Statistical Hypothesis : A statistical hypothesis is an assumption/assertion/statement about a pop^n parameter. This assumption may or may not be true. [also about the values of several parameters/about form of an entire prob^r dist^n]

Simple Hypothesis: The hypothesis which completely specifies the dist^n of the parameters is called simple hypothesis.

for example;

$H_0 : \theta = \theta_0$ vs $H_1 : \theta = \theta_1$ is a simple hypothesis. "সরাসরি dist^n specified হবে?"

Composite Hypothesis: The hypothesis which completely doesn't specify the dist^n of the parameter is called composite hypothesis.

example;

$H_0 : \boxed{\theta = \theta_0}$ vs $H_1 : \theta > \theta_0$ or $\theta < \theta_0$ | or $\theta \neq \theta_0$

There are two Contradictory hypotheses.
(1) Null, (2) Alternative

ans 4 **Null Hypothesis:** The null hypothesis is a statement about the popⁿ parameters which is to be tested. It is denoted by H_0.

value, effect, conserv.

Alternative Hypothesis: It is a complementary hypothesis to the null; i.e. the logical opposite of null hypothesis is called alternative hypothesis. It is denoted by H_1 or H_A.

zify ... called ... searchers ... thesis.

One tailed test: A statistical hypothesis test where the alternative hypothesis is one sided such as

$$H_0: \mu \leq \mu_0 \quad vs \quad H_1: \mu > \mu_0$$

or $$H_0: \mu \geq \mu_0 \quad vs \quad H_1: \mu < \mu_0$$

Two tailed test: A statistical hypothesis test where the alternative hypothesis is two sided is called two tailed test.

Example: $H_0 : \theta = \theta_0$ vs $H_1 : \theta \neq \theta_0$

H_1 [$\theta \neq \theta_0$ মানে $< \theta_0$ অথবা $> \theta_0$ যে কোনোটাই হতে পারে].

07/03/17

* A manufacturer claims that the mean $\boxed{\text{thickness}}$ of the spearmint gum it produces is 2.5 cm in a inch. A quality control specialist regularly check this claim. From the production run, he took a r.s. of pieces of gum and measured their thickness. [Continuous variable এর ক্ষেত্রে usually mean দিয়ে denote করা parameter]

$H_0 : \mu = 2.5$

$H_1 : \mu \neq 2.5$

[যেহেতু বলেনি তাই normally distributed]

* A biologist was interested in determining whether a new treatment resulted in a lower ~~ag~~ average height of sunflower than the standard height of 15.2 cm

$H_0 : \mu = 15.2$ [চিহ্ন/Claim থাকলে

vs $H_1 : \mu < 15.2$ alternative এ যাবে]

[Null Hypothesis এ অবশ্যই 'equal' থাকবে, না থাকলেই Alternative]

* The scores of a r.s. of 8 students on hypothesis test are as follows -

60, 62, 65, 69, 70, 72, 75 and 78

One researcher wants to test whether the average is greater than 65. → Pop^n এর

$H_0 : \mu = 65$ [ধরেছে 65 নয় এর দিকের Value]

vs $H_1 : \mu > 65$

Reject $H_0 | H_0$ is true = type 1 error

Accept $H_0 | H_0$ is false = type 2 error

The error of rejecting null hypothesis when null hypothesis is true, is called type 1 error.

The error of accepting null hypothesis when null hypothesis is false, is called type 2 error.

α = level of significance = P(rejecting $H_0 | H_0$ is true)

The prob^y of type 1 error is called level of significance.

The prob^y of type 2 error is called ~~lev~~

$\beta = P$(accepting $H_0 | H_0$ is false)

	Ho is True	Ho is False
Reject Ho	Type 1 error α	Correct $(1-\beta)$ — Power of the test
Don't reject Ho	Correct $1-\alpha$	Type 2 Error β

Let, $H_0: \mu = 20$

vs $H_1: \mu > 20$

$\alpha = 5\%$

$\leftarrow 0.95 \rightarrow$ $0.05 = \alpha$

<u>Power of the test:</u> The probability of rejecting a false hypothesis is called power of the test.

$$P(\text{reject } H_0 \mid H_0 \text{ is false}) = 1-\beta$$

08/03/17

Among Type 1 error and Type 2 error – which is more serious ?? Ans; depends on situation

suppose.

