06/01/23

Math

Probability: The term probability is an estimate of the proportion of one or more uncertain ~~event~~ experimental outcomes when the experiment is performed at random.

Some ~~demm~~ examples:

[winning a lottery, precipitation estimation, winter temperature fall estimation, winning a number on dice] → Real life

(i) What is the likelihood that the new vaccine will be more effective than the old one in controlling covid-19?

(ii) Any kind of gambling (stock market price কোথায় যাবে,

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২৫০,

Set theory & set Notation :

Set : A set is simply a well-defined list or collection of distinct objects.

why well-defined ?

=)

why well defined instead of pre-defined or ill-defined ?

=)

why distinct objects ?

=)

- The individual objects of a set are elements or numbers.
- The set is the collection of its elements.
- The elements of a set must be distinct, i.e. each element must occur

and only one.

⊛ Universal Set :

A universal set is the set of all elements that may possibly be considered in a particular discussion.

Examples :

$$U = \{x : x \text{ is the sum of points upper-most side and the two dices}\}$$

$$= \{2, 3, 4, 5, 6, \ldots, 12\}$$

⊛ NULL Set :

The null set which is also important may seen like it is not a set at all.

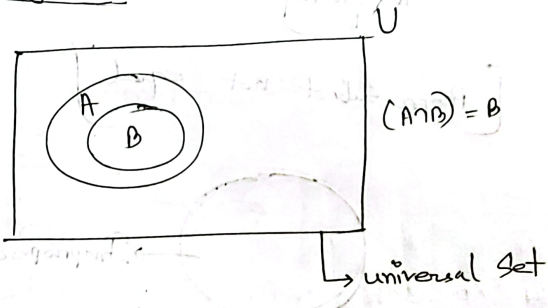
→ {list থাকবে কিন্তু list এর মধ্যে কোনো entry থাকবে না ; No element}

Denoted by,

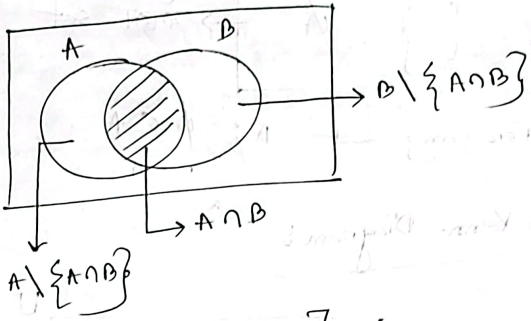
$\{\ \}$, ϕ → ফাঁকা set.

For any set A; $\phi \subset A$.

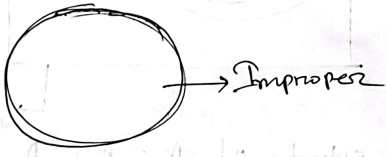
Venn-Diagram:



→ B is subset of A; $B \subset A$



[Proper subset not equal]



$A = \{1, 2, 3\}$ $B = \{2, 3\}$

proper subset of A

যদি $B \subset A$

A = super - set
B = subset of A

$B \subseteq A$
$A \subseteq B$

$A = B$

→ Improper ~~set~~ subset

Necessary : প্রয়োজনীয়
Sufficient : যথেষ্ট

→ Both side থেকে হলে true হবে

Set Operations :

- **Union :** $A \cup B$ $(x \in A$ or $x \in B)$

- **Intersection :** $A \cap B$ $(x \in A$ and $x \in B)$

- **Complement :** $(A \cup B)^c$

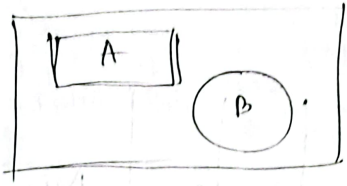
  ~~$(x \neq A$~~

  $(x \notin A$ and $x \notin B)$

complement

Two important properties of collection of sets :

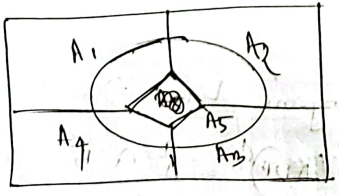
1. Mutually exclusive / Disjoint : দুটি কি নয় সেটি কোনো কমন নেই ,



A collection of events $A_1, A_2 --- A_n$

$A_i \cap A_j = \phi$; $[i \neq j]$

(2) Collectively Exhaustive :



