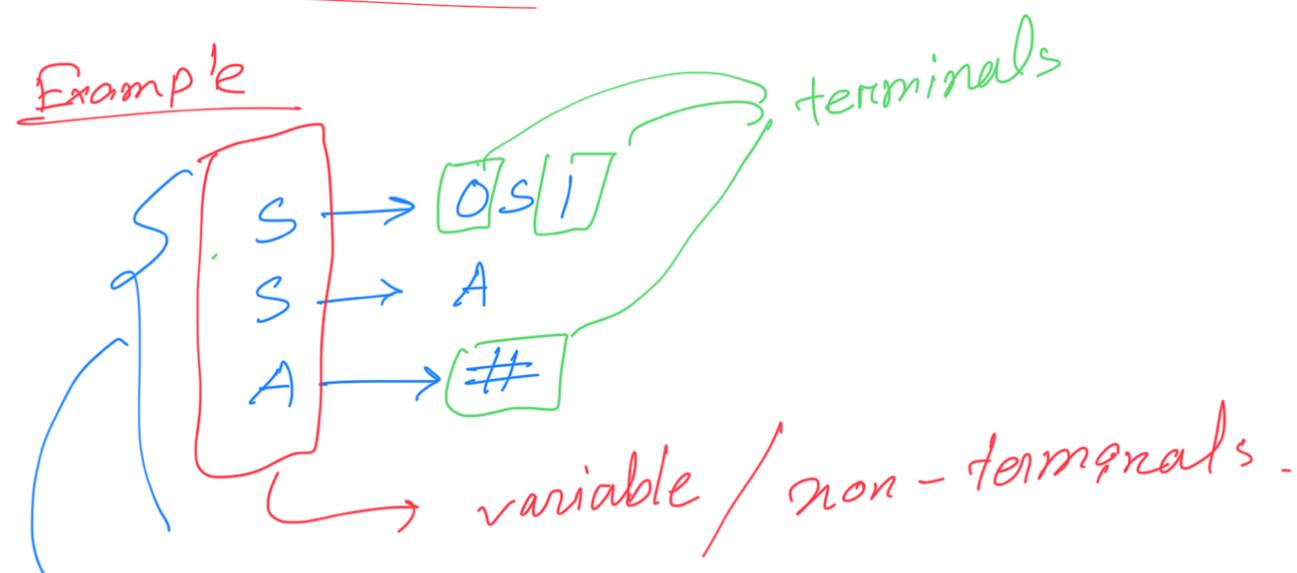
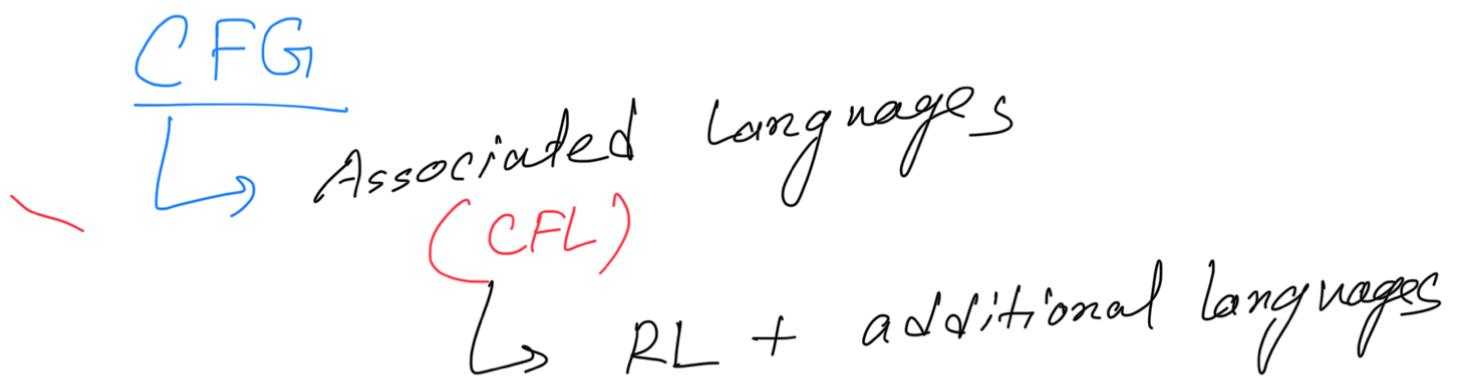
