STAT 333 Summary of Probability Rules

Basic Axioms:

Let S be the sample space for an experiment. Then a probability rule P must satisfy:

(i) $P(S) = 1$

(ii) $0 \le P(A) \le 1$ for all $A \subseteq S$

(iii)* If $A_1, A_2, \ldots, A_m, \ldots$ is a sequence (finite or countably infinite) of disjoint events (i.e. $A_j \cap A_k = \emptyset$), then

$$P(\bigcup_n A_n) \;=\; \sum_n P(A_n) \qquad \text{(general additivity axiom)}$$

If S is finite, then (iii)* can be replaced by:

(iii) If $A_1, A_2, \ldots, A_m$ are disjoint events (i.e. $A_j \cap A_k = \emptyset$), then

$$P(\bigcup_{n=1}^{m} A_n) \;=\; \sum_{n=1}^{m} P(A_n) \qquad \text{(finite additivity axiom)}$$

Rules Which Follow From The Axioms:

1. If $A \subseteq B$ then $P(A) \le P(B)$ and $P(B \setminus A) = P(B) - PA)$.
2. $P(\bar{A}) = 1 - P(A)$; in particular $P(\emptyset) = 0$.
3. For any sequence of events $A_1, A_2, \ldots$ (finite or countably infinite),

$$P(\bigcup_n A_n) \;\le\; \sum_n P(A_n)$$

   Equality holds if the events are disjoint.

4. $P(A \cup B) \;=\; P(A) + P(B) - P(A \cap B)$ for any events A and B.
5. $P(A \cup B \cup C) \;=\; P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$.

Conditional Probability, Independence, and Multiplication Rules:

- **conditional probability** $P(A|B) = P(A \cap B)/P(B)$.
- **basic multiplication rule** $P(A \cap B) = P(A)P(B|A)$.
- **extended multiplication rule** $P(A_1 \cap A_2 \cap \ldots \cap A_m) = P(A_1)P(A_2|A_1) \cdots P(A_m|A_1 \cap \ldots \cap A_{m-1})$.
- A and B are **independent** if $P(A \cap B) = P(A)P(B)$.
- If an experiment consists of a sequence of **independent trials,** and $A_1, \ldots, A_m$ are events such that A_j depends *only* on the j^{th} trial, then $A_1, \ldots, A_m$ are independent, and

$$P(A_1 \cap A_2 \cap \ldots \cap A_m) \;=\; P(A_1)P(A_2) \cdots P(A_m)\,.$$