we see that this critical values of Z are ± 2.58, that means the critical regions are $Z<-2.58$, $Z>2.58$.

Now, we have to calculate: $z=\dfrac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$.

Here, $\bar{x}=197.5, \mu_0=200, \sigma=20$ and $n=75$, we have

$$z=\frac{\bar{x}-\mu_0}{s/\sqrt{n}}=\frac{197.5-200}{20/\sqrt{75}}=-1.08.$$

The observed value of Z is greater than the lower critical value -2.58. Since the observed value of test statistic does not fall within the critical region, so we fail to reject the null hypothesis at 1% level of significance. That means the mean efficiency rating is still 200.

- **Confidence interval.** 99% confidence limits are given by

$$\bar{x} \pm z_{0.005}\frac{\sigma}{\sqrt{n}}=197.5 \pm 2.58\frac{20}{\sqrt{75}}=(191, 203),$$

that means, we are 99% confident that true average efficiency rating of all employees will between 191 and 203.

Example 16.6.11. Suppose in Example 16.6.3, instead of checking if the price is different from standard price, CAB decided to verify whether the specified price of the product is more than standard price.

Solution. It is obviously a one-tailed test, because, if average price obtained from the sample less than or equal to the standard price, the claim of the producer will be established. Hence, we have to consider a composite hypothesis defined as

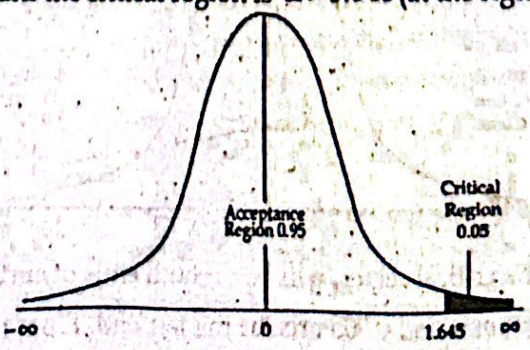
Null hypothesis: $H_0: \mu \leq 1500$

Alternative hypothesis: $H_1: \mu > 1500$.

The test statistic for testing the null hypothesis is: $Z=\dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$.

Here $\alpha=0.05$ (given).

It is a right- tailed test; the critical region will be on right side of curve of Z that will comprise 5% or 0.05 area at the right end. From the table of area of normal curve, we see that this critical value of Z is 1.645 that means the critical region is $Z>1.645$ (at the right end).



We have found $z=2.22$ (from Example 16.6.3).

Hence, the observed value of $Z=2.22$ which is greater than the critical value 1.645, hence, observed value of test statistic falls in the critical region, we fail to accept the null hypothesis 5% level of significance.

The claim of the producer is not right, that means, the average price of the products of the producer is more than the standard price.

p-value. The p-value is given by $P(Z>2.22)=0.0132$ that means the smallest level of significance at which the hypothesis is rejected is 0.0132 or 1.3%.

Example 16.6.12. The average petrol consumption of existing auto engines is 10.5 km. per liter. An auto company decided to introduce a new six-cylinder car whose mean petrol consumption is claimed to be lower than that of the existing auto engine. In order to verify company's claim, a sample of 50 new cars was randomly selected and it was found that the mean petrol consumption is 10 km. per liter with a standard deviation of 3.5 km. per liter. Test whether the claim of the company is acceptable at 5% level of significance.

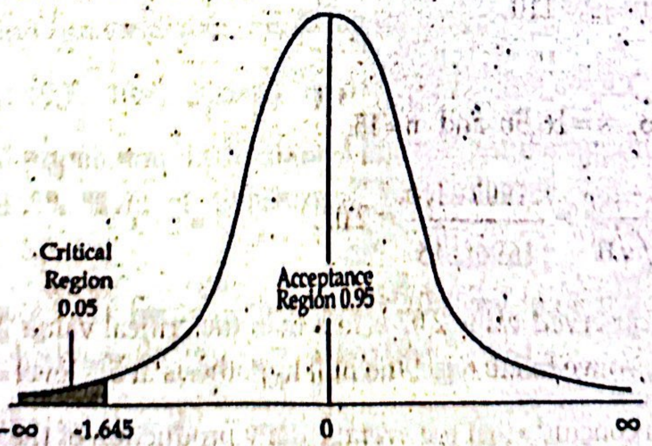
Solution. Here we have to consider the following hypotheses

$$H_0: \mu=10.5 \text{ and}$$

$$H_1: \mu<10.5.$$

Although the population variance is not known, since sample size is large, the appropriate test statistic is $Z=\dfrac{\overline{X}-\mu}{s/\sqrt{n}}$ where s^2 is sample variance and n is the sample size.

Since it is a left tailed test, the critical region lies in the left end of the curve given by $z<-1.645$



Given, $\overline{x}=10$ km., $s=3.5$, $n=50$, so the value of test statistic Z under H_0 is given by

$$z=\frac{\overline{x}-\mu_0}{s/\sqrt{n}}=\frac{10-10.5}{3.5/\sqrt{50}}=-1.01.$$

This observed value of Z does not lie in the critical region, so we fail to reject the null hypothesis. That means the claim of the company is not acceptable.

Example 16.6.13. Suppose the daily number of items produced by a firm for randomly selected 15 days is as foll...

110, 118, 130, 140, 142, 146, 112, 100, 95, 98, 96, 122, 123, 124, 130.

Can we conclude at 5% level of significance that the average daily production of items of firm is 110?

Solution. It is a two-tailed test because if the average hourly number of items produced by company is more or less than 110, then the statement that the average daily production of will be proved as false, then the null hypothesis that the average daily production of items would be rejected.

So, we the null hypothesis and alternative hypothesis are as follows

Null hypothesis: $H_0 : \mu = 110$

Alternative hypothesis: $H_a : \mu \neq 110.$

Level of significance as given in the problem is $\alpha = 0.05$, since the variance is unknown sample size is small, the appropriate test statistic is

$$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} \text{ which is distributed as Student's t with } n-1 \text{ df.}$$

Here, $n = 11$, so the $df = 15 - 1 = 14$.

Here $\alpha = 0.05$, and it a two tailed-test, the critical region will be on both sides distribution, in such a way that the critical region will comprise 2.5% or 0.025 area at the end and 2.5% at the left end. From the table of t-distribution, we find for $df = 14$, $\alpha = 0.02$ values of t are ± 2.145, that means the critical regions are $t < -2.145$ (at the left end) and $t >$ (at the right end).

Here, $\bar{x} = 119.07$ $\mu = \mu_0 = 110$.

$= 287.78$, so, $s = 16.96$ and $n = 15$.

So, we have, $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{119.07 - 110}{16.96/\sqrt{15}} = 2.07.$

It is found that the observed value 2.07 is less than the critical value 2.145; hence it does fall in the critical region, so we fail to reject the null hypothesis at 5% level of significance.

- **Conclusion.** We can conclude that the average daily production of the items of the given can be accepted as 110.

