Review Exercise 1

Solution:

a) In F.A., $V_{BE} = 0.7\ V$. Here, $V_E = 0$, so, $V_B = V_{BE} = 0.7\ V$.

$$I_B = \frac{10 - V_B}{100k} = \frac{10 - 0.7}{100k} = 0.093\ mA$$

b) $I_C = \beta \times I_B = 50 \times 0.093 = 4.65\ mA$

c) $V_C = 20 - I_C \times 10 = 20 - 4.65 \times 10 = -26.5\ V$

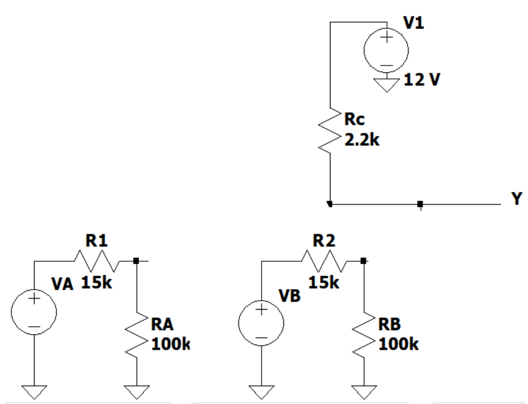
d) $V_{CE} = V_C - V_E = -26.5 - 0 = -26.5 < 0$

Exercise 1

Solution:

a) We assume cutoff mode for input logic '0' & saturation mode for input logic '1'.

Case (0,0):



Due to the open circuit, we see, $V_Y = 12\ V$

Verification:

$V_{E_{Q_1}} = V_{E_{Q_2}} = 0\ V$

$V_A = V_B = 0.2\ V$

Applying KCL at the base of Q_1,

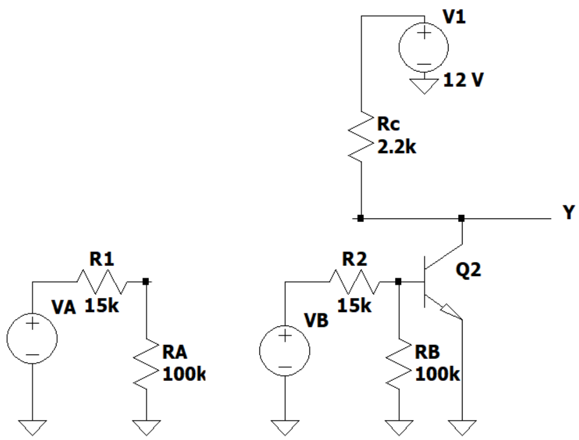
$$\frac{V_A - V_{B_{Q_1}}}{15} = \frac{V_{B_{Q_1}} - 0}{100k} \rightarrow V_{B_{Q_1}} = 0.17\ V$$

$\therefore V_{BE_{Q_1}} = 0.17 - 0 = 0.17\ V < 0.5\ V$

$Hence\ Q_1 will\ be\ off$

Similarly, we can show Q_2 will be off too, as it has the same parameters.

Case (0,1):



Since Q_2 will be on, we can assume it will be in saturation.

$\therefore V_{CE_{Q_2}} = 0.2\ V$

$\therefore V_Y = V_{CE_{Q_2}} = 0.2\ V$

Verification:

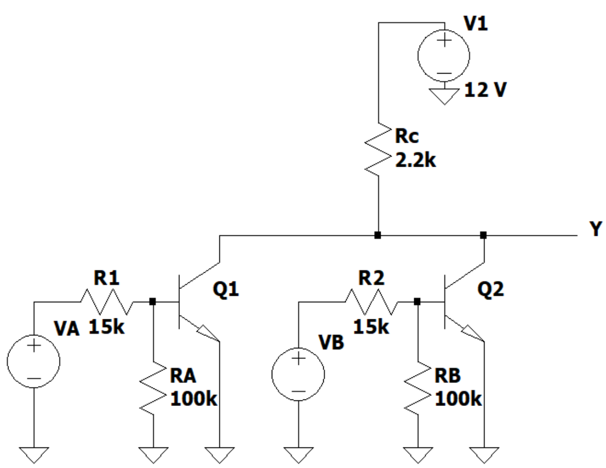
$$I_{C_{Q_2}} = \frac{12 - V_Y}{2.2} = 5.363\ mA$$

