Artificial Intelligence
CSE 4617

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What is Search for?

- Assumptions about the world
  - Single agent → No adverseries
  - Deterministic actions
  - Fully observed state
  - Discrete state space

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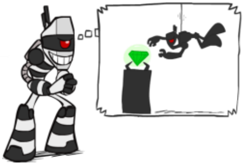
  - The path to the goal is the important thing
  - Paths have various costs, depths
  - Heuristics give problem-specific guidance



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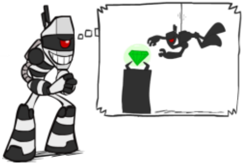
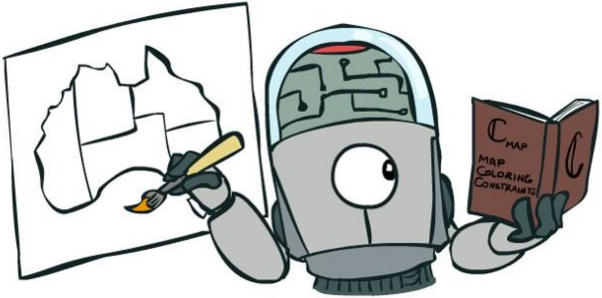
  - Heuristics give problem-specific guidance
- Identification: assignments to variables
  - The goal itself is important, not the path
  - All paths at the same depth (for some formulations)
  - CSPs are a specialized class of identification problems

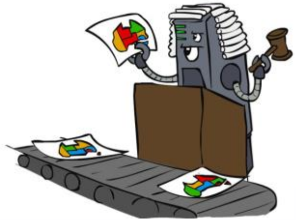


Constraint Satisfaction Problems



Constraint Satisfaction Problems

- Standard search problems
  - State is a "black box": arbitrary data structure
  - Goal test can be any function over states
  - Successor function can also be anything



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  - A special subset of search problems
  - State is defined by variables X_i with values from a domain D (sometimes D depends on i)
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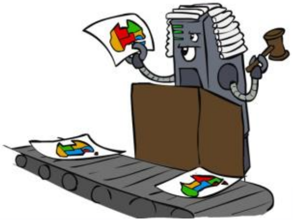

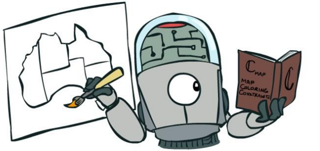

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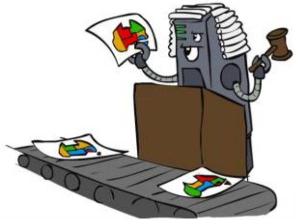

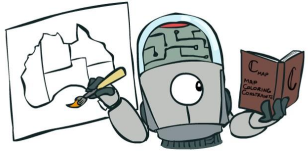

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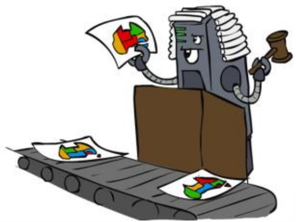
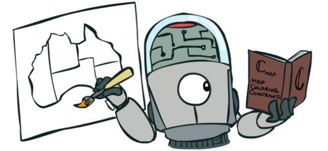
- Simple example of a *formal representation language*
- Allows useful general-purpose algorithms with more power than standard search algorithms

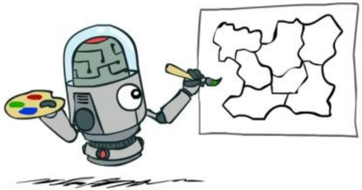




CSP Examples

Example: Map Coloring

- Variables:



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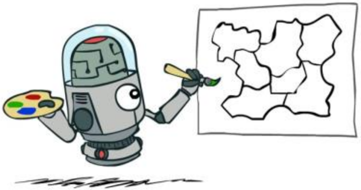

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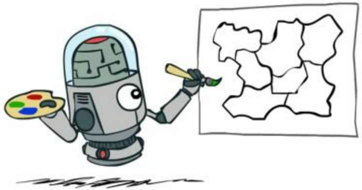

