

## Exercise 1

### Solution:

For transistor T to be in saturation, at least 0.5 V must be between the base & emitter. For  $D_3$  &  $D_4$  both to be on,  $0.6+0.6=1.4$  V is required. Thus, the required voltage for  $D_3, D_4$  & T to be on, at P,

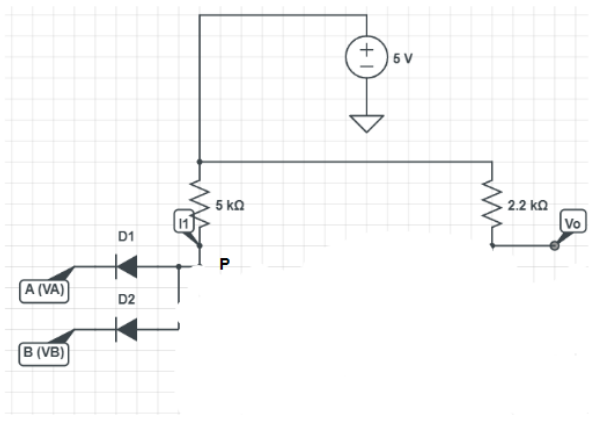
$$V_P = 0.6 \times 2 + 0.5 = 1.7 \text{ V}$$

### Case (0,0):

Assuming  $D_1$  &  $D_2$  to be on (since low voltage 0.2 V at cathode), we get,

$$V_P = 0.2 + 0.7 = 0.9 \text{ V} < 1.7 \text{ V}$$

Thus,  $D_3, D_4$  & T will be off. This leads to an open circuit-



Thus,  $V_o = 5 \text{ V}$

$$I_{5k\Omega} = \frac{5 - V_P}{5} = \frac{5 - 0.9}{5} = 0.82 \text{ mA}$$

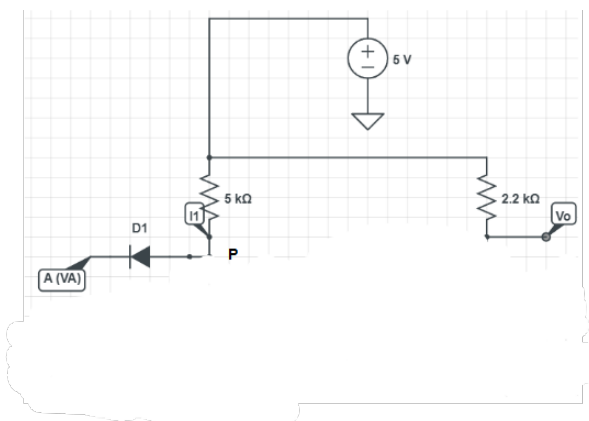
$$I_{D_1} = I_{D_2} = \frac{I_{5k\Omega}}{2} = 0.41 \text{ mA}$$

### Case (0,1):

Assuming  $D_1$  on &  $D_2$  off,

$$V_P = 0.2 + 0.7 = 0.9 < 1.7 \text{ V again.}$$

Thus,  $D_3, D_4$  & T will be off. This leads to an open circuit-



$V_o = 5 \text{ V}$

$$I_{5k\Omega} = \frac{5 - 0.9}{5} = 0.82 \text{ mA}$$

$$I_{D_1} = I_{5k\Omega} = 0.82 \text{ mA}$$

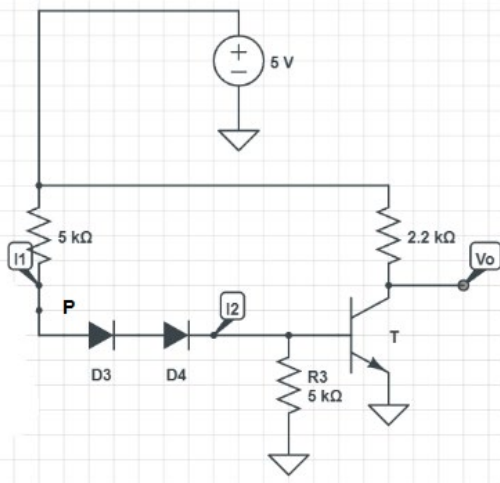
**Case (1,0):** Same as (0,1)

**Case (1,1):**

Assuming both  $D_1$  &  $D_2$  off,  $D_3$ ,  $D_4$  &  $T$  can be assumed on, since there is no bound of  $V_P$  from the input diodes. Each diode will have 0.7 V & there will be 0.8 V between the base and emitter of T.

