

CSE260 Practice Sheet

Q: Are all of these enough to get full marks in the exam?

A: NO. This is a practice sheet. Meaning, you can practice all you want using the questions from this sheet. However, doing well in exams depends upon your ability to understand a question, formulate an answer, and express it correctly. You see, these are humane skills which cannot be guaranteed from completing a practice sheet only. But yeah, Best of luck anyways.

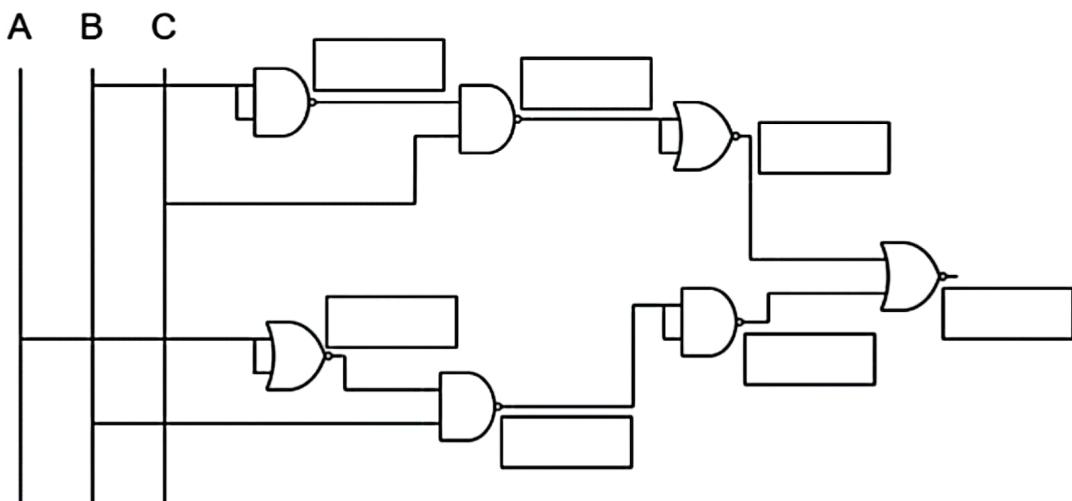
- Convert the following binary numbers to equivalent decimal numbers.
 - (a) $(101110001001)_2$
 - (b) $(11011.101)_2$
- Convert the following decimal number to equivalent binary numbers: $(4195)_{10}$
- Convert the following decimal number to equivalent binary numbers: $(3785.65625)_{10}$
- Convert the following decimal number to equivalent binary numbers: $(4785.150263)_{10}$ [for infinite fractional part, just do 6-7 steps and use dots for the rest]
- Convert the following decimal number to equivalent base 5 numbers: $(4123)_{10}$
- Convert the following decimal number to equivalent hexadecimal numbers: $(513)_{10}$
- Convert the following decimal number to equivalent base 9 numbers: $(813)_{10}$
- Perform the following base conversions
 - a) $(29)_{12} = (?)_7$
 - b) $(10110111)_5 = (?)_4$
- Perform addition, subtraction and multiplication for the pair of following base-9 numbers. Verify your results by converting the problem into decimal.

412
134

- Subtract 13 from 27 in 7 bits using 2's complement number system and justify whether there is an overflow or not.
- Subtract 45 from 98 in 12 bits using 2's complement number system and justify whether there is an overflow or not.
- Add 13 with 27 in 6 bits using 2's complement number system and justify whether there is an overflow or not.
- Add -57 with 63 in 11 bits using 2's complement number system and justify

whether there is an overflow or not.

- Perform the following arithmetic operations using 13-bit two's complement and one's complement systems . State if there is an overflow in each case.
 - a) $91 - 499$
 - b) $379 + 98$
- You are a computer engineer and you want to buy two 8 GB DDR4 RAMs. Each RAM costs $(1C2)_{16}$ dollars. You also want to buy a graphics card RTX4070Ti which costs $(10010110000)_2$ dollars. However, you don't have that much money with you and you are afraid to ask your parents about it. Suddenly, one of your generous friends agreed to give you the money you need. He decided to give you $(4064)_8$ dollars. How much will you have left after buying those components? (Show the answer in decimal)
- Simplify the following boolean expression to minimum number of literals:
$$(A + B)(A + \bar{B})(\bar{A} + C)$$
- Find the complement of the following expression:
$$(x' + y + z')(x' + y')(x + z')$$
- Determine the logic function of the above circuit. Use the boxes in the diagram to write your answer step by step.



- Draw the following functions using NAND gates only:
$$F(A,B,C,D) = (A'B'CD' + A'D + (B+D'))$$

NB: Please draw horizontally on your script.

NB: You can't simplify the above functions and then draw using NAND gate. You have to draw based on the function given in question
- Draw the following functions using NOR gates only:
$$F(A,B,C,D) = (AB'C'D' + AD + (B+D'))$$

NB: Please draw horizontally on your script.

NB: You can't simplify the above functions and then draw using NAND gate. You have to draw based on the function given in question

- Find out SOP and POS for the following:
 1. $F(A,B,C) = AB + BC'$
 2. $F(A,B,C,D) = A + B'CD'$
 3. $F(A,B,C,D,E) = AB + CDE$
- Simplify the following boolean equation using the laws of boolean algebra and implement the simplified function using only NOR gates:
 $F(a,b,c,d) = \Sigma(8, 9, 10, 11, 13, 15)$
- Find the simplified function using K-map:
 1. $F(a,b,c,d) = \Sigma(8, 9, 10, 11, 13, 15)$
 2. $F(a,b,c,d) = \Sigma(0, 1, 2, 11, 13, 15)$
 3. $F(a,b,c) = \Sigma(0,3,5,7)$
 4. $F(a,b,c,d) = \Sigma(8, 9, 10) + d(0,1,2)$
 5. $F(a,b,c,d) = \Sigma(1, 9, 10) + d(0,8,11,12)$
 6. $F(a,b,c,d) = \pi(1, 9, 10) + d(0,8,11,12)$
 7. $F(a,b,c,d) = \Sigma(0, 1, 2,3,4,5,6,7,8,9) + d(10,11,12,13,14)$
 8. $F(a,b,c,d) = \Sigma(0, 1, 2,3,4,5,6,7,8,9) + d(10,11,12,13,14,15)$
 9. $F(a,b,c,d) = \Sigma() + d(4,5,9)$
 10. $F(a,b,c,d) = \Sigma() + d()$

K Map Scenario Based Practice Problems

*For all problems, if you are asked to draw the circuit, you will need to draw the circuit too.

