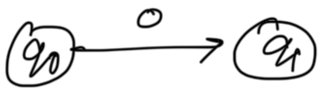
Non - deterministic Finite Automata

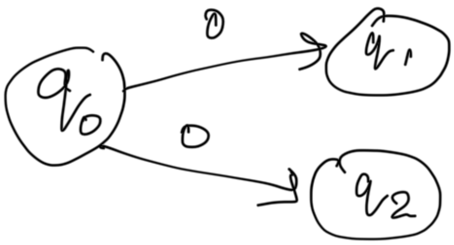
* For any particular input symbol, the machine can move to multiple state.

Previously,



This is not the case fore NFA. For getting 0, there might be multiple transitions.

* At the same time, we don't have to show transition fore all input symbols.

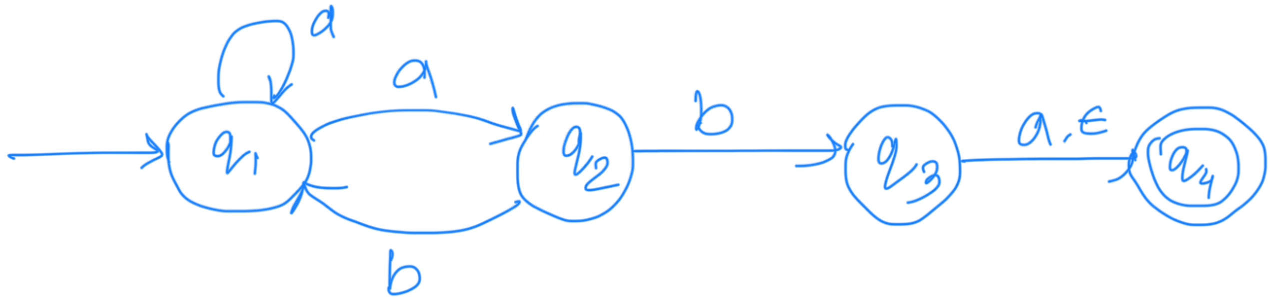


* We can go to other state without giving any input.

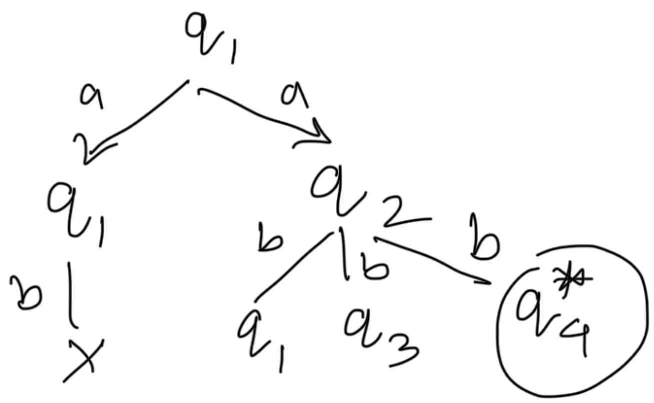
$q_0 \longrightarrow q_1, q_2$

* However, NFA can't be used for machine.
* We use NFA for $\boxed{\text{design purpose}}$.
* To use this for machine we need to convert the NFA to DFA.

