CSE423 - Computer Graphics

<u>Midterm Practice Sheet [Spring 2025]</u>

N.B. ***This is merely a reference to the problems inclusive and exclusive to the questions that will be set in the midterm. This practice sheet does not include all kinds of questions that may come into the examination.***

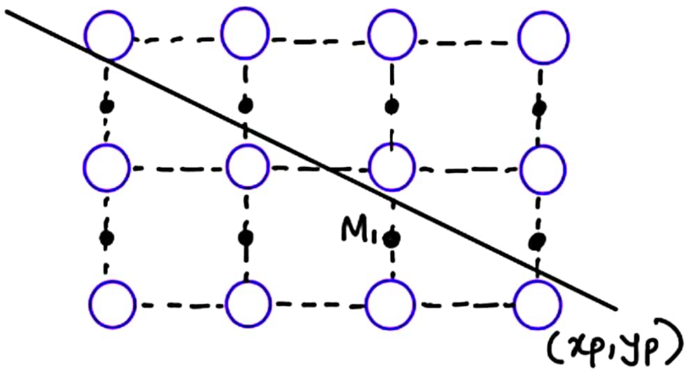
1. How does a computer transform a 3D model into 2D images? State each stage of this process.
2. A GPU can process 80,000 pixels per millisecond. If the frame time at 30 FPS is approximately 33.33 milliseconds, estimate the maximum number of pixels that can be rendered per frame.
3. Given a screen resolution of 2560 x 1440 and a frame rate of 45 FPS, calculate the total number of pixels processed per second.
4. Suppose a line segment starts at (120, 23) and ends at (423, 428). If we draw this problem using the DDA algorithm, determine the number of times x will be increased. How many pixels will be needed to draw this line?
5. Using the Digital Differential Analyzer (DDA) algorithm, find out the pixels between the lines whose endpoints are given:
    a. (5, 16) to (13, 10)
    b. (0, 0) to (4, 6)
    c. (4, -8) to (-1, 7)
    d. (14, -8) to (0, 14)
    e. (5, 6) to (8, 12)
6. Which problem persists in the DDA algorithm and how can it be solved?
7. Why is the value of y incremented/decremented by 1 instead of x, when the gradient is < -1 or > 1?
8. Suppose a line segment starts from P (s, r) and ends at Q (m, n). This line segment falls in Zone 1. Given that s < m and r < n, derive the formulas for determining the pixels' coordinates if DDA Algorithm is used.
9. Using the Slope Independent Line Drawing Algorithm, find the first 6 missing pixel values starting from: (Show the present value of d as well as Δs at each stage.)
    a. (20, 10) to (30, 18)
    b. (0, -14) to (-45, 11)
    c. (-1, -2) to (-15, 82)
    d. (-18, 11) to (0, -3)

e. (5, 10) to (-20, 5)

$$\frac{x}{7} - \frac{y}{12} = 5$$

10. Suppose a line segment has the equation above, and the starting point is at y = 12 and the ending point is at y = 0. Find the first 6 pixels using the Midpoint Line Algorithm.
11. Describe the significance of 8 Way Symmetry in the Midpoint Line Algorithm.
12. A line segment has the following orientation.

Derive starting/initial deviation ('d') and its derivatives (Δs)/decision parameters using mid-point line drawing algorithm.



13. Do you think the Mid Point Line Drawing Algorithm is better than DDA? Provide reasoning.
14. There is a line with the equation, y = -5x - 20. The line segment starts from where it cuts the y-axis and ends at where it cuts the x-axis. Find the first 8 pixels using the Midpoint Line Drawing Algorithm.
15. Given (-30, -30) and (30, 30) are the corners of the clip rectangle. The endpoints of a line are given as (35, 37) and (-38, -20). Using the Cohen-Sutherland line clipping algorithm, identify whether the lines are Accepted, Rejected, or Partial (partially accepted) only from the outcodes. Does Cyrus-Beck line clipping algorithm produce the same output?
16. Given (0, 0) and (400, 300) are the corners of the clip rectangle. The endpoints of a line are given as (−50, 350) and (450, 350). Using the Cohen-Sutherland line clipping algorithm, identify whether the lines are Accepted, Rejected, or Partial (partially accepted) only from the outcodes. Does Cyrus-Beck line clipping algorithm produce the same

output?

17. Given (-30, -30) and (30, 30) are the corners of the clip rectangle. The endpoints of a line are given as (-80, 35) and (35, -80). Using the Cohen-Sutherland line clipping algorithm, identify whether the lines are Accepted, Rejected, or Partial (partially accepted) only from the outcodes. Does Cyrus-Beck line clipping algorithm produce the same output?
18. The top-right corner of a clipping rectangle is (20, 60). The window is 60 units wide and 40 units tall. The endpoints of a line segment are (7, 16) and (85, 75). Apply Cohen-Sutherland Algorithm, and identify whether the line is "partially inside", "completely inside", or "completely outside". If it is the first case, run the algorithm to calculate new endpoints for the line segment so that it is inside the clipping window. Does Cyrus-Beck line clipping algorithm produce the same output?
19. A clipping window is 70 units wide and 30 units tall, and has its center at (0, 35). The endpoints of a line segment are (-45, 65) and (23, -10). Apply Cohen-Sutherland Algorithm, and identify whether the line is "partially inside", "completely inside", or "completely outside". If it is the first case, run the algorithm to calculate new endpoints for the line segment so that it is inside the clipping window. Does Cyrus-Beck line clipping algorithm produce the same output?
20. What is 2D Region Outcode and 3D Region Outcode?
21. What are the disadvantages of the Cohen-Sutherland line clipping algorithm?
22. Which algorithm is suitable for polygonal or circular clipping region?
23. Given a line segment from (10, 60) to (25, 30). Construct the parametric equation P(t) of the line. Using the parametric equation, determine the coordinates of the point where t = 3/5.
24. Does Cyrus-Beck algorithm work for both Concave and Convex polygonal regions? Explain why or why not with a figure.
25. For the following value of t_E and t_L, comment whether the following lines are accepted, rejected or partially clipped:

