Math 4512 Numerical Analysis Methods

Quiz 01, Set A

Time: 30 min **Full Marks: 15**

1. Find the 4^{th} degree Taylor polynomial, $\boldsymbol{P_4(x)}$, centered at $\frac{\pi}{2}$ for the function $\cos(x)$. DO NOT simplify your answer. Use $\boldsymbol{P_4(x)}$ found above to approximate $\mathbf{cos(100^\circ)}$. Use the Taylor remainder theorem $\boldsymbol{R_4(x)}$, of find an upper bound for the absolute error of the approximation of $\mathbf{cos(100^\circ)}$. 8

2. Solve the given nonlinear function $f(x) = x - 0.2\sin(x) - 0.8$. Use Newton-Raphson method to approximate a solution of the equation given with initial guess $x_0 = \frac{\pi}{4}$. Only find x_1, x_2 *and* x_3. Clearly indicate your answers and show all of your work. 7

QUIZ/CLASS TEST/LAB TEST SCRIPTS

STUDENT NO	TEST NO	SET
170012032	01	B

COURSE NO	MARKS OBTAINED
Math 4543	

PROGRAMME	DATE	SIGNATURE OF THE TEACHER
CSE	10.2.2020	

9

Ans to Q.8.1

$f(x) = \sin x \rightarrow f\left(\frac{\pi}{2}\right) = 1$

$f'(x) = \cos x \rightarrow f'\left(\frac{\pi}{2}\right) = 0$

$f''(x) = -\sin x \rightarrow f''\left(\frac{\pi}{2}\right) = -1$

$f'''(x) = -\cos x \rightarrow f'''\left(\frac{\pi}{2}\right) = 0$

$f''''(x) = \sin x = f(x)$, keeps repeating from here

5

so, for taylor series $\rightarrow x+h = 100°$

$\Rightarrow x+h = \frac{\pi}{2} + \frac{\cancel{10}\pi}{18}$

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)h^2}{2!} + \frac{f'''(x)(h)^3}{3!} + \frac{f''''(x)h^4}{4!} + \cdots$$

$$= 1 + 0 \times \frac{\pi}{18} + \frac{(-1)\left(\frac{\pi}{18}\right)^2}{2!} + 0 + \frac{1 \times \left(\frac{\pi}{18}\right)^4}{4!} + \cdots$$

$$= 1 - \frac{\pi^2}{18^2 \times 2!} + \frac{\pi^4}{18^4 \times 4!} - \frac{\pi^6}{18^6 \times 6!} + \frac{\pi^8}{18^8 \times 8!} - \cdots$$

generally $= 1 - \frac{h^2}{2!} + \frac{h^4}{4!} - \frac{h^6}{6!} + \frac{h^8}{8!} - \cdots$

According to remainder theorem

$$R_n = f^{n+1}(x) \times \frac{h^{n+1}}{n!}$$

$n=4$, so

$$R_4 = f^4(x) \times \frac{h^4}{4!} = \sin x \times \frac{10^4\pi^4}{18^4 \times 4!}$$

$$\cancel{= \frac{10^4\pi^4}{18^4 4!}}$$

for $x = \frac{\pi}{2}$

$$R_4 = \frac{\cancel{10^4}\,\pi^4}{18^4 \times 4!}$$

$180 - \pi$
$10 - \frac{10\pi}{180}$

for $x = x+h = \frac{10\pi}{18}$

$$R_4 = \frac{10^4\pi^4}{18^4 \times 4!}$$

so upper bound is $\frac{10^4\pi^4}{18^4 \times 4!}$

you omitted the other terms.

Ans: 4th deg $P_4(x) = 1 - \frac{h^2}{2!} + \frac{h^4}{4!} - \frac{h^6}{6!} + \cdots$

for $\frac{\pi}{2}$ $P_4(x) = 1 - \frac{\pi^2}{18^2 \times 2!} + \frac{\pi^4}{18^4 \times 4!} - \frac{\pi^6}{18^6 \times 6!} + \cdots$

upper bound $= \frac{10^4 \pi^4}{18^4 \times 4!}$

Ans to Q. 2

$f(x) = x - 0.2\cos x - 0.8$ $\quad x_0 = \frac{\pi}{3}$

$f(x_0) = \frac{\pi}{3} - 0.2 \times \frac{1}{2} - 0.8$

$= \frac{\pi}{3} - 0.9 = 0.1472$

$f'(x_0) = 1 + 0.2\sin x$

$= 1 + \frac{\sqrt{3}}{10} = 1.1738$

f

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \frac{\pi}{3} - \frac{\frac{\pi}{3} - 0.9}{1.1738}$$

$$x_1 = \cancel{0.08}\ 0.9148\ \frac{\pi}{3} = 0.95798$$

$$\boxed{x_1 = 0.92179}$$

$\cancel{x_2 = x_1 -}$ $\cancel{f}$ $f(x_1) = -0.0783$

$f'(x_1) = 1.0032$

$$x_2 = x_1 - \frac{f(x_1)}{f'x_1} = \boxed{0.99975}$$

$f(x_2) = -2.196 \times 10^{-4}$

$f'(x_2) = 1.0035$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.99975 - \frac{-2.196 \times 10^{-4}}{1.0035}$$

$$\boxed{x_3 = 0.9999688}$$

Ans: $x_1 = 0.95798$

$x_2 = 0.99975$

$x_3 = 0.999969$