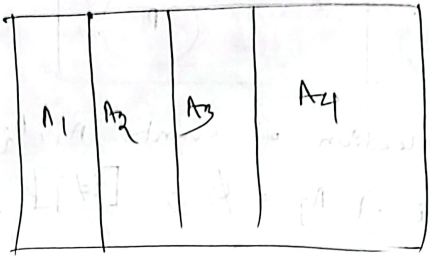
A collection of sets $A_1, A_2 --- A_n$ are collectively exhaustive iff

$A_1 \cup A_2 \cup A_3 --- \cup A_n = S$ (Same as $\cap A$)

event-so set applicable

$= \bigcup A_i$ Rotation Sign

(3)



Partition ⇒ M.E + C.E

(যা একসাথে পূর্ণ partition বলব)

(*) De Morgan's Law ⇒

i) $(A \cup B)^c = A^c \cap B^c$

ii) $(A \cap B)^c = A^c \cup B^c$

(4) Applying Set Theory to Probability :

Probability ~~based~~ is based on a repeatable experiment that consists of a procedure and observations.

- An outcome is an observation
- An event is a set of observation.

Outcome : An outcome of an experiment is any possible observation of that experiment.

Random Experiment : An experiment that can result in different outcomes even though it is repeated in the same manner every time is called random experiment.

Example: Run Rate Calculation,

ব্যাটার একবার ৬ Run, পরেরবার ২ ও Run
↳ different outcomes

- An experiment consists of the following procedure, observation and model.

→ <u>Procedure</u> : Monitor activity at a supershop.

→ <u>Observation</u> : Observe what type of products are being purchased by the next customers

→ <u>Model</u> :

<u>Sample Space</u> : (যে space এর ভিতরে sample collect করতে পারব তাই sample space)

Set Theory :		Probability Theory
Set	⟷	Event
Universal Set	⟷	Sample Space
Element	⟷	~~Observation~~ Outcome

Def^n of Sample Space :

The sample space of an experiment is the finest grain, mutually exclusive collectively exhaustive set of all possible outcomes.

Another one ⇒

" The set of all possible outcomes of a random experiment is called sample space of that experiment ".

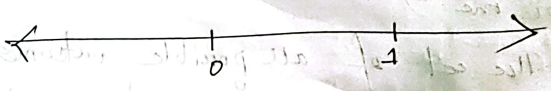
Discrete Sample Space:

A sample space is a discrete if it consists of a finite or countable infinite set of outcomes.

Continous Sample Space:

If a sample space contains a infinite number of probabilities equal to the number of points on a line segment, is called Continous Sample Space.

(1) A Sample Space is a continous, if it contains an interval of $\mathbb{R}$ (Real Number)



Flag flag → 36 bit

64 bit

MQM

18 Jan, 2023

Tanmoy Copy

18 Jan, 2023

Math

Event: An event is a subset of the Sample ~~Sp~~ space of a random experiment.

Union: The union of two events is the event that consists of all outcome that are ~~contain~~ contained either of the two events. If E_1 & E_2 are two events, than their union is denoted by $E_1 \cup E_2$.

Intersection: The intersection of two events is the event that ~~cont~~ consists of all outcomes that are contained in both of the two events. If E_1 & E_2 are two events, then their intersection is denoted by $E_1 \cap E_2$.

Complement: The ~~compl~~ complement of an event in a sample space is the set of outcomes in the sample space that are not ~~the~~ in the event. We denote the complement of an event E by E' or E^c.

Equally likely events: Two or more events are said to be equally likely, if they have the same chances of occurance. In other words, whenever a sample space consists of N possible outcomes that are equally likely then the probability of each outcome is $1/N$.

* For a discrete sample space, probability of an event E, denoted as $P(E)$, equals the sum of the probabilities of the outcomes in E.

Axioms of Probability:

If S is the sample space and E is any event in a random experiment, ~~then the probability~~

(i) $P(S) = 1$; (ii) $0 \leq P(E_i) \leq 1$;

(iii) For any, two events E_1 & E_2 with $E_1 \cap E_2 = \phi$; $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

[$n(A \cup B) = n(A) + n(B) - n(A \cap B)$]

Laws of sets:

(i) $A \cup B = B \cap A$; $A \cap B = B \cap A$ [commutative Law]

(ii) $(A \cup B) \cup C = A \cup (B \cup C)$
~~$(A \cup B)$~~
$(A \cap B) \cap C = A \cap (B \cap C)$ } [Associative Law]

(iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ } [Distributive Law]

(iv) $A \cup \phi = A$; $A \cap U = A$ | $A \cap \phi = \phi$; $A \cup U = U$ } [Identity Law]

