# Artificial Intelligence
## CSE 4617

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# What is Search for?

- Assumptions about the world
  - Single agent → No adverseries
  - Deterministic actions
  - Fully observed state
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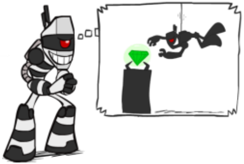

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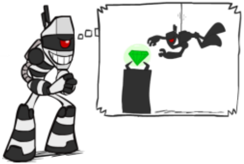
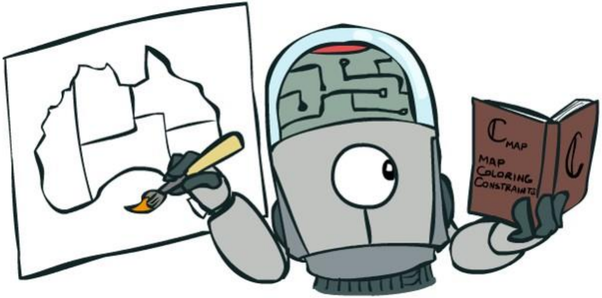
  - Heuristics give problem-specific guidance
- Identification: assignments to variables
  - The goal itself is important, not the path
  - All paths at the same depth (for some formulations)
  - CSPs are a specialized class of identification problems



# Constraint Satisfaction Problems



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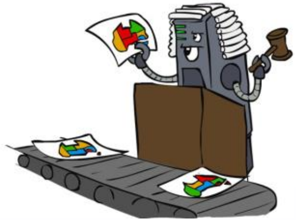
  - State is a “black box”: arbitrary data structure
  - Goal test can be any function over states
  - Successor function can also be anything



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  - A special subset of search problems
  - State is defined by variables $X_i$ with values from a domain $D$ (sometimes $D$ depends on $i$)
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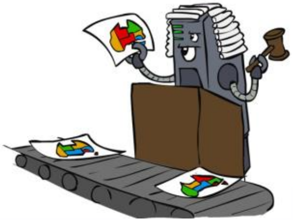

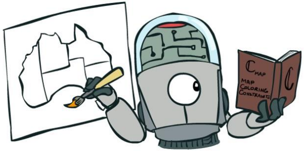

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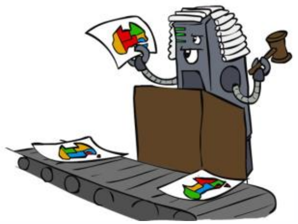

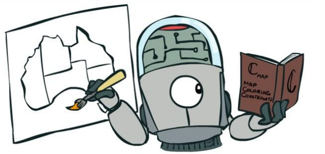

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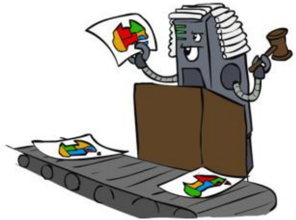
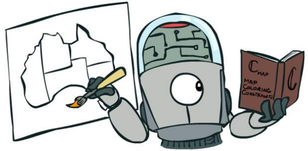
- Simple example of a *formal representation language*
- Allows useful general-purpose algorithms with more power than standard search algorithms

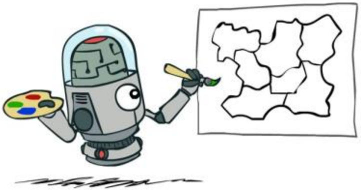




# CSP Examples

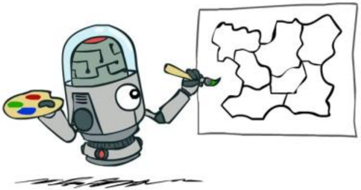
# Example: Map Coloring

- Variables:

# Example: Map Coloring

- Variables:
  - $WA, NT, Q, NSW, V, SA, T$

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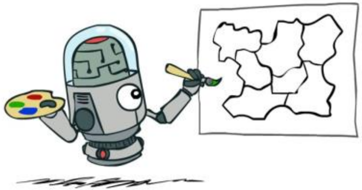

# Example: Map Coloring

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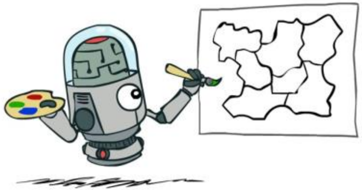
- Domains:
  - $D = \{\text{red}, \text{green}, \text{blue}\}$

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- Variables:
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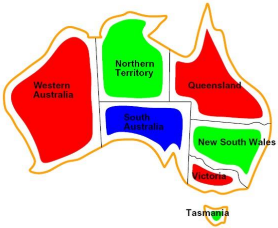