Example 16.6.14. (Small sample with unknown variance) A gas station repair shop claims the average time it takes to do a lubrication job and oil change is maximum 30 minutes consumer protection department wants to test the claim. A sample of six cars was sent station for oil change and lubrication. The job took an average of 34 minutes with a standard deviation of 4 minutes. Assuming that the population is normal, do you think that the job takes average of time more than 30 minutes? (use $\alpha = 0.05$)

...ution. It is obviously a one-tailed test, because, the claim will not be rejected if the average ...e taken on a car for the oil change and lubrication job is considerably less than 30 minutes. So, ...this case we have to consider a composite hypothesis given by (although in practice we use ...ple hypothesis)

$$H_0: \mu = \mu_0 \leq 30 \text{ against the alternative } H_1: \mu = \mu_1 > 30.$$

Since the sample size is small and population variance is unknown, t statistic will be used.

Under the null hypothesis the value of t is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \text{ which is distributed as Student's t with } n-1 \text{ df.}$$

Here, $n=6$, $\bar{x}=34$, $s=4$ and $\hat{\sigma}(\bar{x}) = s/\sqrt{n} = 1.63$.

Therefore, we get: $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{34-30}{1.63} = 2.45$

The critical value of t at $\alpha = 0.05$ and $df = n-1 = 5$, for a right tailed test is given by ...$_{.05} = 2.02$, since the computed value of $t = 2.45$ is higher than the critical value of $t = 2.02$, we ...ject the null hypothesis, that means the claim of the shop is not considered to be correct.

p-value: In this case p-value is given by $P(t_5 > 2.45) = 0.03$(from the table of critical value of t distribution), hence p-value is 0.03 or 3%.

...ample 16.6.15. A process that produces bottles of shampoo, when operating correctly, ...oduces bottles whose contents weigh, on average, 20 ounces. A random sample of nine bottles ...m a single production run yielded the following weights

21.4, 19.7, 19.7, 20.6, 20.8, 20.1, 19.7, 20.3, 20.9.

Assuming that the population distribution is normal, test the hypothesis that the process is ...erating correctly at 5% level of significance. Also calculate 95% confidence interval for ...pulation mean.

...lution. It is a two-tailed test, because, the claim will be rejected if the average weight deviates ...m 20 ounces in any direction. So, the hypothesis is given by

$$H_0: \mu = \mu_0 = 20 \text{ against the alternative } H_1: \mu \neq 20.$$

Since the sample size is small and population variance is unknown, t statistic will be used.

Under the null hypothesis the value of t is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \text{ which is distributed as Student's t with } n-1 \text{ df.}$$

Here, $n=9$, $\bar{x}=20.36$, $\mu_0 = 20$, $s=0.61$ and $\hat{\sigma}(\bar{x}) = s/\sqrt{n} = 0.203$

Hence, $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{20.36 - 20}{0.203} = 1.77.$

The critical values of t at $\alpha = 0.05$ with $n-1 = 8$ df for a two- tailed test are given by

$$\pm t_{8,0.025} = \pm 2.316.$$

Since the computed value of t is 1.77 which does not fall in the critical region, so we reject the null hypothesis, that means the process is operating correctly.

95% confidence interval for population mean μ is given by

$$\bar{x} \pm t_{n-1,\alpha/2} \frac{s}{\sqrt{n}} = 20.36 \pm 2.316 \times \frac{0.61}{\sqrt{9}}.$$

So, the lower and upper confidence limits are 19.89 and 20.83 respectively.

16.7. Test of Hypothesis Concerning Two Population Means

Suppose $X_{11}, X_{12}, \ldots\ldots, X_{1n_1}$ be a random sample of size n_1 drawn from normal popul with mean μ_1 and variance σ_1^2, and $X_{21}, X_{22}, \ldots\ldots, X_{2n_2}$ be another sample of size n_2 dr from normal population with mean μ_2 and variance σ_2^2. Suppose, the observed sample m are $\bar{X}_1$ and $\bar{X}_2$. In the earlier chapter, it is mentioned that the distribution of the differ between two sample means follows normal distribution when variances are known fo possible sample sizes, that means

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,\ 1).$$

The possible null and alternative hypotheses considered for testing the significanc difference between two population means are

i) $H_0: \mu_1 = \mu_2$ against $H_a: \mu_1 \neq \mu_2$ (for a two tailed alternative);

ii) $H_0: \mu_1 = \mu_2$ against $H_a: \mu_1 > \mu_2$ (for a right tailed alternative);

iii) $H_0: \mu_1 = \mu_2$ against $H_a: \mu_1 < \mu_2$ (for a left tailed alternative).

Again, the following alternative situations may arise in testing the above mentioned null alternative hypotheses

i) Independent samples with known population variances, sample sizes are large or sm
ii) Independent samples with unknown population variances, sample sizes are large.
iii) Independent populations for small sample sizes ($\leq$ 29) with unknown but e variances.
iv) Independent populations for small sample sizes ($\leq$ 29) with unknown and une variances.
v) Correlated sample or matched sample (the sample obtained from a bi-variate nor population or paired observations).

The test statistic to be used for testing the simple hypothesis $H_0: \mu_1 - \mu_2 = 0$ against a one-tailed or two-tailed alternative is given by

Under situation (i): $$Z = \frac{(X_1 - X_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1).$$

Under situation (ii): $$Z = \frac{(X_1 - X_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim N(0, 1).$$

where $s_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1}$ and $s_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1}$.

Under situation (iii): $$t = \frac{(X_1 - X_2) - (\mu_1 - \mu_2)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

which is distributed as Student's t with $n_1 + n_2 - 2$ degrees of freedom, where, s^2 is pooled-estimate of variance, given by

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

The value of $(\mu_1 - \mu_2) = 0$ for all Z and t defined above under null hypothesis $H_0: \mu_1 - \mu_2 = 0$, but if any other value is specified (say, $\mu_1 - \mu_2 = \theta_0$) by null hypothesis for the difference between means, that means if

$$H_0: \mu_1 - \mu_2 = \theta_0$$

then the value of $(\mu_1 - \mu_2)$ will be θ_0 instead of zero.

For testing the hypothesis regarding difference between two means under situation (i) to (iii) it is desirable to test the equality of two population variance to check if the assumptions of equal variances are valid or not. If the variances are found not to be equal, then, Student's t statistic can not be applied. Hence, under situation (iv), under null hypothesis, the test statistic is given by:

$$t' = \frac{(\overline{X}1 - \overline{X}2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}.$$

Here, t' is not a Student's t statistic. The critical values of t' at $100\alpha\%$ level of significance are computed using the formula:

$$t'_\alpha = \frac{\frac{s_1^2 t_1}{n_1} + \frac{s_2^2 t_2}{n_2}}{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

where t_1 and t_2 are Student's with $(n_1 - 1)$ and $(n_2 - 1)$ df respectively at $100\alpha\%$ l... significance.