H_0: no disease
H_1: disease present
$\alpha = Pr[\text{reject } H_0 \mid H_0 \text{ true}]$

Medical Test
→ +ve / no disease
→ -ve / disease → [Type 1 more serious]

H_0:

H_0: innocent
H_1: criminal
$\beta = \text{Prob}^r[\text{accept } H_0 \mid H_1 \text{ true}]$

Judge decision
→ Punishment / didn't commit murder → [Type 2]
→ no punishment / committed murder

Central Limit Th^m:

Let, $x_1, x_2, \ldots, x_n$ denote the observations of a r.s. from a $dist^n$ that has mean μ and variance $\sigma^2 (< \infty)$. Then the r.v.

$$Y_n = \frac{\sum_{i=1}^{n} x_i - n\mu}{\sqrt{n}\cdot\sigma} = \frac{\bar{x}-E(\bar{x})}{\sqrt{V(\bar{x})}} = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \sqrt{n}(\bar{x}-\mu)/\sigma$$

has a standard normal $dist^n$ when n is large.

~~Test Statistic~~

Procedures of a test:

① State the null and alternative hypothesis

② Choose the level of significance

③ State the assumptions such as normally distributed or equal variance

④ Collect data

⑤ Choose appropriate test statistic based on assumptions.

⑥ construct acceptance/rejection region or calculate ~~p-value~~ p-value

⑦ Based on ⑤ ~~and~~ and ⑥, take a decision.

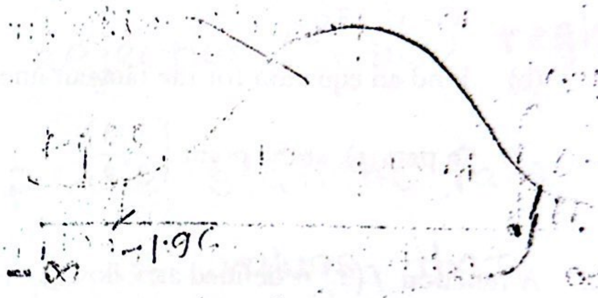
Test Statistic: A test statistic is calculated from sample data during a hypothesis test. It is used to determine whether to reject the null hypothesis. It is also used to calculate ~~procedure~~ p-value

Ex To test $H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$

test statistic, $Z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$

$\alpha = 5\%$

procedure



critical value / procedure

Rejection region

Mean = Mode = Median

(b) t-dist^n / st. normal

$H_1: \mu \neq \mu_0$ vs

$H_1: \mu > \mu_0$

normal Dist^n

symmetric

Normal Dist^n

symmetric about zero

	5%	1%
(i) $H_1: \mu \neq \mu_0$	1.96	
one tailed upper tail test $H_1: \mu > \mu_0$	1.645	
one tailed (lower tail) test $H_1: \mu < \mu_0$	-1.645	

Region of rejection: From the sample data, the researcher computes a test statistic. The specified range of values in which the statistic falls within and leads the researcher to reject the null hypothesis is called region of rejection (critical region).

Region of acceptance: If the statistic falls within a specified range of values that leads the researcher to accept the null hypothesis is called the region of acceptance.

Critical value: The particular ~~value~~ point which seperates the rejection region from the acceptance region is called the critical value.

