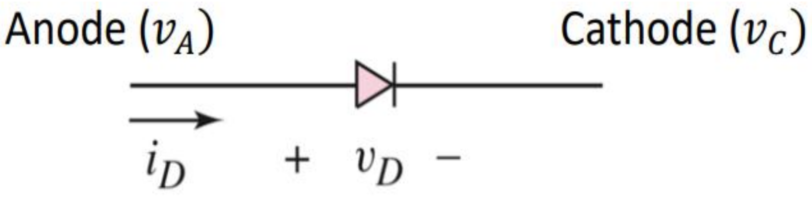
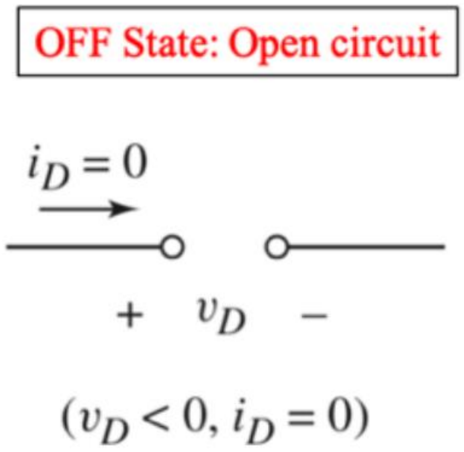
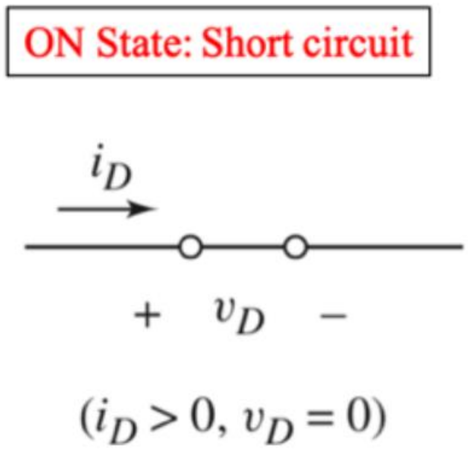
Lecture 5

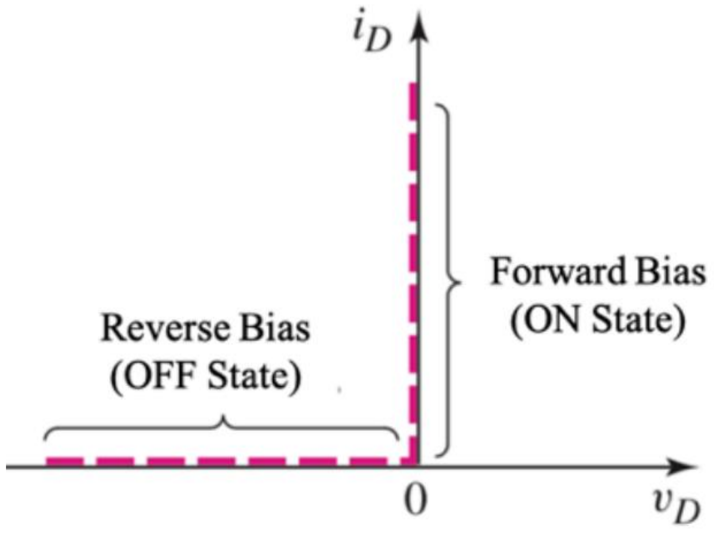
Diode models & Rectifiers

Review: Ideal Diode Model

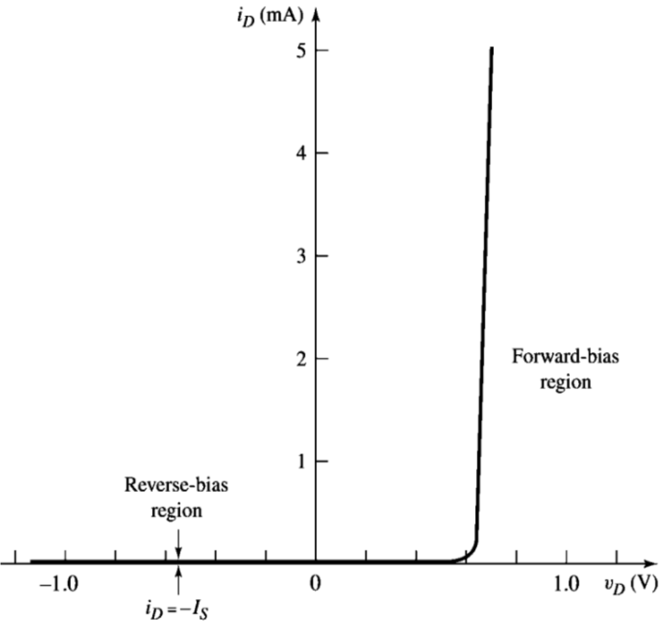








Real diode



I-V characteristics of a real diode

Relation between diode current and diode voltage:

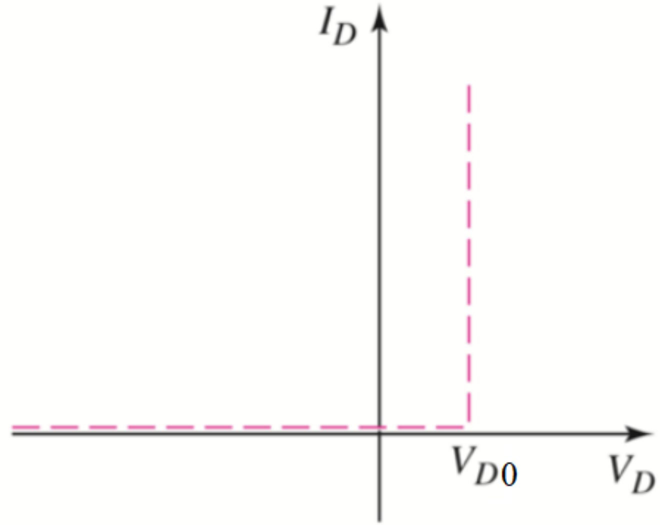
$$i_D = I_S\left(e^{\frac{v_D}{\eta V_T}} - 1\right)$$

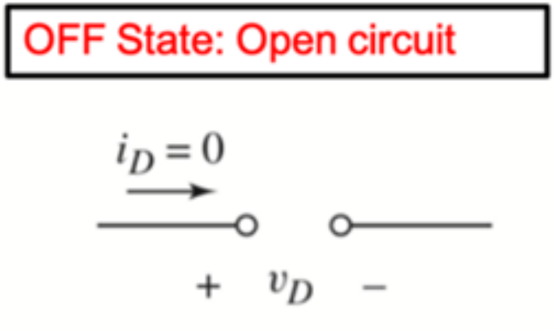
where v_D $(= v_A - v_C)$ is the voltage across the diode, i_D is the current through the diode (from anode to cathode) and V_T, called the thermal voltage, is a temperature dependent constant. For temperature $T = 300K$, $V_T = 25\ mV$.

η is called the ideality factor (try to recall, you measured this in the lab!)

