Probability: The term "Probability" is an estimate of the proportion of one or more uncertain experimental outcomes when the experiment is performed at random.

Some examples: (i) What is the likelihood that the new vaccine will be more effective than the old one in controlling COVID-19?

Set theory & Set Notation:

Set: A set is simply a well-defined list or collection of distinct objects.

- The individual objects of a set are elements or members
- The set is the collection of its elements
- The elements of a set must be distinct, i.e., each element must occur once & only once.

Universal Set: A universal set is the set of all elements that may possibly be considered in a particular discussion.

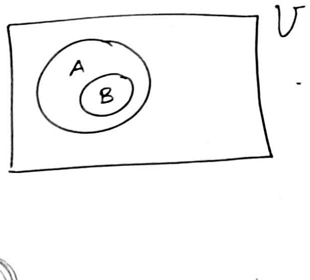
$$U = \{x : x \text{ is the sum of points uppermost side on the two dices}\}$$
$$= \{2, 3, 4, 5, 6, \ldots, 12\}$$

Null Set: The null set, which is also important, may seen like it is not a set at all.

$$\{\ \}, \quad \phi$$

For any set A, $\phi \subset A$

Venn Diagram:



$B \subset A$

$A = \{1, 2, 3\}$

$B = \{1\}$

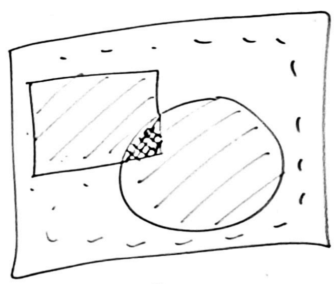
$B \subset A$

$\left.\begin{matrix} B \subseteq A \\ A \subseteq B \end{matrix}\right\} A = B$

Union: $A \cup B$

Intersection:

Complement:



$(A \cup B)^c$

$x \in A$ or $x \in B$

$x \in A$ and $x \in B$

Two important Properties of collection of sets:

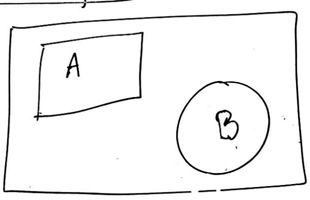
Sum

$\sum A_1 + A_2 + \cdots + A_n$

Product

$\prod A_i = A_1 \cdot A_2 \cdot A_3 \cdots A_n$

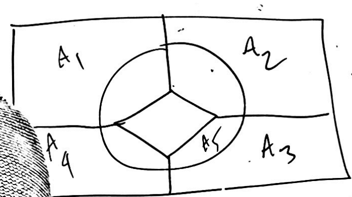
① Mutually Exclusive / Disjoint:



A collection of events $A_1, A_2, \ldots, A_n$

: $A_i \cap A_j = \phi$; $[i \neq j]$

② Collectively Exhaustive:



→ A collection of sets $A_1, A_2, \ldots, A_n$ is collectively exhaustive iff

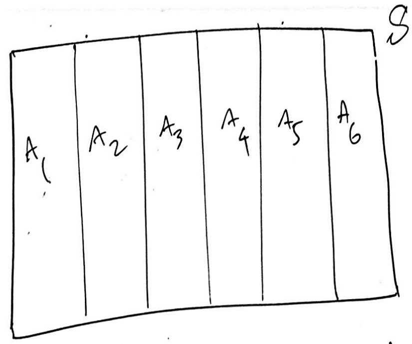
$A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n = S$

$= \bigcup_i A_i$ $\quad \bigcap_i A_i$

d) <u>De Morgan's Law</u>:

i) $(A \cup B)^C = A^C \cap B^C$

ii) $(A \cap B)^C = A^C \cup B^C$



<u>Applying Set Theory to Probability</u>:

Probability is based on a repeatable experiment that consists of a <u>procedure</u> and <u>observations</u>.

- An outcome is an observation
- An event is a set of observations

<u>Partition</u> ⇒ Mutually Exclusive + Collectively exhaustive

↓

Process

Outcome: An outcome of an experiment is any possible observation of that experiment.

Random Experiment: An experiment that can result in different outcomes, even though it is repeated in the same manner every time, is called random experiment.

An experiment consists of the following procedure, observation & model.

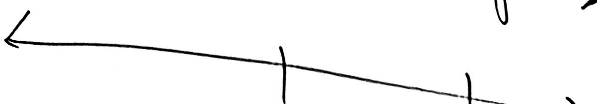
- √ Procedure: Monitor activity in a supershop.
- √ Observation: Observe what type of products are being purchased by the next customer
- √ Model:

∘ The elements of a set must be distinct, i.e., each element must occur once & only once.

Discrete Sample space : A sample space is discrete if it consists of a finite or countable infinite set of outcomes.

Continuous Sample Space : If a sample space contains an infinite number of probabilities equal to the number of points on a line segment, is called a continuous Sample space.

A sample space is cont' if it contains an interval of real numbers.



Event: An event is a subset of the sample space of a random experiment.

Union: The union of two events is the event that consists of all outcomes that are contained in either of the two events. If E_1 & E_2 are two events, then their union is denoted by $E_1 \cup E_2$.

Intersection: The intersection of two events is the event that consists of all outcomes that are contained in both of the two events. If E_1 & E_2 are two events, then their intersection is denoted by $E_1 \cap E_2$.

Complement: The complement of an event in a sample space is the set of outcomes in the sample space that are not in the event. We denote the complement of an event E by E' or E^C.

Equally Likely Events:

in both of the two events. If E_1 & E_2 are two events, then their intersection is denoted by $E_1 \cap E_2$.

Complement: The complement of an event in a sample space is the set of outcomes in the sample space that are not in the event. We denote the complement of an event E by E' or E^C.

Equally Likely Events: Two or more events are said to be equally likely if they have the same chances of occurance. In other words, whenever a sample space consists of N possible outcomes that are equally likely, then the probability of each outcome is $1/N$.

For a discrete sample space, the probability of an event E, denoted as P(E), equals the sum of the probabilities of the outcomes in E.

	H	T
H	HH	HT
T	TH	TT

Axioms of Probability: If S is the sample space and E is any event in a random experiment,

(I) $P(S) = 1$; (II) $0 \le P(E_i) \le 1$, (III) For any two events E_1 & E_2 with $E_1 \cap E_2 = \phi$, $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Laws of sets:

i) $A \cup B = B \cup A$; $A \cap B = B \cap A$ [Commutative laws]

ii) $(A \cup B) \cup C = A \cup (B \cup C)$
$(A \cap B) \cap C = A \cap (B \cap C)$ [Associative laws]

iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [Distributive laws]

iv) $A \cup \phi = A$, $A \cap U = A$; $A \cap \phi = \phi$, $A \cup U = U$ [Identity laws]

(v) $A \cup A = A$ & $A \cap A = A$ [Idempotent laws]

(vi) $A \cup A^c = U$ & $A \cap A^c = \phi$
$(A^c)^c = A$ [Complementary laws]

vii) De Morgan's laws :

$(A \cap B)^c = A^c \cup B^c$
$(A \cup B)^c = A^c \cap B^c$

Sample

$P(S) = 1$

$\Rightarrow P(E \cup E^c) = 1$

$\Rightarrow \boxed{P(E) + P(E^c) = 1}$

Joint Probability : Two or more events form a joint event if all of the occur simultaneously and the probability of these joint events are called joint probabilities.