t_E	t_L	Comments
0.3	0.7	
-0.3	1.2	
-0.3	-0.33	
-0.5	0.5	

1.1	2.4	
0.3	1.3	

26. Define a homogeneous coordinate system. Why does computer graphics prefer a homogeneous coordinate system?
27. Make a simple classification tree of transformations/motions.
28. Derive |4x4| rotation matrix for a 3D point while;
    a. Rotation across the X-axis and the center of rotation (a, b, c).
    b. Rotation across the Y-axis and the center of rotation (a, b, c).
    c. Rotation across the Z-axis and the center of rotation (a, b, c).
29. Derive |3x3| transformation matrix for reflection about any line L.
30. Determine the coordinate of a 3D point P(100, -60, 80)
    a. After rotating 60^0 across the Y-axis, given that the center of rotation is (50, 20, 45).
    b. After rotating 60^0 across the X-axis, given that the center of rotation is (50, 20, 45).
    c. After rotating 60^0 across the Z-axis, given that the center of rotation is (50, 20, 45).
31. Find the following composite transformation matrices as instructed:
    a. A 3D rotation of 90^0 clockwise about the y-axis with respect to the point (a, b, c) followed by a translation of (a, b, c).
    b. A reflection about the line ax - by + c = 0 followed by a scaling "e" times with respect to the point (a, b).
    c. A 3D rotation of 45° counterclockwise about the z-axis with respect to point (d, e, f) followed by a uniform scaling of factor 3 with respect to point (d, e, f) and lastly followed by a translation of (a, b, c). [Here a, b, c, d, e, f are arbitrary values]
32. Arya drew a triangle on a coordinate plane with vertices A(3, 2), B(5, 1), and C(4, 3). She performed the following transformations in sequence: At first, she translated the triangle by (−2, 3). Then, rotate the triangle 90^0 counterclockwise about the origin. After that, she reflected the triangle on the x-axis. Finally, she scaled the triangle uniformly by a factor of 2 about the point (1, −1). After applying these transformations, the new location of one vertex was found to be (a, b).
    a. Now, identify the position of the vertices before only the reflection is applied. (That means only translation and rotation are applied to each of the vertices).
    b. Write the composite matrix formulation for all transformations applied to the triangle. (You do not need to perform matrix multiplication.)
    c. Which geometric properties are preserved after each

transformation?

33. A 3D composite transformation is defined by a shearing along the X axis by a shearing factor of (2, 4) about point (423, 0, 0), followed by a uniform scaling by factor 3 again, followed by a 30-degree counterclockwise rotation on the X axis about point (2, 10, 12) and finally followed by a translation of (-4, -2, -3). A point P is transformed into P’ with the above transformation.
    a. Now, write down the composite transformation matrix representation for P’ in the correct sequence of matrix multiplications. [N.B. the shearing factor will be $Sh_{y\,=\,2}$ and $Sh_{z\,=\,4}$. Go sequentially if it is not mentioned]
    b. Also, find out the inverse composite transformation matrix representation for P’ in the correct sequence of matrix multiplications.

34. A 3D point M is transformed to M’ applying shearing along the Y Axis by a shearing factor of (12, 21) with respect to a point (-33, 44, -55). If the coordinate of M’ is (125, 255, -674). What was the original coordinate of M?

35. A composite transformation is defined by a scaling on the X-axis and Y-axis (first transformation) followed by a second transformation, via the following equations:

$$x' = 5x - 11$$

$$y' = 10y + 22$$

$$z' = 33 + z$$

    a. Find the composite transformation matrix. What was the second transformation after scaling?
    b. Write down the scaling matrix from the above composite transformation.
    c. If a 3D point M has the coordinate (4, 2, 3), what is the new coordinate of the point, M’ after the transformation defined by the equations?

423 Midterm Practice Sheet Solve by Azmari

① How does a computer transform a 3D model into 2D images? State each stage of this process.

– A computer transforms a 3D model into a 2D image through a 3D rendering pipeline, which consists of multiple stages.

i) Vertex processing and clipping

ii) Rasterization

iii) Fragment processing

iv) Image composition / Output Merging

② A GPU can process 80000 pixels per millisecond. If the frame time at 30 FPS is approximately 33.33 milliseconds, estimate the maximum number of pixels that can be rendered per frame.

Pixels per frame = Pixels per millisecond $\times$ frame time

$= 80000 \times 33.33$

$= 2666400$ pixels.

(3) Given a screen resolution of 2560 x 1440 and a frame rate of 45 FPS, calculate the total number of pixels processed per second.

Total pixels per second = pixels per frame × FPS pixels per second

$= 2560 \times 1440 \times 45$

$= 165888000$

(4)

DDA selects the maximum of Δx and Δy as the number of steps :

$$\text{steps} = \max(\Delta x, \Delta y) = \max(423-120, 428-23)$$
$$= \max(303, 405)$$
$$= 405$$

The line will be drawn using 405 pixels.

Number of times x is increased $= 423-120$
$= 303$

(5) DDA algorithm

a) (5, 16) to (13, 10)

$$m = \frac{10-16}{13-5} = -0.75$$

x	y	y (around)	pixel
5	16	16	(5, 16)
6	15.25	15	(6, 15)
7	14.5	15	(7, 15)
8	13.75	14	(8, 14)
9	13	13	(9, 13)
10	12.25	12	(10, 12)
11	11.5	12	(11, 12)
12	10.75	11	(12, 11)
13	10	10	(13, 10)

b) (0, 0) to (4, 6)

$$m = \frac{6-0}{4-0} = 1.5$$

$$\frac{1}{m} = 0.67$$

y	x	x (round)	pixel
0	0	0	(0, 0)
1	0.67	1	(1, 1)
2	1.34	1	(1, 2)
3	2.01	2	(2, 3)
4	2.68	3	(3, 4)
5	3.35	3	(3, 5)
6	4.02	4	(4, 6)

c) (4, −8) to (−1, 7)

$$m = \frac{7+8}{-1-4} = -3$$

$$\frac{1}{m} = -\frac{1}{3} = -0.33$$

y	x	x (round)	pixel
−8	4	4	(4, −8)
−7	3.67	4	(4, −7)
−6	3.34	3	(3, −6)
−5	3.01	3	(3, −5)
−4	2.68	3	(3, −4)
−3	2.35	2	(2, −3)
−2	2.02	2	(2, −2)
−1	1.69	2	(2, −1)
0	1.36	1	(1, 0)
1	1.03	1	(1, 1)

