Ambiguity

same string ← way 1, way 2 → different ways to generate a string from a CFG

different parse tree ✓
different derivation ✗

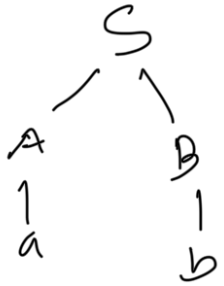
Derivation ← leftmost derivation, right most derivation

$S \to AB$
$A \to a$
$B \to b$

→ CFG
string : ab

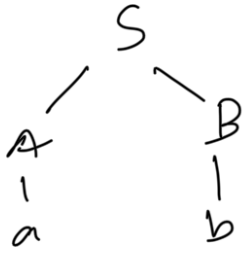
Leftmost derivation

$S \to AB \to aB \to ab$



Rightmost derivation

$S \to AB \to Ab \to ab$



different parse trees
different leftmost derivations
→ The CFG is ambiguous

Example 1

$$S \longrightarrow S+S \mid S*S \mid a \mid b$$

Given the context free grammar, answer the following questions.

a) Give a leftmost derivation for the string $a+a*b$

b) sketch a parse tree corresponding to the derivation you gave in (a)

c) Give a rightmost derivation for the string $a+a*b$.

d) sketch a parse tree corresponding to the derivation you gave in (c)

e) Demonstrate one more parse tree (apart from the one you already found in

(b) & (d)

$S \rightarrow S+S \mid S*S \mid a \mid b$

a) Leftmost derivation of $a+a*b$ (1: +, 2: *)

$S \rightarrow S+S$

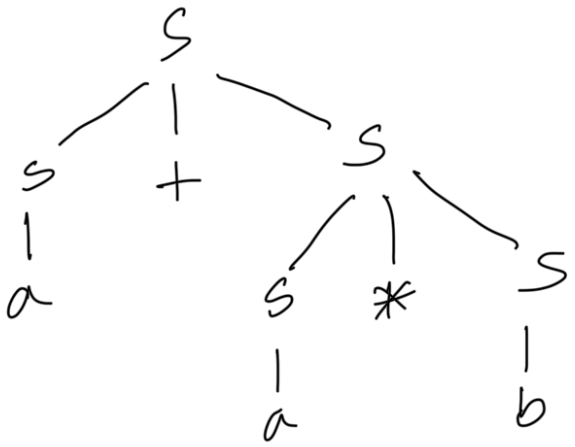
$\rightarrow a+S$

$\rightarrow a+S*S$

$\rightarrow a+a*S$

$\rightarrow a+a*b$

b)



c) Rightmost derivation of $a+a*b$ (1: +, 2: *)

$S \rightarrow S+S$

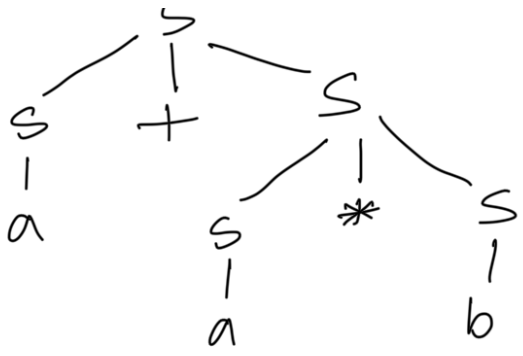
$\rightarrow S+S*S$

$\rightarrow S+S*b$

$\rightarrow S+a*b$

$\rightarrow a+a*b$

d)



e) $a \underset{2}{+} a \underset{1}{*} b$

$S \rightarrow S*S$

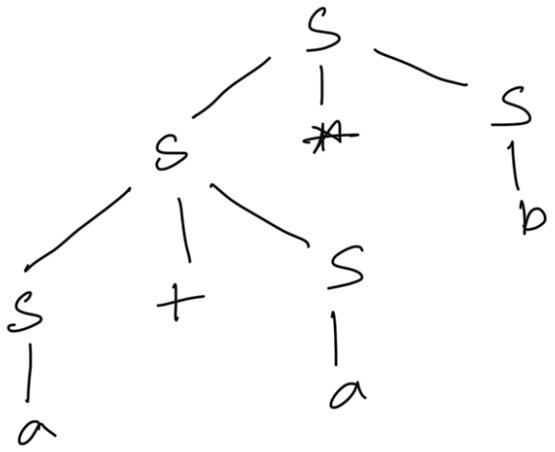
$\rightarrow S+S*S$

$\rightarrow a+S*S$

$\rightarrow a+a*S$

$\rightarrow a+a*b$

} leftmost derivation



Example 2

$S \rightarrow BIB$

$B \rightarrow OB \mid 1B \mid \epsilon$

Unambiguous grammar

$0^*1\,(0+1)^*$

$S \rightarrow A1B$

$A \rightarrow 0A \mid \epsilon$

$B \rightarrow 0B \mid 1B \mid \epsilon$

... grammar, answer the

Given the context free grammar, answer the following questions.

a) Give a leftmost derivation for the string 0101

b) sketch a parse tree corresponding to the derivation you gave in (a)

c) Demonstrate one more parse tree (apart from the one you already found in (b)) for the string 0101

d) Find a string w of length six such that w has exactly one parse tree in the grammar above.

e) Design an unambiguous grammar for the language represented by the given ambiguous grammar.