Example

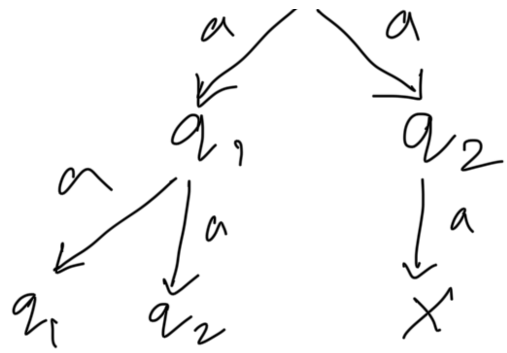


input: ab

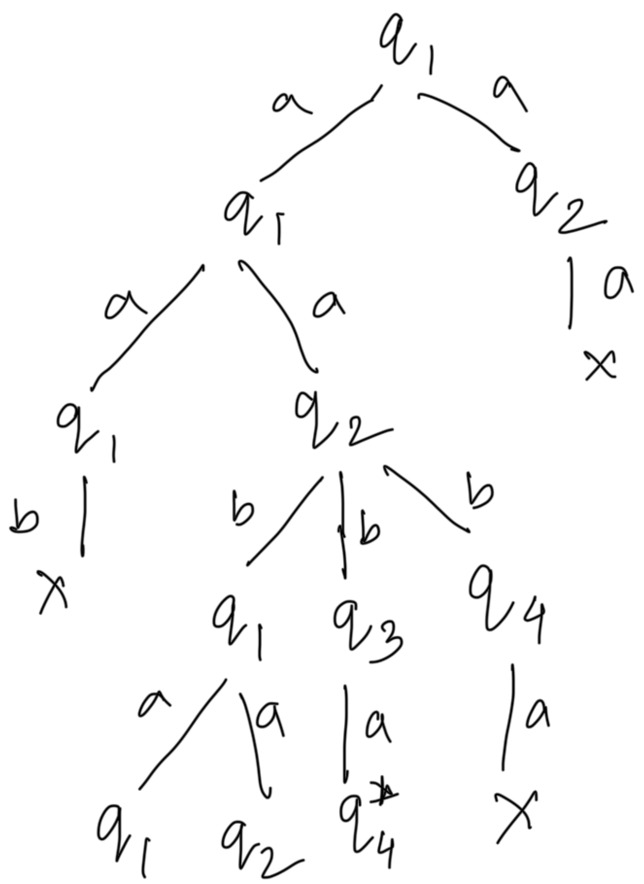


input: aa

q_1



input : aaba

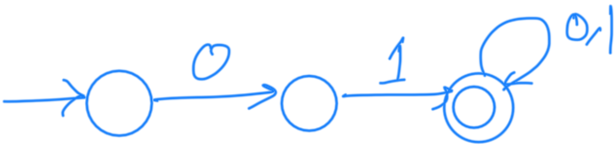


* NFA has some choices. From those choices it always selects or makes the right choice.

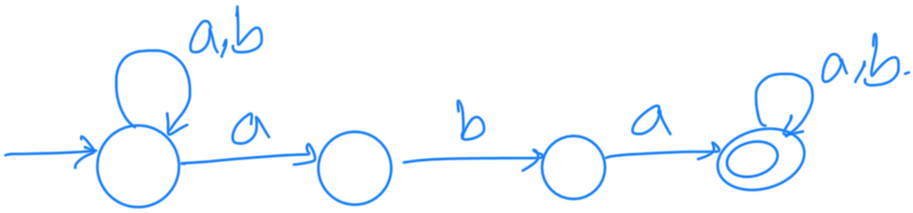
↳ You may ask, if there exists an accepting choice, how does the NFA makes the right choice? $\Longrightarrow$ It simples does that.

Some Examples

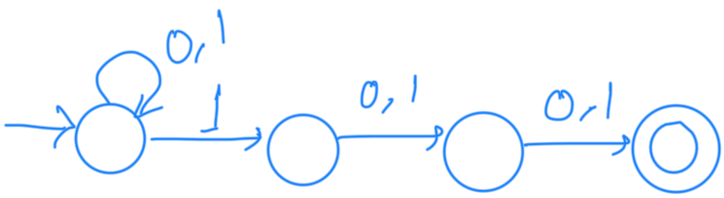
1. $L = \{ w \in \{0,1\}^* :$ w starts with 01 $\}$



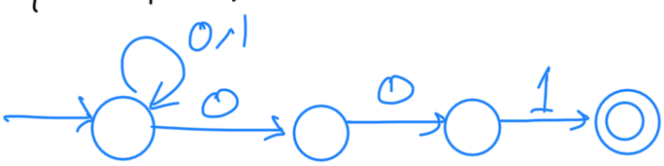
2. $L = \{ w \in \{a,b\} :$ w contains 'aba' as a substring $\}$