Example: Map Coloring

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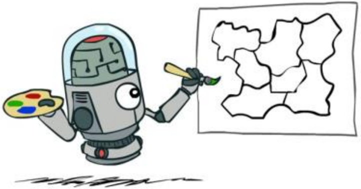

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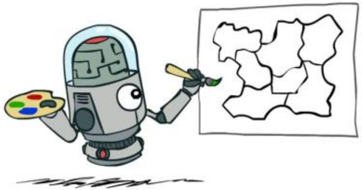

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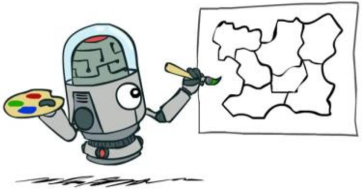

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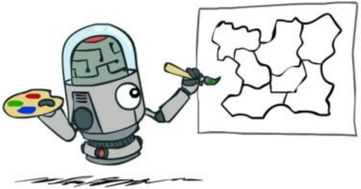

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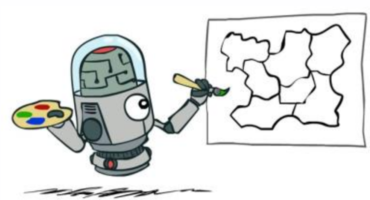

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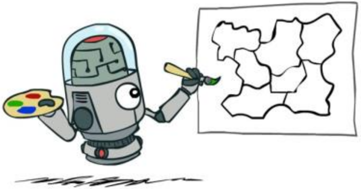
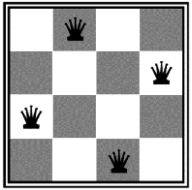


Example: N-Queens



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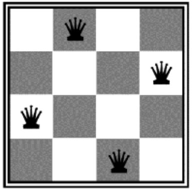

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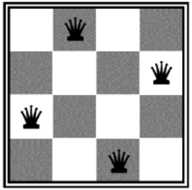

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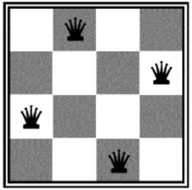

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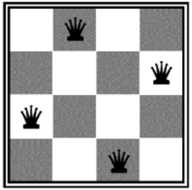

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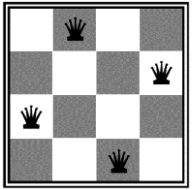

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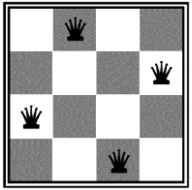

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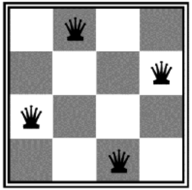

$$\forall i,j,k\ (X_{ij}, X_{ik}) \in \{(0,0),(0,1),(1,0)\}$$
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$$\forall i,j,k\ (X_{ij}, X_{i+k,j+k}) \in \{(0,0),(0,1),(1,0)\}$$
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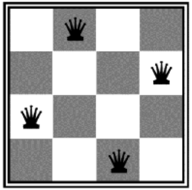

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$$\sum_{i,j} X_{ij} = N$$

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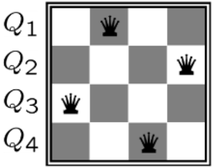
- Formulation 2:
  - Variables: Q_k
  - Domains: $\{1, 2, 3, \ldots, N\}$
  - Constraints:



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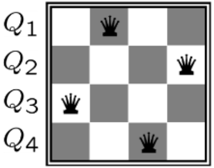

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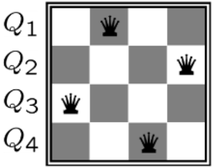

- Implicit: $\forall i,j$ non-threatening (Q_i, Q_j)
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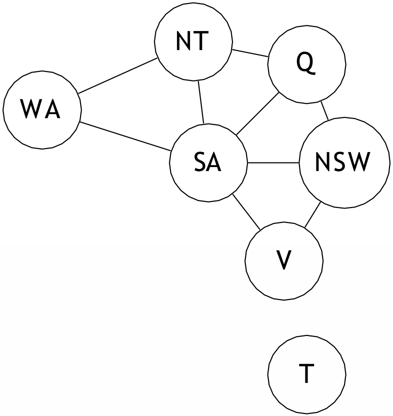
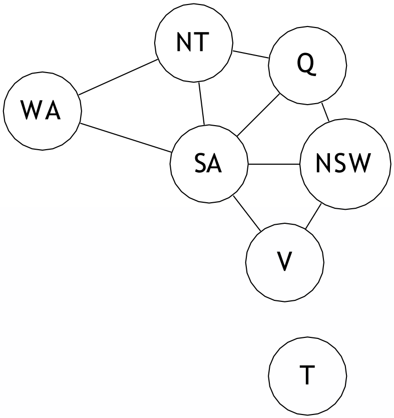
  $\ldots$

