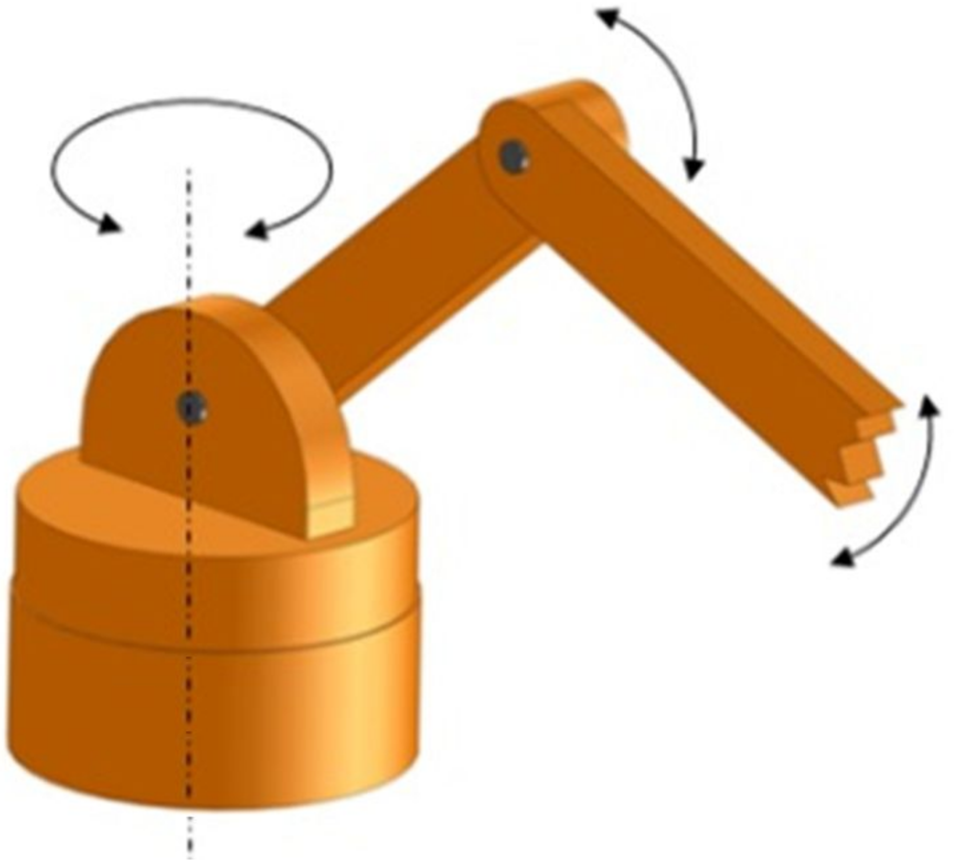
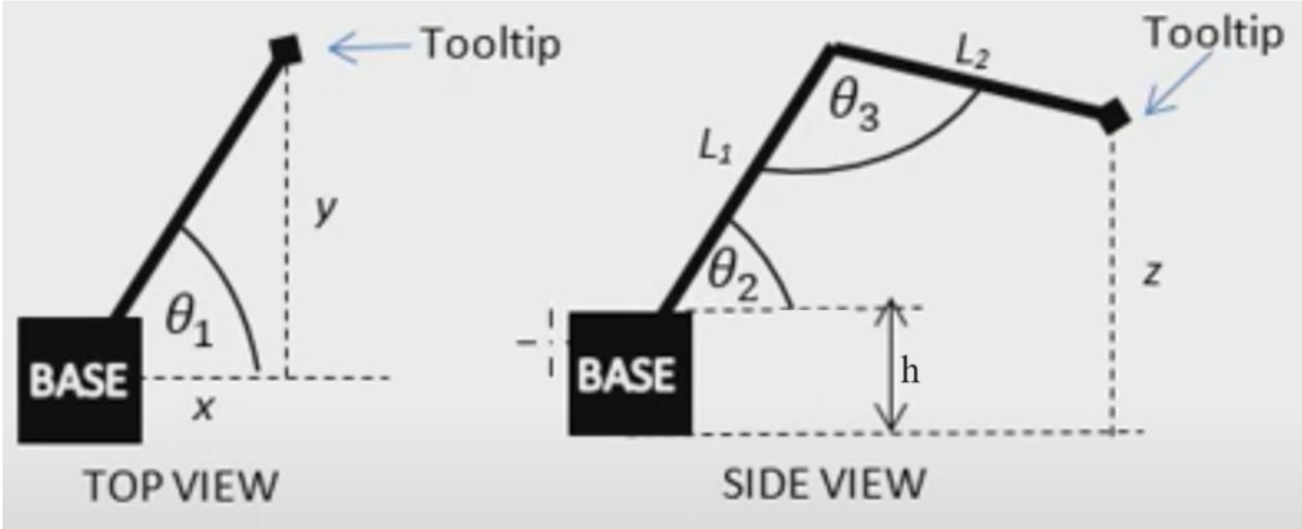
Kinematics

Md. Khalilur Rhaman

Forward Kinematics

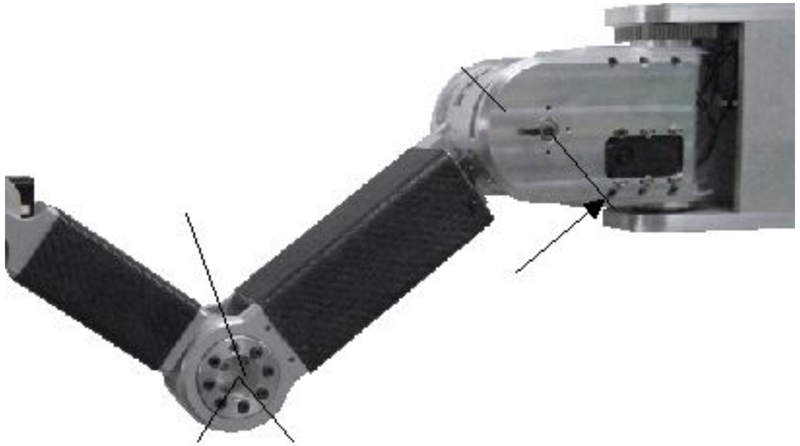




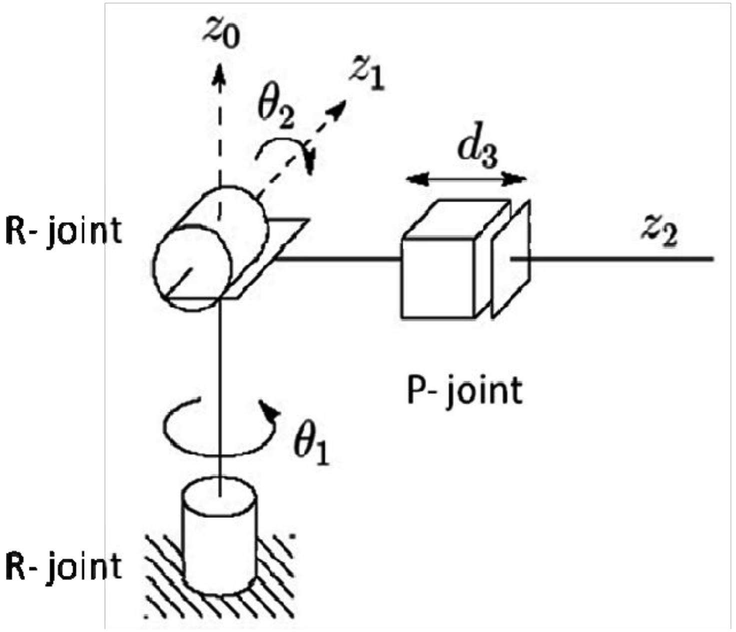
https://www.youtube.com/watch?v=NRgNDlVtmz0

Type of Joint

- Active Joint
- Passive Joint

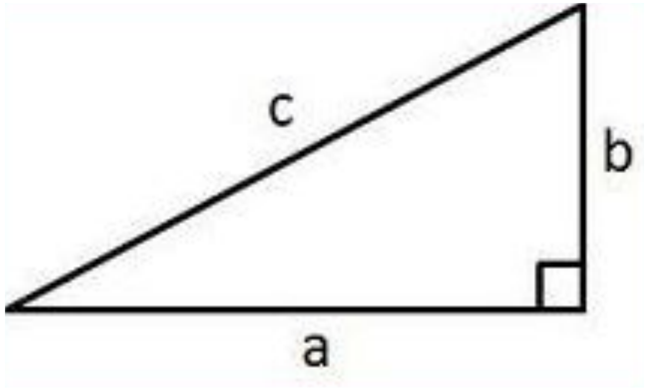


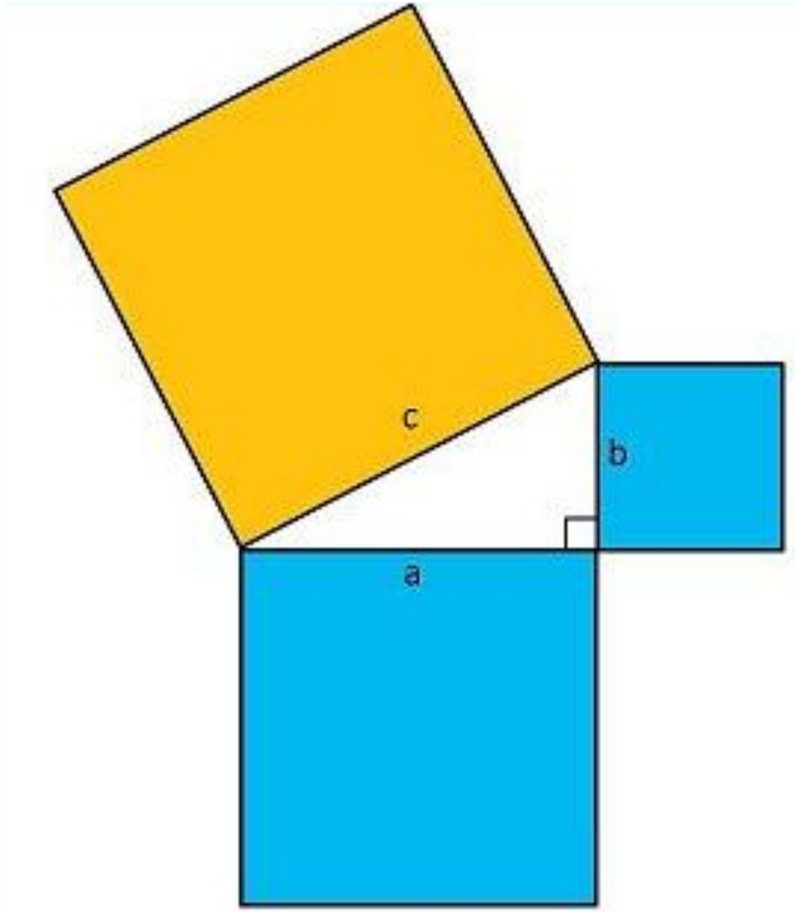
- Revolute
- Prismatic



Forward kinematics using trigonometry

Pythagoras' Theorem for right triangle

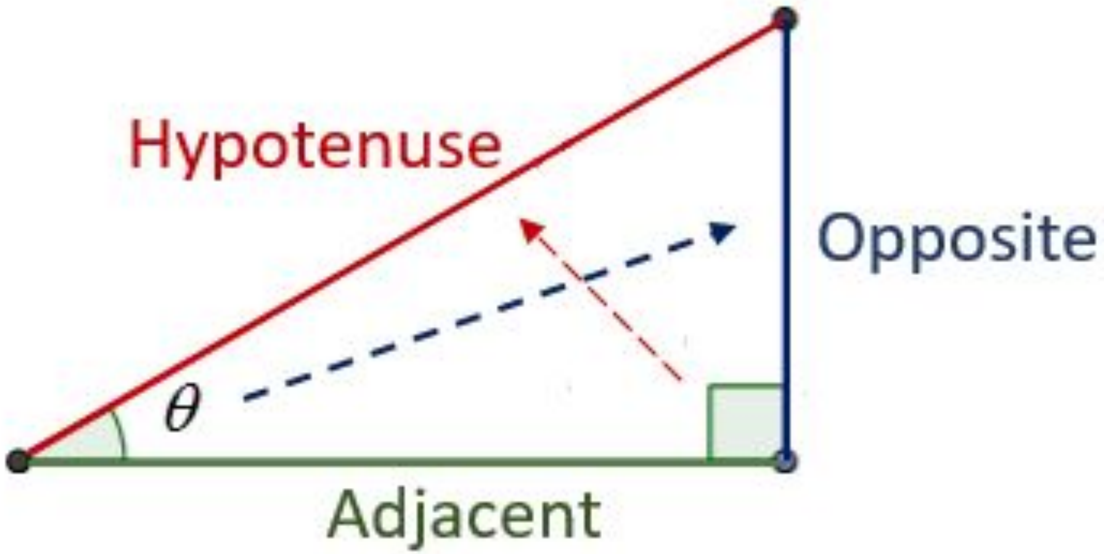
- 



- $c^2 = a^2 + b^2$

Basic Trigonometric Functions

SOHCAHTOA



SOH $\quad \sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$

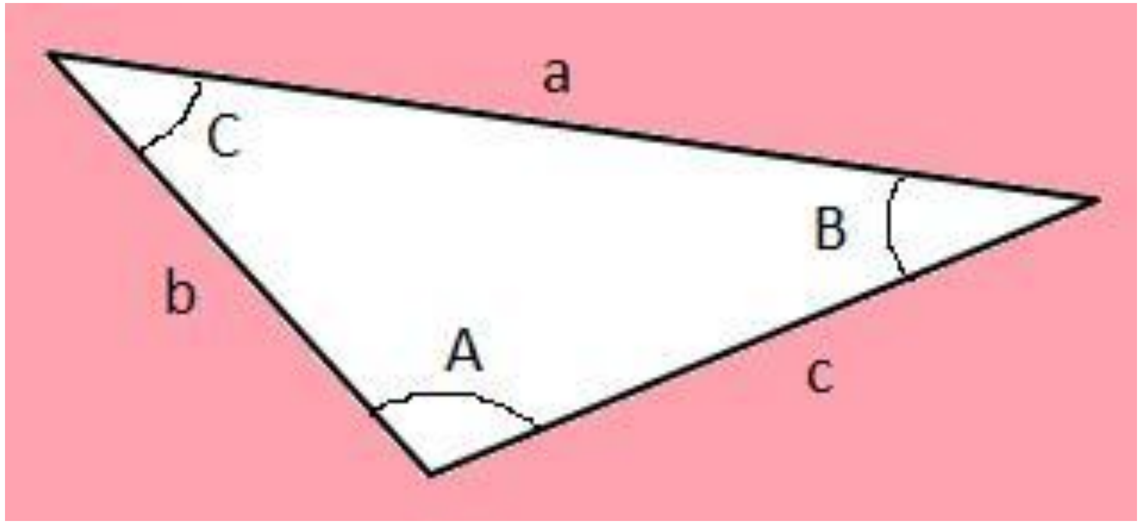
CAH $\quad \cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

