

CSE330- Numerical Methods
Quiz 05: Fall'24 [CO3]

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Marks: 15 points

Time: 20 minutes

Instructions: Answer all questions on the space provided below for each.

A linear system is described by the following equations:

$$2x_1 + 6x_2 + 2x_3 = 6$$

$$4x_1 + 2x_2 + 3x_3 = 10$$

$$2x_1 + 5x_2 = 15$$

Based on these equations, answer the questions below.

- (a) From the given linear equations, identify the matrices **A**, **x** and **b**. Examine if the **matrix A** has any pivoting problem? If yes, solve the pivoting problem. [5 marks]
- (b) Construct the **Frobenius matrices** $F^{(1)}$ and $F^{(2)}$ from this system. [3 marks]
- (c) Compute the **unit lower triangular matrix L**. [3 marks]
- (d) Now find the **solution** of the linear system using the LU decomposition method. Use the unit lower triangular matrix found in the previous question. [4 marks]

Set $\rightarrow \Lambda$
a

$$\begin{bmatrix} 2 & 6 & 2 \\ 4 & 2 & 3 \\ 2 & 5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 15 \end{bmatrix}$$

A x b

$$R_1 \rightleftharpoons C_3,$$

$$A_1 = \begin{bmatrix} 2 & 6 & 2 \\ 3 & 2 & 4 \\ 0 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 15 \end{bmatrix}$$

b

$$m_{21} = \frac{a_{21}}{a_{11}} = \frac{3}{2} \quad \left| \quad m_{31} = \frac{0}{2} = 0 \right.$$

$$F^1 = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 6 & 2 \\ 3 & 2 & 4 \\ 0 & 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 & 2 \\ 0 & -7 & 1 \\ 0 & 5 & 2 \end{bmatrix} A_2$$

$$m_{32} = \frac{a_{32}}{a_{22}} = \frac{-5}{-7}$$

$$F^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{5}{7} & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{5}{7} & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 6 & 2 \\ 0 & -7 & 1 \\ 0 & 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 & 2 \\ 0 & -7 & 1 \\ 0 & 0 & \frac{19}{7} \end{bmatrix} \quad \text{U}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ 0 & -\frac{5}{7} & 1 \end{bmatrix}$$

$$Ly = b$$

$$\left| \begin{array}{ccc|c} 1 & 0 & 0 & y_1 \\ \frac{3}{2} & 1 & 0 & y_2 \\ 0 & -\frac{5}{7} & 1 & y_3 \end{array} \right| = \left| \begin{array}{c} 6 \\ 10 \\ 15 \end{array} \right|$$

$$y_1 = 6 \quad \left| \begin{array}{l} \frac{3}{2}y_1 + y_2 = 10 \\ \Rightarrow y_2 = 1 \end{array} \right.$$

$$-\frac{5}{7}y_2 + y_3 = 15$$

$$y_3 = \frac{110}{7}$$

d

$$Ux = y$$

$$\begin{bmatrix} 2 & 6 & 2 \\ 0 & -7 & 1 \\ 0 & 0 & \frac{19}{7} \end{bmatrix} \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \frac{110}{7} \end{bmatrix}$$

$$\left| \begin{array}{l} \frac{19}{7}x_1 = \frac{110}{7} \\ x_1 = \frac{110}{19} \end{array} \right| \quad \left| \begin{array}{l} -7x_2 + x_1 = 1 \\ x_2 = \frac{13}{19} \end{array} \right.$$

$$2x_3 + 6x_2 + 2x_1 = 6$$

$$x_3 = \frac{6 - 6x_2 - 2x_1}{2} = -\frac{22}{19}$$