Artificial Intelligence
CSE 4617

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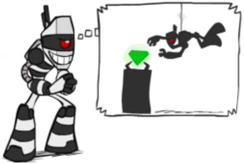
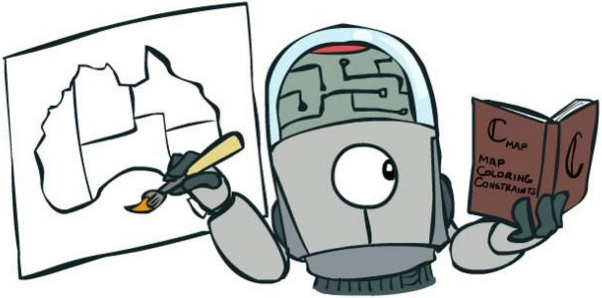
What is Search for?

- Assumptions about the world
  - Single agent → No adverseries
  - Deterministic actions
  - Fully observed state
  - Discrete state space
- Planning: sequences of actions
  - The path to the goal is the important thing
  - Paths have various costs, depths
  - Heuristics give problem-specific guidance
- Identification: assignments to variables
  - The goal itself is important, not the path
  - All paths at the same depth (for some formulations)
  - CSPs are a specialized class of identification problems

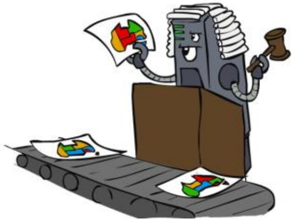


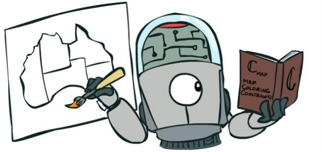
Constraint Satisfaction Problems



Constraint Satisfaction Problems

- Standard search problems
  - State is a "black box": arbitrary data structure
  - Goal test can be any function over states
  - Successor function can also be anything
- Constraint satisfaction problems (CSPs):
  - A special subset of search problems
  - State is defined by variables X_i with values from a domain D (sometimes D depends on i)
  - Goal test is a set of constraints specifying allowable combinations of values for subsets of variables
- Simple example of a *formal representation language*
- Allows useful general-purpose algorithms with more power than standard search algorithms

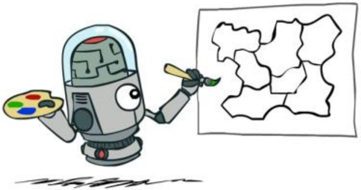




CSP Examples

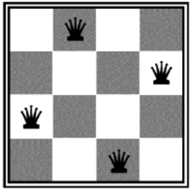
Example: Map Coloring

- **Variables:**
  - WA, NT, Q, NSW, V, SA, T
- **Domains:**
  - $D = \{\text{red}, \text{green}, \text{blue}\}$
- **Constraints:** adjacent regions must have different colors
  - Implicit: $WA \neq NT$
  - Explicit:
    $(WA, NT) \in \{(\text{red}, \text{green}), (\text{red}, \text{blue}), \ldots\}$
- **Solutions are assignments satisfying all constraints**
  - $\{WA = \text{red}, NT = \text{green}, Q = \text{red}, NSW = \text{green}, V = \text{red}, SA = \text{blue}, T = \text{green}\}$



Example: N-Queens

- Formulation 1:
  - Variables: X_{ij}
  - Domains: $\{0,1\}$
  - Constraints:



$$\forall i,j,k \; (X_{ij}, X_{ik}) \in \{(0,0), (0,1), (1,0)\}$$

$$\forall i,j,k \; (X_{ij}, X_{kj}) \in \{(0,0), (0,1), (1,0)\}$$

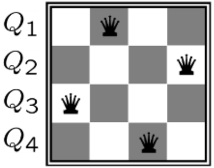
$$\forall i,j,k \; (X_{ij}, X_{i+k,j+k}) \in \{(0,0), (0,1), (1,0)\}$$

$$\forall i,j,k \; (X_{ij}, X_{i+k,j-k}) \in \{(0,0), (0,1), (1,0)\}$$

$$\sum_{i,j} X_{ij} = N$$

Example: N-Queens

- Formulation 2:
  - Variables: Q_k
  - Domains: $\{1, 2, 3, \ldots, N\}$
  - Constraints:

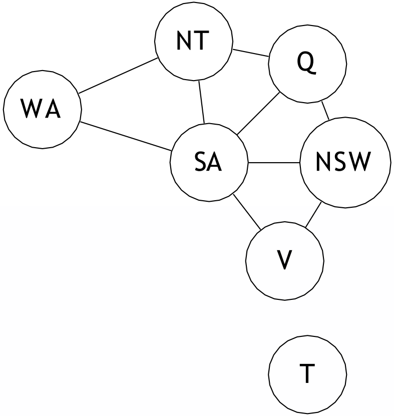


- Implicit: $\forall i,j$ non-threatening (Q_i, Q_j)
- Explicit: $(Q_i, Q_j) \in \{(1,3), (1,4), \ldots\}$

  $\ldots$

Constraint Graphs

- Binary CSP: each constraint relates (at most) two variables
- Binary constraint graph: nodes are variables, arcs show constraints
- General-purpose CSP algorithms use the graph structure to speed up search. E.g., Tasmania is an independent subproblem!