Again, under situation (v), let us consider a random sample of n matched pa... observations (x_i, y_i) from a bi-variate normal population, then the test of the hypo... $H_0: \mu_1 - \mu_2 = 0$ requires to compute a statistic d defined as $d = x_i - y_i$, and the testing pro... is the same as testing the significance of a single mean of the observations obtained fro... difference of two variables assuming that small sample size and unknown population va... The statistic used for testing this type of hypothesis is called paired-t test, defined as

$$t = \frac{\bar{d}}{se(\bar{d})} \sim t_{n-1} \quad \text{under situation (v)}$$

which is distributed as t with $n-1$ df.

where, $\bar{d} = \frac{\Sigma d}{n}$, $s_d^2 = \frac{\Sigma(d - \bar{d})^2}{n-1}$ and $se(\bar{d}) = \frac{s_d}{\sqrt{n}}$.

- **Decision rule.** The decision rules in all of the above cases are the same as that of testin... significance of a single mean using Z or t statistic.

Example 16.7.1. It is wanted to investigate if male and female typists earn comparable wage... sample data for daily wages of male and female provide with the following information.

Table 16.6. Sample mean and variance of male and female typists

	Male	Female
Sample size	60	60
Mean wage	Taka 158.50	Taka 141.60
SD (Population)	Taka 18.20	Taka 20.60

Test whether the mean wages of male typists is more than female typists at 5% and 1% of significance.

Solution. Let the wages of male and female are normally and independently distribute... means μ_1 and μ_2, and known variances σ_1^2 and σ_2^2 respectively. It is a one-tailed test... consider the following hypothesis

$$H_0: \mu_1 = \mu_2 \text{ against } H_1: \mu_1 > \mu_2.$$

As the sample sizes are large, under null hypothesis, the value of test statistic is given b...

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$$z=\frac{(\bar{x}_1-\bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}.$$

For $\alpha=0.05$, the critical region is $Z>1.645$, and for $\alpha=0.01$, the critical region is $Z>2.33$.

Here, $\bar{x}_1=158.50$, $\bar{x}_2=141.6$, $\sigma_1=18.20$, $\sigma_2=20.60$, $n_1=n_2=60$.

Thus, the computed value of Z is: $z=\frac{(158.50-141.6)}{\sqrt{\frac{(18.20)^2}{60}+\frac{(20.60)^2}{60}}}=4.76.$

Conclusion. The computed value of Z is greater than critical values at both the level of significance, hence in the given city, male typists have on the average higher earnings than their female counterpart at 1% and 5% levels of significance. Here, the value z is highly significant.

Example 16.7.2. (Large sample sizes with unknown population variances) A potential buyer of electric bulbs bought 100 bulbs each of two famous brands A and B. Upon testing both these sample, he found that brand A had a mean life of 1500 hours with standard deviation of 50 hours whereas brand B had an average life of 1530 hours with standard deviation of 60 hours. Can it be concluded at 5% level of significance that the bulbs of two brands differ significantly in quality?

Solution. We assume that the parent population of these two lifetimes are independently distributed with means μ_1 and μ_2 and unknown variances σ_1^2 and σ_2^2. We also assume that there is no significant difference in the quality of both brands so that brand A is as good as brand B in terms of operating hours. Hence, we have to test

$$H_0:\mu_1=\mu_2 \text{ against } H_1:\mu_1\neq\mu_2.$$

Since sample sizes are large, so under the null hypothesis, the value of test statistic is

$$z=\frac{(\bar{x}_1-\bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}\sim N(0,1).$$

Again, $\alpha=0.05$ and since it is a two tailed test, the critical region is

$$|Z|>1.96.$$

Here, $\bar{x}_1=1500$, $\bar{x}_2=1530$, $s_1=50$ and $s_2=60$, $n_1=n_2=100$.

Now, $$\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}=\sqrt{\frac{(50)^2}{100}+\frac{(60)^2}{100}}=7.81.$$

Thus, $$z=\frac{(1500-1530)}{7.81}=-3.841$$

Since the observed value of Z is greater than the critical value, the null hypothesis may be rejected at 5% level of significance.

- **p-value.** From the table of Z, we have $P(Z<-3.00)=P(Z>3.00)=0.00$, hence the null hypothesis may be rejected even at 0% level of significance, so, $p=0.00$. It is said that value z is highly significant since the p-value is 0.00.
- **Confidence Interval.** 95% confidence limits for the difference of two population means $(\mu_1-\mu_2)$ are given by

$$(\bar{x}_1-\bar{x}_2)\pm 1.96\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}=(-30)\pm 7.81=(22.19,\ 37.81).$$

Example 16.7.3. (Large sample sizes with unknown population variances) A professor taught two sections of an introductory marketing course using very different styles. In the first section approach was extremely formal and rigid, while in the second section an independent, more relaxed and informal attitude was adopted. At the end of the course, a common final examination was administered. In the first section the seventy-two students obtained a mean score of 71.03 and the sample standard deviation was 22.91. In the second section there were sixty-four students, with mean score 80.92 and standard deviation 23.11. Assume that these two groups of students can be regarded as independent random samples from the populations of all students who might be exposed to these teaching methods. Test at 5% level of significance (i) whether the performance of these two methods is the same, (ii) whether second method is better than first one.

Solution. We assume that the parent populations are independently distributed with means μ_1 and μ_2 and unknown variances σ_1^2 and σ_2^2.

(i) We have to test that there is no significant difference in two method, so that first method is as good as second method as far as the teaching system is concerned. Hence, we have to test

$$H_0:\mu_1=\mu_2 \text{ against } H_1:\mu_1\neq\mu_2.$$

Since sample sizes are large, so under null hypothesis the value of test statistic is

$$z=\frac{(\bar{x}_1-\bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}\sim N(0,1).$$

Again, $\alpha=0.05$ and since it is a two tailed test, the critical region is

$$|Z|>1.96.$$

Here, $\bar{x}_1=71.03$, $\bar{x}_2=80.92$, $s_1=22.91$ and $s_2=23.11$, $n_1=72$, $n_2=64$.

Now, $$\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}=\sqrt{\frac{(22.91)^2}{72}+\frac{(23.11)^2}{64}}=3.95.$$

Thus, $z = \frac{(71.03 - 80.92)}{3.95} = -2.50.$

Since the computed absolute value of Z is greater than the critical value, the null hypothesis may be rejected at 5% level of significance. The sample supports that there is significant difference between average performances of the two methods.

p-value. From the table of Z, we have $P(Z < -2.50) = P(Z > 2.50) = 0.007$, hence the null hypothesis may be rejected even at 1.4% (since $0.007 \times 2 = 0.014$ level of significance, so $p = 0.014$.

