MID QUESTION SOLUTION

1(a) relative approximate error can help us calculate accuracy even when we don't know the true value. if the value of R.A.E is less than a specified tolerance of error then we can stop iterations. we can also calculate number of ~~d~~ significant digit.

Remainder theorem: $R_n = \frac{h^{n+1}}{(n+1)!} \times f(c)$ | $|\epsilon_a| \leq 0.5 \times 10^{2-n}$

1(b) truncation error: when we limit the terms in our expansion, that gives us truncation error

round off error: occurs when digits after decimal points are limited.

$$e^{1.5} = 1 + 1.5 + \frac{(1.5)^2}{2!} + \frac{(1.5)^3}{3!} + \frac{(1.5)^4}{4!} = 4.39844$$

~~for 4 significant~~ digits. for 4 digits to be right,

$|\epsilon_a| \leq 0.5 \times 10^{2-4} = 5 \times 10^{-3}$

~~$|\epsilon_a| = 0.5 \times 10^{2-4}$~~

~~$= 5 \times 10^{-3}$~~

$$\text{error} = e^{1.5} - 1 + 1.5 + \frac{1.5^2}{2!} + \frac{(1.5)^3}{3!}$$

$$= \overset{4.4816889}{} - 4.3984375$$

$$= 0.0833$$

the error occured because of truncation and of terms & limiting digits after decimal point.

1 (c) $f(3) = 6$

$f'(3) = 8$

$f''(3) = 11$

$f'''(3) = 16$

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)h^2}{2!} + \frac{f'''(x)h^3}{3!}$$

$$f(3+1.3) = 6 + 8\times 1.3 + \frac{11\times(1.3)^2}{2!} + \frac{16\times(1.3)^3}{3!}$$

$$= 31.554$$

2(a) in case of interpolation, we can only find ~~o~~ data points that are within the dataset range. ~~Regress~~ and data needs to fit in eqn line. In regression it does not need over-fit. and any point outside the range of dataset can also be calculated.

Example :

2(b) using lagrange interpolation, 2nd order $\rightarrow i = 0, 1, 2$

$x = 4$

$x_0 = 2$

$x_1 = 4.25$

$x_2 = 5.25$

$$f(x) = \sum L_i y_i$$

$$= L_0 y_0 + L_1 y_1 + L_2 y_2$$

$$L_0 = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} = \frac{(4 - 4.25)(4 - 5.25)}{(2 - 4.25)(2 - 5.25)} = 0.04274$$

$$L_1 = \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} = \frac{(4 - 2)(4 - 5.25)}{(4.25 - 2)(4.25 - 5.25)} = 1.1111$$

$$L_2 = \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} = \frac{(4 - 2)(4 - 4.25)}{(5.25 - 2)(5.25 - 4.25)} = 0.1538 \quad \cancel{= 0.01923}$$

$$f(x) = (0.04274 \times 7.2) + (1.11111 \times 7.1) + (\overset{0.1538}{\cancel{0.01923}} \times 6)$$

$$\cancel{= 8.311989} \quad 7.2738$$

$x = 4$

$t_0 = 2.00$

$t_1 = 4.25$

$$\begin{array}{r} 5.25 \\ 2.00 \\ \hline 3.25 \end{array}$$

1st order $\rightarrow i = 0, 1$

$$L_0 = \frac{x - x_1}{x_0 - x_1} = \frac{4 - 4.25}{2 - 4.25} = 0.11111$$

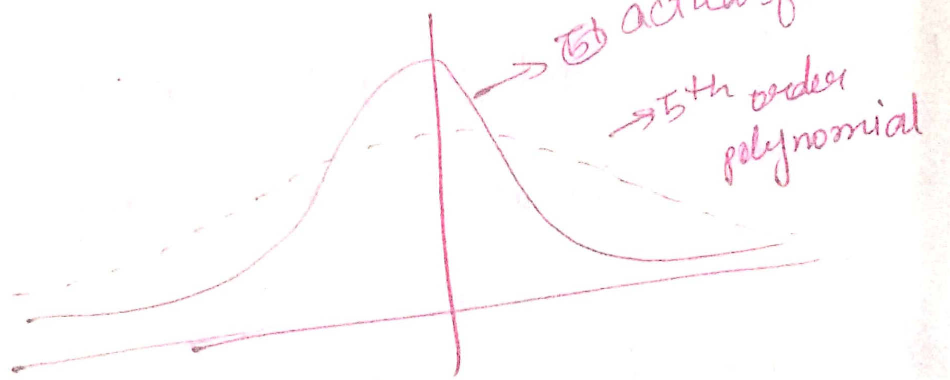
$$L_1 = \frac{x - x_0}{x_1 - x_0} = \frac{4 - 2}{4.25 - 2} = 0.88888$$

