Final

TOPIC NAME: Chapter 4 — Fixed point Iteration

DATE: 8/4/25

## Non-linear equation

$f(x) = 2x+5$ → linear equation

$f(x) = x^3 + 2x^2 + 5$ → non linear equation

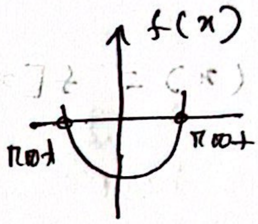
↓

$x$ এর power $> 1$

$f(x) = x^2 - 4 = 0$

$\Rightarrow x^2 = 4$

$\therefore x = \pm 2$ → root




## Fixed point Iteration → multiple root finding algorithm

$f(x) = 0 \rightarrow x_1, x_2$

↳ $g(x) = x$

↓

fixed point function

$g(x_1) = x_1$

$(2, 2) \rightarrow g(2) = 2$

input == output

$f(x) = x^2 - 2x - 3 = 0$

$x^2 - 3 = 2x$

$\Rightarrow x = \frac{x^2-3}{2}$ → fixed point function → $g_1(x)$

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$f(x) = x^2 - 2x - 3 = 0$

$\Rightarrow x^2 = 2x + 3$

$\Rightarrow x = \sqrt{2x+3} \rightarrow g_2(x)$

$x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - x - x - 3 = 0$

$\Rightarrow x = x^2 - x - 3 \rightarrow g_3(x)$

Root: $x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - 3x + x - 3 = 0$

$\Rightarrow x(x-3) + 1(x-3) = 0$

$\Rightarrow (x-3)(x+1) = 0$

$x - 3 = 0$ $\quad\quad$ $x + 1 = 0$

$\Rightarrow x = 3$ $\quad\quad$ $\therefore x = -1$

fixed point function

* $g(x) = \sqrt{2x+3}$ ; $x_0 = 0$ ← starting point

7 iteration

$g(x_0 = 0) = \sqrt{3}$

$g(\sqrt{3}) = \sqrt{2\sqrt{3}+3} = 2.5425$

$g(2.5425) = 2.834$

$g(2.834) = 2.94$

$g(2.94) = 2.98$

$g(2.98) = 3$

$g(3) = \sqrt{2\times3+3} = 3 \rightarrow$ root

input == output

error bound = $1 \times 10^{-3} = 0.001$

$g(x) = 2.98$

$g(x) = 2.998$

$g(x) = 2.99998$ → root

$|3 - 2.99| = 0.01$

$|3 - 2.99998| = 0.00002$

↳ original error tollerance থেকে কম।

*

$f(x) = x^2 - x = 0$

$x^2 = x$

$g(x) = x^2$

$g(x) = x^2, \quad x_0 = 3$

$g(3) = 9$

$g(9) = 81$

$g(81) = 6561$

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$\gamma$ = convergence rate

$f(x) = x^2 - 2x - 3 = 0$ ; $x = -1, 3$ ← roots

$x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - x - x - 3 = 0$

$\Rightarrow x = x^2 - x - 3 = g(x)$

$\gamma = |g'(x)|$
↳ roots

$g(x) = x^2 - x - 3$

$g'(x) = 2x - 1$

if $\gamma = 0$ ; superlinear convergent (fast convergent) → <100, you get root

$\gamma < 1$ ; linear convergent (slow convergent) → >1000+, you get root

$\gamma \geq 1$ ; divergent ↓ you never get roots

$|g'(-1)| = 3 > 1$ → divergent

$|g'(3)| = 5$ → divergent

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1. a) $f(x) = x^3 - 2x^2 - x + 2$

find out the roots.

$$x^3 - 2x^2 - x + 2 = 0$$

$$\Rightarrow x^2(x-2) - 1(x-2) = 0$$

$$\Rightarrow (x-2)(x^2-1) = 0$$

$$\therefore x_* = 2, -1, 1$$

b) Create 2 or 3 fixed point function.

$$f(x) = x^3 - 2x^2 - x + 2 = 0$$

$$\Rightarrow x^3 - 2x^2 - x = -2$$

$$\Rightarrow x(x^2 - 2x - 1) = -2$$

$$\Rightarrow x = -\frac{2}{x^2 - 2x - 1}$$

$$g_1(x) = -\frac{2}{x^2 - 2x - 1}$$

$$f(x) = x^3 - 2x^2 - x + 2 = 0$$

$$\Rightarrow x = x^3 - 2x^2 + 2$$

$$g_2(x) = x^3 - 2x^2 + 2$$

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$f(x) = x^3 - 2x^2 - x + 2 = 0$

$\Rightarrow 2x^2 = x^3 - x + 2$

$\Rightarrow x = \sqrt{\dfrac{x^3 - x + 2}{2}}$

$$g_3(x) = \sqrt{\frac{x^3 - x + 2}{2}}$$

c) Determine if the created $g(x)$ are convergent or divergent.

$g_1(x) = \dfrac{2}{x^2 - 2x - 1} = 2(x^2 - 2x - 1)^{-1}$

$g_1'(x) = 2(x^2 - 2x - 1)^{-2}(2x - 2)$

$$= \frac{2(2x-2)}{(x^2 - 2x - 1)^2}$$

$|g_1'(-1)| = 2 > 1$ ; for $x = -1 \rightarrow$ divergent

$|g_1'(1)| = 0$ ; for $x = 1 \rightarrow$ superlinear convergent

$|g_1'(2)| = 4 > 1$ ; for $x = 2 \rightarrow$ divergent

$g_2(x) = x^3 - 2x^2 + 2$

$g_2'(x) = 3x^2 - 4x$

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$|g_2'(-1)| = 7$ ; for root $x = -1 \rightarrow$ divergent

