Question 01

(c) $f(x) = x^3 - x^2 - 4x - 4$

Solving $f(x) = 0$, we get,

$x_1 = -1$

$x_2 = -2$

$x_3 = 2$

$\therefore f(x) = 0$

$\Rightarrow x^3 + x^2 - 4x - 4 = 0$

$\Rightarrow x^2 = -x^3 + 4x + 4$

$\Rightarrow x = \sqrt{-x^3 + 4x + 4}$

$\Rightarrow g_1(x) = \sqrt{-x^3 + 4x + 4}$

$\therefore f(x) = 0$

$\Rightarrow x^3 + x^2 - 4x - 4 = 0$

$\Rightarrow 4x = x^3 + x^2 - 4$

$\Rightarrow x = \dfrac{x^3 + x^2 - 4}{4}$

$\Rightarrow g_2(x) = \dfrac{x^3 + x^2 - 4}{4}$

(d) Here,

$\lambda_1 = |g_1'(\text{root})|$

$\lambda_2 = |g_2'(\text{root})|$

$g_1(x) = \sqrt{-x^3 + 4x + 4}$

$\therefore g_1'(x) = \dfrac{4 - 3x^2}{2\sqrt{-x^3 + 4x + 4}}$

$g_2(x) = \dfrac{x^3 + x^2 - 4}{4}$

$g_2'(x) = \dfrac{3x^2 + 2x}{4}$

Now, for roots $-1, -2$ and 2, we get,

$\lambda_1 = |g_1'(-1)| = 0.5 \; [0 \leq \lambda < 1]$

$\lambda_1 = |g_1'(-2)| = 2 \; [\lambda \geq 1]$

$\lambda_1 = |g_1'(2)| = 2 \; [\lambda \geq 1]$

$\lambda_2 = |g_2'(-1)| = 0.25 \; [0 \leq \lambda < 1]$

$\lambda_2 = |g_2'(-2)| = 2 \; [\lambda \geq 1]$

$\lambda_2 = |g_2'(2)| = 4 \; [\lambda \geq 1]$

Root -1 is converging for both $g_1(x)$ and roots -2 and 2 are diverging.

Question 02

ⓐ $f(x) = xe^{x} - 1$ and $x_0 = 1.5$ $\therefore f'(x) = e^{x} + xe^{x}$

<u>Iteration-1</u>

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.5 - \frac{f(1.5)}{f'(1.5)} = 0.9891$$

<u>Iteration-2</u>

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.9891 - \frac{f(0.9891)}{f'(0.9891)} = 0.6787$$

<u>Iteration-3</u>

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.6787 - \frac{f(0.6787)}{f'(0.6787)} = 0.5766$$

↓ Iteration continues like this.

ⓑ Given, $g(x) = \frac{2x+1}{\sqrt{x+1}}$

$$\therefore g'(x) = \frac{2x+3}{2(x+1)^{3/2}}$$

To be super linear convergent, $\lambda = 0$.

$\therefore \lambda = |g'(x)| = 0$

$\Rightarrow \frac{2x+3}{2(x+1)^{3/2}} = 0$

$\Rightarrow 2x+3 = 0$ or, $x = -3/2$ (showed)

Question 03

ⓐ Given, $f(x) = 2x^3 - 2x - 5$ $\therefore f'(x) = 6x^2 - 2$

Using newton's method,

$$g(x) = x - \frac{f(x)}{f'(x)}$$

$$= x - \frac{2x^3 - 2x - 5}{6x^2 - 2}$$

$$= \frac{6x^3 - 2x - 2x^3 + 2x + 5}{6x^2 - 2}$$

$$= \frac{4x^3 + 5}{6x^2 - 2}$$