Thus,  $V_P = 0.7 + 0.7 + 0.8 = 2.2 \text{ V}$

The circuit looks like:



$$V_o = V_{CE_T} = 0.2 \text{ V}$$

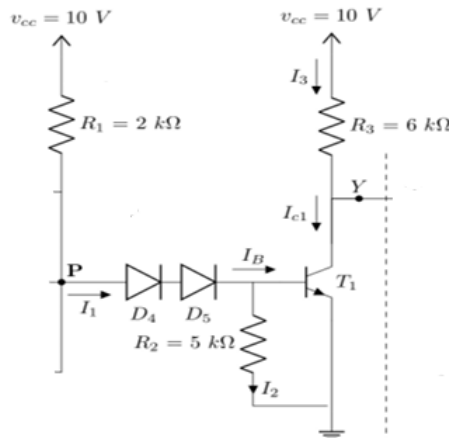
$$I_{5V} = I_1 + I_{2.2k\Omega} = \frac{5-2.2}{5} + \frac{5-0.2}{2.2} = 2.74 \text{ mA}$$

$$I_{R_3} = \frac{0.8}{5} = 0.16 \text{ mA}$$

## Exercise 2

### Solution:

a)  $D_1, D_2, D_3$  all off.  $V_P = 0.7 \times 2 + 0.8 = 2.2 \text{ V}$



$$I_{R_1} = \frac{10 - V_P}{2} = \frac{10 - 2.2}{2} = 3.9 \text{ mA}$$

$$I_2 = \frac{0.8}{5} = 0.16 \text{ mA}$$

$$\therefore I_B = I_{R_1} - I_{E1} = 3.9 - 0.16 = 3.74 \text{ mA}$$

$$I_C = I_{R_3} = \frac{10 - 0.1}{6} = 1.65 \text{ mA}$$

$$\therefore \beta_{min} = \frac{I_C}{I_B} = 0.441$$

This is the minimum value of  $\beta$  to keep T in saturation.

b) Let  $V_N$  be the noise voltage at A, that would cause the malfunction.

Thus, total voltage at input A =  $10 + V_N$

It will malfunction if  $D_A$  turns on. The marginal voltage across the diode for this situation is the cut-in voltage (0.6 V) of the diode.

Thus,  $V_{D_A} = 0.6 = V_P - (10 + V_N)$

Now, we found in (a),  $V_P = 2.2 \text{ V}$  when all inputs are high.

So,  $V_N = 2.2 - 0.6 - 10 = -8.4 \text{ V}$

Magnitude of  $V_N = |-8.4| = 8.4 \text{ V}$

c) If at least one input is low, it will cause  $D_4, D_5$  &  $T_1$  to be off.

Thus,  $V_P = 0.1 + 0.7 = 0.8 \text{ V}$  when no noise is present.

Let,  $V_N$  is the noise voltage.

Then, malfunction will happen when due to  $V_N$ ,  $V_P$  will be high enough to turn  $D_4, D_5$  &  $T_1$  on.

$$V_{P_{malfunction}} = (V_{D_1})_{cut-in} + (V_{D_2})_{cut-in} + V_{T_{1Y}} = 0.6 \times 2 + 0.5 = 1.7 \text{ V}$$

$$V_P = V_N + 0.1 + 0.7 = V_N + 0.8$$

Considering marginal condition,

$$V_P = V_{P_{malfunction}} \rightarrow V_N + 0.8 = 1.7 \rightarrow V_N = 0.9$$

### Exercise 3

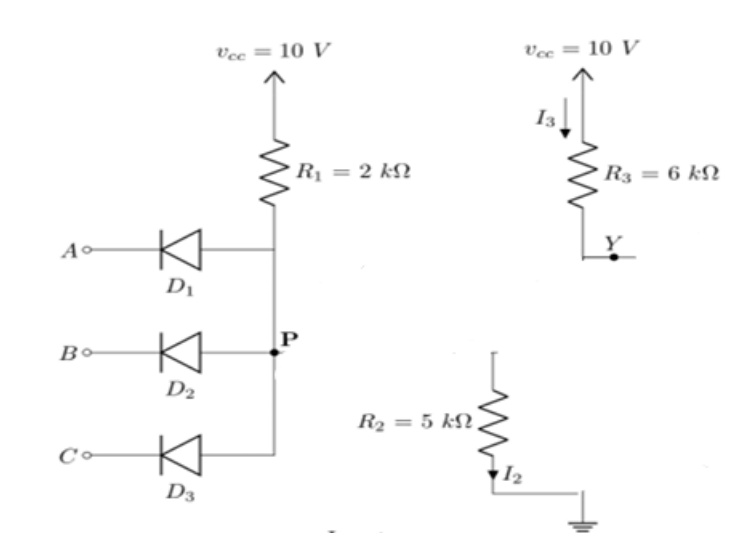
#### Solution:

- a) Case (at least one input low) [(0,0,0), (0,0,1), (0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0)]:

As we found in **Exercise 2**,

$$V_P = 0.1 + 0.7 = 0.8 \text{ V}$$

$D_4, D_5, T_1$  all off

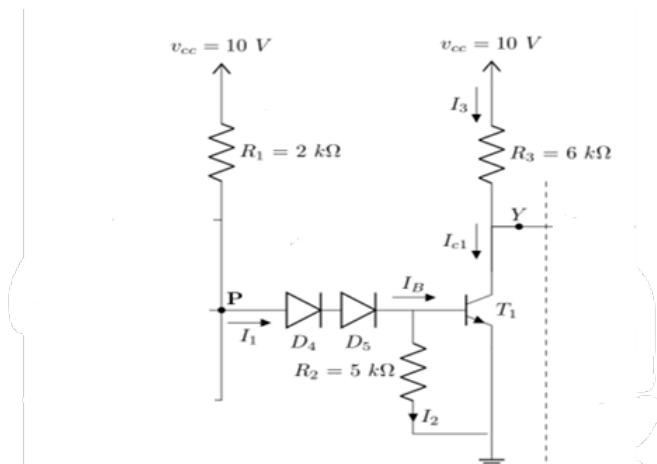


$$P = (10 - 0.1) \times I_{R_1} = 9.9 \times \frac{10 - 0.8}{2} = 45.54 \text{ mW}$$

Case (1,1,1):

From **Exercise 2**, we found that for this case,  $V_P = 2.2\text{ V}$

$D_4, D_5, T_1$  all on



$$P = (10 - 0) \times I_{R_1} + (10 - 0) \times I_{R_3} \\ = 10 \times \frac{10-2.2}{2} + 10 \times \frac{10-0.1}{6} = 55.5\text{ mW}$$

b)  $P_{max} = \max(55.5, 45.54) = 55.5\text{ mW}$

$$P_{avg} = \frac{7 \times 45.54 + 55.5}{8} = 46.785\text{ mW}$$

## Exercise 4

**Solution:**

a) **Step 1:**

Current flows from loads toward the driver circuit. Thus, the condition for maximum fanout is the marginal case, when  $T_1$  transitions from saturation to forward active.

**Step 2:**

Case (at least 1 input low at the driver):

Output  $V_Y$  is high, which turns  $D_6$  of loads off. Thus, no current enters the driver circuit.

$$\therefore \text{Fanout} = \infty$$

Case (all inputs high):

Output  $V_Y$  is low,  $D_6$  is on. KCL at Y,

$$I_{c1} = I_3 + I_R = \frac{10-0.1}{6} + \frac{10-0.8}{2} \times N = 1.65 + 4.6N$$

Here, we have considered the case (1,0,0) at all N loads connected. This is because maximum current will flow through  $D_6$  in this situation, as all other diodes &  $T_2$  will be off. The anode node voltage of  $D_6$  will be 0.8 V for the same reasons described in previous examples (0.1+0.7).

$$I_B = I_1 - I_2 = \frac{10-2.2}{2} - \frac{0.8}{5} = 3.74 \text{ mA}$$

$$\text{Now, } \beta_{forced} = \frac{I_C}{I_B} = \beta_F = 30 \rightarrow \frac{1.65+4.6N}{3.74} = 30 \rightarrow N = \text{floor} \left( \frac{3.74 \times 30 - 1.65}{4.6} \right) = 24$$

**Step 3:**

$$\therefore \text{Maximum fanout} = \min(\infty, 24) = 24$$

b) We see, the difference in this case from the previous problem is that, the load currents will add to the collector current of  $T_1$ . The load currents  $I_R$  will flow between  $V_Y$  & ground. Thus,

$$\begin{aligned} P_{max} &= (10 - 0) \times I_1 + (10 - 0) \times I_3 + (V_Y - 0) \times I_R \times N \\ &= 10 \times \frac{10-2.2}{2} + 10 \times \frac{10-0.1}{6} + (0.1 - 0) \times 24 \times \frac{10-0.8}{2} = 66.54 \text{ mW} \end{aligned}$$