2	0.7	1	(1,2)
3	0.37	0	(0,3)
4	0.04	0	(0,4)
5	-0.29	0	(0,5)
6	-0.62	-1	(-1,6)
7	-0.95	-1	(-1,7)

d) (14,-8) to (0,14)

$$m = \frac{14+8}{0-14} = -1.57$$

$$\frac{1}{m} = -0.64$$

y	x	x (round)	pixel
-8	14	14	(14,-8)
-7	13.36	13	(13,-7)
-6	12.72	13	(13,-6)
-5	12.08	12	(12,-5)
-4	11.44	11	(11,-4)
-3	10.8	11	(11,-3)
-2	10.16	10	(10,-2)
-1	9.52	10	(10,-1)
0	8.88	9	(9,0)
1	8.24	8	(8,1)
2	7.6	8	(8,2)

3	6.96	7	(7,3)
4	6.32	6	(6,4)
5	5.68	6	(6,5)
6	5.04	5	(5,6)
7	4.4	4	(4,7)
8	3.76	4	(4,8)
9	3.12	3	(3,9)
10	2.48	2	(2,10)
11	1.84	2	(2,11)
12	1.2	1	(1,12)
13	0.56	1	(1,13)
14	-0.08	0	(0,14)

e) (5,6) to (8,12)

$$m = \frac{12-6}{8-5} = 2$$

$$\frac{1}{m} = 0.5$$

y	x	x (round)	pixel
6	5	5	5,6
7	5.5	6	6,7
8	6	6	6,8
9	6.5	7	7,9
10	7	7	7,10
11	7.5	8	8,11
12	8	8	8,12

⑥ Which problem persists in the DDA algo and how can it be solved?

- DDA uses floating point operations, leading to rounding errors. It uses incremental addition, which may not always choose the best pixel. It also uses multiplication and division in every iteration. These are the problems.

In this case, midpoint line algo solves DDA's problems. It uses integer calculations, making it more accurate and efficient. It evaluates a midpoint decision function to select the closest pixel, reducing jagged edges (aliasing). It uses only addition and subtraction, making it computationally efficient.

⑦ Why is the value of y incremented / decremented by 1 instead of x, when the gradient is < -1 or > 1?

i) If the slope is steep ($|m| > 1$), changing x by 1 would result in a large change in y, skipping pixels and causing gaps. Instead, we increment / decrement y by 1 and calculate x accordingly to maintain smoothness.

ii) If we always increment x, steep lines would appear disconnected.

(8)

Line segment falls in zone 1

so $|dy| > |dx|$

In that case: $-1 > m > 1$ $\xleftarrow{}$ slope

So,

$y_{new} = r + 1$

$x_{new} = s + \frac{1}{m}$, here $m = \text{slope}$

(9)

a) (20, 10) to (30, 18)

$dx = 30 - 20 = 10$

$dy = 18 - 10 = 8$

$|dx| > |dy|$

$dx > 0$, $dy > 0$

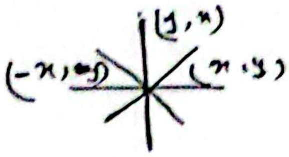
$\therefore$ zone 0

$d_{init} = 2dy - dx = 2 \times 8 - 10 = 6$

$\Delta d_{NE} = 2dy - 2dx = -4$

$\Delta d_E = 2dy = 16$

x	y	d	NE/E 1,1 / 1,0	d updating	pixel
20	10	6	NE	2	(20, 10)
21	11	2	NE	-2	(21, 11)
22	12	-2	E	14	(22, 12)
23	12	14	NE	10	(23, 12)
24	13	10	NE	6	(24, 13)
25	14	6	NE	2	(25, 14)
26	15	2	NE	-2	(26, 15)
27	16	-2	E	14	(27, 16)



b) $(0,-14)$ to $(-15, 11)$

$dx = -15 - 0 = -15$

$dy = 11 + 14 = 25$

$dx < 0, \; dy > 0$

$|dx| > |dy|$

zone = 3

$(0,-14)$ to $(45, 11)$

$dx' = 45$

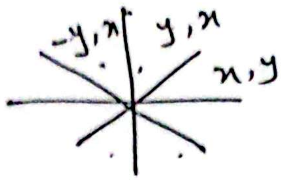
$dy' = 11 + 14 = 25$

$d_{init} = 2dy - dx = 5$

$\Delta d_{NE} = 2dy - 2dx = -40$

$\Delta d_E = 2dy = 50$

x	y	d	NE/E	d update	pixel zone 0	pixel zone 3
0	−14	5	NE	−35	(0, −14)	(0, −14)
1	−13	−35	E	15	(1, −13)	(−1, −13)
2	−13	15	NE	−25	(2, −13)	(−2, −13)
3	−12	−25	E	25	(3, −12)	(−3, −12)
4	−12	25	NE	−15	(4, −12)	(−4, −12)
5	−11	−15	E	35	(5, −11)	(−5, −11)
6	−11	35	NE	−5	(6, −11)	(−6, −11)



c) $(-1,-2)$ to $(-15,82)$

$dx = -15+1 = -14$

$dy = 82+2 = 84$

$|dy| > |dx|$

$dy > 0,\ dx < 0$

zone 2

$(-1,-2) \Rightarrow (-2,1)\ ,\ (-15,82) \Rightarrow (82,15)$

$dx = 82+2 = 84$

$dy = 15-1 = 14$

$d_{init} = 2dy - dx = -56$

$\Delta d_{NE} = 2dy - 2dx = -140$

$\Delta d_E = 2dy = 28$

x	y	d	NE/E	d update	pixel zone 0	pixel zone 2
-2	1	-56	E	-28	(-2,1)	-1,-2
-1	1	-28	E	0	(-1,1)	-1,-1
0	1	0	E	28	(0,1)	-1,0
1	1	28	NE	-112	(1,1)	-1,1
2	2	-112	E	-84	(2,2)	-2,2
3	2	-84	E	-56	(3,2)	-2,3
4	2	-56	E	-28	(4,2)	-2,4

