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CSE330
Section : 17
Assignment 06

① a)

$f(0) = 3$

$f(4) = -2$

$f(-1) = 2$

$f(1) = 1$

$P_2(0) = x_0 + x_1(0)^1 + x_2(0)^2 = 3$

$P_2(4) = x_0 + x_1(4)^1 + x_2(4)^2 = -2$

$P_2(-1) = x_0 + x_1(-1)^1 + x_2(-1)^2 = 2$

$P_2(1) = x_0 + x_1(1)^1 + x_2(1)^2 = 1$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 4 & 16 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} x_0 \\ x_1 \\ x_2 \end{vmatrix} = \begin{vmatrix} 3 \\ -2 \\ 2 \\ 1 \end{vmatrix}$$

$A \qquad x \qquad b$

b) $A^T A = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 4 & -1 & 1 \\ 0 & 16 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 1 & 4 & 16 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$

$$= \begin{vmatrix} 4 & 4 & 18 \\ 4 & 18 & 64 \\ 18 & 64 & 258 \end{vmatrix}$$

$$\det(A^TA) = 4(258\times18 - 64\times64) - 4(4\times258 - 64\times18) + 18(64\times4 - 18\times18) = 1448$$

c)

$$A^TA\,x = A^Tb$$

$$\Rightarrow \begin{vmatrix} 4 & 4 & 18 \\ 4 & 18 & 64 \\ 18 & 64 & 258 \end{vmatrix} \begin{vmatrix} x_0 \\ x_1 \\ x_2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 4 & -1 & 1 \\ 0 & 16 & 1 & 1 \end{vmatrix} \begin{vmatrix} 3 \\ -2 \\ 2 \\ 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} x \\ x_1 \\ x_2 \end{vmatrix} = \begin{vmatrix} 4 & 4 & 18 \\ 4 & 18 & 64 \\ 18 & 64 & 258 \end{vmatrix}^{-1} \begin{vmatrix} 4 \\ -9 \\ -29 \end{vmatrix}$$

$$= \begin{vmatrix} 137/362 & 15/181 & -17/362 \\ 15/181 & 177/362 & -23/181 \\ -17/362 & -23/181 & 7/181 \end{vmatrix} \begin{vmatrix} 4 \\ -9 \\ -29 \end{vmatrix}$$

$$= \begin{vmatrix} 771/362 \\ -139/362 \\ -30/181 \end{vmatrix}$$

Best fit polynomial :

$$f(x) = \frac{771}{362} - \frac{139}{362}x - \frac{30}{181}x^2$$

② a)

$f(0) = 1$

$f(0.5) = 1.4$

$f(1) = 1.7$

$f(1.5) = 2$

$P_1(0) = x_0 + x_1(0) = 1$

$P_1(0.5) = x_0 + x_1(0.5) = 1.4$

$P_1(1) = x_0 + x_1(1) = 1.7$

$P_1(1.5) = x_0 + x_1(1.5) = 2$

$$\begin{vmatrix} 1 & 0 \\ 1 & 0.5 \\ 1 & 1 \\ 1 & 1.5 \end{vmatrix} \begin{vmatrix} x_0 \\ x_1 \end{vmatrix} = \begin{vmatrix} 1 \\ 1.4 \\ 1.7 \\ 2 \end{vmatrix}$$

A x b

b) $u_1 = \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$ $u_2 = \begin{vmatrix} 0 \\ 0.5 \\ 1 \\ 1.5 \end{vmatrix}$

$$p_1 = u_1 = \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$$

$$q_1 = \frac{p_1}{|p_1|} = \begin{vmatrix} 1/\sqrt{4} \\ 1/\sqrt{4} \\ 1/\sqrt{4} \\ 1/\sqrt{4} \end{vmatrix}$$

$$p_2 = u_2 - (u_2^T q_1) q_1$$

$$= \begin{vmatrix} 0 \\ 0.5 \\ 1 \\ 1.5 \end{vmatrix} - \left(\begin{vmatrix} 0 & 0.5 & 1 & 1.5 \end{vmatrix} \begin{vmatrix} 1/\sqrt{4} \\ 1/\sqrt{4} \\ 1/\sqrt{4} \\ 1/\sqrt{4} \end{vmatrix} \right) \begin{vmatrix} 1/\sqrt{4} \\ 1/\sqrt{4} \\ 1/\sqrt{4} \\ 1/\sqrt{4} \end{vmatrix}$$

$$= \begin{vmatrix} 0 \\ 0.5 \\ 1 \\ 1.5 \end{vmatrix} - 1.5 \begin{vmatrix} 1/\sqrt{4} \\ 1/\sqrt{4} \\ 1/\sqrt{4} \\ 1/\sqrt{4} \end{vmatrix} = \begin{vmatrix} 0 \\ 0.5 \\ 1 \\ 1.5 \end{vmatrix} - \begin{vmatrix} 3/4 \\ 3/4 \\ 3/4 \\ 3/4 \end{vmatrix}$$