$A \cap B$, $A \cup B$

$A \cap B \cap C$

Prob^m: Suppose a sample space consists of 500 persons and are distributed according to their sex & employment status as shown in the following table

Sex \ Employment status	Employed (E)	Unemployed (U)	Total
Male (M)	255 $(M \cap E)$	20 $(M \cap U)$	275
Female (F)	80 $(F \cap E)$	145 $(F \cap U)$	225
Total	335	165	500

One of these 500 persons was selected at random

Let us define the following simple events:

M: The selected person is Male

F: " " " " female

E: " " " " employed

U: " " " " unemployed.

The joint events that can be formulated as follows:

$M \cap E$: The selected person is male & employed

$M \cap U$: " " " " " & un "

$F \cap E$: " " " " female & employed

$F \cap U$: " " " " " & un "

Since the totals $n(M)$, $n(F)$, $n(E)$, $n(U)$ all appear in the margins of the table. These values are called marginal totals and the corresponding probabilities $P(M)$, $P(F)$, $P(E)$, $P(U)$ are called marginal probabilities.

Here, $n(M) = 275$; $n(E) = 335$ $n(S) = 500$

$n(F) = 225$; $n(U) = 165$

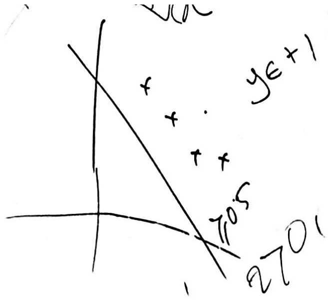
Thus, the marginal probability that a randomly selected person will be a male is

$$P(M) = \frac{n(M)}{n(S)} = \frac{275}{500} = 0.55$$

$$P(E) = \frac{n(E)}{\ldots} = \frac{335}{500} = 0.67$$

$$P(F) = \frac{225}{500} = 0.45$$

$$P(U) = \frac{165}{500} = 0.33$$



$0 \sim 1$ $\quad h_\theta(x)$

$$\frac{P(y=1 \mid x;\theta) \equiv h_\theta(x)}{P(y=0 \mid x;\theta) \equiv 1 - h_\theta(x)}$$

Prob^m: Suppose a sample space consists of 500 per as shown in the following table

Employment Status / Sex	Employed (E)	Unemployed (U)	Total
Male (M)	255 ($M \cap E$)	20 ($M \cap U$)	275 $n(M)$
Female (F)	80 ($F \cap E$)	145 ($F \cap U$)	225 $n(F)$
Total	335 $n(E)$	165 $n(U)$	500 $n(S)$

What is the probability that a randomly chosen person is a male and at the same time employed.

$$P(M \cap E) = \frac{n(M \cap E)}{n(S)} = \frac{255}{500} = 0.51$$

$$P(M \cap U) = \frac{20}{500} = 0.04$$

Also,

$$P(M) = P(M \cap E) + P(M \cap U) = 0.51 + 0.04 = 0.55$$

Probⁿ: In an office of 100 employees 75 reads English, 50 reads Bangla newspapers and 40 reads both. Now if an employee is selected at random, what is the probability that

(a) he reads English newspaper?

(b) reads at least one of them?

(c) reads none?

(d) reads English but not Bangla?

Solⁿ: Let us first define the events as follows:

E: Reads English newspaper

B: Reads Bangla newspaper

	E	$\bar{E}$	Total
B	$n(B \cap E) = 40$	$n(B \cap \bar{E}) = ? = 10$	$n(B) = 50$
$\bar{B}$	$n(\bar{B} \cap E) = ? = 35$	$n(\bar{B} \cap \bar{E}) = ? \; 15$	$n(\bar{B}) = 50$
Total	$n(E) = 75$	$n(\bar{E}) = 25$	100

$\bar{E} \cap \bar{B}$: Reads none

$B \cap \bar{E}$: Reads Bangla but not English

(a) $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{75}{100} = 0.75$

(b) $P(E \cup B) = P(E) + P(B) - P(E \cap B)$

$= \dfrac{n(E)}{n(S)} + \dfrac{n(B)}{n(S)} - \dfrac{n(E \cap B)}{n(S)}$

$= \dfrac{75}{100} + \dfrac{50}{100} - \dfrac{40}{100}$

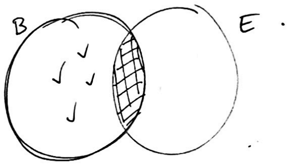
$= \dfrac{85}{100} = 0.85$

(c) $P(\overline{E \cup B}) = 1 - P(E \cup B) = 1 - 0.85 = 0.15$

(d) $P(B \cap \bar{E}) = \dfrac{n(B) - n(B \cap E)}{n(S)}$

$= \dfrac{50-40}{100}$

$= 0.10$



online classes after this 20 and 22 Jan

Random Variable: A variable whose values are any definite numbers or quantities that arise as a result of chance factors such that they can not exactly be predicted in advance, is called a random variable. (r.v.).

A r.v is a real-valued function defined over a sample space.

Ex:

A school → 7 teachers

4 Males

3 Females

2 Members Committee

R.V ← Y: No. of male teachers selected.

$Y = y$

Events	Sequence of Events	$Y = y$
E_1	Male, male	2
E_2	Male, female	1
E_3	Female, female	0

A r.v is a real-valued function defined over a sample space.

Ex:

Events	Sequence of Events	Y = y
E_1	Male, male	2
E_2	Male, female	1
E_3	Female, female	0

A school → 7 teachers

4 Males

3 Females

2 Members Committee

R.V ← Y: No. of male teachers selected.

$Y = y$

Types of random variables:

R.V

- Discrete
  - No. of telephone calls received
  - No. of correct answers in 100 marks MCQ Exam
  - No. of defective bulbs produced in a day
- Continuous
  - Time taken to serve a customer in a bank counter
  - Weight of a six-month old baby
  - Rate of interest offered by a bank.
  - Longevity of an electric bulb.
  - Temperature recorded by the Meteorological Dept.

Probability Distribution:

Frequency Distribution:

20, 2, 11, 14, 33, . . .

11 - 20			
21 - 30	
31 - 40	
41 - 50	

Def^n: Any statement of a function associating each of a set of mutually exclusive and collectively exhaustive class or classes of intervals with its probability is a probability distribution.

A pr. distribution will be either discrete or continuous according as the r.v is discrete or continuous.