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d) $(-18, 11)$ to $(0, -3)$

$dx = 18$

$dy = -3 - 11 = -14$

$|dx| > |dy|$

$dx > 0$, $dy < 0$ $\therefore$ zone 7

$(-18, 11) \to (-18, -11)$, $(0, -3) \to (0, 3)$

$dx = 18$, $dy = 3 + 11 = 14$

$d_{init} = 2dy - dx = 28 - 18 = 10$

$\Delta NE = 2dy - 2dx = 28 - 36 = -8$

$\Delta E = 2dy = 28$

x	y	d	NE/E	d updated	pixel zone 0	pixel zone 7
-18	-11	10	NE	2	(-18, -11)	(-18, 11)
-17	-10	2	NE	-6	(-17, -10)	(-17, 10)
-16	-9	-6	E	22	(-16, -9)	(-16, 9)
-15	-9	22	NE	14	(-15, -9)	(-15, 9)
-14	-8	14	NE	6	(-14, -8)	(-14, 8)
-13	-7	6	NE	-2	(-13, -7)	(-13, 7)
-12	-6	-2	E	26	(-12, -6)	(-12, 6)

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e) (5, 10) to (−20, 5)

$dx = -20 - 5 = -25$

$dy = 5 - 10 = -5$

$|dx| > |dy|$

$dx < 0, dy < 0 \therefore$ zone 4

$(5, 10) \rightarrow (-5, -10)$, $(-20, 5) \rightarrow (20, -5)$

$dx = 20 + 5 = 25$

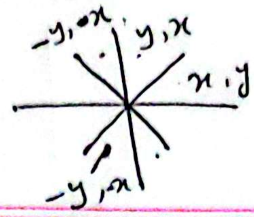
$dy = -5 + 10 = 5$

$d_{init} = 2dy - dx = 10 - 25 = -15$

$\Delta NE = 2dy - 2dx = 10 - 50 = -40$

$\Delta E = 2dy = 10$

x	y	d	NE/E	d updated	pixel zone0	pixel zone4
−5	−10	−15	E	−5	(−5, −10)	(5, 10)
−4	−10	−5	E	5	(−4, −10)	(4, 10)
−3	−10	5	NE	−35	(−3, −10)	(3, 10)
−2	−9	−35	E	−25	(−2, −9)	(2, 9)
−1	−9	−25	E	−15	(−1, −9)	(1, 9)
0	−9	−15	E	−5	(0, −9)	(0, 9)
1	−9	−5	E	5	(1, −9)	(−1, 9)



(10)

$$\frac{x}{7} - \frac{y}{12} = 5$$

$y = 12;$

$$\frac{x}{7} - \frac{12}{12} = 5$$

$$\Rightarrow \frac{x}{7} = 5 + 1$$

$$\Rightarrow x = 42$$

$(x, y) = (42, 12)$

$y = 0;$

$$\frac{x}{7} - \frac{0}{12} = 5$$

$$\Rightarrow x = 35$$

$(x, y) = (35, 0)$

$dx = 35 - 42 = -7$

$dy = 0 - 12 = -12$

$|dy| > |dx|$

$dx < 0, \; dy < 0$

zone $= 5$

$(42, 12) \Rightarrow (-12, -42)$, $(35, 0) \Rightarrow (0, -35)$

$dx = 0 + 12 = 12$

$dy = -35 + 42 = 7$

$d_{init} = 2dy - dx = 2$

$\Delta d_{NE} = 2dy - 2dx = -10$

$\Delta d_E = 2dy = 14$

x	y	d	NE/E	d update	Pixel(0)	Pixel(5)
−12	−42	2	NE	−8	(−12, −42)	(42, 12)
−11	−41	−8	E	6	(−11, −41)	(41, 11)
−10	−41	6	NE	−4	(−10, −41)	(41, 10)
−9	−40	−4	E	10	(−9, −40)	(40, 9)
−8	−40	10	NE	0	(−8, −40)	(40, 8)
−7	−39	0	E	14	(−7, −39)	(39, 7)
−6	−39	14	NE	4	(−6, −39)	(39, 6)

(11) Describe the significance of 8 way symmetry in the midpoint line algorithm.

i) It reduces computation. Instead of computing every pixel in a shape, the algorithm computes only one octant and mirrors it.

ii) It ensures accuracy. Here symmetric reflection ensures smooth and uniform shapes.

iii) It optimized for hardware. It makes circle and ellipse drawing algo faster and more efficient.

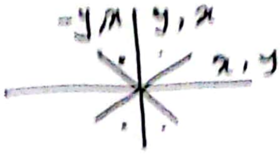
(13) Do you think the mid point line drawing algorithm is better than DDA? Provide reasoning.

- Yes, the midpoint line drawing algorithm is better than the DDA algo.

i) DDA uses floating point arithmetic, but midpoint uses integer calculations, ensuring better precision without floating point errors.

ii) DDA uses multiplication and division in every step, making it computationally expensive. But mid point line uses addition and subtraction, which is faster.

iii) DDA rounds off computed values, sometimes leading to uneven pixel distribution (jagged edges). But MPL uses a decision parameter to choose the next pixel, reducing aliasing and making lines smoother.



(11)

$y = -5x - 20$

$x = 0$

$\Rightarrow y = -5 \times 0 - 20$

$\therefore y = -20$

$(x, y) = (0, -20)$

$y = 0$

$0 = -5x - 20$

$\Rightarrow x = -4$

$(-4, 0) = (x, y)$

starts at $(0, -20)$ and ends at $(-4, 0)$.