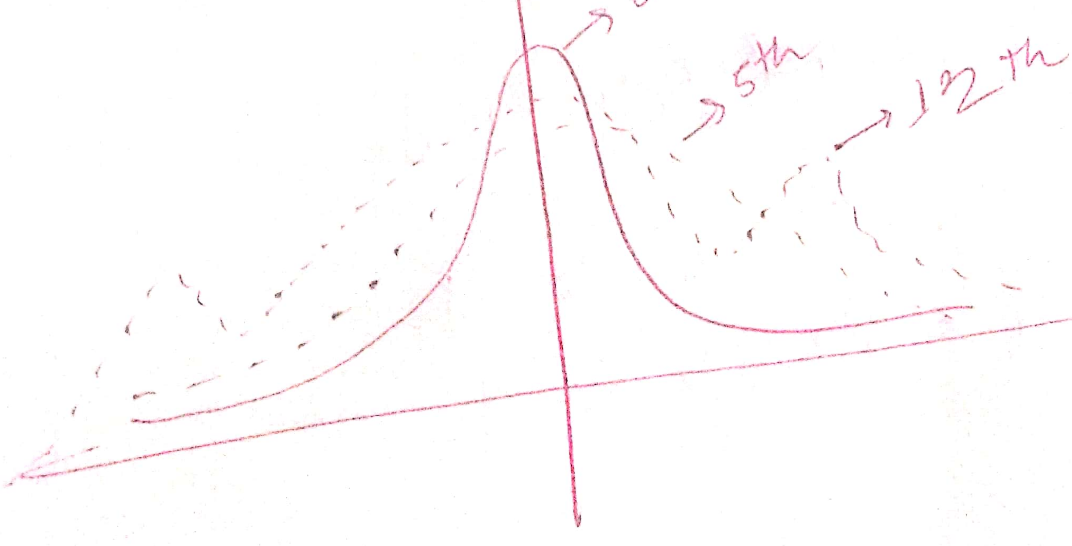
$$f(x) = (0.11111 \times 7.2) + (0.88888 \times 7.1) = 7.11107$$

$$R.A.E = \left| \frac{\cancel{8.311989}\ 7.27343 - 7.11109}{\cancel{8.311989\ 7.11109}\ 7.27343} \right| \times 100\%$$

