If p-value is between 0.05 and 0.10, the results are regarded as only tending toward statistical significance.

If p-value is greater than 0.10, the results are regarded as not statistically significant.

Example:

Suppose that the null hypothesis and its alternative are respectively

$$H_0: \mu = 55$$

$$H_1: \mu \neq 55$$

and the sample observations yielded a mean $\bar{x} = 50$ and the standard error of the mean was computed to be 2. The calculated value of z is

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{50 - 55}{2} = -2.5$$

The p-value is determined using the standard normal table. Looking at the table, we see that probability of observing a $z \leq -2.5$ or $z \geq 2.5$ when the null hypothesis is true is $P(z \leq -2.5) + P(z \geq 2.5)$

$= 0.0062 + 0.0062 = 0.0124$, which is

the p-value. Since the p-value is less than $\alpha = 0.05$ the calculated z is significant ($p < 0.05$) and the null hypothesis is rejected at 5% level. Notice that the two-tailed test p-value is actually twice the tail area corresponding to the calculated value of the test statistic. For two-tailed test, the p-value is actually the area to the right or left tail area of the calculated value of the statistic, depending on the direction of the alternative.

Confidence interval:

Def^n: A confidence interval for some unknown parameter is an interval which consists of two numerical values defining a range of values such that the parameter may be expected to be included by the interval with a specified degree of confidence.

If the ~~interval~~ interval includes the parameter with probability 0.95, we call the interval a 95% confidence interval and the limits defining the interval are called the confidence limits.

To obtain a confidence interval for some parameter we draw a random sample from the population,

calculate the corresponding statistic from the sample and then construct the confidence interval from the knowledge of the distribution of that statistic.

When the population is assumed to be normally distributed and the population standard deviation σ is known, we can calculate the sample mean $\bar{x}$ from a sample size n to construct the confidence interval for the population mean μ. For example, a $100(1-\alpha)\%$ confidence interval for the population mean μ is given by

$$\bar{x} - Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Here $Z_{\alpha/2}$ is the value of the standard normal distribution leaving an area of $\alpha/2$ to the right. For $\alpha = 0.05$, $Z_{\alpha/2} = 1.96$ and the interval referred to as above is called the 95% confidence interval for the population mean μ.

The quantity $1-\alpha$ is called the confidence coefficient while the quantities $\bar{x} - Z_{\alpha/2}(\sigma/\sqrt{n})$ and $\bar{x} + Z_{\alpha/2}(\sigma/\sqrt{n})$ are called lower confidence limit and upper confidence limit, respectively.

Example:

The mean and standard deviation of marks obtained from a sample of 36 students of IUT were 2.6 and 0.3 respectively. Find the 95% confidence interval for the entire group of the students of IUT.

Soln:

Given, the sample mean, $\bar{x} = 2.6$ & $\alpha = 0.05$

Sample standard deviation, $\sigma_{\bar{x}} = 0.3$

Also, we have $Z_{\alpha/2} = 1.96$

Hence, the 95% confidence interval is

$$2.6 - (1.96)\left(\frac{0.3}{\sqrt{36}}\right) < \mu < 2.6 + (1.96)\cdot\left(\frac{0.3}{\sqrt{36}}\right)$$

$$\Rightarrow \quad 2.5 < \mu < 2.7$$

Thus we may be confident 95% that the true mean μ will be between 2.5 and 2.7.

Note:

The importance of estimating confidence interval can never be over emphasized. It places a bound on the error of estimation of the population parameter and hence reflects the reliability of the estimate.

or non-normal populations provided $n \geq 30$ and in that instance, the test statistic is modified to

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

where is the standard deviation calculated from the sample data. As before, z is a $N(0,1)$ under H_0.

The decision rule for one-sided test with $H_1: \mu < \mu_0$ is to reject H_0 if $\bar{x} < \mu_0 - z_\alpha (\sigma/\sqrt{n})$.

For $H_1: \mu > \mu_0$, reject H_0 if $\bar{x} > \mu_0 + z_\alpha (\sigma/\sqrt{n})$.

In both the cases, the critical region lies entirely on one side of the distribution.