3. $L = \{ w \in \{0,1\}^* :$ The 3red last symbol in w is 1 $\}$

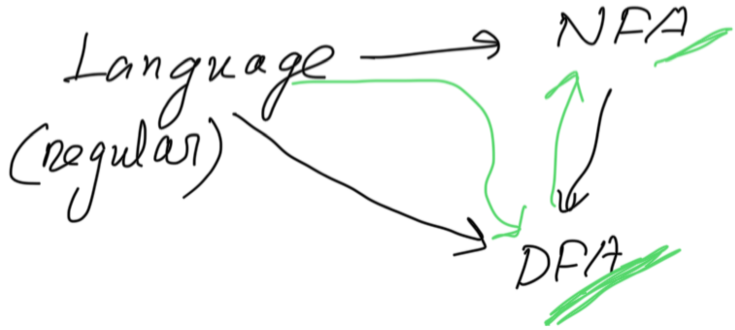


4. $L = \{ w \in \{0,1\}^* :$ w ends with 001 $\}$



Every NFA has an equivalent DFA.

NFA to DFA



power set

set $\rightarrow$ n elements

powerset $\rightarrow$ 2^n "

$\{\_\ \_\}$

$\{a, b\} \rightarrow 2^2 = 4$

a: T / F

b: T / F

FF	$\{F, F\}$ $\leftarrow$	$\{\ \}$	$\rightarrow$	$\{\_, \_\}$
TF	$\{T, F\}$ $\leftarrow$	$\{a\}$	$\rightarrow$	$\{a, \_\}$
FT	$\{F, T\}$ $\leftarrow$	$\boxed{\{b\}}$	$\rightarrow$	$\{\_, b\}$
TT	$\{T, T\}$ $\leftarrow$	$\{a, b\}$	$\rightarrow$	$\{a, b\}$

