B.Sc. Engg. (SWE) 6th Semester **17 January 2022 (Afternoon)**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **SUMMER SEMESTER, 2020-2021**
DURATION: 1 HOUR 30 MINUTES **FULL MARKS: 75**

MATH 4643: Probability and Statistics II

Programmable calculators are not allowed. Do not write anything on the question paper.
Answer **all <u>3 (three)</u>** questions. Marks of each question and corresponding CO and PO are written in the right margin.

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1. a) Define probability mass function (PMF) with necessary conditions. A discrete random variable X has the following probability distribution — 8 (CO1) (PO1) (CO2) (PO2)

$$P(X=x) = \binom{3}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{3-x} \ ; \ x = 0,1,2,3.$$

   i. Check whether $P(X)$ is a PMF or not.

   ii. Find its CDF, and hence evaluate $P(X \le 2)$.

   b) i. The organization that Jones works for is running a father–son dinner for those employees having at least one son. Each of these employees is invited to attend along with his youngest son. If Jones is known to have two children, what is the conditional probability that they are both boys given that he is invited to the dinner? — 8 (CO2) (PO2)

   ii. Three coins are tossed. Show that the events "heads on the first coin" and the events "tails on the last two" are independent.

   c) i. Write down the statement of Bayes' Theorem with its mathematical expression. — 9 (CO2) (PO2)

   ii. Suppose there will be three candidates for the post of principal at Notre Dame College. They are Fr. Costa, Fr. Hemanta, and Fr. Adam. The chances that they will get selected for the post are 4:3:2. The probability that Fr. Costa if gets selected will introduce co-education is 0.3. The probabilities of Fr. Hemanta, and Fr. Adam doing the same are 0.5 and 0.8, respectively. What is the probability that co-education will be introduced this year in the college? If co-education is introduced then what will be the probability that it will be introduced by Principal Fr. Adam?

2. a) Define joint probability distribution for two discrete random variables. Also write down the properties of a joint probability distribution. — 5 (CO1) (PO1)

   b) A discrete random variable X follows the rule: — 8 (CO2) (PO2)

$$f(x) = \begin{cases} me^{-2x}, & x > 0 \\ 0, & otherwise \end{cases}$$

   Find m so that $f(x)$ is a probability distribution. Also determine the probability $P(1 < X < 2)$.

c) Define conditional probability distributions for two random variables X and Y considering both discrete and continuous cases. The joint probability distributions of the discrete random variable X and Y are given as follows: 12 (CO3) (PO2)

X \ Y	1	2	3	4
1	0.1	0	0.1	0
2	0.3	0	0.1	0.2
3	0	0.2	0	0

Now, evaluate $f(x|2), f(y|1)$, and $P(X=1|Y=2)$.

3. a) Define mathematical expectation and variance of a random variable. For the following probability distribution, find $E[X]$ and $V[X]$. 8 (CO1) (PO1)

$$f(x) = \binom{3}{x}\left(\frac{1}{4}\right)^{x}\left(\frac{1}{4}\right)^{3-x} \; ; \; x = 0,1,2,3.$$

b) i. Suppose that an ideal coin is tossed twice. If it turns up tails both times, you win nothing. If head turns up only once, you win \$1. If both tosses result in heads, you win \$4. Based on the given condition find the expected number of heads to turn up. Also evaluate the expected amount of money you would win. 11 (CO2) (PO2)

ii. Let X denotes the number of spots showing on the face of a well-balanced dice after it is rolled. Given that $Y = X + 2X + 3$, find $E[X]$, $E[Y]$, $E[X^2]$, $E[Y^2]$, $V[X]$, and $V[Y]$.

c) Define a Bernoulli distribution. Evaluate the mean, variance of a Bernoulli distribution. 6 (CO1) (PO1)