Discrete Pr. Dist^n: If a r.v X has a discrete pr. dist^n, the pr. dist^n of X is defined as the function of "f" such that any real number x,

$$\boxed{f(x) = P(X = x)}$$

PMF:

1. $f(x) \geq 0$
2. $\sum_x f(x) = 1$
3. $P(X=x) = f(x)$

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

X: No. of heads

Value of $(X=x)$ x	0	1	2	3
$P(X=x)$ $=f(x)$	$1/8$	$\frac{3}{8}$	$3/8$	$1/8$

$= 1$

$$f(x) = {}^{3}C_{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{3-x}$$

A r.v is a real-valued function defined over a sample space.

Ex:

Events	Sequence of Events	$Y=y$
E_1	Male, male	2
E_2	Male, female	1
E_3	Female, female	0

A school → 7 teachers

4 Males

3 Females

2 Members Committee

R.V ← Y: No. of male teachers selected.

$Y=y$

$$f(x) = \frac{x+1}{16}; \quad x = 0,1,2,3$$

$$S_\infty = \frac{a}{1-r}$$

Ex: The probability function (PMF) of a discrete r.v X is given by

$$f(x) = \alpha\left(3/4\right)^x ; \quad x = 0,1,2,\ldots,\infty$$

$$= 0 \quad ; \text{otherwise}$$

Evaluate α and find $P(X \le 3)$

$$P(X\le 3) = f(0)+f(1)+f(2)+f(3)$$

$$= \frac{1}{4} + \frac{3}{16} + \frac{9}{64} + \frac{27}{256}$$

$f(x) = ?$

$$\sum_x f(x) = 1$$

$$\Rightarrow \sum_x \alpha\left(3/4\right)^x = 1$$

$$\Rightarrow \alpha\left[1+\frac{3}{4}+\left(3/4\right)^2+\left(3/4\right)^3+\ldots\right]=1$$

$$\Rightarrow \alpha.\left[\frac{1}{1-3/4}\right]=1$$

$f(0) = \alpha.(3/4)^0 = \alpha$

$f(1) = \alpha(3/4) = \alpha.\frac{3}{4}$

$f(2) = \alpha.(3/4)^2$

$f(3) = \alpha.(3/4)^3$

$\boxed{ar^{n-1}}$

$\boxed{a+(n-1)d}$

Cumulative Dist^n function: The cumulative dist^n function (CDF), $F(x)$ of a discrete r.v X with probability function $f(x)$ defined over all real numbers x is the cumulative probability upto and including the point x.

$$F(x) = P(X \leq x)$$

The function $F(x)$ is monotonically increasing function.

$$\left.\begin{array}{l} \max F(x) = 1 \\ \min F(x) = 0 \end{array}\right\}$$

$$\left.\begin{array}{l} \lim\limits_{x \to -\infty} F(x) = 0 \\ \lim\limits_{x \to +\infty} F(x) = 1 \end{array}\right\}$$

Ex :

Events	Sequence of Events	$Y=y$
E_1	Male, male	2
E_2	Male, female	1
E_3	Female, female	0

A school $\longrightarrow$ 7 teachers

4 Males

3 Females

2 Members Committee

R.V $\leftarrow$ Y : No. of male teachers selected.

$Y=y$

Ex: $f(x) \longrightarrow$

$X:x$	0	1	2	3
$P(X=x)=f(x)$	$20/120$	$60/120$	$36/120$	$4/120$

$F(0) = P(X=0) = f(0) = \frac{20}{120}$

$F(1) = P(X\le 1) = f(0)+f(1) = \frac{20}{120}+\frac{60}{120} = \frac{80}{120}$

$F(2) = P(X\le 2) =$

$= \frac{116}{120}$

$= \frac{120}{120} = 1$

$F(3)$

$X:x$	<0	0	1	2	3	$3>0$
$P(X=x)=f(x)$	0	$\frac{20}{120}$	$60/120$	$36/120$	$4/120$	0
$F(x)$	0	$\frac{20}{120}$	$80/120$	$\frac{116}{120}$	1	1

$$f(x) = \begin{cases} 0, & x<0 \\ \frac{20}{120}; & 0\le x<1 \\ \frac{80}{120}; & 1\le x<2 \\ \frac{116}{120}; & 2\le x<3 \\ 1; & x\geq 3 \end{cases}$$

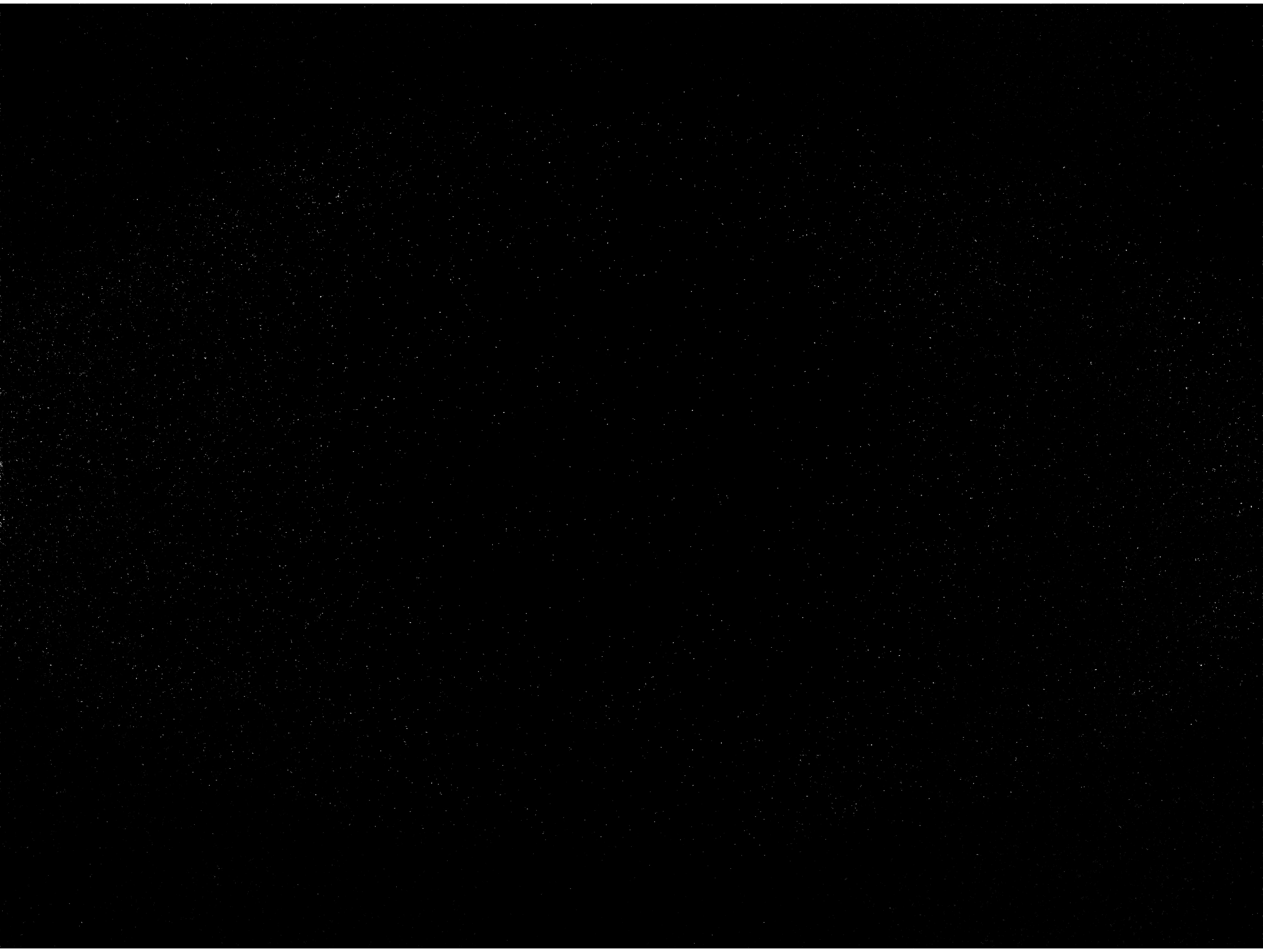
$\underline{Ex}$ # $P(Y=y) = {}^{2}C_{y} \left(\frac{1}{2}\right)^{y} \left(\frac{1}{2}\right)^{2-y}$; $y = 0, 1, 2$

