Artificial Intelligence

CSE 4617

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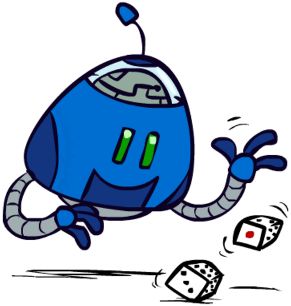
Islamic University of Technology

Uncertain Outcomes



Recap: Probabilities

- **Random variable** → Event whose outcome is unknown
- **Probability distribution** → Assignment of weights to outcomes
- Example: Traffic on freeway
  - Random variable: $T =$ whether there's traffic
  - Outcome: $T \in$ {none, light, heavy}
  - Distribution: $P(T =$ none$) = 0.25$, $P(T =$ light$) = 0.50$, $P(T =$ heavy$) = 0.25$
- Some laws of probability (more later):
  - Non-negative
  - Sum of probabilities over all possible outcomes: 1
- As we get more evidence, probabilities may change:
  - $P(T =$ heavy$) = 0.25$, $P(T =$ heavy$|H =$ 8 a.m.$) = 0.60$

0.25

0.50

0.25

Recap: Expectations

- Expected value of a function of random variable
- Average, weighted by the probability distribution over outcomes

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- Expected value of a function of random variable
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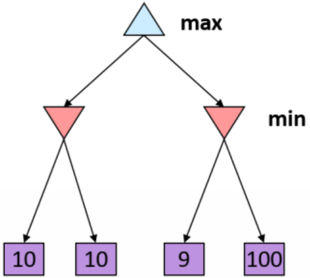
- Example: How long to get to the airport?

Time:	20 min	30 min	60 min
Probability:	0.25	0.50	0.25



Worst-Case vs. Average Case



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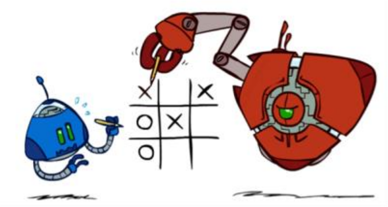

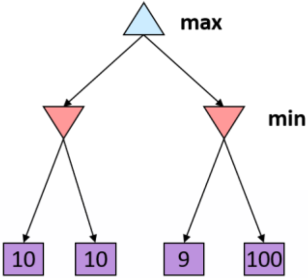

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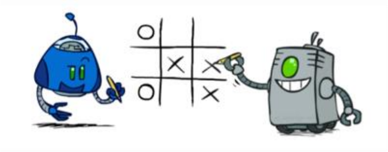

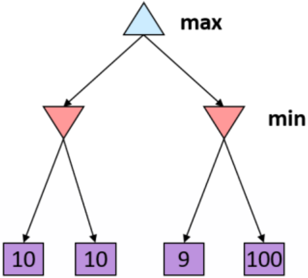

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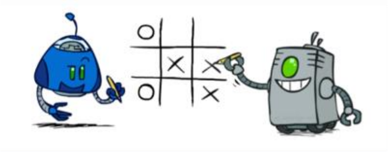

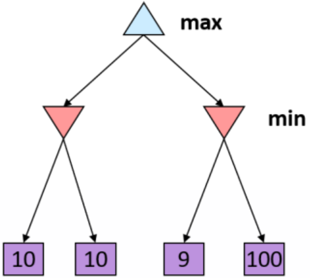

Idea: Uncertain outcomes controlled by chance, not an adversary!

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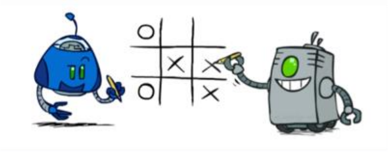

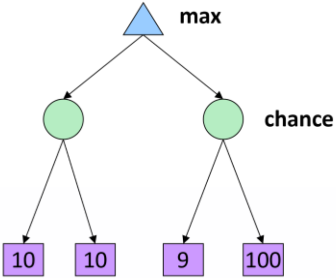

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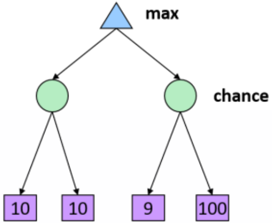
Expectimax Search

- Why wouldn't we know what the result of an action will be?