TOA $\quad \tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$

The Sine and Cosine Rules

- *Sine Rule:*
- $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$

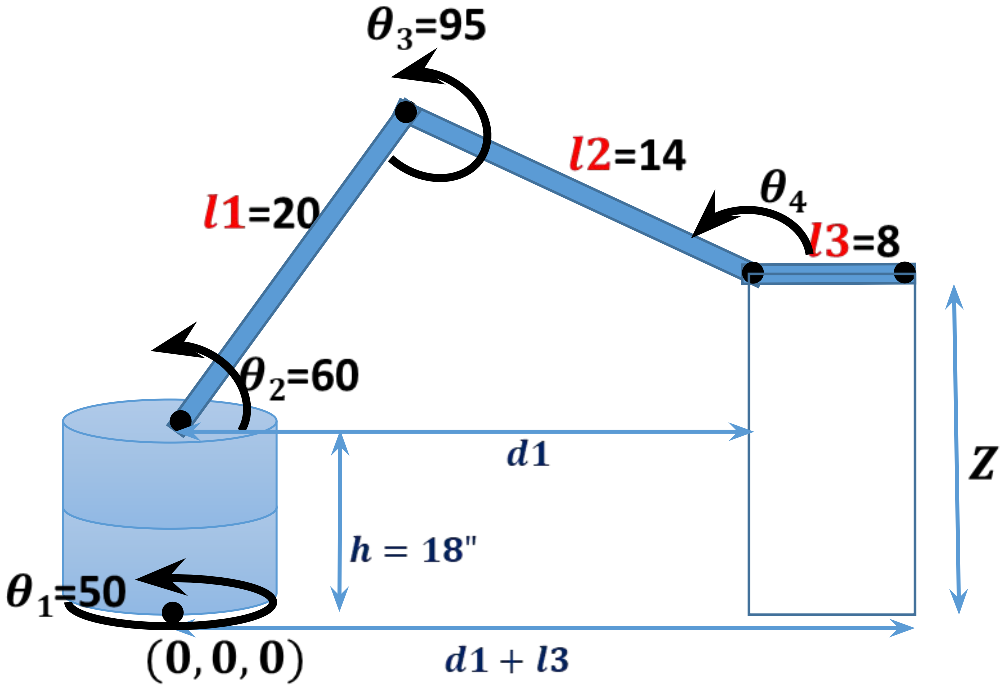
- *Cosine Rule:*
- $a^2 = b^2 + c^2 - 2bcCosA$
- *or*
- $CosA = \frac{b^2+c^2-a^2}{2bc}$



Forward Kinematics

- Angles → coordinate
- $(\boldsymbol{\theta_1}, \boldsymbol{\theta_2}, \boldsymbol{\theta_3})$ → (X, Y, Z)
- Where Height, length of arm-1 and Length of Arm-2 are fixed

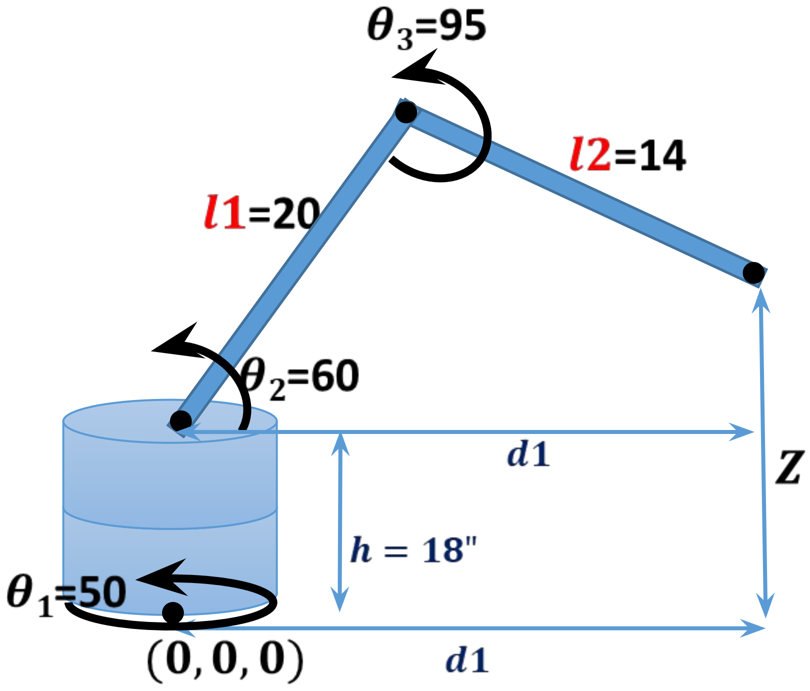
4 DOF Arm Calculation



- *l1=Length of first arm*
- *l2=Length of 2nd Arm*
- *l3=Length of 3rd Arm (end effector)*
- *h=height of base*

- $\theta_1 = Angle\ of\ base\ rotation$
- $\theta_2 = Angle\ of\ first\ arm\ from\ horizon$
- $\theta_3 = Angle\ between\ 1st\ and\ 2nd\ arm$
- $\theta_4 = Angle\ between\ 2nd\ arm\ and\ end\ effector$

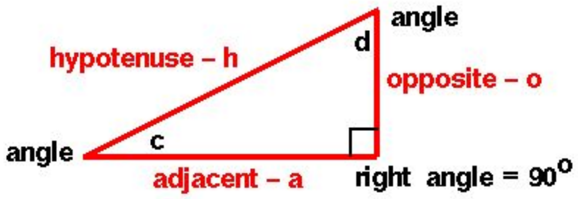
Example with a Value



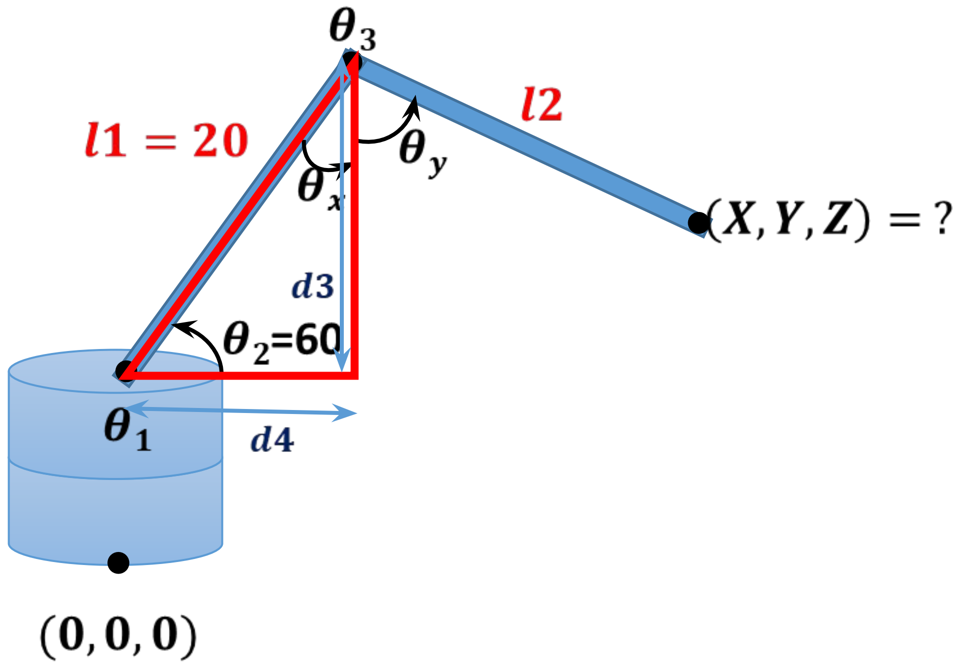
- *l1=20”*
- *l2=14”*
- *l3=8”*
- *h=18”*

- $\theta_1 = 50$
- $\theta_2 = 60$
- $\theta_3 = 95$

Forward Kinematics Calculation

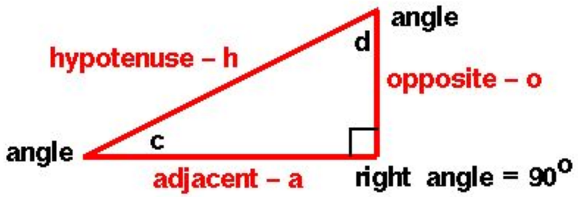


$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$

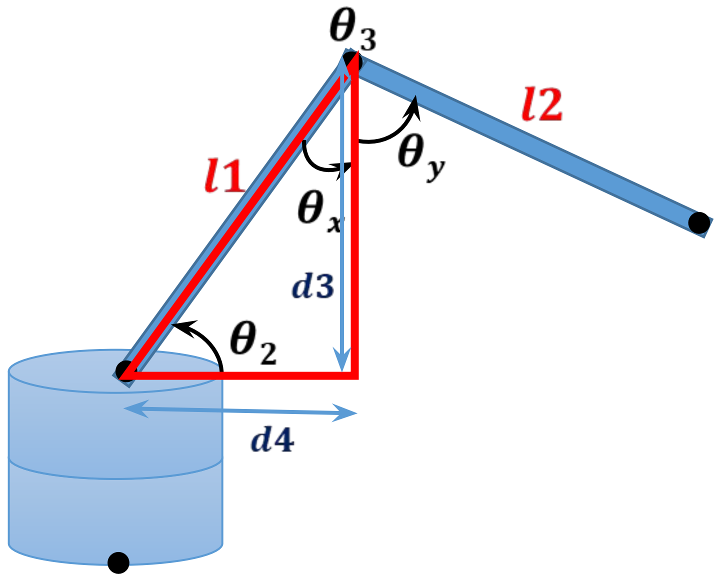


- $If\ we\ draw\ a\ right\ angle\ triangle\ with\ first\ arm,$
- $\theta_x\ is\ the\ angle\ with\ opposit - d3$
- $\theta_y\ is\ the\ angle\ with\ Adjacent - d4$
- $So, \theta_3 = \theta_x + \theta_y$
- $Here\ l1\ is\ hypotenus$
- $So, sin\theta = \frac{O}{H};\ sin\theta_2 = \frac{d3}{l1}$
- $d3 = sin\theta_2 * l1$

Example continuation…



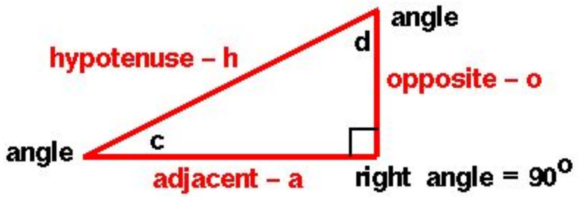
$$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$$



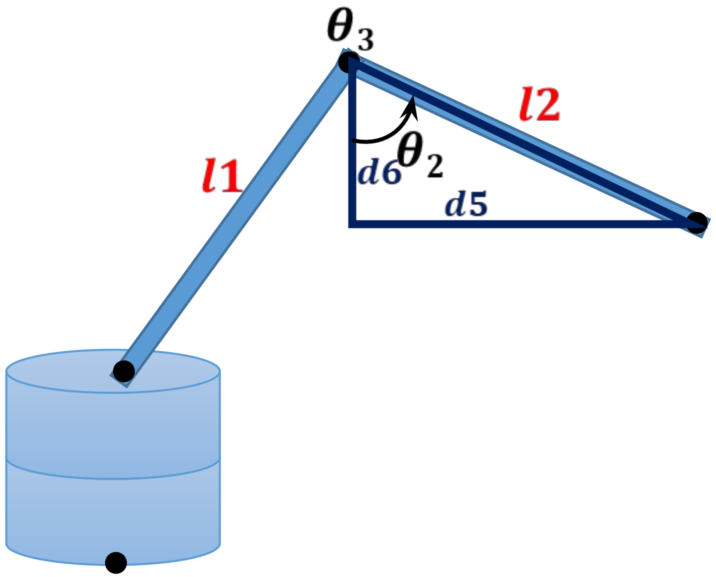
- $\theta_x = 180 - 90 - \theta_2$
- $\theta_x = 180 - 90 - 60 = 30$
- $\theta_3 = \theta_x + \theta_y$
- $\theta_y = (\theta_3 - \theta_x)$
- $\theta_y = 95 - 30 = 65$
- $d3 = sin\theta_2 * l1$
- $d3 = sin60 * l1$
- $d3 = 0.866 * 20 = 17.32$
- $d4 = cos\theta_2 * l1$
- $d4 = cos60 * l1$
- $d4 = .5 * 20 = 10$

l1=20”
l2=14”
l3=8”
h=18”
$\theta_1 = 50$
$\theta_2 = 60$
$\theta_3 = 95$

Forward Kinematics Calculation

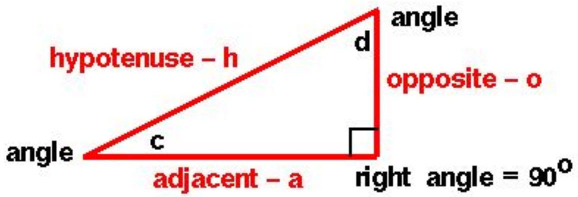


$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$

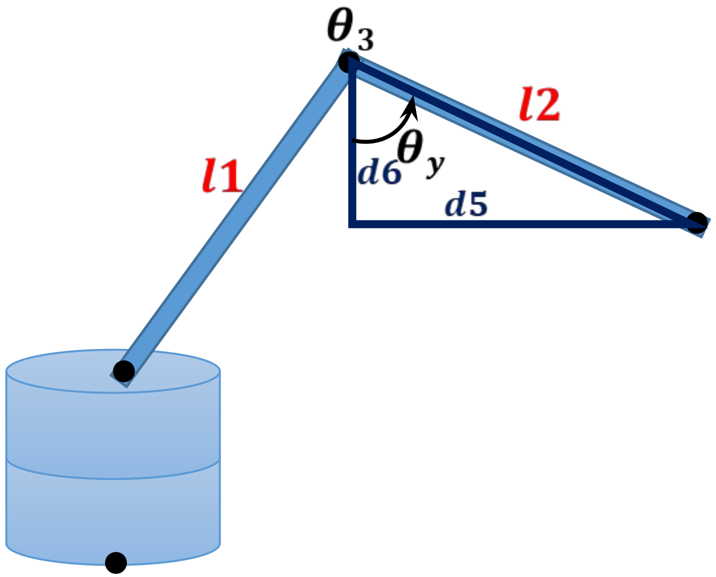


- $If\ we\ draw\ a\ right\ angle\ triangle\ with\ 2nd\ arm\ l2$
- $\theta_y\ is\ the\ angle\ with\ opposit - d6$
- $l2\ is\ hypotenus$
- $So,\ cos\theta = \frac{A}{H};\ cos\theta_y = \frac{d6}{l2};$
- $d6 = cos\theta_y * l2;$
- $sin\theta = \frac{O}{H};\ sin\theta_y = \frac{d5}{l2};$
- $d5 = sin\theta_y * l2$

