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CSE 330

Assignment 5

Section: 17

Ans no 1

a) $$\begin{vmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 10 \\ 6 \\ 9 \end{vmatrix}$$

A $\quad$ x $\quad$ b

b) $$A^1 = \begin{vmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{vmatrix}$$

$$m_{21} = \frac{a_{21}}{a_{11}} = \frac{3}{1} = 3$$

$$m_{31} = \frac{a_{31}}{a_{11}} = \frac{4}{1} = 4$$

$$F^1 = \begin{vmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{vmatrix}$$

$$A^2 = F^1 A^1 = \begin{vmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -19 & -6 \end{vmatrix}$$

$$m_{32} = \frac{a_{32}}{a_{22}} = \frac{-19}{-16} = \frac{19}{16}$$

$$F^2 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -19/16 & 1 \end{vmatrix}$$

$$A^3 = F^2 A^2 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -19/16 & 1 \end{vmatrix} \begin{vmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -19 & -6 \end{vmatrix} = \begin{vmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -1/16 \end{vmatrix}$$

c) $$L = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 19/16 & 1 \end{vmatrix}$$

d) $Ly = b$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 19/16 & 1 \end{vmatrix} \begin{vmatrix} y_1 \\ y_2 \\ y_3 \end{vmatrix} = \begin{vmatrix} 10 \\ 6 \\ 9 \end{vmatrix}$$

$\Rightarrow y_1 = 10$ | $3y_1 + y_2 = 6$ | $4y_1 + \frac{19}{16} y_2 + y_3 = 9$

$\Rightarrow y_2 = 6 - 30 = -24$ | $\Rightarrow 10 + \frac{19}{16}(-24) + y_3 = 9$

$\Rightarrow y_3 = -5/2$

$Ux = y$

$$\Rightarrow \begin{vmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -1/16 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 10 \\ -24 \\ -5/2 \end{vmatrix}$$

$-\frac{1}{16}x_3 = -5/2$ | $-16x_2 - 5x_3 = -24$ | $x_1 + 6x_2 + 2x_3 = 10$

$\Rightarrow x_3 = 40$ | $\Rightarrow -16x_2 - 5\times 40 = -24$ | $\Rightarrow x_1 + 6(-11) + 2\times 40 = 10$

$\Rightarrow x_2 = -11$ | $\Rightarrow x_1 = -4$

$$x = \begin{vmatrix} -4 \\ -11 \\ 40 \end{vmatrix}$$

Ans no 2

a) $$\begin{vmatrix} 0 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 10 \\ 6 \\ 9 \end{vmatrix}$$

$A \qquad x \qquad b$

b) $$m_{21} = \frac{a_{21}}{a_{11}} = \frac{3}{0} = \infty$$

It causes pivoting problem.

Pivoting problem arises when Gaussian elimination encounters a zero, which can lead to errors or even algorithm failure.

So, A matrix has pivoting problem because the top-left element is 0.

c)

$$Aug(A) = \begin{vmatrix} 0 & 6 & 2 & 10 \\ 3 & 2 & 1 & 6 \\ 1 & 5 & 2 & 9 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 1 & 5 & 2 & 9 \end{vmatrix} \quad R_1 \rightleftharpoons R_2$$

$$m_{31} = \frac{a_{21}}{a_{11}} = \frac{1}{3} \quad ; R_3' \to R_3 - R_1 m_{31}$$

$$A^2 = \begin{vmatrix} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 0 & 7/3 & 2/3 & 1 \end{vmatrix}$$

$$m_{32} = \frac{a_{32}}{a_{22}} = \frac{7/3}{6} = \frac{7}{18} \quad ; R_3' \to R_3 - R_2 m_{32}$$

$$A^3 = \begin{vmatrix} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 0 & 0 & -1/9 & -26/9 \end{vmatrix} = U$$

d) $-1/9 x_3 = -26/9$

$\therefore x_3 = 26$

$6x_2 + 2x_3 = 10$

$\Rightarrow 6x_2 + 2 \times 26 = 10$

$\therefore x_2 = -7$

$3x_1 + 2x_2 + x_3 = 6$

$\Rightarrow 3x_1 + 2(-7) + 26 = 6$

$\therefore x_1 = -2$

$$\therefore x = \begin{vmatrix} -2 \\ -7 \\ 26 \end{vmatrix}$$