$$I_{B_{Q_2}} = I_{R_2} - I_{R_B} = \frac{12 - V_{B_{Q_2}}}{15} - \frac{V_{B_{Q_2}} - 0}{100} = \frac{12 - 0.8}{15} - \frac{0.8}{100} = 0.738\ mA$$

$$\therefore \beta_{Q_2} = \frac{I_{C_{Q_2}}}{I_{B_{Q_2}}} = \frac{5.363}{0.738} = 7.27 < \beta_F(30)$$

Case (1,0): Same as Case (0,1) since both BJTs' parameters are identical.

Case (1,1):



$V_Y = V_{CE_{Q_1}} = V_{CE_{Q_2}} = 0.2\ V$

Verification:

KCL at Y,

$$I_{R_C} = I_{C_{Q_1}} + I_{C_{Q_2}} \rightarrow \frac{12 - 0.2}{2.2} = 2 \times I_{C_{Q_1}} \rightarrow$$
$$I_{C_{Q_1}} = 2.681\ mA$$

$$I_{B_{Q_1}} = I_{B_{Q_2}} =$$
$$0.738\ mA\ [Similarly\ as\ in\ Case\ (0,1)]$$

$$\beta_{Q_1} = \beta_{Q_2} = \frac{I_{C_{Q_1}}}{I_{B_{Q_1}}} = 3.63 < \beta_F(30)$$

Case	V_Y
0,0	12
0,1	0.2
1,0	0.2
1,1	0.2

b) High threshold = min (12) = 12 V, Low threshold = max (0.2, 0.2, 0.2) = 0.2 V

c) Applying KCL at the base of Q_2,

$I_{B_{Q_2}} = I_{R_2} - I_{R_B}$

$= \frac{12-0.8}{15} - \frac{0.8}{2\times100} = 0.742\ mA$

Now, $\beta_{0,1} = \frac{\left(I_{C_{Q_2}}\right)_{0,1}}{0.742} = \frac{5.363}{0.742} = 7.23 < \beta_F(30)\ \&\ \beta_{1,1} = \frac{\left(I_{C_{Q_2}}\right)_{1,1}}{0.742} = \frac{2.681}{0.742} = 3.61 < \beta_F(30)$

Thus, it will still satisfy saturation mode conditions for (1,1) & (0,1).

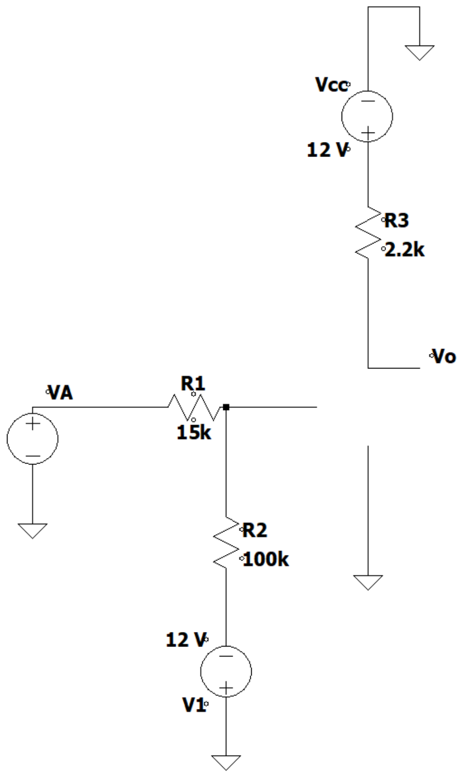
Exercise 2

Solution:

Step 1:

Case	BJT Q1
0	Cutoff
1	Saturation

Case 0:



Step 2:

$I_B = I_C = I_E = 0$

$V_o = V_{cc} = 12\ V$

$I_{R1} = I_{R2} = \frac{V_A-(-12)}{R_1+R_2} = \frac{0.2+12}{15+100}\ mA$

$= 0.106\ mA$

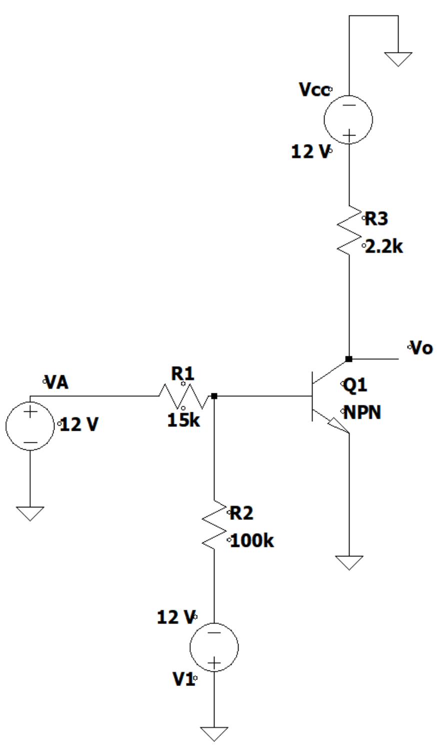
$V_A = 0.2\ V, V_1 = -12\ V$

Current flows only through R_1& R_2

Step 3:

$P_0 = (V_A - V_1) \times I_{R1} = \left(0.2 - (-12)\right) \times$
$0.106\ mW$
$= 1.2932\ mW$

Case 1:



Step 2:

$V_o = V_{CE} = 0.2\ V, V_B = V_{BE} = 0.8\ V$

$I_{R1} = \frac{V_A-V_B}{15} = \frac{12-0.8}{15}\ mA = 0.747\ mA$

$I_{R2} = \frac{V_B-(-12)}{100} = \frac{0.8+12}{100}\ mA = 0.128\ mA$

Using KCL, $I_B = I_{R1} - I_{R2} = 0.619\ mA$

$I_c = \frac{V_{cc}-V_o}{2.2} = \frac{12-0.2}{2.2}\ mA = 5.363\ mA$

I_{R_1} flows between V_A&V_B

I_{R2} flows between V_B& $-$ 12 V

I_B flows between V_B& 0 V

I_C flows between V_{cc}& 0 V

Step 3:
$\therefore P_1 = (V_A - 0.8) \times I_{R1} + \big(V_B - (-12)\big) \times$
$I_{R2} + (0.8 + (V_B - 0) \times I_B + (V_{cc} - 0) \times I_C$
$= (12 - 0.8) \times 0.747 + (0.8 + 12) \times 0.128 +$
$(0.8 - 0) \times 0.619 + (12 - 0) \times 5.363$
$= 74.856\, mW$

$\therefore$ Maximum power dissipation $= P_1 = 74.856\, mW$

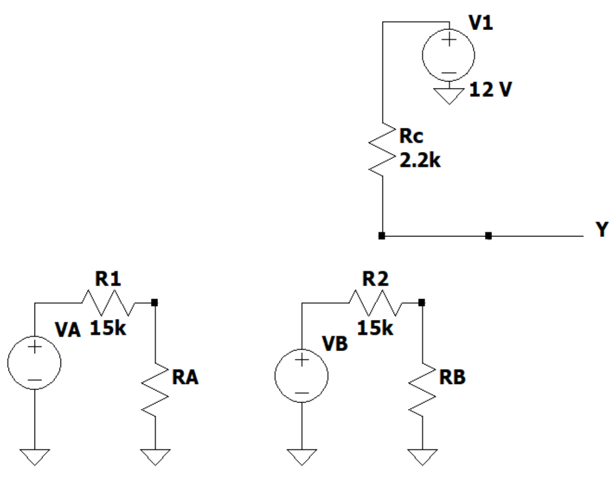
$\therefore$ Average power dissipation $= \frac{P_0 + P_1}{2} = 38.0746\, mW$

Exercise 3

Solution:

Both $Q_1 \& Q_2$ will be off for (0,0).