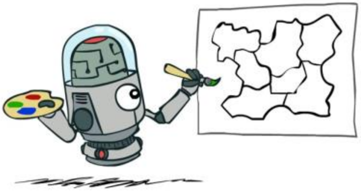

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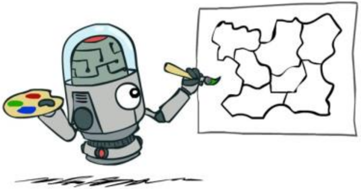

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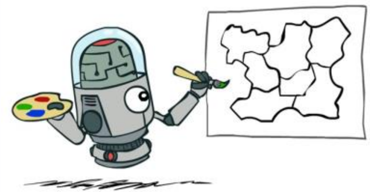

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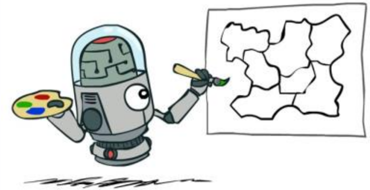
- Solutions are assignments satisfying all constraints

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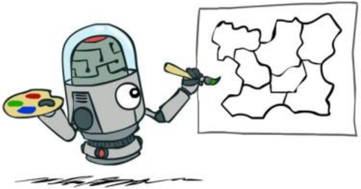
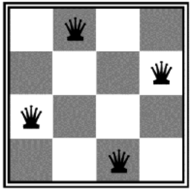
- Solutions are assignments satisfying all constraints
  - $\{WA = \text{red}, NT = \text{green}, Q = \text{red}, NSW = \text{green}, V = \text{red}, SA = \text{blue}, T = \text{green}\}$

# Example: N-Queens



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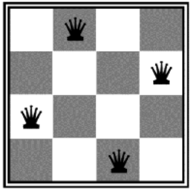

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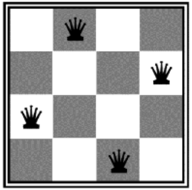

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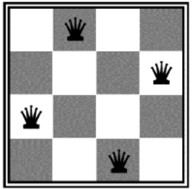

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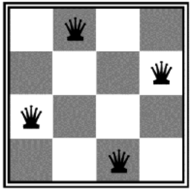

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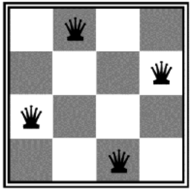

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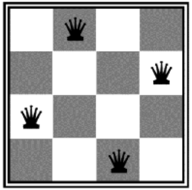

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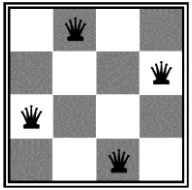

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$$\forall i, j, k \ (X_{ij}, X_{i+k, j+k}) \in \{(0,0), (0,1), (1,0)\}$$

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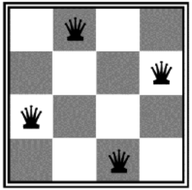

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$$\sum_{i,j} X_{ij} = N$$

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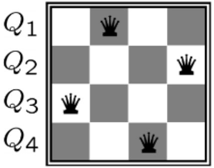
- Formulation 2:
  - Variables: $Q_k$
  - Domains: $\{1, 2, 3, \ldots, N\}$
  - Constraints:



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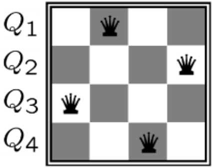


- Implicit: $\forall i, j$ non-threatening $(Q_i, Q_j)$

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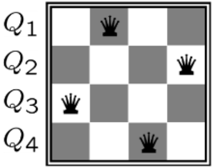

- Implicit: $\forall i,j$ non-threatening $(Q_i, Q_j)$
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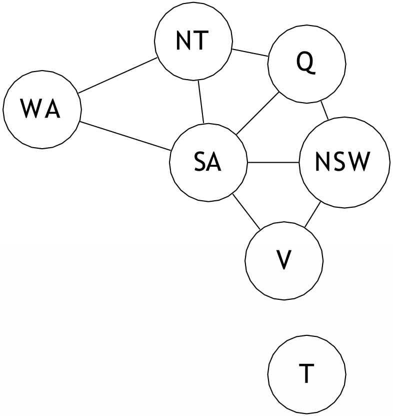
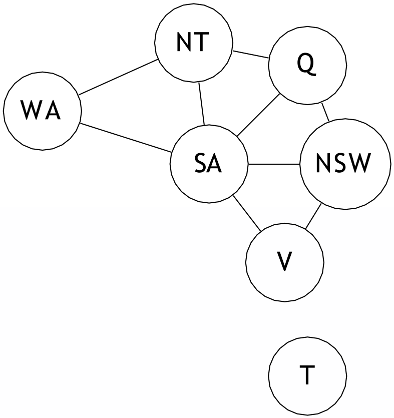
  $\ldots$

