ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **SUMMER SEMESTER, 2018-2019**

DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

Math 4641: Numerical Methods

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 4 (four) questions. Answer any 3 (three) of them.

Figures in the right margin indicate marks.

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1. a) When approximating any mathematical model using numerical methods, how can you use relative approximate errors to minimize the error? Explain your answer with the help of Tailor's Remainder Theorem. 5

   b) What do you understand by truncation error and round-off error? Explain both of them with the Maclauren Series expansion of $e^{1.5}$. Your error should be calculated up to four significant digits. 8

   c) Given that $f(3) = 6$, $f'(3) = 8$, $f''(3) + 1 = 12$, $f'''(3) = f''(3) + 5$, and that all other higher order derivatives of $f(x)$ are zero at $x = 3$, and assuming the function and all its derivatives exist and are continuous between $x = 3$ and $x = 4.5$. Find out the value of $f(4.3)$. 12

2. a) Explain the difference between interpolation and regression with appropriate examples. 4

   b) A robot arm with a rapid laser scanner is doing a quick quality check on holes drilled in a 15" x 10" rectangular plate. The centers of the holes in the plate describe the path the arm needs to take, and the hole centers are located on a Cartesian coordinate system (with the origin at the bottom left corner of the plate) given by the specifications in Table 1. 14

   Table 1: The coordinates of the holes on the plate.

x (in)	y(in)
2.00	7.2
4.25	7.1
5.25	6.0
7.81	5.0
9.20	3.5
10.60	5.0

   If the laser is traversing from $x = 2.00$ to $x = 4.25$ to $x = 5.25$ in a quadratic path, what is the value of y at $x = 4.00$ using a second order Lagrange polynomial? Find the absolute relative approximate error for the second order polynomial approximation.

   c) Why do we need to use Spline interpolation over Lagrange interpolation for higher order approximation? Explain with appropriate logic. Use figure if necessary. 7

3. a) What do you understand by Riemann's sum? Find the area of the region between the function $f(x) = x^3 + x^2$ on the interval $[0, 4.5]$ using Riemann's sum. 12

b) In order to find out the values of $3n$ number of unknowns, you need $3n$ number of equations. How can you get $3n$ number of simultaneous equations from $(n-1)$ data points in Quadratic Spline method of interpolation? 8

c) Derive the trapezoidal rule of integration from Calculus using Newton's Divided Difference method. 5

4. a) Explain why we minimize the sum of square of the residuals instead of absolute value of the residuals in regression with sound mathematical reasoning. 6

b) The progress of a homogeneous chemical reaction is followed and it is desired to evaluate the rate constant and the order of the reaction. The rate law expression for the reaction is known to follow the power function form 13

$$-r = kC^n.$$

From the given data in the Table 2, Calculate the value of n and k.

Table 2: Chemical Kinetics

C_A (gmol/l)	4	2.25	1.45	1.0	0.65	0.25	0.006
$-r_A$ (gmol/l·s)	0.398	0.298	0.238	0.198	0.158	0.098	0.048

c) Which method is better between Newton-Raphson and Secant method for finding out the root of a non-linear equation? Justify your answer with appropriate reason. 6

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SEMESTER FINAL EXAMINATION **SUMMER SEMESTER, 2018-2019**
DURATION: 3 Hours **FULL MARKS: 150**

MATH 4641: Numerical Analysis

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There are **8 (eight)** questions. Answer any **6 (six)** of them.
Figures in the right margin indicate marks.