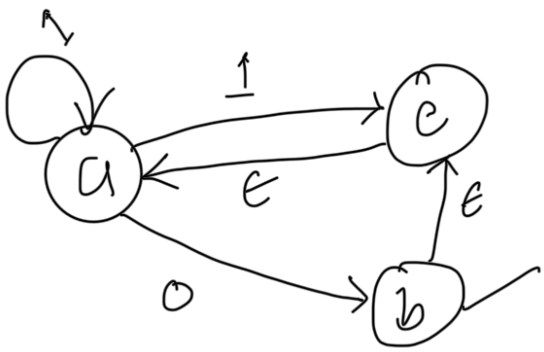
T/F T/F

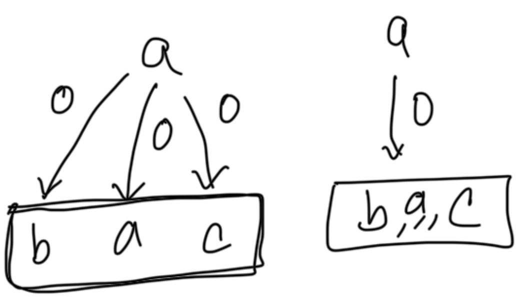
$2 \times 2 = 2^2$

$\{a, b, c\} \longrightarrow \{a, c\} \rightarrow$ TFT

$2 \times 2 \times 2 = 2^3$

0/1 0/1 0/1



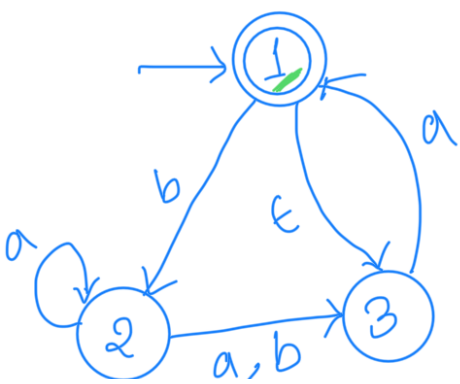


10

$a \xrightarrow{0} \boxed{b,c,a}$

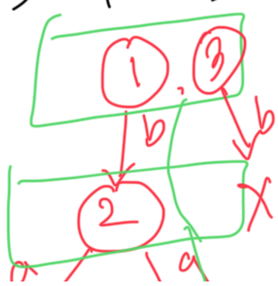
$a \xrightarrow{1} a,c$

