STAT 333 Summary of Expectation, Variance, and Covariance Rules

Variance and Covariance definitions:

1. $\text{Var}(X) = E\left((X - E(X))^2\right) = E(X^2) - E(X)^2.$
2. $\text{Cov}(X, Y) = E\left((X - E(X))(Y - E(Y))\right) = E(XY) - E(X)E(Y).$

Expectation Rules:

1. $E(a) = a$ for any constant a.
2. $E(aX) = aE(X)$ for any constant a.
3. $E(X + Y) = E(X) + E(Y)$ for any two r.v.'s.

generalization: $E(\sum_{j=1}^{n}(a_j X_j + b_j)) = \sum_{j=1}^{n} a_j E(X_j) + \sum_{j=1}^{n} b_j.$

Variance Rules:

1. $\text{Var}(a) = 0$ and $\text{Var}(X + a) = \text{Var}(X)$ for any constant a.
2. $\text{Var}(aX) = a^2 \text{Var}(X)$ for any constant a.
3. $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\,\text{Cov}(X, Y)$ for any two r.v.'s.

generalizations:

1. $\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) + 2ab\,\text{Cov}(X, Y).$
2. $\text{Var}(\sum_{j=1}^{n}(a_j X_j + b_j)) = \sum_{j=1}^{n} a_j^2 \text{Var}(X_j) + 2 \sum_{i=1}^{n-1} \sum_{j>i} a_i a_j\,\text{Cov}(X_i, X_j).$

Covariance Rules:

1. $\text{Cov}(X, Y) = \text{Cov}(Y, X)$ for any two r.v.'s.
2. $\text{Cov}(X, Y) = 0$ if X and Y are independent.
3. $\text{Cov}(X, a) = 0$ for any constant a.
4. $\text{Cov}(aX, bY) = ab\,\text{Cov}(X, Y)$ for any constants a and b.
5. $\text{Cov}(X, Y + Z) = \text{Cov}(X, Y) + \text{Cov}(X, Z).$
   Similarly $\text{Cov}(X + Y, Z) = \text{Cov}(X, Z) + \text{Cov}(Y, Z).$

generalization: $\text{Cov}(aX + bY, cW + dZ) = ac\,\text{Cov}(X, W) + ad\,\text{Cov}(X, Z) + bc\,\text{Cov}(Y, W) + bd\,\text{Cov}(Y, Z).$