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CSE423

Section: 14

Assignment 04

CamScanner

Ans no 1

a) Diffuse reflection coefficient, $K_d = 0$ of a mirror. A mirror does not scatter light diffusely. Diffuse reflection is typical of rough, matte surfaces, which a mirror is not. So, mirror has no diffuse reflection.

Again, Specular reflection coefficient, $k_s = 1$ for a mirror. A mirror exhibits pure specular reflection, meaning it reflects light in a single direction.

b)

$I_P = 30$

$L = (5, -9, 15)$

$N = (0, 0, 1)$

$V = (-10, 6, 11)$

$k_a = 0.5$

$k_d = 0.8$

$k_s = 0.7$

$I_a = 10$

$K = 1.2$

$P = (-4, 0, 3)$

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$$\vec{L} = L - P = (5+4, -9-0, 15-3)$$
$$= (9, -9, 12)$$

$$|\vec{L}| = \sqrt{9^2 + (-9)^2 + 12^2} = \sqrt{306}$$

$$\hat{L} = \left(\frac{9}{\sqrt{306}}, \frac{-9}{\sqrt{306}}, \frac{12}{\sqrt{306}} \right)$$

$$\hat{N} \cdot \hat{L} = 1 \cdot \frac{12}{\sqrt{306}} = \frac{12}{\sqrt{306}}$$

$$\vec{V} = V - P = (-10+4, 6-0, 11-3) = (-6, 6, 8)$$

$$|\vec{V}| = \sqrt{(-6)^2 + 6^2 + 8^2} = \sqrt{136}$$

$$\hat{V} = \left(\frac{-6}{\sqrt{136}}, \frac{6}{\sqrt{136}}, \frac{8}{\sqrt{136}} \right)$$

$$\vec{R} = 2(\hat{N} \cdot \hat{L}) N - \hat{L}$$

$$= 2 \cdot \frac{12}{\sqrt{306}} (0, 0, 1) - \left(\frac{9}{\sqrt{306}}, \frac{-9}{\sqrt{306}}, \frac{12}{\sqrt{306}} \right)$$

$$= \left(0, 0, \frac{24}{\sqrt{306}}\right) - \left(\frac{9}{\sqrt{306}}, \frac{-9}{\sqrt{306}}, \frac{12}{\sqrt{306}} \right)$$

$$= \left(\frac{-9}{\sqrt{306}}, \frac{9}{\sqrt{306}}, \frac{12}{\sqrt{306}} \right)$$

$$R \cdot \hat{V} = \frac{-9}{\sqrt{306}} \cdot \frac{-6}{\sqrt{136}} + \frac{9}{\sqrt{306}} \cdot \frac{6}{\sqrt{136}} + \frac{12}{\sqrt{306}} \cdot \frac{8}{\sqrt{136}}$$

$$= 1.0004$$

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$I = I_a k_a + I_P (K_d \max(\hat{L}\cdot\hat{N}, 0) + k_s \max(\hat{R}\cdot\hat{V}, 0)^n)$

$= 10 \times 0.5 + 30 \left(0.8 \times \frac{12}{\sqrt{306}} + 0.7 \times 1.0004^{1.2}\right)$

$= 12.47$

c)

$$\hat{L} = \left(\frac{9}{\sqrt{306}}, \frac{-9}{\sqrt{306}}, \frac{12}{\sqrt{306}}\right)$$

$$\hat{V} = \left(-\frac{6}{\sqrt{136}}, \frac{6}{\sqrt{136}}, \frac{8}{\sqrt{136}}\right)$$

$$\hat{L} + \hat{V} = (0.522 - 0.514, -0.522 + 0.514, 0.686 + 0.686) = (0.008, -0.008, 1.372)$$

$$|\vec{H}| = \sqrt{0.008^2 + (-0.008)^2 + 1.372^2}$$

$$= 1.372$$

$$\vec{H} = \frac{\hat{L} + \hat{V}}{\|\hat{L} + \hat{V}\|}$$

$$= \left(\frac{0.008}{1.372}, \frac{-0.008}{1.372}, \frac{1.372}{1.372}\right)$$

$$= (0.00583, -0.00583, 1)$$

$$\hat{N} \cdot \hat{H} = 1.1 = 1$$

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$$I = I_a k_a + I_p \left(k_d \max(\hat{L} \cdot \hat{N}, 0) + k_s \max(\hat{N} \cdot \hat{H}, 0)^k \right)$$

$$= 10 \times 0.5 + 30 \left(0.8 \times \frac{12}{\sqrt{306}} + 0.7 \times 1^{1.2} \right)$$

$$= 42.47$$

Since the view vector $\vec{V}$ and light vector $\vec{L}$ are almost symmetrical about the surface normal, the reflection vector and the halfway vector align closely.

Therefore $\hat{R} \cdot \hat{V} = \hat{N} \cdot \hat{H} = 1$, so the specular component ends up the same.

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d) Yes, I agree with this statement in the content of question b.

