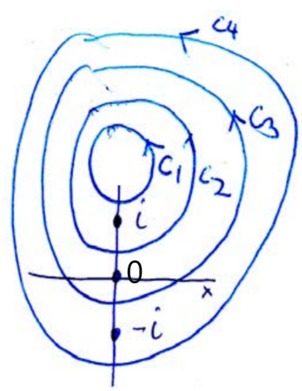
Final practice problem

1 PART-1: Integral

1. Evaluate
$$\int_{(0,1)}^{(2,5)} (3x+y)dx + (2y-x)dy$$
(i) along the straight lines from $(0,1)$ to $(2,1)$ and then from $(2,1)$ to $(2,5)$, and (ii) along the parabola $y = x^2 + 1$.

2. Evaluate the following integral using cauchy integral theorem:
$$\int_{|z|=3} \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)} dz$$

3. Let
$$f(z) = \frac{\sin(z^2) + \cos(\pi z)}{z(z^2+1)(z+1)}$$
Compute $\int f(z)dz$ over each of the contours/closed curves C_1, C_2, C_3 and C_4 shown below.



4. Verify the Cauchy-Goursat theorem for the function $f(z) = z^2 + 5z$ around the closed curve C defined by a half circle $|z| = 1$ from the point $(1,0)$ to $(-1,0)$ in the counterclockwise direction and then the straight line from $(-1,0)$ to $(1,0)$.

5. Evaluate the integral $\oint_C \overline{z}^2 dz$ where C is the boundary of the triangle with vertices $(1,1), (2,1)$ and $(2,3)$.

6. Let

$$f(z) = \frac{z+1}{z^3(z^2+1)}$$

find the integral $\int_C f(z)dz$ where $C : |z| = 0.5$.

7. Find,

$$\mathcal{L}\{\sin(at)\}, \mathcal{L}\{\cos(at)\}, \mathcal{L}\{\sinh(at)\}, \mathcal{L}\{\cosh(at)\}$$

2 PART-2: Laplace Transform

8. Find the Laplace transform of the function,

$$f(t) = e^{-2t}t[\sin(t)\cos(t)u(t-\pi)]$$

9. Find the Laplace transform of the function,

$$f(t) = \begin{cases} 0, & 0 < t < \pi \\ \cos(2t), & \pi < t < 3\pi \\ 4-2t, & t > 3\pi \end{cases}$$

10. Find the Laplace transform of the function,

$$f(t) = \begin{cases} \sin(t), & t < \pi \\ \cos(t), & t \geq \pi \end{cases}$$

11. Find the Laplace transform of the function using the definition,

$$\sin(t)e^t$$

12. Find the Laplace transform of the function,

$$\frac{\sin(3t)}{t}e^{-2t}$$

13. Find the Inverse Laplace transform of,

$$\frac{6s-4}{s^2-8s-9}$$

14. Find the Inverse Laplace transform of,

$$\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}e^{-3\pi s}$$

15. Find the Inverse Laplace transform of,

$$\frac{-s}{(s^2+1)(s+1)}e^{-\pi s}$$

16. Solve the given differential equation:

$$y'' + 4y = \sin(t)u(t-2\pi), \quad y(0) = 1, \quad y'(0) = 0$$

Given,

$$\frac{1}{(x^2+1)(x^2+4)} = \frac{1/3}{x^2+1} + \frac{-1/3}{x^2+4}.$$

17. Solve the given differential equation:

$$y'' + 9y = \cos(2t), \quad y(0) = 1, \quad y'\left(\frac{\pi}{2}\right) = -1.$$

18. Solve the given differential equation:

$$y' + y = f(t), \quad y(0) = 5, \text{ where } f(t) = \begin{cases} 0, & 0 \le t < \pi \\ \cos(t), & t \ge \pi \end{cases}$$

19. Solve the given differential equation:

$$y''' - 3y'' + 3y' - y = e^t t^2, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = -2$$

20. Solve the given system of differential equations:

$$\begin{aligned} x' &= -x + y, \quad x(0) = 0 \\ y' &= 2x, \qquad\quad y(0) = 1 \end{aligned}$$

Best of Luck!