Example continuation…



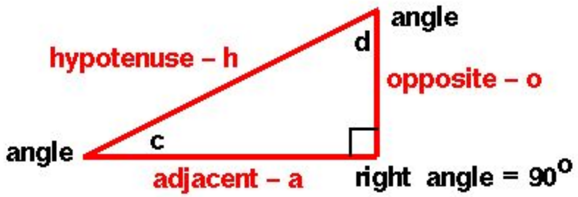
$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$



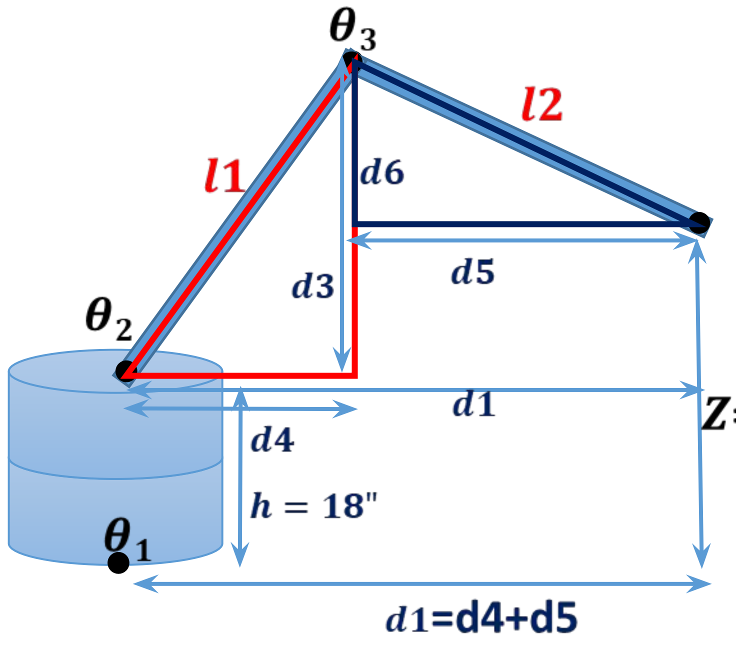
- $d6 = cos\theta_y * l2;$
- $d6 = cos65 * l2;$
- $d6 = 0.4226 * 14 = 5.91$
- $d5 = sin\theta_y * l2$
- $d5 = sin65 * l2$
- $d5 = .9 * 14$
- $d5 = 12.69$

l1=20”
l2=14”
l3=8”
h=18”
$\theta_1 = 50$
$\theta_2 = 60$
$\theta_3 = 95$
$\theta_x = 30$
$\theta_y = 65$
$d3 = 17.32$
$d4 = 10$

Forward Kinematics Calculation



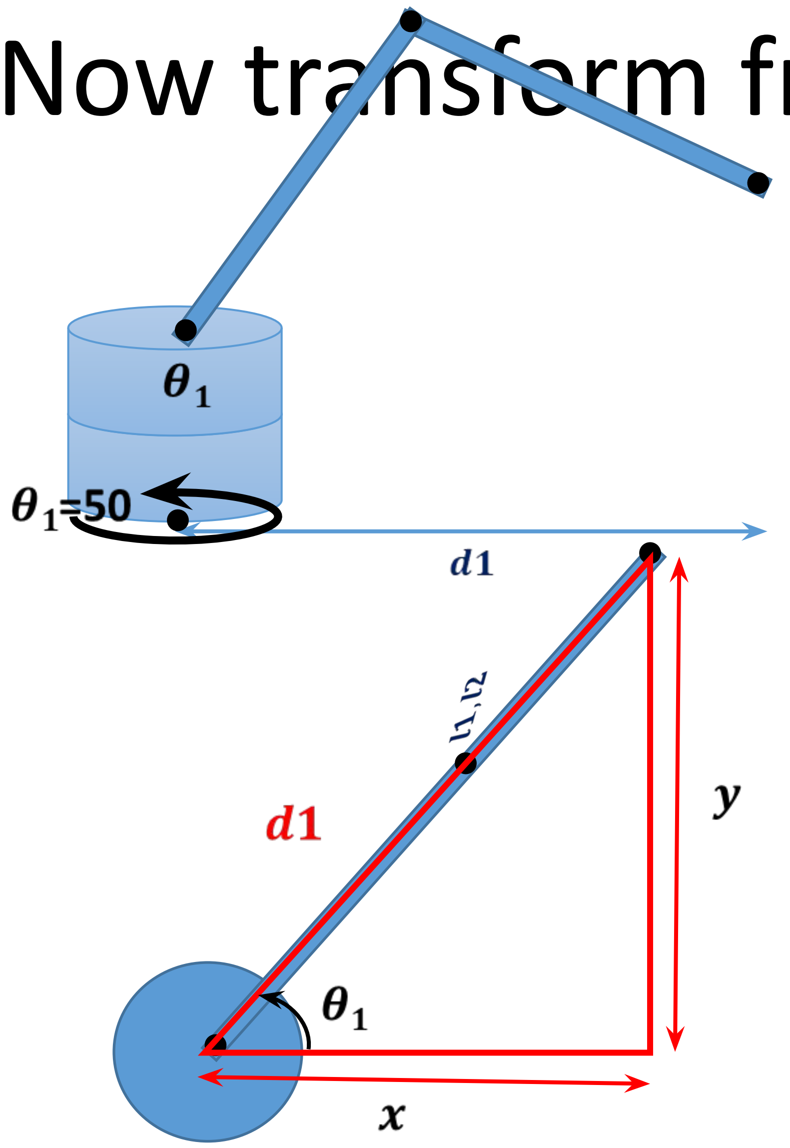
$$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$$

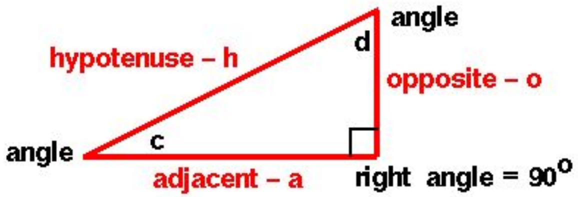


- $Z = h + (d3 - d6)$
- $d1 = d4 + d5$

- *So, d1 is the current length of arm from base*
- *Z is the height of arm endpoint*

Now transform from top view

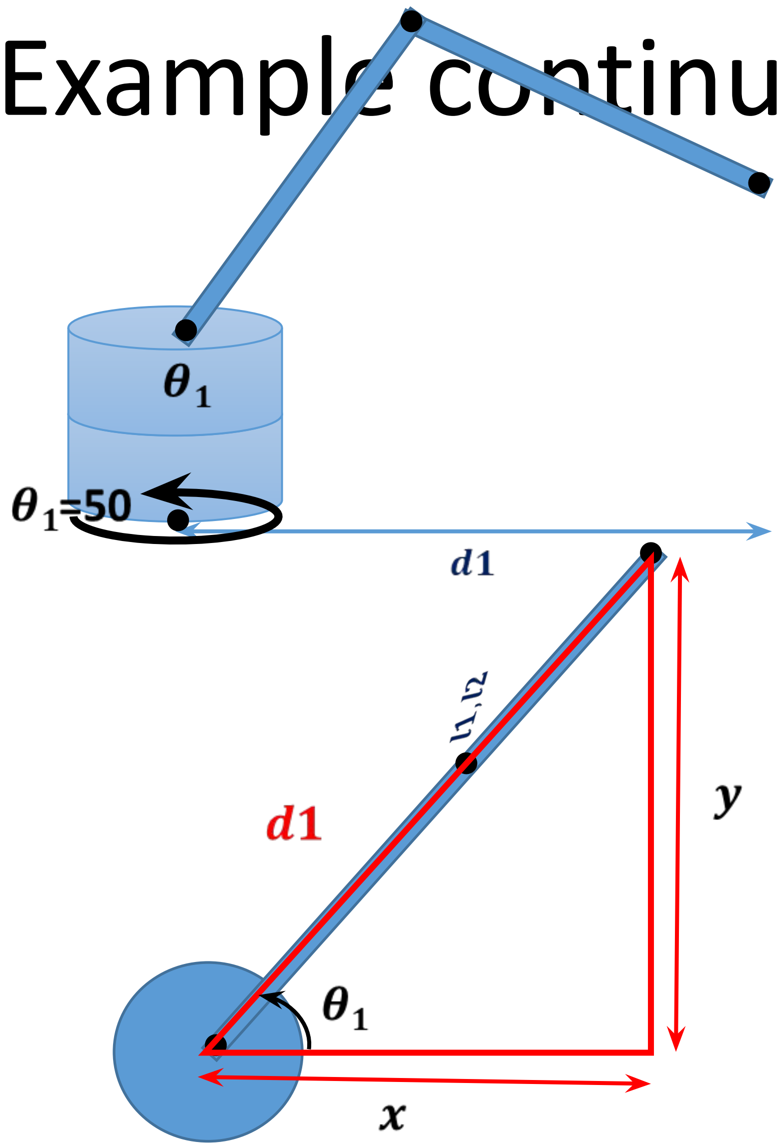


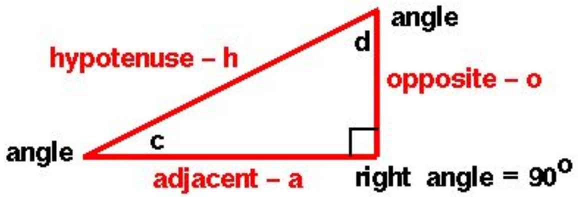


$$\cos\theta = \frac{A}{H}; \sin\theta = \frac{O}{H}; \tan\theta = \frac{O}{A}$$

- $\cos\theta_1 = \dfrac{x}{d1};$
- $x = \cos\theta_1 * d1$
- $\sin\theta_1 = \dfrac{y}{d1};$
- $y = \sin\theta_1 * d1$
- $Position\ is:\ (x, y, z)$

Example continuation…





$$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$$

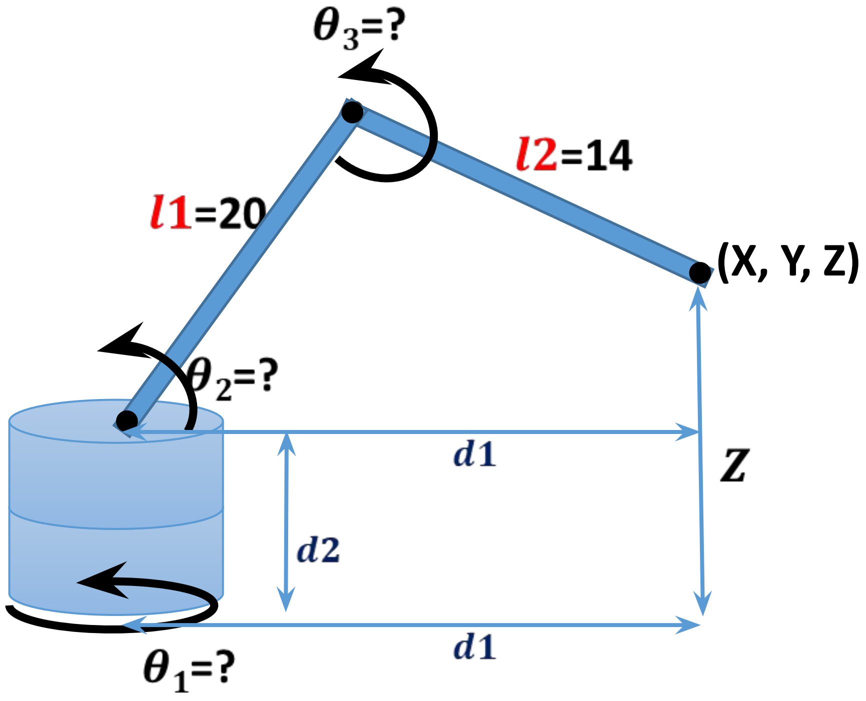
- $Z = d2 + (d3 - d6)$
- $Z = 18 + 17.32 - 5.916 = 29.4$
- $d1 = d4 + d5 = 10 + 12.69 = 22.69$
- $x = cos\theta_1 * d1(d1 + l3)$
- $x = cos50 * 22.69 = 14.58$
- $y = sin50 * 22.69 = 17.38$
- *Position is*:
- $(x, y, z) = (14.58, 17.38, 29.4)$

$l1$=20"
$l2$=14"
$l3$=8"
h=18"
$\theta_1 = 50$
$\theta_2 = 60$
$\theta_3 = 95$
$\theta_x = 30$
$\theta_y = 65$
$d3 = 17.32$
$d4 = 10$
$d6 = 5.91$
$d5 = 12.69$

