CSE 260
Digital Logic Design

Text Books:

Core Texts:

- **Digital Logic And Computer Design, Morris Mano**

Reference material:

- Digital Design Principles and Practices, John F. Wakerly

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Number System & Calculations

Objective	Analog vs. Digital
• Distinguish between analog and digital system • Understand the advantage and limitation of digital system • Understand the meaning of digital logic	• Analog data can vary over a continuous range of values. Example: speedometer • Digital quantities can take on only discrete values (0 and 1, high and low). Example: Digital Computer, Decimal Digits, Alphabets

Digital System

- A digital system is a combination of devices designed to manipulate physical quantities or information that are represented in digital form.

- “A discreet information processing system”
- Signals: Discreet information

Digital Technology

Advantage	Limitation
• Greater accuracy or precision • Easier to design (generality) • Easier information storage • Programmability (instructions) • Speed • Economical	• The real world is mainly analog **How to Overcome** • Convert the real world analog input data into digital one • Process this digital data • Then again convert into analog form

Digital logic

- Design logic is a term used to denote the design and analysis of digital system

- Digital logic is concerned with the interconnection among digital components and modules

- Digital logic design is engineering and engineering means problem solving

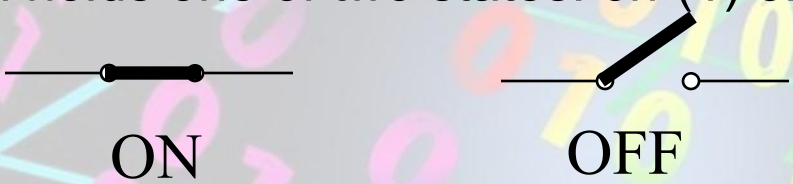
Number systems and codes

Digital Systems are built from circuits that process binary digits. BUT very few real-life problems are based on binary numbers.

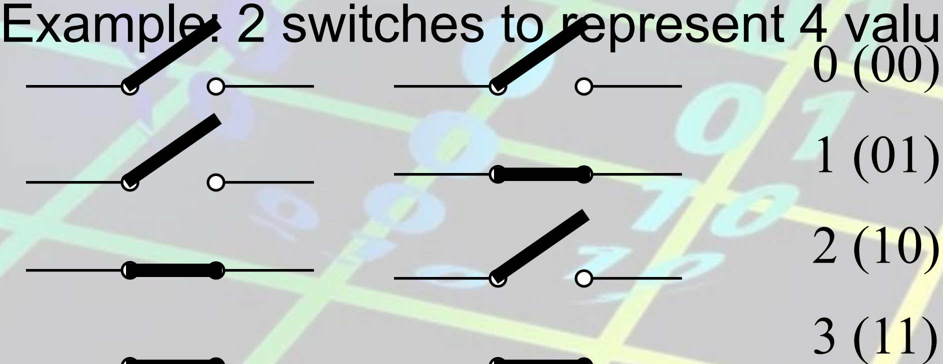
SO a digital system designer must establish some correspondence between the binary digits processed by digital circuits and real-life numbers, events and conditions.

Information representation

- Human decisions tends to be binary i.e. Yes or No
- Elementary storage units inside computer are *electronic switches*. Each switch holds one of two states: *on* (1) or *off* (0).



- We use a *bit* (*bi*nary digi*t*), 0 or 1, to represent the state.
- Storage units can be grouped together to cater for larger range of numbers. Example: 2 switches to represent 4 values.



Information representation

- In general, N bits can represent 2^N different values.
- For M values, $\lceil \log_2 M \rceil$ bits are needed.

1 bit → represents up to 2 values (0 or 1)
2 bits → rep. up to 4 values (00, 01, 10 or 11)
3 bits → rep. up to 8 values (000, 001, 010. …, 110, 111)
4 bits → rep. up to 16 values (0000, 0001, 0010, …, 1111)

32 values → requires 5 bits
64 values → requires 6 bits
1024 values → requires 10 bits
40 values → requires 6 bits
100 values → requires 7 bits