Video: minimax, expectimax

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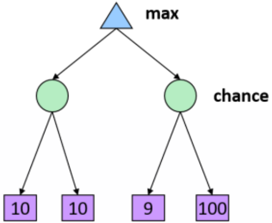

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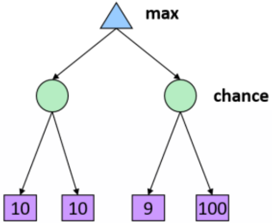

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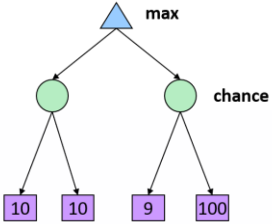

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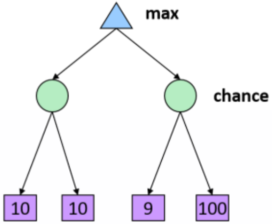

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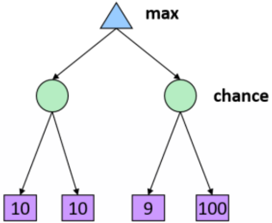
- Values should now reflect average-case (expectimax) outcomes, not worst-case (minimax) outcome
- Compute the average score under optimal play
  - Max nodes as in minimax search
  - Chance nodes are like min nodes but the outcome is uncertain
  - Calculate their **expected utilities**
    i.e. take weighted average (expectation) of children



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Expectimax Pseudocode

def value(*state*):
  if the state is a terminal state: return the state's utility
  if the next agent is MAX: return max-value(*state*)
  if the next agent is EXP: return exp-value(*state*)

def max-value(*state*):
  initialize $v = -\infty$
  for each successor of *state*:
    $v = \max(v, value(successor))$
  return v

def exp-value(*state*):
  initialize $v = 0$
  for each successor of *state*:
    $p = probability(successor)$
    $v \mathrel{+}= p \times value(successor)$
  return v

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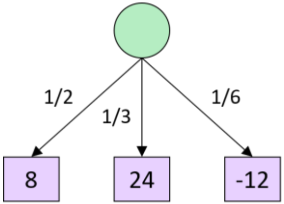
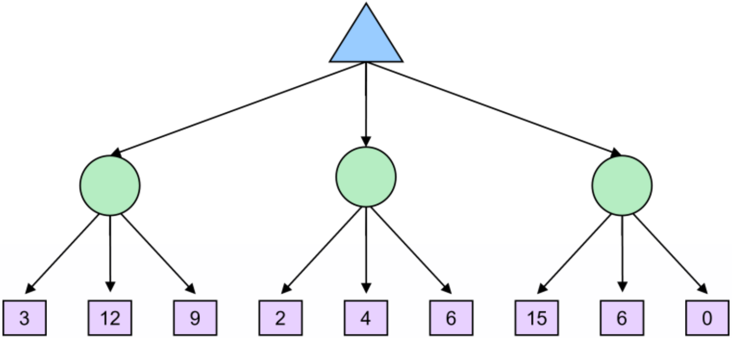
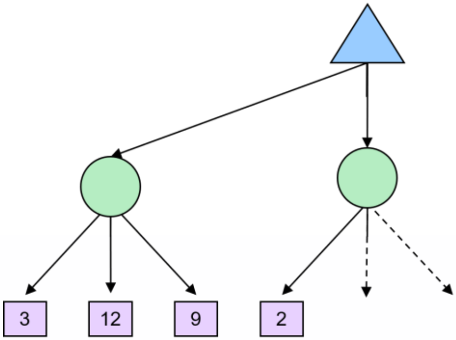


Expectimax Quiz



Expectimax Pruning?



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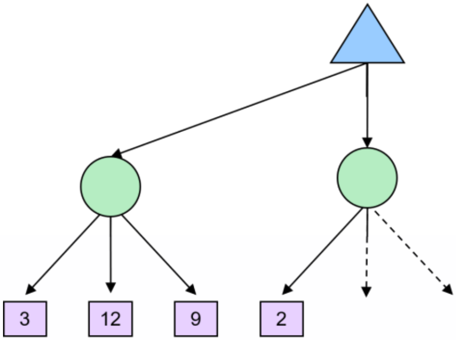
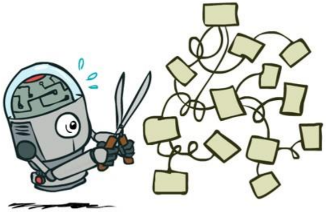
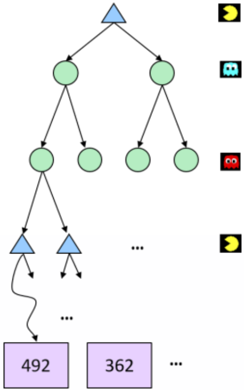