Constraint Graphs



Constraint Graphs

- Binary CSP: each constraint relates (at most) two variables
- Binary constraint graph: nodes are variables, arcs show constraints
- General-purpose CSP algorithms use the graph structure to speed up search. E.g., Tasmania is an independent subproblem!



Example: Cryptarithmetic

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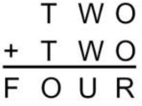

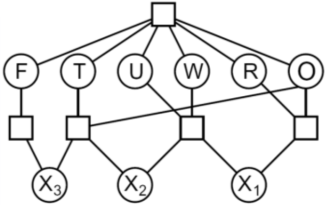

Example: Sudoku

- Variables
  - Each (open) square
- Domains
  - $\{1, 2, \ldots, 9\}$
- Constraints

					8			4
	8	4		1	6			
			5			1		
1		3	8			9		
6		8				4		3
		2			9	5		1
		7			2			
			7	8		2	6	
2			3					

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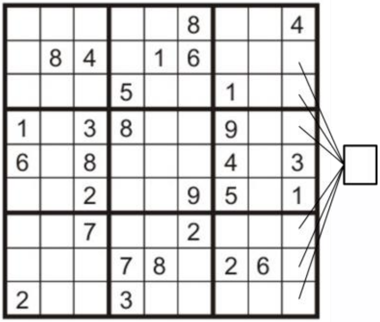

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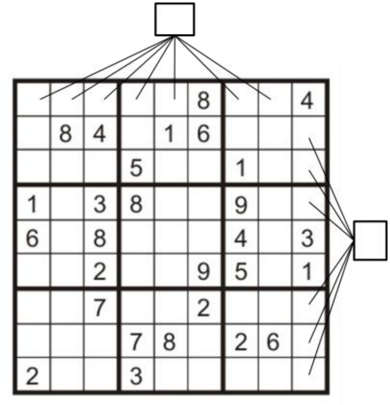

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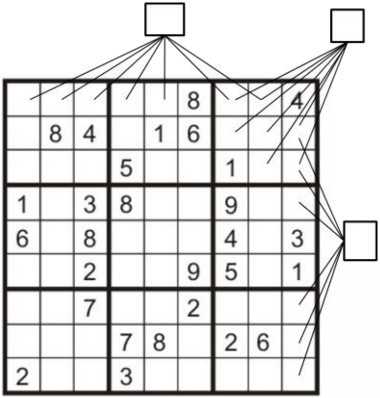

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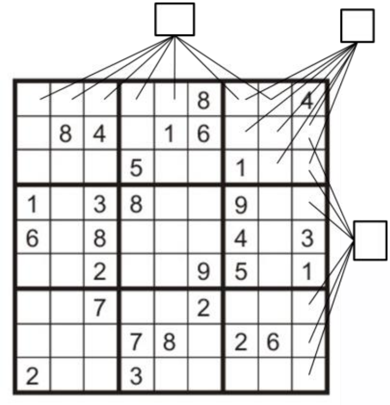
  - Can also have a bunch of pairwise inequalities



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- Continuous Variables
  - E.g., start/end times for Hubble Telescope observations

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- Higher-order constraints involve 3 or more variables, e.g., cryptarithmetic column constraints