# Constraint Graphs



# Constraint Graphs

- Binary CSP: each constraint relates (at most) two variables
- Binary constraint graph: nodes are variables, arcs show constraints
- General-purpose CSP algorithms use the graph structure to speed up search. E.g., Tasmania is an independent subproblem!



# Example: Cryptarithmetic

- Variables
- Domains:
- Constraints:

$$\begin{array}{r} T\ W\ O \\ +\ T\ W\ O \\ \hline F\ O\ U\ R \end{array}$$

# Example: Cryptarithmetic

- Variables
  - $F, T, U, W, R, O, X_1, X_2, X_3$
- Domains:
- Constraints:

$$\begin{array}{cccc} & T & W & O \\ + & T & W & O \\ \hline F & O & U & R \end{array}$$

# Example: Cryptarithmetic

- Variables
  - $F,\ T,\ U,\ W,\ R,\ O,\ X_1,\ X_2,\ X_3$
- Domains:
  - $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- Constraints:

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  alldiff($F, T, U, W, R, O$)

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  $\text{alldiff}(F, T, U, W, R, O)$

  $O + O = R + 10 \times X_1$

  $\ldots$

$$
\begin{array}{ccccc}
 & & T & W & O \\
+ & & T & W & O \\
\hline
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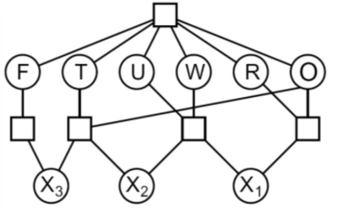

# Example: Sudoku

- Variables
  - Each (open) square
- Domains
  - $\{1, 2, \ldots, 9\}$
- Constraints

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | 8 | | | 4 |
| | 8 | 4 | | 1 | 6 | | | |
| | | | 5 | | | 1 | | |
| 1 | | 3 | 8 | | | 9 | | |
| 6 | | 8 | | | | 4 | | 3 |
| | | 2 | | | 9 | 5 | | 1 |
| | | 7 | | | 2 | | | |
| | | | 7 | 8 | | 2 | 6 | |
| 2 | | | 3 | | | | | |

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- Variables
  - Each (open) square
- Domains
  - $\{1, 2, \ldots, 9\}$
- Constraints
  - Unary constraints for given values

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | 8 | | | 4 |
| | 8 | 4 | | 1 | 6 | | | |
| | | | 5 | | | 1 | | |
| 1 | | 3 | 8 | | | 9 | | |
| 6 | | 8 | | | | 4 | | 3 |
| | | 2 | | | 9 | 5 | | 1 |
| | | 7 | | | 2 | | | |
| | | | 7 | 8 | | 2 | 6 | |
| 2 | | | 3 | | | | | |

# Example: Sudoku

- Variables
  - Each (open) square
- Domains
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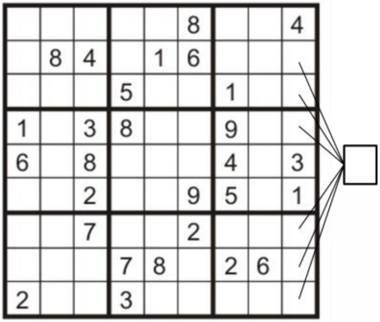
- Constraints
  - Unary constraints for given values
  - 9-way alldiff for each column



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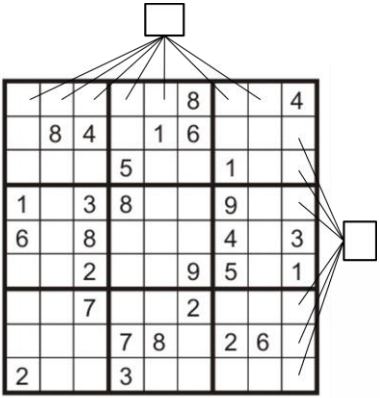