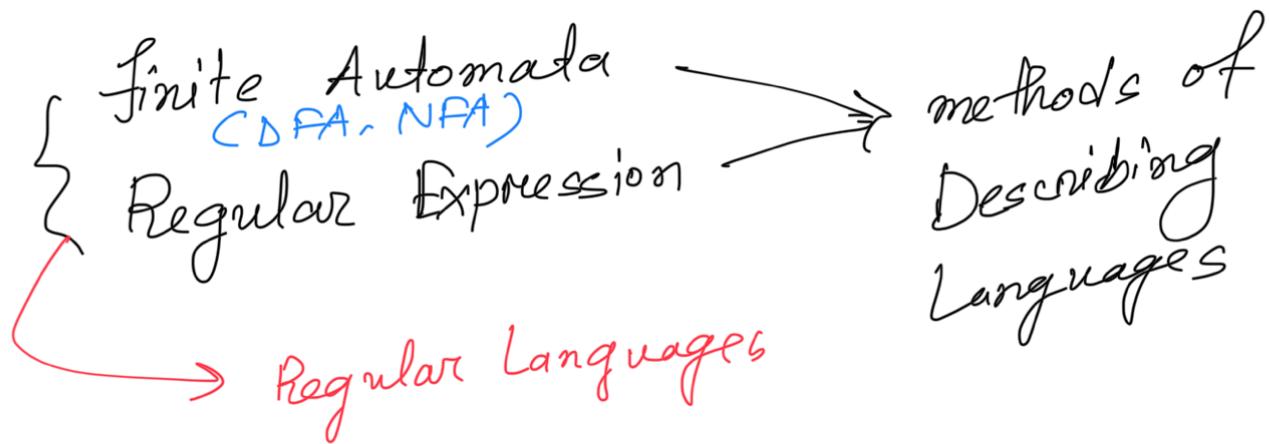
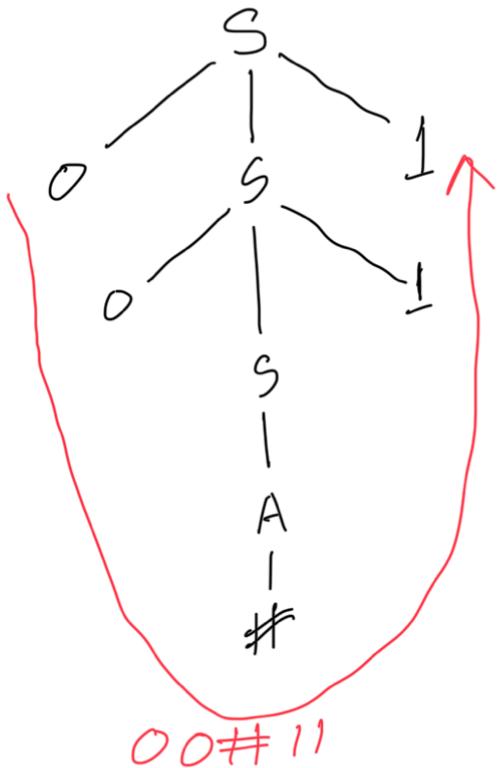


