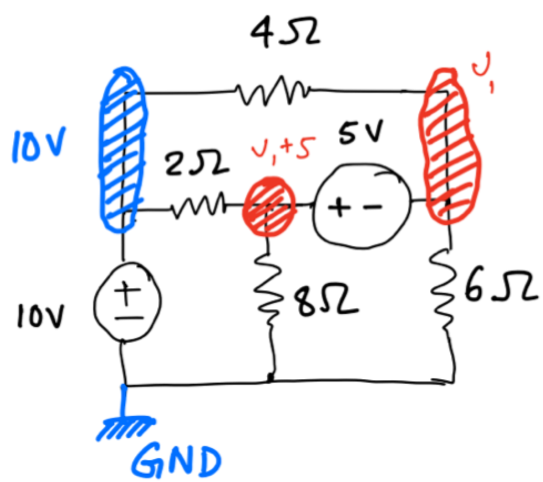
$\alpha = 40$

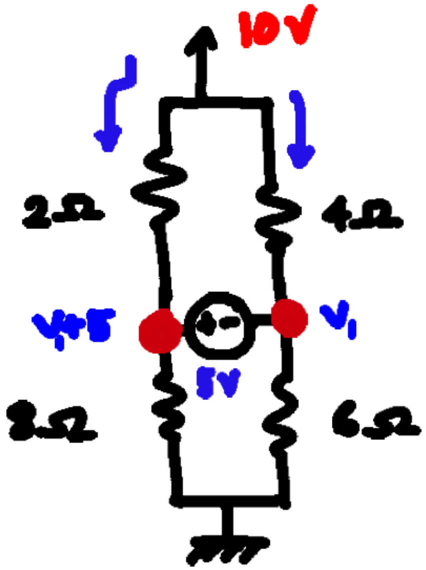
(2) Question :




Write down the node equations.

Solution:

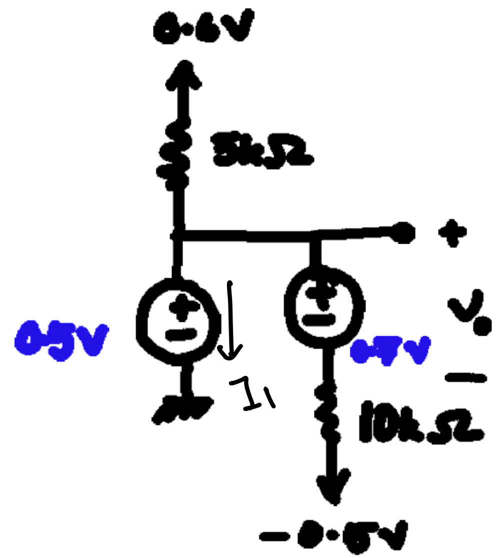





Node equation for node $\boldsymbol{v_1}$

$$v_1\left(\frac{1}{4}+\frac{1}{6}\right)+(v_1+5)\left(\frac{1}{2}+\frac{1}{8}\right)-10\left(\frac{1}{2}+\frac{1}{4}\right)=0$$

## (3) Question: find $I_1$

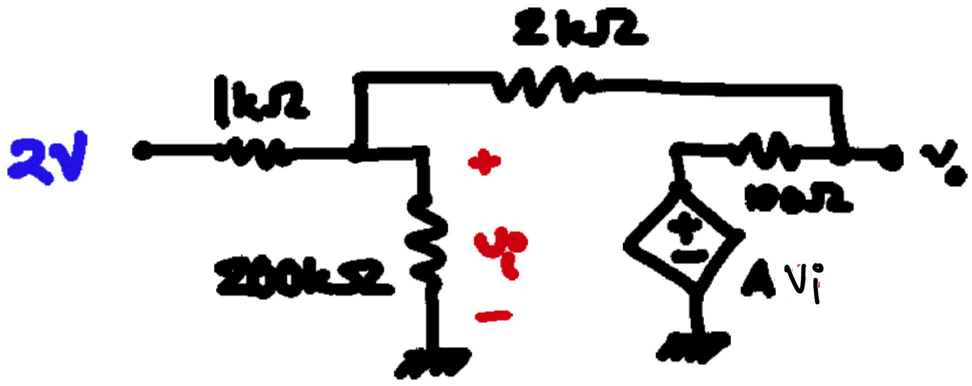



KCL at node $\boldsymbol{v_o}$

$$\frac{0.6-0.5}{5}=\frac{(0.5-0.7)-(-0.5)}{10}+\boldsymbol{I_1}$$

$$\boldsymbol{I_1}=-\mathbf{0.01}\ \mathrm{mA}$$

(4) Question:




$A = 2 \times 10^5$

Solution:

KCL at node $\boldsymbol{v_i}$

$$\frac{2 - v_i}{1} = \frac{v_i - v_o}{2} + \frac{v_i}{200}$$

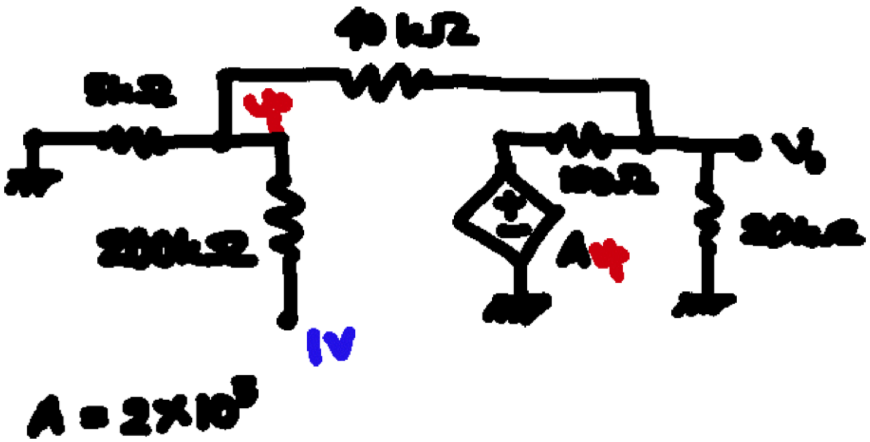
$$\frac{301}{200} v_i - \frac{1}{2} v_o = 2$$

KCL at node $\boldsymbol{v_o}$

$$\frac{v_i - v_o}{2} + \frac{Av_i - v_o}{0.1} = 0$$

$$(2 \times 10^6 + 0.5) v_i - 10.5 v_o = 0$$

5 Question:




KCL at node $v_i$

$$\frac{0-v_i}{5}=\frac{v_i-v_o}{40}+\frac{v_i-1}{200}$$

$$\frac{23}{100}v_i-\frac{1}{40}v_o=\frac{1}{200}$$

KCL at node $v_o$

$$\frac{v_i-v_o}{40}+\frac{Av_i-v_o}{0.1}=\frac{v_o}{20}$$

$$(2\times10^6+0.025)v_i-10.075v_o=0$$

6 Question:

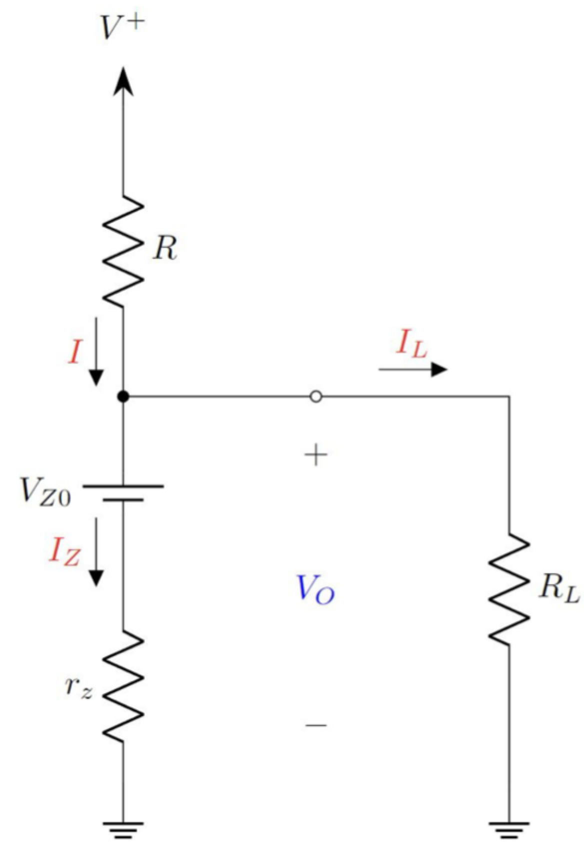
For $\boldsymbol{R} = 100\ \Omega$, $\boldsymbol{R_L} = 10\ \mathrm{k}\Omega$, $\boldsymbol{r_z} = 20\ \Omega$, $\boldsymbol{V_{ZO}} = 3\ \mathrm{V}$, and $\boldsymbol{I_Z} = 1\ \mathrm{mA}$.