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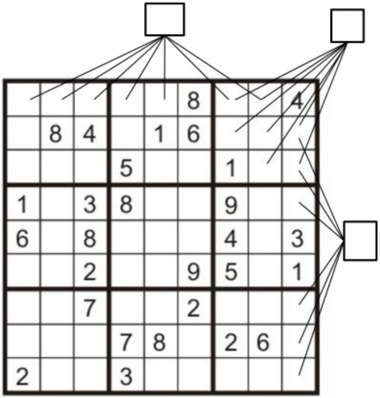

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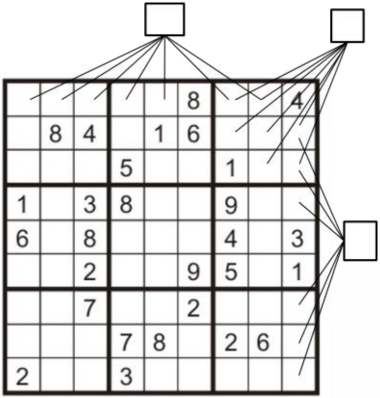
  - Can also have a bunch of pairwise inequalities



# Varieties of CSPs and Constraints

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- Discrete Variables
  - Finite domains
    - Size $d$ means $O(d^n)$ complete assignments

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- Discrete Variables
  - Finite domains
    - Size $d$ means $O(d^n)$ complete assignments
  - Infinite domains (integers, strings, etc.)
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# Varieties of CSPs

- **Discrete Variables**
  - Finite domains
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- **Continuous Variables**
  - E.g., start/end times for Hubble Telescope observations

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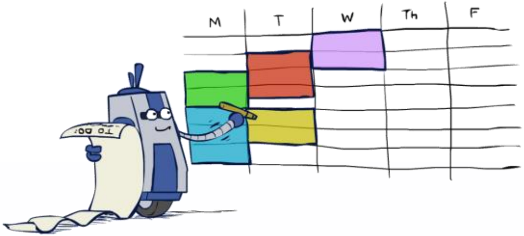
- Binary constraints involve pairs of variables, e.g.: $SA \neq WA$
- Higher-order constraints involve 3 or more variables, e.g., cryptarithmetic column constraints
- Preferences (soft constraints)
  - E.g., red is better than green
  - Often representable by a cost for each variable assignment
  - Gives constrained optimization problems
  - (We'll ignore these until we get to Bayes' nets)

# Real-World CSPs

- Scheduling problem
- Timetabling problem
- Assignment problem
- Hardware configuration
- Transportation scheduling
- Factory scheduling Circuit
- layout
- Fault diagnosis
- ... lots more!

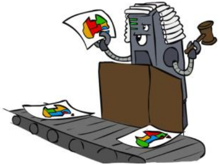



- Many real-world problems involve real-valued variables...

# Solving CSPs

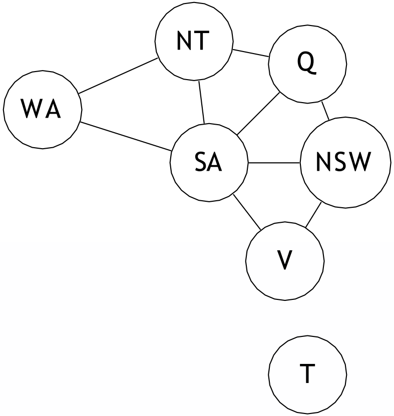
# Standard Search Formulation

- States defined by the values assigned so far (partial assignments)
  - Initial state: the empty assignment, {}
  - Successor function: assign a value to an unassigned variable
  - Goal test: the current assignment is complete and satisfies all constraints



# Search Methods

- What would BFS do?




Website: [simple-naive]

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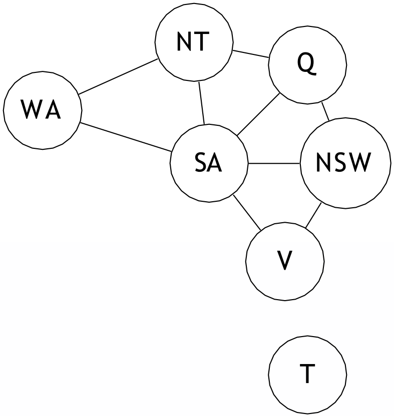