Example: Cryptarithmetic

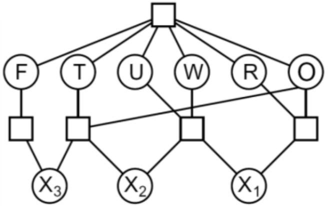
- Variables
  - $F, T, U, W, R, O, X_1, X_2, X_3$
- Domains:
  - $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- Constraints:

  alldiff(F, T, U, W, R, O)

  $O + O = R + 10 \times X_1$

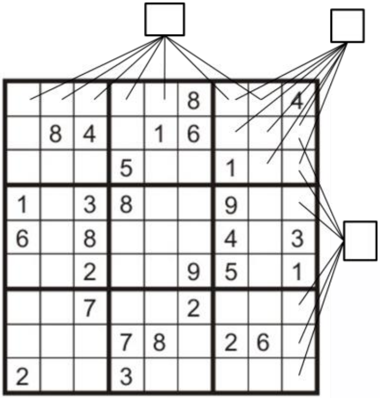
  $\ldots$

```
    T W O
  + T W O
  -------
  F O U R
```



Example: Sudoku

- Variables
  - Each (open) square
- Domains
  - $\{1, 2, \ldots, 9\}$
- Constraints
  - Unary constraints for given values
  - 9-way alldiff for each column
  - 9-way alldiff for each row
  - 9-way alldiff for each region
  - Can also have a bunch of pairwise inequalities



Varieties of CSPs and Constraints

Varieties of CSPs

- Discrete Variables
  - Finite domains
    - Size d means $O(d^n)$ complete assignments
  - Infinite domains (integers, strings, etc.)
    - E.g., job scheduling, variables are start/end times for each job

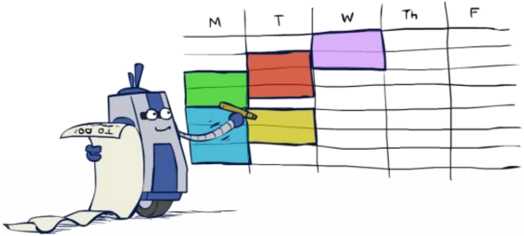
- Continuous Variables
  - E.g., start/end times for Hubble Telescope observations

Varieties of Constraints

- Unary constraints involve a single variable (equivalent to reducing domains), e.g.: $SA \neq$ green
- Binary constraints involve pairs of variables, e.g.: $SA \neq WA$
- Higher-order constraints involve 3 or more variables, e.g., cryptarithmetic column constraints
- Preferences (soft constraints)
  - E.g., red is better than green
  - Often representable by a cost for each variable assignment
  - Gives constrained optimization problems
  - (We'll ignore these until we get to Bayes' nets)

Real-World CSPs

- Scheduling problem
- Timetabling problem
- Assignment problem
- Hardware configuration
- Transportation scheduling
- Factory scheduling Circuit
- layout
- Fault diagnosis
- … lots more!

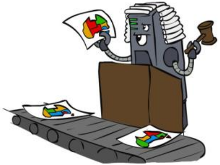


- Many real-world problems involve real-valued variables…

Solving CSPs

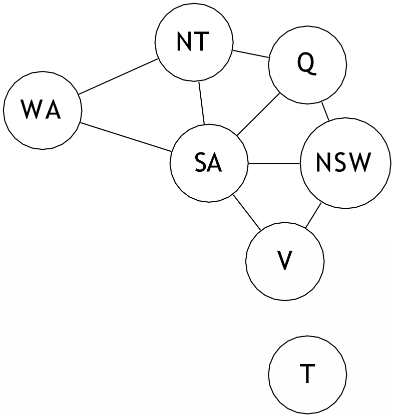
Standard Search Formulation

- States defined by the values assigned so far (partial assignments)
  - Initial state: the empty assignment, {}
  - Successor function: assign a value to an unassigned variable
  - Goal test: the current assignment is complete and satisfies all constraints