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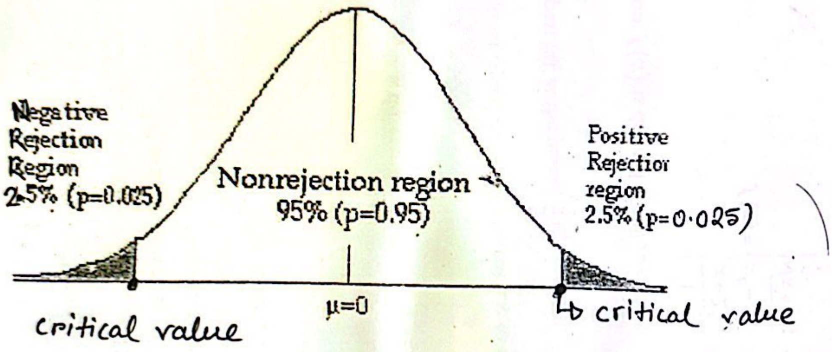
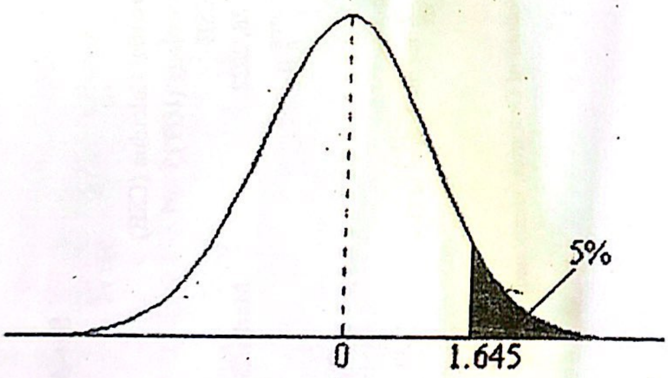
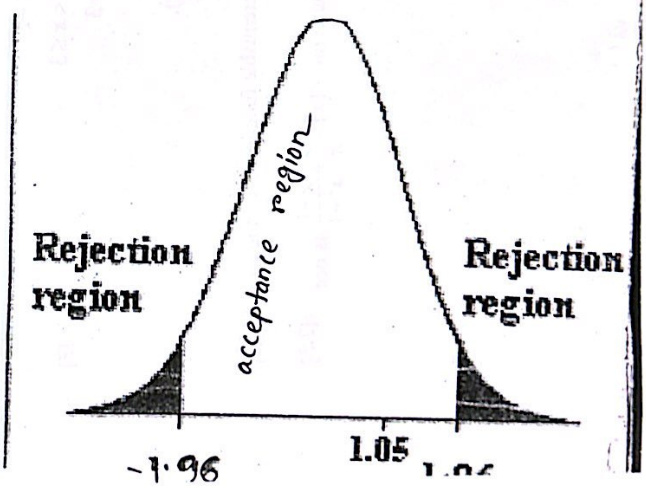




(a) One-tailed test

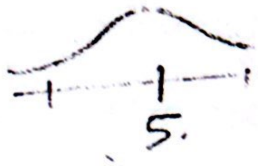


H_0 True ধরি শুরুতে কারণ –

i) যেহেতু Null এ '=' থাকে, এর জন্য চিত্র আঁকা যায়,

$H_0 : \mu = 5$

$H_A : \mu > 5$

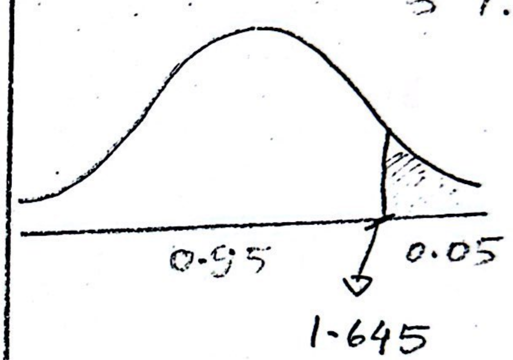


Critical region: From the data, one computes a test statistic. If the value of the test statistic falls within a specified range of values; the null hypothesis (H_0) is rejected; the region constituted by those speified range of values is called critical region.

$H_0: \mu = \mu_0$

vs $H_1: \mu > \mu_0$

t / z $= \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$



P-value

~~1.675~~

P-value < 0.05, H_0 rejected

P-value > 0.05, H_0 can not be rejected.