Depth-Limited Expectimax



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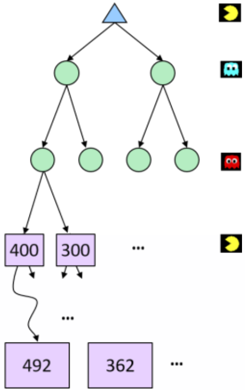

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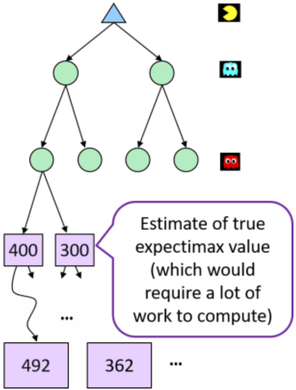
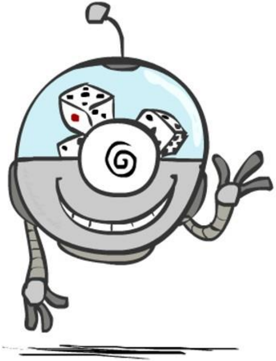
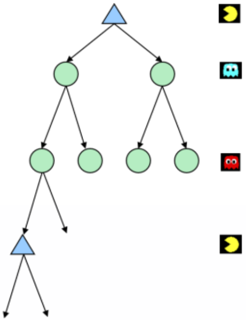


Probabilities



What Probabilities to Use?

- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated and require a great deal of computation
  - We have a chance node for any outcome out of our control: opponent or environment
  - The model might say that adversarial actions are likely!
- For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes



Having a probabilistic belief about another agent's action does not mean that the agent is flipping any coins!

Informed Probabilities

- Let's say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise
- Question: What tree search should you use?

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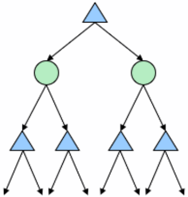

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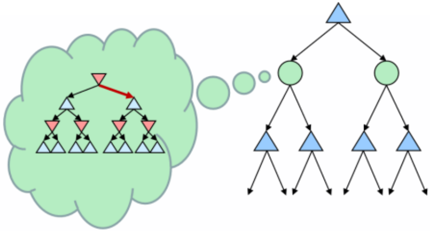

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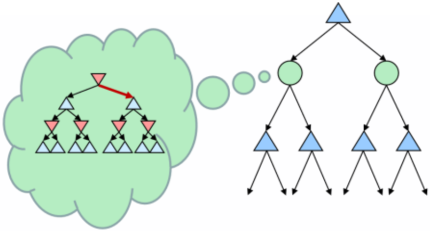

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  - This kind of things gets very slow very quickly
  - Even worse if you have to simulate your opponent simulating you...

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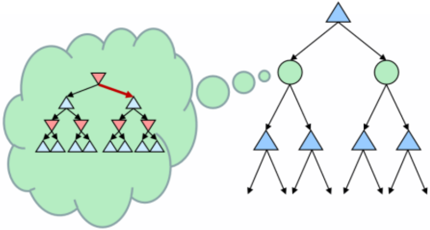

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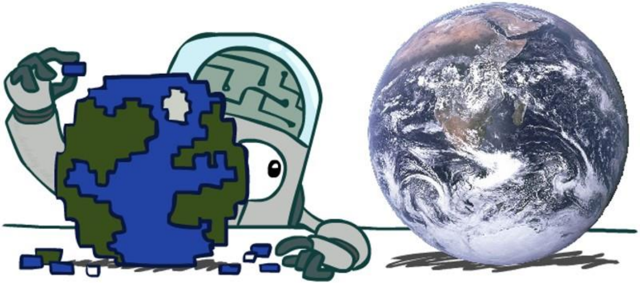
  - This kind of things gets very slow very quickly
  - Even worse if you have to simulate your opponent simulating you…
  - …except for minimax, which has the nice property that it all collapses into one game tree.