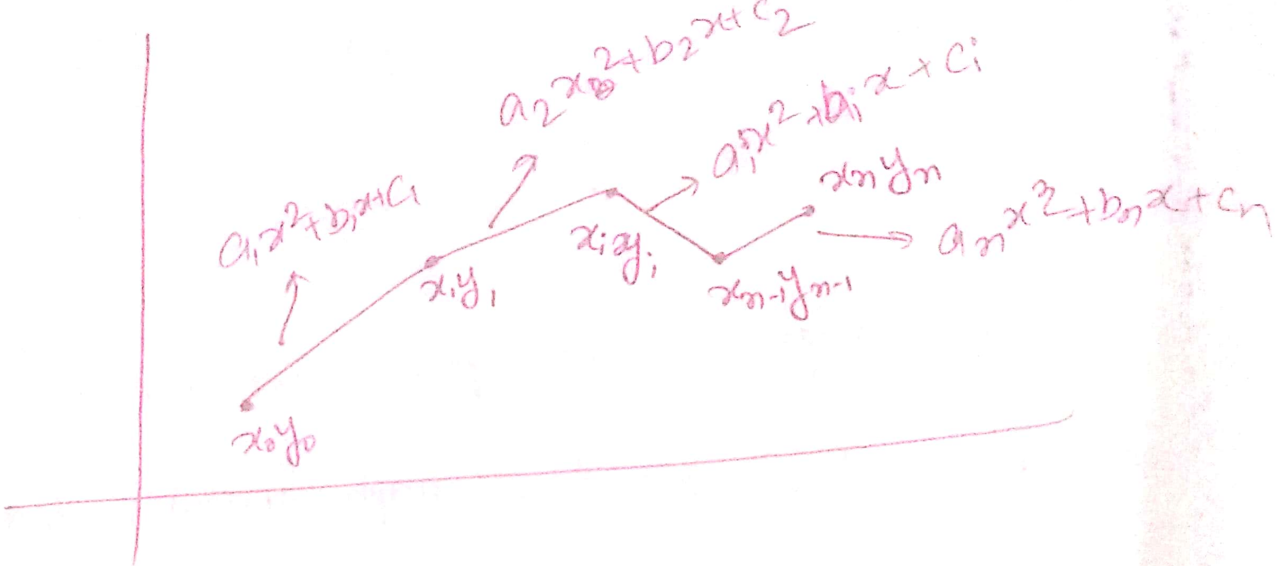
$= 2.23\%$

2(c) for higher order polynomial, we use spline method because in that case the graph diverges drastically.





2b) for spline method,



for (n+1) points, we get n splines each spline has 3 unknowns of quadratic eqn ax^2+bx+c

1) one spline goes through 2 consecutive points

$$\left[\begin{array}{l} a_1x_0^2+b_1x_0+c_1=y_0 \\ \cancel{a_2x_0}\ a_1x_1^2+b_1x_1+c_1=y_1 \end{array}\right.$$

$$\left[\begin{array}{l} a_2x_1^2+b_2x_1+c_2=y_1 \\ a_2x_2^2+b_2x_2+c_2=y_2 \end{array}\right.$$

this will give 2n equations.

2) each point has 2 splines connected to it. derivative at that point would be equal.

$$2a_1x_1 + b_1 = 2a_2x_1 + b_2$$

$$2a_2x_2 + b_2 = 2a_3x_2 + b_3$$

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this will give us ~~2n~~ (n-1) eqn.

3) the last eqn can be assumed where

$a_1 = 0$ so eqn becomes linear.

1(a) at the derivation of regression ~~p~~ function, when we use absolute ~~fa~~ value of residual value ~~of~~ we get $-n=0$ or, $n=0$, both of these cases are invalid because n is the number of data points which can never be 0. square residual can ~~ab~~ avoid this problem.

4(a) ~~newton~~ secant method is better than newton ~~ra~~ rhapson because

1) it converges fast (if converges)

2) ~~has~~ uses two values that do not bracket the ~~rule~~ root.

4(b) $-r = kC^n$

~~take ln~~

$$\Rightarrow \underbrace{\ln(-r)}_{y} = \cancel{kC^n}\ \underbrace{\ln k}_{a_0} + \underbrace{n}_{a_1}\underbrace{\ln C}_{x}$$

$-r$	C	$\ln(-r) \to y$	$\ln C \to x$	x^2	y^2	xy
0.398	4	-0.92	1.38	1.9044	0.846	-1.2696
0.298	2.25	-1.21	0.8109	0.658	1.46	-0.981
0.238	1.45	-1.43	0.3718	0.138	2.05	-.531
~~0.158~~ 0.198	1.0	-1.61	0	0	2.59	0
~~0.098~~ 0.158	0.65	-1.84	-0.43	0.185	3.386	0.79
0.098	0.25	-2.32	-1.38	1.904	5.38	3.20
0048	0.006	-3.037	-5.11	26.11	9.22	15.51

$\Sigma x = -4.3575$

$\Sigma y = -12.367$

$\Sigma \cancel{xy}\, x^2 = 30.8994$

$\Sigma y^2 = 24.932$

$\Sigma xy = 16.7189$

$(\Sigma x)^2 = 18.98878$

$$a_1 = \frac{\Sigma x \Sigma y - n\Sigma xy}{n\Sigma x^2 - (\Sigma x)^2} = \frac{-(-4.3575)(-12.367) + (16.7189 \times 7)}{(7 \times 30.8999) - 18.9878}$$

$= -0.32$

$$a_0 = \frac{-\Sigma x \Sigma xy - \Sigma x^2 \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = \frac{(-4.3575) \times (16.7189) - (30.899 \times -12.3}{(7 \times 30.8999) - 18.9848}$$

$= -2.30$

$r = 0.32$

$R = 0.0997$