## Context Free Grammar



↙  
00#11

parse tree



Derivation

$$S \Rightarrow OS1 \Rightarrow 0OS11 \Rightarrow 00A11$$

↓  
00#11

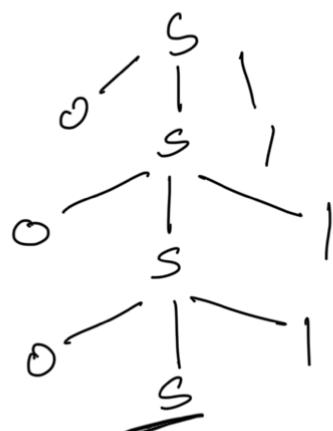
Example 1

$L = \{ w \in \{0,1\}^*: w = 0^m 1^n, \text{ where } m=n, m \geq 0 \}$

ε    01    0011    000111

✓ S → OS1  
S → ε

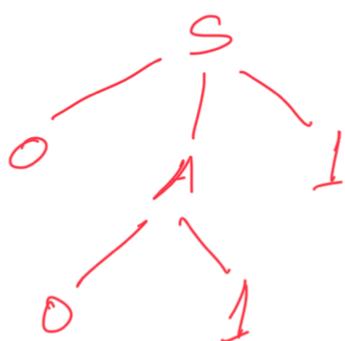
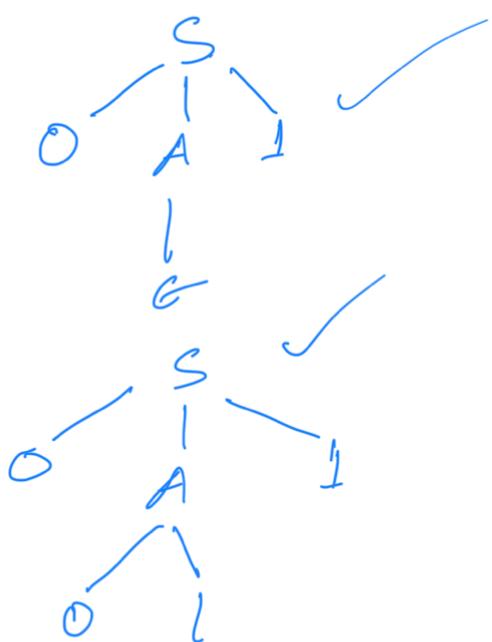
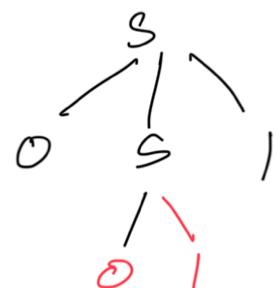
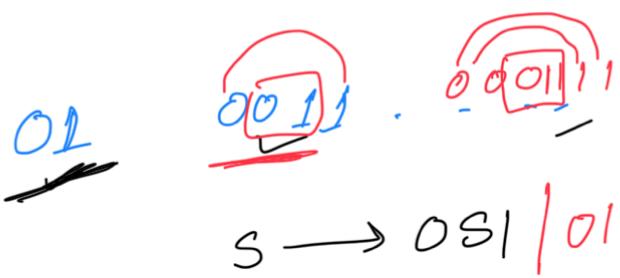
✓ S → OS1 | ε



Example 2

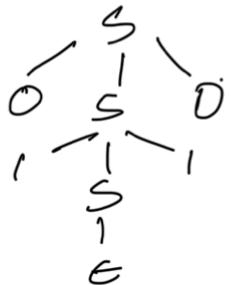
?

L = {w ∈ {0,1}\*: w = 0<sup>m</sup>1<sup>n</sup>, where m=n, m ≥ 1}



### Example 3

$L = \{ w \in \{0, 1\}^*: w \text{ is a valid palindrome} \}$



$S \rightarrow 0 \underline{S} 0 \mid 1 \underline{S} 1 \mid 0 \mid 1 \mid \epsilon$

Annotations: "even length" points to the middle symbol "0" in "0110". "odd length" points to the middle two symbols "11" in "0110".

### Example 4

$L = \{ w \in \{0, 1\}^*: w = 1^n, \text{ where } n \geq 0 \}$

$S \rightarrow \underline{1} \underline{S} \mid \epsilon$

$S \rightarrow \underline{S} \underline{1} \mid \epsilon$

$S \rightarrow 1 \underline{S} 1 \mid 1 \mid \epsilon$

Annotations: "even length" points to the terminal symbol "1" in "11". "odd lengths" points to the terminal symbol "1" in "1". "pairwise" is written below the first "1".