Confidence Interval. 95% confidence limits for the difference of two population means $(\mu_1 - \mu_2)$ are given by

$$(\bar{x}_1 - \bar{x}_2)) \pm 1.96\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (-9.89) \pm 7.81 = (-17.70,\ 2.08).$$

(ii) Since the second method would be proved to be better than the first method, if the average score obtained by first method is significantly smaller than that of second method, so, in order to test whether second method is better than the first method, we have to consider a one-tailed test defined as

$$H_0 : \mu_1 = \mu_2 \text{ against } H_1 : \mu_1 < \mu_2.$$

Here, since the sample sizes are large, the test statistic is also Z as defined in (i), but the critical region will cover the 5% area only in the left tail, thus the critical region is $Z < -1.645$.

The observed value of Z is -2.50, as found in (i), falls in the critical region, so the null hypothesis may be rejected at 5% level of significance. That means, the sample supports that the second method is on an average better than the first method.

Example 16.7.4. (Large sample sizes with known variances) A firm believes that the tires produced by process I on an average last longer than tires produced by process II. To test this belief, random samples of tires produced by two processes were tested and the results are as,

Table 16.7. Mean and standard deviation of lifetimes of tires

	Process I	Process II
Sample size	50	50
Average lifetime (in km.)	22,400	21,800
Population Standard deviation (in km.)	1000	1000

Is there any evidence at 5% level of significance that the firm is correct in its belief?

Solution. We have to consider the following null and alternative hypothesis

$$H_0 : \mu_1 = \mu_2 \text{ against } H_1 : \mu_1 > \mu_2.$$

Since the sample sizes are large, the value of test statistic under null hypothesis is

$$z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}.$$

Since it is a one tailed test, the critical region is: $Z > 1.645$.

Given, $\bar{x}_1 = 22400, \bar{x}_2 = 21800, \sigma_1 = 1000, \sigma_2 = 1000, n_1 = n_2 = 50$.

So, the value of Z is: $z = \frac{(22400 - 21800)}{\sqrt{\frac{(1000)^2}{50} + \frac{(1000)^2}{50}}} = 3.00.$

Since the calculated value of Z is more than its critical value at 5% level, therefo hypothesis may be rejected. Hence, we can conclude that the tires produced by proces longer life than process II.

- p-value. From the table of area under the standard normal probability distribution, we $P(Z > 3.00) = 0.001$, so the smallest critical value for which null hypothesis may be reje 0.001, thus the p-value is 0.001.

Example 16.7.5. (Small sample sizes with unknown and equal population variance) Manag factory I claims that the average wage of its workers is higher than that of factory II. A conducted a survey on daily wages of workers of two factories to see if the claim of

Table 16.8. Mean and standard deviation of lifetimes of tires

	Factory I	Factory II
Sample size	16	11
Sample mean wage	Taka 290	Taka 250
Sample standard deviation	15	50

Solution. Let us assume that the wages of corresponding population of two factori independent and normally distributed with common unknown variance σ^2. Thus, the nu alternative hypothesis are

$$H_0 : \mu_1 = \mu_2 \text{ against } H_1 : \mu_1 > \mu_2.$$

Since the sample sizes are small and variances are unknown but equal, the test statistic i

$$t = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

which is distributed as Student's t with $n_1 + n_2 - 2$ degrees of freedom,

where, $s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$ is an estimate of common variance σ^2.

Here, $n_1 = 16$, $n_2 = 11$, so the degrees of freedom is $n_1 + n_2 - 2 = 25$.

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It is a one tailed test, thus from the table of t-distribution we have the critical values of t at 5% level of significance with 25 df is 1.708, that means, $t_{25,0.05}=1.708$, in other words, the critical region is: $t>1.708$

Given, $\bar{x}_1=290$, $\bar{x}_2=250$, $s_1=15$, $s_2=50$, $s^2=1135$ and $s=33.69$

So, under the null hypothesis, the value of t is

$$t=\frac{\bar{x}_1-\bar{x}_2}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}=\frac{290-250}{13.19}=3.03.$$

Decision. The observed value of t is greater than the critical value, it lies in the critical region, so the null hypothesis may be rejected at 5% level of significance.

Conclusion. The claim of manager of factory-I is justified.

Example 16.7.6. (Small sample sizes with unknown and equal population variances) The residence of Dhaka city complains that traffic speeding fines given in their city are higher than the traffic speeding fines that are given in Chittagong city. The appropriate authority agreed to study the problem. To check if the complaints were reasonable, independent random samples of the amounts paid by the residents for speeding fines in each of two cities over the last three months were obtained and shown in following table.

Table 16.9. Amounts of traffic speeding fines

Dhaka city	100	125	135	128	140	142	128	137	156	142
Chittagong city	95	87	100	75	110	105	85	95		

Assuming an equal population variance, test

i). whether there is any significant difference in the mean cost of speeding in these two cities and find the 95% confidence interval.

ii) whether the mean speeding cost in Dhaka city is higher than Chittagong city at 1% level of significance.

Solution. (i) Let X_1 be the speeding cost in Dhaka city and X_2 be the speeding cost in Chittagong city. Assuming the samples have been drawn independently from two normal populations with means μ_1 and μ_2 respectively with common variance σ^2. We have to test the following hypothesis

$$H_0:\mu_1=\mu_2 \text{ against } H_1:\mu_1\neq\mu_2.$$

It is a two tailed test, the sample sizes are small and variances are also unknown, so, the appropriate test statistic is given by

$$t=\frac{\bar{x}_1-\bar{x}_2-(\mu_1-\mu_2)}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

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which is distributed as Student's t with n_1+n_2-2 degrees of freedom, where

$$s^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$$ is an estimate of common variance σ^2.

Given, $n_1=10$, $n_2=8$, so the degrees of freedom is: $n_1+n_2-2=16$.

From the table of t-distribution we have the critical values of t at 5% level of significance, 16 df are ± 2.12, that means, $t_{16,0.025}=2.12$, in other words, the critical region is

$$|t|>2.12.$$

From the given observations, we have,

$$\bar{x}_1=133.30, \bar{x}_2=94.00, s_1=18.20, s_2=20.60, s^2=371.98$$

So, under null hypothesis, the calculated value of t is

$$t=\frac{\bar{x}_1-\bar{x}_2}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}=\frac{133.30-94.00}{9.14}=4.30.$$

- **Decision.** The observed absolute value of t is greater than the critical value, it lies in the critical region, so the null hypothesis may be rejected at 5% level of significance.
- **Conclusion.** There is significant difference between the average traffic fines in two cities.

Again, the 95% confidence interval for $(\mu_1-\mu_2)$ is given by

$$(\bar{x}_1-\bar{x}_2)-t_{16,0.025}\times s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}<(\mu_1-\mu_2)<(\bar{x}_1-\bar{x}_2)+t_{16,0.025}\times s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

or, $39.30\pm 2.12\times 4.30=39.30\pm 9.12=(30.18, 48.42)$.

- **p-value.** From the table of critical values of t-distribution, we find that $t_{16,0.005}=2.921$, that means $2\times 0.005=0.010=1\%$ can be considered as the smallest level of significance at which the null is still rejected, so the required p-value is 0.01.