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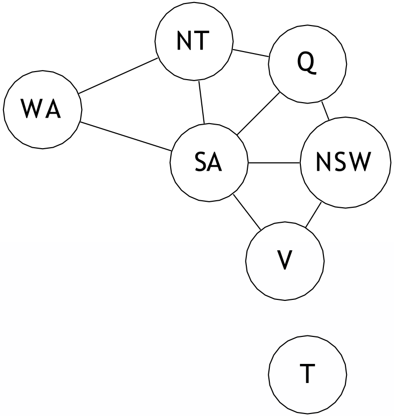
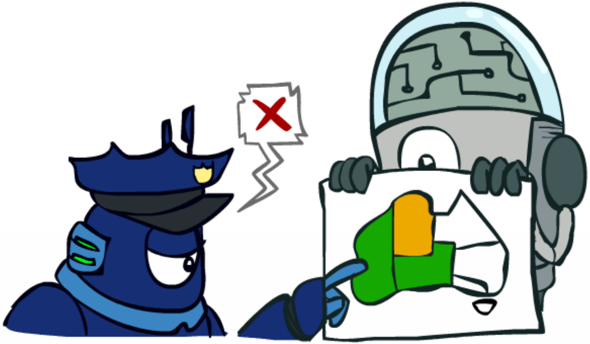
# Search Methods

- What would BFS  do?
- What would DFS do?
- What problems does naïve search have?




Website: [simple -naive]

# Backtracking Search



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  - "Incremental goal test"
- Depth-first search with these two improvements is called *backtracking search*

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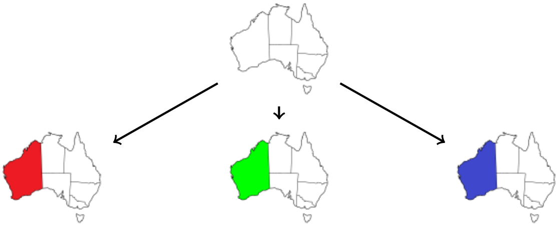

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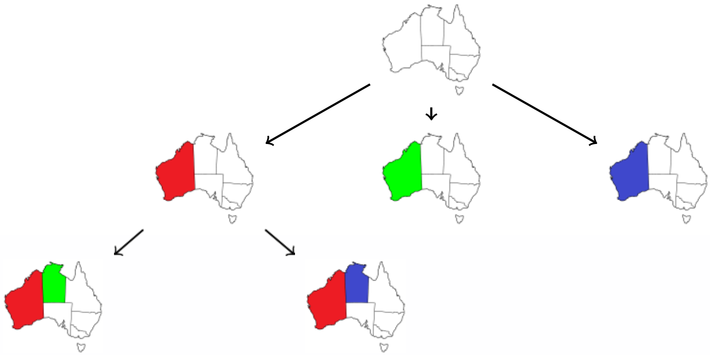

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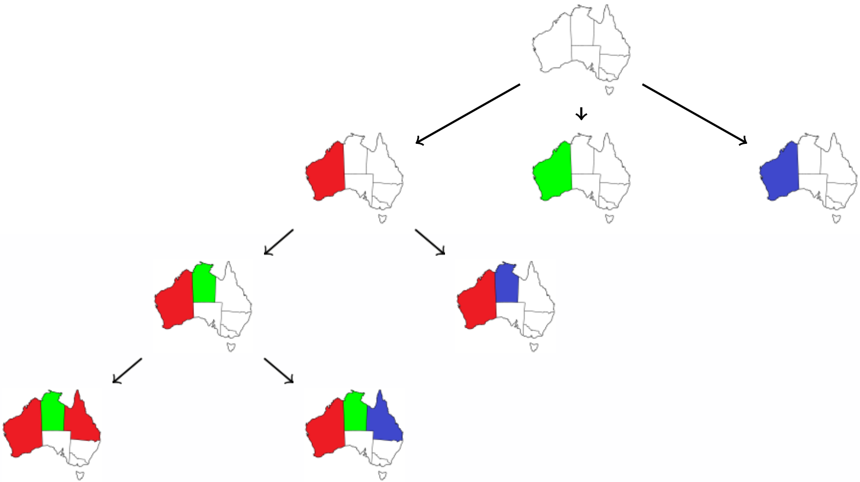


# Backtracking Search

```
function BACKTRACKING-SEARCH(csp) returns a solution, or failure
   return RECURSIVE-BACKTRACKING({}, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns a solution, or failure
   if assignment is complete then return assignment
   var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
   for each value in ORDER-DOMAIN-VALUE(var, assignment, csp) do
      if value is consistent with assignment given CONSTRAINTS[csp] then
         add {var = value} to assignment
         result ← RECURSIVE-BACKTRACKING(assignment, csp)
         if result ≠ failure then return result
         remove {var = value} from assignment
   return failure
```

- Backtracking = DFS + variable-ordering + fail-on-violation

Website: simple -backtracking

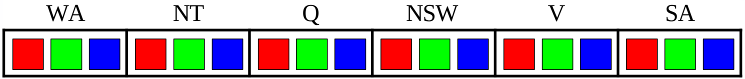
# Improving Backtracking

- General-purpose ideas give huge gains in speed
- Filtering: Can we detect inevitable failure early?
- Ordering:
  - Which variable should be assigned next?
  - In what order should its values be tried?
- Structure: Can we exploit the problem structure?