i) Check if $P(Y)$ is a PMF

ii) Find $F(y)$ & hence find $F(-2)$ & $\underline{\underline{P(Y \leq 1)}}$

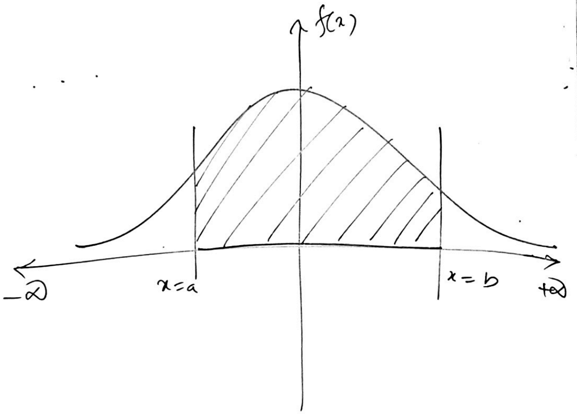
$$\Downarrow$$

$$F(1)$$



Continuous Probability Distribution:

Probability Density function (PDF): A probability density function . . . .



① $f(x) \geq 0$

② $\int_{-\infty}^{\infty} f(x)\,dx = 1$

③ $P(a < X < b) = \int_{a}^{b} f(x)\,dx = \text{Area}$

Ex.: A r.v X

$$f(x) = Kx; \quad 0 < x < 4$$
$$= 0; \quad \text{otherwise}$$

(i) $K = ?$

(ii) $P(1 < X < 2)$

(iii) $P(X > 2)$

Sol^n: For $f(x)$ to be a PDF, we must have

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_0^4 Kx\,dx = 1 \Rightarrow K = 1/8$$

$$f(x) = \frac{1}{8}x; \quad 0 < x < 4$$
$$\boxed{= 0 \quad ; \text{otherwise}}$$

$$P(1 < X < 2) = \frac{1}{8}\int_1^2 \frac{1}{8}x\,dx = \frac{3}{16}.$$

$$P(X > 2) = \frac{1}{8}\int_2^{\infty} \frac{1}{8}x\,dx$$

$$= \frac{1}{8}\int_2^4 x\,dx + \int_4^{\infty} 0\,dx$$

$$= 3/4.$$

Cumula

(ii) $P(1<X<2)$

(iii) $P(X>2)$

Sol^n: For $f(x)$ to be a PDF, we must have

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{0}^{4} kx\,dx = 1 \Rightarrow k = 1/8$$

$$P(X>2) = \frac{1}{8}\int_{2} \frac{1}{8}x\,dx$$

$$= \frac{1}{8}\int_{2}^{4} x\,dx + \int_{4}^{\infty} 0\,dx$$

$$= 3/4.$$

Cumulative Distribution Function for a continuous R.V.:

Def^n: $$F(x) = P(X \le x) = \int_{-\infty}^{\infty} f(t)\,dt.$$

$$F(x) = \int_{-\infty}^{\infty} f(t)\,dt$$

$$f(x) = \frac{d}{dx}\left[F(x)\right] = F'(x)$$

$$\boxed{f(x) = F'(x)} \Rightarrow f(x) = \frac{dF}{dx} \Rightarrow \boxed{dF = f(x)\,dx} \rightarrow \text{Differential Form}$$

(I) $\frac{dF}{dx} = f(x) > 0$

(II) $F(-\infty) = 0$

(III) $F(+\infty) = 1$

(IV) $F(x)$ is defined at every point in a continuous range and is itself continuous.

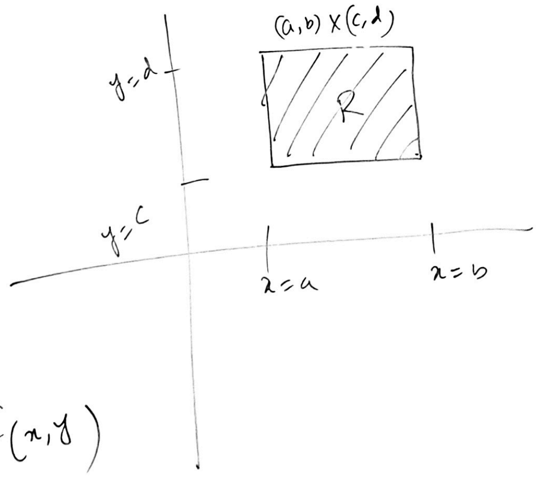
$$P(a < X < b) = \int_{-\infty}^{b} f(x)\,dx - \int_{-\infty}^{a} f(x)\,dx = \boxed{F(b) - F(a)} = \int_{a}^{b} f(x)\,dx$$

$$F'(x) = f(x)$$

Joint Probability Distribution:

X & Y D. R.V:

$$f(x,y) = P(X=x, Y=y)$$



(i) $f(x,y) \geq 0$ for all (x,y)

(ii) $\sum_x \sum_y f(x,y) = 1$

(iii) $P\left[(X,Y) \in R\right] = \sum\sum_{(x,y)\in R} f(x,y)$

① $x = ?$

② $P(1 < X < 2)$

③ $P(X > 2)$

n: For $f(x)$ to be a PDF, we must have

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{0}^{4} kx\,dx = 1 \Rightarrow k = 1/8$$

$$P(X>2) = \frac{1}{8}\int_{2}^{\infty} \frac{1}{8}x\,dx$$

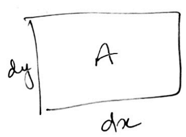
$$= \frac{1}{8}\int_{2}^{4} x\,dx + \int_{4}^{\infty} 0\,dx$$

$$= 3/4.$$

$\forall$ $\exists$ $\Rightarrow$

Joint Distribution for C.R.V.