Step 1:



Step 2:
$I_{R_C} = 0\ (open\ circuit)$
$V_A = V_B = 0.2\ V$

Step 3:
$P_{0,0} = 2 \times (V_A - 0) \times I_{R_A} = 2 \times 0.2 \times \frac{0.2 - 0}{15 + R_A}$
$\rightarrow 0.25 \times 10^{-3} = 0.4 \times \frac{0.2}{15 + R_A}$
$\therefore R_A = R_B = 145\ k\Omega$

Exercise 4

Solution:

Step 1:

$V_{OH} = 11.5\ V, V_{OL} = 0.2\ V$

Step 2:

Determination of V_{IH}:

V_{IH} is the lowest input voltage that would drive the RTL Not gate loads to forward active mode from saturation. Thus, the marginal condition here is,

$\beta_{loads} = \beta_F = 30$

For saturation mode, we know, $V_{BE} = 0.8\ V, V_{CE} = 0.2\ V$

Thus, $I_{B_{loads}} = \frac{V_{IH}-V_B}{0.45} = \frac{V_{IH}-(0.8-0)}{0.45} = \frac{V_{IH}-0.8}{0.45}$

And $I_{C_{loads}} = \frac{V_{cc}-V_{CE}}{0.64} = \frac{3.6-0.2}{0.64} = 5.3125\ mA$

Now, $\beta_{loads} = \frac{I_{C_{loads}}}{I_{B_{loads}}} = 30 \rightarrow \frac{V_{IH}-0.8}{0.45} = \frac{5.3125}{30} \rightarrow V_{IH} = 0.88\ V$

Determination of V_{IL}:

V_{IL} is the highest input voltage that would turn the RTL Not gates on & violate cutoff condition of $V_{BE} < 0.5\ V$ So, the marginal $V_{BE} = V_B = 0.5\ V$

Since $V_E = 0\ V$,

$V_{BE} = V_B = V_{IL} - I_B \times 0.45 = V_{IL} - 0 \times 0.45 = V_{IL} (In\ cutoff\ I_B = 0)$

Thus, $V_{IL} = 0.5\ V$

Step 3:

$N_H = V_{OH} - V_{IH} = 3.5 - 0.88 = 2.62\ V$

$N_L = V_{IL} - V_{OL} = 0.5 - 0.2 = 0.3\ V$

$\therefore N_M = \min(N_H, N_L) = 0.3\ V$

Exercise 5

Solution:

a) **Step 1:**
The current from the driver circuit is going outward to the NOT gate loads. Hence, the condition for maximum fanout is supply & demand current balance.
Step 2:
Case (1,1):
Both T_1 & T_2 on, output = V_{OL}
Supply current from the driver, $I_{R_C} = \frac{V_{CC}-V_{OL}}{R_C} = \frac{3.6-0.2}{0.64} = 5.3125\ mA$
Loads will be cutoff due to low voltage at the base. Thus, demand current, $I_L = 0$.
$\therefore Fanout = \infty$

Case (0,1):
T_1 off & T_2 on, output = V_{OH}
Supply current from the driver, $I_{R_C} = \frac{V_{CC}-V_{OH}}{R_C} = \frac{3.6-1.3}{0.64} = 3.6\ mA$
Loads will be in saturation due to high voltage at the base. Thus, demand current, $I_L = \frac{V_{OH}-V_B}{R_B}$
$= \frac{1.3-0.8}{0.45} = 1.11\ mA$
$\therefore Fanout = floor\left(\frac{I_{R_C}}{I_L}\right) = 3$