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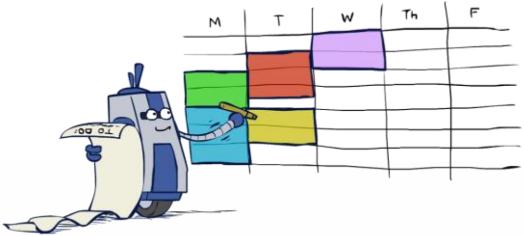
- Unary constraints involve a single variable (equivalent to reducing domains), e.g.: $SA \neq \text{green}$
- Binary constraints involve pairs of variables, e.g.: $SA \neq WA$
- Higher-order constraints involve 3 or more variables, e.g., cryptarithmetic column constraints
- Preferences (soft constraints)
  - E.g., red is better than green
  - Often representable by a cost for each variable assignment
  - Gives constrained optimization problems
  - (We'll ignore these until we get to Bayes' nets)

Real-World CSPs

- Scheduling problem
- Timetabling problem
- Assignment problem
- Hardware configuration
- Transportation scheduling
- Factory scheduling Circuit
- layout
- Fault diagnosis
- ... lots more!

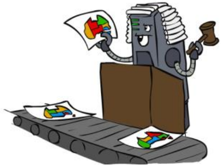
- Many real-world problems involve real-valued variables...



Solving CSPs

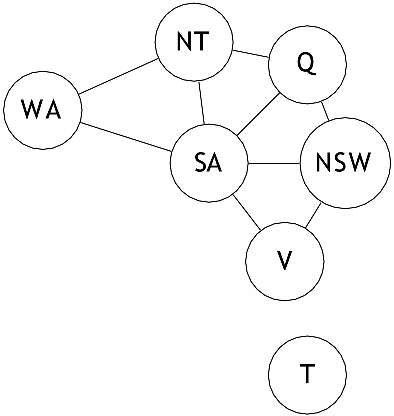
Standard Search Formulation

- States defined by the values assigned so far (partial assignments)
  - Initial state: the empty assignment, {}
  - Successor function: assign a value to an unassigned variable
  - Goal test: the current assignment is complete and satisfies all constraints



Search Methods

- What would BFS do?



Website: simple-naive

Search Methods

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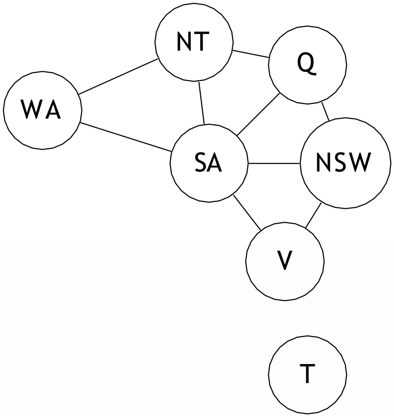
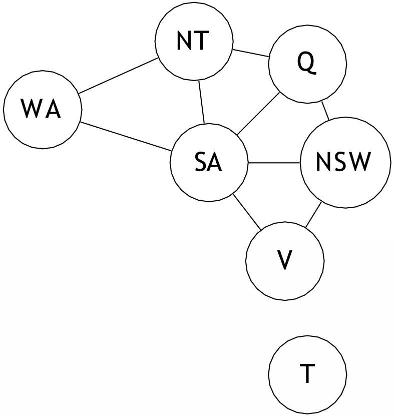
- What would DFS do?



Website: [simple -naive]

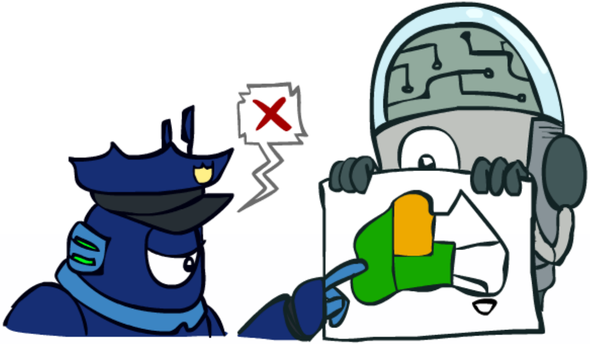
Search Methods

- What would BFS do?
- What would DFS do?
- What problems does naïve search have?