Question 1

In planet Z, there have been two new discoveries of Condition A and Condition B in plants. You have been tasked with creating a system which will detect the chances of development of Conditions A and B in plants. The conditions depend on the following factors:

Inputs (Factors) :

W: High Temperature. ($W=1$ if the temperature is high, $W=0$ otherwise)

X: Oxygen Level ($X=1$ if oxygen level is normal, $X=0$ otherwise)

Y: Presence of UV Light ($Y=1$ if UV light is present, $Y=0$ otherwise)

Z :Humidity ($Z= 1$ if the humidity is high, $Z=0$ otherwise)

Outputs: A : Condition A

B : Condition B

1. A plant will develop Condition A if **presence of UV Light** is accompanied by
 - i) **a high temperature with normal oxygen level**
or
 - ii) **low humidity.**
2. If **UV Light is absent** then the plant will develop Condition A if there is
 - i) **an abnormal level of oxygen**
or
 - ii) **there is high humidity with low temperature**
3. If the **oxygen level is not normal or there is high humidity with low temperature** then the plant will develop Condition B.

Devise a truth table for the system above which will detect the presence of the conditions based on the given factors. In your truth table, the input columns should be in the order WXYZ, where W is the MSB and Z is the LSB.

Draw K-map (s) based on the scenario above and derive simplified output (s).

Question 2

A new bizarre coffee machine has two LED lights, RED and GREEN which light up based on certain inputs. You are asked to derive the internal circuit of the machine which enables the lights to glow. Depending on the inputs both the lights, any one of the lights or none of the lights may glow.

Inputs (Symptoms) :

W: Water Temperature. ($W=1$ if the water temperature is high, $W=0$ otherwise)

X: Amount of coffee beans ($X=1$ if sufficient amount of coffee beans is present, $X=0$ otherwise)

Y: Presence of Milk ($Y=1$ if milk is present, $Y=0$ otherwise)

Z :Sugar Level ($Z= 1$ if sugar level is normal, $Z=0$ otherwise)

Outputs: R : RED LED

G : GREEN LED

1. The RED LED will light up if there is **milk present** and there is
 - i) **an absence of coffee beans**
or
 - ii) **normal sugar level with low water temperature.**
2. If there is **no milk** then the RED LED will still glow up if there is
 - i) **normal sugar level.**
or
 - ii) **a high water temperature with an adequate amount of coffee.**
3. The GREEN LED will light up if the **sugar level is normal** or when there is a **high water temperature with sufficient amount of coffee beans present.**

Devise a truth table for the system above which will detect the presence of the diseases based on the given symptoms. **In your truth table, the input columns should be in the order WXYZ, where W is the MSB and Z is the LSB.**

Question 3

On planet Z, there have been two new outbreaks of Disease X and Disease Y. You have been tasked with creating a system which will detect the presence of Diseases X and Y. A person may have no disease or any one of the diseases or both. The presence of the diseases will depend on the following symptoms:

Inputs (Symptoms) :

- A: Breathing Difficulties. (A=1 if you have breathing difficulties, A=0 otherwise)
- B: Normal Body Temperature (B=1 if you have a normal body temperature, B=0 otherwise)
- C: Fatigue (C=1 if you have fatigue, C=0 otherwise)
- D :Loss of smell (D= 1 if the person has lost his ability of smell, D=0 otherwise)

Outputs: X : Disease X

Y : Disease Y

1. You have Disease X if your **fatigue** is accompanied by
 - i) **breathing difficulties with normal body temperature**
or
 - ii) **no loss of smell.**
2. If you **don't have fatigue** then you will have Disease X if you have
 - i) **an abnormal body temperature**
or
 - ii) **you have loss of smell with no breathing difficulties**
3. If you **don't have a normal body temperature or you have lost your sense of smell with no breathing difficulties** then you have Disease Y.

Devise a truth table for the system above which will detect the presence of the diseases based on the given symptoms. **In your truth table, the input columns should be in the order ABCD, where A is the MSB and D is the LSB.**

Draw K-map (s) based on the scenario above and derive simplified output (s).

Question 4

On planet Z, there have been two new outbreaks of Disease X and Disease Y. You have been tasked with creating a system which will detect the presence of Diseases X and Y. A person may have no disease or any one of the diseases or both. The presence of the diseases will depend on the following symptoms:

Inputs (Symptoms) :

- A: Normal Breathing. (A=1 if you have normal breathing, A=0 otherwise)
- B: Abnormal Body Temperature (B=1 if you have an abnormal body temperature, B=0 otherwise)
- C: Fatigue (C=1 if you have fatigue, C=0 otherwise)
- D :Loss of smell (D= 1 if the person has lost his ability of smell, D=0 otherwise)

Outputs: X : Disease X

1. You have Disease X if your **fatigue** is accompanied by
 - i) **abnormal breathing with normal body temperature**
or
 - ii) **loss of smell.**
2. If you **don't have fatigue** then you will have Disease X if you have
 - i) **an abnormal body temperature**
or
 - ii) **you have loss of smell with normal breathing**
3. If you have **fatigue or you have an abnormal body temperature with loss of sense of smell** then you have Disease Y.

Devise a truth table for the system above which will detect the presence of the diseases based on the given symptoms. **In your truth table, the input columns should be in the order ABCD, where A is the MSB and D is the LSB.**

Draw K-map (s) based on the scenario above and derive simplified output (s).

Question 5

In planet Z, there have been two new discoveries of Condition A and Condition B in plants. You have been tasked with creating a system which will detect the chances of development of Conditions A and B in plants. The conditions depend on the following factors:

Inputs (Symptoms) :

- W: Low Temperature. (W=1 if the temperature is Low, W=0 otherwise)
- X: Oxygen Level (X=1 if oxygen level is abnormal, X=0 otherwise)
- Y: Presence of UV Light (Y=1 if UV light is present, Y=0 otherwise)
- Z :Humidity (Z= 1 if the humidity is high, Z=0 otherwise)

Outputs: A : Condition A

B : Condition B

1. A plant will develop Condition A if **presence of UV Light** is accompanied by
 - i) **high temperature with normal oxygen level**
or
 - ii) **low humidity.**
2. If **UV Light is absent** then the plant will develop Condition A if there is
 - i) **an abnormal level of oxygen**
or
 - ii) **high humidity with low temperature**
3. If there is an **abnormal level of oxygen or UV light is present in high humidity** then the plant will develop Condition B.

Devise a truth table for the system above which will detect the presence of the conditions based on the given factors. In your truth table, the input columns should be in the order WXYZ, where W is the MSB and Z is the LSB.

Draw K-map (s) based on the scenario above and derive simplified output (s).

Question No. 6 : Rock Scissors Game

Rock-paper-scissors is a game usually played between two people, in which each player simultaneously forms one of three shapes with an outstretched hand. These shapes are "rock" (a simple fist), "paper" (a flat hand), and "scissors" (a fist with the index and middle fingers together forming a V). The game has only three possible outcomes other than a tie: a player who decides to play rock will beat another player who has chosen scissors ("rock crushes scissors") but will lose to one who has played paper ("paper covers rock"); a play of paper will lose to a play of scissors ("scissors cut paper"). This time we have decided to slightly modify the game with only two options, Rock and Scissor.

Input (4 variables)

$R1 = 1$ indicates Player 1 has chosen **Rock**.