## Exercise 5

**Solution:**

a) When all inputs are high,

$T_1$  in forward active mode,  $D_2$  on,  $T_2$  in saturation mode, and load diodes are off. Hence, loads are disconnected.

$$\text{Thus, } I_{C_{T_2}} = I'_1 = \frac{12-0.1}{2.2} = 5.41 \text{ mA}$$

$$I_{B_{T_2}} = i_1 - \frac{0.8}{100} = (\beta_F + 1) \times I_{C_{T_1}} - 0.008 = (30 + 1) \times \frac{V_{C_{T_1}} - 2.2}{15} - 0.008$$

Now, KCL at  $V_{C_{T_1}}$ ,

$$\frac{V_{C_{T_1}} - 2.2}{15} + 30 \times \frac{V_{C_{T_1}} - 2.2}{15} = \frac{(12 - V_{C_{T_1}})}{15} \rightarrow V_{C_{T_1}} = 2.51 \text{ V}$$

$$\text{Thus, } I_{B_{T_2}} = 0.625 \text{ mA}$$

$$\therefore \beta_{forced} = \frac{I_{C_{T_2}}}{I_{B_{T_2}}} = 8.654 < \beta_F(30)$$

**b) Step 1:**

The current flows from the loads toward the driver. Thus, the condition for maximum fanout is the marginal condition when  $T_2$  transitions from saturation to forward active.

**Step 2:**

Cases with at least one low input:

$T_1, D_2, T_2$  all off

Output is high, and all load diodes will be off.

$$\therefore \text{Fanout} = \infty$$

Case (1,1,1):

$T_1$  in forward active mode,  $D_2$  on,  $T_2$  in saturation mode

$$I_{C_{T_2}} = I'_1 + N \times I_L = \frac{12-0.1}{2.2} + N \times \frac{12-0.1-0.7}{15+15} = 5.41 + 0.373N$$

$$I_{B_{T_2}} = 0.625 \text{ mA}$$

$$\therefore \frac{5.41+0.373N}{0.625} = 30 \rightarrow N = 35$$

$$\text{c) } P_{\text{at least one input is low cases}} = (12 - 0.1) \times I_1 = 11.9 \times \frac{12-0.7-0.1}{15+15} = 4.4387 \text{ mW}$$

$$P_{1,1,1} = (12 - 0) \times \frac{12-V_{C_{T_1}}}{15} + (0.1 - 0) \times N \times I_1 = 12 \times \frac{12-2.51}{15} + 0.1 \times 35 \times 0.373 = 73.82 \text{ mW}$$

$$\text{d) } V_A + V_{D_A} = V_P = 2.2 \rightarrow 12 + V_N + 0.6 = 2.2 \rightarrow V_N = -10.4 \text{ V}$$

$$\therefore |V_N| = 10.4 \text{ V}$$

**Exercise 6****Solution:**

$$\text{a) } Y = AB$$

**b) Step 1:**

Current flows from driver to loads. Hence, the condition for maximum fanout is supply-demand balance.

**Step 2:**

Case (1,1):

$D_3, D_4, Q_1$  on,  $D_1 \& D_2$  off

Maximum supply current from the driver,  $I_{2.2k} = \frac{12-0.2}{2.2} = 5.363 \text{ mA}$

Demand current by each load,  $I_L = \frac{0.2}{115} = 1.74 \mu A$

$$\therefore \text{Fanout} = \text{floor}\left(\frac{5.363}{1.74 \times 10^{-3}}\right) = 3082$$

Case (0,0)/(0,1)/(1,0):

$D_3, D_4, Q_1$  off,  $D_1$  &  $D_2$  on

Maximum supply current from the driver,  $I_{2.2k} = \frac{12-10}{2.2} = 0.91 \text{ mA}$

Since the output of the driver is high, loads will be in saturation. Thus demand current,

$$I_L = \frac{10-0.8}{15} = 0.613 \text{ mA}$$

$$\therefore \text{Fanout} = \text{floor}\left(\frac{0.91}{0.613}\right) = 1$$

### Step 3:

$$\therefore \text{Maximum fanout} = \min(1, \infty)$$

- c) N=1 load will be in saturation mode when driver output is high. Thus, maximum power dissipation in the single load,

$$P_{max} = (10 - 0) \times I_L + (12 - 0) \times I_{Cload} = 10 \times \frac{(10-0.8)}{15} + 12 \times \frac{12-0.2}{2.2} = 70.5 \text{ mW}$$