CSE330- Numerical Methods

Quiz 05: Fall'24 [CO3]

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Marks: 15 points   **Time: 20 minutes**

A linear system is described by the following equations:

$$x_1 + 2x_3 = 10$$
$$3x_1 = 6$$
$$2x_1 + 5x_2 + 2x_3 = 9$$

Based on these equations, answer the questions below.

(a) From the given linear equations, identify the matrices **A, x and b**. Examine if the **matrix A** has any pivoting problem? If yes, solve the pivoting problem. [5 marks]

(b) Construct the **Frobenius matrices** $F^{(1)}$ and $F^{(2)}$ from this system. [3 marks]

(c) Compute the **unit lower triangular matrix** L. [3 marks]

(d) Now find the **solution** of the linear system using the LU decomposition method. Use the unit lower triangular matrix found in the previous question. [4 marks]

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<u>a</u>

diagonal elements is 0

So pivot problem exists.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 0 & 0 \\ 2 & 5 & 2 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad b = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

1) $R_3 \rightleftharpoons R_1$, then $C_1 \rightleftharpoons C_2$

$$\begin{bmatrix} 5 & 2 & 2 \\ 0 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 10 \end{bmatrix}$$

<u>b</u>

$$m_{21} = \frac{a_{21}}{a_{11}} = \frac{0}{5} = 0$$

$$R_2' = R_2 - R_1 \times 0 = R_2$$

$$m_{31} = \frac{a_{31}}{a_{11}} = \frac{0}{5} = 0$$

$$R_3' = R_3 - R_1 \times 0 = R_3$$

$$A_2 = \begin{bmatrix} 5 & 2 & 2 \\ 0 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$m_{32} = \frac{a_{32}}{a_{22}} = \frac{1}{3}$$

$$R_3'' = R_3' - R_2' \times \frac{1}{3}$$

$$A_3 = \begin{bmatrix} 5 & 2 & 2 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} = U$$

$$F^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad F^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{3} & 1 \end{bmatrix}$$

c

$$L = \begin{bmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{3} & 1 \end{bmatrix}$$

d

$Ly = b$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{3} & 1 \end{vmatrix} \begin{vmatrix} y_1 \\ y_2 \\ y_3 \end{vmatrix} = \begin{vmatrix} 9 \\ 6 \\ 10 \end{vmatrix}$$

y_1		
y_2	=	
y_3		

$y_1 = 9$

$y_2 = 6$

$\frac{1}{3} y_2 + y_3 = 10$

$\frac{1}{3} \times 6 + y_3 = 10$

$y_3 = 8$

$$\begin{array}{ccc} U & x & = y \end{array}$$

$$\begin{bmatrix} 5 & 2 & 2 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

$2x_3 = 8$

$\Rightarrow x_3 = 4$

$3x_1 = 6$

$x_1 = 2$

$5x_2 + 2x_1 + 2x_3 = 9$

$\Rightarrow 5x_2 = 9 - 2x_1 - 2x_3$

$\Rightarrow 5x_2 = 9 - 4 - 8$

$x_2 = -\frac{3}{5}$

So, $x_1 = 2$, $x_2 = -\frac{3}{5}$, $x_3 = 4$.

(Ans).