# Filtering

# Filtering: Forward Checking

- Filtering: Keep track of domains for unassigned variables and cross off bad options
- Forward checking: Cross off values that violate a constraint when added to the existing assignment

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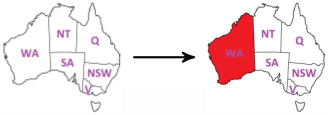


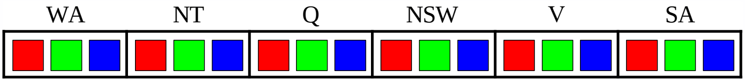

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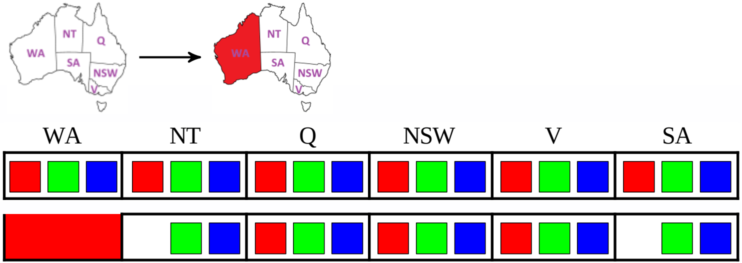

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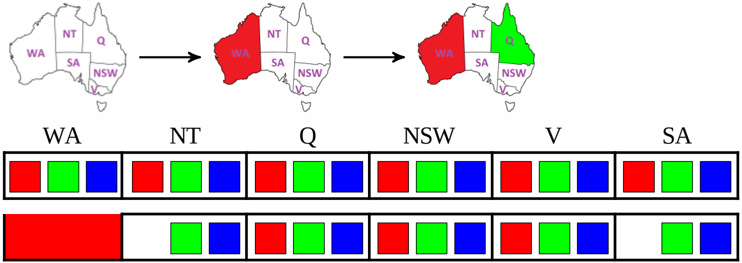


Website: [simple - backtracking, forward]

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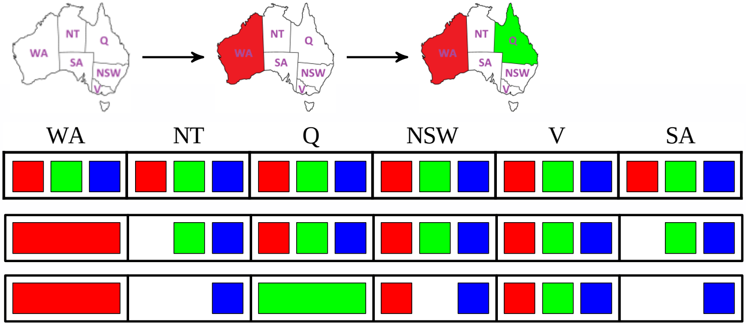

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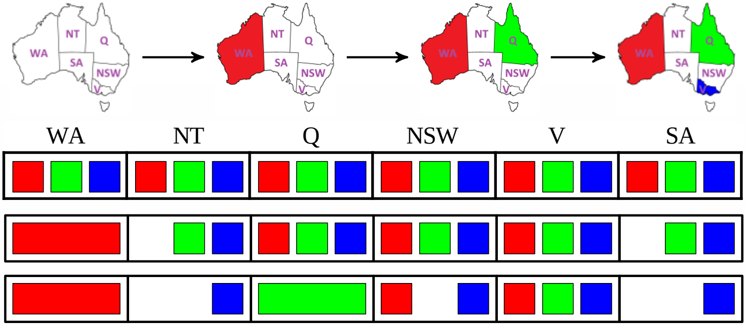