(ii) In this case we have to perform a one-tailed test given by

$$H_0: \mu_1=\mu_2 \text{ against } H_1: \mu_1>\mu_2.$$

Here the critical value of t at 1% level of significance with 16 df is: $t_{16,0.01}=2.583$.

The observed value of t is same as previous one, so $t=4.30$.

In this case too, the observed t falls in the critical region, so the null hypothesis may be rejected at 1% level of significance. We can conclude that on an average the traffic fines at Dhaka city is higher than that of Chittagong city.

- **p-value.** In this case the p-value is 0.005 because, $P(t>4.30)=0.005$.

Example 16.7.7. (Matched observations or paired sample) A study was conducted by a pharmaceutical company to compare the difference in effectiveness of two particular drugs

...sterol levels. The company used paired sample approach to control variation in reduction ...might be due to factors other than the drug itself. Each member of a pair was matched by ...weight, lifestyle, and other pertinent factors. Drug X was given to one person randomly ...ed from each pair, and drug Y was given to the other individual in the pair. After a specific ...d of time each person's cholesterol level was measured again. Suppose a random sample of ...pairs of patients with known cholesterol problems is selected from the large populations of ...cipants. The following table gives the number of points by which each person's cholesterol ...was reduced..

Table 16.10. Reduction levels of cholesterol by drugs

Pair	1	2	3	4	5	6	7	8
Drug X	29	32	31	32	32	29	31	30
Drug Y	26	27	28	27	30	26	33	36

i) Test whether there is any significant difference between the mean reductions of cholesterol levels by two drugs at 1% level of significance.

ii) Find 99% confidence interval for the difference between the population means.

...tion. (i) Let the observations have been selected from a bi-variate normal population. So, it is ...ssary to use paired t-test for testing the difference between the mean reduction levels.

Let us formulate the null and alternative hypothesis as

$$H_0: \mu_x = \mu_y \text{ against } H_1: \mu_x \neq \mu_y.$$

The appropriate test statistic is: $t = \frac{\bar{d}}{se(\bar{d})} \sim t_{n-1}$, which is distributed as t with $n-1$ df,

where, $\bar{d} = \frac{\Sigma d}{n}$, $s_d^2 = \frac{\Sigma(d-\bar{d})^2}{n-1}$ and $se(\bar{d}) = \frac{s_d}{\sqrt{n}}$:

Here, $n=8$, degrees of freedom is $n-1=7$.

It is two-tailed test, so at 1% level of significance the critical region is

$$|t| > t_{n-1;\alpha/2} = t_{7;0.005} = 3.499$$

Now, let us construct the following table for calculation of mean and standard deviation of d.

Table 16.11. Calculation of mean and standard deviation of d

Pair	1	2	3	4	5	6	7	8
Drug X	29	32	31	32	32	29	31	30
Drug Y	26	27	28	27	30	26	33	36
$d=X-Y$	3	5	3	5	2	3	-2	-6
$(d-\bar{d}))$	1.38	3.38	1.38	3.38	0.38	1.38	-3.63	-7.63
$(d-\bar{d})^2$	1.89	11.39	1.89	11.39	0.14	1.89	13.14	58.14

We have $\bar{d} = 1.625$, $s_d^2 = 14.27$, $s_d = 3.777$

So, $t = \frac{\bar{d}}{se(\bar{d})} = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{1.625}{3.777/\sqrt{8}} = 1.22.$

- **Decision.** The observed value of t is smaller than absolute value of critical value, that means it does no fall in the critical region, so we fail to reject null hypothesis.
- **Conclusion.** There is no significant difference between the average reduction of cholesterol level by two drugs, that means, drug X and drug Y are equally effective.

(ii) The 99% confidence interval for the difference of population average reduction of cholesterol levels is given by

$$\bar{d} - t_{n-1,\alpha/2} \cdot se(\bar{d}) < \mu_x - \mu_y < \bar{d} + t_{n-1,\alpha/2} \cdot se(\bar{d}) = (-3.05,\ 6.30).$$

Example 16.7.8. (Matched observations or paired sample) Ten persons were appointed as probationary officers in an office. Their performance was noted by taking a test and the marks were recorded out of 100. After training for a 6-months period, another test was conducted. The marks obtained by the officers before and after training were as follows.

Table 16.12. Marks of employees before and after training

Employees	A	B	C	D	E	F	G	H	I	J
Before training	80	76	92	60	70	56	74	56	70	56
After training	84	70	96	80	70	52	84	72	72	50

Were the employees benefited by the training?

Solution. It would be proved that the employees were not benefited by the training if there is no significant difference between the average score obtained by them before and after training. On the other hand, it would be proved to be benefited if the average score obtained by employees significantly improved after training.

So we have to conduct a one tailed test defined by the null and alternative hypothesis as

$$H_0 : \mu_a = \mu_b \text{ against } H_1 : \mu_a < \mu_b.$$

The appropriate test statistic is $t = \frac{\bar{d}}{se(\bar{d})} \sim t_{n-1}$, which is distributed as t with $n-1$ df,

where, $\bar{d} = \frac{\Sigma d}{n}$, $s_d^2 = \frac{\Sigma(d-\bar{d})^2}{n-1}$ and $se(\bar{d}) = \frac{s_d}{\sqrt{n}}$.

Here, $n=10$, degrees of freedom is: $n-1=9$.

It is one-tailed test, so at 5% level of significance the critical region is

$$t < -t_{n-1,\alpha} = -t_{9,0.05} = -2.62.$$

Now, let us construct the following table for calculation of mean and standard deviation of d

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Table 16.13. Computation of mean and standard deviation of d

Employees	A	B	C	D	E	F	G	H	I	J
Before training (X)	80	76	92	60	70	56	74	56	70	56
After training (Y)	84	70	96	80	70	52	84	72	72	50
$d = X - Y$	-4	6	-4	-20	0	4	-10	-16	-2	6
$(d - \bar{d})$	0.00	10.00	0.00	-16.00	4.00	8.00	-6.00	-12.00	2.00	10.00
$(d - \bar{d})^2$	0.00	100.00	0.00	256.00	16.00	64.00	36.00	144.00	4.00	100.00

We have: $\bar{d} = -40/10 = -4$, $s_d^2 = 80$, $s_d = 8.944$

So, $$t = \frac{\bar{d}}{se(\bar{d})} = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{-4}{8.944/\sqrt{10}} = -1.414.$$

ecision. The observed value of t does not fall in the critical region, so we fail to reject null hypothesis that means the null hypothesis holds true.

Conclusion. It can be concluded that the employees were not benefited by training.

1 Test of Hypothesis Concerning Attributes

In case of attributes, we can only find out the presence or absence of a certain qualitative racteristics. For example, in the study of attribute 'employment', a survey may be conducted the people may be classified as employed and unemployed, in the study of the attribute ctiveness' of a drug, the patients may be classified as cured and not cured and in the study of bute 'size', the items may be classified as small and large. The appearance of an attribute be considered as success and its non-appearance as failure. Obviously, this type of two omes will follow binomial distribution. Thus, when the sample size is large, we can perform following tests with the attributes having two categories

i) Test of a population proportion for a specified value;
ii) Test of the difference between two population proportions;
iii) Tests of independence of two attributes (having two or more categories of each attribute).