Website: [simple -naive]

Backtracking Search



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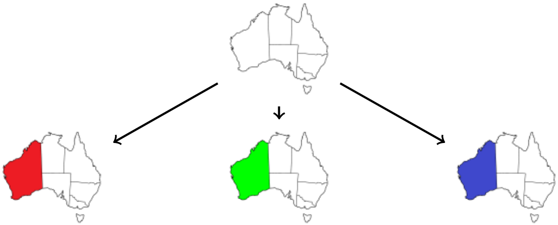
  - "Incremental goal test"
- Depth-first search with these two improvements is called *backtracking search*

Backtracking Search

Backtracking Search



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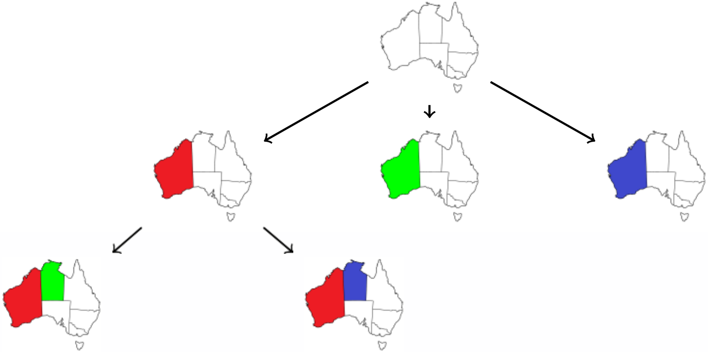

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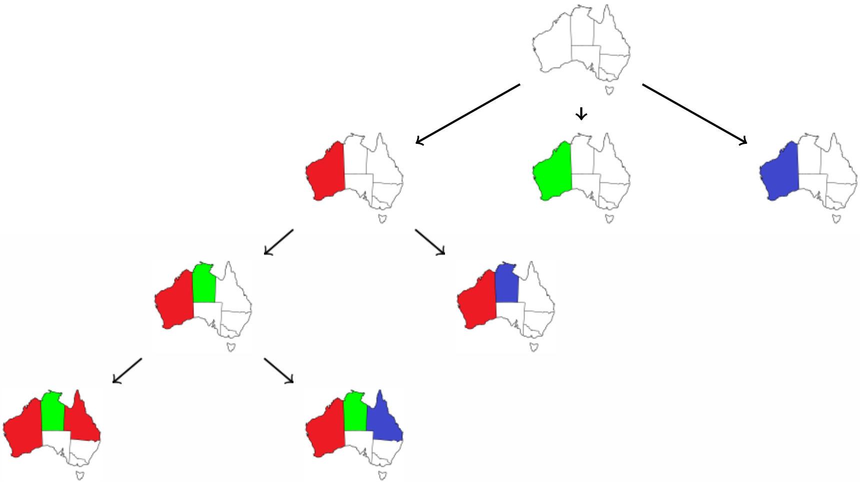


Backtracking Search

```
function BACKTRACKING-SEARCH(csp) returns a solution, or failure
   return RECURSIVE-BACKTRACKING({}, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns a solution, or failure
   if assignment is complete then return assignment
   var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
   for each value in ORDER-DOMAIN-VALUE(var, assignment, csp) do
      if value is consistent with assignment given CONSTRAINTS[csp] then
         add {var = value} to assignment
         result ← RECURSIVE-BACKTRACKING(assignment, csp)
         if result ≠ failure then return result
         remove {var = value} from assignment
   return failure
```

- Backtracking = DFS + variable-ordering + fail-on-violation

Website: simple -backtracking

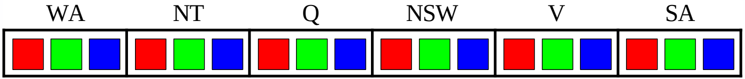
Improving Backtracking

- General-purpose ideas give huge gains in speed
- Filtering: Can we detect inevitable failure early?
- Ordering:
  - Which variable should be assigned next?
  - In what order should its values be tried?
- Structure: Can we exploit the problem structure?