$$P[(X,Y)\in R] = \iint_R f(x,y)\,dA = \int_{x=a}^{b}\int_{y=c}^{d} f(x,y)\,dy\,dx$$



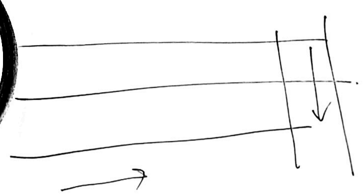
Cond^n's:

① $f(x,y) \geq 0 \quad \forall (x,y)\in R$ (For all) $\quad -\infty < x < \infty$, $-\infty < y < \infty$

② $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dy\,dx = 1$

③ $P[(X,Y)\in R] = P(a \le X \le b,\ c \le Y \le d) = \int_{x=a}^{b}\int_{y=c}^{d}$

$$\int_{y=c}^{d}\left[\int_{x=a}^{b} f(x,y)\,dx\right]dy = \int_{y=c}^{d} g(y)\,dy = \text{constant value}$$



$f(x,y) \longrightarrow X, Y \rightarrow$ Discrete

Def^n: Let $F(x,y)$ be the joint cumulative distribution function of (X,Y), then the marginal CDF of X is

$$F_1(x) = \lim_{y \to \infty} F(x,y) \longrightarrow X \quad \text{M. Dist}^n.$$

Similarly,

$$F_2(y) = \lim_{x \to \infty} F(x,y) \longrightarrow Y \quad \text{M. Dist}^n.$$

Ex:

	X				
Y	x = 0	x = 1	x = 2	x = 3	Row Sum
y = 0	0	1/8	2/8	1/8	4/8
y = 1	1/8	2/8	1/8	0	4/8
Column Sum	1/8	3/8	3/8	1/8	1

Find the marginal distribution of X & Y.

$X \longrightarrow g(x)$ $\quad$ $Y \longrightarrow h(y)$

Solⁿ: For R.V X

$$g(0) = P(X=0) = \sum_{y=0}^{1} f(0,y) = f(0,0) + f(0,1)$$
$$= 0 + \frac{1}{8} = \frac{1}{8}$$

$$g(1) = P(X=1) = \sum_{y=0}^{1} f(1,y) = f(1,0) + f(1,1)$$
$$= 3/8$$

$$g(2) = P(X=2) = 3/8$$

$$g(3) = P(X=3) = 1/8$$

Topic Name :

Day :

Time : Date : / /

Marginal Distribution : X

x	0	1	2	3	Sum
$f(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1

$$\left.\begin{aligned} g(x) &= \sum_{y} f(x,y) \\ h(y) &= \sum_{x} f(x,y) \end{aligned}\right\} \text{Discrete}$$

$$\left.\begin{aligned} g(x) &= \int_{y=-\infty}^{\infty} f(x,y)\,dy \\ h(y) &= \int_{x=-\infty}^{\infty} f(x,y)\,dx \end{aligned}\right\} \text{Continuous}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A|B) \cdot P(B) = \underset{\boxed{\text{Unconditional}}}{P(B)} \cdot \underset{\boxed{\text{Conditional}}}{P(A|B)}$$

Conditional Distribution:

$$\boxed{f(y|x) = \frac{f(x,y)}{g(x)}}$$

$$P(Y=y \mid X=x) = \frac{P(X=x, Y=y)}{P(X=x)} = \frac{f(x,y)}{g(x)} = \frac{\text{Joint Dist}^n \text{ of } (X,Y)}{\text{Marginal Dist}^n \text{ of } X}$$

$$P(X=x \mid Y=y) = \frac{P(X=x, Y=y)}{P(Y=y)} = \frac{f(x,y)}{h(y)} = \frac{\text{Joint Dist}^n \text{ of } (X,Y)}{\text{Marginal Dist}^n \text{ of } Y}$$

$$\Rightarrow \boxed{f(x|y) = \frac{f(x,y)}{h(y)}}$$

Ex:

Y \ X	0	1	2
0	3/8	9/28	3/28
$\boxed{y = 1}$	$\boxed{6/28}$	$\boxed{6/28}$	$\boxed{0}$
2	1/28	0	0

Find $f(x|1) = \dfrac{f(x,1)}{h(1)}$ —— (1)

$f(y|1) = \dfrac{f(1,y)}{g(1)}$

x	0	1	2
$f(x\|1)$	1/2	1/2	0

$f(x|1) = \frac{7}{3} f(x,1)$; $x = 0, 1, 2$

$f(0|1) = \frac{7}{3} f(0,1) = \frac{7}{3} \times \frac{6}{28} = \frac{1}{2}$

$f(1|1) = \frac{7}{3} f(1,1) = \frac{7}{3} \times \frac{6}{28} = \frac{1}{2}$

$f(2|1) = \frac{7}{3} f(2,1) = \frac{7}{3} \times 0 = 0$

Mathematical Expectation: /Expected value of a R.V:

Mean = Expected Value of a R.V

Def^n: If X is a discrete R.V having a PMF $f(x)$, then the expected value of X or the mathematical expectation of X is denoted with $\mathbb{E}[X]$ and is defined by

$$\mathbb{E}[X] = \sum_x x f(x)$$

$X = 0, 1, 2$

$f(0) =$

$f(1) =$

$f(2) =$

$X \longrightarrow$ C. R.V

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x) dx$$

Def. If X is a discrete R.V having a PMF $f(x)$, then the expected value of X or the mathematical expectation of X is denoted with $\mathbb{E}[X]$ and is defined by

$$\mathbb{E}[X] = \sum_x x f(x) = 0.f(0)+1.f(1)+2.f(2) \qquad X = 0, 1, 2$$

$f(0)=$

$f(1)=$

$f(2)=$

$X \longrightarrow$ C. R.V

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x)\,dx$$

Th^m: let X be a D. R.V with probability function $f(x)$ and c be a constant.

$$\mathbb{E}[c] = c. \qquad \mathbb{E}[c] = c\,\mathbb{E}[1] = c.1.$$

Proof:

$$\mathbb{E}[c] = \sum_x c f(x)$$

$$= c \underbrace{\sum_x f(x)}$$

$$= c.\ 1$$

$$= c$$

Variance: let X be a R.V with finite mean $\mu = \mathbb{E}[X]$, then the variance of X is denoted by $\mathbb{V}[X]$ or $Var[X]$, and is defined by

$$\mathbb{V}[X] = \mathbb{E}[X-\mu]^2 \quad \text{where } \mu = \mathbb{E}[X]$$

$$= \mathbb{E}\left[\left(X - \mathbb{E}[X]\right)^2\right]$$

Standard Deviation : The positive square root of the variance is known as the standard deviation (S.D) i.e.

$$\sigma = \sqrt{\mathbb{V}[X]} = \sqrt{\mathbb{E}[X-\mu]^2}$$

Th^m : Let X be a discrete R.V with pmf $f(x)$, then

$$\mathbb{V}[X] = \sigma^2 = \mathbb{E}[X-\mu]^2 = \mathbb{E}[X^2] - \mu^2 = \mathbb{E}[X^2] - \{\mathbb{E}[X]\}^2$$

Pf: Tys

$\ldots] = \mathbb{E}[X^2] - \mu^2 = \mathbb{E}[X^2] - \{\mathbb{E}[X]\}^2$

Pf: Tys

Th^m: X & Y

$$\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$$

$$\mathbb{E}[aX+b] = \mathbb{E}[aX] + \mathbb{E}[b]$$
$$= a\,\mathbb{E}[X] + b$$

Th^m: $\boxed{\mathbb{E}[XY] = \mathbb{E}[X] \cdot \mathbb{E}[Y]}$ iff X & Y are independent.