$S1 = 0$ indicates Player 1 has not chosen **Scissor**.

$R2 = 0$ indicates Player 2 has not chosen **Rock**.

$S2 = 1$ indicates Player 2 has chosen **Scissor**.

Output (3 variables)

$P1 = 1$ indicates **Player 1 is winner**

$P2 = 1$ indicates **Player 2 is winner**

$D = 1$ indicates **there is a tie**

We have to consider the following points when designing the game:

- i) If player 1 or player 2 has not chosen anything, there will be **no winner nor tie**.
- ii) A player cannot choose rock and scissor both at a time. This state can be considered as **DON'T CARE**.
 - a) Using the above specification, prepare a truth table for the game.
 - b) Using 4 variable Karnaugh-Map method, derive SOP expressions for all of the three possible results.

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Practise
Sheet solve
by Azmani

Convert the following binary numbers to equivalent decimal numbers.

a) $(101110001.001)_2$

$$= 1 \times 2^{11} + 0 \times 2^{10} + 1 \times 2^9 + 1 \times 2^8 + 1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 \\ + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= (2953)_{10}$$

b) $(11011.101)_2$

$$= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$$

$$= (27.625)_{10}$$

* decimal to binary

$$(4195)_{10} = (?)_2$$

$$\begin{array}{r} 4195 \\ \hline 2 | 2097 - 1 \\ \hline 2 | 1048 - 1 \\ \hline 2 | 524 - 0 \\ \hline 2 | 262 - 0 \\ \hline 2 | 131 - 0 \\ \hline & 65 - 1 \end{array}$$

$$\begin{array}{r} 65 - 1 \\ \hline 2 | 32 - 1 \\ \hline 2 | 16 - 0 \\ \hline 2 | 8 - 0 \\ \hline 2 | 4 - 0 \\ \hline 2 | 2 - 0 \\ \hline 2 | 1 - 0 \\ \hline & 0 - 1 \end{array}$$

$$(1000001100011)$$

$$(3785 \cdot 65625)_{10} = (?)_2$$

$$\begin{array}{r} 3785 \\ \hline 2 | 1892 - 1 \\ 2 | 946 - 0 \\ 2 | 473 - 0 \\ 2 | 236 - 1 \\ 2 | 118 - 0 \\ 2 | 59 - 0 \\ 2 | 29 - 1 \\ 2 | 14 - 1 \\ 2 | 7 - 1 \\ 2 | 3 - 1 \\ 2 | 1 - 1 \\ \hline 0 - 1 \end{array}$$

$$(11111001001)_2$$

$$0 \cdot 65625 \times 2 = 1.3125$$

$$0.3125 \times 2 = 0.625$$

$$0.625 \times 2 = 1.25$$

$$0.25 \times 2 = 0.5$$

$$0.5 \times 2 = 1.00$$

$$(0.10101)_2$$

$$(11111001001 \cdot 10101.)_2$$

$$(4785.150263)_{10} = (?)_2$$

$$\begin{array}{r} 4785 \\ \hline 2 | 2392 - 1 \\ 2 | 1196 - 0 \\ 2 | 598 - 0 \\ 2 | 299 - 0 \\ 2 | 149 - 1 \\ 2 | 74 - 1 \\ 2 | 37 - 0 \\ 2 | 18 - 1 \\ 2 | 9 - 0 \\ 2 | 4 - 1 \\ 2 | 2 - 0 \\ 2 | 1 - 0 \\ \hline 0 & - 1 \end{array}$$

$$(1001010110001)_2$$

$$0.150263 \times 2 = 0.300526 \quad | \quad (0.0010011)_2$$

$$0.300526 \times 2 = 0.601052$$

$$0.601052 \times 2 = 1.202104$$

$$0.202104 \times 2 = 0.404208$$

$$0.404208 \times 2 = 0.808416$$

$$0.808416 \times 2 = 1.616832$$

$$0.616832 \times 2 = 1.233664$$

$$(1001010110001.0010011)_2$$

$$(4123)_{10} = (?)_5$$

$$\begin{array}{r} 5 \\ \hline 4123 \\ 5 \\ \hline 824 - 3 \\ 5 \\ \hline 164 - 4 \\ 5 \\ \hline 32 - 4 \\ 5 \\ \hline 6 - 2 \\ 5 \\ \hline 1 - 1 \\ 0 - 1 \end{array}$$

$$(412443)_5$$

$$(513)_{10} = (?)_{16}$$

$$\begin{array}{r} 16 \\ \hline 513 \\ 16 \\ \hline 32 - 1 \\ 16 \\ \hline 2 - 0 \\ 0 - 2 \end{array}$$

- A - 10
- B - 11
- C - 12
- D - 13
- E - 14
- F - 15

$$(513)_{10} = (201)_{16}$$

$$(813)_{10} = (?)_9$$

$$\begin{array}{r} 9 \\ \hline 813 \\ 9 \\ \hline 90 - 3 \\ 9 \\ \hline 10 - 0 \\ 9 \\ \hline 1 - 1 \\ 0 - 1 \end{array} = (1103)_9$$

$$*(29)_{12} = (?)_7$$

$$(29)_{12} = 2 \times 12^1 + 9 \times 12^0 \\ = (33)_{10}$$

$$\begin{array}{r} 7 | 33 \\ 7 | \underline{4 - 5} \\ 0 - 4 \end{array} \quad (45)_7$$

$$(29)_{12} = (45)_7$$

$$*(10110111)_5 = (?)_4$$

$$(10110111)_5 = 1 \times 5^7 + 0 \times 5^6 + 1 \times 5^5 + 1 \times 5^4 + 0 \times 5^3 \\ + 1 \times 5^2 + 1 \times 5^1 + 1 \times 5^0 \\ = (81906)_{10}$$

$$\begin{array}{r} 4 | 81906 \\ 4 | \underline{20476 - 2} \\ 4 | \underline{5119 - 0} \\ 4 | \underline{1279 - 3} \\ 4 | \underline{319 - 3} \\ 4 | \underline{79 - 3} \\ 4 | \underline{19 - 3} \\ 4 | \underline{4 - 3} \\ 4 | \underline{1 - 0} \end{array}$$

$$(103333302)_{48}$$

* addition, subtraction, multiplication

son the pair of following base 9 numbers.