<u>Example:</u>

A bulb manufacturing company claims that the average longevity of their bulb is 3.65 years with a standard deviation of 0.16 years. A random sample of 36 bulbs gave a mean longevity of 3.45 years. Does the sample mean justify the claim of the manufacturer? Use a 5% level of significance.

<u>Sol^n:</u>

The test is a two-tailed one with

① $H_0: \mu = 3.65$ against the alternative $H_1: \mu \neq 3.65$

② $\alpha = 0.05$

③ Critical region: $z < -1.96$ or $z > 1.96$, where

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

④ $\bar{x} = 3.45$, $n = 36$, $\mu_0 = 3.65$ and $\sigma = 0.16$

⑤ $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{3.45 - 3.65}{0.16/\sqrt{36}} = -7.5$$

⑥ Conclusion: Since the computed value of the statistic (-7.5) is less than the critical value (-1.96), the result is highly significant $(p < 0.01)$. Hence we reject the null hypothesis and conclude that the mean longevity of the bulb is not 3.65 years; but in fact less than this.

<u>Example</u>:

The mean and standard deviation of GPA scores obtained from a random sample of 40 students of IUT were 2.8 and 0.35, respectively. Can we conclude that the sample has come from the entire group of students which has a mean score of 2.4? Use $\alpha = 0.01$.

Compute 99% confidence ~~least~~ interval for the mean score in the population.

Solⁿ:

(1) $H_0: \mu = 2.4$ against the alternative $H_1: \mu \neq 2.4$

(2) $\alpha = 0.01$

(3) Critical region: $z < -2.58$ or $z > 2.58$, where

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

(4) $\bar{x} = 2.8$, $n = 40$, $\mu_0 = 2.4$ and since sample size is large, the standard deviation σ can be approximated by its estimate $\sigma = 0.35$

(5) $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{2.8 - 2.4}{0.35/\sqrt{40}} = 7.22$$

(6) Conclusion: Since the computed z-value exceeds the critical value (~~1.96~~ 2.58), we can reject the ~~null~~ null hypothesis and conclude that the mean score in the population is not as claimed.

The 99% confidence interval for the population mean μ is

$$\bar{x} - z_{\alpha/2}(\sigma/\sqrt{n}) < \mu < \bar{x} + z_{\alpha/2}(\sigma/\sqrt{n})$$

$$\Rightarrow 2.8 - 2.58\left(\frac{0.35}{\sqrt{40}}\right) < \mu < 2.8 + 2.58\left(\frac{0.35}{\sqrt{40}}\right)$$

$$\Rightarrow 2.66 < \mu < 2.94$$

<u>Example:</u>

The average IQ of working women in Bangladesh is suspected to be more than 110, on average. A random sample of 64 of such women yielded an average IQ of 115.5 and a standard deviation of 20. Can you conclude that the average score of the women in the population is really more than 110? Test this at 5% level of significance.

<u>Sol^n:</u>

The test is a one-tailed test with

(1) $H_0: \mu = 110$ against the alternative $H_1: \mu > 110$

(2) $\alpha = 0.05$

(3) Critical region: $z > 1.64$, where $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

(4) $\bar{x} = 115.5$, $n = 64$, $\mu_0 = 110$, $\sigma = 20$

(5) $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{115.5 - 110}{20/\sqrt{64}} = 2.2$

(6) Conclusion: The computed z-value is significant at 5% level ($P < 0.05$). Hence we reject the null hypothesis and accept the alternative: "the mean of working women is higher than the average (110) for women in the population".

Confidence limits computation shows that the true mean lies between 111.4 & 119.6.

Example:

An internet server claims that its users spend on the average 20 hours per week with a standard deviation of 2.5 hours on the information superhighway. To determine whether this is an overstatement, a competitor conducted a sample survey of 15 customers and found that the average time spent online was 21.8 hours per week. Do the data provide sufficient evidence to indicate that the average hours of use are less than that claimed by the first internet server? Test at 1% level of significance.

Sol^n:

This is a one-tailed test.