$|g_2'(1)| = 1$ ; for root $x = 1 \rightarrow$ divergent

$|g_2'(2)| = 4$ ; for root $x = 2 \rightarrow$ divergent

$$g_3(x) = \sqrt{\frac{x^3 - x + 2}{2}}$$

$$g_3'(x) = \frac{3x^2 - 1}{2\sqrt{2}(x^3 - x + 2)^{1/2}}$$

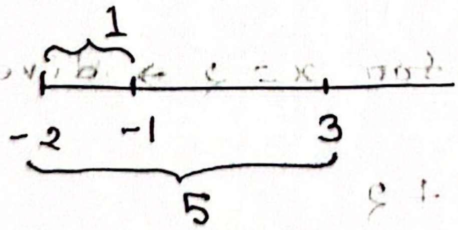
$|g_3'(-1)| = 0.6$ ; for $x = -1 \rightarrow$ linear convergent

$|g_3'(1)| = 0.5$ ; for $x = 1 \rightarrow$ linear convergent

$|g_3'(2)| = 3.75$ ; for $x = 2 \rightarrow$ divergent

② $g(x) \rightarrow -1, 3$

$x_0 = -2$ (starting location)

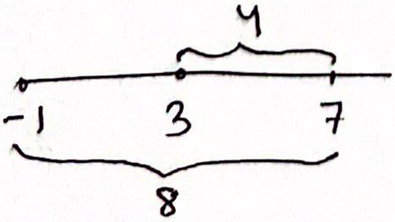



$x_0 = -2$

$\hookrightarrow$ root $= -1$

(took closest root)

$\downarrow$ depends on starting location




$x_0 = 7$

$\hookrightarrow$ root $= 3$

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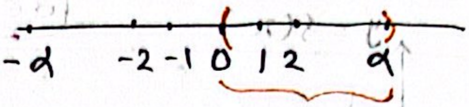
determine the range of values for which we can converge to a root.

③ $g(x) = \sqrt{2x+3}$

superlinear হলে ২০ হতে হবে।

$x_0 \to [-1, \alpha]$




$$g'(x) = \frac{1}{\sqrt{2x+3}} < 1$$ → $< 1$ হলে convergent হবে।

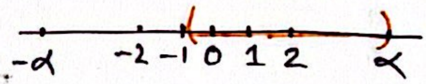
$$\Rightarrow \sqrt{2x+3} > 1$$

$$\Rightarrow 2x + 3 > 1$$

$$\Rightarrow 2x > -2$$

$$\Rightarrow x > -1$$ → $x_0 > -1$

starting point $x_0 > -1$ হলে function root এ convergent হবে




fixed point এর problem:

fixed point iteration এ 3 টা possible output থাকে। এক্ষেত্রে linear convergent slow হয়। এই problem কে solve করে Newton's Raphson.

↓

always fast convergent $(\alpha = 0)$

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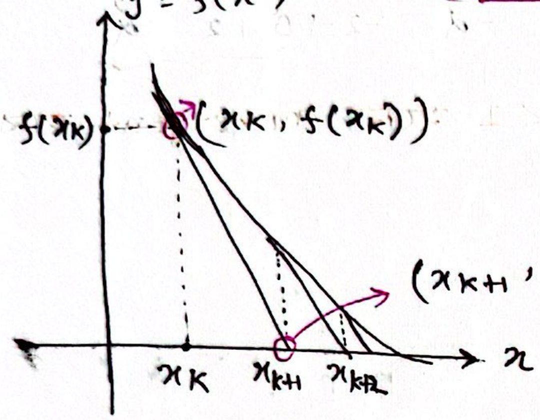
# Newton's Raphson Method:
(derivation)




$$\tan\theta = m = f'(x_k) = \frac{0 - f(x_k)}{x_{k+1} - x_k}$$

$$\Rightarrow x_{k+1} - x_k = \frac{-f(x_k)}{f'(x_k)}$$

$$\Rightarrow x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

superlinear convergent (derivation ⇒ proof)

$$x_{k+1} = \underbrace{x_k - \frac{f(x_k)}{f'(x_k)}}_{g(x)}$$

$$\gamma = |g'(x)| = 0$$

$$g(x) = x - \frac{f(x)}{f'(x)} \qquad \left[\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right]$$

$$g'(x) = 1 - \frac{f'(x)f'(x) - f(x)f''(x)}{(f'(x))^2}$$

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$$= \frac{(f'(x))^2 - (f'(x))^2 + f(x)f''(x)}{(f'(x))^2}$$

$$= \frac{f(x)f''(x)}{(f'(x))^2}$$

$$g'(x_*) = \frac{f(x_*)f''(x_*)}{(f'(x_*))^2} = 0$$ {function এর root বসিয়ে 0 পাওয়া যায়}

Type 1

# $f(x) = \frac{1}{x} - 0.5$ ; $x_* = 2$ ; $x_0 = 1$ ; $\epsilon = 1 \times 10^{-6}$

| $k$ | $x_k$ | $\lvert x_* - x_k \rvert$ | $\frac{\lvert x_* - x_k \rvert}{\lvert x_* - x_{k-1} \rvert} < \epsilon$ |
|---|---|---|---|
| 0 | $x_0 = 1$ | $\lvert x_* - x_0 \rvert = 1$ | X |
| 1 | $x_1 = 1.5$ | $\lvert 2 - 1.5 \rvert = 0.5$ | $\frac{\lvert x_* - x_1 \rvert}{\lvert x_* - x_0 \rvert} = \frac{0.5}{1} = 0.5 \nless \epsilon$ |
| 2 | $x_2 = 1.875$ | $\lvert 2 - 1.875 \rvert = 0.125$ | $\frac{0.125}{0.5} = 0.25 \nless \epsilon$ |
| 3 | $x_3 = 1.992$ | $\lvert 2 - 1.992 \rvert = 0.008$ | $\frac{0.008}{0.125} = 0.064 \nless \epsilon$ |
| 4 | $x_4 = 1.999$ | $\lvert 2 - 1.999 \rvert = 0.001$ | $\frac{0.001}{0.008} = 0.125 \nless \epsilon$ |
| 5 | $x_5 = 2$ | $\lvert 2 - 2 \rvert = 0$ | $\frac{0}{0.001} = 0 < \epsilon$ |