P-value $= P[Z \geq z_{cal}]$ (one tailed) [when $H_1: \mu > \mu_0$]

P-value $= P[Z > z_{cal}] + P[Z < -z_{cal}]$ (two tailed)

$= P[Z > |z_{cal}|] + P[Z < -|z_{cal}|]$

[0.05 এর সাথে compare করছি becaus $\alpha = 5\%$]

p-value : Calculated value থেকে extreme value পাবার সম্ভাবনা,

<u>P-value</u>: The probability of getting the observed or more extreme value when Ho is true, is called p-value.

(two tailed)

$H_0: \mu = \mu_0$

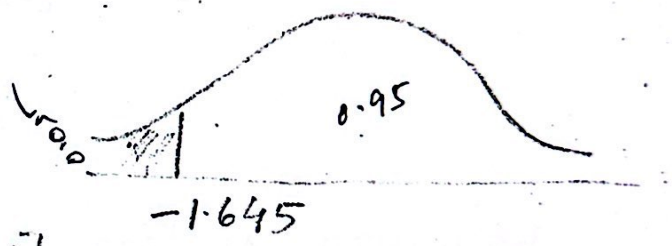
$H_1: \mu \neq \mu_0$

$\alpha = 5\%$

$H_0: \mu = \mu_0$

$H_1: \mu < \mu_0$

$\alpha = 5\%$

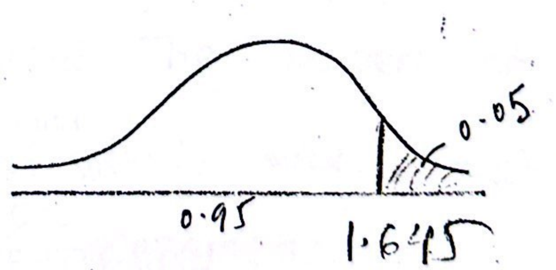


p-value $= P[Z \leq -|Z_{cal}|]$

$H_0: \mu = \mu_0$

$H_1: \mu > \mu_0$

$\alpha = 5\%$



p-value $= P[Z \geq |Z_{cal}|]$

Tests of Significance

After collecting data (sample), statistical inference requires to make assumptions about the popⁿ from which sample is drawn. This particular assumption depends on the $\boxed{\text{statistical test}}$ being used. In view of requirements, significant tests can be ~~used~~ broadly classified to fall into one of the two groups.

(i) Parametric Test.

(ii) Non-parametric Test

If prob^y distⁿ of the popⁿ is required and known, then parametric tests are used. If the prob^y distⁿ is unknown then non-parametric tests are used. The power of ~~tests s~~ non-parametric tests are lower than parametric tests. So, parametric tests should be used if possible.

Power = P [Reject H_0 / ~~H_0~~ is (+)ve] ??

Uniformly most powerful test (UMP test)

An UMP test is a hypothesis test that has the greatest power among all possible tests of a given size α. [say $\alpha = 0.05$]

* Single Mean Test

(i) Pop^n is normal with known variance.

Suppose $x_1, x_2, \dots, x_n$ be a r.s. of size n which is drawn from a normal pop^n with mean μ and known variance σ^2. Let us define $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

① Hypothesis: $H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Under the above assumption, the test statistic is given by-

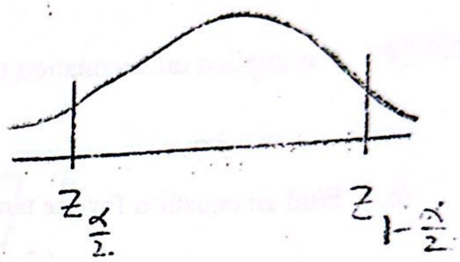
$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

Level of significance $= \alpha$

Decision:

Reject Ho if $Z_{cal} < Z_{\alpha/2}$ or $Z_{cal} > Z_{1-\alpha/2}$

2-tailed test.