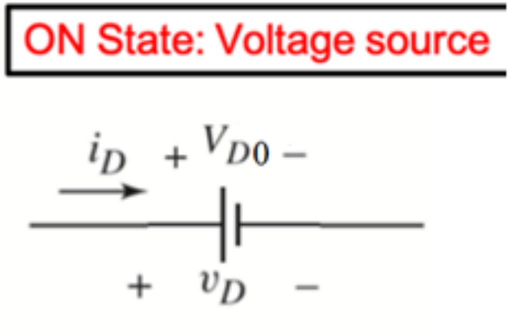
Modeling the real diode

1. Ideal diode model
2. **Constant voltage drop (CVD) model**
3. CVD+R model



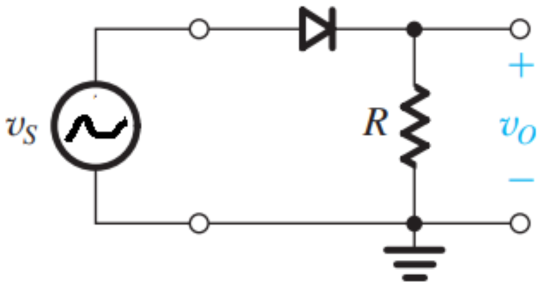


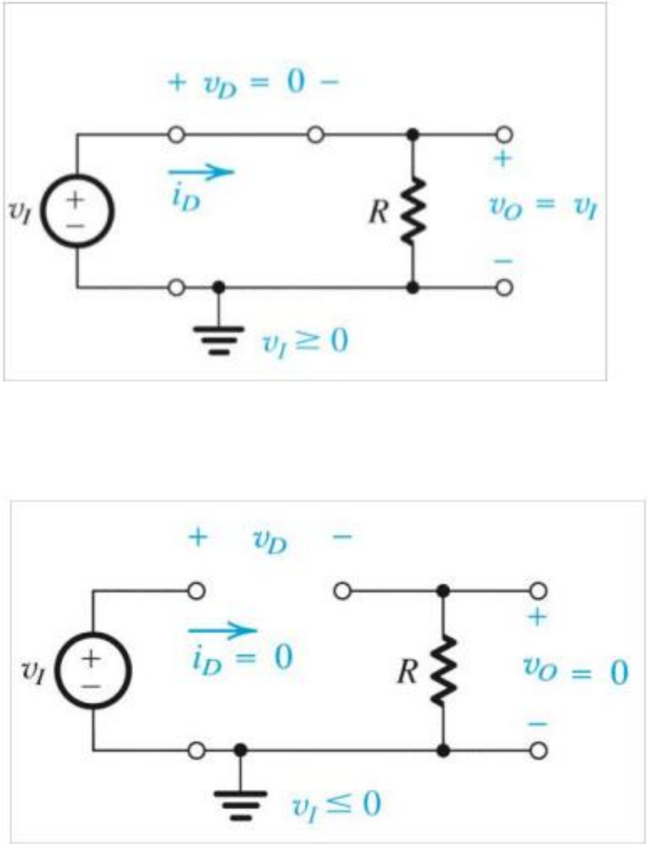
$(v_D < V_{D0}, i_D = 0)$

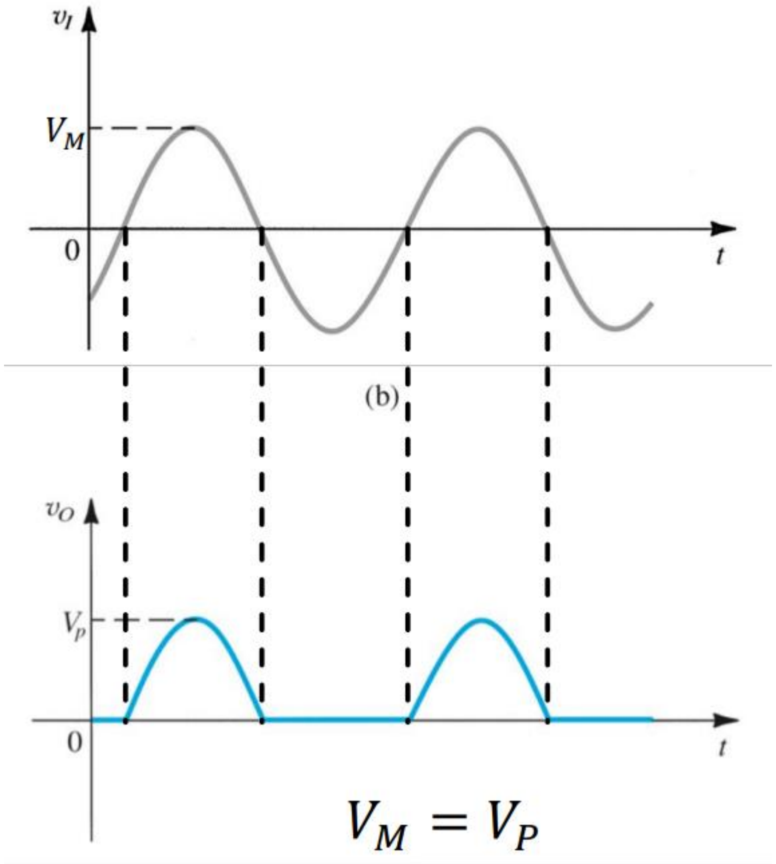


$(i_D > 0, v_D = V_{D0})$

Half-wave rectifier (ideal diode model)







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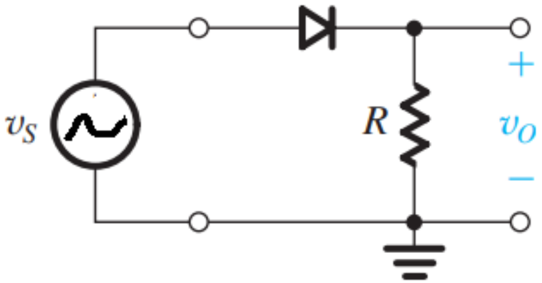
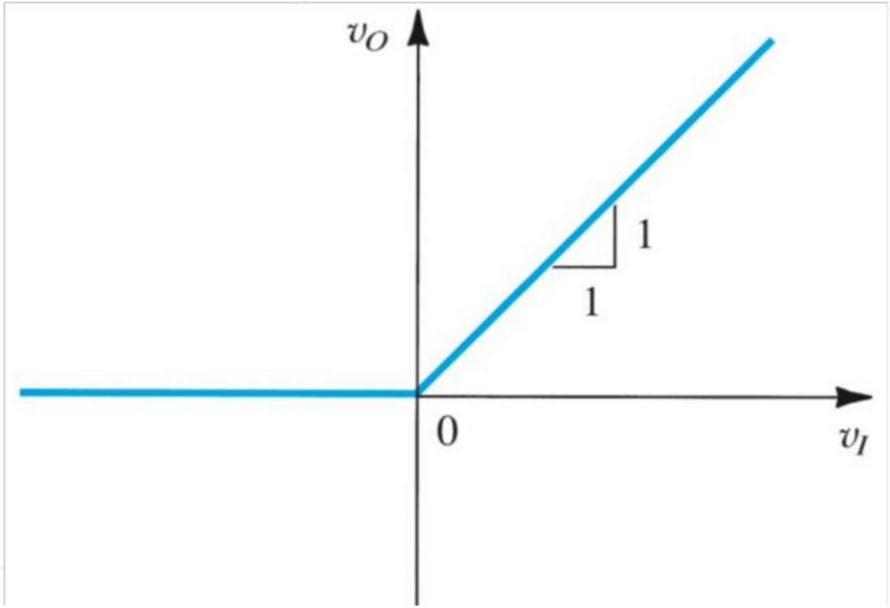
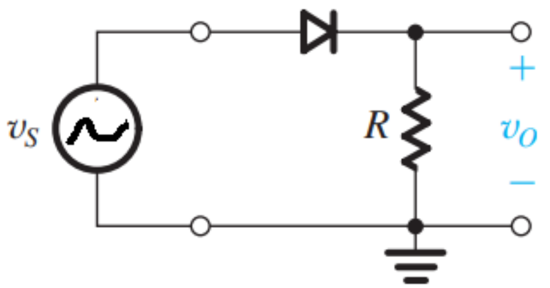
Half-wave rectifier (ideal diode model)

