Quiz-03 Set-0A Solution

① $g(x) = \sqrt{2x+3}$ $\quad g(x) = (2x+3)^{1/2}$

$\lambda = |g'(x)|$

When $x = -1$ and 3,

$g'(x) = 1/2(2x+3)^{-1/2} \cdot 2$

$g'(x) = (2x+3)^{-1/2}$

$g'(-1) = (2(-1)+3)^{-1/2} = (1)^{-1/2}$

$g'(-1) = 1$. When $\lambda = 1$, $g(x)$ is diverging for root -1.

$g'(3) = (2(3)+3)^{-1/2} = (9)^{-1/2}$

$g'(3) = 0.3333$. When $\lambda = 0.3333$, $g(x)$ is converging for root 3.

② $g(x) = (9x-1)^3$.

ⓐ $\lambda = |g'(x)|$

$g'(x) = 3(9x-1)^2(9)$

$g'(x) = 27(9x-1)^2$.

$g'(x) \geq 1$ for divergence,

$27(9x-1)^2 \geq 1$.

$$(9x-1)^2 \geqslant \frac{1}{27}.$$

$$(9x-1)(9x-1) \geqslant \frac{1}{27}$$

$$81x^2 - 9x - 9x + 1 - \frac{1}{27} \geqslant 0$$

$$81x^2 - 18x + \frac{26}{27} \geqslant 0$$

$$x_* \geqslant \frac{9+\sqrt{3}}{81}, \quad \frac{9-\sqrt{3}}{81}$$

$$\frac{9-\sqrt{3}}{81} \leq x_* \leq \frac{9+\sqrt{3}}{81}.$$

** If you received the result $x_* \geqslant \frac{9+\sqrt{3}}{81}$ that is correct as well.

(b) $\lambda = |g'(x)| = 0$. super-linear convergence

$$27(9x-1)^2 = 0$$

$$(9x-1)^2 = \frac{0}{27}.$$

$$81x^2 - 18x + 1 = 0$$

$$x_* = 1/9.$$

(c)

	k	x_k	Calculation
	0	$x_0 = 3.5$	$g(3.5) = (9(3.5)-1)^3$
1st	1	$x_1 = 28373$	$g(28373) = (9(28373)-1)^3$
2nd	2	$x_2 = 1.665*10^{16}$	$g(1.665*10^{16}) = (9(1.665*10^{16})-1)^3$
3rd	3	$x_3 = 3.365*10^{51}$	$g(3.365*10^{51}) =$ undefined since function value becomes increasingly large

3.

k	x_k	$f(x_k)$	$f'(x_k)$
0	2.00	8.00	14.0
1	1.43	1.77	8.13
2	1.21	0.194	200.3
3	1.18	0.003	120362

$x_0 = f(2) = (2)^3 + 2(2) - 4$. $f'(2) = 3(2)^2 + 2$

$$x_1 = x_0 - \frac{f(2)}{f'(2)}$$ $f'(1.43) =$

$$x_1 = 2 - \frac{8}{14}$$

$$x_1 = 1.43.$$

calculation continues like this .

Quiz-03 Set-0B Solution.

① Same as Set-0A solution.

② ⓐ $g(x) = (4x-1)^3$

$g'(x) = 3(4x-1)^2(4)$

$g'(x) = 12(4x-1)^2$.

$g'(x) \geq 1$ for divergence,

$12(4x-1)^2 \geq 1$.

$12(16x^2 - 8x + 1) \geq 1$.

$192x^2 - 96x + 12 - 1 \geq 0$

$192x^2 - 96x + 11 \geq 0$

$x_* \geq \frac{6+\sqrt{3}}{24}, \frac{6-\sqrt{3}}{24}$.

$\frac{6-\sqrt{3}}{24} \leq x_* \leq \frac{6+\sqrt{3}}{24}$.

** If you received the result $x_* \geq \frac{6+\sqrt{3}}{24}$ that is correct as well.

ⓑ $\lambda = |g'(x)| = 0$ For super linear convergence

$12(4x-1)^2 = 0$

$(4x-1)^2 = 0$

$16x^2 - 8x + 1 = 0$

$x_* = 1/4$.

(c)

	k	x_k	Calculation
	0	3.5	$g(3.5) = (4(3.5)-1)^3$
1st	1	2197	$g(2197) = (4(2197)-1)^3$
2nd	2	$6.785 * 10^{11}$	$g(6.785 * 10^{11}) = (4(6.785 * 10^{11})-1)^3$
3rd	3	$1.999 * 10^{37}$	$g(1.999 * 10^{37}) = (4(1.999 * 10^{37})-1)^3$ = undefined since function becomes increasingly large.

3.

k	x_k	$f(x_k)$	$f'(x_k)$
0	3.00	29.0	29.0
1	2.00	8.00	14.0
2	1.43	1.77	8.13
3	1.21	0.194	200.3.

$x_0 = 3, \quad f(3) = (3)^3 + 2(3) - 4$

$f'(3) = 3(3)^2 + 2$

$$x_1 = x_0 - \frac{f(3)}{f'(3)}$$

$$x_1 = 3 - \frac{29}{29}$$

$x_1 = 2$.

Calculation continues like this.