It is appropriate here to formulate a hypothesis with alternative hypothesis $H_1: \mu < 20$. The null hypothesis is $H_0: \mu = 20$. The compulation steps are as follows

(1) $H_0: \mu = 20$ against the alternative $H_1: \mu < 20$

(2) $\alpha = 0.01$

(3) Critical region: $z < -2.33$, where $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

(4) $\bar{x} = 21.8$; $n = 15$; $\mu_0 = 20$, & $\sigma = 2.5$

(5) $Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{21.8 - 20}{2.5/\sqrt{15}} = 2.79$

(6) <u>Conclusion</u>: The computed z-value is greater than the critical value. We cannot reject the null hypothesis and conclude that there is sufficient evidence to agree with the claim of the first internet server.

<u>Example</u>:

The weight of a sample of 40 capsules is to be used to examine whether there is sufficient

empirical evidence that the capsules of this type have a mean weight less than 10g. The sample gave a mean weight of 9.77g with a standard deviation 0.50. Perform an appropriate statistical test to provide an evidence in support of the claim at 1% level of significance.

Sol^n:

The appropriate test statistic here is the normal test with $H_0: \mu = 10$ against the alternative $H_1: \mu < 10$

The computational steps are as follows:

(1). $H_0: \mu = 10$ against the alternative $H_1: \mu < 10$

(2) $\alpha = 0.01$

(3) Critical region: $z < -2.33$, where $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

(4) $\bar{x} = 9.77$; $n = 40$; $\mu_0 = 10$; $\sigma = 0.50$

(5) $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{9.77 - 10}{0.50/\sqrt{40}} = -2.91$$

(6) Conclusion: The observed value of the test statistic z is less than the critical value. The data therefore provide sufficient evidence to reject the null hypothesis in support of the alternative.

Example:

A random sample of 64 observations from a normal population provided a standard deviation of 20. Find the critical values of the test for which the null hypothesis $H_0 : \mu = 100$ against the alternative $H_1 : \mu \neq 100$ will be rejected at 5% level of significance. Determine also the acceptance region.

Sol^n:

Since the sample is large and σ can be replaced by $s = 20$, the appropriate test statistic is normal. The other required values are:

$$\mu_0 = 100, \quad n = 64, \quad Z_{\alpha/2} = 1.96 \quad \text{[Since two-tailed test]}$$

The critical values are

$$\bar{x}_1 = \mu_0 - Z_{\alpha/2}(s/\sqrt{n}) = 100 - 1.96(20/\sqrt{64}) = 95.1$$

$$\& \quad \bar{x}_2 = \mu_0 + Z_{\alpha/2}(s/\sqrt{n}) = 100 + 1.96(20/\sqrt{64}) = 104.9$$

The critial regions are

$$\bar{x} \geq 104.9 \quad \& \quad \bar{x} \leq 95.1$$

Hence, the acceptance region is

$$95.1 < \bar{x} < 104.9$$

A guideline to take decision for normal test statistic.

A statistical test may be attempted at several levels of significance, the most common levels being 1% and 5%. We provide below a table to guide you to choose between the null hypothesis $H_0: \mu = \mu_0$ and each of the alternative hypothesis $H_0: \mu \neq \mu_0$ at two different levels of significance, i.e., $\alpha = 0.05$ and $\alpha = 0.01$.

	Decision Rule	
H_1	$\alpha = 0.05$ Reject H_0 iff	$\alpha = 0.01$ Reject H_0 iff
$\mu > \mu_0$	$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \geq 1.64$	$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \geq 2.33$
$\mu < \mu_0$	$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \leq -1.64$	$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \leq -2.33$
$\mu \neq \mu_0$	$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \geq 1.96$ or, $Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \leq -1.96$	$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \geq 2.58$ or $Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \leq -2.58$

Statistic.

Inferences about the difference between two means:

[I] Independent samples:

The need to use two independent samples tests is often encountered in many research areas. For example —

We might compare the smoking habit of a sample of women with that among the men to see if there is any difference between sexes with respect to smoking habit.

We might compare the purchasing predisposition of a sample of subscribers from two magazines to discover if they are from the same population.

A manager of an industrial firm may study the workers' productivity from two groups or two different samples.