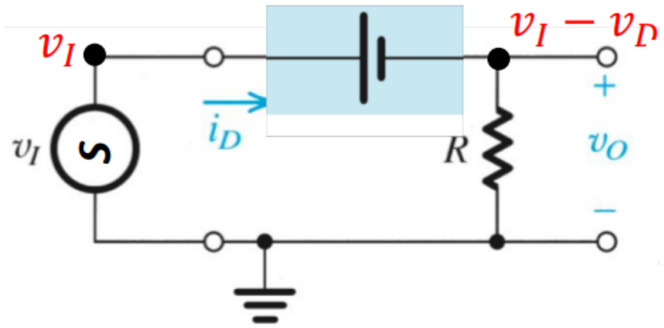


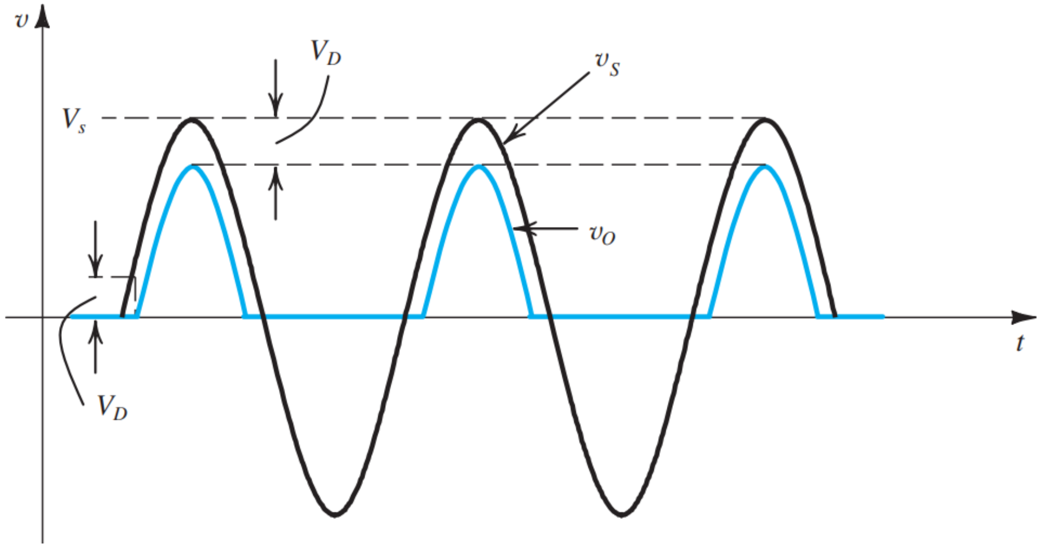
Transfer Characteristics



Half-wave rectifier (CVD model)







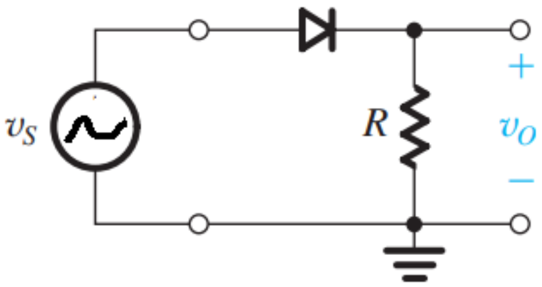
$$v_I = V_M \sin \omega t$$

$$v_o = V_M \sin \omega t - V_D$$

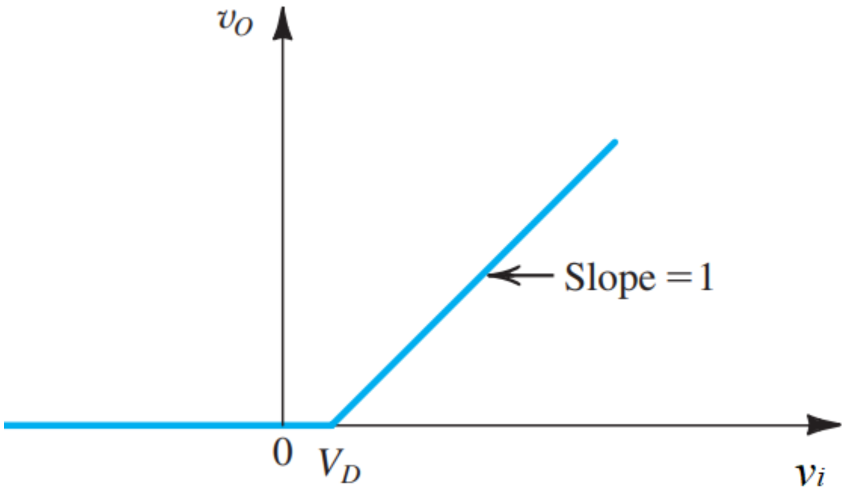
$$V_p = peak\ of\ output$$

$$= V_M - V_D$$

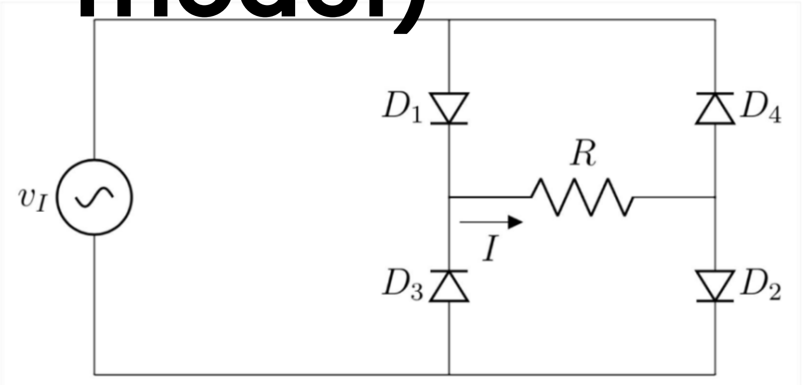
Half-wave rectifier (CVD model)

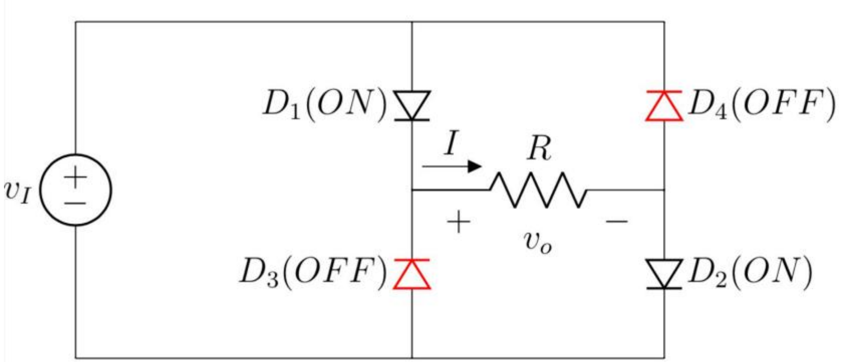


Transfer Characteristics

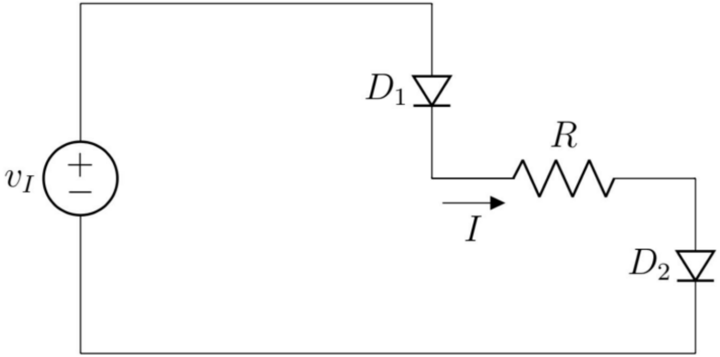


Full-wave rectifier (ideal diode & CVD model)