P-value : $P\left[Z > |Z_{cal}|\right] + P\left[Z < -|Z_{cal}|\right]$

$= 2 * P\left[Z > |Z_{cal}|\right]$

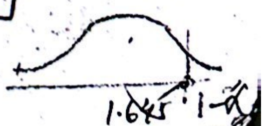
$= 2 * \left[1 - P\left[Z \leq |Z_{cal}|\right]\right]$

One ~~tailed~~ test

Case (2) $\quad H_0 : \mu = \mu_0 \quad vs \quad H_1 : \mu > \mu_0$

Decision: Reject Ho if $|Z_{cal}| > Z_{\cancel{\alpha/2}\, 1-\alpha}$

P- value $= P\left[Z \geq |Z_{cal}|\right] = 1 - P\left[Z < |Z_{cal}|\right]$



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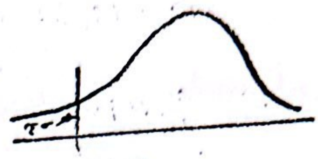
Case (3) $H_0 : \mu = \mu_0$ vs

$H_1 : \mu < \mu_0$



-1.645

Decision: Reject Ho if

$|z_{cal}| < z_\alpha$

$P.\text{value} = P\left[z \le -|z_{cal}|\right]$

2nd situation

(ii) Pop^n is normal with variance unknown and sample size > 30

Suppose $x_1, x_2, \ldots, x_n$ be n independent random sample from normal pop^n with mean μ and unknown variance σ^2.

Let us define $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

e-1) **To test:** $H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Test statistic $z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

level of significance $= \alpha$

Decision: Reject H_0 if $z_{cal} > z_{1-\alpha/2}$ or
$z_{cal} < z_{\alpha/2}$

$$\begin{aligned} P\text{-value} &= P[z > |z_{cal}|] + P[z < -|z_{cal}|] \\ &= 2 * P[z \geq |z_{cal}|] \\ &= 2 * [1 - P[z < |z_{cal}|]] \end{aligned}$$

(3rd situation)

Pop^n is normal with variance unknown and sample size ≤ 30

Suppose $x_1, x_2, \ldots, x_n$ be n iid r.s. from normal pop^n with mean μ and unknown variance σ^2.

Let us define, $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$ and $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$

To test -

$$H_0 : \mu = \mu_0 \quad vs \quad H_1 : \mu \neq \mu_0$$

test statistic $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim t_{n-1}$

level of significance $= \alpha$

Decision:

Reject H_0 if $t_{cal} > t_{n-1, 1-\frac{\alpha}{2}}$ or
upper tailed

$t_{cal} < t_{n-1, \alpha/2}$

P-value $= P[t \geq |t_{cal}|] + P[t < -|t_{cal}|]$

$= 2 * P[t > |t_{cal}|] = 2 * [1 - P[t \leq |t_{cal}|]]$

Problem: A new test was designed to have a mean of 80 and SD 10. A r.s. of 20 students take the test and the mean score turns out to be 85. Does this score significantly differ from 80?

Sol^n: Here, $\bar{x} = 85$, $\sigma = 10$, $n = 20$

We are to test $H_0: \mu = 80$ vs
$H_1: \mu \neq 80$

test statistic, $Z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{85-80}{\frac{10}{\sqrt{20}}} = \sqrt{5}$
$= 2.236$

$\sim N(0,1)$ Ho True
under Ho True

Let, level of significance $\alpha = 0.05$

critical value $Z_{\alpha/2} = -1.96$ and $Z_{1-\alpha/2} = 1.96$

Decision: Since, $Z_{cal} = 2.236 > Z_{1-\frac{\alpha}{2}} = 1.96$

so we may reject Ho at 5% level of significance.

P-value $= 2 * [1 - P[Z < Z_{cal}]] = 2 * 0.0127$
$= 0.025$

since p-value = 0.025 < α = 0.05 we may reject H_0 at 5% level of significance. one

Hence, this test score significantly differs from 80.

Two Means Test (২ টা Pop^n নিয়ে কাজ করে)

The common test statistic is -

$$\text{Test statistic} = \frac{\bar{x} - \bar{y} - D_0}{\text{standard error}}$$

where $\bar{x}$ and $\bar{y}$ are sample means of two populations and D_0 is the value of difference under H_0.