Inverse or Reverse kinematics using trigonometry

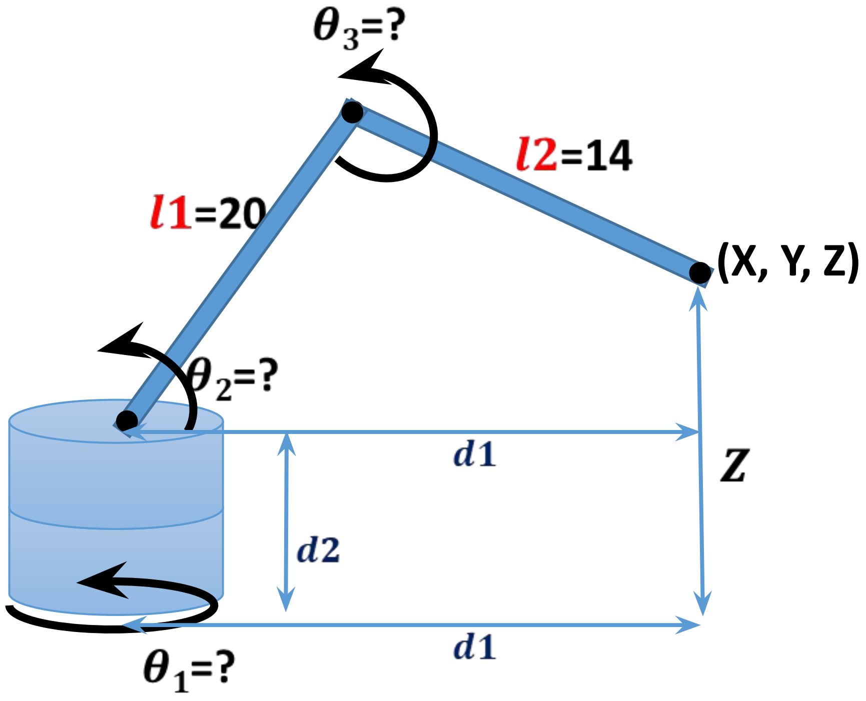
Coordinate → Angle
(X, Y, Z) → $(\theta_1, \theta_2, \theta_3)$
Where Height, length of arm-1 and Length of Arm-2 are fixed

Reverse Kinematics (coordinate to angle)



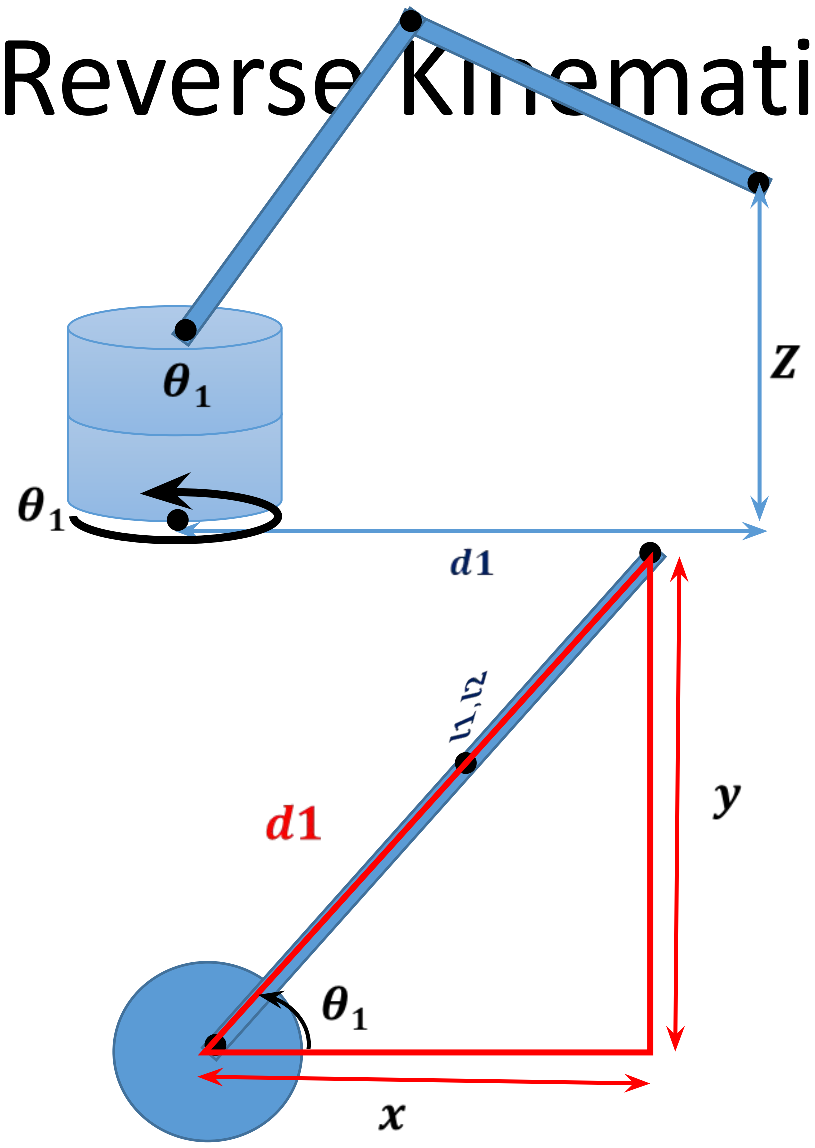
- $x = Position\ on\ front$
- $y = Position\ on\ left\ or\ right$
- $z = Position\ on\ height$
- $l1 = Length\ og\ first\ arm$
- $l2 = Length\ of\ second\ arm$
- $l3 = Length\ of\ third\ arm$
- $d2 = Height\ of\ base\ from\ ground$

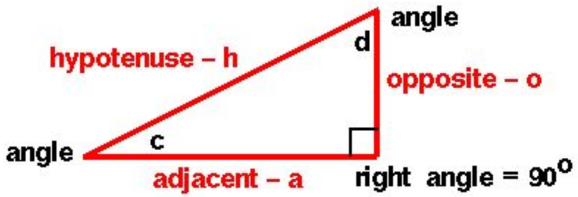
Reverse Kinematics Example value



- $x = 14.58$
- $y = 17.38$
- $z = 29.4$
- $l1 = 20$
- $l2 = 14$
- $l3 = 8$
- $d2 = 18$

Reverse Kinematics

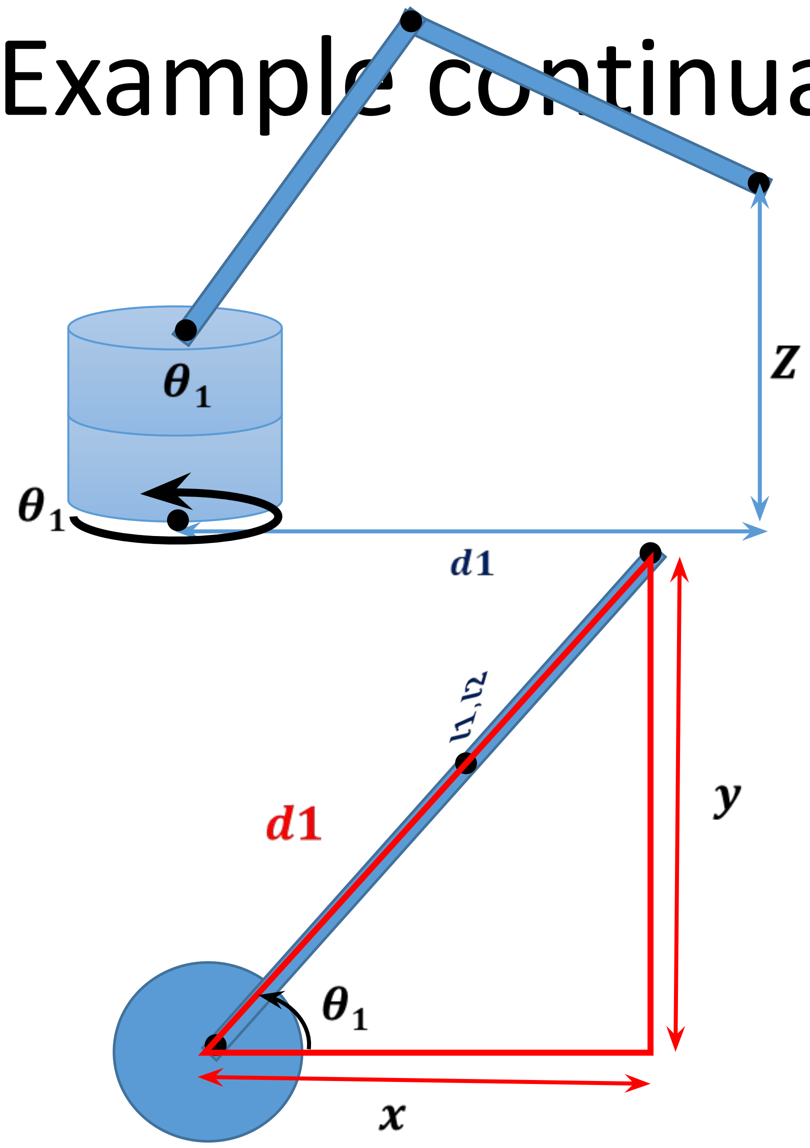


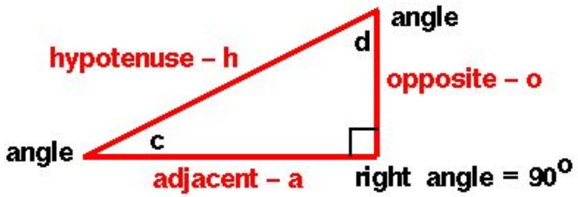


$$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$$

- $(d1 + l3)^2 = x^2 + y^2$
- $d1 = \sqrt{x^2 + y^2}$
- $Cos\theta_1 = \frac{x}{d1}$
- $\theta_1 = Cos^{-1}(\frac{x}{d1})$

Example continuation…



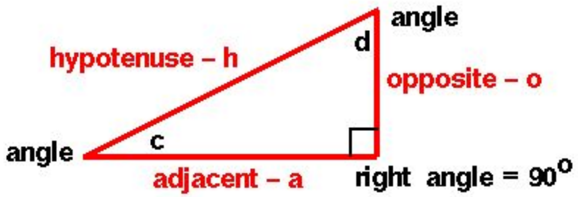


$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$

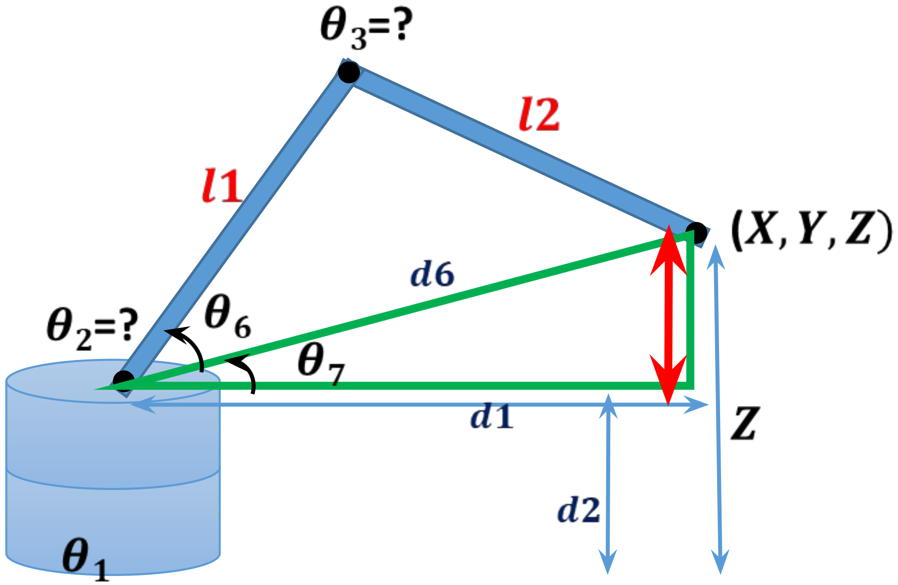
- $d1 = \sqrt{x^2 + y^2}$
- $d1 = \sqrt{14.58^2 + 17.38^2}$
- $d1 = 22.69$
- $\theta_1 = Cos^{-1}\left(\frac{x}{d1}\right)$
- $\theta_1 = Cos^{-1}\left(\frac{14.58}{22.69}\right)$
- $\theta_1 = 50$

$x = 14.58$
$y = 17.38$
$z = 29.4$
$l1 = 20$
$l2 = 14$
$l3 = 8$
$d2 = 18$

Reverse Kinematics

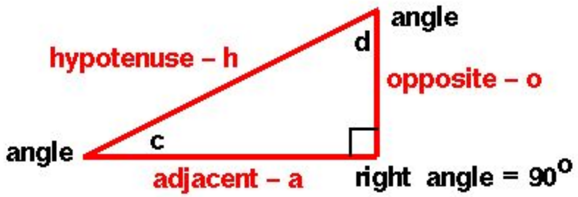


$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$

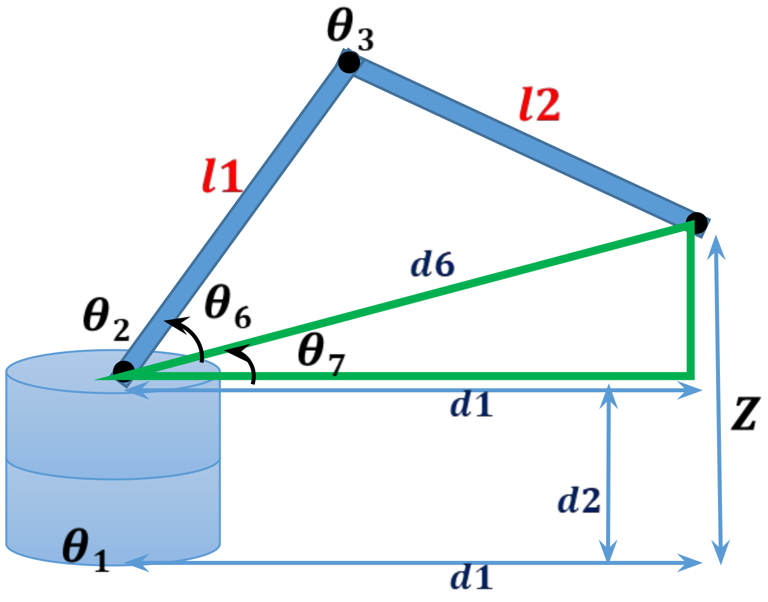


- $d6^2 = d1^2 + (z - d2)^2$
- $d6 = \sqrt{d1^2 + (z - d2)^2}$
- $Cos\theta_7 = \frac{d1}{d6};$
- $\theta_7 = Cos^{-1}(\frac{d1}{d6})$