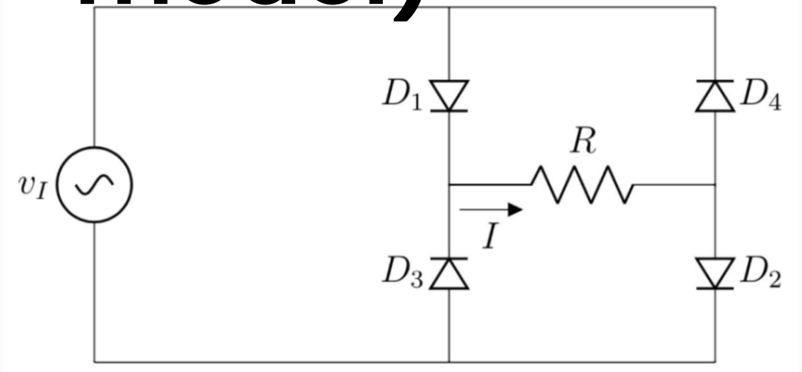


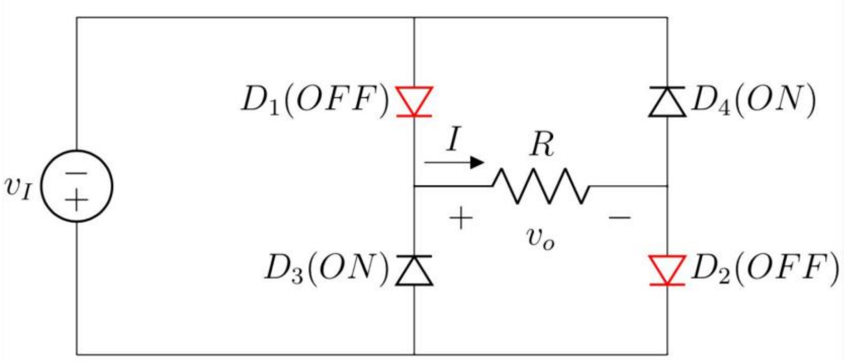
(+) half-cyc



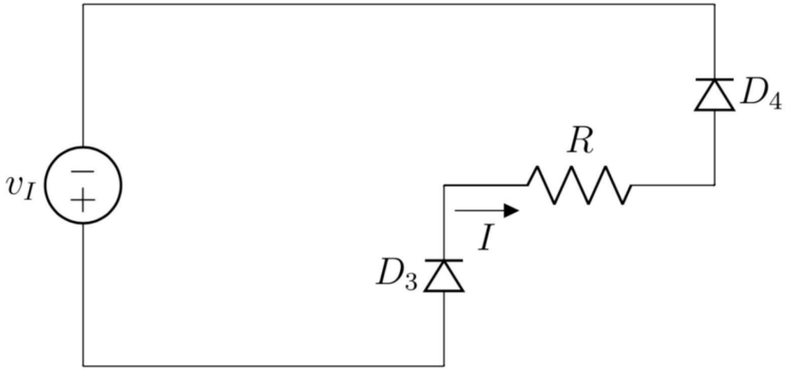
$$v_o = v_I - 2V_D$$

Full-wave rectifier (ideal diode & CVD model)



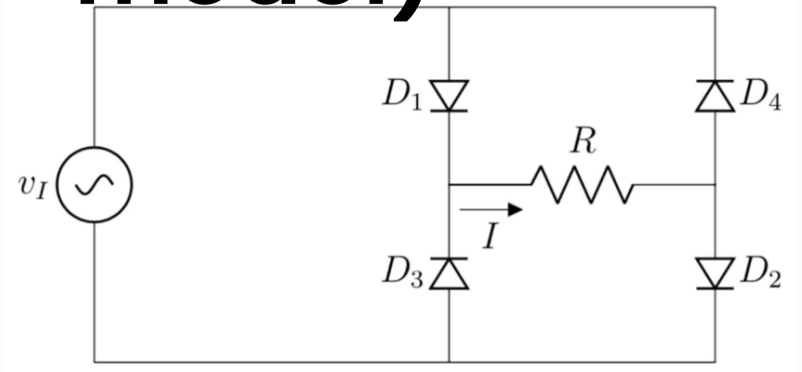


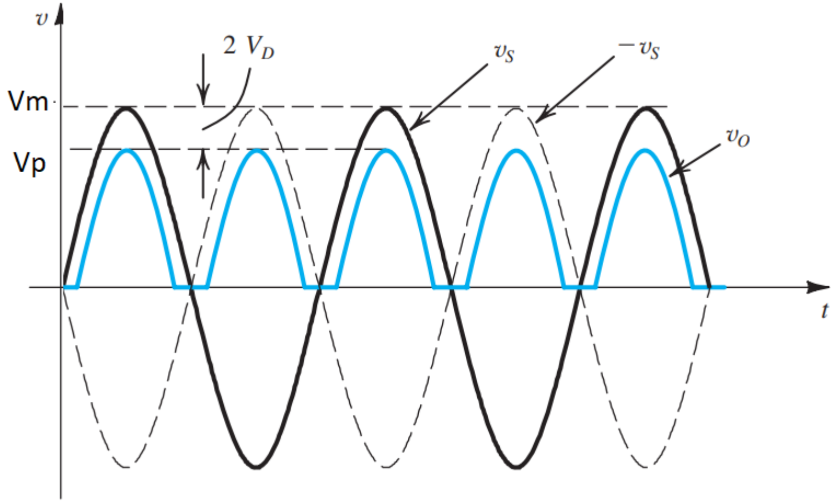
(-) half-cy



$$v_o = -v_I - 2V_D$$

Full-wave rectifier (ideal diode & CVD model)





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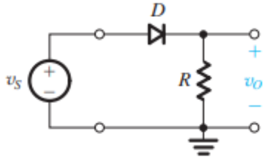
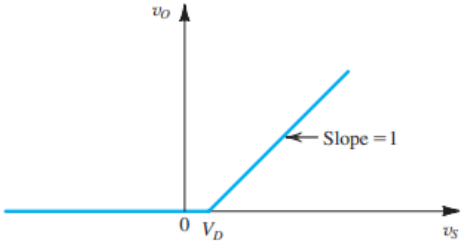
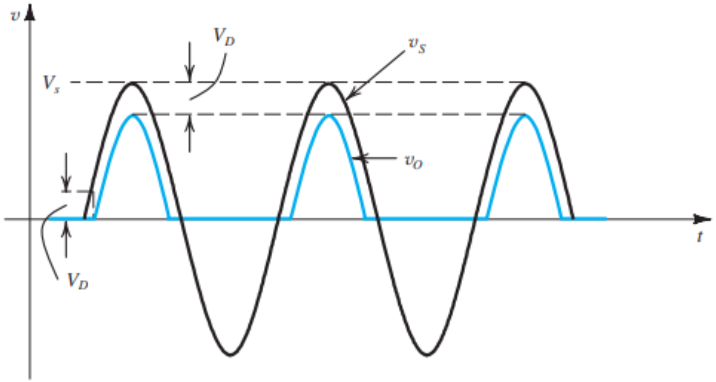
Half-wave and Full-wave rectifier



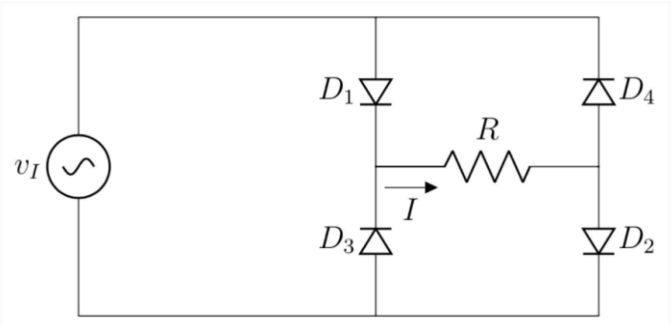
(a)



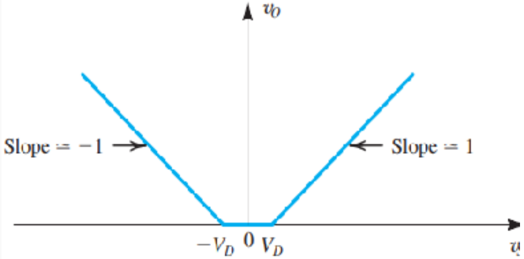
(b)