Modeling Assumptions



The Dangers of Optimism and Pessimism

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Dangerous Optimism

Assuming chance when the world is adversarial

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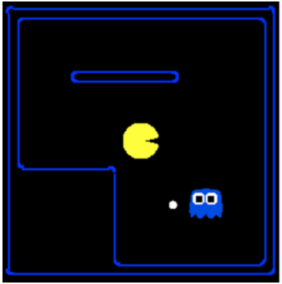
Dangerous Optimism
Assuming chance when the world is adversarial

Dangerous Pessimism
Assuming the worst case when it's not likely

Assumptions vs. Reality

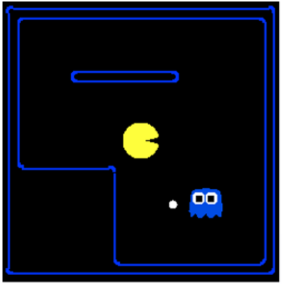


	Adversarial Ghost	Random Ghost
Minimax Pacman		
Expectimax Pacman		

Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

Videos: randGhostExpPac, advGhostMiniPac, miniGhostExpPac, randGhostMiniPac

Assumptions vs. Reality



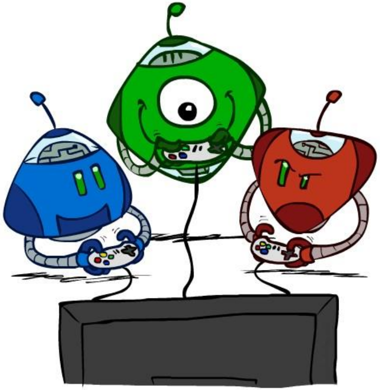
	Adversarial Ghost	Random Ghost
Minimax Pacman	Won 5/5 Avg. Score: 483	Won 5/5 Avg. Score: 493
Expectimax Pacman	Won 1/5 Avg. Score: -303	Won 5/5 Avg. Score: 503

Results from playing 5 games

Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

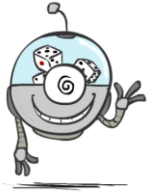
Videos: randGhostExpPac, advGhostMiniPac, miniGhostExpPac, randGhostMiniPac

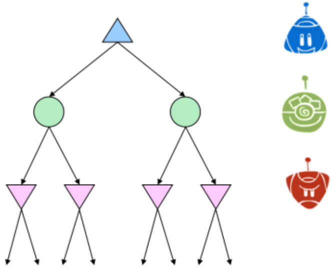
Other Game Types



Mixed Layer Types

- E.g. Backgammon
- Expectiminimax
  - Environment is an extra "random agent" player that moves after each min/max agent
  - Each node computes the appropriate combination of its children





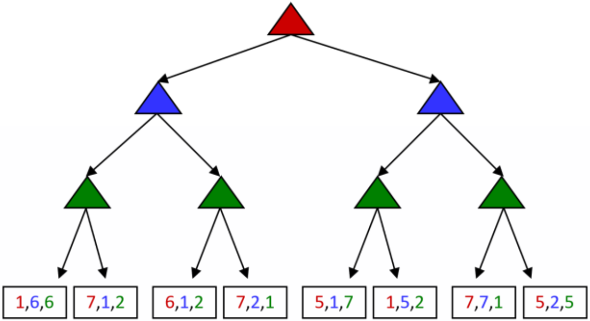
Example: Backgammon

- Dice rolls increase b: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
  - Depth 2 = $20 \times (21 \times 20)^3 = 1.2 \times 10^9$
- As depth increases, probability of reaching a given search node shrinks
  - Usefulness of search is diminished
  - Limiting depth is less damaging
  - Pruning is trickier...
- Historic AI: TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play
- 1st AI world champion in any game!

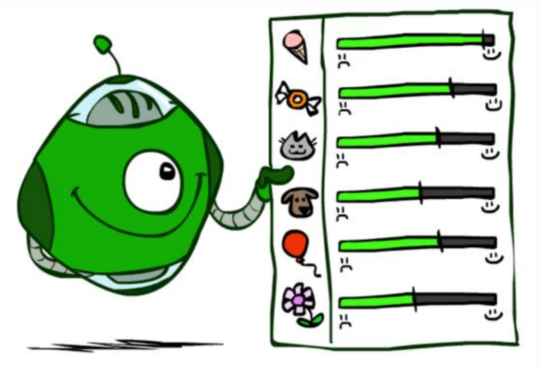
Multi-Agent Utilities

- What if the game is not zero-sum, or has multiple players?

- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component
  - Can give rise to cooperation and competition dynamically...