Filtering

Filtering: Forward Checking

- Filtering: Keep track of domains for unassigned variables and cross off bad options
- Forward checking: Cross off values that violate a constraint when added to the existing assignment



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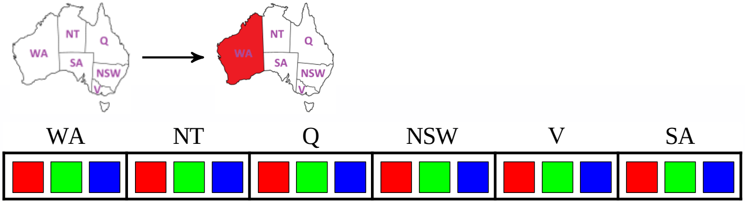

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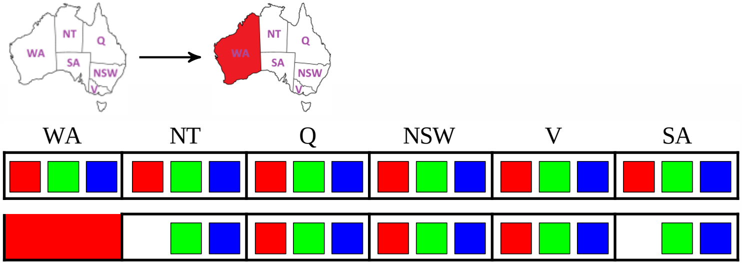

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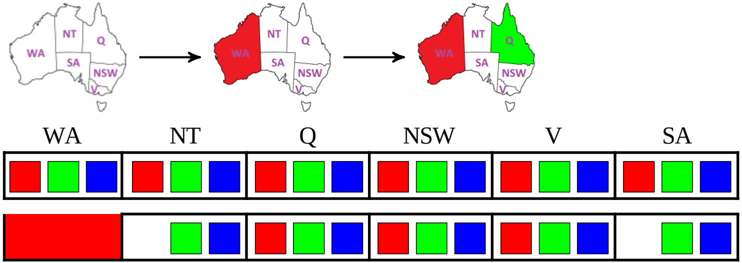

Website: [simple - backtracking, forward]()

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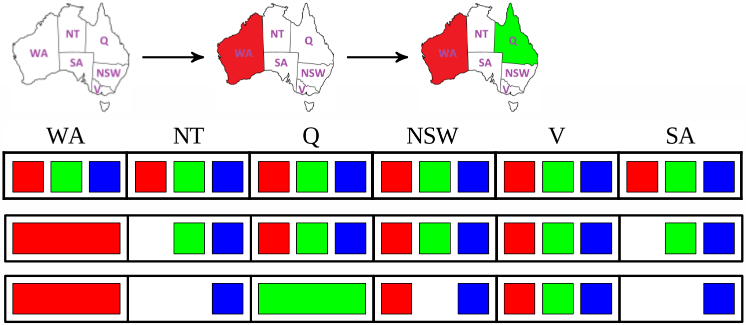

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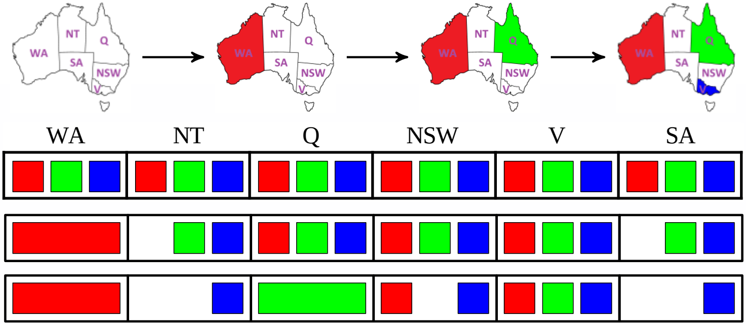

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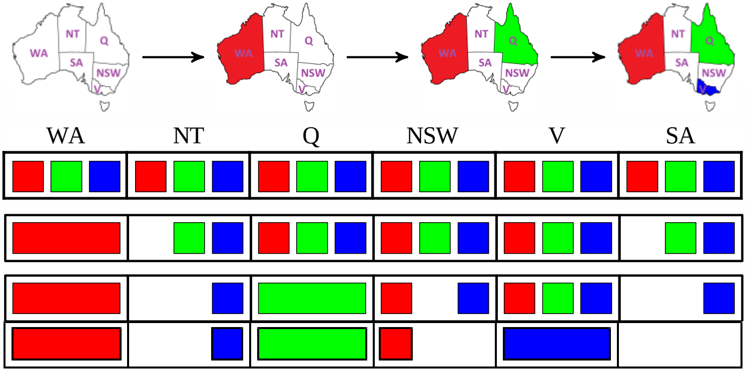