Example continuation…



$$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$$

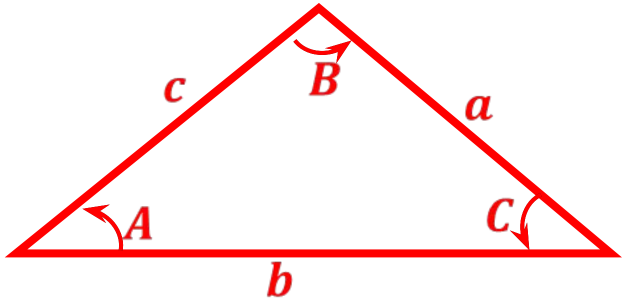


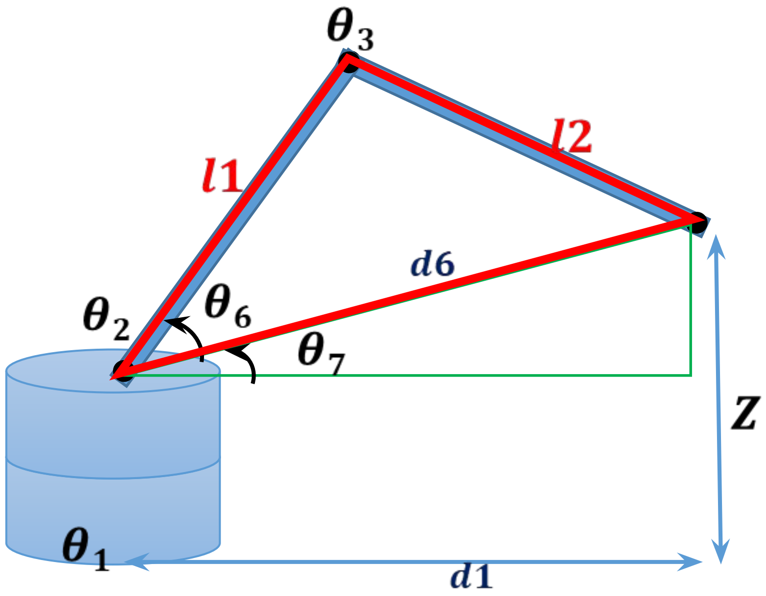
$x = 14.58$
$y = 17.38$
$z = 29.4$
$l1 = 20$
$l2 = 14$
$l3 = 8$
$d2 = 18$
$d1 = 22.69$
$\theta_1 = 50$
$d6 = 25.4$
$\theta_7 = 26.7$

- $d6 = \sqrt{d1^2 + (z - d2)^2}$
- $d6 = \sqrt{22.69^2 + (29.4 - 18)^2}$
- $d6 = 25.4$
- $\theta_7 = Cos^{-1}(\frac{d1}{d6})$
- $\theta_7 = Cos^{-1}(\frac{22.69}{25.4})$
- $\theta_7 = 26.7$

$x = 14.58$
$y = 17.38$
$z = 29.4$
$l1 = 20$
$l2 = 14$
$l3 = 8$
$d2 = 18$
$d1 = 22.69$
$\theta_1 = 50$

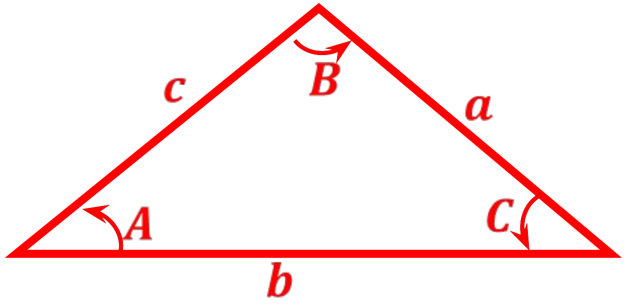
Reverse Kinematics

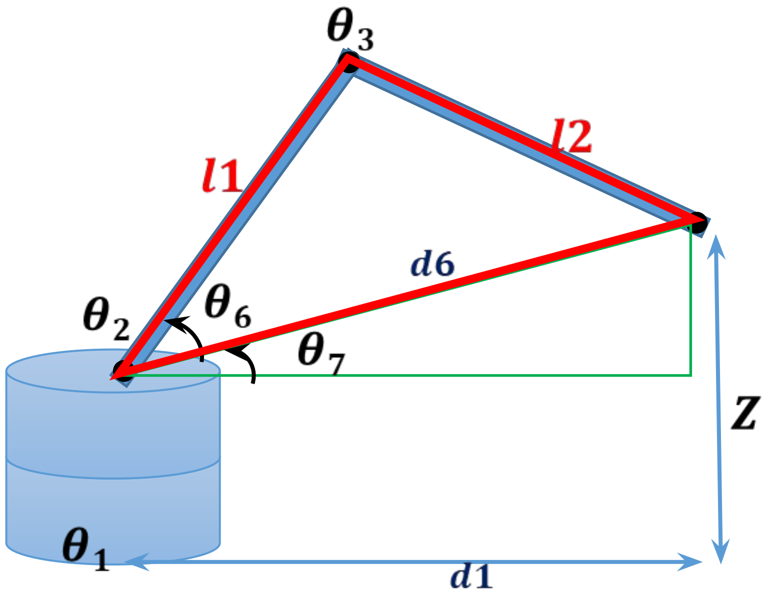




- $b^2 = a^2 + c^2 - 2acCosB$
- $Cos\theta_3 = \left(\frac{l1^2+l2^2-d6^2}{2*l1*l2}\right);$
- $\theta_3 = Cos^{-1}\left(\frac{l1^2+l2^2-d6^2}{2*l1*l2}\right)$
- $Cos\theta_6 = \left(\frac{l1^2+d6^2-l2^2}{2*l1*d6}\right);$
- $\theta_6 = Cos^{-1}\left(\frac{l1^2+d6^2-l2^2}{2*l1*d6}\right)$
- $\theta_4 = \theta_2 + \theta_3$

Example continuation…

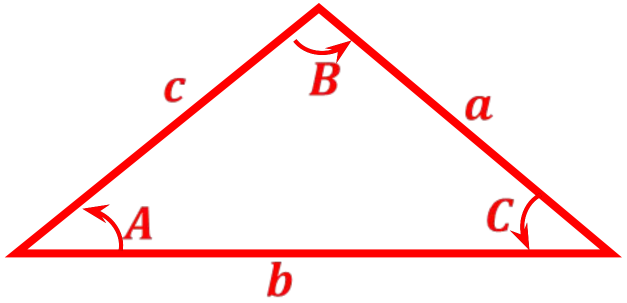


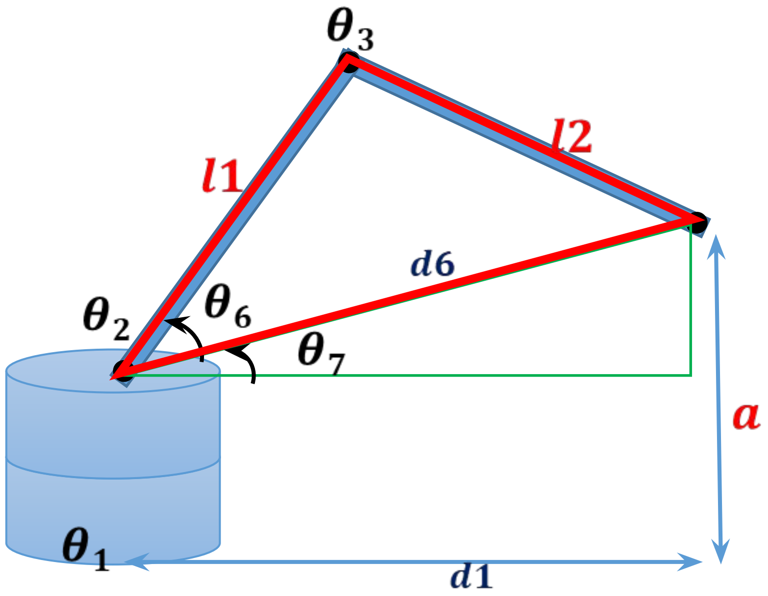


- $\theta_3 = Cos^{-1}\left(\frac{l1^2 + l2^2 - d6^2}{2 * l1 * l2}\right)$
- $\theta_3 = Cos^{-1}\left(\frac{20^2 + 14^2 - 25.4^2}{2 * 20 * 14}\right)$
- $\theta_3 = 95$

$x = 14.58$
$y = 17.38$
$z = 29.4$
$l1 = 20$
$l2 = 14$
$l3 = 8$
$d2 = 18$
$d1 = 22.69$
$\theta_1 = 50$
$d6 = 25.4$
$\theta_7 = 26.7$

Example continuation…

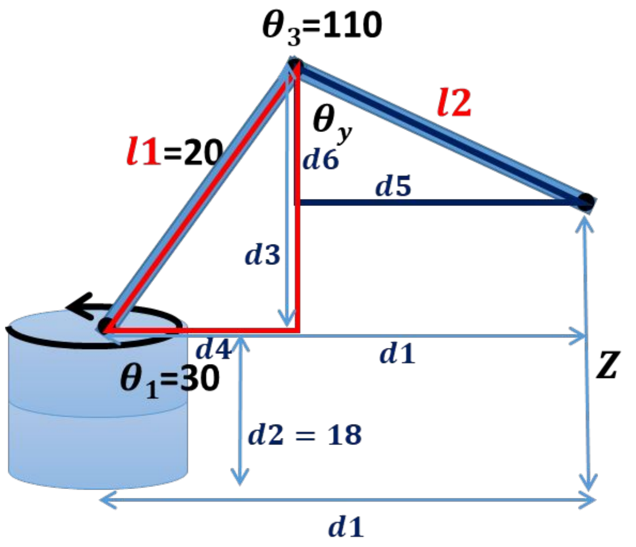


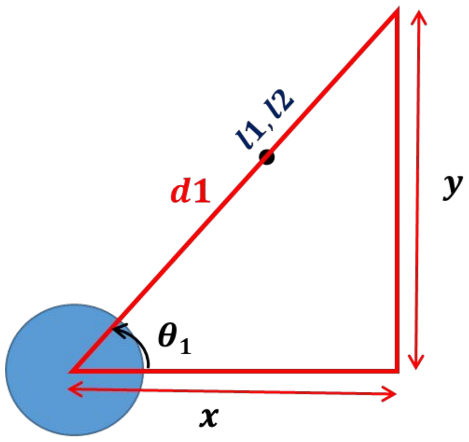


- $\theta_6 = Cos^{-1}\left(\frac{l1^2+d6^2-l2^2}{2*l1*d6}\right)$
- $\theta_6 = Cos^{-1}\left(\frac{20^2+25.4^2-14^2}{2*20*25.4}\right)$
- $\theta_6 = 33.3$
- $\theta_2 = 33.3 + 26.7$
- $\theta_2 = 60$
- $(\theta_1, \theta_2, \theta_3, \theta_4)$
- $(50, 60, 95)$

$x = 14.58$
$y = 17.38$
$z = 29.4$
$l1 = 20$
$l2 = 14$
$l3 = 8$
$d2 = 18$
$d1 = 22.69$
$\theta_1 = 50$
$d6 = 25.4$
$\theta_7 = 26.7$

Example-2

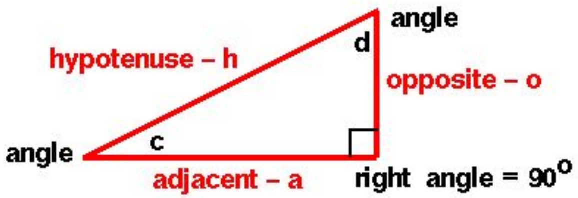




- *Costh=a/h*
- *d4 =Cos45*20=14.14*
- *d3=Sin45*20=14.14*
- θ_x= 180-(90+45)=45
- θ_y= θ_3- θ_x=110-45=65
- d6=Cos65*14=5.91
- d5=Sin65*14=12.68
- d1=d4+d5=14.14+12.68=26.8
- Z=d2+(d3-d6)=18+(14.14-5.91)=26.23
- X=cos30*d1=cos30*26.8=23.2
- Y=sin30*26.8=13.41
- (X,Y,Z)= (23.2, 13.41, 26.23)

- *l1=20"*
- *l2=14"*
- *l3=8"*
- *d2=18"*

- $\theta_1 = 30$
- $\theta_2 = 45$
- $\theta_3 = 110$



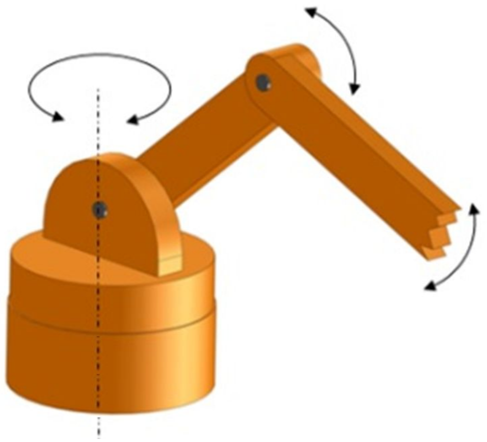
$$\cos\theta = \frac{A}{H}; \sin\theta = \frac{O}{H}; \tan\theta = \frac{O}{A}$$

End of Forward and Inverse Kinematics using trigonometry

https://www.youtube.com/watch?v=1-FJhmey7vk

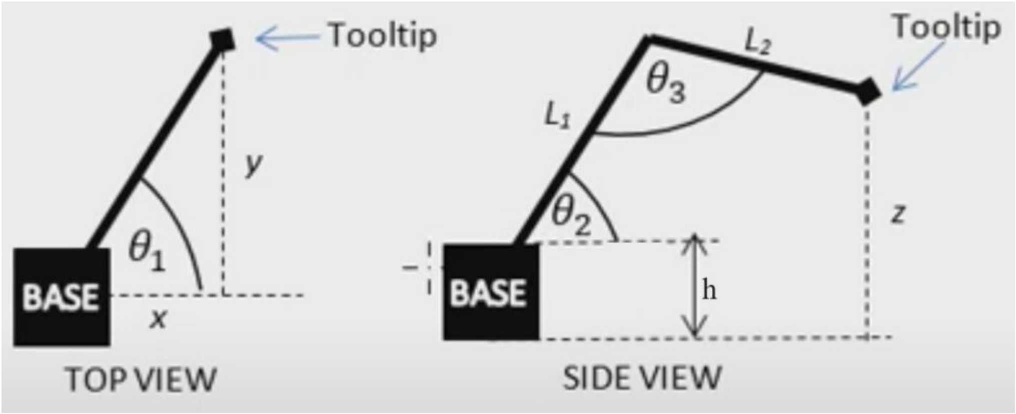
Forward kinematics using DH parameter and Homogeneous transformation matrix

Forward Kinematics



Forward kinematics refers to process of obtaining position and velocity of end effector, given the known joint angles and angular velocities.