$$412 = 4 \times 9^2 + 1 \times 9^1 + 2 \times 9^0 = (335)_{10}$$

$$134 = 1 \times 9^2 + 3 \times 9^1 + 4 \times 9^0 = (112)_{10}$$

Addition

$$\begin{array}{r} 412 \\ \times 134 \\ \hline (546), \end{array}$$

$$\Rightarrow 5 \times 9^2 + 4 \times 9^1 + 6 \times 9^0 \\ = (147)_{10}$$

Subtraction

$$\begin{array}{r} 412 \\ \times 134 \\ \hline (367), \end{array}$$

$$\Rightarrow 2x^9 + 6x^9 + 7x^9 \\ = (233)_{10}$$

Multiplication

$$\begin{array}{r}
 412 \\
 \times 34 \\
 \hline
 1748 \\
 13360 \\
 41200 \\
 \hline
 \underline{(56408)} \\
 56418
 \end{array}$$

$$\begin{array}{r} 9) 16(1 \\ \underline{-9} \\ \hline 7 \end{array}$$

$$\Rightarrow 5x^4 + 6x^3 + 1x^2 + 10x^1 + 8x^0$$

$$= (37520)_{10}$$

* Subtract 13 from 27 in 7 bits using 2's complement number system and justify whether there is an overflow or not.

$$27 - 13 = 27 + (-13)$$

$$27 = 11011$$

$$+ 27 = 011011$$

$$= 0011011 \text{ (7 bit)}$$

$$13 = 1101$$

$$+ 13 = 01101$$

$$= 0001101 \text{ (7 bit)}$$

$$\begin{array}{r} 1110010 \\ + 1 \\ \hline - 13 = 1110011 \end{array}$$

$$0011011$$

$$1110011$$

$$\hline 10001110$$

extra

Ans : 0001110

no overflow, because we add 2 different sign numbers.

* Subtract 45 from 98 in 12 bits

using 2's complement number system
and justify whether there is an overflow
or not.

$$98 - 45 = 98 + (-45)$$

$$98 = 1100010$$

$$+ 45 = 0101101$$

$$= 000001100010 \text{ (12 bit)}$$

$$45 = 101101$$

$$+ 45 = 0101101$$

$$= 000000101101 \text{ (12 bit)}$$

$$\begin{array}{r} 111111010010 \\ + 1 \\ \hline 111111010011 \end{array}$$

$$-45 = 111111010011$$

$$\begin{array}{r} 000001100010 \\ 111111010011 \\ \hline 1000000110101 \end{array}$$

extra

$$\text{Ans} = 000000110101$$

no overflow

Add 13 with 27 in 6 bits using 2's comp.

$$13 = 1101$$

$$+13 = 01101$$

$$= 001101 \text{ (6 bit)}$$

$$27 = 11011$$

$$+27 = 011011$$

$$\begin{array}{r} 001101 \\ 011011 \\ \hline 101000 \end{array}$$

Overflow, because we add two same sign number and got different sign number.

*Add -57 with 63 in 11 bits using 2's comp.

$$57 = 111001$$

$$+57 = 0111001$$

$$= 00000111001 \text{ (11 bit)}$$

$$\begin{array}{r} 11111000110 \\ + 1 \\ \hline -57 = 11111000111 \end{array}$$

$$63 = 111111$$

$$+63 = 0111111 = 00000111111 \text{ (11 bit)}$$

$$\begin{array}{r}
 11111000111 \\
 00000111111 \\
 \hline
 10000000110
 \end{array}$$

$$1000 = 80$$

$$1000 = 80$$

$$Am = 00000000110$$

$$(mid) 10000 =$$

no overflow

$$1000 = 80$$

2's comp (13 bit)

$$1000 = 80$$

$$a) 91 - 499$$

$$101100$$

$$= 91 + (-499)$$

$$110100$$

$$1000101$$

$$91 = 1011011$$

$$+ 91 = 01011011 = 0000001011011 (13 bit)$$

$$499 = 111110011$$

$$+ 499 = 0111110011 = 000011110011 (13 bit)$$

$$\begin{array}{r}
 111100001100 \\
 1001110 + 1 \\
 \hline
 111100001101
 \end{array}$$

$$0000001011011$$

$$1111000001101$$

$$\hline 1111001101000$$

no overflow

1's complement

$$91 + (-199)$$

$$91 = 1011011$$

$$+91 = 01011011 = 0000001011011 \text{ (13 bit)}$$

$$199 = 111110011$$

$$+199 = 0111110011 = 0000111110011 \text{ (13 bit)}$$

$$; (0000111110011 - 199) = 1111000001100$$

$$0000001011011$$

$$\underline{1111000001100}$$

$$\underline{\underline{1111001100111}}$$

no overflow

2's complement / 1's comp

$$b) 379 + 98$$

$$379 = 101111011$$

$$+379 = 0101111011 = 0000101111011 \text{ (13 bit)}$$

$$98 = 1100010$$

$$+98 = 01100010 = 0000001100010 \text{ (13 bit)}$$

$$0000101111011$$

$$\underline{0000001100010}$$

$$\underline{\underline{0000111011101}}$$

no overflow

*

$$\text{cost} = (1 \text{ C } 2)_{16}$$

$$= 1 \times 16^2 + 12 \times 16^1 + 2 \times 16^0$$

$$\therefore \text{cost} = (450)_{10} \quad (10010100000_2 = 1101010 = 100)$$

$$\therefore 8 \text{ GB RAM cost} = 450 \times 2 = 900$$

$$\text{graphics cost} = (10010110000)_2$$

$$= 1 \times 2^{10} + 0 \times 2^9 + 0 \times 2^8 + 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5$$

$$+ 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$$

$$= (1200)_{10}$$

$$\text{Total} = 900 + 1200 = (2100)_{10}$$

$$(4064)_8 = 4 \times 8^3 + 0 \times 8^2 + 6 \times 8^1 + 4 \times 8^0$$

$$= (2100)_{10}$$

$$\text{left} = 2100 - 2100 = 0$$

* Simplify the following boolean expression to minimum number of literals :

$$\begin{aligned}& (A+B)(A+\bar{B})(\bar{A}+C) \\& = (AA + AB + A\bar{B} + B\bar{B}) (\bar{A} + C) \\& = (A + AB + A\bar{B} + 0) (\bar{A} + C) \\& = A\bar{A} + A\bar{A}B + A\bar{A}\bar{B} + AC + ABC + A\bar{B}C \\& = 0 + 0 + 0 + AC(1+B) + A\bar{B}C \\& = AC + A\bar{B}C \\& = AC(1+\bar{B}) \\& = AC\end{aligned}$$

* find the complement

$$\begin{aligned}& (x'+y+z') (x'+y') (x+z') \\& = (x'y'z') + (x'y') + (xz') \\& = xy'z + xy + x'z\end{aligned}$$

Using De-morgan

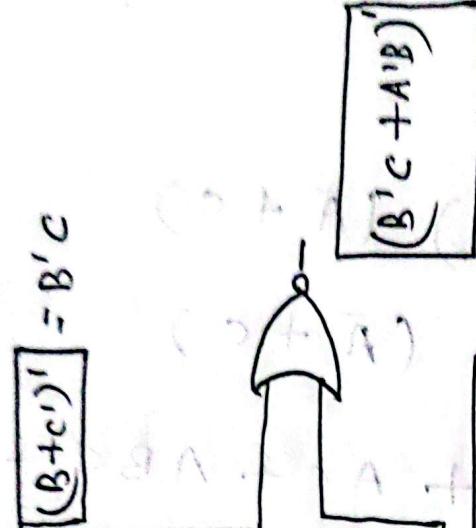
$$\begin{aligned}& ((x'+y+z') (x'+y') (x+z'))' \\& = (x'+y+z')' + (x'+y')' + (x+z')' \\& = xy'z + xy + x'z\end{aligned}$$

