CSE330- Numerical Methods
Quiz 02; Spring'25
Set B

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Marks: 15 points **Time: 30 minutes**

<u>Instructions:</u> Answer all questions on the space provided below for each.

Question 1: CO2 (6+1 points): The velocity of a test object measured at different times has been given below:

Time (seconds)	Velocity (m/s)
8	40
10	52
15	90

- Using Vandermonde Matrix, construct a polynomial that goes through the above nodes.
- Use the polynomial to find the approximate velocity at Time = 19 seconds.

Question 2: CO3 (3+4+1 points): The following nodes come from the function $f(x) = \ln(5x + 9)$:

x	f(x)
-0.5	1.87
0	2.20

- Using Newton's divided difference method, find the equation of a first degree polynomial which fits the above data points.
- Add another node '0.5' to the above nodes and find out the interpolating polynomial of appropriate degree.
- Find out the relative error at x = 3.

Q1

$$P_2(x) = a_0 + a_1 x^1 + a_2 x^2$$

$$\begin{bmatrix} 1 & 8 & 64 \\ 1 & 10 & 100 \\ 1 & 15 & 225 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 52 \\ 90 \end{bmatrix}$$

$V \qquad a \qquad b$

$$a = V^{-1}b = \begin{bmatrix} 1 & 8 & 64 \\ 1 & 10 & 100 \\ 1 & 15 & 225 \end{bmatrix}^{-1} \begin{bmatrix} 40 \\ 52 \\ 90 \end{bmatrix} = \begin{bmatrix} 10.285 \\ 1.8857 \\ 0.2285 \end{bmatrix}$$

$$P_2(x) = 10.285 + 1.8857x + 0.2285x^2$$

at $x/t = 19$, $P_2(19) = 10.285 + 1.8857 \times 19 + 0.2285 \times 19^2$

$$= 128.6018 \text{ m/s}$$

$x_0 = -0.5$ $f[x_0] = 1.87$

$x_1 = 0$ $f[x_1] = 2.20$ $\quad f[x_0, x_1] = \dfrac{2.20 - 1.87}{0 + 0.5} = 0.66$

$x_2 = 0.5$ $f[x_2] = 2.44$ $\quad f[x_1, x_2] = \dfrac{2.44 - 2.20}{0.5 - 0} = 0.48$

$f[x_0, x_1, x_2] = \dfrac{0.48 - 0.66}{0.5 + 0.5} = -0.18$

a) $P_1(x) = f[x_0] + f[x_0, x_1](x - x_0) = 1.87 + 0.66(x + 0.5)$

b) $P_2(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$

$= 1.87 + 0.66(x + 0.5) - 0.18(x + 0.5)(x - 0)$

c) $f(3) = \ln(5 \times 3 + 9) = 3.17805383$

d) $P_2(3) = 1.87 + 0.66(3 + 0.5) - 0.18(3 + 0.5)(3 - 0) = 2.29$

$$\text{relative error} = \frac{|f(3) - P_2(3)|}{|f(3)|} = 0.27943$$