Filtering: Constraint Propagation

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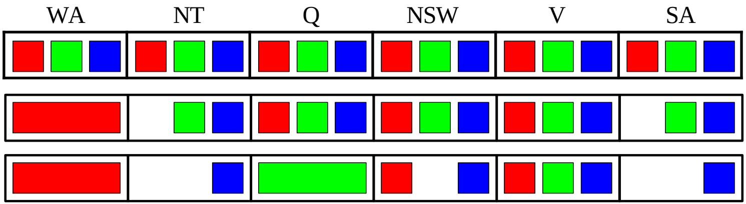

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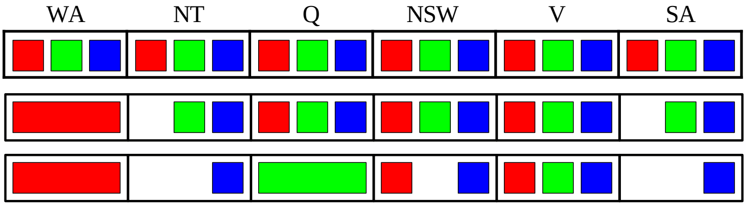

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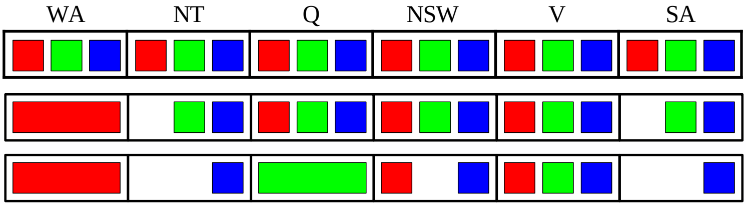

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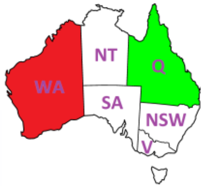
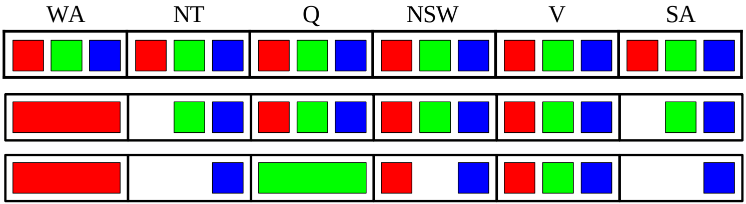
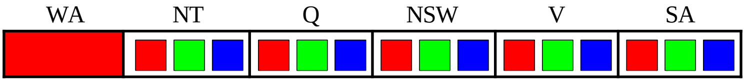




- NT and SA cannot both be blue!
- Why didn't we detect this yet?
- *Constraint propagation:* reason from constraint to constraint

Consistency of A Single Arc

- An arc $X \rightarrow Y$ is consistent iff for *every* x in the tail there is *some* y in the head which could be assigned without violating a constraint



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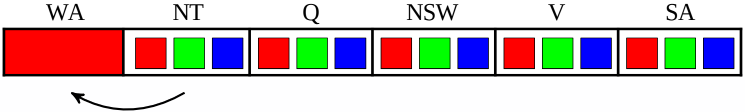

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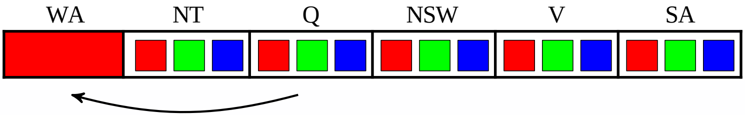

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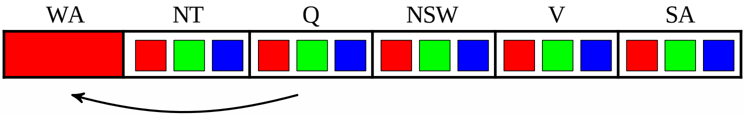