a. Find $\boldsymbol{V_O}$

b. Find $\boldsymbol{I_L}$

c. Find $\boldsymbol{I}$

d. Find $\boldsymbol{V^+}$



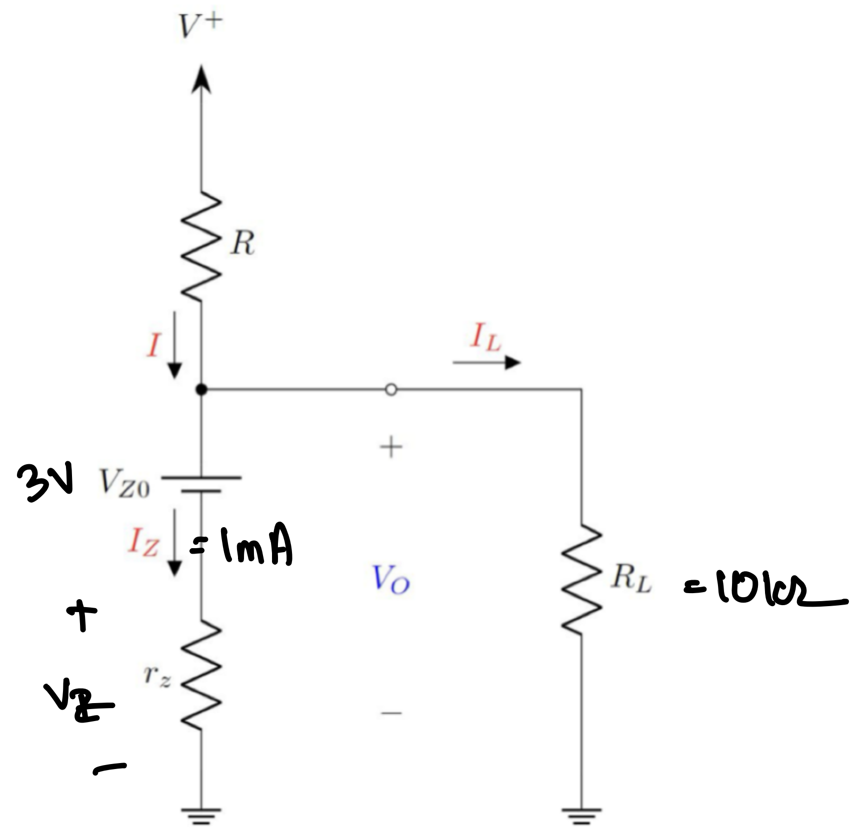
$$V_z = 20 \times 1m = 20mV$$

$$V_o = 3V + V_z = 3.002V$$

$$I_L = \frac{V_o}{R_L} \quad mA$$

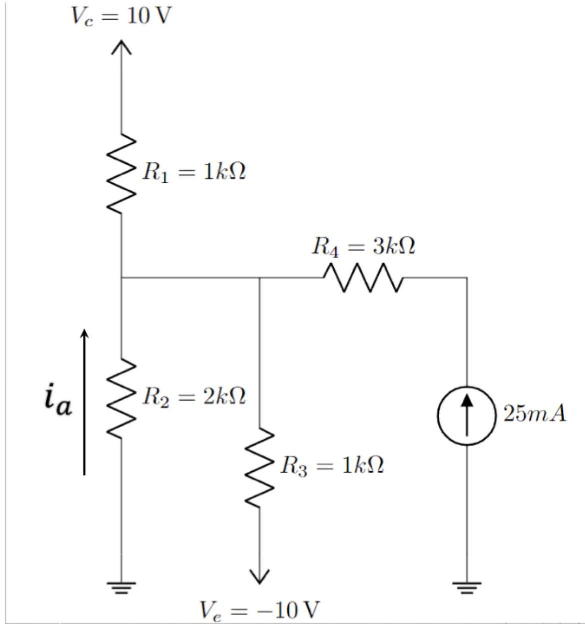
$$I = I_Z + I_L$$

$$\frac{V_+ - V_o}{100} = I$$

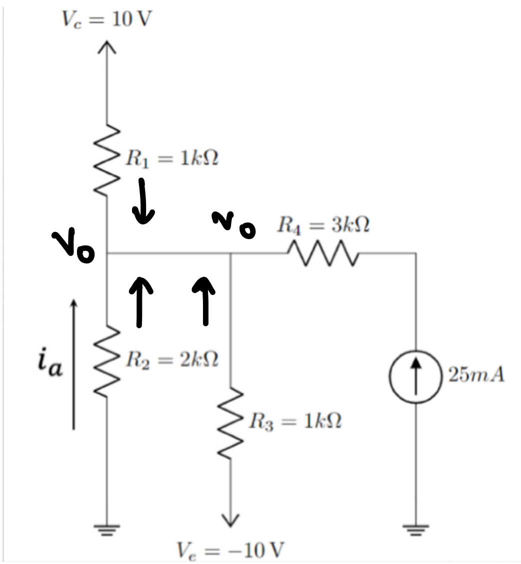


⑦ **Question:**

find $i_a$




**Solution:**




$$\frac{10 - V_o}{1k} + \frac{0 - V_o}{2k} + \frac{-10 - V_o}{1k} + 25mA = 0$$

$$i_a = \frac{0 - V_o}{2k}$$