The test regarding independence of attributes is discussed in section 16.12.

1. Test of hypothesis about a population proportion. Suppose we have a sample of n rvations from a population, a proportion π of population follow a particular attribute. Then, e number of sample observations n is large, and the observed sample proportion is P. For ng the significance of this population proportion for a given value π_0, we consider the wing null and alternative hypothesis

$H_0 : \pi = \pi_0$, against the alternative $H_1 : \pi \neq \pi_0$ (two tailed test).

We know, the sampling distribution of P is normal with mean π and variance $\sigma_P^2 = \frac{\pi(1-\pi)}{n}$ and under H_0 it is $\frac{\pi_0(1-\pi_0)}{n}$.

So, under the null hypothesis, the test statistic is

$$z = \frac{P-\pi_0}{se(P)} = \frac{P-\pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \sim N(0, 1).$$

The decision rule is: Reject H_0 in favor of H_1 at $100\alpha\%$ level of significance if

$$z = \frac{P-\pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} > z_{\alpha/2} \text{ or, } \frac{P-\pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} < -z_{\alpha/2} \text{ for a two-tailed test,}$$

or, if $|Z| > z_{\alpha/2}$

Similarly, for a right tailed test, the decision rule is

Reject H_0 in favor of H_1 at $100\alpha\%$ level of significance if $\frac{P-\pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} > z_\alpha$

and for a left-tailed test, the decision rule is

Reject H_0 in favor of H_A at $100\alpha\%$ level of significance if $\frac{P-\pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} < -z_\alpha$.

Example 16.8.1. For the following questions carry out the test of significance of population proportions at 5% level of significance (where x represents the number of things of particular category).

i) $H_0: \pi = 0.25,\ H_1: \pi \neq 0.25,\ n=100,\ x=40;$

ii) $H_0: \pi = 0.40\ H_1: \pi > 0.40,\ n=200,\ x=100;$

iii) $H_0: \pi = 0.30\ H_1: \pi < 0.30,\ n=400\ x=100.$

Solution: The statistic to be used for testing the given hypothesis is given by

$$Z = \frac{P-\pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \text{ where, } P \text{ is the estimate of proportion.}$$

(i) This is a two tailed test, so the decision rule is

Reject H_0 in favour of alternative at 5% level if $|Z| > z_{0.025}$ or, $|Z| > 1.96$.

Here, $n=100$, so, $P = \frac{40}{100} = 0.40$ and $\pi_0 = 0.25$, so the computed value of test statistic is

$$z = \frac{P-\pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} = \frac{0.40-0.25}{\sqrt{\frac{0.25\times(1-0.25)}{100}}} = 3.46.$$

Decision. $|Z|>1.96$, the computed value of Z falls in the critical region, so we fail to accept null hypothesis.

This is a right tailed test, so the decision rule is

Reject H_0 in favour of alternative at 5% level if $Z > z_{0.05} = 1.645$.

Here, $n=200$, so, $P=\frac{100}{200}=0.50$ and $\pi_0=0.40$, so the computed value of Z is

$$z = \frac{P-\pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} = \frac{0.50-0.40}{\sqrt{\frac{0.40\times(1-0.40)}{200}}} = 2.89.$$

Decision. The observed value of $Z>1.645$, which falls in the critical region, so we fail to accept null hypothesis.

This is a left tailed test, so the decision rule is

Reject H_0 in favour of alternative at 5% level if $Z<-z_{0.05}=-1.645$

Here, $n=400$, so, $P=\frac{100}{400}=0.25$ and $\pi_0=0.30$, so the computed value of Z is

$$z = \frac{P-\pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} = \frac{0.25-0.30}{\sqrt{\frac{0.30\times(1-0.30)}{400}}} = -2.18.$$

Decision. The o bserved value of $Z<-1.645$, which falls in the critical region, so we fail to accept null hypothesis.

ample 16.8.2. Forecasts of corporate earnings per share are made on a regular basis by many ancial analysts. In a random sample of 600 forecasts, it was found that 382 of these forecasts ceeded the actual outcome for earnings. Test against a two tailed alternative the null pothesis that the population proportion of forecasts that are higher than actual outcomes is 50 at 5% level of significance.

lution. Let π denotes the population proportion and P denotes the sample proportion of recasts that are above the actual outcomes. We are interested to test the hypothesis

$$H_0: \pi = 0.50, \text{ against the alternative } H_1: \pi \neq 0.50$$

The appropriate test statistic is: $Z = \dfrac{P-\pi_0}{\sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}}$.

The decision rule is: Reject H_0 in favour of alternative if $|Z|>z_{\alpha/2}$ or $|Z|>1.96$.

Here, $n=600$, $P=\dfrac{382}{600}=0.637$ and $\pi_0=0.50$.

So, the computed value of test statistic is:

$$z=\frac{P-\pi_0}{\sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}}=\frac{0.637-0.50}{\sqrt{\dfrac{0.50(1-0.50)}{600}}}=6.71.$$

Since 6.71 is much bigger than 1.96, the null hypothesis is clearly rejected. That means, forecasts of corporate earnings that exceed the actual values are significantly different from 0.50.

Example 16.8.3. A manufacturer claims that at least 95% of the equipments which he supplied to a factory conformed to the specification. An examination of the sample of 200 pieces of equipment revealed that 18 were faulty. Test the claim of manufacturer at 5% level of significance.

Solution. According to the statement of the problem, it is better if we consider a composite hypothesis such that at least 95% if the equipments supplied by the company conformed to the specification, that means:

$H_0: \pi \geq \pi_0 = 0.95$, against the alternative $H_1: \pi < 0.95$ (one tailed test).

The appropriate test statistic is: $Z=\dfrac{P-\pi_0}{\sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}}$

$\alpha=0.05$, the decision rule is : Reject H_0 in favour of alternative if $Z<-z_{\alpha/2}$ or $Z<-1.645$, (for this left-tailed test).

Here, $n=200$, out of 200, 18 were found faulty, that means $(200-18)=182$ equipments conform to the specification, so $P=\dfrac{182}{200}=0.91$ and $\pi_0=0.95$.

So, the computed value of test statistic is: $z=\dfrac{P-\pi_0}{\sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}}=\dfrac{0.91-0.95}{\sqrt{\dfrac{0.95(1-0.95)}{200}}}=-2.67.$

Since the observed value of $z=-2.67$ is less than the critical value -1.645, it lies in the critical region, so the null hypothesis is clearly rejected. That means, the proportion of equipments conforming to the specification is greater than 95%.