Case (1,0): Same as Case (0,1)
Case (1,1): Same as Case (0,1)
Step 3: $\therefore Maximum\ fanout = \min(\infty, 3) = 3$

b) Applying KCL at output node,
$I_{R_C} = N \times I_L \rightarrow \frac{V_{CC}-V_O}{R_C} = 5 \times \frac{V_O-V_B}{R_B} \rightarrow \frac{3.6-V_O}{0.64} = 5 \times \frac{V_O-0.8}{0.45} \rightarrow V_o = 1.145\ V$

c) For $(\beta_F)_{min}$, $V_{CE} = 0.2\ V, V_{BE} = 0.8\ V$

$$I_B = \frac{V_O - V_B}{R_B} = \frac{1.145 - 0.8}{0.45} = 0.767\ mA$$

$$I_C = \frac{V_{CC} - V_C}{R_C} = \frac{3.6 - 0.2}{0.64} = 5.3125\ mA$$

$$\therefore (\beta_F)_{min} = \frac{I_C}{I_B} = 6.93$$

$$P_{loads} = 5 \times (V_{cc} - 0) \times I_C = 5 \times 3.6 \times 5.3125 = 95.625\ mW$$

d) $P_{driver} = \Delta V_{(base-ground)_A} \times I_{B_A} + \Delta V_{(base-ground)_B} \times I_{B_B} + \Delta V_{cc-ground} \times I_{R_C}$

$$= (V_A - 0) \times I_{B_A} + (V_B - 0) \times I_{B_B} + (V_{cc} - 0) \times I_{R_C}$$

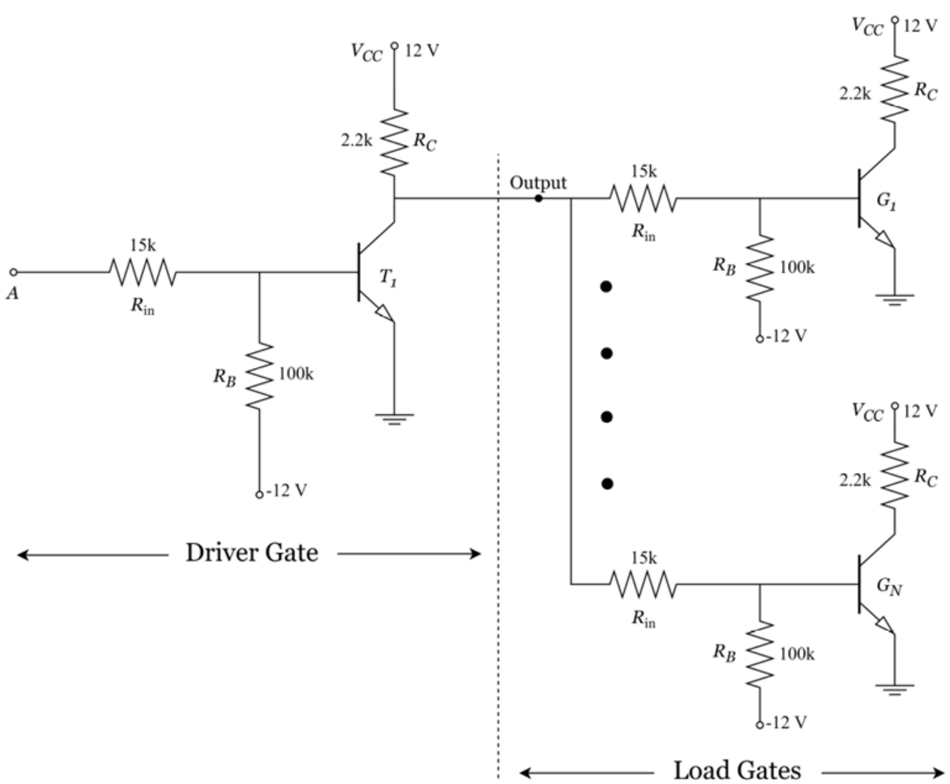
$$= (3.6 - 0) \times \frac{3.6 - 0.8}{0.45} + (3.6 - 0) \times \frac{3.6 - 0.8}{0.45} + (3.6 - 0) \times \frac{3.6 - V_{OL}}{R_C}$$

$$= 2 \times 3.6 \times \frac{2.8}{0.45} + 3.6 \times \frac{3.6 - 0.2}{0.64} = 63.925\ mW$$

e) $Y = \overline{A + B}$

Exercise 6

Solution:



a) **Step 1:**

The current from the driver circuit is going outward to the NOT gate loads. Hence, the condition for maximum fanout is supply & demand current balance.