A B C

$$= B + C'$$

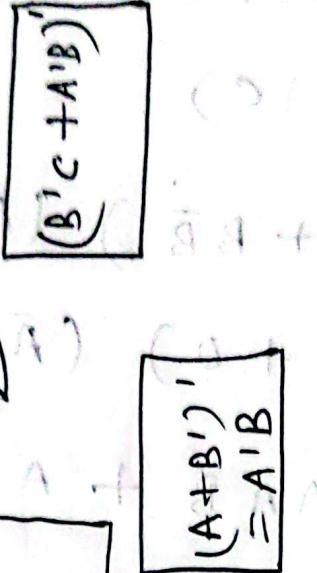
$$B'$$

$$(B + C')' = B' C$$

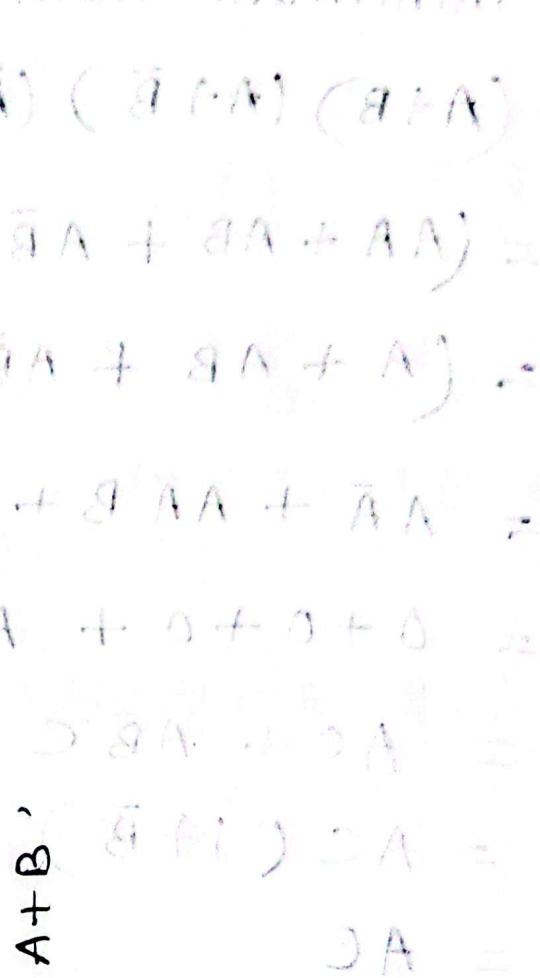


$$A'$$

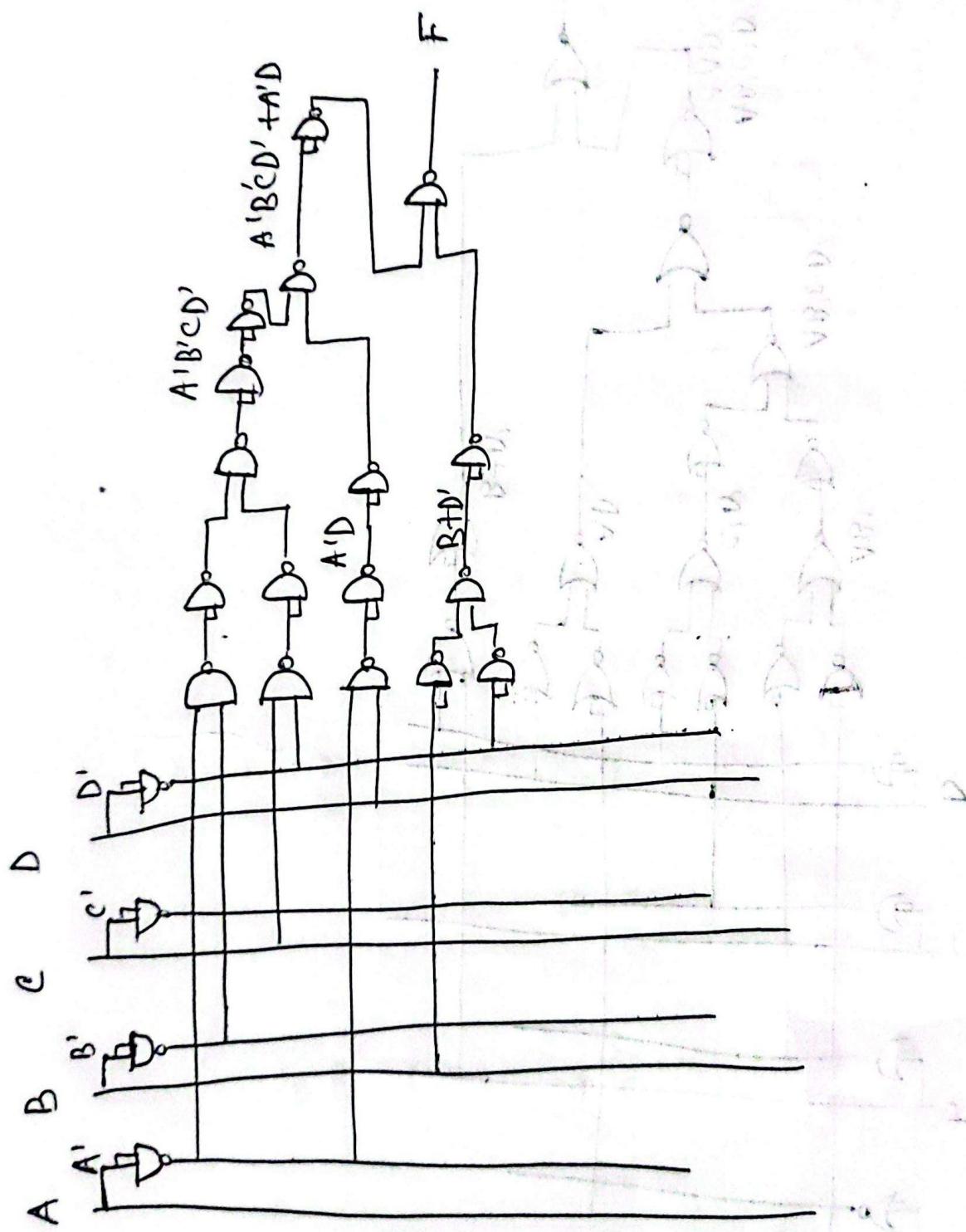
$$(A + B')'$$



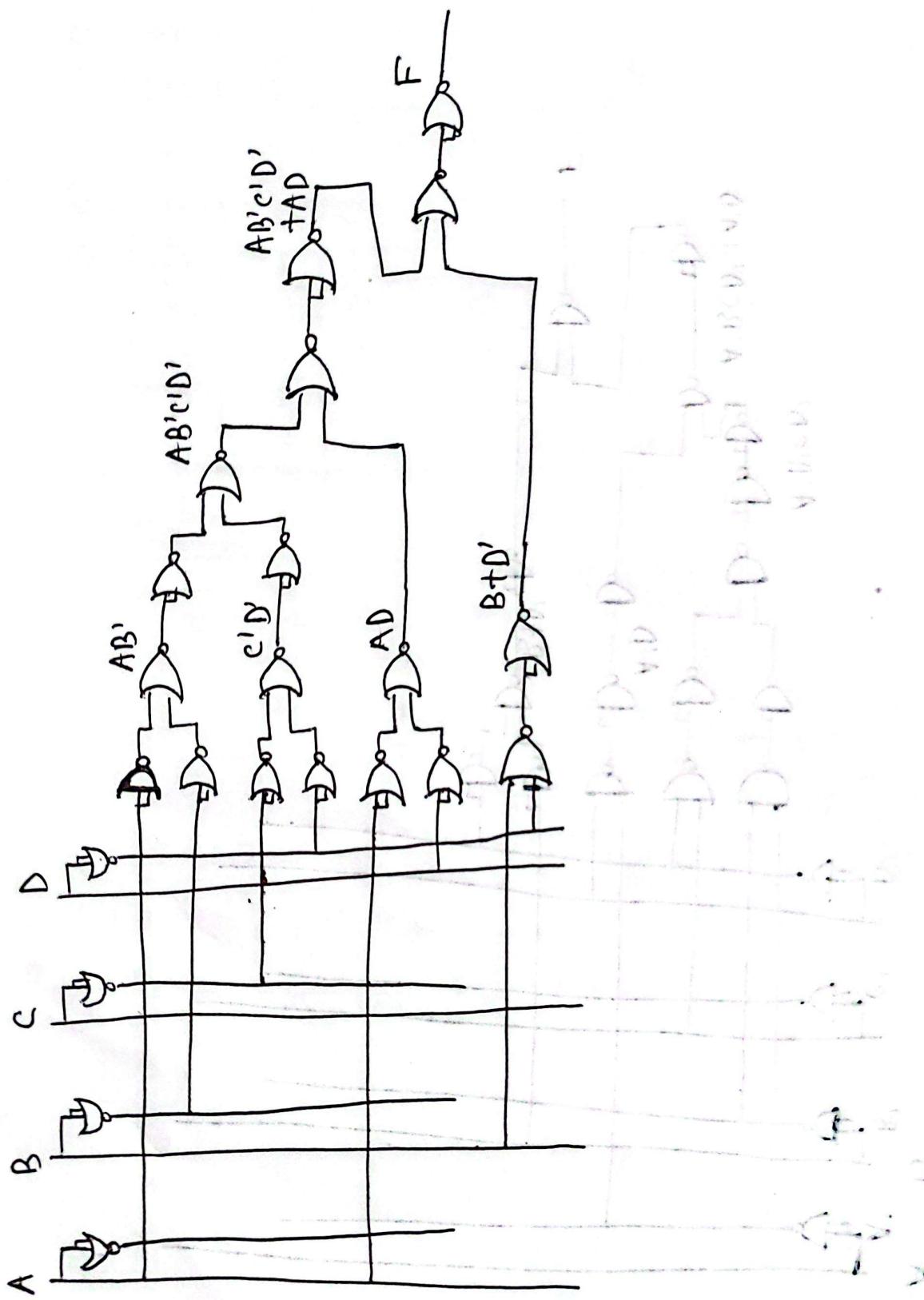
$$= A + B'$$



NAND gate $\Rightarrow F(A, B, C, D) = (A'B'C'D' + A'D + (B + D'))$



$$\text{NOR gate} \Rightarrow F(A, B, C, D) = (AB'C'D' + AD + (B + D'))$$



Find out SOP and POS:

$$1. F(A, B, C) = AB + BC'$$

$$= AB(C+C') + (A+A')BC'$$

$$= ABC + ABC' + A'BC' + A'BC$$

$$= \Sigma(7, 6, 6, 2)$$

$$= \Sigma(2, 6, 7)$$

$$= \Pi(0, 1, 3, 4, 5)$$

SOP \rightarrow POS

$$F(A, B, C) = AB + BC'$$

$$= (AB+B)(AB+C')$$

$$= \cancel{AB}(A+B)(B+B)(A+C')(B+C')$$

$$= (A+B)(B)(A+C')(B+C')$$

$$= (A+B+CC')(AA''+B+CC')(A+BB'+C')(AA'+B+C')$$

$$= (A+B+C)(A+B+C')(A+B+C)(A'+B+C')(A'+B+C)$$

$$(A+B+C')(A+B+C')(A+B'+C')(A+B+C)(A'+B+C)$$