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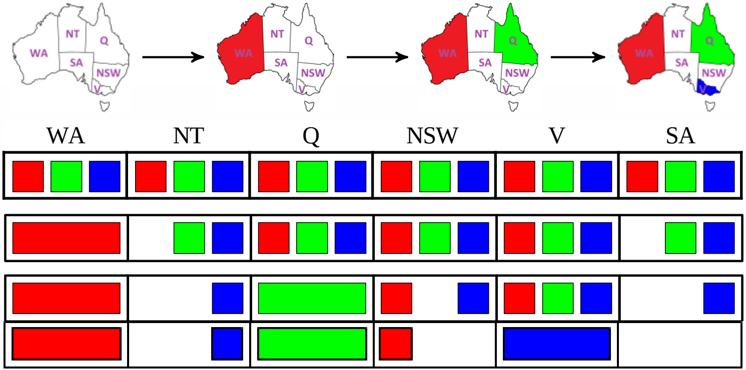

# Filtering: Constraint Propagation

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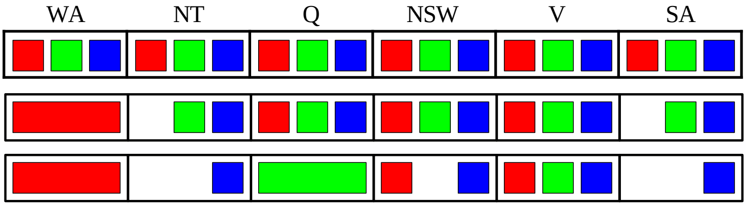

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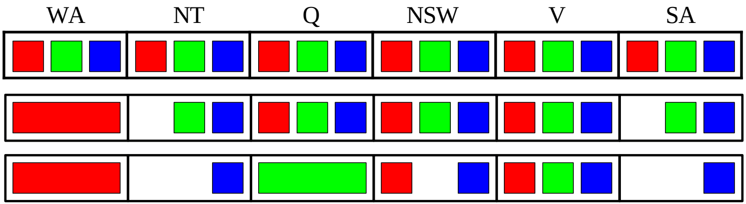


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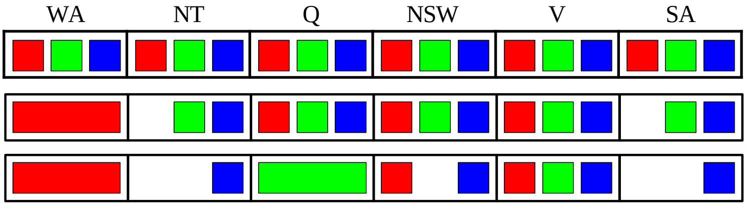


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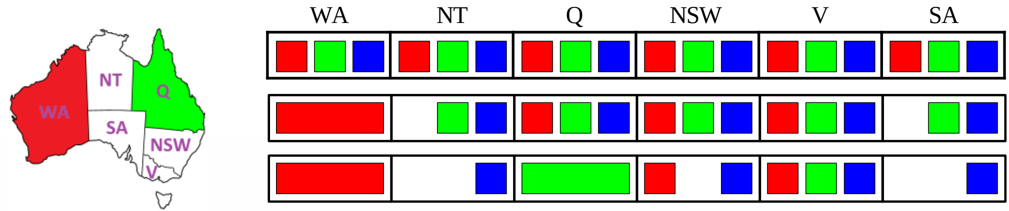
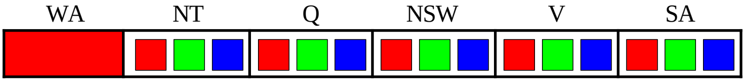



- NT and SA cannot both be blue!
- Why didn't we detect this yet?
- *Constraint propagation:* reason from constraint to constraint

# Consistency of A Single Arc

- An arc $X \to Y$ is consistent iff for *every* $x$ in the tail there is *some* $y$ in the head which could be assigned without violating a constraint

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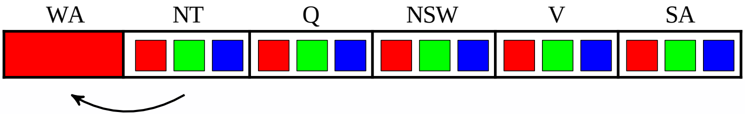

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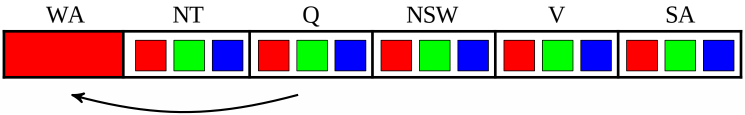

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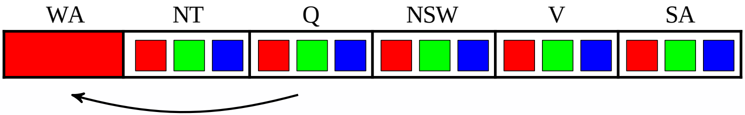