↳ root

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$$f(x) = \frac{1}{x} - 0.5 = (x^{-1}) - 0.5$$

$$f'(x) = -1 \cdot x^{-2} = -\frac{1}{x^2}$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{f(1)}{f'(1)} = 1.5$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.5 - \frac{f(1.5)}{f'(1.5)} = 1.875$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.875 - \frac{f(1.875)}{f'(1.875)} = 1.992$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.992 - \frac{f(1.992)}{f'(1.992)} = 1.999$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 1.999 - \frac{f(1.999)}{f'(1.999)} = 1.9999995 \approx 2$$

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Type 2

* $f(x) = x^2 - 2xe^{-x} + e^{-2x}$ ; $x_0 = 1$ ; $\epsilon = 1 \times 10^{-5}$

$f'(x) = 2x - [e^{-x} 2 - 2xe^{-x}] - 2e^{-2x}$

| k | $x_k$ | $f(x_k) < \epsilon$ |
|---|---|---|
| 0 | $x_0 = 1$ | $f(1) = 0.3995$ |
| 1 | $x_1 = 0.7689$ | $f(0.7689) = 0.093$ |
| 2 | $x_2 = \cancel{0.0226}$ 0.6648 | $\cancel{f(0.0226) =}$ $f(0.6648) = 0.0226$ |
| 3 | $x_3 = 0.6151$ | $f(0.6151) = 5.55 \times 10^{-3}$ |
| 4 | $x_4 = 0.5909$ | $f(0.5909) = 1.37 \times 10^{-3}$ |
| 5 | $x_5 = 0.5789$ | $f(0.5789) = 3.38 \times 10^{-4}$ |
| 6 | $x_6 = 0.5730$ | $f(0.5730) = 8.41 \times 10^{-5}$ |
| 7 | $x_7 = 0.5700$ | $f(0.5700) = 2.00 \times 10^{-5}$ |
| 8 | $x_8 = 0.56$ | $f(0.56) = 0.49 \times 10^{-5} < \epsilon$ |

$x_* = 0.56$

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$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.7689$$

$$x_2 = 0.7689 - \frac{0.093}{f'(0.7689)} = 0.6648$$

$$x_3 = 0.6648 - \frac{f(0.6648)}{f'(0.6648)} = 0.6151$$

$$x_4 = 0.6151 - \frac{f(0.6151)}{f'(0.6151)} = 0.5909$$

$$x_5 = 0.5909 - \frac{f(0.5909)}{f'(0.5909)} = 0.5789$$

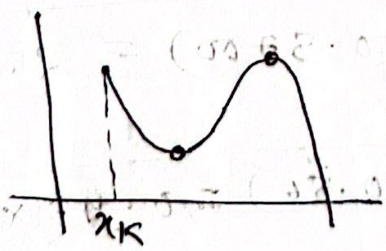
$$x_6 = 0.5789 - \frac{f(0.5789)}{f'(0.5789)} = 0.5730$$

$$x_7 = 0.5730 - \frac{f(0.5730)}{f'(0.5730)} = 0.5700$$

$$x_8 = 0.5700 - \frac{f(0.5700)}{f'(0.5700)} = 0.5685$$

Disadvantage of Newton's Raphson Method:

1. $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$




if $f'(x_k) = 0$, the whole thing has math error.

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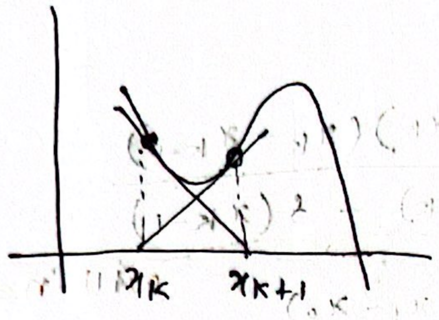
2.




if there is a turning point between the starting point and root, it'll stuck in a loop :

example

$f(x) = x^3 - 2x + 2$

$f'(x) = 3x^2 - 2$

$x_0 = 0$

$x_1 = 0 - \frac{f(0)}{f'(0)} = 1$

$x_2 = 1 - \frac{f(1)}{f'(1)} = 0$

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## Secant Method

$x_0 \qquad x_1$

$$x_{k+1} = x_k - \frac{f(x_k)(x_k - x_{k-1})}{\underbrace{f(x_k) - f(x_{k-1})}}$$

will never be 0, so no error

$x_k \ne x_{k-1}$ ; $f(x_k) \ne f(x_{k-1})$

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

বাকীটা as same as newton's Raphson method

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

can not get stuck in a loop

## Chapter 5

## Linear System:

$$x_1 + 2x_2 + x_3 = 0$$
$$x_1 - 2x_2 + 2x_3 = 4$$
$$2x_1 + 12x_2 - 2x_3 = 4$$

3 equation
3 unknown
variable

number of equation = number of unknown variable

$$\begin{vmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 4 \\ 4 \end{vmatrix}$$

A; coefficient matrix $\qquad x \qquad b$

$$A x = b$$

$$\Rightarrow x = A^{-1} b$$

$A^{-1} \rightarrow$ square matrix
$\rightarrow \det(A) \neq 0$

unique solution

But we need to ignore inverse.

So,

Gaussian Elimination:

↳ lower / upper triangular matrix

combine A & b

$$A_1 = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 0 \\ 1_{\rightarrow 0} & -2 & 2 & 4 \\ 2_{\rightarrow 0} & 12_{\rightarrow 0} & -2 & 4 \end{array}\right]$$

Augmented matrix

$$\left\{ m_{ij} = \frac{a_{ij}}{a_{jj}} \right.$$

$$m_{21} = \frac{a_{21}}{a_{11}} = \frac{1}{1} = 1$$

$$R_2' \rightarrow R_2 - R_1 m_{21}$$

$$m_{31} = \frac{a_{31}}{a_{11}} = \frac{2}{1} = 2 \,;\; R_3' \rightarrow R_3 - R_1 m_{31}$$

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$$A_2 = \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & -4 & 1 & 4 \\ 0 & 8 & -4 & 4 \end{vmatrix}$$

$$m_{32} = \frac{a_{32}}{a_{22}} = \frac{8}{-4} = -2$$

$$R_3' \rightarrow R_3 - R_2 m_{32}$$

$$A_3 = \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & -4 & 1 & 4 \\ 0 & 0 & -2 & 12 \end{vmatrix}$$

$$-2x_3 = 12$$
$$\Rightarrow x_3 = -6$$

$$-4x_2 + x_3 = 4$$
$$\Rightarrow -4x_2 = 4 - x_3 = 4 + 6$$
$$\Rightarrow x_2 = \frac{10}{-4} = -5/2$$

$$x_1 + 2x_2 + x_3 = 0$$
$$\Rightarrow x_1 = -2x_2 - x_3$$
$$\Rightarrow x_1 = -2 \times -\frac{5}{2} + 6 = 11$$

$$x_1 = 11, \quad x_2 = -5/2, \quad x_3 = -6$$

Disadvantage: If we change the values of resulting matrix, we've to calculate the whole thing all over again from the very start.