(v) $A \cup A = A$ & $A \cap A = A$ [Idempotent Law]

(vi) $A \cup A^c = U$ & $A \cap A^c = \phi$ } [Complementary Laws]
$(A^c)^c = A$

(vip)

$$(A \cap B)^c = A^c \cup B^c$$
$$(A \cup B)^c = A^c \cap B^c$$

[De Morgan's Law]

$$\boxed{\begin{aligned} &P(\phi) = 1 \\ &P(E \cup E^c) = 1 \\ &P(E) + P(E^c) = 1 \end{aligned}}$$

{see how this proof comes}

Joint Probability: Two or more events form a joint event if all of them occurs simultenously and the probability of these joint events ~~called~~ are called joint probabilities.

$A \cap B$, $A \cup B$

$A \cap B \cap C$

Problem: Suppose, a sample space consists of 500 persons and are distributed according to their sex and employed employement status as shown in the following table.

Sex \ Employement status	Employed (E)	Unemployed (U)	Total
Male (M)	225 $(M \cap E)$	20 $(M \cap U)$	275 $n(M)$
Female (F)	80 $(F \cap E)$	145 $(F \cap U)$	225 $n(F)$
Total	305 $n(E)$	165 $n(U)$	500 $n(S)$

One of these 500 persons was selected at random.

Soln:

Let us define the following sample ~~space~~ events:

M : The ~~state~~ selected person is male.

F : " " " " female.

E : " " " " employed

U : " " " " unemployed

The joint events that can be formulated as follows:

$M \cap E$: The selected person is male & employed.

$M \cap U$: " " " " & unemployed

$F \cap E$: " " " " female & employed.

$F \cap U$: " " " " " " & unemployed

Shown the [illegible]

n(U) [illegible] the [illegible]

of the [illegible] these values are called

marginal ~~[illegible]~~ totals and the

corresponding probabilities P(m), P(E)

P(E), P(U) are called marginal

probabilities

Here,

$n(m) = 275$, $n(E) = 225$, $n(S) = 500$

$n(F) = 225$, $n(U) = 165$

Thus, marginal probability that a randomly selected person'll be a male is

$$P(m) = \frac{n(m)}{n(S)} = \frac{275}{500} = 0.55$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{225}{500} = 0.45$$

$$P(F) = \frac{225}{500} = 0.45$$

$$P(U) = \frac{165}{500} = 0.33$$

Ques: what is the probability that a randomly selected person is a male and at same time unemployed.

$$P(m \cap E) = \frac{n(m \cap E)}{n(S)} = \frac{255}{500} = 0.51$$

$$P(m \cap U) = \frac{20}{500} = 0.04$$

Also,

$$P(m) = P(m \cap E) + P(m \cap U)$$

$$= 0.51 + 0.04 = 0.55$$

Problem: In an office of 100 employees, 75 reads English, 50 reads Bangla newspaper and 40 reads both. Now if an employee is selected at random, whats the probability that

(a) he reads English newspaper?

(b) reads at best one of them?

(c) reads none?

(d) reads English but not Bangla?

Assumption:

	E	$\bar{E}$	Total
B	$n(B \cap E) = 40$	$n(B \cap \bar{E}) = ? = 10$	$n(B) = 50$
$\bar{B}$	$n(\bar{B} \cap E) = ? = 35$	$n(\bar{B} \cap \bar{E}) = ? = 15$	$n(B) = 50$
Total	$n(E) = 75$	$n(\bar{E}) = 25$	100

Sol^n: Let's first define the events as follows:

E: Reads English newspaper

B: " Bangla "

$\bar{E} \cap \bar{B}$ = Reads none

$B \cap \bar{E}$ = Reads Bangla but not English

$$P(E) = \frac{n(E)}{n(S)} = \frac{75}{100} = 0.75$$

$$P(E \cup B) = P(E) + P(B) - P(E \cap B)$$

$$= \frac{n(E)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(E \cap B)}{n(S)}$$

$$= \frac{75}{100} + \frac{50}{100} - \frac{40}{100}$$

$$= \frac{85}{100} = 0.85$$

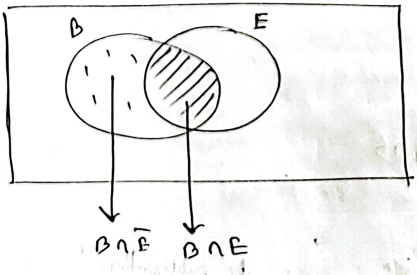
(c) $P(\overline{E \cap B}) = 1 - P(E \cup B)$