Discrete R.V:

Bernoulli Distribution. A random variable X is said to have a Bernoulli distribution with parameter p $(0 \le p \le 1)$ if X can take on only the values 0 & 1, and the probabilities are:

$$\left.\begin{aligned} P(X=1) &= p \\ P(X=0) &= 1-p \end{aligned}\right\} \quad (1)$$

since the distribution involves only two cases of events, it is known as the two-point probability distribution.

If we take $q = 1-p$, then the probability function of X can be written as follows:

$$f(x;p) = \begin{cases} p^x q^{1-x} \ ; & \text{for } x = 0, 1 \\ 0 \ ; & \text{otherwise} \end{cases}$$

Properties of Bernoulli Distribution:

Mean: $\mathbb{E}[X] = \sum_{x} x\, f(x;p) = \underline{0 * p^0 (1-p)^{1-0}} + 1 * p^1 (1-p)^{1-1} = 1.p.1 = p$

$\mathbb{E}[X] = p$

$$\mathbb{V}[X] = \mathbb{E}[X^2] - \{\mathbb{E}[X]\}^2 = \sum_{x} x^2 f(x;p) - p^2$$

$$= 0^2 * p^0 (1-p)^{1-0} + 1^2 * p^1 (1-p)^{1-1} - p^2$$

$$= p - p^2$$

$$\boxed{\mathbb{V}[X] = p(1-p)}$$

$$= pq$$

$$\mathbb{V}[X] = \mathbb{E}[X^2] - \{\mathbb{E}[X]\}^2 = \sum_x \ldots$$
$$= 0^2 \times p^0 (1-p)^{1-0} + 1^2 \times p^1 (1-p)^{1-1} - p^2$$
$$= p - p^2$$
$$\boxed{\mathbb{V}[X] = p(1-p)}$$
$$= pq$$

Moment Generating Function (MGF).

X be a R.V with probability function $f(x)$

$$M_X(t) = \mathbb{E}[e^{tX}] = \underbrace{\sum_x e^{tx} f(x)}_{\text{Expectation}} \longrightarrow \text{Discrete.}$$

Def^n

$$M_X(t) = \mathbb{E}[e^{tX}] = \int_{-\infty}^{\infty} e^{tx} f(x)\,dx \longrightarrow \text{Continuous}$$

x	$P(X=x)$
0	$P(X=0) = 1-p$
1	$P(X=1) = p$

$$\mathbb{E}[X] = \sum x f(x)$$
$$\mathbb{E}[X^2] = \sum x^2 f(x)$$
$$\mathbb{E}[Y] = \sum y f(y)$$

Discrete R.V.

Bernoulli Distribution:

MGF. $M_X(t) = \mathbb{E}\left[e^{tX}\right] = \sum_x e^{tx} f(x,p) = \sum_x e^{tx} p^x (1-p)^{1-x} = \sum_x (pe^t)^x (1-p)$

$= (pe^t)^0 (1-p)^{1-0} + (pe^t)^1 (1-p)^{1-1}$

$= (1-p) + pe^t \cdot 1$

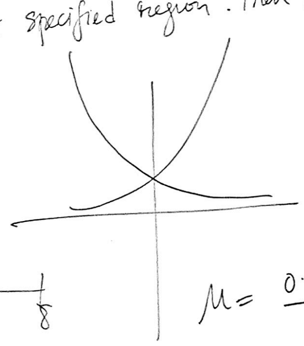
$$\boxed{M_X(t) = pe^t + q}$$

$$= (1-p) + qe^{t}$$

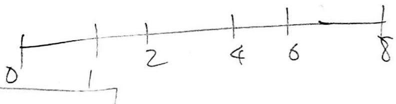
$$\boxed{M_X(t) = pe^{t} + q}$$

Poisson Distribution:

Let μ be the mean or expected number of successes in a specified time or space and the r.v X designates the number of successes in a given time interval or specified region. Then the probabi[lity] distribution of X is a Poisson distribution and is given by



$$\boxed{f(x, \mu) = \frac{e^{-\mu}\mu^{x}}{x!}}, \quad x = 0, 1, 2, 3, \ldots, \infty$$



$$\boxed{\mu =}$$

$\mu = 0.5$ cals/min

$$\mathbb{V}[X] = \mathbb{E}[X^2] - \{\mathbb{E}[X]\}^2 = \sum x^2$$

$$= 0^2 * p^0 (1-p)^{1-0} + 1^2 * p^1 (1-p)^{1-1} - p^2$$

$$= p - p^2$$

$$\boxed{\mathbb{V}[X] = p(1-p)}$$

$$= pq$$

Mean: $\mathbb{E}[X] = \sum_{x=0}^{\infty} x f(x; \mu)$

$$= \sum_{x=1}^{\infty} \frac{x e^{-\mu} \mu^x}{x!}$$

$$= \mu \sum_{(x-1)=0}^{\infty} \frac{e^{-\mu} \mu^{x-1}}{(x-1)!}$$

let, $\boxed{y = x - 1}$

$$\mathbb{E}[X] = \mu \sum_{y=0}^{\infty} \frac{e^{-\mu} \mu^y}{y!} = \mu . \sum_{y=0}^{\infty} f(y; \mu) = \mu . 1 = \mu$$