$$= \begin{vmatrix} -3/4 \\ -0.25 \\ 1/4 \\ 3/4 \end{vmatrix}$$

$$|p_2| = \sqrt{(-3/4)^2 + (-0.25)^2 + (1/4)^2 + (3/4)^2}$$

$$= \sqrt{5}/2$$

$$q_2 = \frac{p_2}{|p_2|} = \begin{vmatrix} -3\sqrt{5}/10 \\ -\sqrt{5}/10 \\ \sqrt{5}/10 \\ 3\sqrt{5}/10 \end{vmatrix}$$

$$Q = \begin{vmatrix} 1/\sqrt{4} & -(3\sqrt{5})/10 \\ 1/\sqrt{4} & -\sqrt{5}/10 \\ 1/\sqrt{4} & \sqrt{5}/10 \\ 1/\sqrt{4} & (3\sqrt{5})/10 \end{vmatrix}$$

e) $$R = \begin{vmatrix} u_1^T q_1 & u_2^T q_1 \\ 0 & u_2^T q_2 \end{vmatrix}$$

$$u_1^T q_1 = \begin{vmatrix} 1 & 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{vmatrix} = 2$$

$$u_2^T q_1 = \begin{vmatrix} 0 & 0.5 & 1 & 1.5 \end{vmatrix} \begin{vmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{vmatrix} = 1.5$$

$$u_2^T q_2 = \begin{vmatrix} 0 & 0.5 & 1 & 1.5 \end{vmatrix} \begin{vmatrix} -3\sqrt{5}/10 \\ -\sqrt{5}/10 \\ \sqrt{5}/10 \\ 3\sqrt{5}/10 \end{vmatrix} = 1.118$$

$$R = \begin{vmatrix} 2 & 1.5 \\ 0 & 1.118 \end{vmatrix}$$

d)

$$Rx = Q^T b$$

$$\Rightarrow \begin{vmatrix} 2 & 1.5 \\ 0 & 1.118 \end{vmatrix} \begin{vmatrix} x_0 \\ x_1 \end{vmatrix} = \begin{vmatrix} 1/\sqrt{4} & 1/\sqrt{4} & 1/\sqrt{4} & 1/\sqrt{4} \\ -3/\sqrt{5}/10 & -\sqrt{5}/10 & \sqrt{5}/10 & 3\sqrt{5}/10 \end{vmatrix}$$

$$\begin{vmatrix} 1 \\ 1.4 \\ 1.7 \\ 2 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} x_0 \\ x_1 \end{vmatrix} = \begin{vmatrix} 2 & 1.5 \\ 0 & 1.118 \end{vmatrix}^{-1} \begin{vmatrix} 3.05 \\ 0.7379 \end{vmatrix}$$

$$= \begin{vmatrix} 1/2 & -375/559 \\ 0 & 500/559 \end{vmatrix} \begin{vmatrix} 3.05 \\ 0.7379 \end{vmatrix}$$

$$= \begin{vmatrix} 1.03 \\ 0.66 \end{vmatrix}$$

Best fit linear polynomial :

$$f(x) = 1.03 + 0.66x$$

(3) a)

$$f(x) = e^{0.5x} + \sin x$$

$$\int_0^2 (e^{0.5x} + \sin x)\,dx$$

$$= \int_0^2 e^{0.5x}\,dx + \int_0^2 \sin x\,dx$$

$$= \left[2e^{0.5x}\right]_0^2 + \left[-\cos x\right]_0^2$$

$$= 2(e^1 - 1) - \cos(2) + \cos(0)$$

$$= 4.8527$$

b) $n = 2$; $[0, 2]$

$$h = \frac{2-0}{2} = 1$$

$x_0 = 0$

$x_1 = x_0 + h = 0 + 1 = 1$

$x_2 = x_1 + h = 1 + 1 = 2$

$f(x) = e^{0.5x} + \sin x$

$f(0) = 1 + 0 = 1$

$f(1) = e^{0.5} + \sin(1) = 2.4902$

$f(2) = e^1 + \sin(2) = 3.6276$

$$\int_0^2 f(x)\,dx$$

$$= \frac{b-a}{6}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

$$= \frac{2-0}{6}\left[f(0) + 4f(1) + f(2)\right]$$

$$= \frac{1}{3}\left[1 + 4(2.4902) + 3.6276\right]$$

$$= 4.8628$$

c) $h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{2}{4} = \frac{1}{2}$

$x_0 = 0$

$x_1 = x_0 + h = 1/2$

$x_2 = x_1 + h = 1/2 + 1/2 = 1$

$x_3 = x_2 + h = 1 + \frac{1}{2} = \frac{6}{4} = \frac{3}{2}$

$x_4 = x_3 + h = \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2$

~~$e_{1,4} = \frac{h}{2}$~~

$f(x_0) = f(0) = 1$

$f(x_1) = f(0.5) = 1.7653$

$f(x_2) = f(1) = 2.4902$

$f(x_3) = f(3/2) =$ ~~3.1812~~ 3.1145

$f(x_4) = f(2) = 3.6276$

$$C_{1,4} = \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4) \right]$$

$$= \frac{1/2}{2}\left[1 + 2(1.7653) + 2(2.4902) + 2(3.1145) + (3.6276) \right]$$

$$= \cancel{4.87525}\ 4.841$$

$$\text{Error} = \left| \frac{4.841 - 4.8527}{4.8527} \right| \times 100$$

$$= \cancel{0.46\%}\ 0.24\%.$$