For example, if shoulder and elbow joint angles are given for arm in sagittal plane, the goal is to find Cartesian coordinates of wrist/fist.



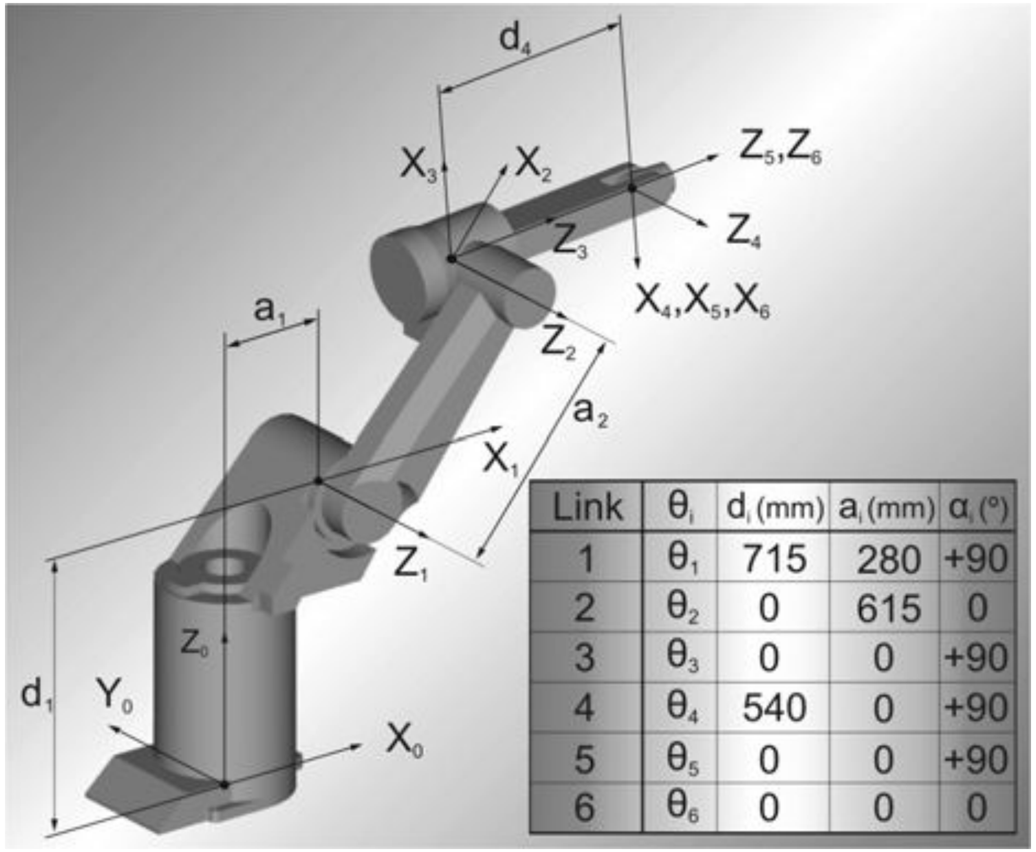
Forward Kinematics

- Angles → coordinate
- $(\boldsymbol{\theta_1}, \boldsymbol{\theta_2}, \boldsymbol{\theta_3})$ → (X, Y, Z)
- Where Height, length of arm-1 and Length of Arm-2 are fixed

https://www.youtube.com/watch?v=D3w3ZANOy3s

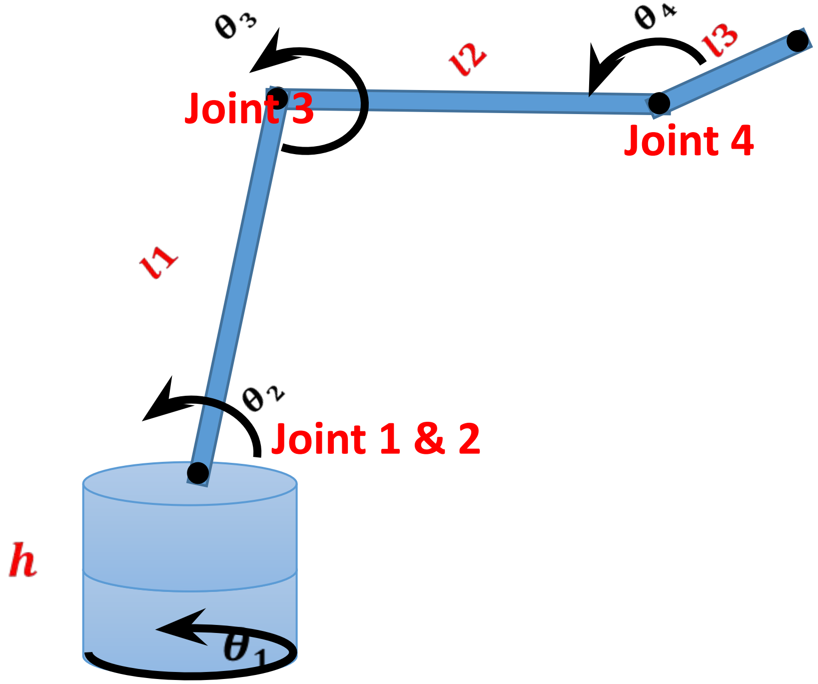
DH (Denavit-Hartenberg) parameters

DH parameters are used in robot manipulator to describe the axis orientations and arm lengths. The 4 DH parameters are called theta, d, alpha and a. DOF stands for degrees of freedom.

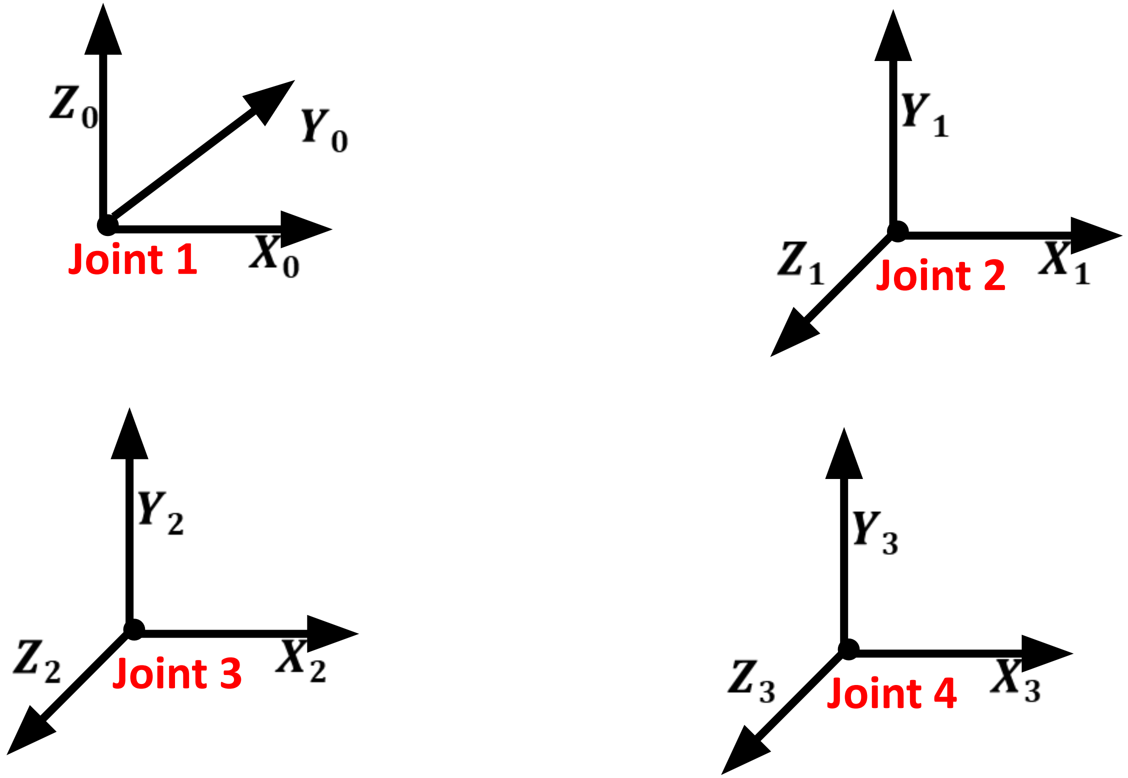


Link	θ_i	d_i (mm)	a_i (mm)	α_i (°)
1	θ_1	715	280	+90
2	θ_2	0	615	0
3	θ_3	0	0	+90
4	θ_4	540	0	+90
5	θ_5	0	0	+90
6	θ_6	0	0	0

Example



θ_3
$l2$
θ_4
$l3$
Joint 3
Joint 4
$l1$
θ_2
Joint 1 & 2
h
θ_1



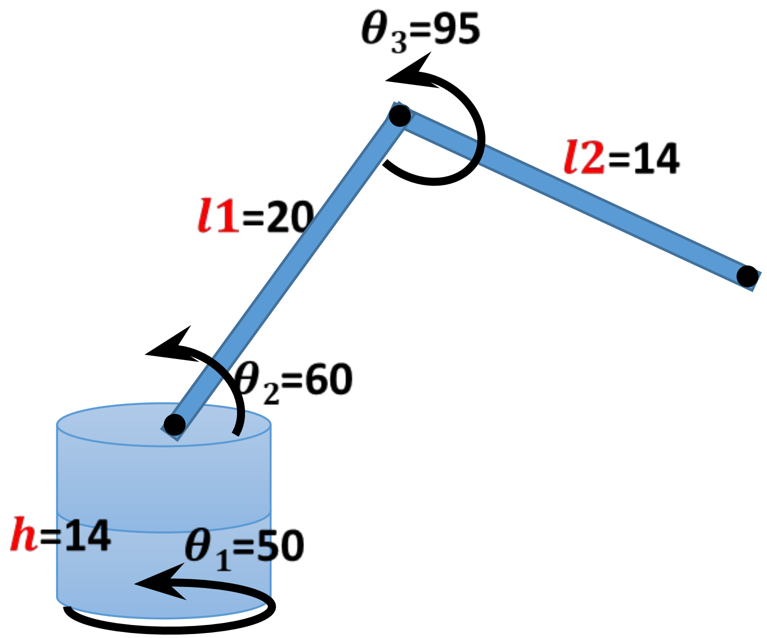
Z_0
Y_0
Joint 1
X_0
Y_1
Z_1
Joint 2
X_1
Y_2
Z_2
Joint 3
X_2
Y_3
Z_3
Joint 4
X_3

DH parameters calculation for two points

- θ_i: Joint angle
- α_i: Link Twist
- a_i: Link length
- d_i: Link offset
- DH convention link with the Homogeneous transformation T_i with Four Basic Transformation
- T_i= Rot$Z\theta_i$, TransZd_i, TransXa_i, Rot $X\alpha_i$

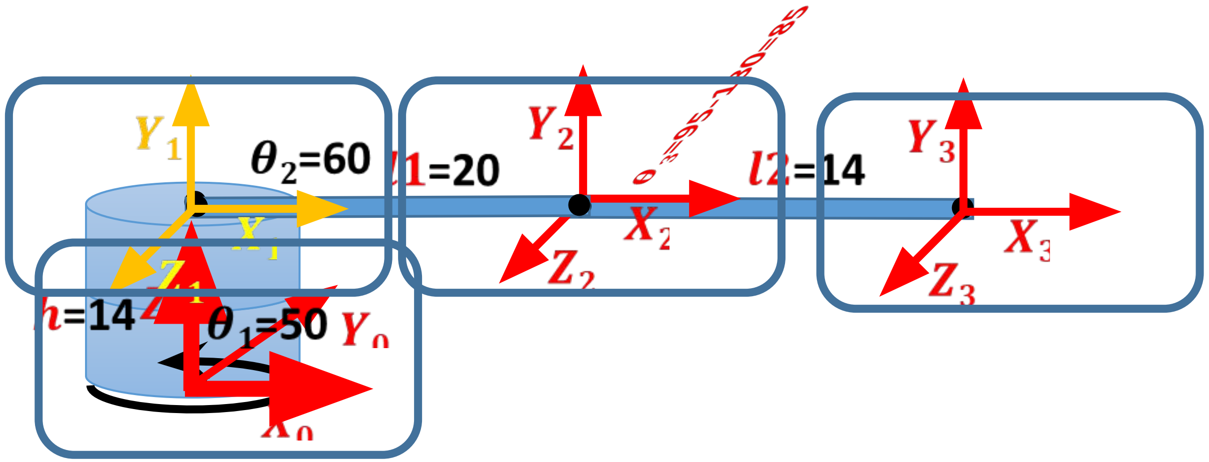
- θ: Alignment of X_n with X_{n+1} with respect to rotating axis Z_n + Rotation of angle of θ_n
- d : Translation along Z axis
- a: Translation along X axis
- α: Alignment of Z_n with Z_{n-1} with respect to rotating axis X_n+ Rotation of angle of θ_n
- Rotation of X for an angle of α_i

Transform arm to find DH parameters



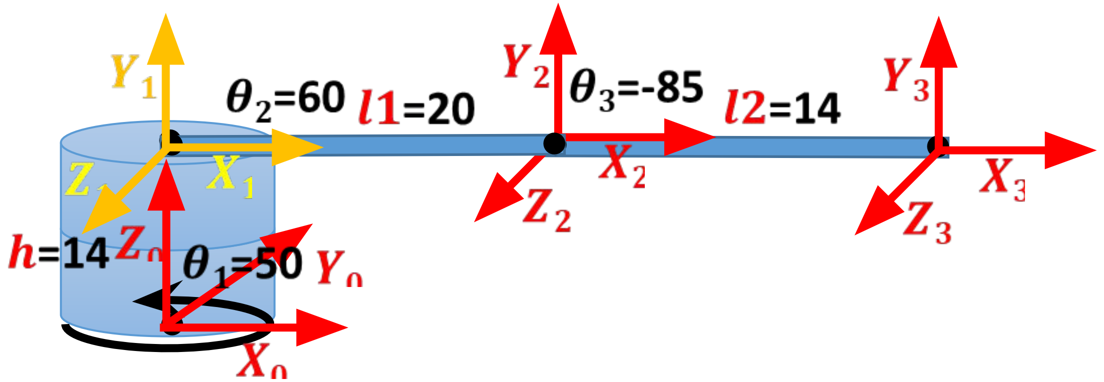
h=18"
$l1$=20"
$l2$=14"
$l3$=8"

$\theta_1 = 50$
$\theta_2 = 60$
$\theta_3 = 95$



There are 4 frames in this arm

Example of DH parameters



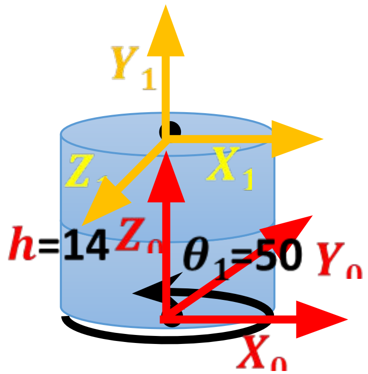
Joint				
1	0+50	90	0	18
2	0+60	0	20	0
3	0-85	0	14	0

Let us consider two frames: frame 0 and frame 1

- θ: defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α: defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a: is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d: is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of first Joint

Joint				
1	0+50			
2				
3				



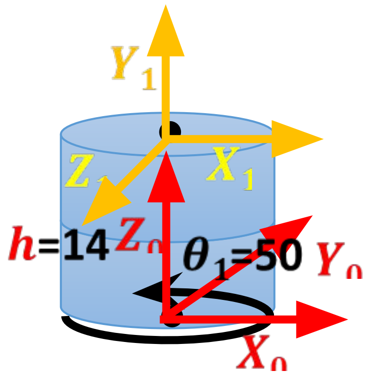
As we can see in the figure, the directions of X_0 and X_1 are same (on right direction) so we do not need to add any rotation around Z_0. So, θ would be the value of only θ_1=50.