Positional Notations

- Decimal number system, symbols = { 0, 1, 2, 3, …, 9 }
- Position is important
- Example: $(7594)_{10} = (7 \times 10^3) + (5 \times 10^2) + (9 \times 10^1) + (4 \times 10^0)$
- In general, $(a_n a_{n-1} \ldots a_0)_{10} = (a_n \times 10^n) + (a_{n-1} \times 10^{n-1}) + \ldots + (a_0 \times 10^0)$
- $(2.75)_{10} = (2 \times 10^0) + (7 \times 10^{-1}) + (5 \times 10^{-2})$
- In general, $(a_n a_{n-1} \ldots a_0 \,.\, f_1 f_2 \ldots f_m)_{10} = (a_n \times 10^n) + (a_{n-1} \times 10^{n-1}) + \ldots + (a_0 \times 10^0) + (f_1 \times 10^{-1}) + (f_2 \times 10^{-2}) + \ldots + (f_m \times 10^{-m})$

Other Number Systems

- Binary (base 2): weights in powers-of-2. Binary digits (bits): ***0,1.***
- Octal (base 8): weights in powers-of-8. Octal digits: ***0,1,2,3,4,5,6,7***
- Hexadecimal (base 16): weights in powers-of-16. Hexadecimal digits: ***0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F***

Note: when base is r, coefficient values range from 0 to r-1.

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Binary	Octal	Decimal	Hexadecimal
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	10	8	8
1001	11	9	9
1010	12	10	A
1011	13	11	B
1100	14	12	C
1101	15	13	D
1110	16	14	E
1111	17	15	F

Base-R to Decimal Conversion

*****Formula= ∑ digit * source_base $^{\text{position}}$**

- $(1101.101)_2 = 1\times2^3 + 1\times2^2 + 1\times2^0 + 1\times2^{-1} + 1\times2^{-3}$
  $= 8 + 4 + 1 + 0.5 + 0.125$
  $= (13.625)_{10}$
- $(572.6)_8 = 5\times8^2 + 7\times8^1 + 2\times8^0 + 6\times8^{-1} = 320 + 56 + 2 + 0.75 = (378.75)_{10}$
- $(2A.8)_{16} = 2\times16^1 + 10\times16^0 + 8\times16^{-1} = 32 + 10 + 0.5 = (42.5)_{10}$
- $(341.24)_5 = 3\times5^2 + 4\times5^1 + 1\times5^0 + 2\times5^{-1} + 4\times5^{-2} = 75 + 20 + 1 + 0.4 + 0.16 = (96.56)_{10}$

Decimal to Base-R Conversion

- Decimal to base-R
  - Whole numbers: repeated division-by-R
  - Fractions: repeated multiplication-by-R

Repeated Division-by-2 Method

- To convert a whole number to binary, use successive division by 2 until the quotient is 0. The remainders form the answer, with the first remainder as the *least significant bit (LSB)* and the last as the *most significant bit (MSB)*.

2	43		
2	21	rem 1	← LSB
2	10	rem 1	
2	5	rem 0	
2	2	rem 1	
2	1	rem 0	
	0	rem 1	← MSB

$(43)_{10} = (101011)_2$

Repeated Multiplication-by-2 Method

- To convert decimal fractions to binary, repeated multiplication by 2 is used, until the fractional product is 0 (or until the desired number of decimal places). The carried digits, or *carries*, produce the answer, with the first carry as the MSB, and the last as the LSB.

$(0.3125)_{10} = (.0101)_2$

	Carry	
$0.3125 \times 2 = 0.625$	0	←MSB
$0.625 \times 2 = 1.25$	1	
$0.25 \times 2 = 0.50$	0	
$0.5 \times 2 = 1.00$	1	←LSB

Note: difference between conversion of integer and fraction

Integer

- Division by base of target no. system
- Remainders are accumulated
- By division we obtain LSB to MSB

Fraction

- Multiplication by base of target no. system
- Integers are accumulated
- By multiplication we obtain MSB to LSB

Binary-Octal/Hexadecimal Conversion

- Binary → Octal: Partition in groups of 3

  $(10\ 111\ 011\ 001\ .\ 101\ 110)_2 = (2731.56)_8$

- Octal → Binary: reverse

  $(2731.56)_8 = (10\ 111\ 011\ 001\ .\ 101\ 110)_2$

- Binary → Hexadecimal: Partition in groups of 4

  $(101\ 1101\ 1001\ .\ 1011\ 1000)_2 = (5D9.B8)_{16}$

- Hexadecimal → Binary: reverse

  $(5D9.B8)_{16} = (101\ 1101\ 1001\ .\ 1011\ 1000)_2$

Binary-Octal/Hexadecimal Conversion

- $(1100100.10)_2 = (144.4)_8$

  $= (64.8)_{16}$

Exercise:

(1)Try converting this to
$(10110001101011.111100000110)_2$
a)octal b) hexadecimal

(2) Try converting these to binary
a) $(673.124)_8$
b) $(306.D)_{16}$

Answers:

(1) a) $(26153.7406)_8$
(1) b) $(2C6B.F06)_{16}$

(2) a) $(110\ 111\ 011\ .\ 001\ 010\ 100)_2$

(2) b) $(0011\ 0000\ 0110\ .\ 1101)_2$

Binary operations: Addition: Addition Rules w/Carries

For 2 bit

- 0 + 0 = 0 0 (0 with a 0 carry)
- 0 + 1 = 0 1 (1 with a 0 carry)
- 1 + 0 = 0 1 (1 with a 0 carry)
- 1 + 1 = 1 0 (0 with a 1 carry)

For 3 bit

- 0+0+0 = 0 0 (0 WITH 0 CARRY)
- 0+0+1 = 0 1 (1 WITH 0 CARRY)
- 0+1+1 = 1 0 (0 WITH 1 CARRY)
- 1+1+1 = 1 1 (1 WITH 1 CARRY)

Adding Binary Numbers

		0 1 1 1 0 0 0
28	→	00011100
+ 43	→	+ 00101011
71		01000111

Exercise

3(a) Add $(101101)_2$ with $(100111)_2$

Solution

- 3(a) $(1010100)_2$

Working:

Augend:	101101
Addend:	+100111
Sum:	1010100

Addition of base-r

- Example:
  $(34)_5 + (41)_5 + (24)_5$

1+3+4+2= 10
10%5=0
10/5=2 (carry)

21

$(34)_5$
$(41)_5$
$+ (24)_5$

$(2\ 0\ 4)_5$

4+1+4= 9
9%5=4
9/5=1 (carry)

Exercise

- Try:
  2 (a) $(FF)_{16}+(F1)_{16}$
  2 (b) $(66)_7 + (55)_7$

Solution

- 2(a) $(1F0)_{16}$
- 2(b) $(154)_7$

Binary Multiplication

- The multiplication of two binary numbers can be carried out in the same manner as the decimal multiplication.
- Unlike decimal multiplication, only two values are generated as the outcome of multiplying the multiplication bit by 0 or 1 in the binary multiplication. These values are either 0 or 1.
- The binary multiplication can also be considered as repeated binary addition. Therefore, the binary multiplication is performed in conjunction with the binary addition operation.

Exercise

6 (a) Multiply 1011 with 101

Multiplicand	**1011**
Multiplier	**× 101**
Partial Products	**1011**
	0000 -
	1011 - -
Product	**110111**

Multiplication with base-r

- 2A3C
  xB7

  127A4
  1D094

  1E30E4

Working

7*C = 7*12 = 84 =(write 4, carry 5)
7*3+5 = 26 = 1A (write A, carry 1)
7*A+1 = 71 = 0x47 (write 7, carry 4)
7*2+4 = 18 = 0x12
This completes the 7*2A3C = 127A4 partial product.

B*C = 11*12 = 132 = (write 4, carry 8)
B*3+8 = 11*3+8 = 41 = (write 9, carry 2)
B*A+2 = 11*10+2 = 112 = (write 0, carry 7)
B*2+7 = 11*2+7 = 29 = 1D
This completes the B[0]*2A3C = 1D094[0] partial product, where I'm noting the [0] digits to remind us this is in the 16s column.

Adding the partial products: 127A4 + 1D0940
4+0 = 4
A+4 = E
7+9 = 16 = 0x10 (write 0, carry 1)
2+0+1 = 3
1+D = E
1 = 1

Exercise

7 (a) Multiply $(34)_5$ with $(42)_5$
7 (b) Multiply $(25)_9$ with $(36)_9$

Solution

7 (a) $(3133)_5$
7 (b) $(1033)_9$

Binary operations: Subtraction Rules w/Carries

- For 2 bit

  - 0 - 0 = 0 0 (0 with a 0 carry)
  - 1 - 1 = 0 0 (0 with a 0 carry)
  - 1 - 0 = 0 1 (1 with a 0 carry)
  - 0 - 1 = ? (???)

