

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION

DURATION: 1 Hour 30 Minutes

WINTER SEMESTER, 2019-2020
FULL MARKS: 75

Math 4543: Numerical Methods

Programmable calculators are not allowed. Do not write anything on the question paper.
There are **4 (four)** questions. Answer any **3 (three)** of them.
Figures in the right margin indicate marks.

1. a) When approximating any mathematical model using numerical methods, how can you use relative approximate errors to minimize the error? Explain your answer with the help of Taylor's Remainder Theorem. 5
- b) What do you understand by truncation error and round off error? Explain both of them with the MacLaurin Series expansion of $\sin(\frac{\pi}{3})$. Your error should be calculated up to four significant digits. 10
- c) Given that $f(3) = 7$, $f'(3) = 4$, $f''(3) + 1 = 11$, $f'''(3) = f''(3) + 3$ and that all other higher order derivatives of $f(x)$ are zero at $x = 3$, and assuming the function and all its derivatives exist and are continuous between $x = 3$ and $x = 4.5$. Find out the value of $f(4.3)$. Also provide an upper bound for the error with Taylor's remainder theorem if you used only 2 terms of the series. 10
- a) Explain the advantages and drawbacks of bisection method for solving nonlinear equations. 6
- b) Solve the following non-linear equation within the range $[-1, 0]$ and $[0, 1]$ with at least 3 iterations.

$$f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$$
In each of the iterations, calculate the relative approximate error. Your answer should be correct up to 4th digit. 12
- c) Why do we need to use Spline interpolation over Lagrange interpolation for higher order approximation? Explain with appropriate logic. Use figure if necessary. 7
- a) In order to find out the values of $3n$ number of unknowns, you need $3n$ number of equations. How can you get $3n$ number of simultaneous equations from $(n+1)$ data points in the Quadratic Spline method of interpolation? 10
- b) A thermistor is a special kind of thermometer that is able to calculate the temperature of a body using the value of the resistance attached to it. A manufacturer of thermistors makes several observations with a thermistor which are given in Table 1.

Table 1: Observations

R (ohm)	T (°C)
1101.0	25.113
911.3	30.131
636.0	40.120
451.1	50.128

Determine the temperature corresponding to 758.8 ohms using a second order Lagrange polynomial. Your result should be correct up to 3 digits.

- c) Generalize the n^{th} order form of Newton's Divided difference method of interpolation. Use figures or sketches if necessary.

4. a) Derive the coefficients of the linear regression model $y = a_0 + a_1x$ using root mean square minimization of the residuals.
- b) The progress of a homogeneous chemical reaction is followed and it is desired to evaluate the rate constant and the order of the reaction. The rate law expression for the reaction is known to follow the power function form:

$$-r = kC^n.$$

From the given data in Table 2. Calculate the value of n and k .

Table 2: Chemical Kinetics

$C_A(\text{gmol/l})$	4	2.25	1.45	1.0	0.65	0.25	0.006
$-r_A(\text{gmol/l}\cdot\text{s})$	0.398	0.298	0.238	0.198	0.158	0.098	0.048

- c) Convert the following nonlinear regression model into linear regression problem.
- $y = Ae^{\lambda t}$ Where A and e are the regression coefficients.
 - $y = \frac{ax}{b+x}$ Where a and b are the regression coefficients.