(c)



(a)



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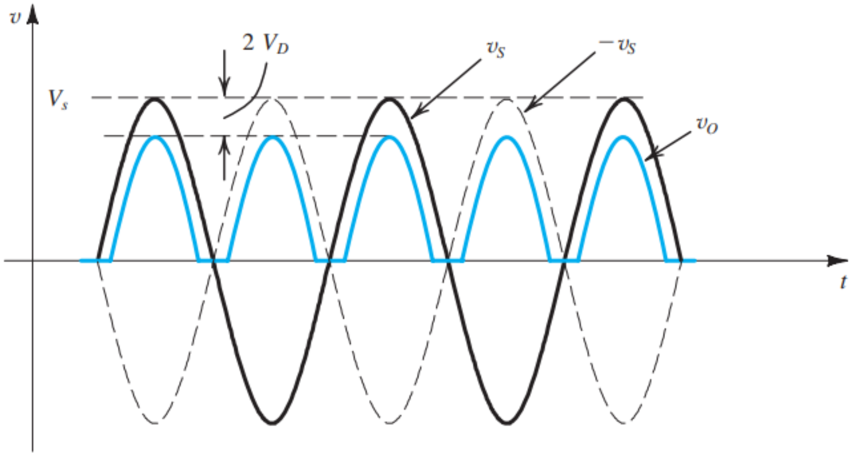
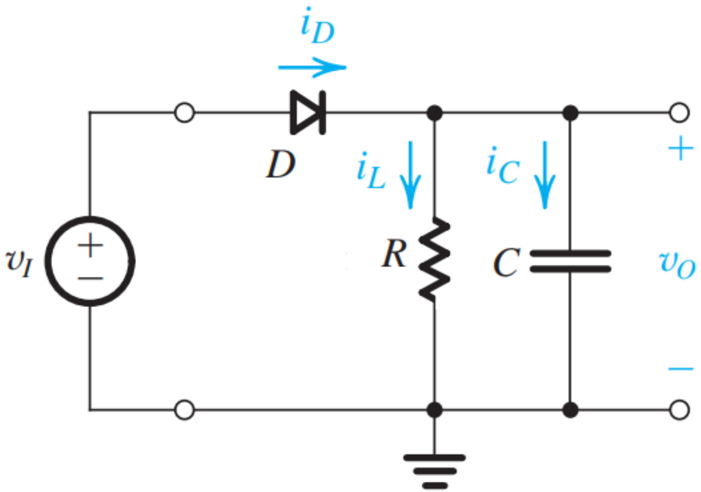
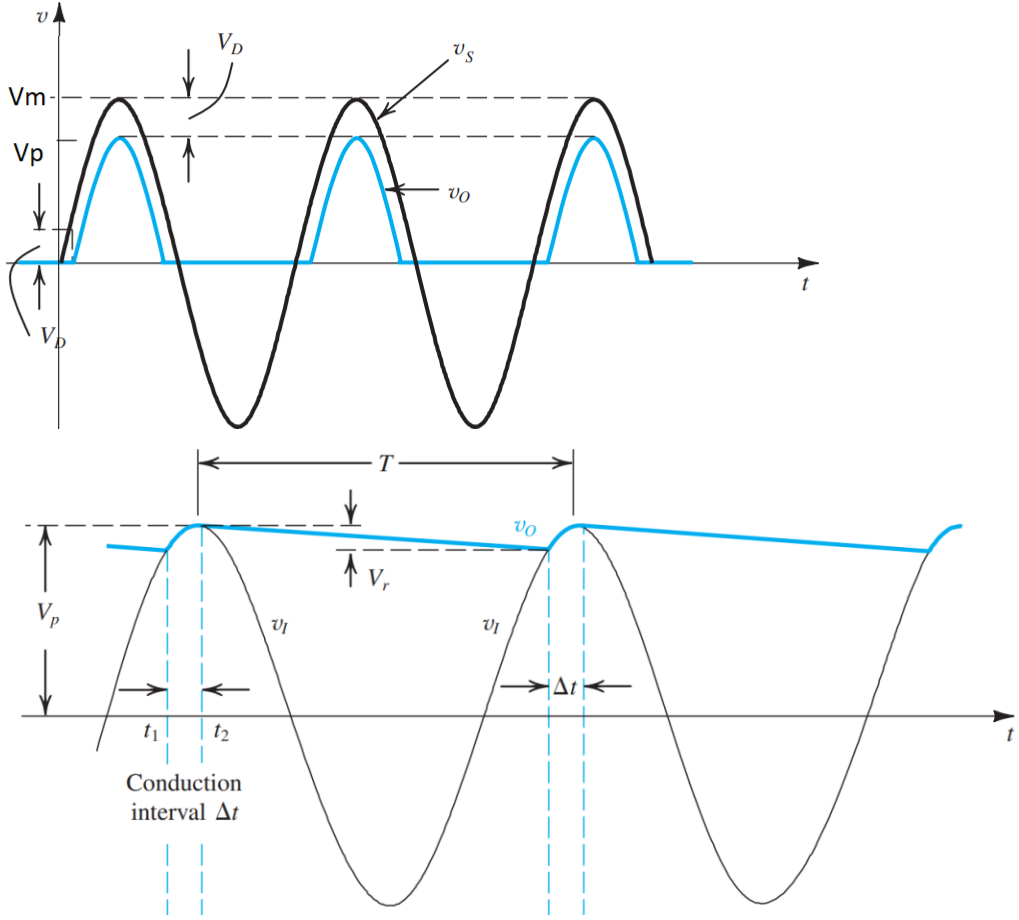


