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CSE330

Sec: 17

Assignment 2

Ans no 1

a)

t	v
2	10
4	20
6	25

$n = 3 - 1 = 2$

$P_2(t) = a_0 t^0 + a_1 t^1 + a_2 t^2$

$2^0 a_0 + 2a_1 + 2^2 a_2 = 10$ — (i)

$4^0 a_0 + 4a_1 + 4^2 a_2 = 20$ — (ii)

$6^0 a_0 + 6a_1 + 6^2 a_2 = 25$ — (iii)

$$\begin{vmatrix} 1 & 2 & 4 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{vmatrix} \begin{vmatrix} a_0 \\ a_1 \\ a_2 \end{vmatrix} = \begin{vmatrix} 10 \\ 20 \\ 25 \end{vmatrix}$$

$$a = v^{-1} b = \begin{vmatrix} 3 & -3 & 1 \\ -5/4 & 2 & -3/4 \\ 1/8 & -1/4 & 1/8 \end{vmatrix} \begin{vmatrix} 10 \\ 20 \\ 25 \end{vmatrix}$$

$$= \begin{vmatrix} -5 \\ 35/4 \\ -5/8 \end{vmatrix}$$

$$P_2(t) = -5 + \frac{35}{4} t - \frac{5}{8} t^2$$

$$p'(t) = \frac{35}{4} - \frac{10}{8}t$$

$$p''(t) = -\frac{10}{8} = -\frac{5}{4}$$

acceleration at $t = 8$ sec:

$$p''(8) = -\frac{5}{4} = -1.25 \ ms^{-2}$$

b)

$$P_2(t) = l_0(t) f(t_0) + l_1(t) f(t_1) + l_2(t) f(t_2)$$

$$l_0(t) = \frac{(t-t_1)(t-t_2)}{(t_0-t_1)(t_0-t_2)} = \frac{(t-4)(t-6)}{(2-4)(2-6)} = \frac{(t-4)(t-6)}{8}$$

$$l_1(t) = \frac{(t-t_0)(t-t_2)}{(t_1-t_0)(t_1-t_2)} = \frac{(t-2)(t-6)}{(4-2)(4-6)} = \frac{(t-2)(t-6)}{-4}$$

$$l_2(t) = \frac{(t-t_1)(t-t_0)}{(t_2-t_0)(t_2-t_1)} = \frac{(t-2)(t-4)}{(6-2)(6-4)} = \frac{(t-2)(t-4)}{8}$$

$$P_2(t) = \frac{(t-4)(t-6)}{8} \times 10 + \frac{(t-2)(t-6)}{-4} \times 20$$

$$+ \frac{(t-2)(t-4)}{8} \times 25$$

c) If a new data point is added in the above scenario, we need to use Newton's divided difference method. Because we don't need to calculate whole thing again in this method, just calculate the things for newly added data point.

New polynomial degree, $n = 4 - 1 = 3$

Ans no 2

a) $x_0 = -\frac{\pi}{2}$

$x_1 = 0$

$x_2 = \frac{\pi}{2}$

$f(x) = x\sin(x)$

$f(x_0) = -\frac{\pi}{2}\sin\left(-\frac{\pi}{2}\right) = -\frac{\pi}{2}(-1) = \frac{\pi}{2}$

$f(x_1) = 0$

$f(x_2) = \frac{\pi}{2}\sin\left(\frac{\pi}{2}\right) = \frac{\pi}{2}(1) = \frac{\pi}{2}$

x	$f(x)$
$-\pi/2$	$\pi/2$
0	0
$\pi/2$	$\pi/2$

$n = 3 - 1 = 2$

$P_2(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2)(x - x_0)(x - x_1)$

$= \frac{\pi}{2} - 1\left(x + \frac{\pi}{2}\right) + \frac{2}{\pi}\left(x + \frac{\pi}{2}\right)x$

$= \frac{\pi}{2} - x - \frac{\pi}{2} + \frac{2}{\pi}x^2 + x$

$= \frac{2}{\pi}x^2$

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$x_0 = -\pi/2 \qquad f[x_0] = \pi/2$

$x_1 = 0 \qquad f[x_1] = 0$

$x_2 = \pi/2 \qquad f[x_2] = \pi/2$

$x_3 = \pi \qquad f[x_3] = 0$

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{0 - \pi/2}{0-(-\pi/2)} = -1$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{\pi/2 - 0}{\pi/2 - 0} = 1$$

$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} = \frac{0 - \frac{\pi}{2}}{\pi - \pi/2} = -1$$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{1-(-1)}{\pi/2 + \frac{\pi}{2}} = \frac{2}{\pi}$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} = \frac{-1-1}{\pi - 0} = \frac{-2}{\pi}$$

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0} = \frac{-\frac{2}{\pi} - \frac{2}{\pi}}{\pi + \frac{\pi}{2}}$$

$$= -\frac{8}{3\pi^2}$$

b) $x = \frac{\pi}{3}$

$$P_2\left(\frac{\pi}{3}\right) = \frac{2}{\pi}\left(\frac{\pi}{3}\right)^2$$
$$= \frac{2}{\pi} \cdot \frac{\pi^2}{9}$$
$$= \frac{2\pi}{9}$$

c) new node $= \pi$

$f(\pi) = \pi \sin\pi = 0$

$n = 4 - 1 = 3$

$$P_3(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)$$

$$= \frac{\pi}{2} - \left(x + \frac{\pi}{2}\right) + \frac{2}{\pi}\left(x + \frac{\pi}{2}\right)x - \frac{8}{3\pi^2}\left(x + \frac{\pi}{2}\right)(x + 0)\left(x - \frac{\pi}{2}\right)$$

$$= \frac{2}{\pi}x^2 - \frac{8x}{3\pi^2}\left(x^2 - \left(\frac{\pi}{2}\right)^2\right)$$

Ans no 3

$$f(x) = e^{3x} - e^{-3x}$$

$$x_0 = -1, \quad x_1 = 0, \quad x_2 = 1$$

$$\xi\,[-1.5, 1.5]$$

$$f'(x) = 3e^{3x} + 3e^{-3x}$$

$$f''(x) = 9e^{3x} - 9e^{-3x}$$

$$f'''(x) = 27e^{3x} + 27e^{-3x}$$

$$= 27\left(e^{3x} + e^{-3x}\right)$$

$$f'''(1.5) = 27\left(e^{3\times 1.5} + e^{-3(-1.5)}\right)$$

$$= 4860.925$$

$$\omega(x) = (x - x_0)(x - x_1)(x - x_2)$$

$$= (x+1)(x-0)(x-1)$$

$$= x(x^2 - 1)$$

$$= x^3 - x$$

$$\omega'(x) = 3x^2 - 1$$

$$x_1 = \frac{1}{\sqrt{3}}$$

$$x_2 = -\frac{1}{\sqrt{3}}$$

x	$\omega(x)$
$1/\sqrt{3}$	-0.3849
$-1/\sqrt{3}$	0.3849
1.5	1.875
-1.5	-1.875

$$\text{error bound} = \left|\frac{f^3(\xi)}{3!}\right| \left|(x-x_0)(x-x_1)(x-x_2)\right|$$

$$= \frac{4860.925}{6} \times 1.875$$

$$= 1519.039063$$

$$= 1519.0 \ [5 \text{ sig figure}]$$