Utilities



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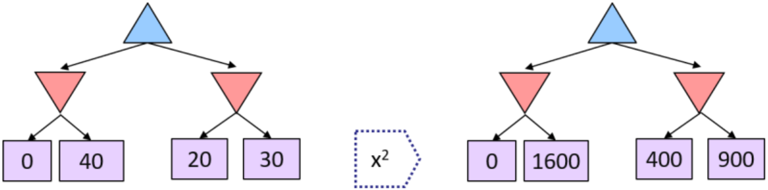
Maximum Expected Utility

- Why should we average utilities? Why not minimax?
- Principle of maximum expected utility:
  - A rational agent should chose the action that maximizes its expected utility, given its knowledge
- Questions:
  - Where do utilities come from?
  - How do we know such utilities even exist?
  - How do we know that averaging even makes sense?
  - What if our behavior (preferences) can't be described by utilities?

What Utilities to Use?



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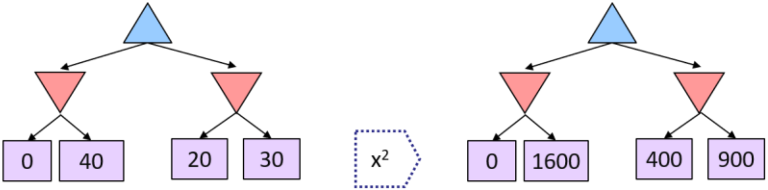

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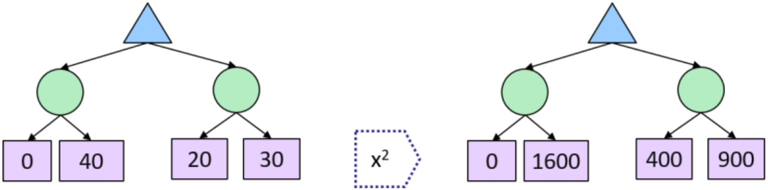

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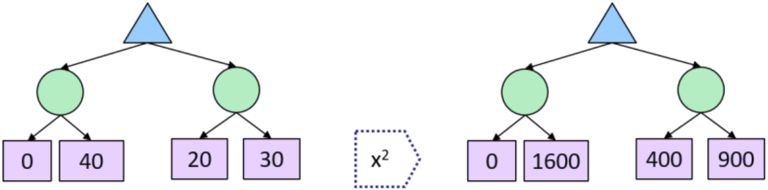

- For worst-case minimax reasoning, terminal function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - We call this **insensitivity to monotonic transformations**
- For average-case expectimax reasoning, we need *magnitudes* to be meaningful

Utilities (Revisited)

- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences
- Where do utilities come from?
  - In a game, may be simple (+1/-1)
  - Utilities summarize the agent's goals
  - Theorem: any "rational" preferences can be summarized as a utility function

Utilities: Uncertain Outcomes

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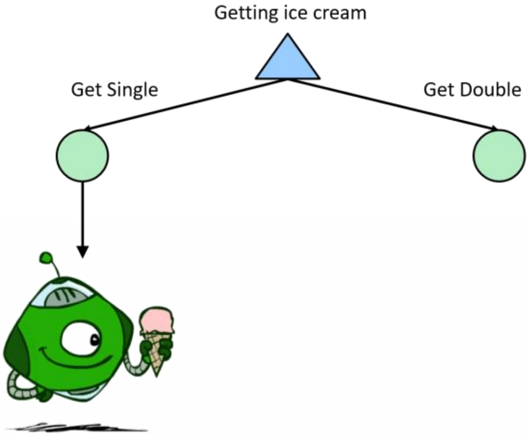

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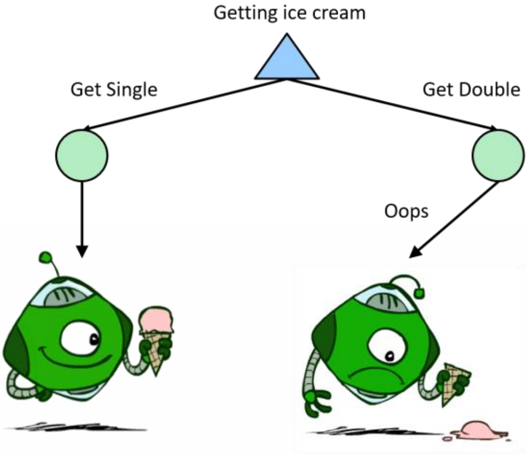