A nutritionist might be interested to study if the consumption of fatty foods have declined over the last ten years.

Let us consider that the first population, from which a sample of size n_1 has been drawn, has an unknown mean μ_1 and a known variance σ_1^2.

For the second sample of size n_2, the corresponding population parameters are μ_2 and σ_2^2, respectively. If $\bar{x}_1$ and $\bar{x}_2$ are the sample means for the first and second samples, respectively, then the difference between the sample means $\bar{x}_1 - \bar{x}_2$ will have a normal distribution with mean $\mu_1 - \mu_2$ and variance $\sigma_1^2/n_1 + \sigma_2^2/n_2$.

Under the normality and independence assumption, the quantity z is defined as

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

will have a standard normal distribution.

Testing the equality of two population means:

To test the null hypothesis of no difference between two population means with two-tailed alternatives, i.e., $H_0: \mu_1 - \mu_2 = 0$ against the alternative $H_1: \mu_1 - \mu_2 \neq 0$, the test statistic is

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Under the null hypothesis, z follows a standard normal distribution.

The decision rule is to reject the null hypothesis if $|z| > z_{\alpha/2}$ for a level of significance α. With one-sided alternative we reject the null hypothesis if $z > z_{\alpha}$ when the alternative hypothesis is $H_1: \mu_1 > \mu_2$. If $H_1: \mu_1 < \mu_2$, the decision rule is to reject H_0 if $z < -z_{\alpha}$.

The use of the test procedures just described assumes that the populations are normal with known but unequal variances and further that the samples have been drawn independently from the populations.

<u>Example:</u>

Suppose we want to see if there is any difference between the average number of children born to the tribal women and the non-tribal women of a certain Division in Bangladesh. A random sample of 12 women from tribal population and 15 women from non-tribal population were drawn and their number of children recorded. The average number

of children born to these two groups of women were respectively 4.5 and 3.4. The population variances in the number of children computed from the census data were 1 and 1.5 respectively. Use 5% level of significance to see if the sample data reflect any difference in the mean number of children born in the population

Sol^n:

Since the variances are known, the normal test could well be applied. The computation steps are as follows:

(1) $H_0: \mu_1 = \mu_2$ against the alternative $H_1: \mu_1 \neq \mu_2$

(2) $\alpha = 0.05$

(3) Critical region: (a) $z < -1.96$ (b) $z > 1.96$

where $z = \dfrac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

(4) $\bar{x}_1 = 4.5$; $\bar{x}_2 = 3.4$; $n_1 = 12$; $n_2 = 15$;
$\sigma_1^2 = 1$; $\sigma_2^2 = 1.5$

(5) $z = \dfrac{4.5 - 3.4}{\sqrt{\dfrac{1}{12} + \dfrac{1.5}{15}}} = 2.57$

(6) <u>Conclusion</u>: The H_0 is rejected at 5% level and hence we may conclude that there is a difference between the true average number of children born to tribal women and non-tribal women.

- If the variances are unknown but the samples are large (at least 30 in each case), σ_1^2 and σ_2^2 can be replaced by s_1^2 and s_2^2, respectively. Then the test statistic is modified as follows:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

- This statistic is approximately normally distributed with zero mean and unit standard deviation. The result remain valid even for non-normal population so long as sample sizes are large. The decision rule regarding the significance of the values remain the same.

The 95% confidence interval for the difference $\mu_1 - \mu_2$ is

$$(\bar{x}_1 - \bar{x}_2) - 1.96\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + 1.96\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Example:

The sample data show that 120 adult males born in rural area have a mean height of 62.7 inches with a standard deviation of 2.50 inches, and that 150 adult males born in the urban area have a mean height of 61.8 inches with a standard deviation of 2.62 inches. Test the hypothesis that the mean heights in the two areas from which the samples have been drawn do not differ. Compute also the 95% confidence interval for the difference in population means.