$L = \{ w \in \{0, 1\}^*: w = 0^n, \text{ where } n \text{ is even} \}$

$\dots \underline{n} \underline{s} \mid \epsilon \text{ or}$

$$S \rightarrow UVU^{-1}$$


---

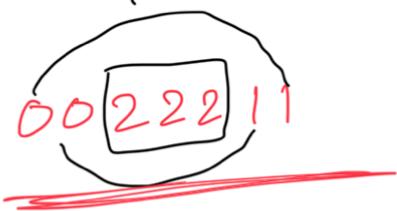

$$S \rightarrow SOO | \epsilon \quad \text{or}$$


---


$$S \rightarrow OSO | \epsilon$$

### Example 5

$L = \{ w \in \{0,1\}^*: w = 0^i 1^j 2^k; \text{ where } i = k, \text{ and } i, j, k \geq 0 \}$

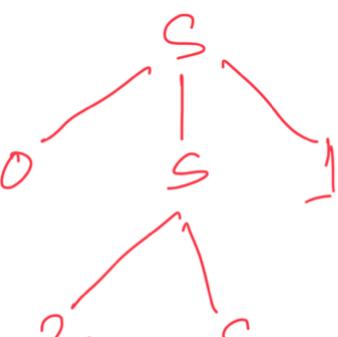


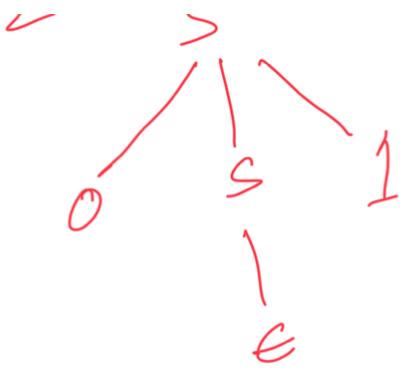
0 1 pair ✓  
Count of 2 ✓

$$S \rightarrow OS1 | A$$

$$A \rightarrow 2A | \epsilon$$

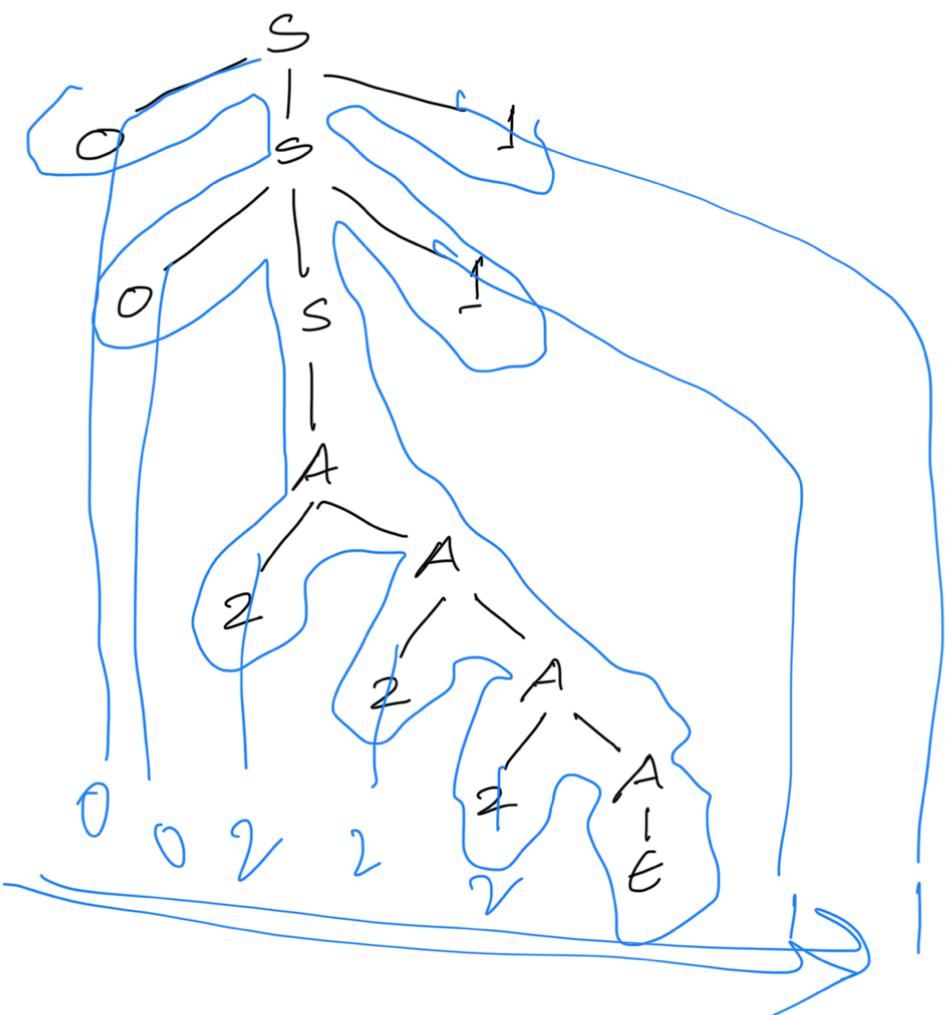
$$S \rightarrow OS1 | \underline{2S} | \epsilon \quad \times$$