Subtracting Binary Numbers:

$$\begin{array}{r} 2 \\ -\ 1 \\ \hline 1 \end{array} \quad \longrightarrow \quad \begin{array}{r} \overset{0}{\cancel{1}}\,\overset{2}{0} \\ -\ 0\cancel{1} \\ \hline 01 \end{array}$$

$$\begin{array}{r} 4 \\ -\ 1 \\ \hline 3 \end{array} \quad \longrightarrow \quad \begin{array}{r} 100 \\ -\ 001 \\ \hline 011 \end{array}$$

Exercise

5(b) Subtract $(100111)_2$ from $(101101)_2$

Solution

5 (b) $(000110)_2$

Working:

5(b)Subtraction

Minuend: 101101

Subtrahend: −100111

Difference: 000110

Subtraction of Base-r

$$\begin{array}{r} {}^{3\ \ 16} \\ (\cancel{4}\ A\ 6)_{16} \\ -(1\ B\ 3)_{16} \\ \hline (2\ F\ 3)_{16} \end{array}$$

$$\begin{array}{r} {}^{4\ 6} \\ (\cancel{5}4)_{6} \\ -\ (35)_{6} \\ \hline (15)_{6} \end{array}$$

Exercise

4 (a) $(71)_8$-$(56)_8$

4(b) $(21)_3$-$(12)_3$

Solution

- 4 a) $(13)_8$
- 4 b) $(2)_3$

Division of Base-r

- Dividend: $x_1x_2x_3...x_m$; Divisor: $y_1y_2...y_n = y$;
  Perform $101_2 / 10_2$; Dividend = 101 and Divisor = 10

Step1: Start with the first digit of the dividend. (x_1)

Step2: Compare it with the divisor;

If (it is smaller, than the divisor): ($x_1 < y$)

2.a append 0 to the quotient.

else: ($x_1 >= y$)

proceed with division.

2.a Find the largest multiple of the divisor that fits.

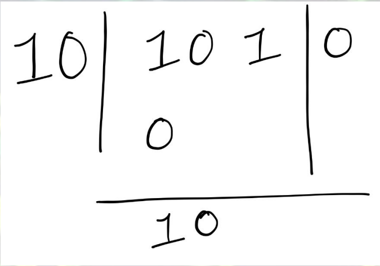
2.b Append the multiplier to the quotient.

Step3: Subtract and bring down the next digit.

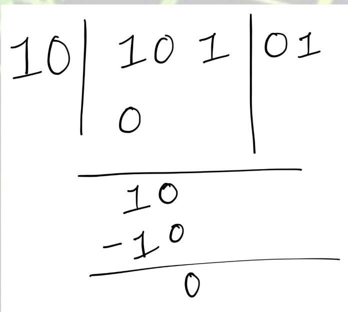
Step4: From now on compare the newly formed number in step3 with the divisor while repeating step2.

Step5. Repeat this step 2, 3 & 4 until all digits of the dividend ($x_1x_2x_3...x_m$) are processed.

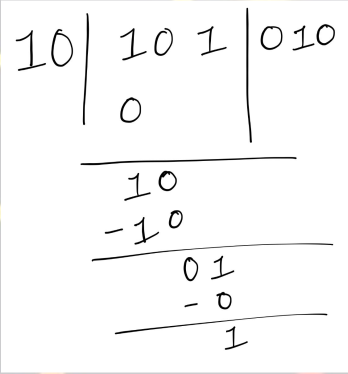
Iteration-1: (1 < 10)



Iteration-2: (10 == 10)