Sol^n:

This is a two-tailed test. The computation steps are as follows:

(1) $H_0: \mu_1 = \mu_2$ against the alternative $H_1: \mu_1 \neq \mu_2$

(2) $\alpha = 0.05$

(3) Critical region: (a) $z < -1.96$ (b) $z > 1.96$,

where $$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

(4) $\bar{x}_1 = 62.7$; $\bar{x}_2 = 61.8$; $n_1 = 120$; $n_2 = 150$;

$s_1 = 2.50$; $s_2 = 2.62$

(5) $$z = \frac{62.7 - 61.8}{\sqrt{\frac{(2.50)^2}{120} + \frac{(2.62)^2}{150}}} = 2.87.$$

(6) <u>Conclusion</u>: The H_0 is rejected at 5% level. In other words, the sample data show that there is a difference between the true average heights of adult males in two areas.

The 95% confidence interval for the difference between two means is

$$(62.7 - 61.8) \pm 1.96\sqrt{\frac{(2.50)^2}{120} + \frac{(2.62)^2}{150}}$$

or $$0.92 \le \mu_1 - \mu_2 \le 1.51.$$

<u>Example</u>:

It has been observed that in Bangladesh, the proportion of women who are delivered through Caesarean sections is very high. A study is therefore conducted to investigate why this is the case. As small height is known to be one

of the risk factors related to difficult deliveries, the investigator may want to find out if there is a difference between the mean height of women in Bangladesh who had normal deliveries and of those who had Caesarean sections. The data were as follows:

Type of delivery	No. of Women	Mean height in (cm)	Standard deviation
Normal	60	156	3.1
Caesarean	52	154	2.8

Test the hypothesis that there is no difference between the mean heights of the two groups of women. Use a 5% level of significance.

<u>Solution:</u>

Since the sample sizes are large, we employ a normal test and use the sample standard deviations as estimates of population variance. The given problem is a two-tailed test. The computation steps are as follows:

(1) $H_0 : \mu_1 = \mu_2$ against the alternative $H_1 : \mu_1 \neq \mu_2$

(2) $\alpha = 0.05$

(3) Critical region: (a) $Z < -1.96$, (b) $Z > 1.96$,

where $$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

(4) $\bar{x}_1 = 156$; $\bar{x}_2 = 154$; $n_1 = 60$, $n_2 = 52$,

$s_1 = 3.1$; $s_2 = 2.8$;

(5) $$Z = \frac{156 - 154}{\sqrt{\frac{(3.1)^2}{60} + \frac{(2.8)^2}{52}}} = 3.6$$

(6) <u>Conclusion</u>: The computed value of the statistic $Z = 3.6$ exceeds the critical value ($Z = 1.96$). Thus we reject the null hypothesis and conclude that the observed difference of 2 cm between the mean heights of women with normal deliveries and women with Caesarean sections is a statistically significant difference. Alternatively, we can say that the probability that the observed difference of 2 cm

between the two groups is due to chance is less than 5%.

Example:

It is desired to investigate whether male and female file clerks in a city earn comparable wages. The sample data show that 60 male typists earn on the average TK. 158.50 per week with a standard deviation of TK. 18.20, while 60 female typists earn TK. 141.60 per week with a standard deviation of TK. 20.60. Test the null hypothesis $\mu_1 = \mu_2$ against the alternative $\mu_1 > \mu_2$ at the 5% and 1% level of significance.

Solⁿ:

This is a one-tailed test with two different level of significance. The computation steps are as follows:

(1) $H_0: \mu_1 = \mu_2$ against the alternative $H_1: \mu_1 > \mu_2$

(2) (i) $\alpha = 0.05$; (ii) $\alpha = 0.01$

(3) Critical region: (i) $z > 1.645$

(ii) $z > 2.33$; where

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

(4) $\bar{x}_1 = 158.50$; $\bar{x}_2 = 141.60$; $n_1 = 60$; $n_2 = 60$;

$s_1 = 18.20$; $s_2 = 20.60$;

(5) $$Z = \frac{158.50 - 141.60}{\sqrt{\frac{(18.2)^2}{60} + \frac{(20.6)^2}{60}}} = 4.76$$

(6) <u>Conclusion</u>: The difference is significant at both the levels of significance. In the given problem the city male typists have on the average higher earnings than their female counterparts.