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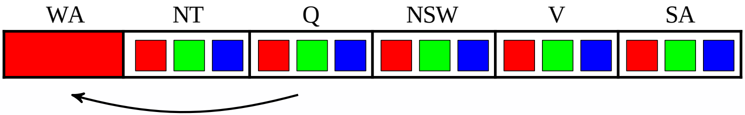

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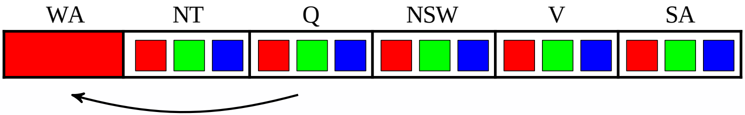

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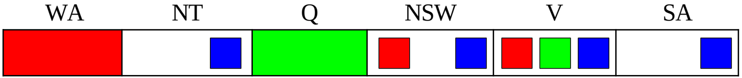
- Forward checking: Enforcing consistency of arcs pointing to each new assignment

Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:



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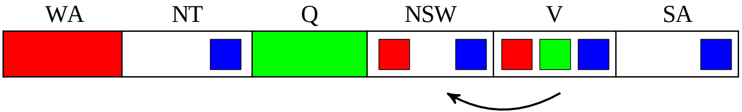

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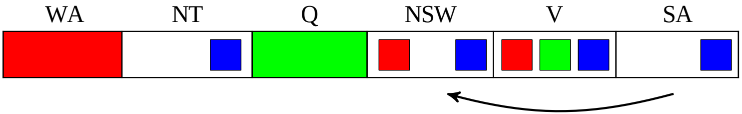

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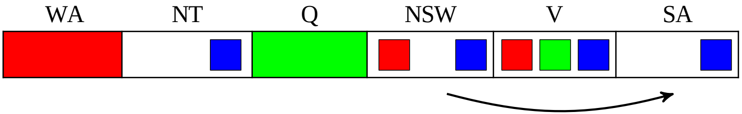

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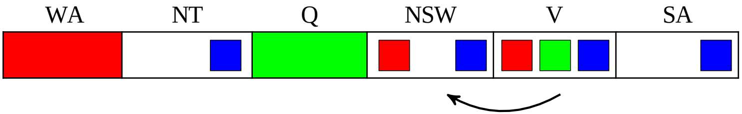

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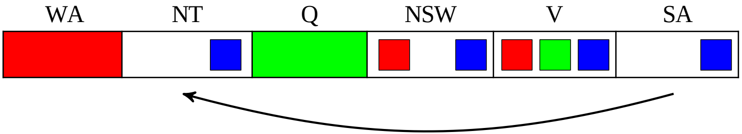

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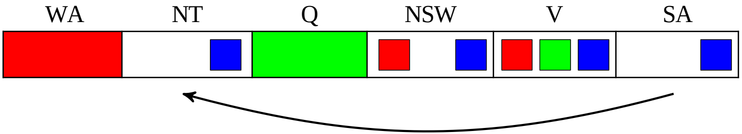

- Important: If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment
- What's the downside of enforcing arc consistency?

Remember: Delete from the tail!

Enforcing Arc Consistency in a CSP

function AC-3(*csp*) **returns** the CSP, possibly with reduced domains
  inputs: *csp*, a binary CSP with variables $\{X_1, X_2, \ldots, X_N\}$
  local variables: *queue*, a queue of arcs, initially all the arcs in *csp*
  while *queue* is not empty **do**
    $(X_i, X_j) \leftarrow$ REMOVE-FIRST(*queue*)
    if REMOVE-INCONSISTENT-VALUES(X_i, X_j) **then**
      for each X_k in NEIGHBORS[X_i] **do**
        add (X_k, X_i) to *queue*

function REMOVE-INCONSISTENT-VALUES(X_i, X_j) **returns** true iff succeeds
  removed ← *false*
  for each x in DOMAIN[X_i] **do**
    if no value y in DOMAIN[X_j] allows (x, y) to satisfy the constraint $X_i \leftrightarrow X_j$
      then delete x from DOMAIN[X_i]; *removed* ← *true*
  return *removed*

Applet: CSP - fiveQueens

Suggested Reading

- Russell & Norvig: Chapter 6.1