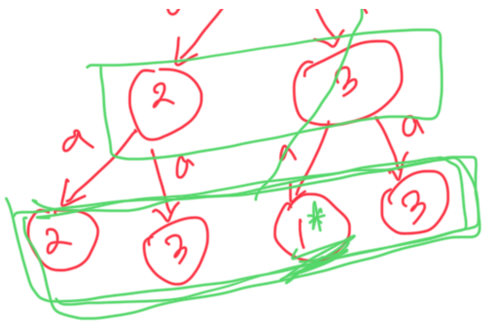
NFA



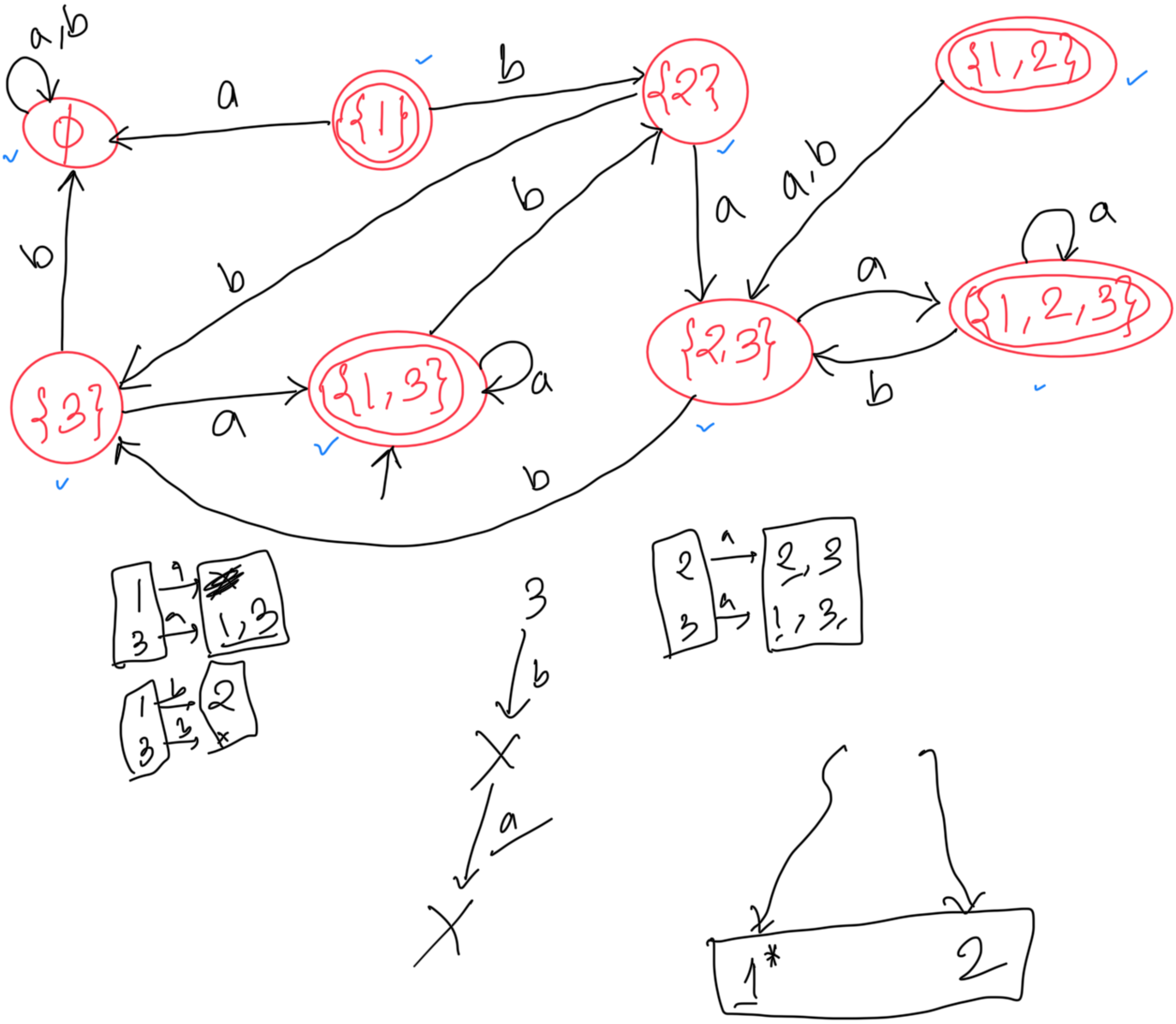
$E(1) = \{1, 3\}$

baa





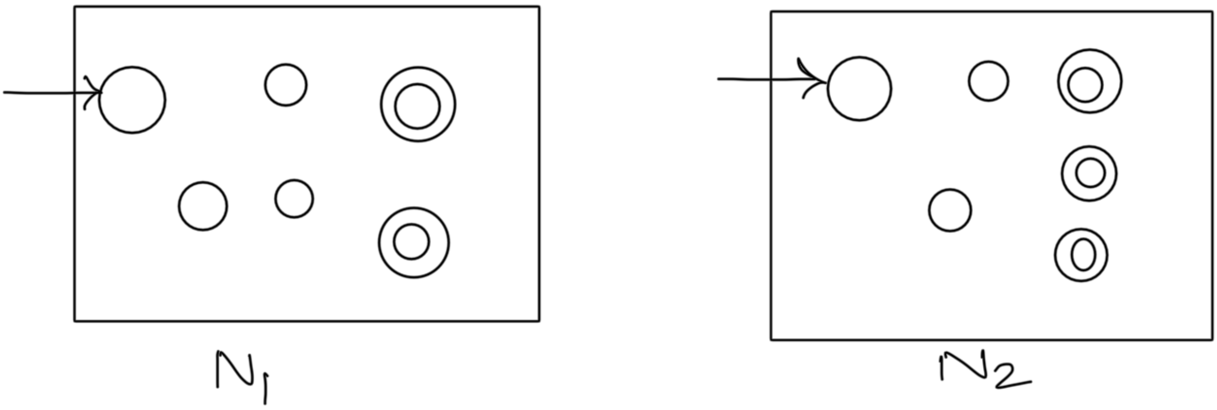
Equivalent DFA



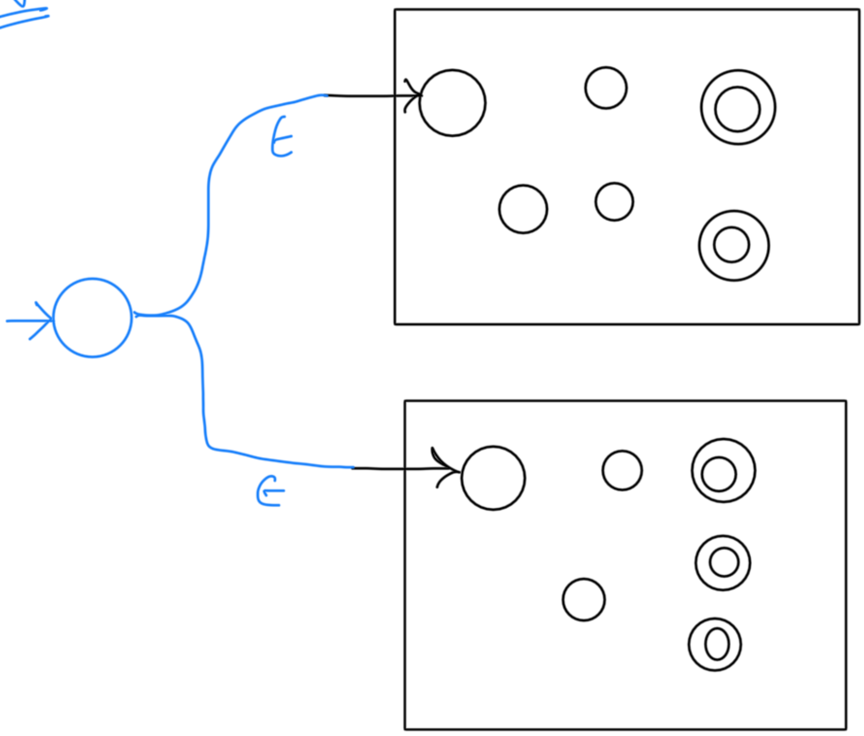
Regular Operations

Union

$$\underset{\downarrow}{L} = \underset{\downarrow}{L_1} \cup \underset{\downarrow}{L_2}$$
$$N \qquad N_1 \qquad N_2$$



