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CSE 330

Section: 17

Assignment 04

① a)

$$f(x) = x^3 + x^2 - 4x - 4$$

$$x^3 + x^2 - 4x - 4 = 0$$

$$\Rightarrow x^2(x+1) - 4(x+1) = 0$$

$$\Rightarrow (x+1)(x^2 - 4) = 0$$

$$\Rightarrow (x+1)(x+2)(x-2) = 0$$

$$\therefore x = -1, -2, 2$$

Roots of $f(x) = -1, -2, 2$

$$f(x) = x^3 + x^2 - 4x - 4 = 0$$

$$\Rightarrow x^3 + x^2 - 4 = 4x$$

$$\Rightarrow x = \frac{x^3 + x^2 - 4}{4}$$

$$\therefore g_1(x) = \frac{x^3 + x^2 - 4}{4}$$

Again,

$$x^3 + x^2 - 4x - 4 = 0$$

$$\Rightarrow x(x^2 + x - 4) = 4$$

$$\Rightarrow x = \frac{4}{x^2 + x - 4}$$

$$\therefore g_2(x) = \frac{4}{x^2 + x - 4}$$

b) $$g_1(x) = \frac{x^3 + x^2 - 4}{4}$$

$$= \frac{x^3}{4} + \frac{x^2}{4} - 1$$

$$g_1'(x) = \frac{1}{4} 3x^2 + \frac{1}{4} 2x$$

$$= \frac{3}{4} x^2 + \frac{x}{2}$$

$g_1'(-1) = \frac{1}{4} < 1$; linear convergent

$g_1'(-2) = 2 > 1$; divergent

$g_1'(2) = 4 > 1$; divergent

$$g_2(x) = \frac{4}{x^2 + x - 4} = 4(x^2 + x - 4)^{-1}$$

$$\cancel{= 4x^{-2} + 4x^{-1} - 1}$$

$$\cancel{g_2'(x) = 4(-2)x^{-3} + 4(-1)x^{-2}}$$

$$\cancel{= -8x^{-3} - 4x^{-2}}$$

$$g_2'(x) = 4 \cdot (-1)(x^2 + x - 4)^{-2}(2x+1) = -\frac{4(2x+1)}{(x^2+x-4)^2}$$

$g_2'(-1) = \frac{1}{4}$ $\cancel{> 1}$ < 1 ; ~~divergent~~ linear convergent

divergent

$g_2'(-2) = 3 > 1$, ~~superlinear convergent~~

$g_2'(2) = -5 < 1$; linear convergent

② a)

$f(x) = xe^x - 1$

$f'(x) = xe^x + e^x$

k	x_k	$f(x_k)$
0	$x_0 = 1.5$	$f(1.5) = 5.722$
1	$x_1 = 0.9892$	$f(0.9892) = 1.660$
2	$x_2 = 0.6788$	$f(0.6788) = 0.3382$
3	$x_3 = 0.5765$	$f(0.5765) = 0.02605$
4	$x_4 = 0.5672$	$f(0.5672) = 0.0001567$
5	$x_5 = 0.5671$	$f(0.5671) = -0.0001196$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.5 - \frac{f(1.5)}{f'(1.5)} = 0.9892$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.9892 - \frac{f(0.9892)}{f'(0.9892)} = 0.6788$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.6788 - \frac{f(0.6788)}{f'(0.6788)} = 0.5765$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.5765 - \frac{f(0.5765)}{f'(0.5765)} = 0.5672$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 0.5672 - \frac{f(0.5672)}{f'(0.5672)} = 0.5671$$

$\therefore x_* = 0.5671$

b)

$$g(x) = \frac{2x+1}{\sqrt{x+1}}$$

$$g'(x) = \frac{2(x+1)^{1/2} - (2x+1)\cdot\frac{1}{2}(x+1)^{-1/2}}{(\sqrt{x+1})^2}$$

$$= \frac{2(x+1)^{1/2} - \frac{2x+1}{2(x+1)^{1/2}}}{x+1}$$

$$= \frac{\frac{4(x+1)-(2x+1)}{2(x+1)^{1/2}}}{x+1}$$

$$= \frac{2x+3}{2(x+1)^{3/2}}$$

$|g'(x)| = 0$ [to be superlinear convergent]

$$g'(x) = \frac{2x+3}{2(x+1)^{3/2}}$$

$$\Rightarrow \frac{2x+3}{2(x+1)^{3/2}} = 0$$

$$\Rightarrow 2x+3 = 0$$

$$\Rightarrow x = -\frac{3}{2}$$

$\therefore$ To be super-linearly convergent, the root must satisfy $x_* = -\frac{3}{2}$

(3)

$$f(x) = 2x^3 - 2x - 5$$

$$f'(x) = 6x^2 - 2$$

$$g(x) = x - \frac{f(x)}{f'(x)}$$

$$= x - \frac{2x^3 - 2x - 5}{6x^2 - 2}$$

$$= \frac{6x^3 - 2x - 2x^3 + 2x + 5}{6x^2 - 2}$$

$$= \frac{4x^3 + 5}{6x^2 - 2}$$