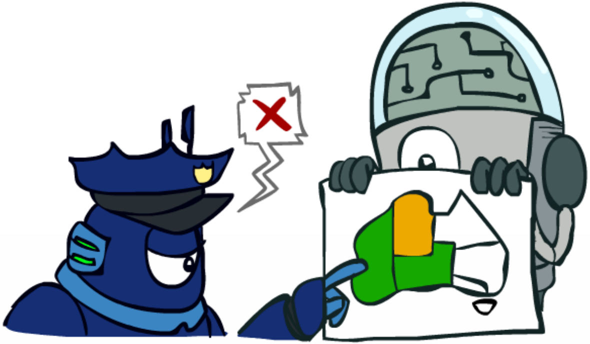
Search Methods

- What would BFS do?
- What would DFS do?
- What problems does naïve search have?



Website: [simple -naive]

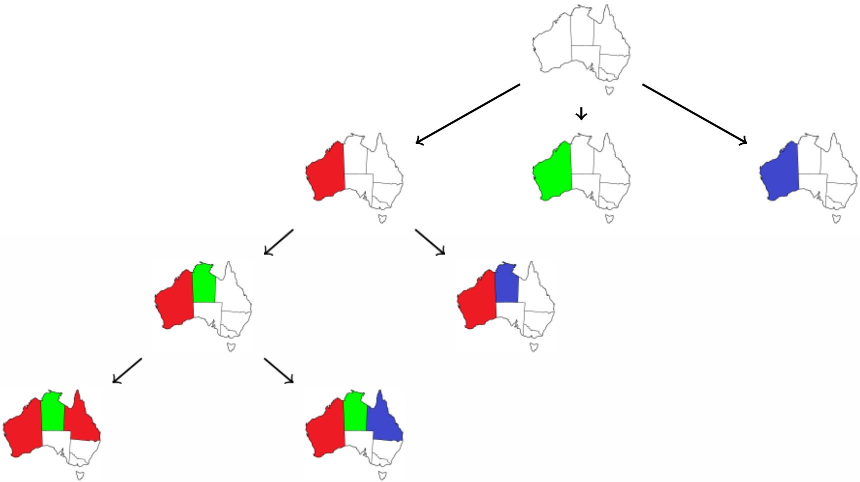
Backtracking Search



Backtracking Search

- Backtracking search is the basic uninformed algorithm for solving CSPs
- Idea 1: One variable at a time
  - Variable assignments are commutative $\rightarrow$ Any ordering is OK!
  - i.e., [$WA =$ red then $NT =$ green] same as [$NT =$ green then $WA =$ red]
  - Only need to consider assignments to a single variable at each step
- Idea 2: Check constraints as you go
  - i.e. consider only values which do not conflict with previous assignments
  - Might have to do some computation to check the constraints
  - "Incremental goal test"
- Depth-first search with these two improvements is called *backtracking search*

Backtracking Search



Backtracking Search

```
function BACKTRACKING-SEARCH(csp) returns a solution, or failure
   return RECURSIVE-BACKTRACKING({}, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns a solution, or failure
   if assignment is complete then return assignment
   var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
   for each value in ORDER-DOMAIN-VALUE(var, assignment, csp) do
      if value is consistent with assignment given CONSTRAINTS[csp] then
         add {var = value} to assignment
         result ← RECURSIVE-BACKTRACKING(assignment, csp)
         if result ≠ failure then return result
         remove {var = value} from assignment
   return failure
```

- Backtracking = DFS + variable-ordering + fail-on-violation

Website: simple -backtracking

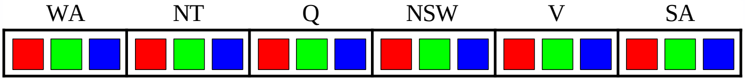
Improving Backtracking

- General-purpose ideas give huge gains in speed
- Filtering: Can we detect inevitable failure early?
- Ordering:
  - Which variable should be assigned next?
  - In what order should its values be tried?
- Structure: Can we exploit the problem structure?