$$= \cancel{\Pi}(0, 1, 0, 5, 4, 1, 1, 3, 1, 5)$$

$$= \cancel{\Pi}(0, 1, 3, 4, 5)$$

$$\begin{aligned}
 ② F(A, B, C, D) &= A + B'C'D' \\
 &= A(B+B')(C+C')(D+D') + (A+A')B'C'D' \\
 &= (AB+AB')(CD+C'D+CD'+C'D') + AB'C'D' + A'B'C'D' \\
 &= ABCD + ABC'D + ABCD' + ABC'D' + AB'C'D \\
 &\quad + AB'C'D + AB'C'D' + AB'C'D' + AB'C'D' + A'B'C'D' \\
 &= \Sigma(15, 13, 14, 12, 11, 9, 10, 8, 10, 2) \\
 &= \Sigma(2, 8, 9, 10, 11, 12, 13, 14, 15) \\
 &= \Pi(0, 1, 3, 4, 5, 6, 7)
 \end{aligned}$$

SOP \rightarrow POS

$$\begin{aligned}
 F(A, B, C, D) &= A + B'C'D + B(C+D) + A(C+D) \\
 &= (A+B')(A+C'D') + A(C+D) \\
 &= (A+B')(A+C)(A+D') \\
 &= (A+B'+CC'+DD')(A+BB'+C+DD')(A+BB'+CC'+DD') \\
 &= (A+B'+C+D)(A+B'+C'+D')(A+B'+C'+D)(A+B'+C+D) \\
 &\quad (A+B+C+D)(A+B'+C+D')(A+B'+C+D)(A+B+C+D) \\
 &\quad (A+B+C+D')(A+B'+C'+D')(A+B'+C+D')(A+B+C'+D') \\
 &= \Pi(4, 7, 6, 5, 0, 5, 4, 1, 1, 7, 5, 3) = \Pi(0, 1, 3, 4, 5, 6)
 \end{aligned}$$

