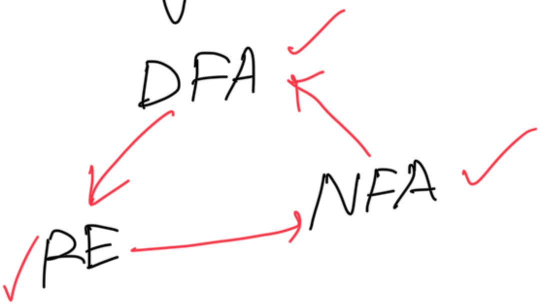
Pumping Lemma

For Regular language.



Special Property of Regular language.

All strings, with at least a certain length, in the language can be "pumped".

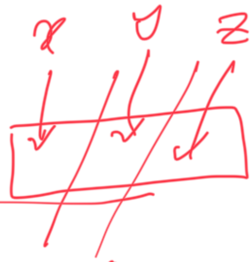
pumping length

pumping length P = The number of states in the DFA.

DFA 5 → $p = 5$

Lemma:

If A is a regular language, then there is a number P (pumping length) where if 's' is any string in A of length at least P, then 's' may be divided into "three" pieces, $s = xyz$, satisfying the following conditions,



1. for each $i \geq 0$, $xy^iz \in A$
2. $|y| > 0$
3. $\underline{|xy| \leq P}$

$y \neq$ " "

$i=1$ $\quad xyz = s \in A$

$i=2$ $\quad s' = xyyz \in A$

$i=0$ $\quad s'' = xz \in A$

*

1. for each $i \geq 0$, $xy^iz \in A$

RL $\longrightarrow$ DFA $=$ 5 $\longrightarrow$ $\underline{q_1, q_2, q_3, q_4, q_5}$

$P = 5$,

$S = s_1 s_2 s_3 s_4 s_5$

states : q_1 q_2 q_2 q_4 q_3 q_5

Seq of states → q_1 q_2 q_3 ...

$\underline{s_1}$ $\underline{s_2}$ $\underline{s_3}$ $\underline{s_4}$ $\underline{s_5}$

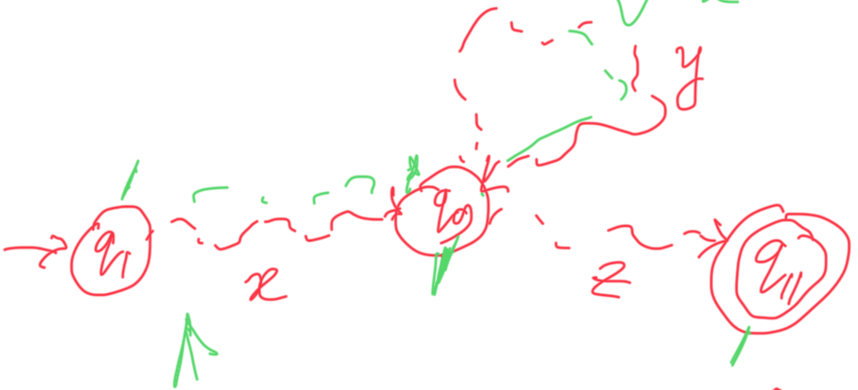
RL, DFA = 11 ↪ $q_1, \ldots\ldots, q_{11}^*$

$s = xyz$

$P = 11$

$S = s_1 s_2 \ldots\ldots s_{11}$

Seq of state : $\underbrace{q_1 \ldots\ldots q_9}_{x} \underbrace{\ldots\ldots q_9}_{y} \underbrace{\ldots\ldots q_{11}^*}_{z}$



$i=1$ $S = xyz \in A$

$i=2$ $S' = xyyz \in A$

$i=3$ $S'' = xyyyz \in A$

$i=0$ $S''' = xz \in A$

② $|y| > 0$

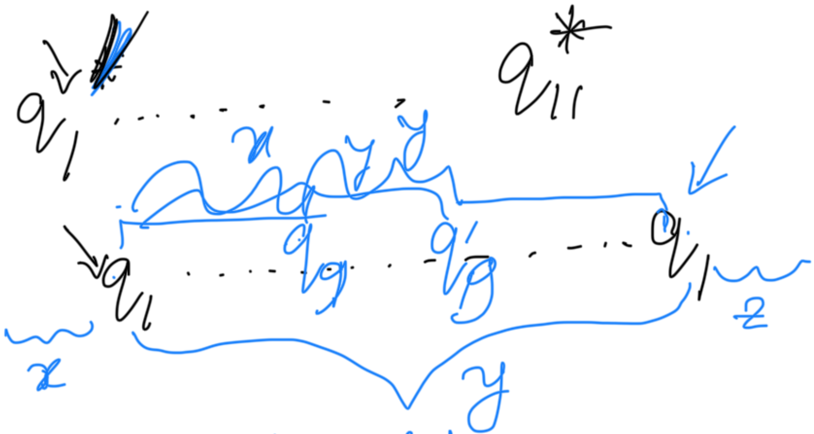
y → minimum 1 length

Seq of state : $\underbrace{q_1 \ldots\ldots q_9}_{x} \underbrace{q_9 \ldots q_9}_{y} \underbrace{\ldots\ldots q_{11}^*}_{z}$