$S \rightarrow B1B$

$B \rightarrow 0B \mid 1B \mid \epsilon$

a) 0101

Leftmost derivation

$S \rightarrow B1B$

$\rightarrow 0B1B$

$\rightarrow 0\epsilon 1B$

$\rightarrow 01B$

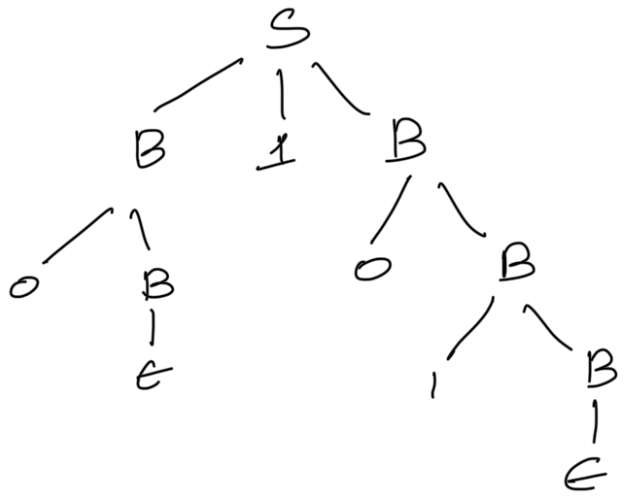
$\rightarrow 010B$

$\rightarrow 0101B$

$\rightarrow 0101\epsilon$

$\rightarrow 0101$

b)



c) 0101

Another leftmost derivation

$S \rightarrow B1B$

$\rightarrow 0B1B$

$\rightarrow 01B1B$

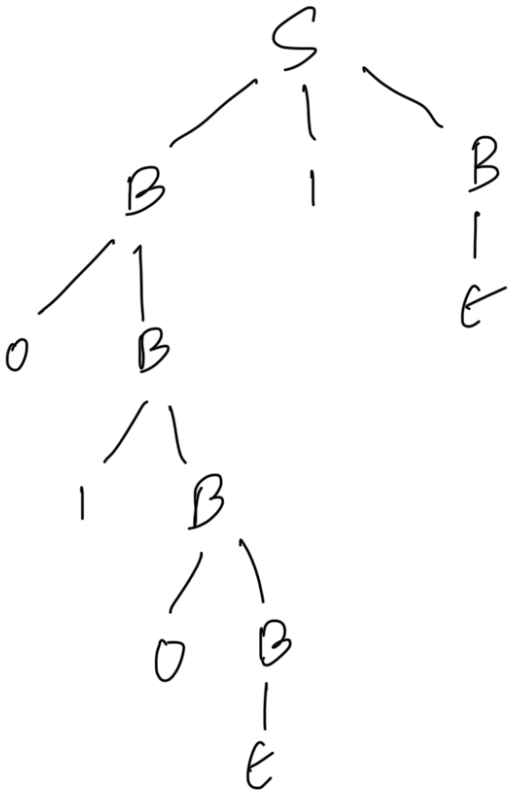
$\rightarrow$ 0 1 0 B 1 B

$\rightarrow$ 0 1 0 ϵ 1 B

$\rightarrow$ 0 1 0 1 B

$\rightarrow$ 0 1 0 1 ϵ

$\rightarrow$ 0 1 0 1 .



d)

0 0 0 1 0 0 0 1

1 0 0 0 0 0

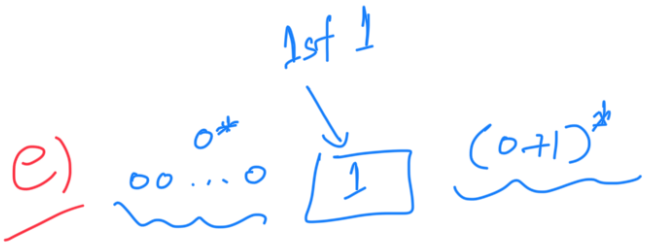
0 1 0 0 0 0

0 0 1 0 0 0

0 0 0 1 0 0

0 0 0 0 1 0

0 0 0 0 0 1

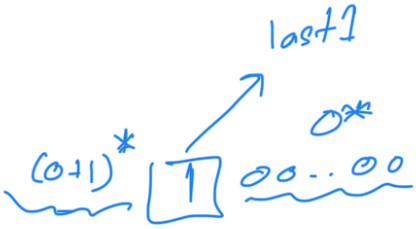
e) 

$S \rightarrow A1B$

$A \rightarrow 0A \mid \epsilon$

$B \rightarrow 0B \mid 1B \mid \epsilon$

or



$S \rightarrow B1A$

$A \rightarrow 0A \mid \epsilon$

$B \rightarrow 0B \mid 1B \mid \epsilon$