Step 2:

Case (0):

T_1 off, loads on due to high voltage at base.

Supply current from driver, $I_{R_C} = \frac{12-V_{OH}}{R_C} = \frac{12-10}{2.2} = 0.91\ mA$

Demand current, $I_L = \frac{V_{OH}-V_B}{R_{in}} = \frac{10-0.8}{15} = 0.61\ mA$

$\therefore Fanout = floor\left(\frac{0.91}{0.61}\right) = 1$

Case (1):

T_1 on, loads off due to low voltage at base.

Supply current from driver, $I_{R_C} = \frac{12-V_{OL}}{2.2} = 5.363\ mA$

Demand current, $I_L = \frac{V_{OL}-(-12)}{15+100} = 0.106\ mA$

$\therefore Fanout = floor\left(\frac{5.363}{0.106}\right) = 50$

Step 3:

$\therefore Maximum\ fanout = \min(50,1) = 1$

b) KCL at output of driver,

$$I_{R_C} = \frac{12-V_o}{R_C} = N \times I_L = 2 \times \frac{V_o-V_B}{R_{in}} \rightarrow \frac{12-V_o}{2.2} = 2 \times \frac{(V_o-0.8)}{15} \rightarrow V_o = 9.46\ V$$

c) If the input is high, $V_{CE} = 0.2\ V$. Thus, we have,

$$P_{driver} = (V_{cc} - 0) \times I_{R_C} + (12 - 0) \times I_B + \left(12 - (-12)\right) \times I_{R_B}$$

$$= (V_{cc} - 0) \times (I_{R_C} - 2 \times \frac{0.2}{115}) + (12 - 0) \times I_B + \left(12 - (-12)\right) \times I_{R_B}$$

$$= 12 \times (\frac{12-0.2}{2.2} - 2 \times \frac{0.2}{115}) + 12 \times \left(\frac{12-0.8}{15} - \frac{12.8}{100}\right) + 12.8 \times \frac{12.8}{100}$$

$$= 73.384\ mW$$

d) $P = (V_{cc} - V_o) \times I_{R_C} + 2 \times \left[(V_o - 0.8) \times I_{R_{in}} + \left(0.8 - (-12)\right) \times I_{R_B} + (12 - 0) \times I_{R_C} + (0.8 - 0) \times I_B\right] + \left(0.2 - (-12)\right) \times I_{R_{in}}$

$$= (12 - 9.46) \times \frac{12-9.46}{2.2} + 2 \times [(9.46 - 0.8) \times \frac{9.46-0.8}{15} + 12.8 \times \frac{12.8}{100} +$$
$$12 \times \frac{12-0.2}{2.2} + 0.8 \times (\frac{9.46-0.8}{15} - \frac{12.8}{100})] + 12.2 \times \frac{0.2-(-12)}{15+100} = 146.95\ mW$$

e) **Step 1:**

$V_{OH} = 10\ V, V_{OL} = 0.2\ V$

Step 2:

Determination of V_{IH}:

V_{IH} is the lowest input voltage that would drive the RTL NOT gate loads to forward active mode from saturation. Thus, the marginal condition here is,

$\beta_{loads} = \beta_F = 30$

Now, $I_{C_{load}} = \frac{V_{CC}-V_{OL}}{R_C} = \frac{12-0.2}{2.2} = 5.363\ mA$

$I_{B_{load}} = \frac{V_{IH}-0.8}{15} - \frac{0.8-(-12)}{100}$

$\therefore \beta_F = 30 \rightarrow \frac{5.363}{\frac{V_{IH}-0.8}{15} - \frac{12.8}{100}} = 30 \rightarrow V_{IH} = 5.4015\ V$

Determination of V_{IL}:

V_{IL} is the highest input voltage that would turn the RTL NOT gate on & violate cutoff condition of $V_{BE} < 0.5\ V$. So, the marginal $V_{BE} = V_B = 0.5\ V$

KCL at base,

$\frac{V_{IL}-V_B}{15} = \frac{V_B+12}{100} \rightarrow V_{IL} = 2.375\ V$

Step 3:

$N_H = V_{OH} - V_{IH} = 4.5985\ V$

$N_L = V_{IL} - V_{OL} = 2.175\ V$

$N_M = \min(N_H, N_L) = 2.175\ V$

Exercise 7

Solution:

a) $Y = \overline{A + B}.C$

b) **Step 1:**

Current from load flows toward driver circuit. Thus, the condition for maximum fanout is determined by driving driver BJTs into forward active mode.

Maximum current from load comes when C=0. Thus, we will ignore C=1 cases.

Step 2:

Case (0,0):

High voltage at the negative terminal of D_1, thus it remains off.

$\therefore Fanout = \infty$

Case (0,1):

$V_X = V_{OL} = 0.2\,V, D_1$ on

Now, $I_{C_{Q_2}} = I_{C_{R_C}} + N \times I_{D_1} = \frac{12-0.2}{2.2} + N \times \frac{12-0.2-0.7}{100+0.25} = 5.363 + 1.11N$

$I_{B_{Q_2}} = \frac{12-0.8}{15} - \frac{0.8-0}{100} = 0.739\,mA$

Thus, $\beta_{max} = \frac{I_{C_{Q2}}}{I_{B_{Q2}}} = \beta_F = 25 \rightarrow 5.363 + 1.11N = 25 \times 0.739$

$\rightarrow N = floor(13.112) = 13$

Case (1,0): Same as Case (0,1)

Case (1,1):

$V_X = V_{OL} = 0.2\,V, D_1$ on

$I_{C_{Q_1}} + I_{C_{Q_2}} = 2I_{C_{Q_1}} = I_{C_{R_C}} + N \times I_{D_1} = \frac{12-0.2}{2.2} + N \times \frac{12-0.2-0.7}{100+0.25}$

$= 5.363 + 0.11N$

$\therefore I_{C_{Q_1}} = \frac{1}{2}(5.363 + 0.11N)$

$I_{B_{Q_1}} = I_{B_{Q_2}} = \frac{12-0.8}{15} - \frac{0.8-0}{100} = 0.739\,mA$

Thus, , $\beta_{max} = \frac{I_{C_{Q2}}}{I_{B_{Q2}}} = \beta_F = 25 \rightarrow \frac{1}{2}(5.363 + 0.11N) = 25 \times 0.739$

$\rightarrow N = floor(287.15) = 287$

Step 3:

$\therefore\ Maximum\ fanout = \min(\infty, 13, 287) = 13$

c) Maximum power will be dissipated in the case (1,1,0). This case turns on all the diodes & BJTs.

$P_{max} = P_{driver} + P_{load} = (12 - 0.2) \times I_{R_C} + (12 - 0) \times I_{R_5}$

$= 12 \times [\frac{12-0.2}{2.2} + \frac{12-V_Y}{100}]$

KCL at Y,

$\frac{(12-V_Y)}{100} = \frac{V_Y-0.2-0.7}{.25} + \frac{V_Y-0-0.7}{.25} \rightarrow V_Y = 0.81\ V$