In question b, we computed the specular reflection at the point (−4, 0, 3) and found that the view vector and the reflection vector or halfway vector were nearly perfectly aligned with the surface normal.

This resulted in:

$$\vec{R} \cdot \vec{V} = 1 \quad \text{or} \quad \vec{N} \cdot \vec{H} = 1$$

These values mean that the specular highlight is maximized at that point for the given light and view positions.

② a)

$$Q(t) = (1-3t+3t^2-t^3)P_1 + (3t-6t^2+3t^3)P_2 + (3t^2-3t^3)P_3 + t^3 P_4$$

$$A'(t) = (-3+6t-3t^2)P_1 + (3-12t+9t^2)P_2 + (6t-9t^2)P_3 + 3t^2 P_4$$

$$B'(t) = (-3+6t-3t^2)P_4 + (3-12t+9t^2)P_5 + (6t-9t^2)P_6 + 3t^2 P_7$$

$$C'(t) = (-3+6t-3t^2)P_7 + (3-12t+9t^2)P_8 + (6t-9t^2)P_9 + 3t^2 P_{10}$$

$$A'(1) = (-3+6-3)P_1 + (3-12+9)P_2 + (6-9)P_3 + 3P_4$$

$$= -3P_3 + 3P_4$$

$$\therefore A'(1) = 3(P_4 - P_3)$$

$$B'(0) = (-3)P_4 + 3P_5 = 3(P_5 - P_4)$$

$$A'(1) = B'(0)$$

$$\Rightarrow 3(P_4 - P_3) = 3(P_5 - P_4)$$

$$\Rightarrow \boxed{P_5 = 2P_4 - P_3}$$

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$$B'(1) = (-3+6-3)P_4 + (3-12+9)P_5 + (6-9)P_6 + 3P_7$$

$$= -3P_6 + 3P_7 = 3(P_7 - P_6)$$

$$C'(0) = -3P_7 + 3P_8$$

$$= 3(P_8 - P_7)$$

$$B'(1) = C'(0)$$

$$\Rightarrow 3(P_7 - P_6) = 3(P_8 - P_7)$$

$$\therefore \boxed{P_8 = 2P_7 - P_6}$$

$\therefore$ For the spline to be $C(1)$ continuous, we ne

$$P_5 = 2P_4 - P_3$$

$$P_8 = 2P_7 - P_6$$

b)

$$A''(t) = (6-6t)P_1 + (-12+18t)P_2 + (6-18t)P_3 + 6tP_4$$

$$B''(t) = (6-6t)P_4 + (-12+18t)P_5 + (6-18t)P_6 + 6tP_7$$

$$C''(t) = (6-6t)P_7 + (-12+18t)P_8 + (6-18t)P_9 + 6tP_{10}$$

$A''(1) = B''(0)$

$\Rightarrow 6P_2 - 12P_3 + 6P_4 = 6P_4 - 12P_5 + 6P_6$

$\Rightarrow 6P_2 - 12P_3 + 6P_4 - 6P_4 = -12(2P_4 - P_3) + 6P_6$

$\Rightarrow 6P_2 - 12P_3 = -24P_4 + 12P_3 + 6P_6$

$\Rightarrow 6P_6 = 6P_2 - 24P_3 + 24P_4$

$$\Rightarrow P_6 = \frac{6P_2 - 24P_3 + 24P_4}{6}$$

$\Rightarrow P_6 = P_2 - 4P_3 + 4P_4$

$\therefore \boxed{P_6 = P_2 + 4(P_4 - P_3)}$

$B''(1) = C''(0)$

$\Rightarrow 6P_5 - 12P_6 + 6P_7 = 6P_7 - 12P_8 + 6P_9$

$\Rightarrow 6P_5 - 12P_6 = -12P_8 + 6P_9$

$\Rightarrow 6P_5 - 12P_6 = -24P_7 + 12P_6 + 6P_9$

$\Rightarrow 6P_5 - 12P_6 + 24P_7 - 12P_6 = 6P_9$

$\Rightarrow 6P_5 - 24P_6 + 24P_7 = 6P_9$

$\Rightarrow P_5 - 4P_6 + 4P_7 = P_9$

$\therefore \boxed{P_9 = P_5 + 4(P_7 - P_6)}$

For the spline to be $C(2)$ continuous, we need

$P_6 = P_2 + 4(P_4 - P_3)$

$P_9 = P_5 + 4(P_7 - P_6)$

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e)

$$P_5 = 2P_4 - P_3$$

$$P_6 = P_2 + 4(P_1 - P_3)$$

$$P_8 = 2P_7 - P_6$$

$$P_9 = P_5 + 4(P_7 - P_6)$$

To ensure C(2) continuity, the following control points must be locked or dependent:

At P_4:

P_5 and P_6 must be dependent (based on P_3 and P_4).

At P_7:

P_8 and P_9 must be dependent (based on P_6 and P_7).

The control points P_5, P_6, P_8 and P_9 must be locked or dependent to ensure C(2) continuity.

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