$dx = -4 - 0 = -4$

$dy = 0 + 20 = 20$

$|dy| > |dx|$

$dx < 0, \; dy > 0$

zone 2

$(0, -20) \Rightarrow (-20, 0), \quad (-4, 0) \Rightarrow (0, -4)$

$dx = 0 + 20 = 20$

$dy = 4 - 0 = 4$

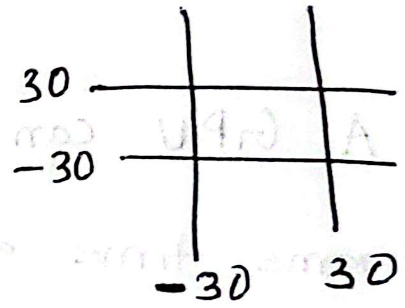
$d_{init} = 2dy - dx = -12$

$\Delta d_{NE} = 2dy - 2dx = -32$

$\Delta d_E = 2dy = 8$

x	y	d	NE/E	d update	pixel zone 0	pixel zone 2
−20	0	−12	E	−4	(−20, 0)	(0, −20)
−19	0	−4	E	4	(−19, 0)	(0, −19)
−18	0	4	NE	−28	(−18, 0)	(0, −18)
−17	1	−28	E	−20	(−17, 1)	(−1, −17)
−16	1	−20	E	−12	(−16, 1)	(−1, −16)
−15	1	−12	E	−4	(−15, 1)	(−1, −15)
−14	1	−4	E	4	(−14, 1)	(−1, −14)
−13	1	4	NE	−28	(−13, 1)	(−1, −13)
−12	2	−28	E	−20	(−12, 2)	(−2, −12)

(15) $x_{min} = -30$, $y_{min} = -30$



$x_{max} = 30$, $y_{max} = 30$

$x_1 = 35$, $y_1 = 37$, $x_2 = -38$, $y_2 = -20$

For (35, 37)

Outcode 1 : 1010

For (-38, -20)

Outcode 2 : 0001

Outcode 1 AND outcode 2 : 0000

So partially inside.

Outcode 1 has right bit.

Applying right boundary intersection:

$x = x_{max}$

$= 30$

$y = y_1 + m(x_{max} - x_1)$

$= 37 + \frac{-20-37}{-38-35}(30-35)$

$= 33.096$

Outcode 1 : 1000 (recalculated)

Outcode 1 AND outcode 2 = 0000

$x_1 = 30$, $y_1 = 33.096$, $x_2 = -38$, $y_2 = -20$

Outcode 1 has top bit.

$y = y_{max} = 30$

$x = x_1 + \frac{1}{m}(y_{max} - y_1)$

$= 35 + \frac{-38-30}{-20-33.096}(30 - 33.096)$

$= 31$

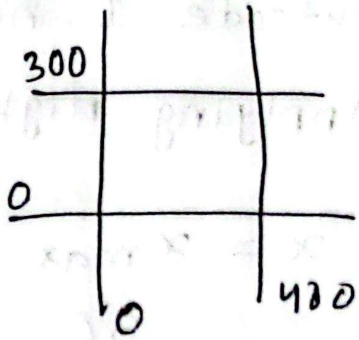
Outcode 1 : 0001

Outcode 1 AND outcode 2 = 0001

So completely outside.

CamScanner

(16)



$x_{max} = 400, \quad x_{min} = 0$

$y_{max} = 300, \quad y_{min} = 0$

$x_1 = -50, \quad y_1 = 350$

$x_2 = 450, \quad y_2 = 350$

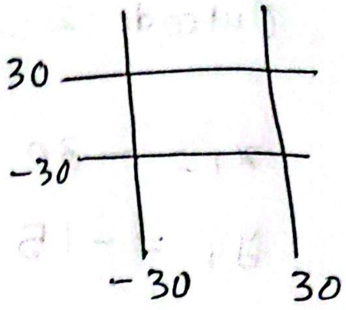
Outcode 1 : 1001

Outcode 2 : 1010

Outcode 1 AND 2 = 1000

So completely outside.

(17)



$x_{min} = -30, \quad y_{min} = -30$

$x_{max} = 30, \quad y_{max} = 30$

$x_1 = -80, \quad y_1 = 35$

$x_2 = 35, \quad y_2 = -80$

Outcode 1 = 1001

Outcode 2 = 0110

Outcode 1 AND outcode 2 = 0000

Outcode 1 has left bit.

$x = x_{min} = -30$

$y = y_1 + m(x_{min} - x_1)$

$= 35 + \frac{-80-35}{35+80}(-30+80)$

$= -15$

Outcode 1 = 0000 (recalculated)

Outcode 1 AND 2 = 0000

Outcode 2 has right bit.

$x = x_{max} = 30$

$y = y_2 + m(x_{max} - x_2)$

$= -80 + \frac{-15+80}{-30-35}(30-35)$

$= -75$

Outcode 2 = 0100 (recalc)

$x_1 = -30$, $x_2 = 30$

$y_1 = -15$, $y_2 = -75$

Outcode 2 has bottom bit.

$y = y_{min} = -30$

$x = x_2 + \frac{1}{m}(y_{min} - y_2)$

$= 30 + \frac{30+30}{-75+15}(-30+75)$

$= -15$

Outcode 2 = 0000 (recalc)

Outcode 1 AND ~~OR~~ = 0000

So completely inside.