$\underline{\underline{N}}$



Example

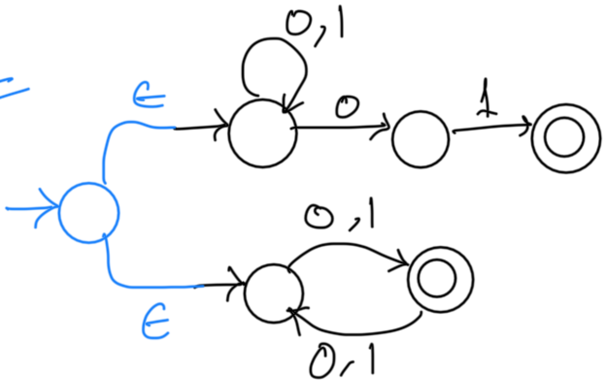
$L_1 = \{ w \in \{0,1\}^* : w \text{ ends with } 01 \}$

$L_2 = \{ w \in \{0,1\}^* : \text{length of } w \text{ is odd} \}$

$\underset{N}{L} = \underset{N_1}{L_1} \cup \underset{N_2}{L_2}$

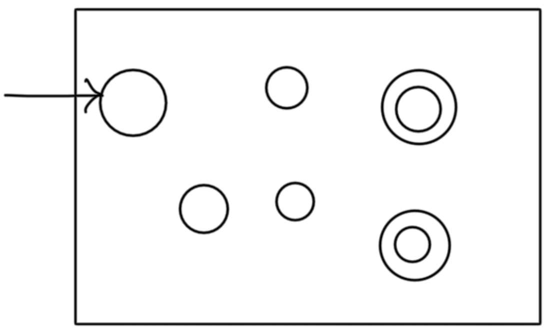
N_1 N_2

N

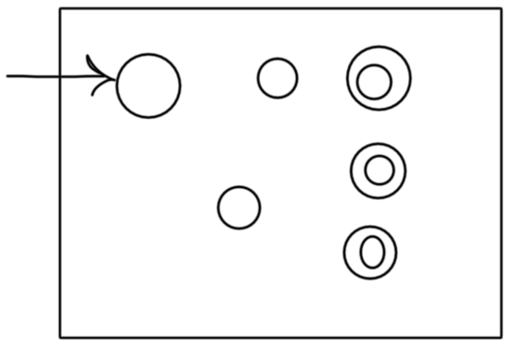


Concatenation

$$L = L_1 \circ L_2$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$N \quad N_1 \quad N_2$$

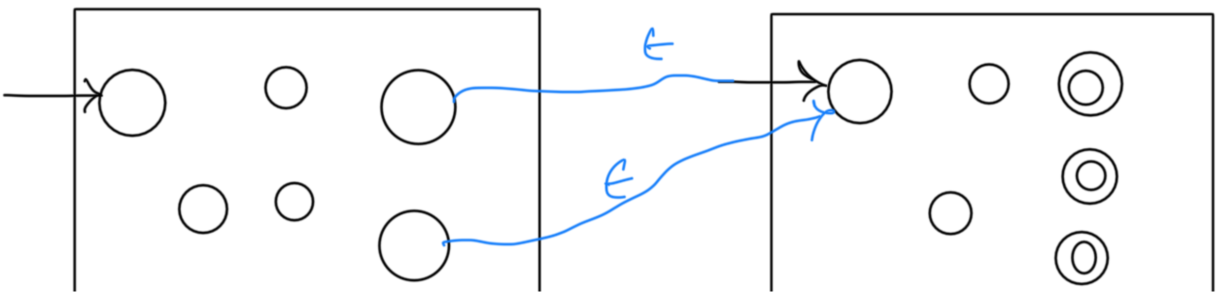


N_1



N_2

N



N_1

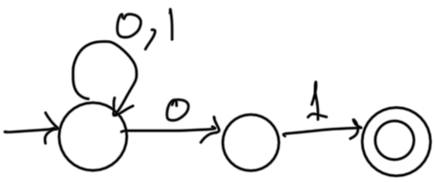
N_2

Example

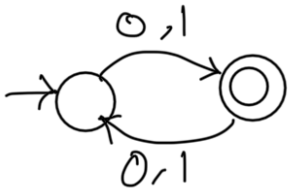
$L_1 = \{ w \in \{0,1\}^* : w \text{ ends with } 01 \}$