$= 1 - 0.85 = 0.15$

(d) $P(B \cap \overline{E}) = \dfrac{n(B)}{n(S)} - \dfrac{n(B \cap E)}{n(S)}$

$= \dfrac{n(B) - n(B \cap E)}{n(S)}$

$= \dfrac{50 - 40}{100} = 0.10$



Defn: Any statement of a function associating each of a set of ~~randomly~~ mutually exclusive and collectively exhaustive class or classes of intervals with its probability is a probability distribution.
P.D

A P.D will be ~~other~~ either discrete or continous according as the r.v is discrete or continous.

Discrete PD: If a r.v X has a discrete Pr. distn; the pr. distn of X is ~~d~~ defined as the function of "f" such that any real number x;
Pr. Distn.

$$\boxed{f(x) = P(X = x)}$$

PMF (probability mass function) is (for discrete variable)

Condn:

① $f(x) \geq 0$

② $\sum_x f(x) = 1$

③ $P(X = x) = f(x)$

→ Normally ~~यहाँ~~ check
~~यहाँ~~ ① & ②

If ~~coin~~ three coin tossed at a time;

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

X : no. of heads } discrete cauz sample points are discrete.

Value of $P(X=x)$	0	1	2	3
$P(X=x) = f(x)$	$1/8$ ✓ 0	$3/8$ ✓ 0	$3/8$ ✓ 0	$1/8$ ✓ 0

} justify ① & ②

Problem: Show that the function,

$$f(x) = \frac{x+1}{16} \quad ; x = 0, 1, 2, 3$$

is a PMF or not.

Problem: The probability fun^n of a discrete r.v X is given by

$$f(x) = \alpha\left(\frac{3}{4}\right)^x \quad ; x = 0, 1, 2, 3 \ldots \infty$$

$$= 0 \quad ; \text{otherwise}$$

Evaluate α and find $P(X \leq 3)$

[PMF মানেই সবগুলো Probability for মিলে 1]

$$P(X \leq 3) = f(0) + f(1) + f(2) + f(3)$$

$$= \frac{1}{4} + \frac{3}{16} + \frac{9}{64} + \frac{27}{256}$$

$$=$$

$f(x) = ?$

$$\sum_x f(x) = 1$$

$$\Rightarrow \sum_x \alpha \left(\tfrac{3}{4}\right)^x = 1$$

$$\Rightarrow \alpha \left[1 + \tfrac{3}{4} + \left(\tfrac{3}{4}\right)^2 + \left(\tfrac{3}{4}\right)^3 + \cdots\right] = 1$$

$$\Rightarrow \alpha \cdot \left[\frac{1}{1 - \left(\tfrac{3}{4}\right)}\right] = 1$$

$$\Rightarrow \boxed{\alpha = \tfrac{1}{4}}$$

[Graphical Representation of Discrete R.V is Bar Diagram]

$f(0) = \alpha \cdot \left(\tfrac{3}{4}\right) =$

$f(1) = \alpha \cdot \left(\tfrac{3}{4}\right) = \alpha(\tfrac{3}{4})$

$f(2) = \alpha \left(\tfrac{3}{4}\right)^2 = \alpha \left(\tfrac{3}{4}\right)^2$

$f(n) = \alpha \cdot \left(\tfrac{3}{4}\right)^n = \alpha \left(\tfrac{3}{4}\right)^n$

[Geometric Progression]

ar^{n-1}

[Arithmetic Progression]

$a + (n-1)d$

$$\boxed{S_n = \frac{a}{1-r}}$$

GP

Cummulative Dist^n function :

The cummulative dist^n function (CDF); $F(X)$ of a discrete r.v X with probability function $f(X)$ defined over all real numbers X is the cummulative probability upto and indicating the point X.

$$F(X) = P(X \leq x)$$

$$\boxed{\begin{aligned} \max \quad & F(X) = 1 \\ \min \quad & F(X) = 0 \end{aligned}}$$

The function $F(X)$ is monotonically increasing function

$$\boxed{\begin{aligned} & \lim_{x \to -\infty} F(X) = 0 \\ & \lim_{x \to +\infty} F(X) = 1 \end{aligned}}$$

Problm:

X: x	0	1	2	3
$P(X=x)=f(x)$	$20/120$	$60/120$	$36/120$	$4/120$

$F(0) = P(X=0) = f(0) = 20/120$

$F(1) = P(X \le 1) = f(0) + f(1) = 20/120 + 60/120 = 80/120$

$F(2) = P(X \le 2) = f(0) + f(1) + f(2) = 116/120$

$F(3) = P(X \le 3) = f(0) + f(1) + f(2) + f(3) = 120/120 = 1$

Can be written as ⇒

X: x	<0	0	1	2	3	>3
$P(X=x)=f(x)$	0	$20/120$	$60/120$	$36/120$	$4/120$	0
$F(x)$	0	$20/120$	$80/120$	$116/120$	1	1

Can be written as precise function ⇒

$$F(x) = \begin{cases} 0 & ; \quad x < 0 \\ 20/120 & ; \quad 0 \le x < 1 \\ 80/120 & ; \quad 1 \le x < 2 \\ 116/120 & ; \quad 2 \le x < 3 \\ 1 & ; \quad x \ge 3 \end{cases}$$

<u>Problm:</u> $P(Y=y) = {}^2C_y \left(\frac{1}{2}\right)^y \left(\frac{1}{2}\right)^{2-y}$; $y = 0, 1, 2$ (discrete r.v.)

i) Check if $P(Y)$ is a PMF.

ii) Find $F(y)$ [CMF] & ~~find~~ hence find $F(-2)$ & $P(Y \le 1)$

$\Downarrow$

$F(1) \longrightarrow$ in CMF (same কথা)

Math 01/09/23

Mathematical ~~Expection~~ Expectation : / (Expected value of a R.V)

mean = Expected value of a R.V

Def^n: If X is a discrete R.V having a pmf $f(x)$, then the expected value of X or the mathematical expectation of X is denoted with $E[X]$ and is defined by $E[X] = \sum_x x \cdot f(x)$

for,

$X \longrightarrow$ C.R.V

$$\mu = E[X] = \int_{-\alpha}^{+\alpha} x \cdot f(x) \cdot dx$$

$X = 0, 1, 2$

$\downarrow \quad \downarrow \quad \downarrow$

$x \quad x \quad x$

Th^m: Let X be a ~~Discrete~~ D.R.V with probability fun^c $f(x)$ and c be a constant

$$E[c] = c$$

$$\boxed{\begin{aligned} E[c] &= cE[1] \\ &= c \cdot 1 \\ &= c \end{aligned}}$$

Proof:

$$\mathbb{E}[c] = \sum_{x} c \cdot f(x)$$

$$= c \underbrace{\sum_{x} f(x)}_{} \quad \text{PMF}$$

$$= c \cdot 1$$

Variance: Let X be a R.V with finite mean $\mu = \mathbb{E}[X]$, then the variance of X is denoted by $\mathbb{V}[X]$, and is defined by

$$\mathbb{V}[X] = \mathbb{E}[X - \mu]^2$$

$$= \mathbb{E}\left[(X - \mathbb{E}[X])^2\right]$$

where, $\mu = \mathbb{E}[X]$

$$\mathbb{V} = \sigma^2$$

(S.D)

Standard Deviation: The positive square root of variance is known as the S.D i.e. denoted by σ

$$\sigma = \sqrt{V[X]}$$

$$= \sqrt{E[X-\mu]^2}$$

Thm: Let X be a discrete R.V with pmf, $f(x)$, then

$$V[X] = \sigma^2 = E[X-\mu]^2 = E[X^2] - \mu^2$$

$$= E[X^2] - \{E[X]\}^2$$

Thm: let R.V be X & Y

$$E[X+Y] = E[X] + E[Y]$$

Thm:

$$E[aX+b] = E[aX] + E[b]$$

$$= aE[X] + b$$

$b \rightarrow$ const

Thm:

$$\boxed{E[XY] = E[X] \cdot E[Y]}$$

iff X & Y are independent.

03/02/23

(for D.R.V)

Bernouli Distribution: A random variable X is said to have a Bernouli distribution with parameter $p\ (0 \le p \le 1)$ if X can take on only the value 0 & 1, and probabilities are:

$$\left.\begin{aligned} P(X=1) &= p \\ P(X=0) &= 1-p \end{aligned}\right\} \text{——— (1)}$$

Since, the distribution involves only two cases of events, it's

known as two-point probability distribution.