Half-wave

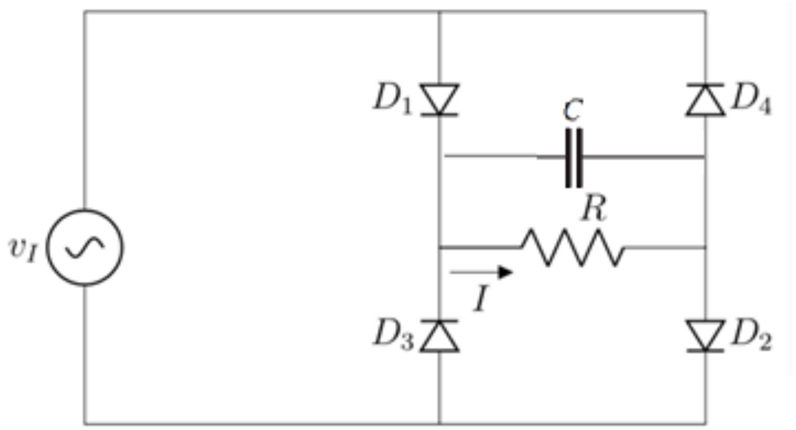
Full-wave

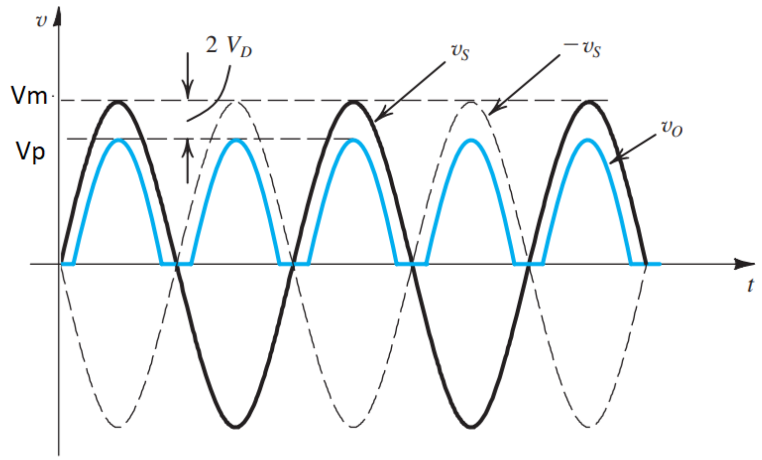
Filtering: Half-wave rectifier

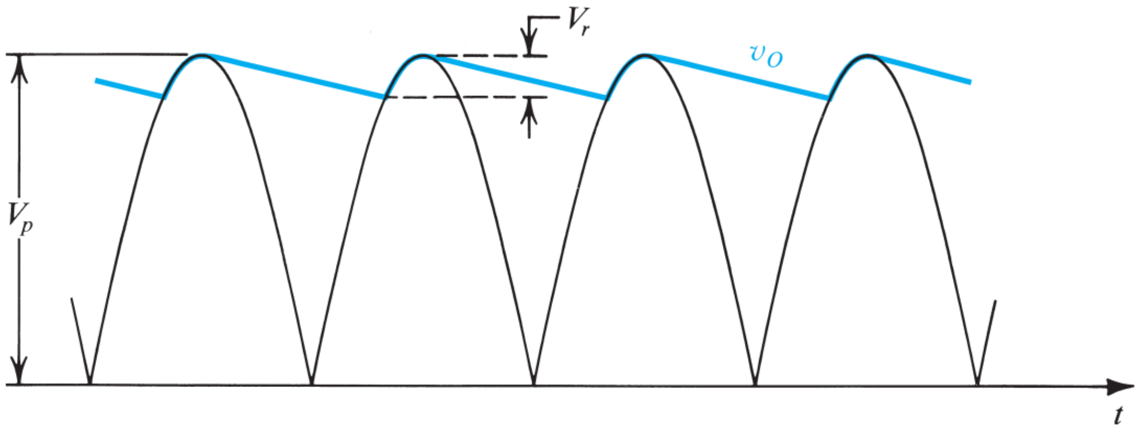




Filtering: Full-wave rectifier







Without capacitor

Rectifier	i/p peak	o/p peak	average
H/W	V_M	V_P $= V_M - V_{DO}$	$V_{avg}=V_{DC}=\frac{1}{\pi}V_M-\frac{1}{2}V_{Do}$
F/W	V_M	V_P $= V_M - 2V_{DO}$	$V_{avg}=V_{DC}=\frac{2}{\pi}V_M-2V_{Do}$

output freq
$f_o = f_i$
$f_o = 2f_i$

With capacitor

with/without capacitor

Rectifier	i/p peak	o/p peak (with/without capacitor)	frequency (with/without capacitor)	Ripple voltage	average	V_{ac} or $V_{r,rms}$
H/W	V_M	$V_P=V_M-V_{Do}$	$f_r=f_i$	$V_r=\frac{V_P}{f_r RC}$	$V_{avg}=V_{DC}=V_P-\frac{1}{2}V_r$	$V_{ac}=V_{r,rms}=\frac{V_r}{2\sqrt{3}}$
F/W	V_M	$V_P=V_M-2V_{Do}$	$f_r=2f_i$	$V_r=\frac{V_P}{f_r RC}$	$V_{avg}=V_{DC}=V_P-\frac{1}{2}V_r$	$V_{ac}=V_{r,rms}=\frac{V_r}{2\sqrt{3}}$

$$I_{o,avg}=V_{o,avg}/R,\ V_{rms}=V_p/\sqrt{2}$$

Example

A voltage waveform $v_i = 8sin(2000\pi t)V$ is input to a full-wave rectifier. A resistance of $R = 50k\ \Omega$ is connected at the load. [Assume that the diodes used in the circuit have a forward drop of 0.8V].

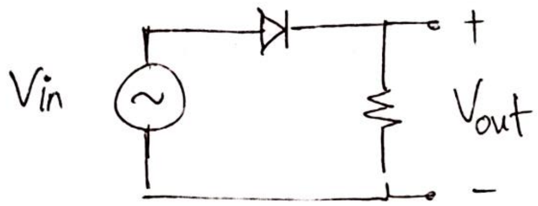
(a) Draw the circuit of the full wave rectifier. Label the input and output voltages properly. [1]

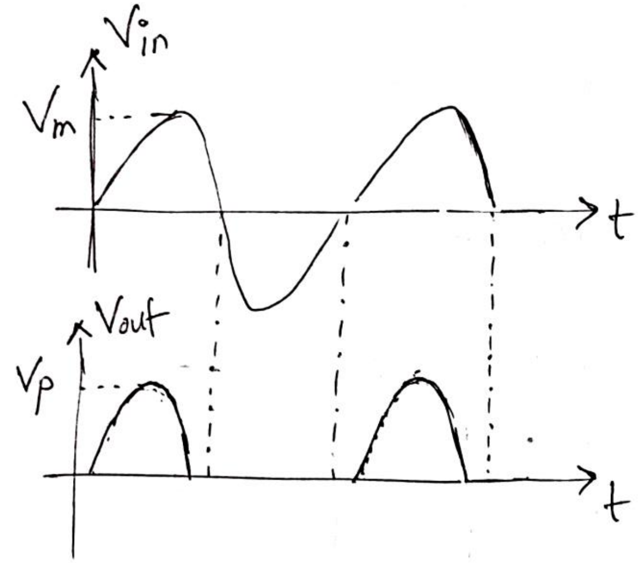
(b) Draw the waveforms of the input and output voltages. What are the peak values of input and output? Show them in the graph. [1+1]

(c) Find the average voltage measured at the output. [1]

Rectifiers: Revisited

Half-Wave Rectifiers:





Suppose, input to the rectifier is $V_m \sin(\omega t)$.

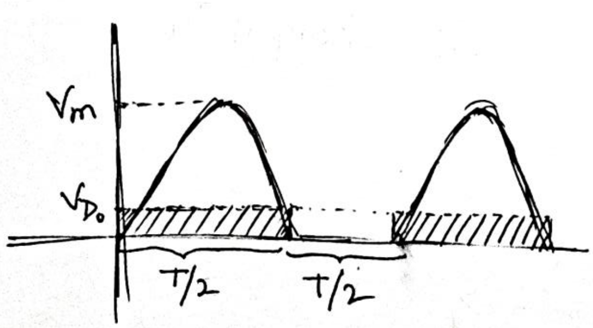
And, peak value of the output is V_p.

Then, V_p will be one diode-drop below V_m, i.e,

$$\boxed{V_p = V_m - V_{D_0}}$$

* Without Cap.

Now, what is the "average" value of the output? It is not actually straightforward, since, the output voltage does not start from $t=0$. We can make an approximation:



Here, if we subtract the area of the shaded portion, from the area of one of the lobes, we can approximately get the area of the output lobe. Then, averaging over 1 time period will give the average output voltage.

Now, Area of one lobe of the sine-wave $= \int_0^{T/2} V_m \sin(\omega t)\,dt$

$$= \frac{2}{\pi} V_m \cdot \left(\frac{T}{2}\right).$$

Area of the shaded region $= V_{D_0} \cdot (T/2)$.