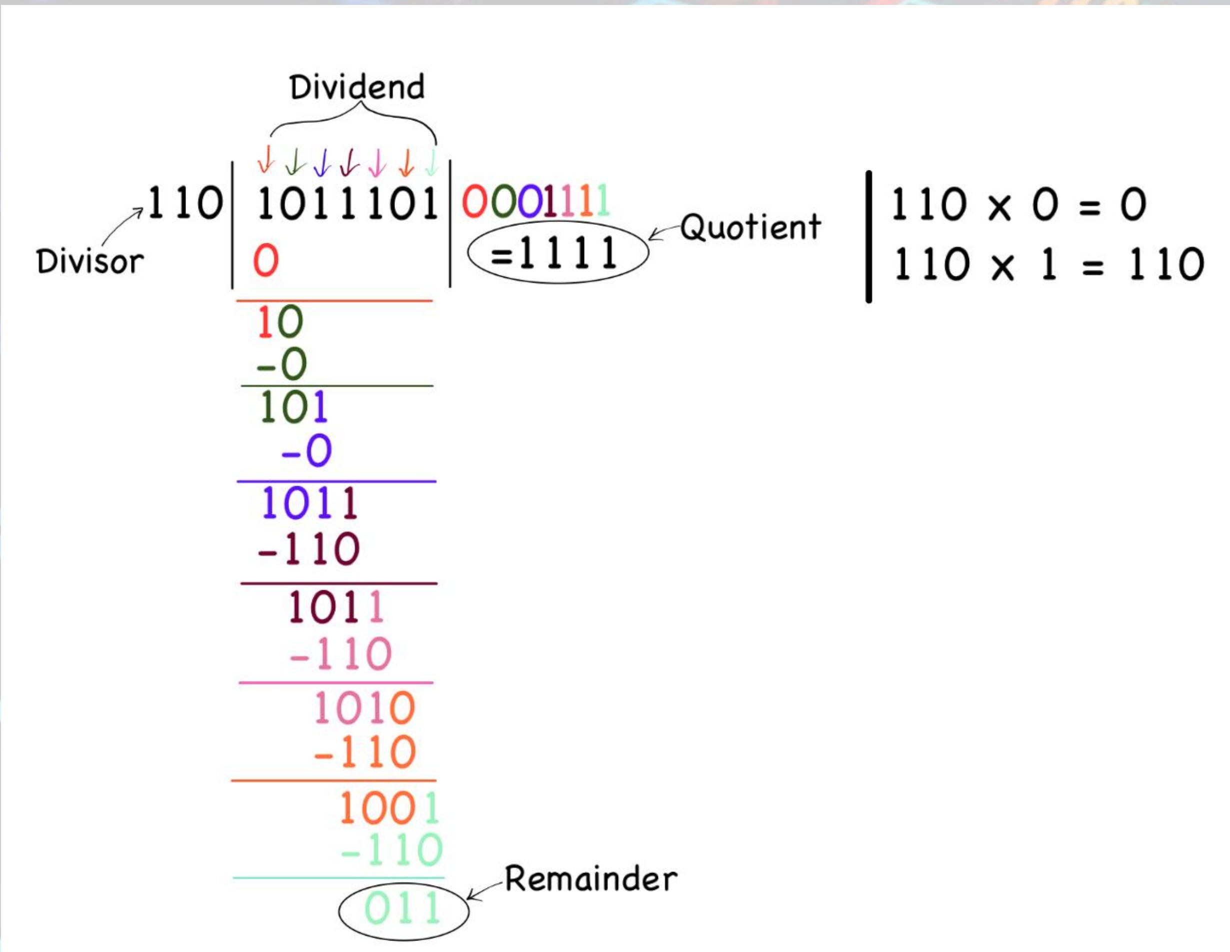


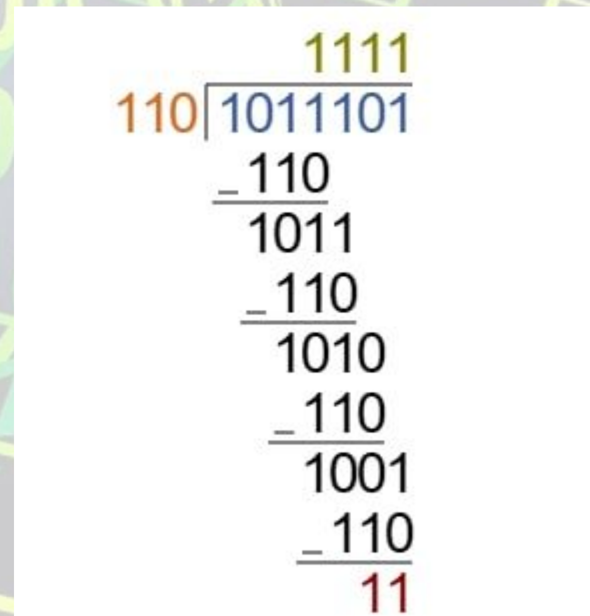
Iteration-3: (01 < 10)