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$x_1 + 2x_2 + x_3 = 0$

$x_1 - 2x_2 + 2x_3 = 4$

$2x_1 + 12x_2 - 2x_3 = 4$

$A = LU$

lower   upper triangular

$$\underbrace{\begin{vmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{vmatrix}}_{A} \underbrace{\begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}}_{x} = \underbrace{\begin{vmatrix} 0 \\ 4 \\ 4 \end{vmatrix}}_{b}$$

$Ux = y \quad \text{...(ii)}$

$y = \begin{vmatrix} a \\ b \\ c \end{vmatrix}$

$LUx = b$

$\Rightarrow Ly = b \quad \text{... (i)}$

$m_{2,1} = \frac{a_{21}}{a_{11}} = 1$, $R_2 = R_2 - R_1 * m_{21}$

$$A^1 = \begin{vmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{vmatrix}$$

$m_{31} = \frac{2}{1} = 2$, $R_3 = R_3 - R_1 * m_{31}$

$$A^2 = \begin{vmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 8 & -4 \end{vmatrix}$$

$m_{32} = \frac{a_{32}}{a_{22}} = \frac{8}{-4} = -2$, $R_3 = R_3 - R_2 * m_{32}$

$$A^3 = \begin{vmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 0 & -2 \end{vmatrix} = U$$

$m_{21} \quad m_{31} \quad m_{32}$

$$L = \begin{vmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -2 & 1 \end{vmatrix}$$

$$A = LU$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -2 & 1 \end{vmatrix} \begin{vmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 0 & -2 \end{vmatrix}$$

$$Ly = b$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -2 & 1 \end{vmatrix} \begin{vmatrix} y_1 \\ y_2 \\ y_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 4 \\ 4 \end{vmatrix}$$

$y_1 = 0$ | $y_1 + y_2 + 0 = 4$, $\Rightarrow y_2 = 4$ | $2y_1 - 2y_2 + y_3 = 4$, $\Rightarrow y_3 = 12$

$$Ux = y$$

$$\begin{vmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 0 & -2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 4 \\ 12 \end{vmatrix}$$

$-2x_3 = 12$, $\Rightarrow x_3 = -6$ | $-4x_2 + x_3 = 4$, $\Rightarrow x_2 = -\frac{5}{2}$ | $x_1 + 2x_2 + x_3 = 0$, $\Rightarrow x_1 + 2 \times -\frac{5}{2} - 6 = 0$, $\Rightarrow x_1 = 11$

$$x = \begin{vmatrix} 11 \\ -5/2 \\ -6 \end{vmatrix}$$

## Frobenius Matrix

$F \rightarrow$ Unit lower triangular matrix

$$A^1 = \begin{vmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{vmatrix}_{3\times 3}$$

A matrix এর dimension = F matrix এর dimension

$$F^1 = \begin{vmatrix} 1 & 0 & 0 \\ -m_{21} & 1 & 0 \\ -m_{31} & 0 & 1 \end{vmatrix}_{3\times 3}$$

$\left\{ \right.$ 1 নং কলামে কাজ হচ্ছে , তাই $m_{32} = 0$

$$A_2 = F^1 A^1$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 8 & -4 \end{vmatrix}$$

$$m_{32} = \frac{8}{-4} = -2$$

$$F^2 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -m_{3/2} & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{vmatrix}$$

$$A^3 = F^2 \cdot A^2$$

$$= \begin{vmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 0 & -2 \end{vmatrix} = U$$

$$L = \begin{vmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ & -2 & 1 \end{vmatrix}$$

$$* \quad A = \begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix}_{4\times 4} \qquad A = LU$$

$$F^1 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ -m_{21} & 1 & 0 & 0 \\ -m_{31} & 0 & 1 & 0 \\ -m_{41} & 0 & 0 & 1 \end{vmatrix}_{4\times 4}$$

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$A^2 = F^1 A^1$

$A^3 = F^2 A^2$

$A^4 = F^3 A^3$

$$F^2 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -m_{32} & 1 & 0 \\ 0 & -m_{42} & 0 & 1 \end{vmatrix}$$

$$L = \begin{vmatrix} 1 & 0 & 0 & 0 \\ m_{21} & 1 & 0 & 0 \\ m_{31} & m_{32} & 1 & 0 \\ m_{41} & m_{42} & m_{43} & 1 \end{vmatrix}$$

$$F^3 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -m_{43} & 1 \end{vmatrix}$$

Number of operation for Backward / forward substitution :

$$\begin{vmatrix} 2 & 0 & 0 \\ 3 & 6 & 0 \\ 5 & 9 & 10 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 15 \\ 22 \\ 31 \end{vmatrix}$$

$n = 3$ { number of unknown variable

$x_1 = \frac{15}{2}$

$3x_1 + 6x_2 = 22$

$\Rightarrow x_2 = \frac{22 - 3x_1}{6}$

$5x_1 + 9x_2 + 10x_3 = 31$

$\Rightarrow x_3 = \frac{31 - 5x_1 - 9x_2}{10}$

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$k=1;\ x_1 = 1$ div $+\ 0\,(1$ mul $+\ 1$ subs$)$