Filtering

Filtering: Forward Checking

- Filtering: Keep track of domains for unassigned variables and cross off bad options
- Forward checking: Cross off values that violate a constraint when added to the existing assignment



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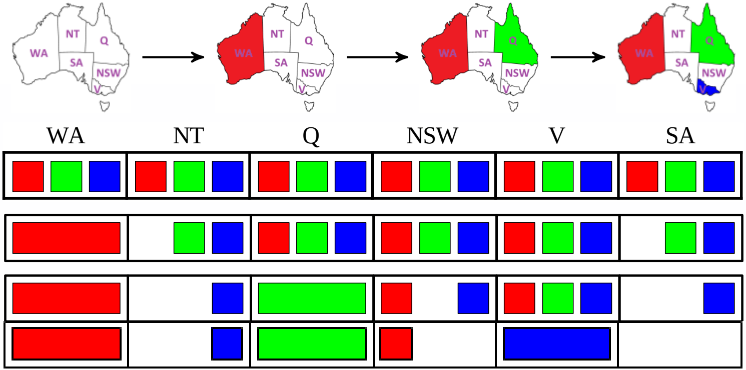
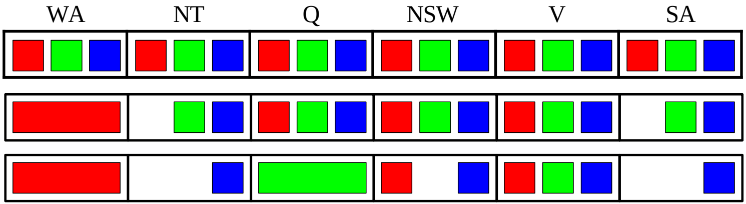


Filtering: Constraint Propagation

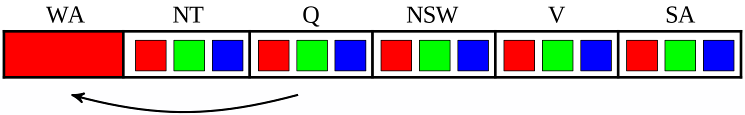
- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:



- NT and SA cannot both be blue!
- Why didn't we detect this yet?
- *Constraint propagation:* reason from constraint to constraint

Consistency of A Single Arc

- An arc $X \rightarrow Y$ is consistent iff for *every* x in the tail there is *some* y in the head which could be assigned without violating a constraint

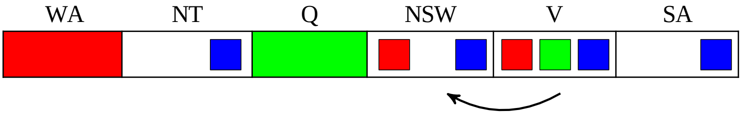


Delete from the tail!

- Forward checking: Enforcing consistency of arcs pointing to each new assignment

Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:



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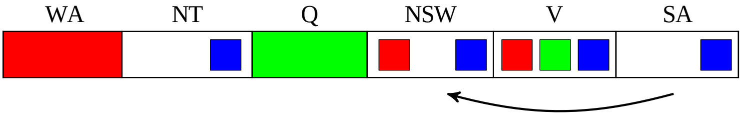

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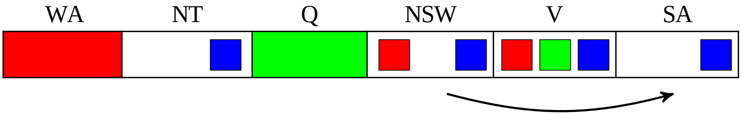

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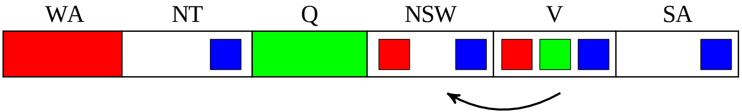

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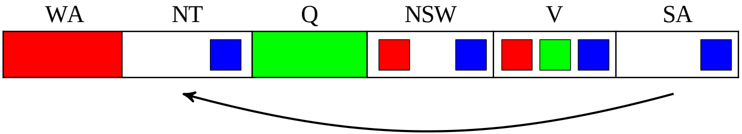

- Important: If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment
- What's the downside of enforcing arc consistency?

Remember: Delete from the tail!

Enforcing Arc Consistency in a CSP

```
function AC-3(csp) returns the CSP, possibly with reduced domains
  inputs: csp, a binary CSP with variables {X_1, X_2, ..., X_N}
  local variables: queue, a queue of arcs, initially all the arcs in csp
  while queue is not empty do
    (X_i, X_j) ← REMOVE-FIRST(queue)
    if REMOVE-INCONSISTENT-VALUES(X_i, X_j) then
      for each X_k in NEIGHBORS[X_i] do
        add (X_k, X_i) to queue
function REMOVE-INCONSISTENT-VALUES(X_i, X_j) returns true iff succeeds
  removed ← false
  for each x in DOMAIN[X_i] do
    if no value y in DOMAIN[X_j] allows (x, y) to satisfy the constraint X_i ↔ X_j
      then delete x from DOMAIN[X_i]; removed ← true
  return removed
```

Applet: CSP - fiveQueens

Suggested Reading

- Russell & Norvig: Chapter 6.1