Let us consider two frames: frame 0 and frame 1

- θ: defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α: defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a: is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d: is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of first Joint

Joint				
1	0+50	90		
2				
3				



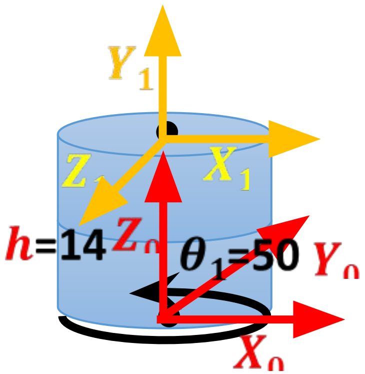
We have to rotate Z_0 anticlockwise 90 degree around X_1 to align with Z_1. So the value of α will be 90.

Let us consider two frames: frame 0 and frame 1

- θ: defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α: defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a: is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d: is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of first Joint

Joint				
1	0+50	90	0	
2				
3				



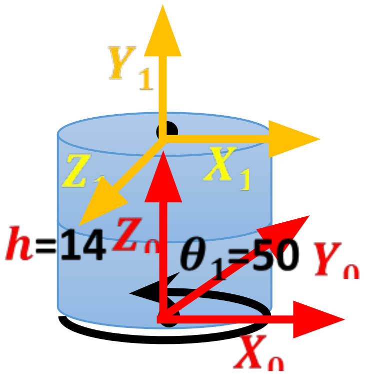
Center of first and second frame are in the same alignment in X_1 direction. So, the value of a will 0.

Let us consider two frames: frame 0 and frame 1

- θ: defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α: defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a: is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d: is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of first Joint

Joint				
1	0+50	90	0	18
2				
3				



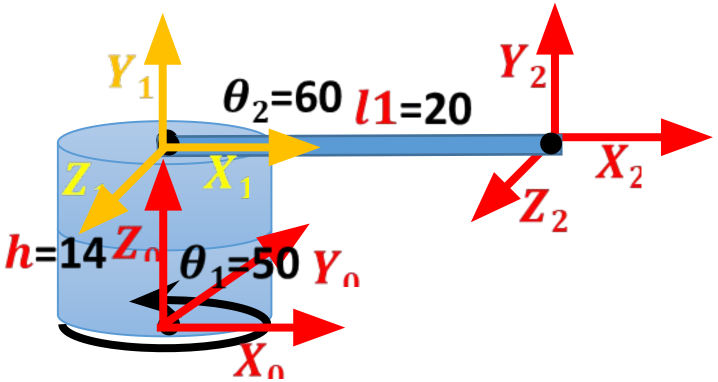
The distance between the center of first and second frame is 18 in Z_1 direction. So, the value of d will 18.

Let us consider two frames: frame 0 and frame 1

- θ: defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α: defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a: is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d: is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of second Joint

Joint				
1	0+50	90	0	18
2	0+60			
3				



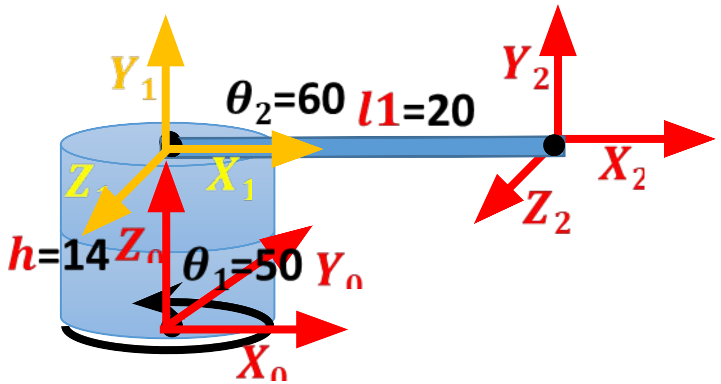
As we can see in the figure, the directions of X_1 and X_2 are same. So again we do not need to add any rotation around Z_1. So, θ would be the value of only θ_2 that is 60.

Let us consider two frames: frame 0 and frame 1

- θ: defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α: defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a: is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d: is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of second Joint

Joint				
1	0+50	90	0	18
2	0+60	0		
3				



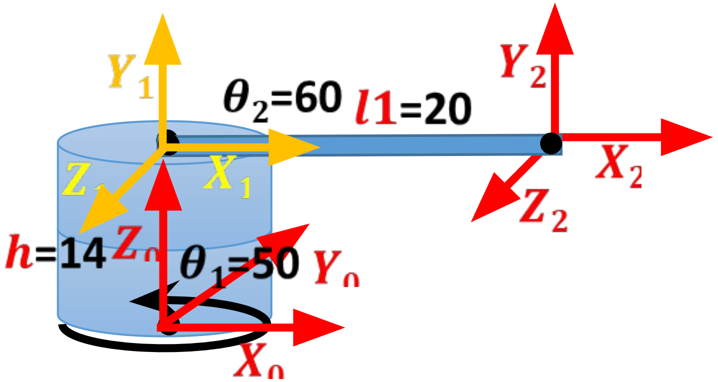
We do not need to rotate Z_1 around X_2 to align with Z_2. So the value of α will be 0.

Let us consider two frames: frame 0 and frame 1

- θ: defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α: defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a: is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d: is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of second Joint

Joint				
1	0+50	90	0	18
2	0+60	0	20	
3				



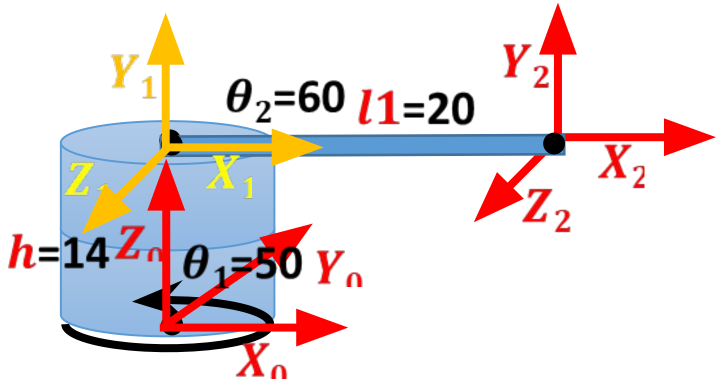
The distance between the center of first and second frame is 20 in X_2 direction. So, the value of a will 20.

Let us consider two frames: frame 0 and frame 1

- θ: defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α: defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a: is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d: is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of second Joint

Joint				
1	0+50	90	0	18
2	0+60	0	20	0
3				



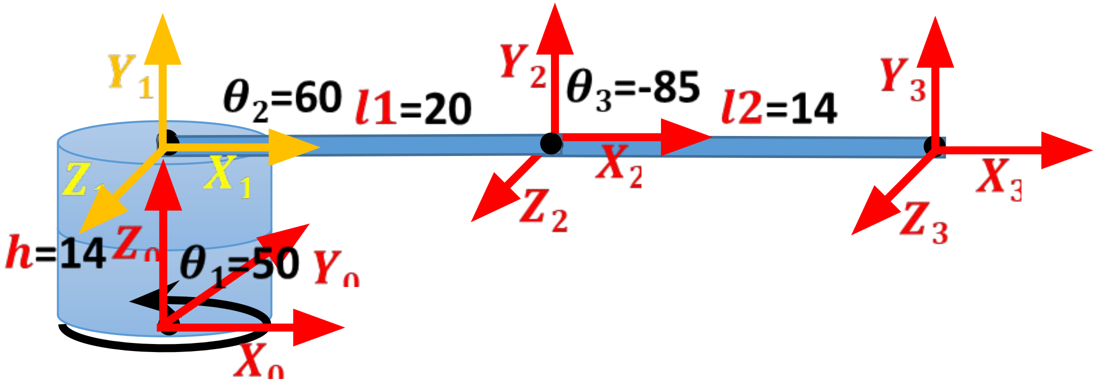
Center of first and second frame are in the same alignment in Z_2 direction. So, the value of d will 0.

Let us consider two frames: frame 0 and frame 1

- θ: defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α: defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a: is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d: is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

Joint				
1	0+50	90	0	18
2	0+60	0	20	0
3	0-85			

DH parameters of third Joint



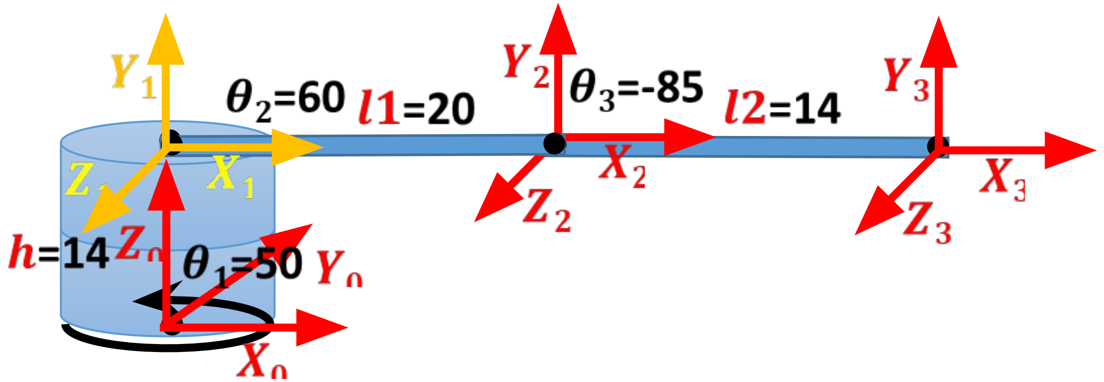
As we can see in the figure, the directions of X_2 and X_3 are same (on right direction) so we do not need to add any rotation around Z_2. So, θ would be the value of θ_3 that is -85.

Let us consider two frames: frame 0 and frame 1

- θ: defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α: defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a: is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d: is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of third Joint

Joint				
1	0+50	90	0	18
2	0+60	0	20	0
3	0-85	0	14	0



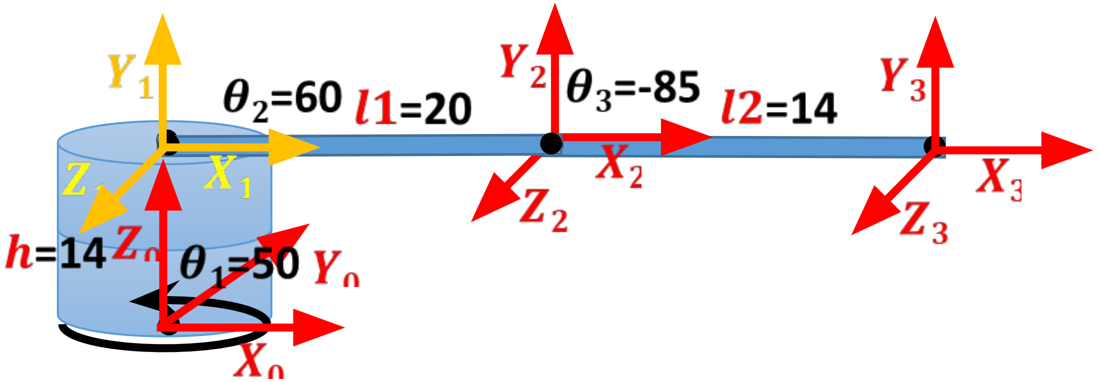
We do not need to rotate Z_2 around X_3 as it is already aligned with Z_3. So the value of α will be 0.

Let us consider two frames: frame 0 and frame 1

- θ**:** defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α**:** defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a**:** is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d**:** is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of third Joint

Joint				
1	0+50	90	0	18
2	0+60	0	20	0
3	0-85	0	14	0



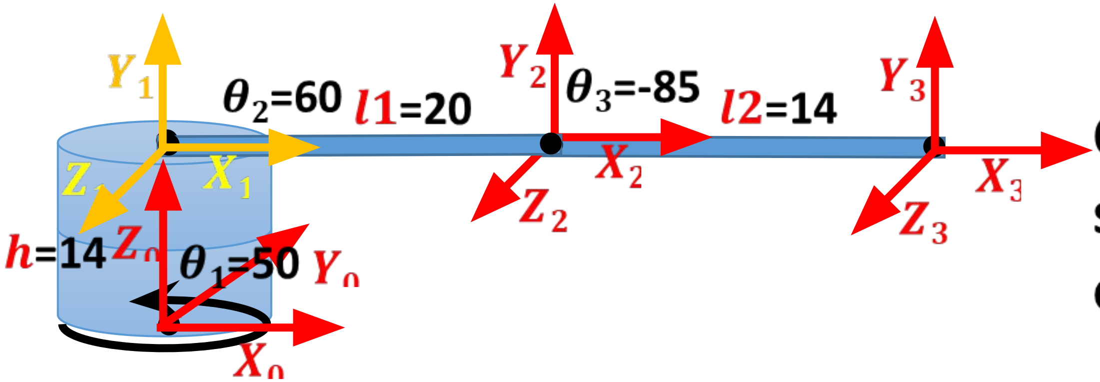
The distance between the center of thired and forth frame is 14 in X_3 direction. So, the value of a will 14.