Example - 1

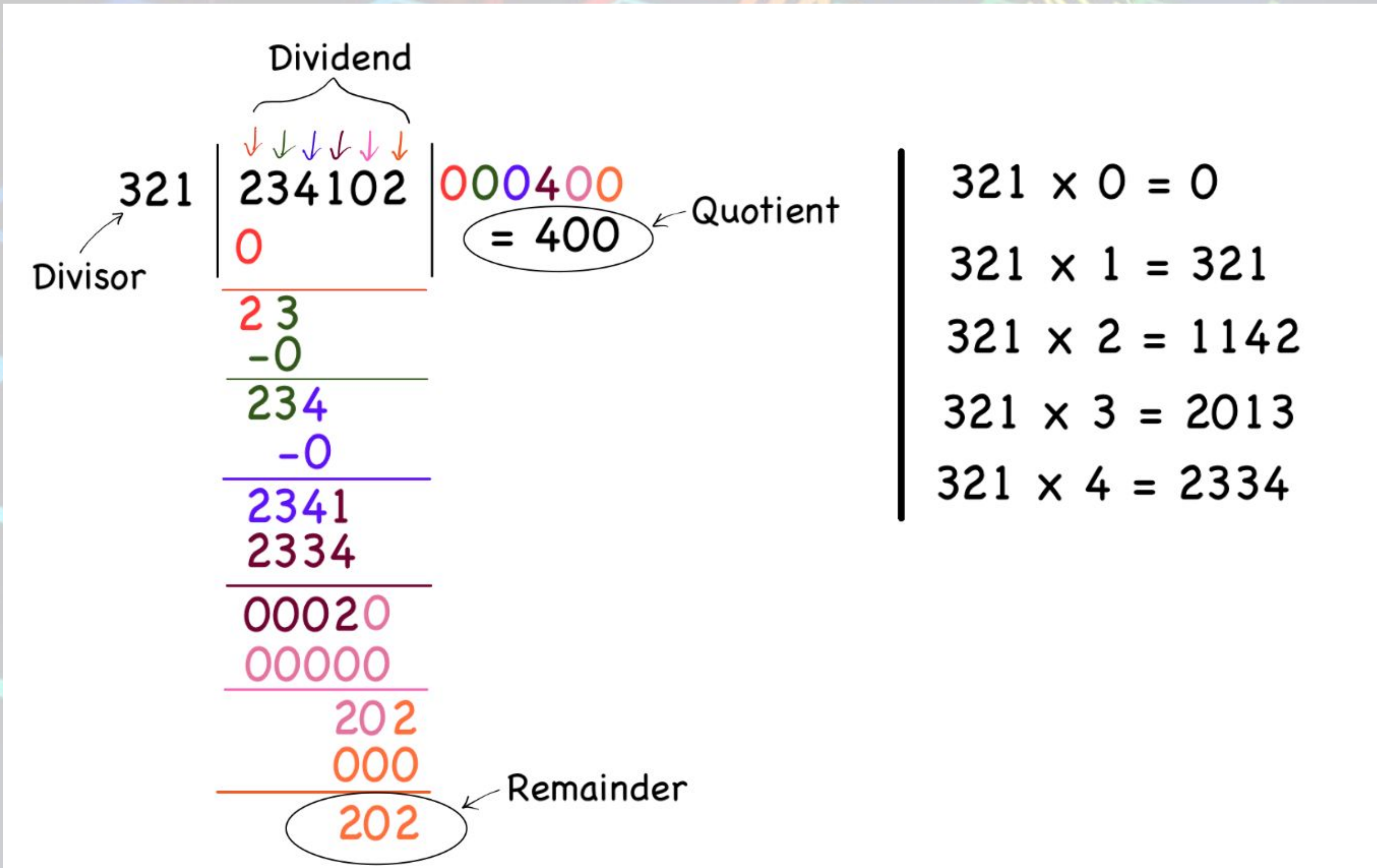
Perform 1011101_2 / 110_2; Find the quotient and remainder.





Example - 2

Perform 234102_5 / 321_5; Find the quotient and remainder.



Exercise

4 (a) $(71)_8 / (56)_8$

4(b) $(21)_3 / (12)_3$

Solution

- 4 a) Quotient = $()_8$, Remainder = $()_8$
- 4 b) Quotient = $()_3$, Remainder = $()_3$