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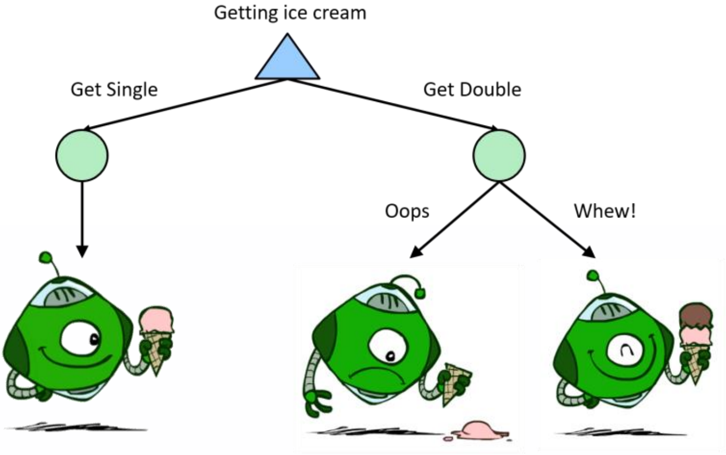
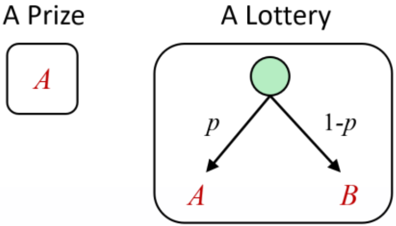
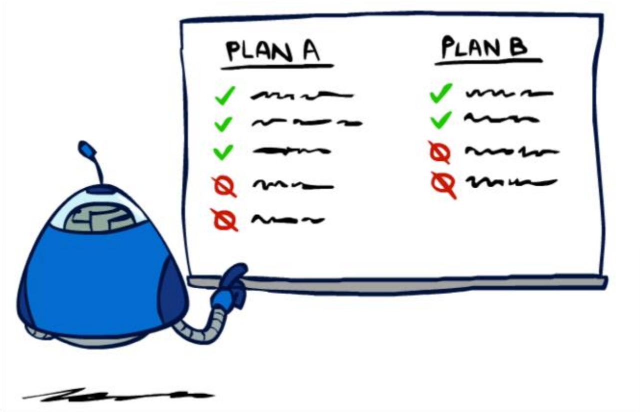


Preferences

- An agent must have preferences among:
  - Prizes: A, B, etc.
  - Lotteries: Situations with uncertain prizes
    $L = [p, A; (1 - p), B]$
- Notation:
  - Preference: $A \succ B$
  - Indifference: $A \sim B$



Rationality



Rational Preferences

- We want some constraints on preferences before we call them rational, such as:

Axiom of Transitivity: $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$

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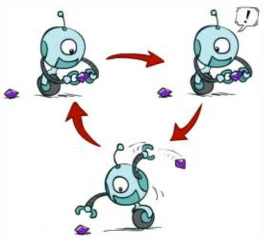
$$\text{Axiom of Transitivity: } (A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

- For example: an agent with **intransitive preferences** can be induced to give away all of its money
  - If $B > C$, then an agent with C would pay (say) 1 cent to get B
  - If $A > B$, then an agent with B would pay (say) 1 cent to get A
  - If $C > A$, then an agent with A would pay (say) 1 cent to get C



Rational Preferences

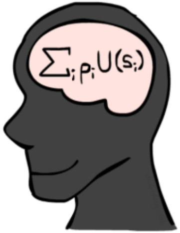
The Axioms of Rationality

- Orderability
  $(A \succ B) \vee (B \succ A) \vee (A \sim B)$
- Transitivity
  $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$
- Continuity
  $A \succ B \succ C \Rightarrow \exists p\,[p, A;\, 1-p, C] \sim B$
- Substitutability
  $A \sim B \Rightarrow [p, A;\, 1-p, C] \sim [p, B;\, 1-p, C]$
- Monotonicity
  $A \succ B \Rightarrow (p \geq q \Leftrightarrow [p, A;\, 1-p, B] \succeq [q, A;\, 1-q, B])$

Theorem: Rational preferences imply behavior describable as maximization of expected utility

MEU Principle

- Theorem [Ramsey, 1931; von Neumann & Morgenstern, 1944]
  - Given any preferences satisfying these constraints, there exists a real-valued function U such that:
    $U(A) \geq U(B) \Leftrightarrow A \succeq B$
    $U([p_1, S_1; \ldots; p_n, S_n]) = \sum_i p_i U(S_i)$
  - i.e. values assigned by U preserve preferences of both prizes and lotteries!