$k=2;\ x_2 = 1$ div $+\ 1\,(1$ mul $+\ 1$ subs$)$

$k=3;\ x_3 = 1$ div $+\ 2\,(1$ mul $+\ 1$ subs$)$

$$\sum_{k=1}^{n} 1 + (k-1)2$$

$$= \sum_{k=1}^{n} 1 + 2k - 2$$

$$= \sum_{k=1}^{n} 2k - 1 = 2\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1$$

$$= 2\frac{n(n+1)}{2} - n = n^2 + n - n = n^2$$

**Pivoting :**

$$\begin{bmatrix} 0 & 3 & 5 \\ 6 & 7 & 9 \\ 10 & 15 & -2 \end{bmatrix}_{3\times 3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} = \begin{bmatrix} 5 \\ 9 \\ 10 \end{bmatrix}_{3\times 1}$$

$m_{21} = \frac{a_{21}}{a_{11}} = \frac{6}{0} = \infty \rightarrow$ pivoting problem

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row swap: $R_1 \rightleftarrows R_2$

$$\begin{vmatrix} 6 & 7 & 9 \\ 0 & 3 & 5 \\ 10 & 15 & 2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 9 \\ 5 \\ 10 \end{vmatrix}$$

column swap: $C_1 \rightleftarrows C_2$

GE/LU

$$\begin{vmatrix} 3 & 0 & 5 \\ 7 & 6 & 9 \\ 15 & 10 & 2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 5 \\ 9 \\ 10 \end{vmatrix}$$

## Chapter 6

$$3x_1 + 3x_2 = 5$$
$$2x_1 + 5x_2 = 15$$
$$3x_1 + 7x_2 = 37$$

3 eq > 2 $\{x_1, x_2\}$

over determine system

$\times$ GE/LU

$$\begin{vmatrix} 3 & 3 \\ 2 & 5 \\ 3 & 7 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} = \begin{vmatrix} 5 \\ 15 \\ 37 \end{vmatrix}$$

To solve this

→ least square approximation

→ QR decomposition

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least square approximation

$$\begin{vmatrix} 3 & 3 \\ 2 & 5 \\ 3 & 7 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} = \begin{vmatrix} 5 \\ 15 \\ 37 \end{vmatrix}$$

$A^T_{2\times3} A_{3\times2}\ x_{2\times1} = A^T_{2\times3}\ b_{3\times1}$ → both side এ $A^T$ নিয়ে

$\Rightarrow A'_{2\times2}\ x_{2\times1} = b'_{2\times1}$

↳ then solve it with LU / GE

is not mentioned, then $x = A^{-1}b$

m equation, n unknow variable :

$A^T_{n\times m}\ A_{m\times n}\ x_{n\times1} = A^T_{n\times m}\ b_{m\times1}$

$A'_{n\times n}\ x_{n\times1} = b'_{n\times1}$

QR decomposition

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## Vector

column matrix

$$x = \begin{vmatrix} 1 \\ 5 \\ 7 \end{vmatrix}$$

$$\vec{x} = (1, 5, 7)^T$$

$$|\vec{x}| = \sqrt{1^2 + 5^2 + 7^2} = \sqrt{75}$$

$$\underbrace{\vec{x} \cdot \vec{x}}_{l_2 \text{ norm}} = |\vec{x}||\vec{x}| \cos\theta$$

$$= 1 \times 1 + 5 \times 5 + 7 \times 7 = 75$$

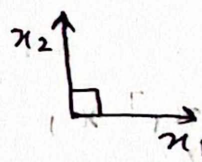
* $\vec{y}$ $l_2$ norm $= y^T y$

$$\vec{y} = (2 \quad 5 \quad 9)^T$$

$$y^T y = [2 \quad 5 \quad 9] \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}$$

## Orthonormality:

S
- Orthogonality
- Normality



$x_1$ & $x_2$ 90° angle এ আছে, তাই orthogonal (perpendicular)

$$S = \{\vec{x_1}, \vec{x_2}, \vec{x_3}\}$$

Orthogonality: $x_1^T x_2 = 0$ ; $x_2^T x_3 = 0$ ; $x_3^T x_1 = 0$

Normality: $x_1^T x_1 = 1$ ; $x_2^T x_2 = 1$ ; $x_3^T x_3 = 1$

দুটো শর্ত fulfill করলে orthonormal.

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$S = \{U_1, U_2\}$

Orthogonal : $U_1^T U_2 = 0$ / $U_2^T U_1 = 0$

Normal : $U_1^T U_1 = 1$ ; $U_2^T U_2 = 1$

column vector

* $S = \{ \frac{1}{\sqrt{5}} (2,1)^T , \frac{1}{\sqrt{5}} (1,-2)^T \}$ ; Prove it's orthonormal

$x_1$ $x_2$

$$x_1 = \begin{vmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{vmatrix} ; \quad x_2 = \begin{vmatrix} \frac{1}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} \end{vmatrix}$$

$$x_1^T x_2 = \begin{vmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{vmatrix} \begin{vmatrix} \frac{1}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} \end{vmatrix} = \frac{2}{5} - \frac{2}{5} = 0$$

So, it's orthogonal.

$$x_1^T x_1 = \begin{vmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{vmatrix} \begin{vmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{vmatrix} = \frac{4}{5} + \frac{1}{5} = 1$$

$$x_2^T x_2 = \begin{vmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \end{vmatrix} \begin{vmatrix} 1/\sqrt{5} \\ -2/\sqrt{5} \end{vmatrix} = \frac{1}{5} + \frac{4}{5} = 1$$

So, it's normal.

So given set of vector is a orthonormal set.