$$\mathbb{E}[X] = \mu$$

$$\frac{5}{5!} = \frac{5}{5 \times 4!}$$

x	$P(X=x)$
0	$P(X=0) = 1-p$
	$P(X=1) = p$

$\mathbb{E}[X] = \sum x f(x)$

$\mathbb{E}[X^2] = \sum x^2 f(x)$

$\mathbb{E}[Y] = \sum y f(y)$

$$= (1-p) + qe^t \cdot 1$$

$$\boxed{M_X(t) = pe^t + q}$$

$X = 0, 1, 2, 3, 4, 6$

$X_1 = a$

$X_2 = b$

$X_3 = c$

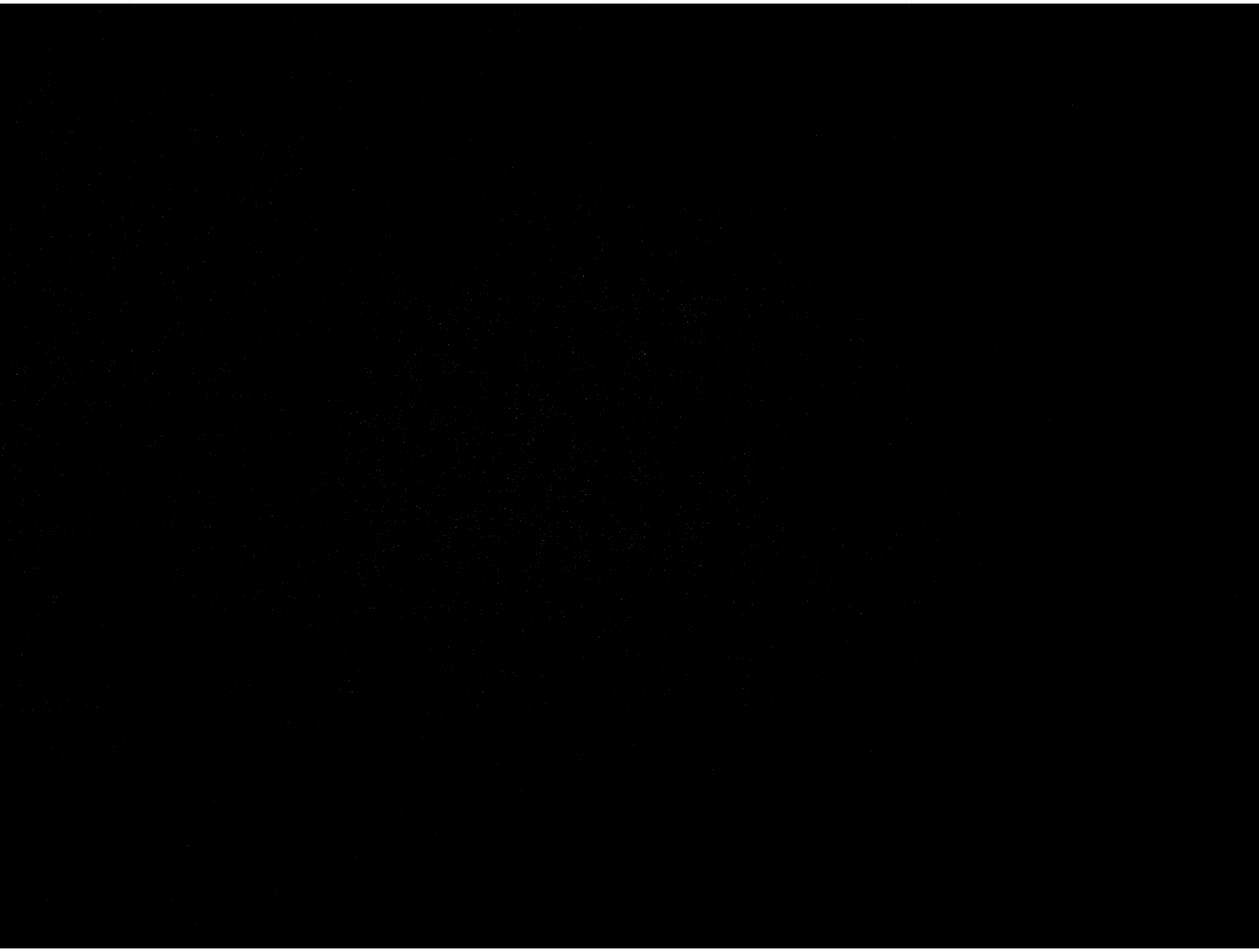
$\bar{x} = [1, 5, 4] = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$\bar{X} = (X_1 \; X_2 \; X_3 \cdots X_n)^T$$

__Random Vector:__ A random vector is a column vector $\bar{X} = [X_1 \; X_2 \cdots X_n]'$. Each X_i is a random variable.

__Vector Sample Value:__ A sample value of a random vector is a column vector $\bar{x} = [x_1 \; x_2 \cdots x_n]'$.

__Random Vector Propability Functions:__



$= \frac{dJ}{dz^2}(a^1)^T$

$dw^1 = dz^1 . a^0$

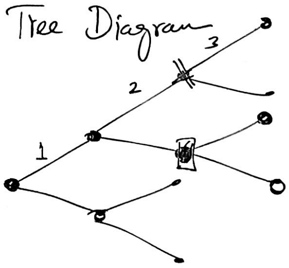
$dw = dz . a$

IF = Ø 1
TF = 0
Push Flag, CS, IP

Ref: David Goodman

Chapter # 13 Stochastic Process (Random)

Tree Diagram



- When we study stochastic process, each observation corresponds to a function of time.
- The word "Stochastic" means "random".
- The word "process" means "function of time"
- In stochastic process, we pay attention to the time sequence of events.

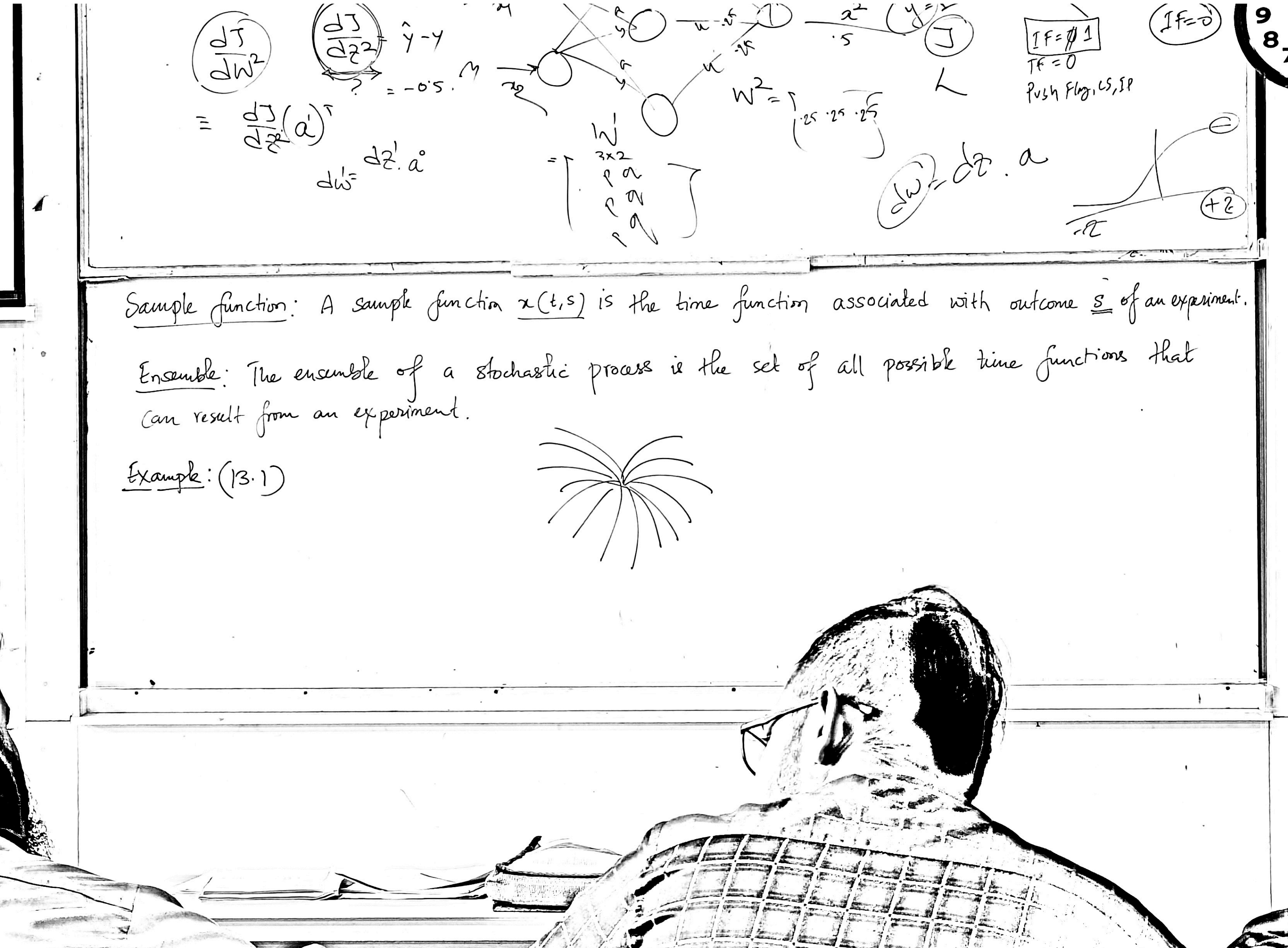
process assigns a sample function to each outcome s.