Cyrus Beck

$x_{min} = -30, \quad y_{min} = -30$

$x_{max} = 30, \quad y_{max} = 30$

$x_0 = -80, \quad y_0 = 35$

$x_1 = 35, \quad y_1 = -80$

$D = (x_1 - x_0, \ y_1 - y_0)$

$= (115, \ -115)$

Initially, $t_E = 0$, $t_L = 1$

Boundary	N_i	$N_i \cdot D$	t	PE/PL	t_E	t_L
Left	$(-1, 0)$	-115	$\frac{-(-80+30)}{35+80} = 0.43$	PE	0.43	1
Right	$(1, 0)$	115	$\frac{-(-80-30)}{35+80} = 0.95$	PL	0.43	0.95
Bottom	$(0, -1)$	115	$\frac{-(35+30)}{-80-35} = 0.56$	PL	0.43	0.56
Top	$(0, 1)$	-115	$\frac{-(35-30)}{-80-35} = 0.04$	PE	0.43	0.56

$t_L > t_E$

P(0.43) and P(0.56) are the true clip intersection.

$$P(0.43) = (x_0, y_0) + 0.43\,(115, -115)$$

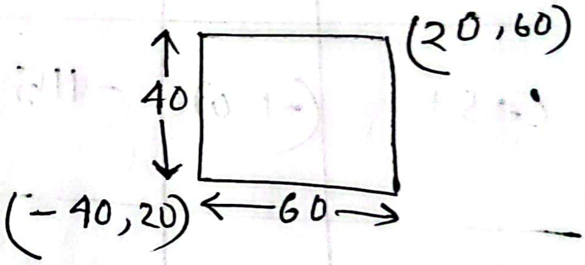
$$= (-80, 35) + 0.43\,(115, -115)$$

$$= (-30.55, -14.45)$$

$$P(0.56) = (-80, 35) + 0.56\,(115, -115)$$

$$= (-15.6, -29.4)$$

(18)

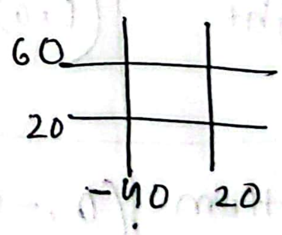


$x_{min} = -40$, $y_{min} = 20$

$x_{max} = 20$, $y_{max} = 60$

$x_1 = 7$, $y_1 = 16$

$x_2 = 85$, $y_2 = 75$



Outcode 1 = 0100

Outcode 2 = 1010

Outcode 1 AND outcode 2 = 0000

Outcode 1 has bottom bit.

$$y = y_{min} = 20$$

$$x = x_1 + \frac{1}{m}(y_{min} - y_1)$$

$$= 7 + \frac{85-7}{75-16}(20 - 16)$$

$$= 12.28$$

Outcode 1 = 0000 (recalculated)

$x_1 = 12.28$, $y_1 = 20$

$x_2 = 85$, $y_2 = 75$

Outcode 2 has right bit.

$$x = x_{max} = 20$$

$$y = y_2 + m(x_{max} - x_2)$$

$$= 75 + \frac{75-20}{85-12.28}(20 - 85)$$

$$= 25.84$$

Outcode 2 = 0000 (recalculated)

So completely inside.

Cyrus Beck

$x_{min} = -40$, $y_{min} = 20$

$x_{max} = 20$, $y_{max} = 60$

$x_0 = 7$, $y_0 = 16$

$x_1 = 85$, $y_1 = 75$

$D = (x_1 - x_0, y_1 - y_0) = (78, 59)$

Initially $t_E = 0$, $t_L = 1$

Boundary	Ni	Ni.D	PE/PL	t	t_E	t_L
Left	(-1,0)	-78	PE	$\frac{-(7+40)}{85-7} = -0.6$	0	1
Right	(1,0)	78	PL	$\frac{-(7-20)}{85-7} = 0.167$	0	0.167
Bottom	(0,-1)	-59	PE	$\frac{-(16-20)}{75-16} = 0.067$	0.067	0.167
Top	(0,1)	59	PL	$\frac{-(16-60)}{75-16} = 0.75$	0.067	0.167

$t_L > t_E$

$P(0.067) = (7, 16) + 0.067(78, 59)$

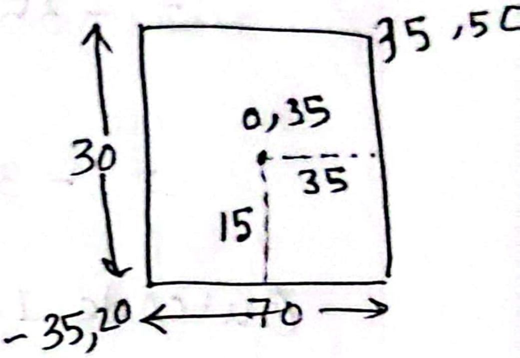
$= (12.226, 19.953)$

$P(0.167) = (7, 16) + 0.167(78, 59)$

$= (20.026, 25.853)$

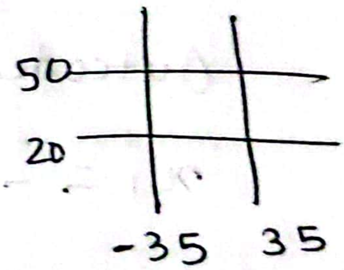
(19) $x_{min} = -35$, $y_{min} = 20$

$x_{max} = 35$, $y_{max} = 50$



$x_1 = -45$, $y_1 = \cancel{23}\ 65$

$x_2 = \cancel{65}\ 23$, $y_2 = -10$



Outcode 1 : 1001

Outcode2 : 0100

Outcode 1 AND outcode 2 = 0000

Outcode 1 has left bit.

$x = x_{min} = -35$

$y = y_1 + m(x_{min} - x_1)$

$= 65 + \frac{-10-65}{23+45}(-35+45) = 53.97$

Outcode 1 = 1000 (recalculated)

Outcode 1 has top bit.

$x_1 = -35, \quad y_1 = 53.97$

$x_2 = 23, \quad y_2 = -10$

$y = y_{max} = 50$

$$x = x_1 + \frac{1}{m}(y_{max} - y_1)$$

$$= -35 + \frac{23+35}{-10-53.97}(50 - 53.97)$$

$$= -31.4$$

Outcode 1 = 0000 (recalculated)

Outcode 2 has bottom bit.

$x_1 = -31.4, \quad y_1 = 50$

$x_2 = 23, \quad y_2 = -10$

$y = y_{min} = 20$

$$x = x_2 + \frac{1}{m}(y_{min} - y_2)$$

$$= 23 + \frac{23+31.4}{-10-50}(20+10) =$$

$$= -4.2$$

Outcode 2 = 0000 (recalculated)

So completely inside.