Example 16.8.4. An auditor claims that 10 percent of the customers' ledger accounts of a bank are carrying mistakes of posting and balancing. A random sample of 600 accounts was taken to test ccuracy of posting and balancing, and 45 accounts were found to have mistakes. Are these ple results consistent with the claim of auditor? (use 5% level of significance).

tion. Let us take the null that the claim of the auditor is valid, that means

$$H_0 : \pi \geq \pi_0 = 0.10, \text{ against the alternative } H_1 : \pi_0 \neq 0.10$$

The appropriate test statistic is: $Z = \dfrac{P - \pi_0}{\sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}}$.

$\alpha = 0.05$, the decision rule is: Reject H_0 in favour of alternative if

$$|Z| > z_{\alpha/2} \text{ or } |Z| > 1.96.$$

Here, $n = 200$, so, $P = \dfrac{45}{600} = 0.075$ and $\pi_0 = 0.10$.

So, the computed value of test statistic is: $z = \dfrac{P - \pi_0}{\sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}} = \dfrac{0.075 - 0.10}{\sqrt{\dfrac{0.10(1-0.10)}{600}}} = -2.049.$

Since the observed value of $|z| = 2.049$ is greater than the critical value 1.96, it lies in the ical region, so the null hypothesis is rejected at 5% level of significance. That means, the claim the auditor is not valid.

mple 16.8.5. Suppose machine produces 12% faulty items. A manufacturer of the same type of chine claims that their machine is better than this machine. In order to test the manufacturer's im, a random sample of 300 items were checked and 30 items were found to be faulty. On the is of the information, comment on the claim of manufacturer.

lution. The manufacturer's claim will be proved to be justified if it produces less proportion of ective items than the existing one. So, let us think that the existing machine is as better as the machine, and consider the null hypothesis as:

$$H_0 : \pi = \pi_0 = 0.12, \text{ against the alternative } H_1 : \pi_0 < 0.12 \text{ (one tailed test).}$$

The appropriate test statistic is: $Z = \dfrac{P - \pi_0}{\sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}}$.

Let $\alpha = 0.05$, the decision rule is: Reject H_0 in favour of alternative if

$$Z < -z_{\alpha/2} \text{ or } Z < -1.96.$$

Here, $n = 300$, so, $P = \dfrac{30}{300} = 0.10$ and $\pi_0 = 0.12$.

So, the computed value of test statistic is: $z=\dfrac{P-\pi_0}{\sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}}=\dfrac{0.10-0.12}{\sqrt{\dfrac{0.12(1-0.12)}{300}}}=-1.066.$

Since the observed value of $z=-1.066$ does not fall in the critical region, so we fail to the null hypothesis at 5% level of significance. That means, the claim of the manufacturer is justified and the new machine is as better as the existing one.

16.8.2. Test of hypothesis about difference between two population proportions. Let P_1, P_2 be the sample proportions obtained from large samples of sizes n_1 and n_2 from resp. population having proportions π_1 and π_2. We are interested to test the hypothesis that the no difference between the population proportions, that means,

$$H_0:\pi_1=\pi_2, \text{ against the alternative } H_1:\pi_1\neq\pi_2.$$

The sampling distribution of difference between sample proportions (P_1-P_2) is no with mean $E(P_1-P_2)=\pi_1-\pi_2$ and variance $\sigma_P^2=\dfrac{\pi_1(1-\pi_1)}{n_1}+\dfrac{\pi_2(1-\pi_2)}{n_1}$.

Since the sample sizes are large, the test statistic is: $Z=\dfrac{(P_1-P_2)-(\pi_1-\pi_2)}{\sqrt{\dfrac{\pi_1(1-\pi_1)}{n_1}+\dfrac{\pi_2(1-\pi_2)}{n_2}}}\sim N(0,1)$

However, since all tests are undertaken under null hypothesis, so if the null hypothesis true, then P_1 and P_2 are two independent unbiased estimators of the same population param $\pi_1=\pi_2=\pi$. Thus the best estimate of the common proportion π is the pooled proportions P two samples. The pooled estimate of π is the weighted mean of two sample proportions, g by:

$$P=\frac{n_1P_1+n_2P_2}{n_1+n_2}$$

The test statistic Z then becomes

$$Z=\frac{(P_1-P_2)-(\pi_1-\pi_2)}{\sqrt{P(1-P)\left[\dfrac{1}{n_1}+\dfrac{1}{n_2}\right]}}=\frac{(P_1-P_2)}{\sqrt{P(1-P)\left[\dfrac{1}{n_1}+\dfrac{1}{n_2}\right]}} \text{ under } H_0.$$

The decision rule is: For a two tailed test, reject H_0 in favor of H_1 at $100\alpha\%$ level of significance if

$$\frac{(P_1-P_2)}{\sqrt{P(1-P)\left[\dfrac{1}{n_1}+\dfrac{1}{n_2}\right]}}>z_{\alpha/2} \text{ or, } \frac{(P_1-P_2)}{\sqrt{P(1-P)\left[\dfrac{1}{n_1}+\dfrac{1}{n_2}\right]}}<-z_{\alpha/2} \text{ or, if } |Z|>z_{\alpha/2}.$$

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Similarly, for a right tailed test, the decision rule is : Reject H_0 in favor of H_1 at $100\alpha\%$ level of significance if $\dfrac{(P_1-P_2)}{\sqrt{P(1-P)\left[\dfrac{1}{n_1}+\dfrac{1}{n_2}\right]}} > z_\alpha$.

And for a left-tailed test, the decision rule is : Reject H_0 in favor of H_1 at $100\alpha\%$ level of significance if

$$\frac{(P_1-P_2)}{\sqrt{P(1-P)\left[\dfrac{1}{n_1}+\dfrac{1}{n_2}\right]}} < z_\alpha.$$

Example 16.8.6. A company is considering two different television advertisements for promotion of a new product. Manager believes that advertisement A is more effective than advertisement B. Two test market areas with virtually identical consumer characteristics are selected: advertisement A is used in one area and advertisement B is used in other area. In a random sample of 60 customers who saw the advertisement A, 18 had tried to buy the product, on the other hand, in a random sample of 100 consumers who saw advertisement B, 22 had tried to buy the product. Does this indicate that the advertisement A is more efficient than advertisement B, if level of significance is 5%?

Solution. Let π_1 and π_2 be the population proportions of customers who had tried to buy the products after seeing the advertisement A and advertisement B respectively, then we consider the null hypothesis as both advertisements are equally effective, that means

$H_0: \pi_1 = \pi_2$, against the alternative $H_1: \pi_1 > \pi_2$.

(One tailed test, because, advertisement A will be considered more effective if proportion of customers who had tried to buy in this case is more than that of advertisement B).

Under the null hypothesis, the appropriate test statistic is

$$Z=\frac{(P_1-P_2)}{\sqrt{P(1-P)\left[\dfrac{1}{n_1}+\dfrac{1}{n_2}\right]}}$$

where, P is the pooled estimate of proportion.

Given, $\alpha = 0.05$ and it is a right-tailed test, so the decision rule is: Reject H_0 in favour of H_1 if $Z > z_{0.05}$ or $Z > 1.645$.