$$\begin{aligned}
 ③ F(A, B, C, D, E) &= AB + CDE \\
 &= AB(C+C')(D+D')(E+E') + (A+A')(B+B')CDE \\
 &= (ABC + ABC') (DE + D'E + DE' + D'E') + (AB + AB' + A'B \\
 &\quad + A'B') CDE \\
 &= ABCDE + ABCD'E + ABCDE' + ABCD'E' + ABC'DE \\
 &\quad + ABC'D'E + ABC'D'E' + ABC'D'E' + ABCDE \\
 &\quad + AB'CDE + A'BCDE + A'B'CDE \\
 &= \Sigma(31, 29, 30, 28, 27, 25, 26, 24, 31, 23, \\
 &\quad 15, 7) \\
 &= \Sigma(7, 15, 23, 24, 25, 26, 27, 28, 29, 30, 31) \\
 &= \Pi(0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, \\
 &\quad 19, 20, 21, 22)
 \end{aligned}$$

$$F(a, b, c, d) = \sum(8, 9, 10, 11, 13, 15)$$

1000 1001 1010 1011 1101 1111

$$= a'b'c'd' + ab'c'd + ab'cd' + ab'cd + abc'd + abcd$$

$$= ab'c'(d+d') + ab'c(d+d') + abd(c+c')$$

$$= ab'c' + ab'c + abd$$

$$= ab'(c+c') + abd$$

$$= ab' + abd$$

$$= a(b' + bd)$$

$$= a(b' + b)(b' + d)$$

$$= a(b' + d)$$

(F)

(Simplification)

SOP \rightarrow POS

$$\begin{aligned} F(w, x, y, z) &= wy + x'z \\ &= (wy + x')(wy + z) \\ &= (w + x')(x' + y)(w + z)(y + z) \\ &= (w + x' + y + z)(w + x' + y + z') \\ &\quad (w + x' + y + z') \\ &= (w + x' + y + z)(w + x' + y' + z') \\ &\quad (w + x' + y' + z) \\ &\quad (w + x' + y + z') \\ &= (w + x + y + z)(w + x' + y' + z) \\ &\quad (w + x' + y + z) \\ &\quad (w + x + y + z') \\ &= \pi(4, 7, 6, 5, 4, 13, 12, 5, 0, 6, 4, 2, 0, \\ &\quad 12, 8, 4) \\ &= \pi(0, 2, 4, 5, 6, 7, 8, 12, 13) \end{aligned}$$

K-map

$$1) F(a, b, c, d) = \Sigma(8, 9, 10, 11, 13, 15)$$

	$c'd'$	$c'd$	cd	cd'
$a'b'$	0	1	3	2
$a'b$	4	5	7	6
ab	12	13	1	5
ab'	1	1	1	1
	8	9	11	10

$$ad + ab'$$

$$2) F(a, b, c, d) = \Sigma(0, 1, 2, 11, 13, 15)$$

	$c'b'$	$c'd$	cd	cd'
$a'b'$	1	1	1	1
$a'b$	4	5	7	6
ab	12	13	1	14
ab'	8	9	11	10

$$a'b'd' + a'b'c'd + abd + acd'$$

$$3) F(a, b, c) = \Sigma(0, 3, 5, 7)$$

	$b'c'$	$b'c$	bc	bc'
a'	1		1	2
a	4	1	1	6

$$a'b'c' + bc + ac$$

$$4) F(a, b, c, d) = \Sigma(8, 9, 10) + d(0, 1, 2)$$

$c'd' c'd cd \bar{c}d'$

$a'b'$	X_0	X_1	3	X_3
$a'b$	4	5	7	6
ab	8	9	10	10
ab'	1	1		1

$$b'c' + b'\bar{c}d'$$

$$5) F(a, b, c, d) = \Sigma(1, 9, 10) + d(0, 8, 11, 12)$$

$c'd' c'd \bar{c}d cd \bar{c}d'$

$a'b'$	X_0	1	3	2
$a'b$	4	5	7	6
ab	X_{12}	13	15	14
ab'	X_8	1	X_{11}	1

$$b'c' + ab'$$

$$\textcircled{6} \quad F(a, b, c, d) = \pi(1, 9, 10) + d(0, 8, 11, 12)$$

$$= \sum(2, 3, 4, 5, 6, 7, 13, 14, 15) + d(0, 8, 11, 12)$$

	$c'd'$	$c'd$	cd	cd'
$a'b'$	X_0	1	1_3	1_2
$a'b$	1_4	1_5	1_7	1_6
ab	X_{12}	1	1_{13}	1_{15}
ab'	X_8	1_9	X_{11}	X_{10}

$b + a'c$

$$\textcircled{7} \quad F(a, b, c, d) = \sum(0, 1, 2, 3, 4, 5, 6, 7, 8, 9) \\ + d(10, 11, 12, 13, 14)$$

	$c'd'$	$c'd$	cd	cd'	x
$a'b'$	1_0	1 ₁	1_3	1_2	
$a'b$	1_4	1_5	1_7	1_6	
ab	X_{12}	X_{13}	1_{15}	X_{14}	
ab'	1_8	1_9	X_{11}	X_{10}	

$a' + c'$

$$⑧ F(a, b, c, d) = \sum (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)$$

$$+ d(10, 11, 12, 13, 14, 15)$$

	$c'd'$	cd'	cd	$c'd$	$a'b'$
$a'b$	1 ₀	1 ₁	1 ₃	1 ₂	1 ₄
ab	X ₁₂	X ₁₃	X ₁₅	X ₁₄	
ab'	1 ₈	1 ₉	X ₁₁	X ₁₀	

$$⑨ F(a, b, c, d) = \sum () + d(4, 5, 9)$$

X	X	
X		

$$⑩ F(a, b, c, d) = \sum () + d()$$

Ans : 0

K-map Scenario Based:

①

$$A = Y(WX + Z')$$

$$A = Y'(X' + ZW')$$

$$A = WXY + YZ' + X'Y' + W'Y'Z$$

$$= WX(Y(Z+Z')) + (W+W')(X+X')YZ' + (W+W')X'Y'$$

$$(Z+Z') + W'(X+X')Y'Z$$

$$= WXYZ + WXYZ' + (WX + W'X + W'X' + W'X)YZ'$$

$$+ (WX'Y' + W'X'Y')(Z+Z') + W'XY'Z + W'X'Y'Z$$

$$= \underset{1111}{WXYZ} + \underset{1110}{WX'YZ'} + \underset{1110}{WX'YZ} + \underset{0110}{W'XYZ'} + \underset{1010}{W'X'YZ}$$

$$+ \underset{0010}{W'X'YZ'} + \underset{1001}{WX'Y'Z} + \underset{1000}{WX'Y'Z'} + \underset{0001}{W'X'Y'Z}$$

$$+ \underset{0000}{W'X'Y'Z} + \underset{0101}{W'XY'Z} + \underset{0001}{W'X'Y'Z}$$