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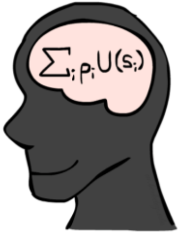
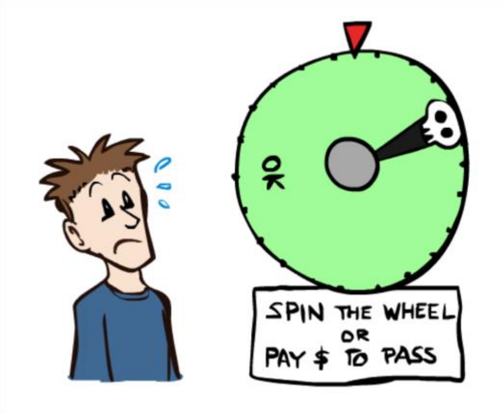
  - i.e. values assigned by U preserve preferences of both prizes and lotteries!
- Maximum Expected Utility (MEU) principle:
  - Choose the action that maximizes expected utility
  - Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
  - E.g., a lookup table for perfect tic-tac-toe, a reflex vacuum cleaner



Human Utilities



Utility Scales

- **Normalized utilities**: $u_{+} = 1.0, u_{-} = 0.0$

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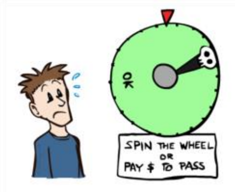
- With deterministic prizes only (no lottery choices), only **ordinal utility** can be determined, i.e., total order on prizes

Human Utilities (Revisited)

- Utilities map states to real numbers



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- Utilities map states to real numbers
- Standard approach to assessment (elicitation) of human utilities:
  - Compare a prize A to a standard lottery L_p between
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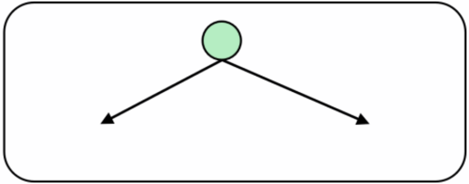

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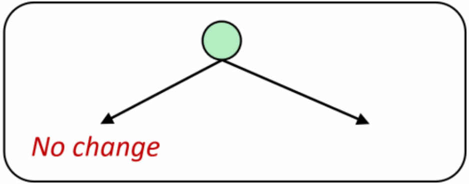

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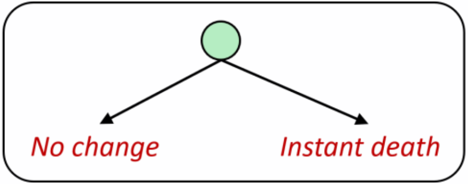

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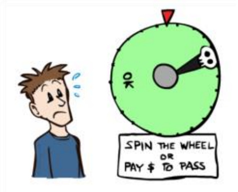

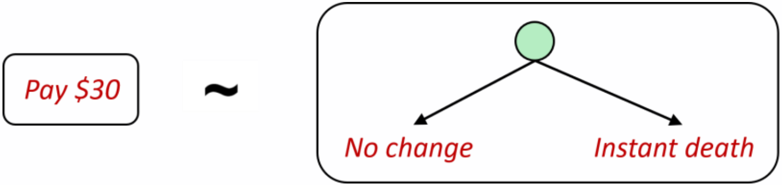

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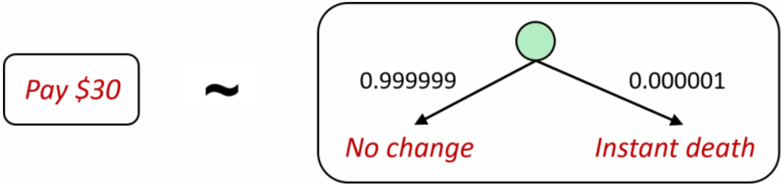

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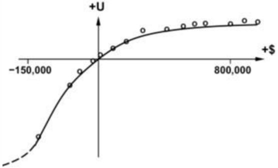

Suggested Reading

- Russell & Norvig: Chapter 5.2-5.5, 16.1-16.3