B.Sc. Engg. CSE 6th Semester 22 October 2018

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **SUMMER SEMESTER, 2017-2018**

DURATION: 3 Hours **FULL MARKS: 150**

Math 4641: Numerical Methods

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **8 (eight)** questions. Answer any **6 (six)** of them.

Figures in the right margin indicate marks.

---

1. a) While solving a mathematical model using numerical methods, how can we use relative approximate errors to minimize the error? If anyone wants at least m significant digits to be correct in the answer, then what you would need to do? 7

   b) What are the dangers of extrapolation? 6

   c) The Taylor series for e^x at point $x = 0$ is given by 12

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \qquad -\infty < x < \infty$$

   i. What is the truncation (true) error in the representation of $e^{2.5}$ if only four terms of the series are used?

   ii. How many terms it would require to get an approximation of $e^{2.5}$ within a magnitude of true error of less than 10^{-6}?

2. a) What are advantages and disadvantages of Bisection method for root finding problem? Briefly explain. 8

   b) Use the Newton-Raphson method to estimate the root of $f(x) = e^{-x} - x$. Employ an initial guess of $x_0 = 0$ and perform at least two iterations. 10

   c) What is *Inflection Point?* Explain with appropriate example. 7

3. a) Why minimize the sum of square of the residuals in case of regression? Briefly explain 8

   b) The upward velocity of a rocket is given as a function of time in Table 1. Determine the value of the velocity at time $t = 16$ and $t = 26$ seconds using linear splines. 9

Table 1: Velocity as a function of time

t (sec)	V(t) (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

   c) From the results obtained in Question 3.b), calculate the distance (in meters) traversed by the rocket from t =16 to t = 26 using 3-segment trapezoidal rule. 8

4. a) How does regression differ from interpolation? Explain with example. 7

   b) What is the motivation behind using Spline method of interpolation instead of direct method of interpolation? Justify your answer with appropriate example. 7

c) The population of Mississippi during three census periods was as follows: 11

Table 2: Data for polynomial regression

Year	1951	1961	1971
Population(in millions)	2.8	3.2	4.5

Interpolate the population at 1966 using quadratic Lagrange Interpolation.

5. a) How can you transform the data to use linear regression formula for the following nonlinear functions? 10

i. $y = ae^{-bx}$

ii. $-r = kC^n$

b) Why are the applications of Taylor's theorem important for numerical methods? Give example to two applications of Taylor's theorem. 6

c) Suppose you are given a uniformly sampled function integration. Consider you have the function value known at equally spaced 6 points. Now design a composite of Simpson's 1/3 rule and 3/8 rule to solve the problem. Justify your solution with appropriate reason. 9

5. a) How can you check the existence of a root of any nonlinear equation between two given points using bisection method? Explain with appropriate diagrams. 8

b) A ball at 1200 K is allowed to cool down in air at an ambient temperature of 300K. Assuming heat is lost only due to radiation, the differential equation for the temperature of the ball is given by 10

$$\frac{d\theta}{dt} = -2.2067x10^{-12}(\theta^4 - 81x10^8)$$

where θ is in Kelvin and t is in seconds. Find the temperature at t= 480 seconds using Runge-Kutta 2nd order method. Assume a step size of h=240 seconds. Use Heun's method for the assignment of constants.

c) Why the Taylor's Polynomial cannot be used for interpolation? Justify your answer. 7

7. a) With the help of necessary figures, derive the formula for Euler's method of solving ordinary differential equations. 10

b) Find an approximate value of the following using Euler's method of solving an ordinary differential equation. Use a step size of $h=1.5$ 8

$$I = \int_5^8 6x^3 dx$$

c) Compare the accuracy of Euler's method, Runge-Kutta 2nd and 4th order method for solving ordinary differential equations. Justify your answer with appropriate mathematical argument. 7

8. a) Why is it useful to decompose matrix A into LU format when a simulation system is given by the format $Ax = b$ with different values of b? Justify your claim with appropriate logic. 5

b) Solve the following system with the help of *LU Decomposition.* 12

$$x_1 + x_2 - x_3 = 4$$
$$x_1 - 2x_2 + 3x_3 = -6$$
$$2x_1 + 3x_2 + x_3 = 7$$

c) Show that in case of calculating numerical differentiation on a discrete dataset, the error in central difference method is always smaller than that both forward and backward difference method. 8