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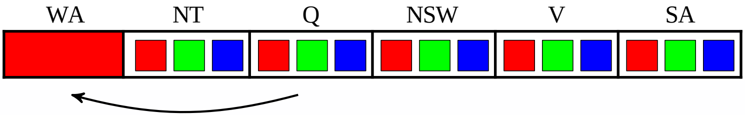


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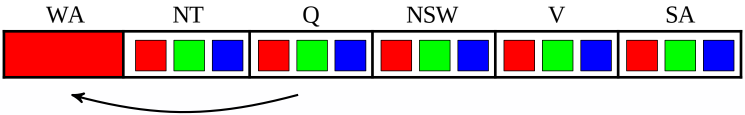


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- Forward checking: Enforcing consistency of arcs pointing to each new assignment

# Arc Consistency of an Entire CSP

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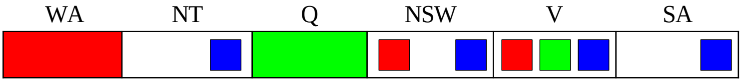


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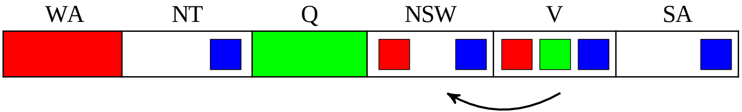


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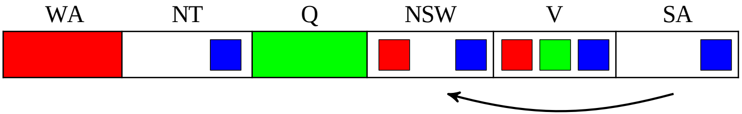


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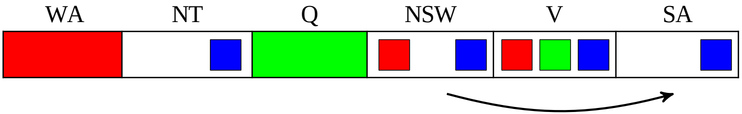


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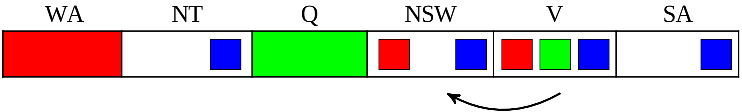


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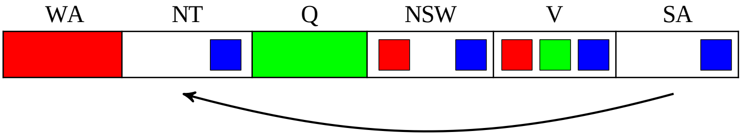


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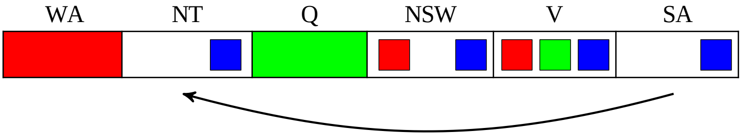


- Important: If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment
- What's the downside of enforcing arc consistency?

*Remember: Delete from the tail!*

# Enforcing Arc Consistency in a CSP

**function** AC-3(*csp*) **returns** the CSP, possibly with reduced domains
  **inputs:** *csp*, a binary CSP with variables $\{X_1, X_2, \ldots, X_N\}$
  **local variables:** *queue*, a queue of arcs, initially all the arcs in *csp*
  **while** *queue* is not empty **do**
    $(X_i, X_j) \leftarrow$ REMOVE-FIRST(*queue*)
    **if** REMOVE-INCONSISTENT-VALUES($X_i, X_j$) **then**
      **for each** $X_k$ in NEIGHTBORS[$X_i$] **do**
        add $(X_k, X_i)$ to *queue*

**function** REMOVE-INCONSISTENT-VALUES($X_i, X_j$) **returns** true iff succeeds
  *removed* $\leftarrow$ *false*
  **for each** $x$ in DOMAIN[$X_i$] **do**
    **if** no value $y$ in DOMAIN[$X_j$] allows $(x, y)$ to satisfy the constraint $X_i \leftrightarrow X_j$
      **then** delete $x$ from DOMAIN[$X_i$]; *removed* $\leftarrow$ *true*
  **return** *removed*

Applet: CSP - fiveQueens

# Suggested Reading

- Russell & Norvig: Chapter 6.1