Here standard error changes depending on whether the pop^n are -

(i) independent with known variance

(ii) " " unknown but assumed equal variances

(iii) ~~indepedent~~ independent with unknown and unequal variances.

(iv) dependent → same individual থেকে 2 বার obs^n নিয়ে ভিন্ন সময়ে

1) independent with known variances: Let $x_1, x_2, \ldots, x_n$ and $Y_1, Y_2, \ldots, Y_m$ be two independent random samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively. Here σ_1^2 and σ_2^2 are known.

Let us define,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \text{and} \quad \bar{y} = \frac{1}{m}\sum_{i=1}^{m} y_i$$

2-① Hypothesis: $H_0: \mu_x - \mu_y = D_0$

vs. $H_1: \mu_x - \mu_y \neq D_0$

Test statistic, $$z = \frac{\bar{x} - \bar{y} - D_0}{S.E.(\bar{x} - \bar{y})}$$

$$= \frac{\bar{x} - \bar{y} - D_0}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}}$$

$$V(\bar{x} - \bar{y}) = V(\bar{x}) + V(\bar{y}) - 2\,Cov(\bar{x}, \bar{y}) = \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m} + 0$$

Let, α = Level of significance

Decision: Reject H_0 if $z_{cal} \leq z_{\alpha/2}$ or $z_{cal} \geq z_{1-\alpha/2}$

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P-value : $2 * P[Z \geq |Z_{cal}|]$

$= 2 * [1 - P[Z \leq |Z_{cal}|]]$

$= 2 * [1 - P[Z \leq |Z_{cal}|]]$

Case (2) $H_0: \mu_x - \mu_y \leq D_0$

$H_1: \mu_x - \mu_y > D_0$

⋮

Decision: reject H_0 if $Z_{cal} > Z_{1-\alpha}$

P-value $= P[Z > |Z_{cal}|] = 1 - P[Z < |Z_{cal}|]$

Case (3) $H_0: \mu_x - \mu_y \geq D_0$

$H_1: \mu_x - \mu_y < D_0$

⋮

Decision: reject H_0 if $Z_{cal} \leq Z_{\alpha}$

P-value $= P[Z < -|Z_{cal}|]$

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Ex. One reason for wage differentials bet^n men and ~~wom~~ women is the difference of their work experience. Given data of experiences as- Dec

Men	Women
$n_x = 100$	$n_y = 80$
$\bar{x} = 13.5$	$\bar{y} = 11.5$
$\sigma_x = 5.2$	$\sigma_y = 3.8$

Test the hypothesis that, "men tend to have ~~at least~~ 4.5 more years of experiences than women."

er than

$D_0 = 0$
যখন

$$H_0: \mu_x - \mu_y \leq 4.5$$

$$\text{vs. } H_1: \mu_x - \mu_y > 4.5$$

$$z = \frac{\bar{x} - \bar{y} - D_0}{SE(\bar{x} - \bar{y})} = \frac{13.5 - 11.5 - 4.5}{\sqrt{\frac{5.2^2}{100} + \frac{3.8^2}{80}}} = -3.72$$

.0005 <.001

Let, $\alpha = 0.05$. Then $Z_\alpha = -1.645$
$= Z_{0.05}$

ision Since $Z_{cal} = -3.72 < Z_{.05} = -1.645$;

so we may reject H_0 at 5% level of significance. (Ans)

$P(Z \geq -P_{cal})$

P-value $= P[Z < -|Z_{cal}|]$ / $1 - P[Z \leq 3.723]$
$= P[Z < -3.72]$ $\quad = 1 - P(Z \leq 3.723)$
$= <0.01$
$= pnorm(-3.72) = 0.0000957$
$= < 0.001$

(2) Independent with unknown but assumed equal variances.

Let, $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_m$ be two samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ where σ_1^2 and σ_2^2 are unknown but $\sigma_1^2 = \sigma_2^2$

Let us define –

$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$ and $\bar{y} = \frac{1}{m} \sum_{i=1}^{m} y_i$