BRAC UNIVERSITY
Inspiring Excellence

Department of Mathematics and Natural Sciences MAT215 : Complex Variables and Laplace Transformations
Assignment 3
Date: December 14, 2024

Deadline : December 22,2024 | Fall 2024 | Total Marks: 80

Name:

Section:

ID:

Use this page as the cover page of your assignment. No late submission will be graded.

1. Evaluate $\int_{(0,1)}^{(2,5)} (3x+y)dx+(2y-x)dy$ along (a) the curve $y=x^2+1$, (b) the straight line joining $(0,1)$ and $(2,5)$, (c) the straight lines from $(0,1)$ to $(0,5)$ and then from $(0,5)$ to $(2,5)$, (d) the straight lines from $(0,1)$ to $(2,1)$ and then from $(2,1)$ to $(2,5)$. [1.25 x 4 = 5]

2. (a) Evaluate $\oint_C (x+2y)dx+(y-2x)dy$ around the ellipse C defined by $x=4\cos\theta, y=3\sin\theta, 0\le\theta<2\pi$ if C is described in a counterclockwise direction.
   (b) What is the answer to (a) if C is described in a clockwise direction? [2 x 2.5 = 5]

3. Evaluate $\oint_C |z|^2 dz$ around the square with vertices at $(0,0),(1,0),(1,1),(0,1)$. [5]

4. Evaluate $\int_i^{2-i} \left(3xy+iy^2\right) dz$ (a) along the straight line joining $z=i$ and $z=2-i$, (b) along the curve $x=2t-2, y=1+t-t^2$. [5]

5. Evaluate $\oint_C \frac{dz}{(z-a)^n}, n=1,2,3,4,\ldots$ where (a) $z=a$ is outside the simple closed curve C, (b) $z=a$ is inside the simple closed curve C. [2 x 2.5 = 5]

6. Evaluate $\oint_C (\bar{z})^2 dz$ around the circles (a) $|z|=1$ and (b) $|z-1|=1$ [5]

7. Evaluate: (a) $\oint_C \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}dz$ (b) $\oint_C \frac{e^{2z}}{(z+1)^4}dz$ where C is the circle $|z|=3$. [5]

8. Evaluate $\frac{1}{2\pi i}\oint_C \frac{e^z}{z-2}dz$ if C is: (a) the circle $|z|=3$, (b) the circle $|z|=1$. [5]

9. Evaluate (a) $\oint_C \frac{\sin 3z}{z+\pi/2}dz$ if C is the circle $|z|=5$. (b) $\oint_C \frac{e^{iz}}{z^3}dz$ where C is the circle $|z|=2$. [5]

10. (a) Let $F(z)$ be analytic inside and on a simple closed curve C except for a pole of order m at $z=a$ inside C. Prove that [6]

$$\frac{1}{2\pi i}\oint_C F(z)dz = \lim_{z\to a}\frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}\{(z-a)^m F(z)\}$$

(b) How would you modify the result in (a) if there were three poles inside C ? [4]

11. Evaluate $\oint_C \frac{e^z}{(z^2+\pi^2)^2}dz$, where C is the circle $|z|=4$. [5]

12. Given C is the circle $|z| = 1$. Find the value of (a) $\oint_C \frac{\sin^6 z}{z-\pi/6} dz$ (b) $\oint_C \frac{\sin^6 z}{(z-\pi/6)^3} dz$ [5]

13. Evaluate (a) $\frac{1}{2\pi i} \oint_C \frac{e^{zt}}{(z^2+1)^2} dz$, (b) $\frac{1}{2\pi i} \oint_C \frac{e^{zt}}{z^2+1} dz$ if $t > 0$ and C is the circle $|z| = 3$. [5]

14. Evaluate $\oint_C \frac{e^{3z}}{z-\pi i} dz$ where C is the circle $|z-1| = 4$. [5]

15. Evaluate $\oint_C \frac{dz}{z-2}$ around (a) the circle $|z-2| = 4$ (b) the circle $|z-1| = 9$. [5]