1. $L_1 = \{ w \in \{0,1\}^* : ww^R$ and w^R means w in reverse $\}$.

Let, L_1 is a regular language. So, the pumping length is P.

Let $s = 0^P 110^P \in L_1$

The length of s is $|s| = 2P+2 \geq P$. So s can be split into xyz such that $|y| > 0$, $|xy| \leq P$ and $xy^iz \in L_1$ for each $i \geq 0$. Since $|xy| \leq P$ and the 1st P characters of s are all 0's, we can conclude that y consists of only 0's. Then for $i=2$, xy^2z will be

$$xy^2z = xyyz = 0^{P+|y|}110^P \notin L_1$$

10^P is not the reverse of $0^{P+|y|}1$ as there is excess 0's. Hence L_1 is not a regular language.

2. $L_2 = \{ w \in \{a,b\}^* : w = b^n a^m, \text{ where } n > m, m \geq 0 \}$

Let, L_2 is a regular language. So, the pumping length is P.

Let $s = b^p a^{p-1} \in L_2$

The length of s is $|s| = 2p-1 \geq p$. So s can be split into xyz such that $|y| > 0$, $|xy| \leq P$ and $xy^iz \in L_2$ for each $i \geq 0$. Since $|xy| \leq P$ and the 1st P characters of s are all b's, we can conclude that y consists of only b's. Then for $i=0$, xy^0z will be

$$xy^0z = xz = b^{P-|y|} a^{P-1} \notin L_2$$

$P-|y|$ can not be greater

As $|y|>0$, so

than $p-1$ here. Hence L_2 is not a regular language

3. $L_3 = \{ \omega \in \{0,1,2,3\}^* : \omega = 1^n 0^m 3^n 2^m, \ n,m \geq 0 \}$

Let, L_3 is a regular language. So, the pumping length is P.

Let $s = 1^p 0^{p+1} 3^p 2^{p+1} \in L_3$

The length of s is $|s| = 4p+2 \geq p$. So s can be split into xyz such that $|y|>0$, $|xy| \leq p$ and $xy^iz \in L_3$ for each $i \geq 0$. Since $|xy| \leq p$ and the 1st p characters of s are all 1's, we can conclude that y consists of only 1's. Then for $i=2$, xy^2z will be

$1^{p+|y|} 0^{p+1} 3^p 2^{p+1} \notin L_3$

$xy^2z = xyyz = 1\ 0\ 3\ 2\ 4$

The count of 1's is greater than the count of 3 here. Hence L_3 is not a regular language

4. $L_4 = \{ w \in \{0,1\}^* : w = 1^n,\ \underset{\hookrightarrow 3^k}{n} \text{ is a power of three} \}$

Let, L_4 is a regular language. So, the pumping length is P.
n is a power of three.
So the string will be in this form 1^{3^k} where $k \geq 0$;
Let $s = 1^{3^P} \in L_4$
The lengths of the strings that are in L_4 will be.

1, 3, 9, 27, ...

Now the length of s is $3^P \geq P$. So we can divide s into xyz, such that $xy^iz \in L_4$ for each i.
$|y| \leq P$ and

$|y|>0$, $|xy| \le p$ and ...

Now

$$|xyz| = 3^p$$

for $i=2$, $|xy^2z| = |xyyz|$

$$= 3^p + p$$

but. $3^p + p < 3^{p+1} = 3 \cdot 3^p = 3^p + 2 \cdot 3^p$

$$\therefore \quad 3^p + p < 3^p + 2 \cdot 3^p$$

So any string that is in L_4 can not have length $3^p + p$. So $xyyz$ in not in L_4. Hence L_4 is not regular.