$L_2 = \{ w \in \{0,1\}^* : \text{length of } w \text{ is odd} \}$

$$\underset{\substack{\downarrow \\ N}}{L} = \underset{\substack{\downarrow \\ N_1}}{L_1} \circ \underset{\substack{\downarrow \\ N_2}}{L_2}$$

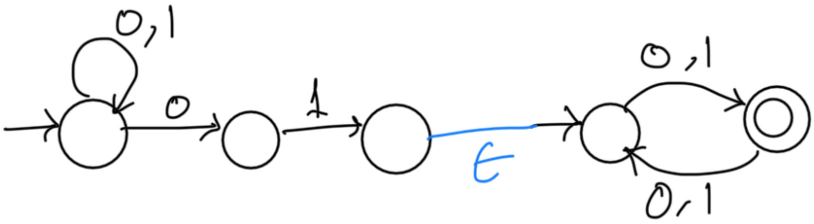


N_1



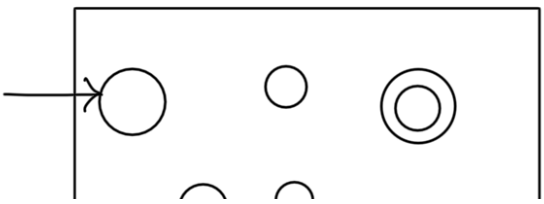
N_2

N



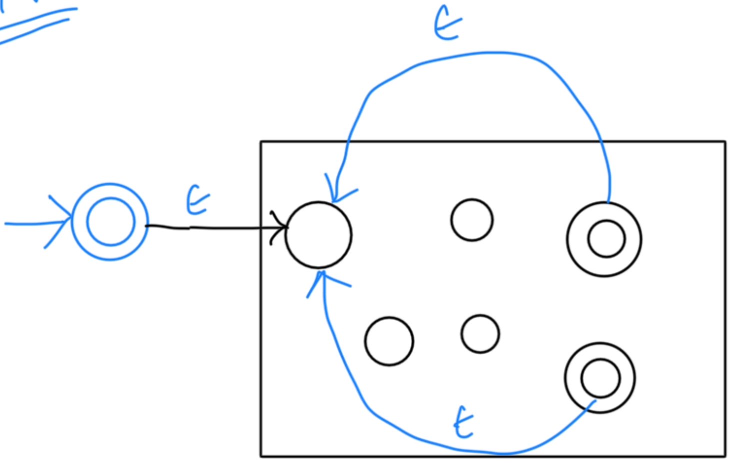
Star

$$\underset{\substack{\downarrow \\ N}}{L} = \underset{\substack{\downarrow \\ N_1}}{L_1^*}$$



N_1

N



N_1

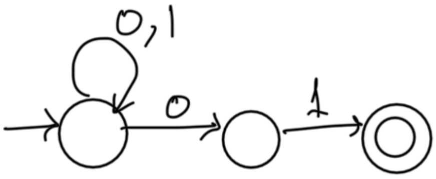
Example

$L_1 = \{ w \in \{0,1\}^* : w \text{ ends with } 01 \}$

$L = L_1^*$

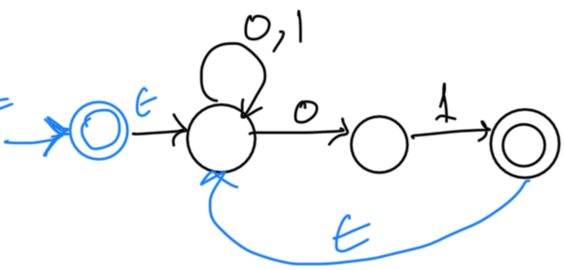
$\downarrow \quad \downarrow$

$N \quad N_1$



N_1

N



Practice Problems

$L_1 = \{ w \in \{a,b\}^* :$ number of a's in w is one more than multiple of two $\}$

$L_2 = \{ w \in \{a,b\}^* :$ w contains "abb" as a substring $\}$

1. $L = L_1 \cup L_2$
2. $L = L_1 \circ L_2 = L_1 L_2$
3. $L = L_1^*$
4. $L = L_2^*$
5. $L = L_2 \circ L_1 = L_2 L_1$