$\therefore$ Approx. area of the output lobe $= \left(\frac{T}{2}\right) \cdot \left[\frac{2}{\pi} V_m - V_{D_0}\right]$

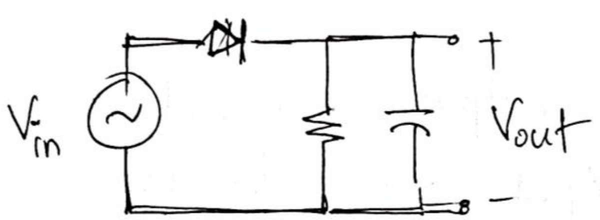
$\therefore$ Average value of voltage (over 1 Time-Period):

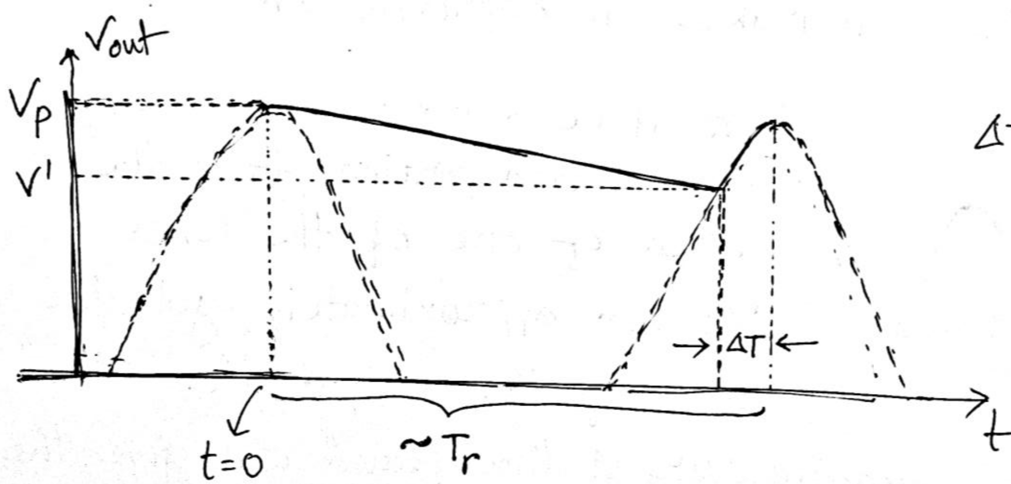
$$V_{avg} \text{, or, } V_{DC} = \frac{1}{T} \times \left(\frac{T}{2}\right)\left[\frac{2}{\pi} V_m - V_{D_0}\right]$$

$$\therefore \boxed{V_{Avg} \text{, or, } V_{DC} = \frac{1}{\pi} V_m - \frac{1}{2} V_{D_0}}$$

With Cap.

If a capacitor is added to the output,





ΔT = Charging Time.

Since the capacitor discharges through the resistor R, the output voltage t seconds after the peak V_p is.

$$V_{out}(t) = V_p e^{-t/RC}$$

Assump^n (1):

If the discharge time $(T_r - \Delta T)$ is much less than RC, we can write,

$$V_{out}(T_r - \Delta T) = V_p e^{-(T_r-\Delta T)/RC}$$

$$= V_p \left(1 - \frac{T_r - \Delta T}{RC} + \frac{(T_r-\Delta T)^2}{2!\,R^2C^2} - \cdots\cdots\right)$$

$$\approx V_p \left(1 - \frac{T_r - \Delta T}{RC}\right)$$

Then, the lowermost point of the voltage wave-form is,

$$V' \approx V_p \left(1 - \frac{T_r - \Delta T}{RC}\right) \qquad [T_r - \Delta T << RC]$$

Here, T_r = Time-period of Ripple

ΔT = Capacitor Charging Time.

Assump^n (2): If the capacitor charging time (ΔT) is much smaller than the ripple time-period (T_r), we may assume,

$$V' \approx V_p \left(1 - \frac{T_r}{RC}\right) \qquad [\Delta T << T_r]$$

~~There~~

Then, the peak-to-peak ripple voltage is,

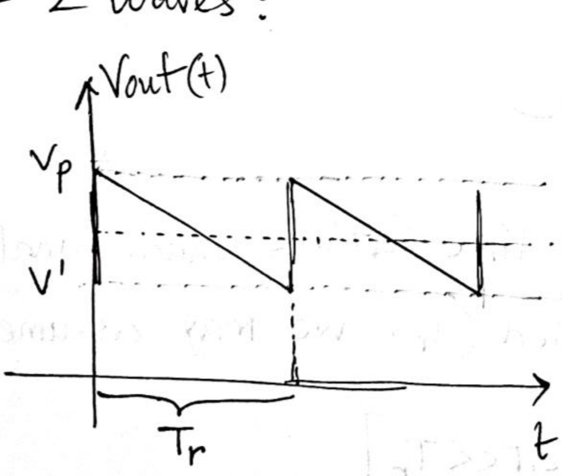
$$V_r(\text{peak-to-peak}) = V_p - V'$$
$$\approx V_p - V_p\left(1 - \frac{T_r}{RC}\right)$$
$$\approx V_p \cdot \frac{T_r}{RC}$$

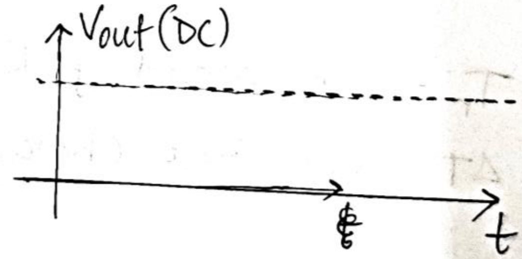
If the ripple-frequency, $f_r = \frac{1}{T_r}$, then,

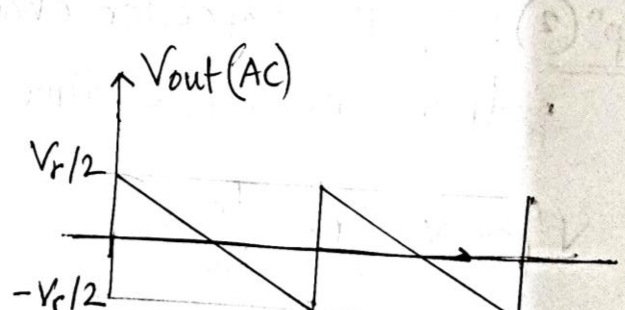
$$\boxed{V_r(p\text{-}p) = \frac{V_p}{f_r \cdot RC}}$$

For a Half-Wave Rectifier, time-period of ripple = time-period of input signal. $\therefore T_r = T_s \Rightarrow f_r = f_s$.

Also, if we neglect the charging time ΔT, the output wave can be approximated as below, which can be shown as the sum of 2 waves:







$$V_{out}(t) = V_{out[DC]} + V_{out[AC]}$$

* [$V_{out}(t)$ has been exaggerated in the figure for clearer understanding. Ripples will not be this big].

Since, the DC-value will go through the middle of the ripple, we have,

$$V_{DC} = \frac{V_p + V'}{2}$$

And, using $V_r(p\text{-}p) = V_p - V'$, we get,

$$\boxed{V_{DC} = V_p - \frac{V_r(p\text{-}p)}{2}}$$

Also, neglecting the charging time ΔT, the ripple waveform, or, the AC waveform will become almost like a triangular wave.

The r.m.s. value of the ripple is:

$$V_{AC} = V_r\,(\text{r.m.s.}) = \sqrt{\frac{1}{T}\int_0^T [V_{AC}(t)]^2\,dt}$$

Assump^n:

Using the eq^n. $V_{AC}(t) = \frac{V_r}{2} - \frac{V_r}{T}\cdot t$ for $0 \leq t \leq T$, → Eq^n for triangular wave.