02011

0022211



Example 6

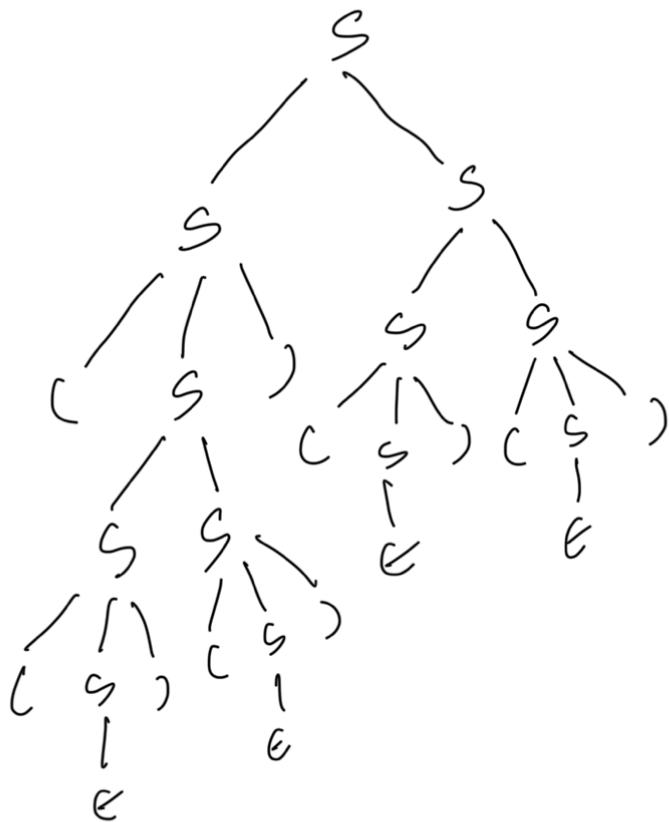
~, -, \*, .., is a valid parentheses }

$L = \{w \in \{c, )\}^*: w \text{ is } s \rightarrow (\bar{s}) \mid \bar{ss} \mid \bar{e}$

(( )) (( )) (( ))

Draw parse tree for

A handwritten diagram consisting of two sets of blue parentheses. The first set is on the left, and the second set is on the right. Each set has a horizontal red line underneath it. Above each set, there is a black arrow pointing upwards, indicating a continuation or a next step.



## Example 7

Example :  $L = \{w \in \{a, b, c\}^*: w = a^i b^j c^k \text{ where } i=j \text{ or } j=k \text{ and } i, j, k \geq 0\}$

S → X | Y ✓

7

$j=K$   $\int Y \rightarrow MN$

$$\begin{array}{c}
 i=j \\
 a^i b^j c^k \\
 \hline
 P \quad Q
 \end{array}
 \left\{
 \begin{array}{l}
 X \rightarrow PQ \\
 P \rightarrow aPb | \epsilon \\
 Q \rightarrow cQ | \epsilon
 \end{array}
 \right\}
 \begin{array}{c}
 a^i b^j c^k \\
 \hline
 M \quad N
 \end{array}
 \left\{
 \begin{array}{l}
 M \rightarrow aM | \epsilon \\
 N \rightarrow bNc | \epsilon
 \end{array}
 \right.$$