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1. a) Define different types of errors in mathematical calculations with appropriate examples. Don't forget to mention specific cases when you calculate those different types of errors. 5

   b) Find out the number of terms required to take in consideration from McLaurin Series for calculating the value of $e^{1.5}$ up to 3 significant digit correctness. 8

   c) Given that $f(4) = 6$, $f'(4) = 8$, $f''(4) + f(4) = 17$, $f'''(4) = f''(4) + 5$ and that all other higher order derivatives of $f(x)$ are zero at $x = 4$, and assuming the function and all its derivatives exist and are continuous between $x = 4.0$ and $x = 4.5$. Find the value of $f(4.3)$. 12

2. a) Define *Inflection Point* with appropriate example. Mention the shortcomings of bisection method of finding roots of non-linear equations with appropriate examples. 7

   b) Derive Newton-Raphson method of finding roots of non-linear equations from Taylor Series. 6

   c) Use the Newton-Raphson method to estimate the root of $f(x) = e^{-x} - x$, employing an initial guess of $x_0 = 0$ and taking at least two iterations. 12

3. a) Solve the following nonlinear equation with false position method: 10

   $$e^{-2x} + 4x^2 - 36 = 0$$

   Consider two initial guesses, $x_l = 1$ and $x_u = 4$

   b) Why do we calculate the sum of square of the residuals in case of regression? Explain with appropriate diagram. 8

   c) Mention the problems associated with using higher order polynomial equations for interpolation. How could you solve those problems? 7

4. a) The upward velocity of a rocket is given as a function of time in Table 1. Determine the value of the velocity at $t = 16$ seconds using the direct method of interpolation with third order polynomial. 12

   Table 1: Velocity as a function of time

t (sec)	V(t) (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

   b) From the answer of the Question 4(a) calculate the absolute relative approximate error for the third order polynomial approximation. 5

   c) Calculate the velocity and acceleration of the rocket from $t = 11$ to $t = 16$ using the third order polynomial interpolant obtained in 4(a) 8

5. a) Why is it useful to decompose matrix A into LU format when a simulation system is given by the format $Ax = b$ with different values of b? Justify your answer with appropriate scenario. 7

b) Solve the following system with the help of *LU Decomposition.* 10

$$x_1 + x_2 - x_3 = 4$$
$$x_1 - 2x_2 + 3x_3 = -6$$
$$2x_1 + 3x_2 + x_3 = 7$$

c) Show that in case of calculating numerical differentiation on a discrete dataset, the error in central difference method is always smaller than that both forward and backward difference method. 8

6. a) With the help of necessary figures, derive the formula for Euler's method of solving ordinary differential equations. 7

b) Human vision has the remarkable ability to infer 3D shapes from 2D images. The intriguing question is: can we replicate some of these abilities on a computer? Yes, it can be done and to do this, integration of vector fields is required. The following integral needs to be integrated. 10

$$I = \int_0^{100} f(x)dx$$

where

$$f(x) = 0, \ 0 < x < 40$$
$$= -9.1688\times10^{-6}x^3 + 2.7961\times10^{-3}x^2 - 2.8487\times10^{-1}x + 9.6778, \ 40 \le x \le 172$$
$$= 0, \ 172 < x < 200$$

Use Simpson's 1/3 Rule to find the integral I.

c) The following equation represents the approximation using Runge-Kutta 4th order of solving ordinary differential equations. 8

$$w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

Describe how the constants are approximated using appropriate diagram

7 a) How can you derive Runge Kutta 2nd order and 4th order method of solving ordinary differential equations from Taylor Series. 8

b) A rectifier-based power supply requires a capacitor to temporarily store power when the rectified waveform from the AC source drops below the target voltage. To properly size this capacitor a first-order ordinary differential equation must be solved. For a particular power supply, with a capacitor of 150 μF, the ordinary differential equation to be solved is 17

$$\frac{dv(t)}{dt} = \frac{1}{150\times10^{-6}}\left\{-0.1 + \max\left(\frac{|18\cos(120\pi(t))| - 2 - v(t)}{0.04}, 0\right)\right\}$$

$v(0) = 0$

Using the Runge-Kutta 2nd order method, find the voltage across the capacitor at t = 0.00004s Use step size h = 0.00002s and follow Heun's method.

8. a) How can you check the existence of a root of any nonlinear equation between two given points using bisection method? Explain with appropriate diagrams. 6

b) Mention at least three applications of Taylor Series in Numerical Approximation methods with appropriate examples. 9

c) Find an approximate value of the following using Euler's method of solving an ordinary differential equation. Use a step size of $h=1.5$ 10

$$I = \int_5^8 6x^3 dx$$