Cyrus - Beck

$x_{min} = -35$, $y_{min} = 20$

$x_{max} = 35$, $y_{max} = 50$

$x_0 = -45$, $y_0 = 65$

$x_1 = 23$, $y_1 = -10$

$D = (x_1 - x_0, y_1 - y_0)$

$= (68, -75)$

$t_E = 0$, $t_L = 1$

Boundary	Ni	D · Ni	PE/PL	t	t_E	t_L
Left	(-1,0)	-68	PE	$\frac{-(-45+35)}{23+45} = 0.147$	0.147	1
Right	(1,0)	68	PL	$\frac{-(-45-35)}{23+45} = 1.176$	0.147	1
bottom	(0,-1)	75	PL	$\frac{-(65-20)}{-10-65} = 0.6$	0.147	0.6
top	(0,1)	-75	PE	$\frac{-(65-50)}{-10-65} = 0.2$	0.2	0.6

$$P(0.2) = (-45, 65) + 0.2(68, -75)$$
$$= (-31.4, 50)$$
$$P(0.6) = (-45, 65) + 0.6(68, -75)$$
$$= (-4.2, 20)$$

(20) What is 2D Regioan outcode and 3D Region Outcode?

- A 2D outcode is a 4 bit binary code that represents the location for a point relative to a rectangular clipping window.

Outcode = TOP BOTTOM RIGHT LEFT

Bit 1 (TOP) → 1 is the point is above the clip rectangle
Bit 2 (Bottom) → 1 〃 〃 〃 〃 below 〃 〃 〃
Bit 3 (Right) → 1 〃 〃 〃 〃 to right 〃 〃 〃
Bit 4 (Left) → 1 〃 〃 〃 〃 to left 〃 〃 〃

3D Region Outcode : In 3D clipping, the outcode extends to 6 bits because there are 6 boundaries

Outcode = TOP Bottom Right Left ~~Near~~ ~~Far~~ Near Far

Bit 5 (Near) = 1 if the point is in front than the near clipping plane.

Bit 6 (Far) = 1 " " " " behind " " far " ".

21) What are the disadvantages of the Cohen-Sutherland line clipping algorithm?

i) Inefficient for non-rectangular clipping.

ii) Partially visible lines require multiple region checks and intersection calculations, which can be computationally expensive.

iii) If a line is completely outside but parallel to the clipping rectangle's edges, the algorithm still performs region code calculations even though it's unnecessary.

iv) Not well-suited for 3D clipping without additional modifications.

(22) Which algorithm is suitable for polygonal or circular clipping region?

- Cyrus-Beck Algo is suitable.

i) it is best for arbitrary convex polygon clipping.

ii) It uses dot product to determine whether a point is inside or outside a polygon efficiently.

iii) It works for any convex shape, including titled rectangles or irregular convex polygons.

(23) $P_0 (10, 60)$

$P_1 (25, 30)$

$$P(t) = P_0 + t(P_1 - P_0)$$

$$= (10, 60) + t(25 - 10, 30 - 60)$$

$$= (10, 60) + t(15, -30)$$

$$= (10 + 15t, 60 - 30t)$$

$$P(3/5) = (10 + 15 \times \frac{3}{5}, 60 - 30 \times \frac{3}{5})$$

$$= (19, 42)$$ inside the line segment

(24) Does cyrus-Beck algorithm work for both concave and convex polygonal regions?

- The cyrus-Beck algo works only for convex polygonal clipping regions, not for concave polygons.

i) It uses dot product for entry and exit points - works when all edge normals point outward, which happens in convex polygons.

ii) The algorithm assumes a single continuous entry and exit, which doesn't hold for concave shapes.

(25)

t_E	t_L	
0.3	0.7	accept
-0.3	1.2	accept
-0.3	-0.33	reject
-0.5	0.5	accept
1.1	2.4	accept
0.3	1.3	accept

CSE423 [Mid]

'Transformation'

Define a homogenous coordinate system. Why does computer graphics presen a homogeneous coordinate system?

- A homogeneous coordinate system is an extension of the cartesian coordinate system where an extra coordinate is introduced, to represent points in space. In a 2D space, a point (x, y) in cartesian coordinates is represented as (x, y, w) in homogeneous coordinates, where $w \neq 0$. Similarly, a point (x, y, z) in 3D space is represented as (x, y, z, w).

Computer graphics extensively uses homogenous coordinates for several reasons:

i) since transformations are represented as 4×4 matrices in 3D, multiple transformations can be combined into a single matrix, reducing computational complexity.

ii) It simplifies and unifies geometric transformations.

iii) It facilitates perspective projection, combining multiple transformation and representing points at infinity.

Derive [4X4] rotation matrix for a 3D point while;

a. Rotation across the X-axis and the center of rotation is (a, b, c)

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -a \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

*** b. Rotation across the Y-axis and the center of rotation is (a, b, c).

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -a \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c. Rotation across the Z-axis and the center of rotation is (a, b, c).