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# Gram - Schmidt process :

$$A = \begin{vmatrix} 1 & 6 \\ 2 & 5 \\ 3 & 4 \end{vmatrix} \rightarrow QR$$

$U_1 \quad U_2$

$\downarrow \quad \downarrow \quad \rightarrow \quad S = \{q_1, q_2\}$

$q_1 \quad q_2$

$QR$

$\hookrightarrow |q_1 \quad q_2|$

1) $P_1 = U_1 = \begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$

$$q_1 = \frac{P_1}{|P_1|} = \frac{\begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}}{\sqrt{1^2+2^2+3^2}} = \frac{1}{\sqrt{14}} \begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$$

$$P_2 = U_2 - (U_2^T q_1) q_1$$

$$q_2 = \frac{P_2}{|P_2|}$$

$$P_2 = \begin{vmatrix} 6 \\ 5 \\ 4 \end{vmatrix} - \left( [6 \quad 5 \quad 4] \begin{vmatrix} \frac{1}{\sqrt{14}} \\ \frac{2}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \end{vmatrix} \right) \begin{vmatrix} \frac{1}{\sqrt{14}} \\ \frac{2}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \end{vmatrix}$$

a

$$Q = |q_1 \quad q_2|$$

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$u_1 \quad u_2 \quad u_3$

$\downarrow q_1 \quad \downarrow q_2 \quad \downarrow q_3$

$$P_3 = U_3 - (U_3^T q_2) q_2 - (U_3^T q_1) q_1$$

$$P_n = U_n - \sum_{k=1}^{n-1} (U_n^T q_k) q_k$$

$$* \quad A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{vmatrix}$$

$\quad U_1 \quad U_2 \quad U_3$

$$A = QR$$

$$A^T A x = A^T \cdot b$$

$$(QR)^T QR x = (QR)^T \cdot b$$

$$\Rightarrow Q^T \cancel{R^T} QR x = Q^T \cancel{R^T} b$$

$$\Rightarrow Q^T QR x = Q^T b \qquad [Q^T Q = I]$$

$$\Rightarrow \boxed{Rx = Q^T b}$$

$R \downarrow$ upper triangular matrix

TOPIC NAME : QR decomposition example DAY :
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## QR decomposition

$f(-3) = 0$
$f(0) = 0$
$f(6) = 2$

1st degree polynomial

$n = 1$, $P_1(x) = a_0 + a_1 x^1$

$P_1(-3) = a_0 + a_1(-3)^1 = 0$

$P_1(0) = a_0 + a_1(0)^1 = 0$

$P_1(6) = a_0 + a_1(6)^1 = 2$

$$\begin{array}{cc} u_1 & u_2 \\ \end{array}$$

$$Q \begin{vmatrix} 1 & -3 \\ 1 & 0 \\ 1 & 6 \end{vmatrix} \begin{vmatrix} a_0 \\ a_1 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 2 \end{vmatrix}$$

$Rx = Q^T b$

$A \quad x \quad = b$

QR

$$u_1 = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} \qquad u_2 = \begin{vmatrix} -3 \\ 0 \\ 6 \end{vmatrix}$$

$$P_1 = u_1 = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} \qquad q_1 = \frac{P_1}{|P_1|} = \begin{vmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{vmatrix}$$

$$p_2 = u_2 - (u_2^T q_1) q_1$$

$$= \begin{vmatrix} -3 \\ 0 \\ 6 \end{vmatrix} - \left( \begin{vmatrix} -3 & 0 & 6 \end{vmatrix} \begin{vmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{vmatrix} \right) \begin{vmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{vmatrix}$$

$$= \begin{vmatrix} -3 \\ 0 \\ 6 \end{vmatrix} - \sqrt{3} \begin{vmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{vmatrix} = \begin{vmatrix} -3 \\ 0 \\ 6 \end{vmatrix} - \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} = \begin{vmatrix} -4 \\ -1 \\ 5 \end{vmatrix}$$

$$|p_2| = \sqrt{(-4)^2 + (-1)^2 + 5^2} = \sqrt{42}$$

$$q_2 = \frac{p_2}{|p_2|} = \begin{vmatrix} -4/\sqrt{42} \\ -1/\sqrt{42} \\ 5/\sqrt{42} \end{vmatrix}$$

$$Q = \begin{array}{c} \begin{array}{cc} q_1 & q_2 \end{array} \\ \begin{vmatrix} 1/\sqrt{3} & -4/\sqrt{42} \\ 1/\sqrt{3} & -1/\sqrt{42} \\ 1/\sqrt{3} & 5/\sqrt{42} \end{vmatrix}_{3\times 2} \end{array} \rightarrow \text{col} = 2$$

$$R = \begin{array}{cc} \begin{array}{cc} u_1 & u_2 \end{array} & \\ \begin{vmatrix} u_1^T q_1 & u_2^T q_1 \\ 0 & u_2^T q_2 \end{vmatrix} & \begin{array}{c} q_1 \\ q_2 \end{array} \end{array}$$

col × col
2 × 2

{ column wise $u_1, u_2$,
row wise $q_1, q_2$ }

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$$u_1^T q_1 = \begin{vmatrix} 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{vmatrix} = \sqrt{3}$$

$$u_2^T q_1 = \begin{vmatrix} -3 & 0 & 6 \end{vmatrix} \begin{vmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{vmatrix} = \sqrt{3}$$

$$u_2^T q_2 = \begin{vmatrix} -3 & 0 & 6 \end{vmatrix} \begin{vmatrix} -4/\sqrt{42} \\ -1/\sqrt{42} \\ 5/\sqrt{42} \end{vmatrix} = \sqrt{42}$$

$$R = \begin{vmatrix} \sqrt{3} & \sqrt{3} \\ 0 & \sqrt{42} \end{vmatrix}$$

$$R x = Q^T b$$

$$\Rightarrow \begin{vmatrix} \sqrt{3} & \sqrt{3} \\ 0 & \sqrt{42} \end{vmatrix} \begin{vmatrix} a_0 \\ a_1 \end{vmatrix} = \begin{vmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -4/\sqrt{42} & -1/\sqrt{42} & 5/\sqrt{42} \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ 2 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} \sqrt{3} & \sqrt{3} \\ 0 & \sqrt{42} \end{vmatrix} \begin{vmatrix} a_0 \\ a_1 \end{vmatrix} = \begin{vmatrix} 2/\sqrt{3} \\ 10/\sqrt{42} \end{vmatrix}$$

$$\sqrt{42}\, a_1 = 10/\sqrt{42}$$

$$\Rightarrow a_1 = \frac{10}{42} = \frac{5}{21}$$

GOOD LUCK

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$$\sqrt{3}a_0 + \sqrt{3}a_1 = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}a_0 + \sqrt{3}\cdot\frac{5}{21} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow a_0 = \frac{3}{7}$$