### Example 8

$L = \{ w \in \{a, b, c\}^*: w = a^i b^j c^k \text{ where } i+j=k \text{ and } i, j \geq 0 \}$

$$\begin{aligned}
 w &= a^i b^j c^k \\
 &= a^i b^j c^{i+j} \\
 &= a^i b^j c^i c^j \\
 &= a^i b^j c^i c^j
 \end{aligned}$$

$$\begin{array}{c}
 a^3 b^2 c^5 \\
 \hline
 \text{aaa} \quad \text{bbcc} \quad \text{ccc}
 \end{array}$$

$$\begin{array}{l}
 S \rightarrow aSc | X \\
 X \rightarrow bXc | \epsilon
 \end{array}$$

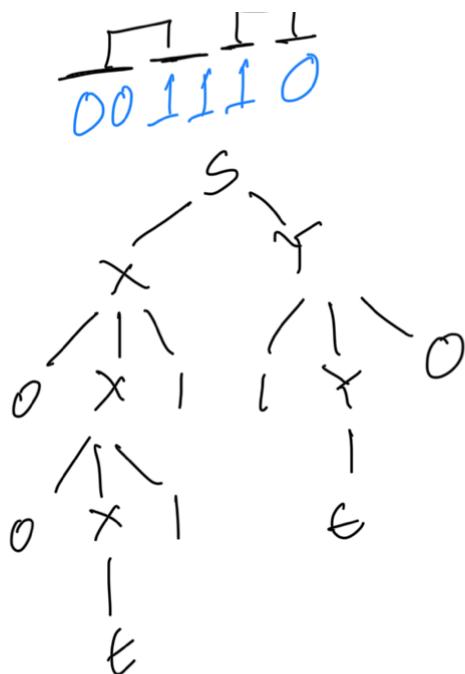
### Example 9

$L = \{ w \in \{0, 1\}^*: w = 0^i 1^j 0^k \text{ where } j = i+k \text{ and } i, k \geq 0 \}$

$$\begin{aligned} w &= 0' 1^{\circ} 0 \\ &= \overset{0}{\underset{0}{\mid}} \overset{1}{\underset{1}{\mid}} \overset{0}{\underset{1}{\mid}} i+k \quad k \\ &= \overset{0}{\underset{0}{\mid}} \overset{1}{\underset{1}{\mid}} \overset{1}{\underset{1}{\mid}} \overset{0}{\underset{0}{\mid}} k \quad k \end{aligned}$$

$S \rightarrow X \vee Y$

$x \rightarrow 0x1e$   
 $y \rightarrow 1y0e$



### Example 10

Example:  
 $L = \{w \in \{0,1\}^n : w = 0^m 1^n, \text{ where } m > n, m \geq 1 \text{ and } n \geq 0\}$

$S \rightarrow OS | X$

$$x \rightarrow ox | o$$

On /

$S \rightarrow X^Y$

$$x \rightarrow 0x10$$

$\gamma \rightarrow 0\gamma l/\ell$

$$m = n + \sum_{i=1}^k \min\{r_i, 1\}.$$

$D^m \perp n$

$$n+D \stackrel{1}{\downarrow} n$$

$\text{ONODI}^*$

D<sup>n</sup> P<sub>1</sub>

## Example 11

L = { $\omega \in \{0,1\}^*$ : the length of  $\omega$  is even}

$S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid \epsilon$

1010  
1001

$S \rightarrow 0oS \mid oIs \mid 1oS \mid 1Is \mid \epsilon$

1010  
1001

### Example 12

$L = \{\omega \in \{0,1\}^*: \omega \text{ starts and ends with different symbols}\}$