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -a \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Derive |3X3| transformation matrix for reflection about any line L.

$$Ax + By + c = 0$$

Let, $-\frac{A}{B} = 1$

$-\frac{c}{B} = 2$

$$y = -\frac{A}{B}x - \frac{c}{B}$$

$$\theta = \tan^{-1}\left(-\frac{A}{B}\right) = \tan^{-1} 1 = 45$$

$$M_{composite} = T_{(0,2)} \times R_{(45)} \times Refl_{(x\ axis)} \times R_{(-45)} \times T_{(0,-2)}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-45) & -\sin(-45) & 0 \\ \sin(-45) & \cos(-45) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

(30) Determine the coordinate of a 3D point $P(100, -60, 80)$.

a: After rotating $60°$ across the Y-axis, given the center of rotation is $(50, 20, 45)$

$$\begin{bmatrix} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 60° & 0 & \sin 60° & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 60° & 0 & \cos 60° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -50 \\ 0 & 1 & 0 & -20 \\ 0 & 0 & 1 & -45 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$P' = M \cdot X P$

$$= \begin{bmatrix} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sqrt{3}/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -50 \\ 0 & 1 & 0 & -20 \\ 0 & 0 & 1 & -45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ -60 \\ 80 \\ 1 \end{bmatrix}$$

b. After rotating 60° across the X-axis, given that the center of rotation is (50, 20, 45).

$$\begin{bmatrix} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 60 & -\sin 60 & 0 \\ 0 & \sin 60 & \cos 60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -50 \\ 0 & 1 & 0 & -20 \\ 0 & 0 & 1 & -45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ -60 \\ 80 \\ 1 \end{bmatrix}$$

c. After rotating 60° across the Z-axis, given that the center of rotation is (50, 20, 45)

$$\begin{bmatrix} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 60 & -\sin 60 & 0 & 0 \\ \sin 60 & \cos 60 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -50 \\ 0 & 1 & 0 & -20 \\ 0 & 0 & 1 & -45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ -60 \\ 80 \\ 1 \end{bmatrix}$$

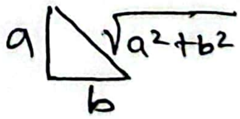
(31) Find the following composite transformation matrices as instructed:

a. A 3D rotation of 90° clockwise about the y-axis with respect to the point (a, b, c) followed by a translation of (a, b, c)

$$\text{Trans} \times T_{(a,b,c)} \times R_{(-90)} \times T_{(-a,-b,-c)}$$

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-90) & 0 & \sin(-90) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-90) & 0 & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -a \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b. a reflection about the line $ax - by + c = 0$ followed by a scaling 'e' times with respect to the point (a, b).



$$y = \frac{a}{b}x + \frac{c}{b}$$

$$m = \frac{a}{b} \quad , \quad b' = \frac{c}{b}$$

$$\theta = \tan^{-1}(a/b)$$
$$= \cos^{-1}\left(b/\sqrt{a^2+b^2}\right)$$
$$= \sin^{-1}\left(a/\sqrt{a^2+b^2}\right)$$

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$

In case of reflection:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & c/b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -c/b \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & c/b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{b}{\sqrt{a^2+b^2}} & -\frac{a}{\sqrt{a^2+b^2}} & 0 \\ \frac{a}{\sqrt{a^2+b^2}} & \frac{b}{\sqrt{a^2+b^2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{b}{\sqrt{a^2+b^2}} & \frac{a}{\sqrt{a^2+b^2}} & 0 \\ -\frac{a}{\sqrt{a^2+b^2}} & \frac{b}{\sqrt{a^2+b^2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -c/b \\ 0 & 0 & 1 \end{bmatrix}$$

In case of scaling:

$$M = T_{(a,b)} \times S_{(e \times e)} \times T_{(-a,-b)}$$

$$= \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -a \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix}$$

Now,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & c/b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{b}{\sqrt{a^2+b^2}} & -\frac{a}{\sqrt{a^2+b^2}} & 0 \\ \frac{a}{\sqrt{a^2+b^2}} & \frac{b}{\sqrt{a^2+b^2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{b}{\sqrt{a^2+b^2}} & \frac{a}{\sqrt{a^2+b^2}} & 0 \\ -\frac{a}{\sqrt{a^2+b^2}} & \frac{b}{\sqrt{a^2+b^2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -c/b \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -a \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix}$$

c. A 3D rotation of 45° counterclockwise about the z-axis with respect to point (d, e, f) followed by a uniform scaling of factor 3 with respect to point (d, e, f) and lastly followed by a translation of (a, b, c).

[trans] × [scaling] × [Rotation]

$$\begin{bmatrix} 1 & 0 & 0 & -d \\ 0 & 1 & 0 & -e \\ 0 & 0 & 1 & -f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & e \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -d \\ 0 & 1 & 0 & -e \\ 0 & 0 & 1 & -f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & e \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(32) a)

$$\begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 5 & 4 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 5 & 4 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & -3 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 5 & 4 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -4 & -6 \\ 1 & 3 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

Position of the vertices :

A $(-5, 1)$, B $(-4, 3)$, C $(-6, 2)$

b)

$$M = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \xcancel{\begin{bmatrix} 3 & 5 & 4 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c) After translation and rotation:

preserves distances, angles

After reflect and scaling: preserves ~~angles~~ parallel lines

(33)

P = [translation] [rotation] [scaling] [Shear]

p' = [Shear] ~~translat~~ [scaling] [rotation] [translation]

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \cos(-30) & -\sin 30 & 0 \\ 0 & \sin(-30) & \cos(-30) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & -12 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 423 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -4 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -423 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/3 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(44)

$$\begin{bmatrix} 1 & 0 & 0 & -33 \\ 0 & 1 & 0 & 44 \\ 0 & 0 & 1 & -55 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -12 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -21 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 33 \\ 0 & 1 & 0 & -44 \\ 0 & 0 & 1 & 55 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 125 \\ 255 \\ -674 \\ 1 \end{bmatrix}$$

(35) a)

$$x' = 5x - 11$$

$$y' = 10y + 22$$

$$z' = z + 33$$

here,

$$[\text{Trans}]\ [\text{scaling}] \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

$$M = \begin{bmatrix} Sx & 0 & 0 & tx \\ 0 & Sy & 0 & ty \\ 0 & 0 & Sz & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 & -11 \\ 0 & 10 & 0 & 22 \\ 0 & 0 & 1 & 33 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

second transformation after scaling :

$$[\text{Trans}] = \begin{bmatrix} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & 22 \\ 0 & 0 & 1 & 33 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b)

$$[\text{Scale}] = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c)

$$M' = \cancel{M}\begin{bmatrix} 5 & 0 & 0 & -11 \\ 0 & 10 & 0 & 22 \\ 0 & 0 & 1 & 33 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 \\ 42 \\ 36 \\ 1 \end{bmatrix}$$