$X = (x = 2)$
$X = (x = x(t,s))$

Stochastic Process (Def^n): The stochastic process $X(t)$ consists of an experiment with a probability measure $P[\cdot]$ defined on a sample space S and a function that assigns a time function $x(t,s)$ to each outcome s in the sample space of experiment.

Essentially, the def^n says that the outcomes of the experiment are all functions of time.

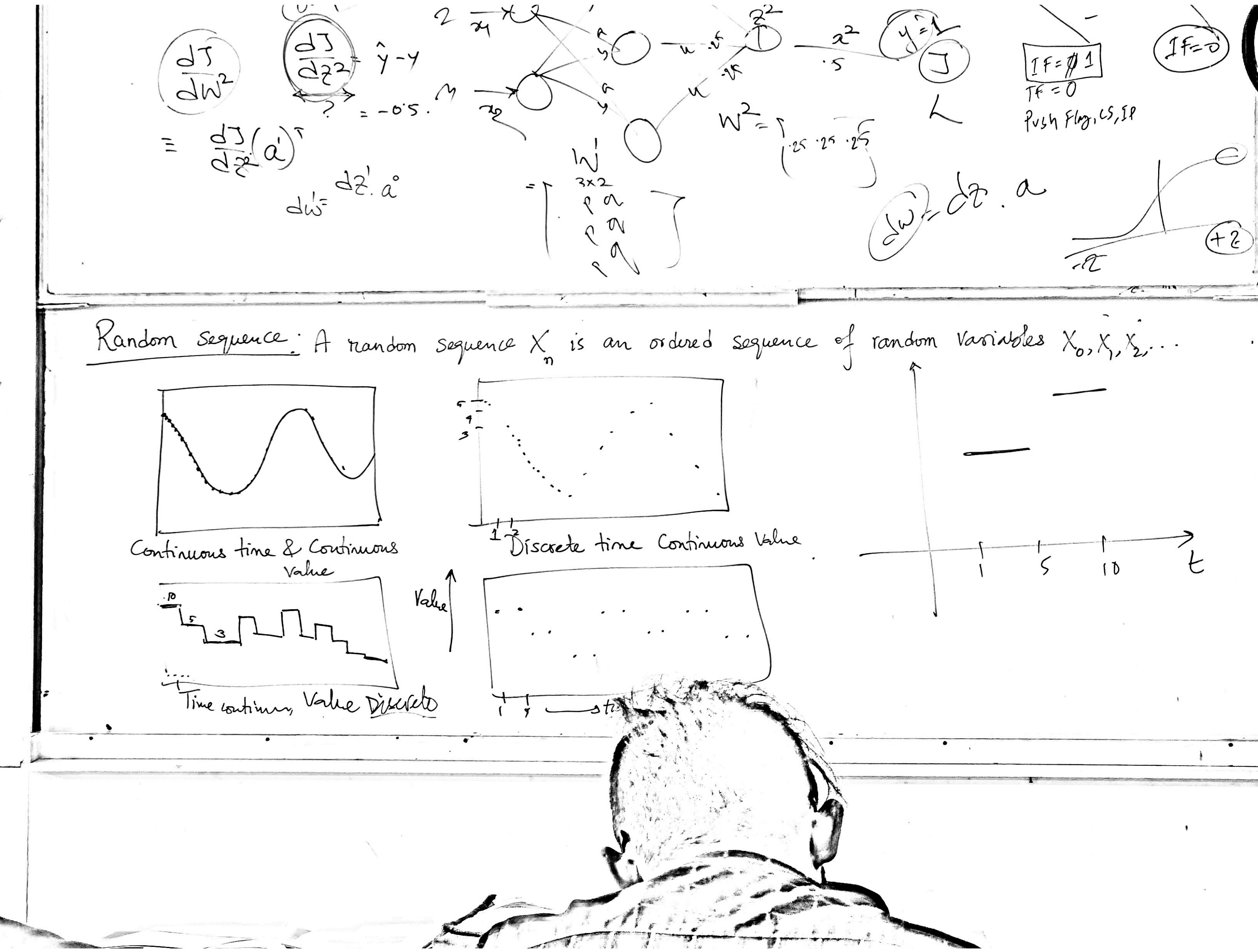
Just as a r.v assigns a number to each outcome s in a sample space S, a stochastic


IF = 1
IF = 0
Push Flag, CS, IP
Sample function: A sample function x(t, s) is the time function associated with outcome s of an experiment.
Ensemble: The ensemble of a stochastic process is the set of all possible time functions that can result from an experiment.
Example: (13.1)

(13.4) Discrete - value & Continuous- value process:

$X(t)$ is a discrete value process if the set of all possible values of $X(t)$ at all time t is a countable set S_X ; otherwise $X(t)$ is continuous - value process.

Discrete - time & Continuous-time process : The stochastic process $X(t)$ is a discrete-time process if $X(t)$ is defined only for a set of time instants, $\boxed{t_n = nT}$, where T is a constant and n is an integer; otherwise $X(t)$ is a continuous-time process.



Random sequence: A random sequence X_n is an ordered sequence of random variables $X_0, X_1, X_2, \ldots$
Continuous time & Continuous Value
Discrete time Continuous Value
Time continuous, Value Discrete
Value
t
IF = 1
IF = 0
Push Flag, CS, IP
IF = 0