If we take $q = 1-p$, then the probability function of X can be written as follows:

$$f(x, p) = \begin{cases} p^x q^{1-x} & ; \text{ for } x = 0, 1 \\ 0 & ; \text{ otherwise} \end{cases}$$

Properties of Bernoulli Distribution

mean : $\mathbb{E}[X] = \sum_x x f(x)$

$$= 0 + p^0 (1-p)^{1-0} + 1 + p^1 \cdot (1-p)^{q-1}$$

$$= 1 \cdot p \cdot 1 = p,$$

$$\mathbb{E}[X] = p$$

$$\mathbb{V}[X] = \mathbb{E}[X^2] - \left\{\mathbb{E}[X]\right\}^2 \rightarrow \text{see proof as well.}$$

$$= \sum_x x^2 f(x, p) - p^2$$

$$= 0^2 + p^0(1-p)^{1-0} + 1^2 \times p(1-p)^{1-1} - p^2$$

$$= p - p^2$$

$$\boxed{\begin{aligned} V[X] &= p(1-p) \\ &= pq \end{aligned}} \rightarrow \left\{ \text{betn moment} \right\}$$

Moment generating fun^c (MGF)

Let X be a R.V with probability fun^c $f(x)$

$$M_X(t) = E\left[e^{tX}\right]$$

Defn MGF

$$= \sum_x e^{tx} f(x)$$

Expectation

Discrete

Suppose,

$E[X] = \sum x f(x)$

$E[X^2] = \sum x^2 f(x)$

$E[Y] = \sum y f(y)$

If x is continuous;

$$m_x(t) = E\left[e^{tx}\right] = \int_{-\infty}^{+\infty} e^{tx} f(x)\,dx$$

[continuous]

$$m_x(t) = E\left[e^{tx}\right] = \sum_x e^{tx} f(x, p)$$

$$= \sum_x e^{tx} p^x (1-p)^{1-x}$$

$$= \sum_x (pe^t)^x (1-p)^{1-x}$$

$$= (pe^t)^0 (1-p)^{1-0} + (pe^t)^1 (1-p)^{1-1}$$

$$= (1-p) + qe^t \cdot 1$$

$$\Rightarrow \boxed{m_x(t) = pe^t + q}$$

Poisson Distribution:

Let, μ be the mean or expected ~~value~~ of number of successes in a specified time or space and the r.v. X designates the number of successes in a given time interval or specified region; Then the probability distribution of X is a Poisson Distribution and is given by

$$\boxed{f(x;\mu) = \frac{e^{-\mu} \cdot \mu^{x}}{x!}}$$

negative exponential

$x = 0, 1, 2, 3 \ldots \infty$

Eg: 0.5 calls per minute

$\mu = 0.5$ calls/min

$\mu = 5 \cdot * 0.5 = 2.5$

$$f(2; 2.5) = \frac{e^{-2.5} * (2.5)^2}{2!}$$

= Req. Probability.

{ Interval mean val → যেটা সময়ের; সেই সময়ের জন্য Poisson Distribution }

Mean :

$$E[X] = \sum_{X=0}^{\infty} X * f(X; \mu)$$

$$= \sum_{X=0}^{\infty} \frac{X * e^{-\mu} \cdot \mu^X}{X!}$$

$$= \mu \sum_{(X-1)=0}^{\infty} \frac{e^{-\mu} \mu^{X-1}}{(X-1)!}$$

Let, $y = X - 1$

$dy = dx$

$$E[X] = \mu \sum_{y=0}^{\infty} \frac{e^{-\mu} \mu^y}{y!}$$

$$= \mu \cdot \sum_{y=0}^{\infty} \underbrace{f(y; \mu)}_{\text{PMF}}$$

$$= \mu \cdot 1 = \mu$$

$\mathbb{E}[X] = \mu$

Variance :

$$\mathbb{V}[X] = \mathbb{E}[X^2] - \underbrace{\{\mathbb{E}[X]\}^2}_{\mu^2}$$

$$= \mathbb{E}[X^2] - \mu^2$$

$$\boxed{\mathbb{V}[X] = \mu}$$

Chapter 08

Random vector

$$\bar{X} = (X_1, X_2, X_3, \ldots X_n)^T$$

X_1 → Random Variable

Random Vector: A random vector is a column vector $\bar{X} = [X_1 X_2 \cdots X_n]'$

Each X_i is a random variable.

Q When a random vector will be called a random variable?

$\Rightarrow$ when $n = 1$.

Vector Sample Value: A sample value of a random vector is a column vector $\bar{X} = [x_1 x_2 \cdots x_n]'$

Random Vector Probability Fun^n: