Department of Mathematics and Natural Sciences
MAT215 : Complex Variables and Laplace Transformations
Problem Sheet on Laplace Transformation

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Section:

ID:

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1. Find Laplace transform of the following functions:

   a. (i) $3e^{-2t}$ (ii) $4t^3 - e^{-t}$ (iii) $\left(t^2+1\right)^2$ (iv) $7\sin 2t - 3\cos 2t$ (v) $\left(4e^{2t}-2\right)^3$ [Hint: Linearity]

   b. (i) t^3e^{-3t} (ii) $5e^{3t}\sin 4t$ (iii) $(t+2)^2e^t$ (iv) $e^{-t}(3\sinh 2t - 5\cosh 2t)$ (v) $e^{-4t}\cosh 2t$ (vi) $e^{2t}(3\sin 4t - 4\cos 4t)$ [Hint: First Translation theorem]

   c. (i) $t\sin t$ (ii) t^2e^{2t} (iii) $t^2\cos at$ [Hint: Multiplication by t^n]

   d. (i) $3\sin 2t\cos 2t$ (ii) $te^{-3t}\sin 3t$ (iii) $te^{-2t}\sin 2t\cos 4t$ (iv) $te^{-3t}\cos 2t\cos 4t$ [Hint: Trigonometric identities]

   e. (i) $(t-1)u(t-1)$ (ii) $(t^2+1)u(t-1)$ (iii) $e^{-2t}u(t-1)$ (iv) $u(t-\pi)\cos 2t$ [Hint: Second Translation theorem]

   f. (i) $f(t) = \begin{cases} 2t+1, & 0 \le t < 2 \\ 3t, & t \ge 2 \end{cases}$ (ii) $f(t) = \begin{cases} 0, & 0 \le t < \pi \\ \sin t, & t \ge \pi \end{cases}$ (iii) $f(t) = \begin{cases} \sin t, & 0 \le t < 2\pi \\ 0, & t \ge 2\pi \end{cases}$

   (iv) $f(t) = \begin{cases} e^{2t}, & 0 \le t < 1 \\ 4, & t \ge 1 \end{cases}$ (v) $f(t) = \begin{cases} 0, & 0 \le x < \pi \\ \sin t, & \pi \le x < 2\pi \\ 0, & x \ge 2\pi \end{cases}$ [Hint: Unit Step function]

2. Find Inverse Laplace transform of the following functions:

   a. (i) $\frac{1}{s^4}$ (ii) $\frac{1}{s^{3/2}}$ (iii) $\frac{s+1}{s^{4/3}}$ (iv) $\frac{12}{4-3s}$ (v) $\frac{23s-15}{s^2+8}$ (vi) $\frac{2s-5}{s^2-9}$ [Hint: Linearity and Laplace transform of elementary functions]

   b. (i) $\frac{6s-4}{s^2-4s+20}$ (ii) $\frac{s/2+5/3}{s^2+4s+6}$ (iii) $\frac{3s+7}{s^2-2s-3}$ (iv) $\frac{4s+12}{s^2+8s+16}$ [Hint: First Translation theorem in inverse form]

   c. (i) $\frac{2s^2-4}{(s+1)(s-2)(s-3)}$ (ii) $\frac{5s^2-15s-11}{(s+1)(s-2)^3}$ (iii) $\frac{4s+12}{s^2+8s+16}$ (iv) $\frac{3s+1}{(s^2+1)(s-1)}$ (v) $\frac{2s+5}{(s-2)^3}$ [Hint: Partial fraction decomposition]

   d. (i) $\frac{1}{s-4}e^{-2s}$ ii) $\frac{s}{s^2+s}e^{-\frac{\pi}{2}s}$ [Hint: Second Translation theorem in inverse form]

3, If $\mathscr{L}\{f(t)\} = F(s)$ then show that

a. $\mathscr{L}\{f'(t)\} = s \cdot F(s) - f(0)$
b. $\mathscr{L}\{f''(t)\} = s^2 \cdot F(s) - s \cdot f(0) - f'(0)$
c. $\mathscr{L}\{f'''(t)\} = s^3 \cdot F(s) - s^2 \cdot f(0) - s \cdot f'(0) - f''(0)$

4. Solve the following initial value problems: [Hint: Laplace transform of derivative]

a. $\dfrac{dy}{dt} + 3y = 13\sin 2t, \quad y(0) = 6.$
b. $y'' - 3y' + 2y = e^{-4t}, \quad y(0) = 1, \quad y'(0) = 5.$
c. $y'' - 6y' + 9y = t^2 e^{3t}, \quad y(0) = 2, \quad y'(0) = 17$
d. $y'' + 4y' + 6y = 1 + e^{-t}, \quad y(0) = 0, \quad y'(0) = 0.$
e. $y' + y = f(t), \quad y(0) = 5, \quad$ where $f(t) = \begin{cases} 0, & 0 \le t < \pi \\ 3\cos t, & t \ge \pi \end{cases}$
f. $y'' + 4y = \sin t \cdot u(t - 2\pi), \quad y(0) = 1, \quad y'(0) = 0$
g. $y'' + 2y' + 5y = e^{-t}\sin(t), \quad y(0) = 0, \quad y'(0) = 1$
h. $y''' - 3y'' + 3y' - y = e^t t^2, \quad y(0) = 0, y'(0) = 1, y''(0) = -2$

5. Solve the boundary value problem $y'' + 9y = \cos 2t, \quad y(0) = 1, \quad y(\pi/2) = -1.$

6. Solve the following system: $\begin{cases} \dfrac{dx}{dt} + y = \sin t \\ \dfrac{dy}{dt} + x = \cos t \end{cases}, \quad x(0) = 2, y(0) = 0.$

7. Evaluate Laplace transform of $F(t) = \begin{cases} 0, 0 \le t < 3 \\ 2, \quad t \ge 3 \end{cases}$ using the definition $\mathscr{L}\{F(t)\} = \int_0^\infty e^{-st} F(t) dt.$

8.Use 2nd translation theorem to evaluate the Laplace transform of $F(t) = 2 - 3u(t-2) + u(t-3)$