Let us consider two frames: frame 0 and frame 1

- θ**:** defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α**:** defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a**:** is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d**:** is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of third Joint

Joint				
1	0+50	90	0	18
2	0+60	0	20	0
3	0-85	0	14	0

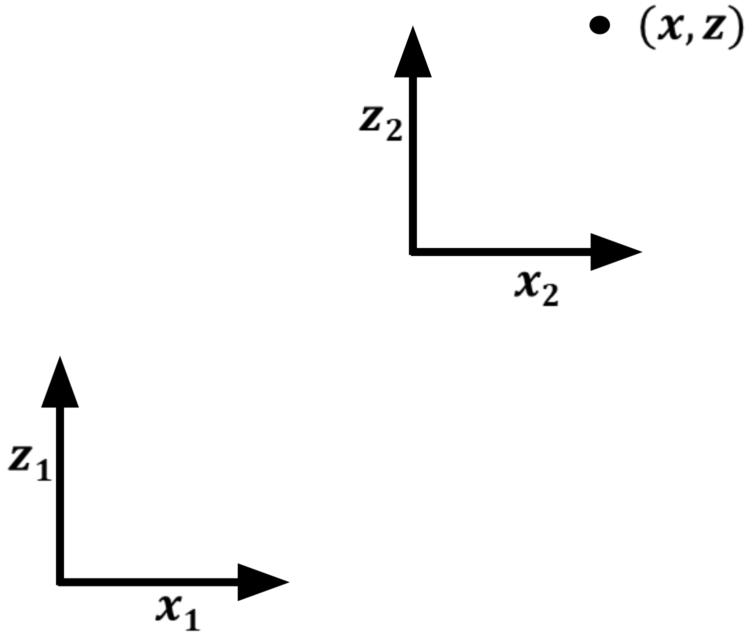


Center of first and second frame are in the same alignment in Z_3 direction. So, the value of d will 0.

Let us consider two frames: frame 0 and frame 1

- θ: defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α: defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a: is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d: is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

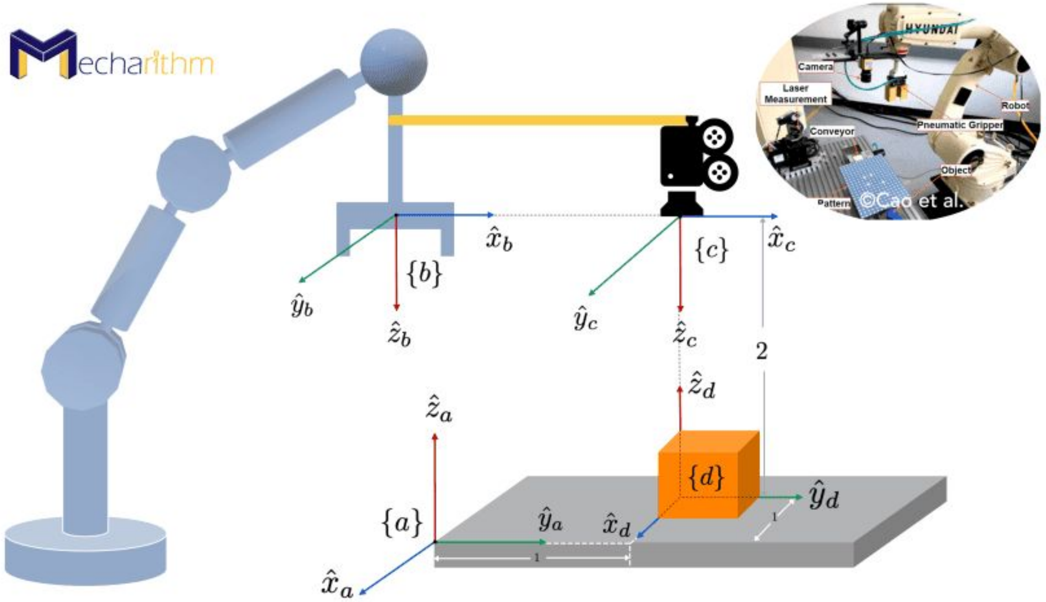
Homogeneous transformation



Homogeneous transformation matrix will transform the axis of pint (x, z) from the (x2, z2) to (x1, z1)

Homogeneous transformation matrix

- The homogenous transformation matrix can act on a vector or a frame and displaces (rotating and translating) it. DH convention link with the Homogeneous transformation T_i is the combination of Four Basic Transformation
- T_i= Rot$Z\theta_i$, TransZd_i, TransXa_i, Rot Xα_i



DH parameters to Homogeneous transformation

- T_i= Rot$Z\theta_i$, TransZd_i, TransXa_i, Rot Xα_i
  - Rotation of angle of θ_i with respect to Z
  - Translation along Z with for value of d_i
  - Translation along X with for value of a_i
  - Rotation of X for an angle of α_i
- Where:
  - α_i: Z axis angle of two points (Link Twist)
  - a_i: Distance in Z axis considering the axis of Joint 4 (Link length)
  - d_i: Distance in Z axis considering the axis of Joint 3 (Link offset)
  - θ_i: X axis angle of two points (Joint angle)

$$\text{Rot}Z\theta_i = \begin{bmatrix} cos\theta_i & -sin\theta_i & 0 & 0 \\ sin\theta_i & cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{TransX}a_i = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{TransZ}d_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

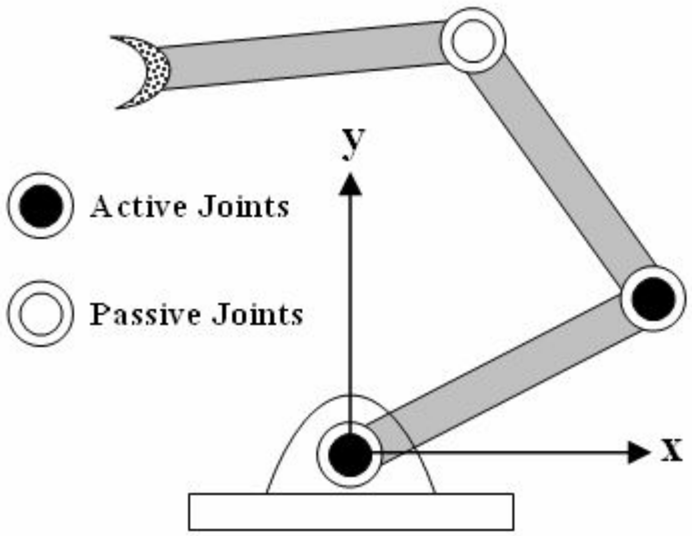
$$\text{Rot X}\alpha_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\alpha_i & -sin\alpha_i & 0 \\ 0 & sin\alpha_i & cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

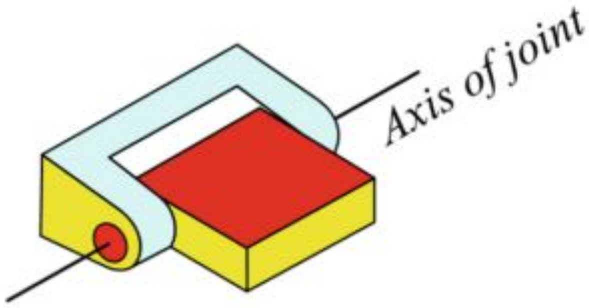
- T_i= Rot$Z\theta_i$, TransZd_i, TransXa_i, Rot Xα_i

$$T_i = \begin{bmatrix} cos\theta_i & -cos\alpha_i sin\theta_i & sin\alpha_i sin\theta_i & a_i cos\theta_i \\ sin\theta_i & cos\alpha_i cos\theta_i & -sin\alpha_i cos\theta_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

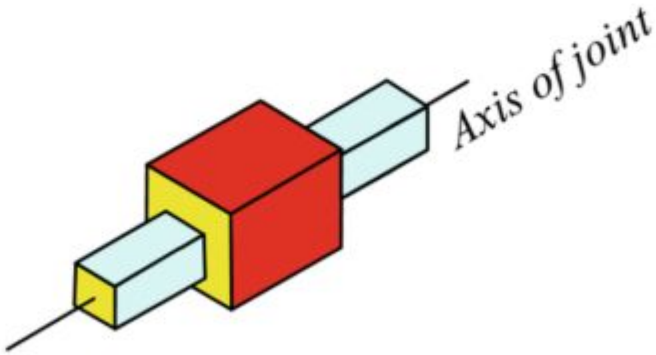
Type of joint

- Passive Joint (No movement)
- Active Joint
  - Revolute Joint (Rotation)
  - Prismatic Joint (Linear Shift)





Revolute joint



Prismatic joint

Homogeneous transformation matrixes for all joints

Joint				
1	0+50	90	0	18
2	0+60	0	20	0
3	0-85	0	14	0

- $$T_i = \begin{bmatrix} cos\theta_i & -cos\alpha_i sin\theta_i & sin\alpha_i sin\theta_i & a_i cos\theta_i \\ sin\theta_i & cos\alpha_i cos\theta_i & -sin\alpha_i cos\theta_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

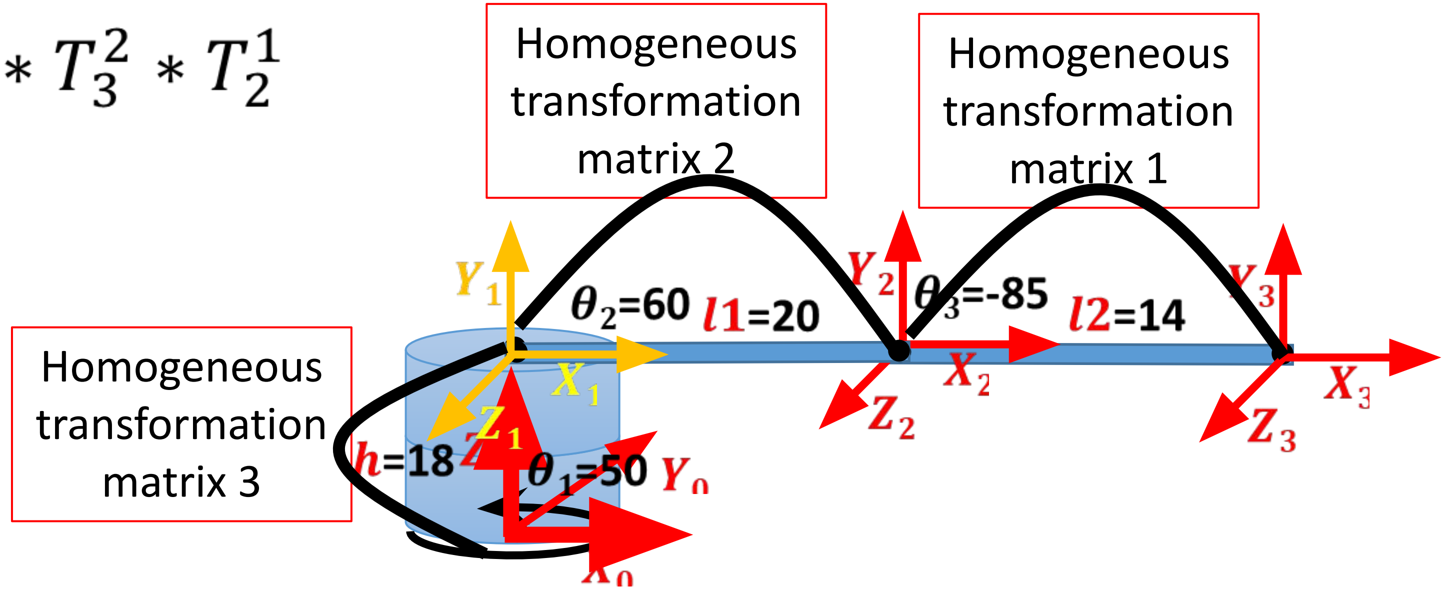
$$T_2^1 = \begin{bmatrix} cos50 & -cos90sin50 & sin90sin50 & 0*cos50 \\ sin50 & cos90cos50 & -sin90cos50 & 0*sin50 \\ 0 & sin90 & cos90 & 18 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0.643 & 0 & 0.766 & 0 \\ 0.766 & 0 & -0.643 & 0 \\ 0 & 1 & 0 & 18 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} cos60 & -cos0sin60 & sin0sin60 & 20*cos60 \\ sin60 & cos0cos60 & -sin0cos60 & 20*sin60 \\ 0 & sin0 & cos0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0.5 & -0.866 & 0 & 10 \\ 0.866 & 0.5 & 0 & 17.321 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} cos95 & -cos0sin95 & sin0sin95 & 20*cos95 \\ sin95 & cos0cos95 & -sin0cos95 & 20*sin95 \\ 0 & sin0 & cos0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0.087 & 0.996 & 0 & 1.22 \\ -0.996 & 0.087 & 0 & -13.947 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Homogeneous transformation matrixes

- $T = T_4^3 * T_3^2 * T_2^1$



$$\begin{bmatrix} 0.087 & 0.996 & 0 & 1.22 \\ -0.996 & 0.087 & 0 & -13.947 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathrm{X} \begin{bmatrix} 0.5 & -0.866 & 0 & 10 \\ 0.866 & 0.5 & 0 & 17.321 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathrm{X} \begin{bmatrix} 0.643 & 0 & 0.766 & 0 \\ 0.766 & 0 & -0.643 & 0 \\ 0 & 1 & 0 & 18 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.583 & 0.272 & 0.766 & 14.584 \\ 0.694 & 0.324 & -0.643 & 17.38 \\ -0.423 & 0.906 & 0 & 29.404 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(X, Y, Z)= (14.584, 17.38, 29.404)

$$\begin{bmatrix} 0.583 & 0.272 & 0.766 & 14.584 \\ 0.694 & 0.324 & -0.643 & 17.38 \\ -0.423 & 0.906 & 0 & 29.404 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$