CFG PRACTICE SHEET

AUTOMATA & COMPUTABILITY (CSE331)

[Give a context-free grammar for each of the following languages.]

1. {w ∈ {0,1}*: 0^n1^n *or* 1^n0^n where $n \geq 0$ }
2. {starts and ends with different symbols}
3. {starts and ends with same symbols}
4. {has exactly two 1's}[SSummer23]
5. {w is a palindrome}
6. {w is not a palindrome}[Spring24]
7. {w is a valid parenthesis}
8. {w ∈ {0,1,2}*: $0^i2^j1^k$ where i=k, i,j,k≥0}
9. {w ∈ {a,b,c}*: $a^ib^jc^k$ where i+j=k, i,j,k≥0}
10. {w ∈ {a,b,c}*: $a^ib^jc^k$ where i=j or j=k, i, j, k ≥0}
11. {w is even length}
12. {w is odd length}
13. {w ∈ {0,1}*: 1^n where $n \geq 0$ }
14. {w ∈ {0,1}*: $0^i1^j0^k$ where j = i+k, $i, k \geq 0$ }
15. {w ∈ {0,1}*: 0^n1^{2n} where $n \geq 0$ }
16. {w ∈ {0,1}*: $0^{3n}1^{5n}$ where $n \geq 0$ }
17. {w ∈ {0,1}*: $0^{n+1}1^n$ where $n \geq 0$ }
18. {w ∈ {0,1,2}*: 0^n12^n where $n \geq 0$ }
19. {w ∈ {0,1,2}*: 0^n1^n2 where $n \geq 0$ }
20. {w ∈ {0,1}*: 0^n1^{n+1} where $n \geq 0$ }
21. {w ∈ {0,1}*: 0^m1^m where m ≥2}
22. {w ∈ {0,1}*: 0^m1^n where m ≥ n}
23. {w ∈ {0,1,2,3}*: $0^m1^n2^k3^j$ where k = m + n + j}

24. {w contains at least 2 0's}[Spring22]
25. {length odd in middle 0}
26. {w ∈ {0,1,2}*: $0^i2^j1^k$ where i=j or j != k , i,j,k≥0}
27. { w contains even number of 0's}
28. {w contains twice as many 1s as 0s}
29. {w contains even number of 0s and 1s}
30. { w where each 0's is followed by at least as many 1's}
31. {w ∈ {a,b}*: a^nb^m where 0 < n < m < 3n}
32. {w contains at least 4 a's}
33. {w ∈ {a,b,c,d}*: $a^nb^mc^md^{2n}$ where $n \geq 0$, $m > 0$}
34. {w ∈ {0,1}*: no of 0's and 1's in w is same}
35. {w ∈ {0,1}*: the parity of 0's and 1's are different}
36. {w ∈ {0,1}*: no of 0's and 1's in w is different}
37. {w starts with 0 and length of w is even}[Spring24 only 2nd part]
38. {every second letter in w is b}
39. {the length of w is divisible by 3}[Spring24]
40. {the length of w is more than multiple of 5}
41. {w ∈ {0,1}*: 0^n1^{2n+3} where $n \geq 0$ }
42. {w ∈ {0,1,2}*: $0^i1^j2^k$ where $j \geq 2i + 3k$ and $i, k \geq 0$ }
43. {w ∈ {0,1,2,3}*: $0^i1^j2^k3^m$ where i=m, j≥ 3k+2 and $m, k \geq 0$ }
44. {w ∈ {0,1,2}*: $0^i1^j2^k$ where $i > 2j + 3k$ and $j, k \geq 0$ }
45. {w = $0^{3i}v2^{2k}$, v ∈ { at least two 0's} and i≥0}[Spring22]
46. {w ∈ {a,b}*:w starts with even no of a's}
47. {w ∈ {a,b}*:w starts with odd no of b's}
48. {w ∈ {0,1,2}*: $1^i02^j1^k$ where i=k, i,j,k≥0}[Fall22]
49. {w ∈ {0,1,2}*: $1^i02^j1^k$ where k=i+2j, i,j,k≥0}[Fall22]
50. {w ∈ {0,1,2}*: $a^i\#b^j$ where i is a multiple of three, i,j,k≥0}[spring23]
51. {w ∈ {0,1,2}*: $a^i\#b^j$ j=2+3i, i,j,k≥0}[spring23]

[Convert the following Regular expressions to a CFG]

1. a(b|c*)
2. 0*1(0 + 1)*
3. (a + b)*(a* + (ba)*)
4. (a+b)* aa (a+b)*
5. a* + a(a|b)*
6. 0(0+1)*1 + 1(0+1)*0
7. (ab+bc) bb (ca+cc)*+(aa)*
8. a*(ba+bb)*+# [Fall23]
9. a(ba)*+#a*b [Fall23]

[Give leftmost and rightmost derivations for the following strings and check parse-tree ambiguity.]

1. *Give a leftmost derivation for string w.*
2. *Sketch the parse tree for the (1) leftmost derivation (w) string.*
3. *If it is ambiguous or not?*
4. *Make a six length string which has only one parse tree.*

a. S → 0A | 1B
   A → 0AA| 1S | 1
   B → 0S |1BB| 0
   String: 001101

b. S → A 1 B
   A → 0A | ε
   B → 0B |1B| ε
   Strings: 10100, 0010101

b. S → A 1 B
   A → 0A | ε
   B → 0B |1B| ε
   Strings: 10100, 0010101

c. D → TL
T → c | Tc
L → L.V | V
V → a | b | 0 | 1| Va | Vb | V0 | V1
Strings: cabb0011.ab1 (Rightmost derivation)

d. S → S + S| Sa | Sb | ε
String: abb + aab + baba

e. S → SA | ε
A → aa | ab | ba | bb
String: aabbba

f. S → aEbS
S → aEbScS | ε
E → d
String: adbadbadbc

g. 00010111
Where S→ ASB|SS|SAS|A
A → ASS|BS|B
B → 00|11|01|10|

h. 001111
Where A → A1|0A1|01