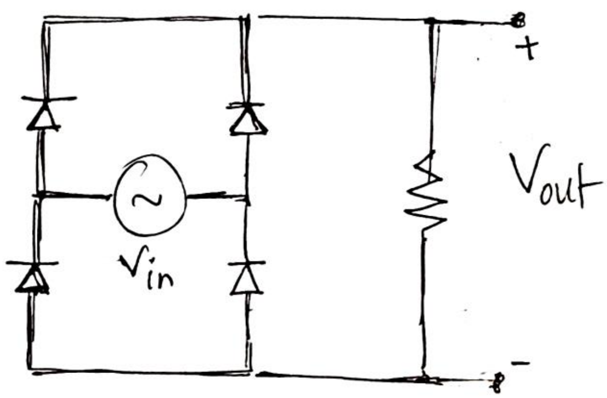
if we perform the integration, we get,

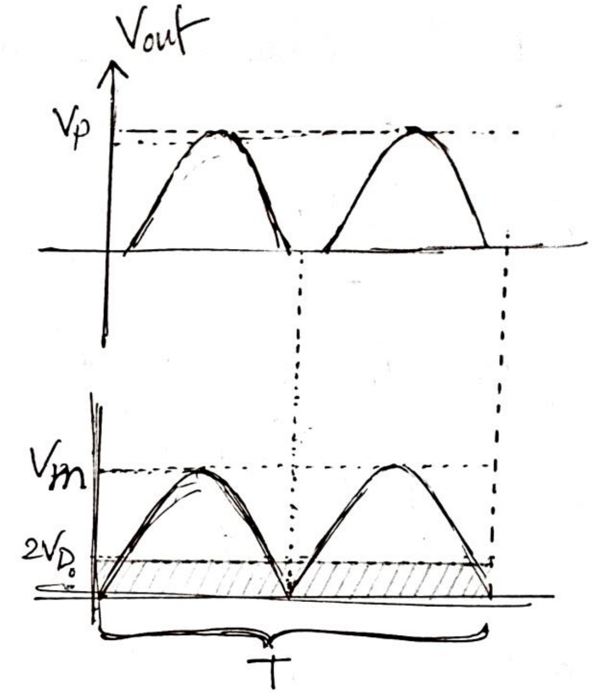
$$V_r(\text{r.m.s.}) = \frac{V_r(p\text{-}p)}{2\sqrt{3}}$$

[$V_r = V_r(p\text{-}p)$ in the above eq^n]

$$\therefore \boxed{V_r(\text{r.m.s.}) = \frac{V_p}{2\sqrt{3}\cdot f_r \cdot RC}}$$

Full-Wave Rectifier :





As before, input to the FW rectifier is,

$$V_{in} = V_m \sin(\omega t).$$

This time, the peak of the output will be 2 diode-drops below the input peak.

$$\therefore \boxed{V_p = V_m - 2V_{D_0}}$$

Without Cap

Like-wise, to find the average value of output, we may approximate by subtracting the area of the shaded portion from the area of the 2 lobes.

$$\text{Area of 2 lobes} = \frac{2}{\pi} V_m \cdot T$$

$$\text{Area of shaded portion} = 2V_{D_0} \cdot T.$$

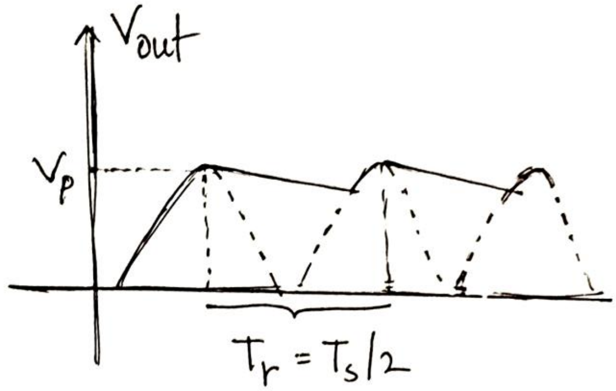
$\therefore$ Approx. Average value of voltage,

$$V_{Avg}, \text{or}, V_{DC} = \frac{1}{T}\left[\frac{2}{\pi} V_m \cdot T - 2V_{D_0} \cdot T\right]$$

$$\therefore \boxed{V_{Avg}, \text{or}, V_{DC} = \frac{2}{\pi} V_m - 2V_{D_0}}$$

With Capacitor

The ripple analysis of an FW rectifier will be ~~&~~ exactly the same as an HW rectifier. Except, one difference.



As can be seen, the ripple period is half the signal period. (Because, the capacitor will start charging if the input reaches its negative peak).

$$\therefore T_r = T_s/2 \Rightarrow f_r = 2f_s.$$

Except this change, all other formulas are same as before.

To sum up.

	HW	FW
V_p	$V_m - V_{D_0}$	$V_m - 2V_{D_0}$
f_r	f_s	$2f_s$
V_{DC} (without Cap)	$\frac{1}{\pi}V_m - \frac{1}{2}V_{D_0}$	$\frac{2}{\pi}V_m - 2V_{D_0}$

With Capacitor

$$V_r(p\text{-}p) = \frac{V_p}{f_r \cdot RC}$$

$$V_{ac} = V_r(rms) = \frac{V_r(p\text{-}p)}{2\sqrt{3}}$$

$$V_{ac} = V_r(rms) = \frac{V_p}{2\sqrt{3}f_r \cdot RC}$$

$$V_{DC} = V_p - \frac{V_r(p\text{-}p)}{2}$$

1.

The input of a full-wave rectifier is a cosine voltage with peak $V_M = 5$ V and frequency 60 Hz, and output load resistance is R = 2 kΩ. Silicon diodes are used in this circuit for which the forward drop is $V_{D0} = 0.7$ V.

(a) Briefly explain the purpose of a rectifier and describe its operation.
(b) Show the input and output waveforms.
(c) Calculate the DC value of the output voltage.

Now after connecting a capacitor in parallel with the load, the output becomes a ripple voltage **Vout =** V_{DC} **±0.2 V**

(d) Calculate the **peak-to-peak ripple voltage**, and from that, the value of the capacitor.
(e) Calculate the average of the output voltage V_{DC} after connecting the capacitor. Compare this with the DC value determined in 'c' and comment on the difference between these two.

c) $V_{DC} = \frac{2}{\pi} V_M - 2V_{DO} = \frac{2}{\pi}(5) - 2(0.7)$

d) $V_r = 0.2 - (-0.2) = 0.4V$

$V_r = \frac{V_P}{f_r RC}$

$\Rightarrow 0.4 =$

2.

The input of a **Half-wave rectifie**r is a sine voltage with peak VM = 10 V and frequency 55 Hz, and output load resistance is R = 2.5 kΩ. Silicon diodes are used in this circuit for which the forward drop is V_{D0} = 0.4 V.

(a) Calculate the DC value of the output voltage.

Now after connecting a capacitor in parallel with the load, the output becomes a ripple voltage **Vout** = V_{DC} ±~~0 V.~~ 0.5V

(d) Calculate the **peak-to-peak ripple voltage**, and from that, the value of the capacitor.

(e) Calculate the average of the output voltage V_{DC} after connecting the capacitor. Compare this with the DC value determined in ‘c’ and comment on the difference between these two.

(f) Draw the **Voltage Transfer Characteristic (VTC) curve**

(g) What is the frequency of the Ripple voltage?

3.

The input of a full-wave rectifier is expressed by, Vs(t)= 7sin(400πt), and output load resistance is R = 5 kΩ. Silicon diodes are used in this circuit for which the forward drop is $V_{D0} = 0.3$ V.

(a) Calculate the input and output wave frequency.
(b) Show the input and output waveforms.
(c) Calculate the DC value of the output voltage.

Now after connecting a capacitor, C= 100 μF in parallel with the load.

(d) Calculate the peak-to-peak ripple voltage,
(e) Calculate the average of the output voltage V_{DC} after connecting the capacitor. Compare this with the DC value determined in 'c' and comment on the difference between these two.
(f) How can you provide better filtering for the output waves?
(g) What is the frequency of the Ripple voltage?

4.

The input of a **Half-wave rectifier** is a **Square** wave voltage with peak $V_M = 15$ V and frequency 0.5 Hz, and output load resistance is R = 5 kΩ. Silicon diodes are used in this circuit for which the forward drop is $V_{D0} = 0.7$ V.

i. Show the input and output waveforms.

ii. Draw the VTC curve

5.

The input of a **full-wave rectifier** is a **Square** wave voltage with peak $V_M = 15$ V and frequency 0.5 Hz, and output load resistance is R = 5 kΩ. Silicon diodes are used in this circuit for which the forward drop is $V_{D0} = 0.7$ V.

i. Show the input and output waveforms.

ii. Draw the VTC curve