$S \rightarrow 0X1 \mid 1X0$   
 $X \rightarrow 0X \mid 1X \mid \epsilon$

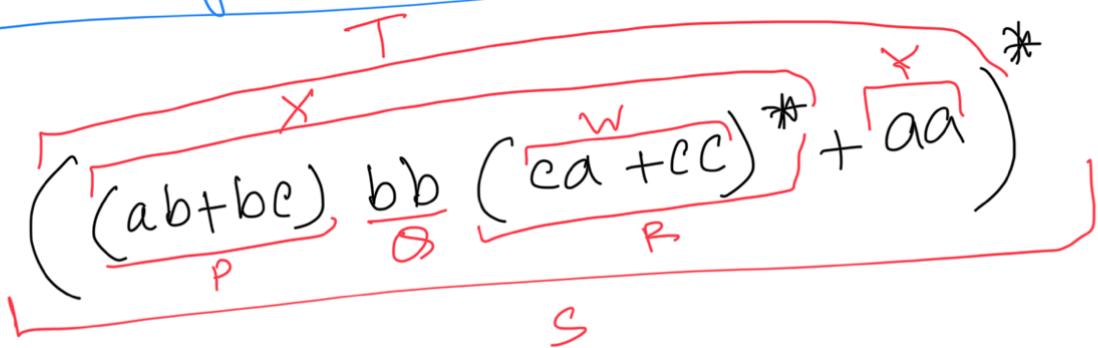
0 [ ] 1  
1 [ ] 0

$\underbrace{0 \overbrace{(0+1)^*}^M}^A 1 + 1 \underbrace{\overbrace{(0+1)^*}^M 0}^B$

$S \rightarrow A \mid B$   
 $A \rightarrow 0X1$   
 $B \rightarrow 1X0$   
 $X \rightarrow MX \mid \epsilon$   
 $M \rightarrow 0 \mid 1$

... into PFG

## Convert Regular expression into CFG



$$S \rightarrow TS \cup \epsilon$$

$$T \rightarrow X \cup Y$$

$$X \rightarrow P \cup Q$$

$$P \rightarrow ab \cup bc$$

$$Q \rightarrow bb$$

$$Y \rightarrow WR \cup \epsilon$$

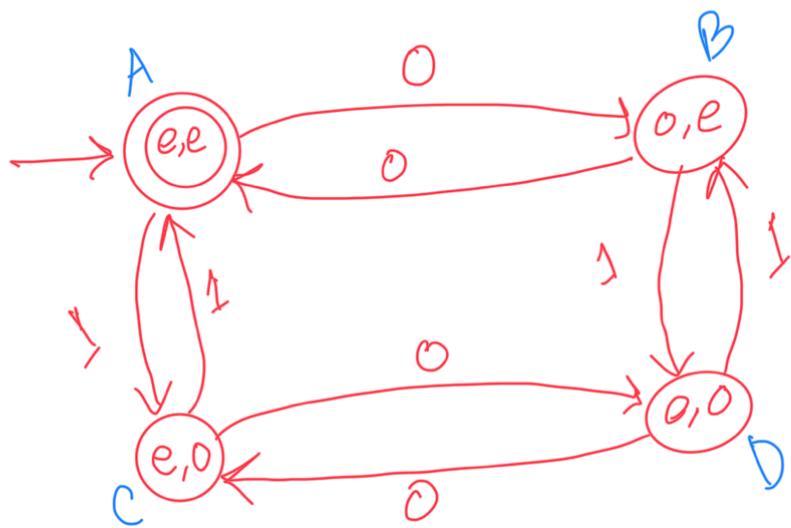
$$W \rightarrow ca \cup cc$$

$$R \rightarrow \epsilon$$

## Convert DFA into CFG

$L = \{ \omega \in \{0,1\}^*: \omega \text{ contains even number of } 0s \text{ and } 1s \}$

0, 1	
e	e
e	0
0	e
~	~



$$R_i^o \xrightarrow{x} R_j^o$$

$$R_i^o \rightarrow x R_j^o$$

if  $R_i^o$  is an accepting state  
 $R_i^o \rightarrow e$

$$S \rightarrow A$$

$$A \rightarrow oB \mid 1C \mid e$$

$$B \rightarrow oA \mid 1D$$

$$C \rightarrow oD \mid 1A$$

$$D \rightarrow oC \mid 1B$$

## Practice

1.  $L = \{w \in \{0,1\}^*: \text{the length of } w \text{ is odd and the mid is } 0\}$
2.  $L = \{w \in \{0,1\}^*: w \text{ contains even numbers of } 0s\}$
3.  $L = \{w \in \{0,1\}^*: w \text{ starts and ends with same symbol}\}$
4.  $L = \{w \in \{0,1\}^*: w \text{ contains exactly three } 1s\}$
5.  $L = \{w \in \{0,1\}^*: w \text{ starts with } 101\}$
6.  $L = \{w \in \{a,b\}^*: w \text{ ends with } "ba"\}$