$$= \Sigma(15, 14, 14, 6, 10, 2, 9, 8, 1, 0, 5, 1)$$

$$= \Sigma(0, 1, 2, 5, 6, 8, 9, 10, 14, 15)$$

$$B = X' + ZW'$$

$$= (W+W')X'(Y+Y')Z + W'(X+X')(Y+Y)Z$$

$$= (WX'+W'X') (YZ+Y'Z+Y'Z+Y'Z) + (W'X+W'X') (YZ+Y'Z)$$

$$= \underset{1011}{WX'}YZ + \underset{1001}{WX'Y'Z} + \underset{1010}{WX'Y'Z} + \underset{1000}{WX'Y'Z} + \underset{0011}{W'X'YZ}$$

$$+ \underset{0001}{W'X'Y'Z} + \underset{0010}{W'X'Y'Z} + \underset{0000}{W'X'Y'Z} + \underset{0111}{W'X'YZ}$$

$$+ \underset{0101}{W'X'Y'Z} + \underset{0011}{W'X'Y'Z} + \underset{0001}{W'X'Y'Z}$$

$$= \Sigma (11, 9, 10, 8, 3, 1, 2, 0, 7, 5, 3, 1)$$

$$= \Sigma (0, 1, 2, 3, 5, 7, 8, 9, 10, 11)$$

minterm	W	X	Y	Z	A	B
0	0	0	0	0	1	1
1	0	0	0	1	1	1
2	0	0	1	0	1	1
3	0	0	1	1	0	1
4	0	1	0	0	0	0
5	0	1	0	1	1	1
6	0	1	1	0	1	0
7	0	1	1	1	0	1
8	1	0	0	0	1	1
9	1	0	0	1	1	1
10	1	0	1	0	1	1
11	1	0	1	1	0	1
12	1	1	0	0	0	0
13	1	1	0	1	0	0
14	1	1	1	0	1	0
15	1	1	1	1	1	0

②

$$R = Y(X' + ZW')$$

$$R = Y'(Z + WX)$$

$$R = X'Y + W'YZ + Y'Z + WX'Y'$$

$$= (W+W')X'Y(Z+Z') + W'(X+X')YZ + (W+W')(X+X')Y'Z + WX'Y'(Z+Z')$$

$$= (WX'Y + W'X'Y)(Z+Z') + W'XYZ + W'X'YZ$$

$$+ (WX + WX' + W'X + W'X')YZ'Z + WX'YZ + WX'Y'Z$$

$$= WX'YZ + W'X'Y'Z + WX'Y'Z' + W'X'YZ' + W'X'YZ$$

$$+ W'X'YZ + WX'Y'Z + W'X'Y'Z' + W'X'YZ' + W'X'YZ$$

$$+ WX'YZ + WX'Y'Z$$

$$= \Sigma 11, 3, 10, 2, 7, 3, 13, 9, 5, 1, 13, 12$$

$$= \Sigma (1, 2, 3, 5, 7, 9, 10, 11, 12, 13)$$

$$\begin{aligned}
 G_7 &= Z + WX \\
 &= (W+W') (X+X') (Y+Y') Z + WX(Y+Y') (Z+Z') \\
 &= (WX+W'X+WX'+W'X') (YZ+Y'Z) + WX(YZ+Y'Z) \\
 &= WXYZ + W'X'YZ + W'X(YZ + W'X'Y'Z) + WXY'Z \\
 &\quad + W'X'Y'Z + WXY'Z + W'X'Y'Z + WXYZ \\
 &\quad + WXY'Z + WXYZ' + WXY'Z + \\
 &= \Sigma(15, 7, 11, 3, 13, 5, 9, 1, 15, 13, 14, 12) \\
 &= \Sigma(1, 3, 5, 7, 9, 11, 12, 13, 14, 15)
 \end{aligned}$$

③

$$X = C(AB + D')$$

$$X = C'(B' + DA')$$

$$X = ABC + CD'(B' + C) + A'C'D$$

$$= ABC(D+D') + (A+A')(B+B')CD' + (A+A')B'C'(D+D')$$

$$+ A'(B+B')C'D$$

$$= ABCD + ABCD' + (AB + A'B + AB' + A'B')CD'$$
$$+ (AB'C + A'B'C)(D+D') + A'BC'D + A'B'C'D$$

$$= ABCD + ABCD' + ABCD' + A'BCD' + AB'C'D'$$

$$+ A'B'C'D' + AB'C'D + AB'C'D' + A'B'C'D$$

$$+ A'B'C'D' + A'BC'D + A'B'C'D$$

$$= \Sigma (15, 14, 14, 6, 10, ?, ?, 8, 1, 0, 5, 1)$$

$$= \Sigma (0, 1, 2, 5, 6, 8, 9, 10, 14, 15)$$

$$Y = B' + A'D$$

$$= (A+A')B'(C+C')(D+D') + A'(B+B')(C+C')D$$

$$= (AB' + A'B')(CD + C'D + CD' + C'D') + (A'B + A'B')(CD + C'D)$$

$$\begin{aligned}
 &= A'B'CD + A'B'C'D + AB'CD' + AB'C'D' \\
 &\quad |011| \quad |001| \quad |010| \quad |000| \\
 &+ A'B'CD + A'B'C'D + A'B'CD' + A'B'C'D' \\
 &\quad |001| \quad |000| \quad |0010| \quad |0000| \\
 &+ A'BCD + A'BC'D + A'B'CD + A'B'C'D \\
 &\quad |011| \quad |010| \quad |001| \quad |000|
 \end{aligned}$$

$$= \Sigma (11, 9, 10, 8, 3, 1, 2, 0, 7, 5, 3, 1)$$

$$= \Sigma (0, 1, 2, 3, 5, 7, 8, 9, 10, 11)$$

(6)

	R ₁	S ₁	R ₂	S ₂	P ₁	P ₂	D
0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0
2	0	0	1	0	0	0	0
3	0	0	1	1	X	X	X
4	0	1	0	0	0	0	0
5	0	1	0	1	0	0	1
6	0	1	1	0	0	1	0
7	0	1	1	1	X	X	X
8	1	0	0	0	0	0	0
9	1	0	0	1	1	0	0
10	1	0	1	0	0	0	1
11	1	0	1	1	X	X	X
12	1	1	0	0	X	X	X
13	1	1	0	1	X	X	X
14	1	1	1	0	X	X	X
15	1	1	1	1	X	X	X

	R ₁ S ₂	R ₁ S ₂ '	R ₂ S ₁	R ₂ S ₁ '
1	0	X ₃	2	
2	1	X ₅	X ₇	6
3	X ₁₂	X ₁₀	X ₁₃	X ₁₁
4	8	X ₉	X ₁₁	10

R ₁ 'S ₁		X	
R ₁ 'S ₁			X
R ₁ S ₁	X	X	X
R ₁ S ₁ '		X	

	X		
1	X	1	
X	X	X	X
	X	1	

$$P_1 = R_1 S_2$$

$$P_2 = R_2 S_1$$

$$D = S_1 S_2 + R_1 R_2$$