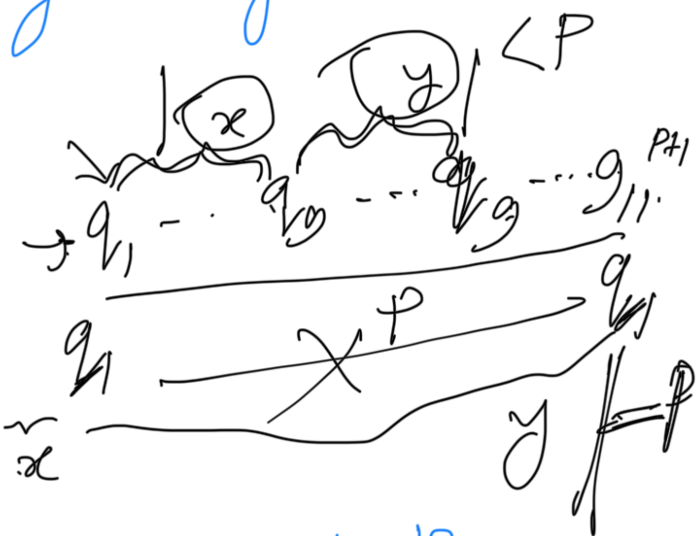
minimum length of $y = 1$.

③ $|xy| \le P$

seq of states:



x and z can be empty string.



highest length $|xy| = P$

lowest length $|xy| = 1$

$x = 0$
$y = 1$

$S = xyz$

$S = x'y'z'$

We can state the Pumping lemma as a

we statement like this:

for $\forall$ long strings ω (length at least P), $\exists\, xyz$, $\omega = xyz$, such that $\forall i$, $xy^iz \in L$

Negation of the statement of pumping lemma:

*You need to prove this to show the contradiction

$\exists$ long string ω, $\forall xyz$ $\omega = xyz$, $\exists i$, $xy^iz \notin L$

$\boxed{S} = x'y'z' \rightarrow i=2 \quad x'y'y'z' \notin L$

$S = xyz \rightarrow i=4 \quad xyyyyz \notin L$.

Examples:

$L = \{\underline{0^n1^n},\ n \geq 0\}$. Prove that L is not a regular language.

Let L is a regular Language.

Pumping Length $= P$

1. for each $i \geq 0$, $xy^iz \in L$
2. $|y| > 0$
3. $|xy| \leq P$

Now we are going to select such a strings in L that can not be pumped, to prove the contradiction.

$S = 0^P 1^P$, length $= 2P$

$\underbrace{0^P 1^P}_{xyz}$

$$\underbrace{\overbrace{0}^{x}\ \overbrace{00}^{y}\ \ldots\ldots 00}_{P}\ \underbrace{111 \ldots\ldots 11}_{P}$$

z

x consists of some 0's

y " " " 0's

$i = 1$, $S = xyz \in L$

$|y| > 0$

$\boxed{i = 2}$ $S' = xyyz$

P $|y|$, P

$|y| > P$

$= 0^{p} 0^{\ldots} 1$

$= \underline{0^{p+|y|} 1^{p}} \notin L$ $\quad p+|y|$

$\therefore$ L is not regular.

$L = \{ \underline{ww},\ w \in \{0,1\}^* \}$ $\qquad w = 110$

$\qquad 110110$

L $\longrightarrow$ regular

pumping length $\longrightarrow$ P

Now we are going to select such a strings in L that can not be pumped to prove the contradiction.

$w = \boxed{0^{P} 1}$

$s = ww$

$= 0^{P} 1 0^{P} 1 \qquad \text{length} = \underline{2P + 2}$

$\underbrace{000\ldots\ldots0}_{P}\ \underbrace{1\ 000\ldots\ldots01}_{P+2}$

$x \longrightarrow$ some 0's

$y \longrightarrow u$ 0's

$\cup$

$i = 1 \quad s = xyz \in L$

$i = 2 \quad s' = xyyz$

$= 0^p 0^{|y|} 1 0^p 1$

$= \underbrace{0^{p+|y|} 1}_{w} \neq \underbrace{0^p 1}_{w} \notin L$

$\cancel{w = 0^p}$

$s = 0^p 0^p$ Can not select this string

$$\overbrace{\underbrace{\overbrace{000 \ldots\ldots 0}^{x}\overbrace{0}^{y}}_{p}}\ \overbrace{\underbrace{000 \ldots 00}_{p}}^{z}$$

xyz

$\hookrightarrow \cancel{i}$

$xy^iz \notin L$

$i = 1 \quad s = xyz \in L$

$i = 2 \quad s' = \underline{x}\,\underline{y}\,\underline{y}z$

$= \underbrace{0^{p-2}\, 00}\, 00\, 0^p$

$= \underbrace{0^p}\, 0\, \underbrace{0\, 0^p}$

$= \underbrace{0^{p+1}}_{w} \underbrace{0^{p+1}}_{w} \in L$

$i = 3$

$s'' = xyyyz = 0^{p-2}\underbrace{000000}\,0^p$

$= 0^{p-2} \ldots$

$= 0^{p+2}\, 0^{p+2} \notin L$

So for $x = 0^{p-2}$, $y = 00$ and $z = 0^p$ there is no i for which $xy^iz \notin L$.

So we cannot prove the statement using this string. Need to choose the long string carefully.