$\therefore\ P_{max} = 65.7\ mW$

Exercise 8

Solution:

a) $Q_A\ off; Q_B\ \&\ Q_C\ in\ saturation$

$V_Y = V_{CE} = 0.2\ V$

b) $I = \frac{V_{CC}-V_Y}{R_C} = \frac{20-0.2}{10} = 1.98\ mA$

c) $I_2 = \frac{I}{2} = 0.99\ mA$

d) $I_3 = I_2 = 0.99\ mA$

Exercise 9

Solution:

a) $v_0 = 0.2\ V$

b) $I_c = \frac{15-0.2}{2.2} = 6.7272\ mA$

c) $I_1 = \frac{15-0.8}{15} = 0.94667\ mA$

d) $I_2 = \frac{0.8-(-15)}{100} = 0.158\ mA$

e) $I_B = I_1 - I_2 = 0.78867\ mA$

Exercise 10

Solution:

a) V_{IL} is the maximum input voltage that would keep the RTL inverter loads from turning on. Here, the condition for turning on RTL loads, $V_{BE} < 0.5\ V$
Thus, KCL at the base of RTL loads gives,
$\frac{V_{IL}-0.5}{15} = \frac{0.5-(-5)}{100} \rightarrow V_{IL} = 1.325\ V$

b) V_{IH} is the minimum input voltage that would keep the RTL loads in saturation and prevent transition into forward active mode. The condition for saturation mode operation is, $\beta_{forced} < \beta_F$
Or, $I_B = \frac{I_C}{\beta_F} = \frac{5-0.2}{2.2} \times \frac{1}{30} = 0.072\ mA$
$\rightarrow \frac{V_{IH}-0.8}{15} - \frac{0.8-(-5)}{100} = 0.072$
$\rightarrow V_{IH} = 2.761\ V$

c) $N_M = \min(N_H, N_L) = \min(V_{OH} - V_{IH}, V_{IL} - V_{OL})$
$= \min(4 - 2.761, 1.325 - 0.2) = 1.125\ V$

d) Case (0):
Driver cutoff, loads in saturation.
$I_{supply} = \frac{V_{cc}-V_{OH}}{2.2} = \frac{5-4}{2.2} = 0.454\ mA$
$I_{demand} = \frac{V_{OH}-0.8}{15} = \frac{4-0.8}{15} = 0.213\ mA$
$\therefore Fanout = floor\left(\frac{I_{supply}}{I_{demand}}\right) = floor\left(\frac{0.454}{0.213}\right) = 2$
Case (1):
Driver in saturation, loads cutoff.
$I_{supply} = \frac{5-0.2}{2.2} = 2.181\ mA$
$I_{demand} = \frac{V_{OL}-(-5)}{15+100} = \frac{0.2+5}{115} = 0.0452\ mA$
$\therefore Fanout = floor\left(\frac{2.181}{0.0452}\right) = 48$
$\therefore Maximum\ Fanout = \min(2, 48) = 2$

Exercise 11

Solution:

$D_A\ on, D_B, D_C\ off. Q\ off.$

a) $I_L + i_2 = i_1 \rightarrow \frac{V_P - 0.7 - 0.1}{2} + \frac{V_P - (-12)}{R_1 + R_B} = \frac{12 - V_P}{R_2} \rightarrow \frac{V_P - 0.8}{2} + \frac{V_P + 12}{15 + 100} = \frac{12 - V_P}{15}$

$\therefore V_P = 1.90\ V$

$\therefore I_L = \frac{V_P - 0.8}{2} = 0.5521\ mA$

b) $i_1 = \frac{12 - V_P}{15} = 0.673\ mA$

c) $i_2 = \frac{V_P + 12}{115} = 0.121\ mA$

d) $I_B = 0\ mA$

e) $I_C = 0\ mA$

f) $P = (12 - 0.1) \times I_L + \big(12 - (-12)\big) \times i_2 = 9.474\ mW$

Exercise 12

Solution:

All diodes off, Q in saturation.

a) $I_L = 0\ mA$

b) $i_1 = \frac{12 - 0.8}{15 + 15} = 0.37334\ mA$

c) $i_2 = \frac{0.8 + 12}{100} = 0.128\ mA$

d) $I_B = i_1 - i_2 = 0.245334\ mA$

e) $I_C = \frac{12 - 0.2}{2.2} = 5.41\ mA$

f) $P = \big(12 - (-12)\big) \times i_2 + (12 - 0) \times I_B + (12 - 0) \times I_C = 70.936\ mW$

g) $\beta_{min} = \frac{I_C}{I_B} = 22.051$