CSE330 Numerical Methods
Quiz 5 - SET[A]

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Question 1: A student, who is a big fan of anime, decided to set up a stall at the BRAC University fair. He draws various anime characters and sells them as posters (P). In addition to posters, he also sells stickers (S) and keyrings (K). His friends came to support his small side business, and the following transactions occurred:

- Friend 1 bought 2 posters, 2 keyrings, and 3 stickers, costing a total of 350 tk. ...(i)
- Friend 2 bought 2 posters, 3 keyrings, and 7 stickers, costing a total of 560 tk. ...(ii)
- Friend 3 bought 6 posters, 5 keyrings, and 1 sticker, costing a total of 430 tk. ...(iii)

In total, the student earned around 2000 tk by selling 45 posters, 15 keyrings, and 27 stickers. ...(iv)

a. **[1.5 mark]** Write down the linear equations that relate the variables P, K & S.
b. **[1.5 mark]** Identify the matrices A, x, and b so that the equations in the previous question can be expressed in the standard matrix equation form Ax = b.
c. **[7 marks]** From matrix A in the previous question, compute the matrices Q and R such that A = QR, where the symbols have their usual meanings.
d. **[5 marks]** Evaluate QTb, and finally solve the system by evaluating x (that is, evaluate P, K & S).

a

$$2p+2k+3s=350$$

$$2p+3k+7s=560$$

$$6p+5k+1s=430$$

$$45p+15k+27s=2000$$

b

$$\begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 7 \\ 6 & 5 & 1 \\ 45 & 15 & 27 \end{bmatrix} \begin{bmatrix} p \\ k \\ s \end{bmatrix} = \begin{bmatrix} 350 \\ 560 \\ 430 \\ 2000 \end{bmatrix}$$

A $\quad x \quad$ b

$$Ax = b$$

c

$$\begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 7 \\ 6 & 5 & 1 \\ 45 & 15 & 27 \end{bmatrix}$$

$u_1 \quad u_2 \quad u_3$

$$P_1 = u_1 = \begin{bmatrix} 2 \\ 2 \\ 6 \\ 45 \end{bmatrix}; \quad |P_1| = 45.48626$$

$$q_1 = \frac{P_1}{|P_1|} = \frac{1}{45.4862} \begin{bmatrix} 2 \\ 2 \\ 6 \\ 45 \end{bmatrix}$$

$$P_2 = U_2 - (U_2^T q_1)\, q_1$$

$$U_2^T q_1 = \begin{bmatrix} 2 & 3 & 5 & 15 \end{bmatrix} \begin{bmatrix} \frac{2}{45.48626} \\ \frac{2}{45.48626} \\ \frac{6}{45.48626} \\ \frac{45}{45.48626} \end{bmatrix} = 15.719031$$

$$= \begin{bmatrix} 2 \\ 3 \\ 5 \\ 15 \end{bmatrix} - 15.719031 \begin{bmatrix} \frac{2}{45.48626} \\ \frac{2}{45.48626} \\ \frac{6}{45.48626} \\ \frac{45}{45.48626} \end{bmatrix}$$

$$= \begin{bmatrix} 1.30884 \\ 2.30884 \\ 2.92653 \\ -0.55099 \end{bmatrix}$$

$$|P_2| = \sqrt{(1.30884)^2 + (2.30884)^2 + (2.92653)^2 + (-0.55099)^2}$$

$$= 3.98898$$

$$q_2 = \frac{P_2}{|P_2|} = \begin{bmatrix} 0.32811 \\ 0.57880 \\ 0.73365 \\ -0.13812 \end{bmatrix}$$

$$P_3 = U_3 - (U_3^T q_2) q_2 - (U_3^T q_1) q_1$$

$$U_3^T q_2 = \begin{bmatrix} 3 & 7 & 1 & 27 \end{bmatrix} \begin{bmatrix} 0.32811 \\ 0.57880 \\ 0.73365 \\ -0.13812 \end{bmatrix}$$

$$= 2.04034$$

$$U_3^T q_1 = \begin{bmatrix} 3 & 7 & 1 & 27 \end{bmatrix} \begin{bmatrix} \frac{2}{45.48626} \\ \frac{2}{45.48626} \\ \frac{6}{45.48626} \\ \frac{45}{45.48626} \end{bmatrix}$$

$$= 27.28296$$

$$= \begin{bmatrix} 3 \\ 7 \\ 1 \\ 27 \end{bmatrix} - 2.04034 \begin{bmatrix} 0.32811 \\ 0.57880 \\ 0.73365 \\ -0.13812 \end{bmatrix} - 27.28296 \begin{bmatrix} \frac{2}{45.48126} \\ \frac{2}{45.48126} \\ \frac{6}{45.48126} \\ \frac{45}{45.48126} \end{bmatrix}$$

$$= \begin{bmatrix} 1.13093 \\ 4.61943 \\ -4.00579 \\ 0.29051 \end{bmatrix} \quad ; |P_3| = 6.28315$$

$$q_3 = \frac{P_3}{|P_3|} = \begin{bmatrix} 0.17999 \\ 0.73520 \\ -0.65186 \\ 0.04623 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.043969 & 0.32811 & 0.17999 \\ 0.043969 & 0.57880 & 0.73520 \\ 0.13190 & 0.73368 & -0.65186 \\ 0.98930 & -0.13812 & 0.04623 \end{bmatrix}$$

$$R = \begin{bmatrix} U_1^T q_1 & U_2^T q_1 & U_3^T q_1 \\ 0 & U_2^T q_2 & U_3^T q_2 \\ 0 & 0 & U_3^T q_3 \end{bmatrix} = \begin{bmatrix} 45.4863 & 15.71903 & 27.2829 \\ 0 & 3.98922 & 2.04034 \\ 0 & 0 & 6.28272 \end{bmatrix}$$

$$\underset{\sim}{d}$$

$$Q^T b = \begin{bmatrix} 0.043969 & 0.043969 & 0.13190 & 0.9893 \\ 0.32811 & 0.57880 & 0.7368 & -0.13812 \\ 0.17999 & 0.7352 & -0.65185 & 0.04623 \end{bmatrix} \times \begin{bmatrix} 350 \\ 560 \\ 430 \\ 2000 \end{bmatrix}$$

$$= \begin{bmatrix} 2075.3287 \\ 478.2089 \\ 286.873 \end{bmatrix}$$

$$Rx = Q^T b$$

$$\begin{bmatrix} 45.4863 & 15.71903 & 27.28296 \\ 0 & 3.98922 & 2.04034 \\ 0 & 0 & 6.28272 \end{bmatrix} \begin{bmatrix} P \\ K \\ S \end{bmatrix} = \begin{bmatrix} 2075.3287 \\ 478.2089 \\ 286.873 \end{bmatrix}$$

$$S = \frac{286.873}{6.28272} = 45.66063$$

$$k = \frac{478.2089 - (2.04034 \times 45.66063)}{3.98922}$$

$$= 96.5215$$

$$P = \frac{2075.3287 - (15.71903 \times 96.5215) - (27.28296 \times 45.66063)}{45.4863}$$

$$= -15.1178$$

$$\begin{bmatrix} P \\ k \\ S \end{bmatrix} = \begin{bmatrix} \cancel{54.43637} \; -15.1178 \\ 96.5215 \\ 45.66063 \end{bmatrix}$$