Quiz 04 solution

1a) Definition of control points.

b) $\boxed{\text{For } C^1 \text{ continuity}}$ solve from notes

c) Yes, since C^1 also ensures same direction of tangent vector → needed for G^1 continuity

2a) ZX plane at $Y = 20$

$$\begin{bmatrix} 14.33 \\ 20 \\ 12.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \lambda\sin\beta & 0 & 0 \\ 0 & 0 & 0 & 20 \\ 0 & \lambda\cos\beta & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 10 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 14.33 \\ 20 \\ 12.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 + 10\lambda\sin\beta \\ 20 \\ 10\lambda\cos\beta + 10 \\ 1 \end{bmatrix} \qquad \lambda = 0.5$$

$$12.5 = (10 \times 0.5\cos\beta) + 10$$

$$\Rightarrow 2.5 = 5\cos\beta$$

$$\therefore \beta = \cos^{-1}\left(\frac{2.5}{5}\right) = 60^\circ$$

Verify

$$14.33 = 10 + (10 \times 0.5\sin\beta)$$

$$4.33 = 5\sin\beta$$

$$\therefore \beta = \sin^{-1}\left(\frac{4.33}{5}\right) = 59.997^\circ \simeq 60^\circ$$

b) $qx = COPx$ $qy = COPy$ $qz = COP_z - z_p$

(50) (10) $(-10-(-110))$

$= 100$

$$= \begin{bmatrix} 1 & 0 & \frac{-50}{100} & -110 \times \frac{50}{100} \\ 0 & 1 & -10/100 & -110 \times \frac{10}{100} \\ 0 & 0 & \frac{110}{100} & -110 + \frac{12100}{100} \\ 0 & 0 & -1/100 & 1 - \frac{110}{100} \end{bmatrix} \times \begin{bmatrix} 10 \\ 10 \\ -250 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 + 0 + 250 \times \frac{50}{100} - \frac{110 \times 50}{100} \\ 0 + 10 + 2500/100 - \frac{1100}{100} \\ 0 + 0 - \frac{250 \times 110}{100} - 110 + \frac{12100}{100} \\ 0 + 0 + \frac{250}{100} + 1 - \frac{110}{100} \end{bmatrix} = \begin{bmatrix} 80 \\ 24 \\ -264 \\ 2.4 \end{bmatrix} \div 2.4$$

$$\downarrow$$

$$\begin{bmatrix} 33.33 \\ 10 \\ -110 \\ 1 \end{bmatrix}$$