Here, $n_1 = 60$, proportions $P_1 = \dfrac{18}{60} = 0.30$ and $n_2 = 100$, $P_2 = \dfrac{22}{100} = 0.22$ and the pooled estimate of proportion is

$$P = \frac{n_1P_1+n_2P_2}{n_1+n_2} = \frac{60 \times 0.30 + 100 \times 0.22}{60+100} = 0.25.$$

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Thus, $$z = \frac{0.30 - 0.22}{\sqrt{0.25(1-0.25)\left\{\frac{1}{60}+\frac{1}{100}\right\}}} = 1.131.$$

Since, observed value z is less than the critical value 1.645, we fail to reject the null hyp... at 5% level of significance.

Hence, we can conclude that there is no significant difference in the effectiveness of th... advertisements.

Example 16.8.7. 800 units from factory A are inspected and 12 are found to be defective, 500 ... from factory B are inspected and 12 are found to be defective. Can it be concluded at 5% le... significance that production at factory A is better than factory B?

Solution. Let π_1 and π_2 be the population proportions of defectives of factory A and fac... respectively, then we consider that null hypothesis that performance of both factories a... same, that means,

$$H_0 : \pi_1 = \pi_2, \text{ against the alternative } H_A : \pi < \pi_2,$$

(it is a one tailed test, because, factory A can be considered as better if the proporti... defectives found in factory A is less than the proportion of defectives found in factory B).

The appropriate test statistic is: $$Z = \frac{(P_1 - P_2)}{\sqrt{P(1-P)\left[\frac{1}{n_1}+\frac{1}{n_2}\right]}}$$

where, P_1 and P_2 are the sample proportions, and P is the pooled estimate of propo... given by

$$P = \frac{n_1P_1 + n_2P_2}{n_1 + n_2}.$$

Given, $\alpha = 0.05$, and it is a left-tailed test, so the decision rule is: Reject H_0 in favour of ... $Z < -z_{0.05}$ or, $Z < -1.645$

Here, $n_1 = 800$, $P_1 = \frac{12}{800} = 0.015$ and $n_2 = 500$, $P_2 = \frac{12}{500} = 0.024$ and the pooled estima... proportion

$$P = \frac{n_1P_1 + n_2P_2}{n_1 + n_2} = \frac{800\times 0.015 + 500\times 0.024}{800+500} = 0.018.$$

Thus, $$z = \frac{0.015 - 0.024}{\sqrt{0.018(1-0.018)\left\{\frac{1}{800}+\frac{1}{100}\right\}}} = -1.184.$$

Since, observed values of Z is not less than the critical value -1.645, we fail to reject the ... hypothesis, that means the null hypothesis holds good at 5% level of significance.

Hence, we cannot conclude that the production at factory A is better than B.

Example 16.8.8. In a random sample of 700 workers from a particular factory of Bangladesh 200 are found to be smokers. In another factory out of 1300 workers 400 were found to be smokers. Can you conclude that there is a significant difference between the two factories with regard to the smoking habit?

Solution. Let π_1 and π_2 be the population proportions of smokers in two factories respectively. Let us take the hypothesis that there is no difference in smoking habits in the two factories, and then the null hypothesis is

$$H_0 : \pi_1 = \pi_2, \text{ against the alternative } H_1 : \pi_1 \neq \pi_2.$$

The appropriate test statistic is: $Z = \dfrac{(P_1 - P_2)}{\sqrt{P(1-P)\left[\dfrac{1}{n_1} + \dfrac{1}{n_2}\right]}}$

where, P_1 and P_2 are the sample proportions, and P is the pooled estimate of proportion is given by $P = \dfrac{n_1P_1 + n_2P_2}{n_1 + n_2}$.

Let $\alpha = 0.05$, and it is a two-tailed test, so the decision rule is: Reject Ho in favour of H_1 if $|Z| > 1.96$.

Here, $n_1 = 700$, $P_1 = \dfrac{200}{700} = 0.2857$ and $n_2 = 500$, $P_2 = \dfrac{400}{1300} = 0.3077$ and

the pooled estimate of proportion: $P = \dfrac{n_1P_1 + n_2P_2}{n_1 + n_2} = \dfrac{200400}{700+1300} = 0.30.$

Thus, $z = \dfrac{0.2857 - 0.3077}{\sqrt{0.30(1-0.30)\left\{\dfrac{1}{700} + \dfrac{1}{1300}\right\}}} = -1.023.$

Since the observed absolute value of Z is less than 1.96, there is no evidence to doubt the hypothesis that means, workers of two factories do not differ significantly with respect to the smoking habit.

16.9. Test of Hypothesis about Correlation Co-efficient

Suppose (x_1, y_1), (x_2, y_2), ..., (x_n, y_n) be n pairs of observations of a random sample drawn from a bi-variate normal population with correlation co-efficient ρ, then the sample correlation co-efficient between n pairs of observations is given by:

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$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2\sum(y-\bar{y})^2}} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\left\{\sum x^2 - \frac{(\sum x)^2}{n}\right\}\left\{\sum y^2 - \frac{(\sum y)^2}{n}\right\}}}$$

The significance of population correlation co-efficient from which the sample has been drawn may be tested under following two assumptions:

(i). $H_0: \rho = 0$, when the population correlation co-efficient is zero (with one-tailed or two-tailed alternative),

(ii) $H_0: \rho = \rho_0$, when the population correlation co-efficient is equal to some specified value ρ_0 (with one-tailed or two-tailed alternative).

16.9.1. Testing the hypothesis when the population correlation co-efficient equal zero. Here the null hypothesis is considered as there is no correlation in the population that means, the relationship between the variables is not linear. The test is undertaken considering the following null hypothesis $H_0: \rho = 0$. (in this case usually a two-tailed test $H_0: \rho \neq 0$ is conducted however, one-tailed test is also conducted when the situation deserves)

Let r be the sample correlation co-efficient (which is the best estimate of population correlation co-efficient ρ). It is to be noted here that, for a sample of size n, the variance of r is given by $\text{var}(r) = \sqrt{\frac{1-r^2}{n-2}}$, then the appropriate test statistic to be used for testing the above mentioned hypothesis is:

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \sim t_{n-2}$$

which follows t-distribution with $n-2$ degrees of freedom.

Thus, if the computed value of r is greater than the tabulated value of t at 100α% level of significance, then the null hypothesis is rejected that means the decision rule is

$$\text{Reject } H_0 \text{ if } |t| > t_{n-2;\alpha/2} \text{ or, } \left|\frac{r\sqrt{n-2}}{\sqrt{1-r^2}}\right| > t_{n-2;\alpha/2}$$

which indicates that the sample data provide sufficient evidence that $\rho \neq 0$.

However, an approximate 'rule of thumb' for considering population correlation co-efficient to be significantly different from zero is given by

$$|r| > \frac{2}{\sqrt{n}}.$$

16.9.2. Testing the hypothesis when the population correlation co-efficient equal some specified value ρ_0. Let us consider the following null and alternative hypothesis

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