$$P_1(x) = a_0 + a_1 x^1$$
$$= \frac{3}{7} + \frac{5}{21}x$$

Home task: Solve it using QR decomposition

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 6 \\ 4 \end{bmatrix}$$

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## Chapter 7

$$I(f) = \int_a^b f(x)\,dx$$

$$I_n(f) = \int_a^b P_n(x)\,dx$$

$n \to$ degree

$$= \int_a^b \sum_{k=0}^{n} f(x_k)\, l_k(x)\, dx$$

$$= \sum_{k=0}^{n} f(x_k) \underbrace{\int_a^b l_k(x)\, dx}_{\sigma_k}$$

$\sigma_k \to$ weight factor

$$\doteq \sum_{k=0}^{n} f(x_k)\,\sigma_k$$

↳ Newton - cotes formula

- → open Newton-cotes formula
- → closed Newton-cotes formula

**Open newton-cotes**

[a, b]

$n = 3$; $x_0, x_1, x_2, x_3$

$a < x_0 < x_1 < x_2 < x_3 < b$ [equidistant]

**closed newton-cotes**

$x_0 = a < x_1 < x_2 < x_3 = b$ [equidistant].

TOPIC NAME

* $[0,6]$, $h = 2$, closed newton-cotes

$x_0 = 0$

$x_1 = 0 + 2 = 2$

$x_2 = 2 + 2 = 4$

$x_3 = 4 + 2 = 6$

## Derivation

$$I_n(f) = \sum_{k=0}^{n} f(x_k)\,\sigma_k$$

Closed Newton-cotes for $n = 1$ [Trapezium rule]

$[a, b]$

$x_0, x_1 \rightarrow$ nodes

$x_0 = a,\ x_1 = b$

$$I_1(f) = f(x_0)\sigma_0 + f(x_1)\sigma_1$$

$$\sigma_0 = \int_a^b l_0(x)\,dx$$

$$= \int_a^b \frac{x - x_1}{x_0 - x_1}\,dx = \int_a^b \frac{x-b}{a-b}\,dx$$

$$= \frac{1}{a-b}\int_a^b (x-b)\,dx = \frac{1}{a-b}\left[\frac{x^2}{2} - bx\right]_a^b$$

$$= \frac{1}{a-b}\left[\frac{b^2}{2} - b^2 - \frac{a^2}{2} + ab\right]$$

$$= \frac{1}{a-b}\left[-\frac{b^2}{2} + ab - \frac{a^2}{2}\right]$$

$$= -\frac{1}{2}\,\frac{1}{a-b}\left[b^2 - 2ab + a^2\right]$$

$$= \frac{1}{2(b-a)}(b-a)^2 = \frac{b-a}{2}$$

$$\sigma_1 = \int_a^b l_1(x)\,dx$$

$$= \int_a^b \frac{x-x_0}{x_1-x_0}\,dx = \int_a^b \frac{x-a}{b-a}\,dx$$

$$= \frac{1}{b-a}\int_a^b (x-a)\,dx = \frac{1}{b-a}\left[\frac{x^2}{2} - ax\right]_a^b$$

$$= \frac{1}{b-a}\left[\frac{b^2}{2} - ab - \frac{a^2}{2} + a^2\right]$$

$$= \frac{1}{b-a}\left[\frac{a^2}{2} - ab + \frac{b^2}{2}\right]$$

$$= \frac{1}{2(b-a)}\left[a^2 - 2ab + b^2\right]$$

$$= \frac{1}{2(b-a)}(a-b)^2$$

$$= -\frac{1}{2(a-b)}(a-b)^2 = \frac{b-a}{2}$$

$$I_1(f) = f(x_0)\,\sigma_0 + f(x_1)\,\sigma_1$$

$$= f(a)\frac{b-a}{2} + f(b)\frac{b-a}{2}$$

$$= \frac{b-a}{2}\left[f(a) + f(b)\right]$$

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* $[a, b] = [0, 2]$ ; $f(x) = e^x$

a) calculate the original integration value.

$$I(f) = \int_0^2 e^x \, dx = \left[e^x\right]_0^2 = e^2 - e^0 = 6.389$$

b) calculate the integration value using Trapezium rule. $\downarrow$ $n = 1$

So, $n = 1$

$$I_1(f) = \frac{b-a}{2}\left[f(a) + f(b)\right]$$

$$= \frac{2-0}{2}\left[f(0) + f(2)\right]$$

$$= e^0 + e^2 = 8.389$$

c) calculate relative error.

$$\text{Relative error} = \frac{|8.389 - 6.389|}{|6.389|} \times 100$$

$$= 31.3\%$$

disadvantage of Trapezium rule : Huge error

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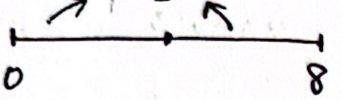
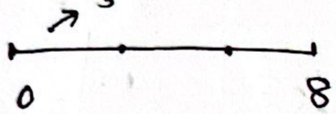
Composite Newton cotes → derivation from book

$\frac{8-0}{2} = 4 = h$



$m = 2$ ; point $3 = x_0, x_1, x_2$

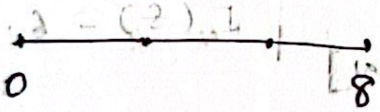
$\frac{8-0}{3} = 8/3$



$m = 3$ ; point $4 = x_0, x_1, x_2, x_3$

$$h = \frac{8-0}{2} = 4 = \frac{b-a}{m}$$

$$\therefore h = \frac{b-a}{m}$$



$$h = \frac{b-a}{m} = \frac{8-0}{3} = 8/3$$

$x_0 = 0$

$x_1 = x_0 + h = 0 + \frac{8}{3} = \frac{8}{3}$

$x_2 = x_1 + h = \frac{8}{3} + \frac{8}{3} = \frac{16}{3}$

$x_3 = x_2 + h = \frac{16}{3} + \frac{8}{3} = \frac{24}{3} = 8$

{ ১ম এবং শেষের 1 multiply, মাঝে সবগুলোর সাথে ২ multipl

$$C_{1,m} = C_{1,3} = \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + f(x_3)\right]$$

