Stat 333 Basic Probability Distributions

Discrete Distributions

distribution	probability mass function	range	$\mathbf{E}(X)$	$\mathbf{Var}(X)$
indicator (p)	$f(0) = 1-p\,,\ f(1) = p$	$k = 0, 1$	p	$p(1-p)$
binomial(n, p)	$f(k) = \binom{n}{k}\, p^k\,(1-p)^{n-k}$	$k = 0, 1, \ldots, n$	np	$np(1-p)$
geometric (p) †	$f(k) = p\,(1-p)^{k-1}$	$k = 1, 2, 3, \ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
neg. binomial(r, p)	$f(k) = \binom{k-1}{r-1}\, p^r\,(1-p)^{k-r}$	$k = r, r+1, \ldots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$
Poisson(λ)	$f(k) = \frac{\lambda^k}{k!}e^{-\lambda}$	$k = 0, 1, 2, \ldots$	λ	λ

† we will always define the geometric as $X =$ number of trials up to (and including) the first Success. Similarly we will always define the negative binomial as $X_r =$ number of trials up to (and including) the first r Successes.

The indicator, binomial, and Poisson random variables are examples of *counting variables*. They count the number of times a certain event E occurs in a fixed number of trials (indicator and binomial) or in a fixed time period (Poisson). The geometric and negative binomial are examples of *waiting time variables*. They count the number of trials (the waiting time) required to obtain a pre-determined fixed number of occurrences of an event E (Success).

Continuous Distributions

distribution	probability density function	range	$\mathbf{E}(X)$	$\mathbf{Var}(X)$
uniform (a, b)	$f(x) = \frac{1}{b-a}$	$a \le x \le b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
exponential(λ)	$f(x) = \lambda e^{-\lambda x}$	$x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
gamma (α, λ)	$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x}$	$x > 0$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$
normal(μ, σ^2)	$f(x) = \frac{\exp[-(x-\mu)^2/2\sigma^2]}{\sqrt{2\pi\sigma^2}}$	$-\infty < x < \infty$	μ	σ^2

The exponential distribution is the continuous counterpart to the geometric distribution and can be viewed as a continuous waiting-time variable. It is the unique continuous distribution possessing the *no-memory property*. The exponential will be the most important continuous distribution in this course.