$$C_{1,4} = \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

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derivation

$$h = \frac{b-a}{m}$$

a c d b

$$[a, c] = \frac{c-a}{2}\left(f(a) + f(c)\right)$$

$$[c, d] = \frac{d-c}{2}\left(f(c) + f(d)\right)$$

$$[d, b] = \frac{b-d}{2}$$

* $f(x) = e^x$ ; $[a, b] \in [0, 2]$

$I_1(f) = 8.389$

$I(f) = 6.389$

$C_{1,m} = ?$

$C_{1,4} = 6.521$ [close to original]

$$h = \frac{b-a}{m} = \frac{2-0}{4} = 2/4$$

$x_0 = 0$

$x_1 = x_0 + h = 2/4$

$x_2 = x_1 + h = \frac{2}{4} + \frac{2}{4} = 1$

$x_3 = x_2 + h = 1 + \frac{2}{4} = \frac{6}{4}$

$x_4 = x_3 + h = \frac{6}{4} + \frac{2}{4} = 2$

$$C_{1,4} = \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

$$= \frac{2/4}{2}\left[f(0) + 2f(2/4) + 2f(1) + 2f(6/4) + f(2)\right]$$

$$= 6.521$$

TOPIC NAME : Derivation

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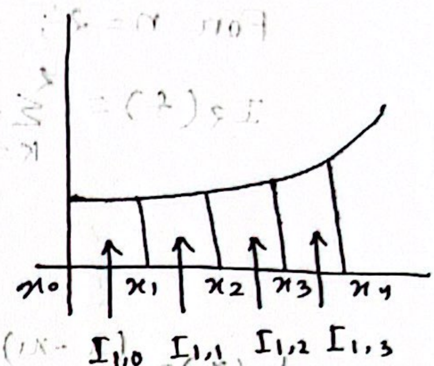
## Composite Newton Cotes

$I_1(f) =$ Trapezium Rule

$$= \frac{b-a}{2}\left[f(a) + f(b)\right]$$

$$= \frac{h}{2}\left[f(a) + f(b)\right]$$




$$I_{1,0} = \frac{h}{2}\left[f(x_0) + f(x_1)\right]$$

$$I_{1,1} = \frac{h}{2}\left[f(x_1) + f(x_2)\right]$$

$$I_{1,2} = \frac{h}{2}\left[f(x_2) + f(x_3)\right]$$

$\vdots$

$\vdots$

$$I_{1,m-1} = \frac{h}{2}\left[f(x_{m-2}) + f(x_{m-1})\right]$$

$$I_{1,m} = \frac{h}{2}\left[f(x_{m-1}) + f(x_m)\right]$$

$$C_{1,m}(f) = \frac{h}{2}\Big[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3)$$
$$+ \cdots \cdots + 2f(x_{m-1}) + f(x_m)\Big]$$

Simpson's rule :

↓

( $n = 2$ ; closed newton-cotes formula )

$$I_2(f) = \int_a^b P_2(x)\,dx$$


$x_0 = a = 3$

$x_1 = \frac{a+b}{2} = 5$

$x_2 = b = 7$

$$\sigma_0 = \int_a^b l_0(x)\,dx$$

$$= \int_a^b \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\,dx$$

$$= \int_a^b \frac{\left(x - \frac{a+b}{2}\right)(x-b)}{\left(a - \frac{a+b}{2}\right)(a-b)}\,dx$$

$$= \frac{b-a}{6}$$

$$\sigma_1 = \frac{2}{3}(b-a)$$

$$\sigma_2 = \frac{b-a}{6}$$

$$\boxed{I_2(f) = \int_a^b p_2(x)\,dx = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]}$$

$$= \frac{2-0}{6}\left[f(0) + 4\left(\frac{2+0}{2}\right) + f(2)\right]$$

$= 6.420$ (closest to the original)

we'll get the best possible value → Simpson's rule

Error bound / upper bound of the error :

$n=1$

$$\frac{f^{n+1}(\xi)}{(n+1)!} \int_a^b (x-x_0)\cdots(x-x_n)$$

$$= \frac{f^2(\xi)}{2!} \left| \int_a^b (x-x_0)(x-x_1) \right|$$

$f(x) = e^x$ ; $[a,b] \in [0,2]$

$f^2(x) = e^x$

$$\int_0^2 (x-0)(x-2)\,dx = \int_0^2 x(x-2)\,dx$$

$$= \int_0^2 (x^2-2x)\,dx = \left[\frac{x^3}{3} - x^2\right]_0^2 = -\frac{4}{3}$$

$$\frac{f^2(\xi)}{2!} \left| \int_a^b (x-x_0)(x-x_1) \right| = \frac{e^2}{2!} \times \frac{4}{3}$$

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## Simpson's Rule :

For $n = 2$;

$$I_2(f) = \sum_{k=0}^{2} \delta_k f(x_k)$$

$h = \frac{b-a}{2}$

$x_0 = a$

$x_1 = (a+b)/2 = m$

$x_2 = b$

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-m)(x-b)}{(a-m)(a-b)}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-a)(x-b)}{(m-a)(m-b)}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-a)(x-m)}{(b-a)(b-m)}$$

$$\delta_0 = \int_a^b l_0(x)\,dx$$

$$\delta_0 = \frac{1}{(a-m)(a-b)} \int_a^b (x-m)(x-b)\,dx$$

$$\delta_1 = \int_a^b l_1(x)\,dx$$

$$\delta_2 = \int_a^b l_2(x)\,dx$$

$$\delta_0 = \frac{1}{6}(b-a)$$

$$\delta_1 = \frac{2}{3}(b-a)$$

$$\delta_2 = \frac{1}{6}(b-a)$$

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$$I_2(F) = \delta_0 f(x_0) + \delta_1 f(x_1) + \delta_2 f(x_2)$$

$$I_2(f) = \frac{(b-a)}{6}\left[f(a) + 4f\frac{(a+b)}{2} + f(b)\right]$$

↳ simpson's rule